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Methodology to Perform Identifiability Analysis for Off-Road Vehicle Tire-Soil Parameter Estimation

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Abstract
The growing trend of model-based design in off-road vehicle engineering requires models that are sufficiently accurate if they are to be used with confidence. Uncertain model parameters are often identified from measured data by using an optimization procedure, but it is important to understand the limitations of such a procedure and to have methods available for assessing the uniqueness and confidence of the results. Model identifiability analysis is used to determine whether system measurements contain enough information to estimate the model parameters. A numerical approach based on the profile likelihood of parameters was utilized to evaluate the local structural and practical identifiability of a tractor and single axle towed implement model with six uncertain tire force model parameters from tractor and implement yaw rate data. The analysis considered simulated data with known model parameter values to examine the effect of measurement error on the identifiability. The accuracy and confidence of identification tended to decrease as the quality of the data decreased, to the point that five of the six parameters were considered practically unidentifiable from the information available. Overall, the study showed that experimental factors such as noise can affect the amount of information available in a dataset for identification and that error in the measured data can propagate to error in model parameter estimates.

Keywords
Tractor and implement model, parameter uncertainty, parameter identifiability, parameter identification, profile likelihood, model-based design

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Abstract. *The growing trend of model-based design in off-road vehicle engineering requires models that are sufficiently accurate if they are to be used with confidence. Uncertain model parameters are often identified from measured data by using an optimization procedure, but it is important to understand the limitations of such a procedure and to have methods available for assessing the uniqueness and confidence of the results. Model identifiability analysis is used to determine whether system measurements contain enough information to estimate the model parameters. A numerical approach based on the profile likelihood of parameters was utilized to evaluate the local structural and practical identifiability of a tractor and single axle towed implement model with six uncertain tire force model parameters from tractor and implement yaw rate data. The analysis considered simulated data with known model parameter values to examine the effect of measurement error on the identifiability. The accuracy and confidence of identification tended to decrease as the quality of the data decreased, to the point that five of the six parameters were considered practically unidentifiable from the information available. Overall, the study showed that experimental factors such as noise can affect the amount of information available in a dataset for identification and that error in the measured data can propagate to error in model parameter estimates.*

Keywords. Tractor and implement model, parameter uncertainty, parameter identifiability, parameter identification, profile likelihood, model-based design

Introduction

Off-road vehicle design and manufacturing companies are continually striving to meet customer needs by providing higher-quality and higher-performance products more quickly and at a lower cost. Advances in computer technology have had a major impact on engineering design and analysis over the last few decades (Dieter and Schmidt, 2009). These technologies have fostered the growing trend of model-based design strategies in the off-road vehicle industry (Prabhu, 2007). In general, a model-based approach utilizes characterizations of system behavior to meet specified design requirements (Wymore, 1993). Model-based design has the potential to reduce reliance on physical prototypes, which can lead to time and cost savings during development (Prabhu, 2007; Lennon, 2008).

However, an ongoing limitation in the advancement of model-based design has been the development of accurate models in which one can put confidence regarding their ability to characterize a system (Radhakrishnan and McAdams, 2005). Without sufficient confidence, the usefulness of a model is restricted, and there will be hesitancy to rely on it to drive decision-making in design. Validation processes can be conducted to ensure that a model can satisfactorily represent a physical system, at least within certain scenarios of interest (Ljung, 1999). Areas of concern include: the appropriateness of the model for the application, the accuracy of the mathematical representation of the model, and the accuracy of the model parameters (Bernard and Clover, 1994).

Off-road vehicle dynamics models are often mathematical models developed based on the principles of on-road vehicle dynamics, which can be found in e.g., Gillespie (1992) and Wong (2008). Off-road vehicle models have been developed for applications such as guidance controller design (Karkee and Steward, 2010a), traction modeling (Book and Goering, 2000), ride evaluation (Ahmed and Goupillon, 1997), handling evaluation (Previati et al., 2007), and real-time driving simulation (Fales, et al., 2005; Hummel et al., 2005; Karimi and Mann, 2006; Karkee et al., 2009). Depending on its level of fidelity, a vehicle model will typically incorporate a set of parameters to describe the physical system, including inertial and geometric properties. Some of the parameter values may be uncertain due to the difficulty or impossibility of direct measurement. In addition, certain parameter values that characterize a system well in one set of conditions may not be appropriate when conditions vary (Kiencke and Nielsen, 2005; Karkee and Steward, 2011). Sensitivity analysis can be used to determine the effects of parameter variation on the model output (Jang and Han, 1997) as well as guide efforts to improve the certainty of specific parameters (Karkee and Steward, 2010b). Additionally, identification approaches can be used to determine vehicle model parameter values by finding the set of values for which the model output most closely represents the actual system output for a given input (Kiencke and Nielsen, 2005).

For off-road vehicle models, the interaction of tires and soil is complex and difficult to characterize accurately (Wong, 1989). In particular, it is difficult to find a widely-accepted tire-soil model for lateral force development, which plays a primary role in steering response and yaw dynamics (Karkee, 2009). However, researchers have used the well-known slip-angle-based tire model from on-road vehicle dynamics to relate tire slip angle to lateral force development in off-road cases as well (Metz, 1993; Bevly et al., 2002; Karimi and Mann, 2006; Karkee and Steward, 2011). In some cases, values for the tire model parameters have been identified from vehicle-level data obtained during field experiments. Bevly et al. (2002) and Karimi and Mann (2006) each used tractor yaw rate data measured with a gyroscopic sensor along with front wheel steering angle data to identify cornering stiffness and relaxation length parameters of the front and rear tires of a linear bicycle model. Karkee and Steward (2011) used tractor yaw rate and heading angle data measured from Global Positioning System (GPS) receivers along with front wheel steering angle data to identify cornering stiffness and tire relaxation length parameters of a linear bicycle model of a tractor and single axle towed implement. In each of these cases, the difficulty in obtaining reliable estimates for the tire model parameters was noted.

It is also important to consider the possible limitations in parameter identification from experimental data. Although parameter sensitivity analysis lends insight into the effects that parameters have on the output, it does not show how uncertainty in the measured outputs propagates to uncertainty in the estimated parameters. Model identifiability analysis is used to determine whether system measurements contain enough information to estimate the model parameters (Walter and Pronzato, 1997).

Identifiability analysis has been conducted in vehicle model identification studies (Serban and Freeman, 2001; Alasty and Ramezani, 2002; Arikan, 2008). Serban and Freeman (2001) developed a local, numerical test that determined if estimated parameters were at an “isolated minimum” of the optimization cost function. That test was demonstrated in the context of parameter identification of a multibody vehicle suspension model. Alasty and Ramezani (2002) tested the structural identifiability, which considered only the model structure but not the effects of experimental data quality, of a nonlinear, vehicle ride model before using genetic algorithm-based optimization to identify 17 parameters from simulated data obtained from a high-fidelity multibody model. The model was linearized about an operating point, and identifiability of the linearized system was used to infer identifiability of the nonlinear system. Arikan (2008) examined the identifiability of a two degree-of-freedom linear vehicle handling model and a three degree-of-freedom nonlinear vehicle handling model prior to identification from data. The structural identifiability of the linear model was analyzed a priori using a transfer function approach for different observed output combinations which guided the sensor configuration for experimental data collection. Structural identifiability of the nonlinear model was examined using a differential algebra technique. The practical identifiability of the nonlinear model, which takes into account the properties of measured data, was also examined based on the Fisher Information Matrix, which was used to ensure that there was not high correlation between parameters to be estimated. As noted by Arikan (2008), high correlation between parameters enables a change in one parameter value to be compensated by a change in another parameter value and limits identifiability.

Identifiability of dynamic models is an active topic of research in the field of systems biology. According to Raue et al. (2009), biochemical reaction networks often permit only a limited number of outputs to be measured, and experimental data is often of insufficient quantity and quality for parameter identification; furthermore, the size and complexity of their mathematical models often renders analytical identifiability methods intractable. The trend has been to utilize growing computational power to perform numerical identifiability analyses (Hengl et al., 2007; Raue et al., 2009). Raue et al. (2009) proposed a numerical approach for local identifiability analysis of arbitrary models by “exploiting” the profile likelihood of model parameters. The approach was able to detect structurally unidentifiable parameters due to functional relations and, since it was data-based, was able to detect practically unidentifiable parameters due to inadequate quality or quantity of data.

Despite its potential importance, a review of the literature shows that many mechanical system parameter identification studies do not seem to consider identifiability. Furthermore, if identifiability is considered, it typically is only structural in nature and does not consider the practical aspects of data collection. In this work, local identifiability analysis was performed for a tractor and single axle towed implement model using a numerical identifiability approach. Specifically, the objective was to investigate identifiability of six tire model parameters from simulated, system-level output data with varying levels of noise in the output. Understanding the ability to uniquely and confidently estimate model parameters from measured data is an important part of a parameter identification experiment.

Materials and Methods

In this research, the identifiability of various tire-soil parameters of a tractor and single axle grain cart steering model was analyzed using the profile likelihood approach. A dynamic bicycle model of the system (Karkee and Steward, 2010a) was adapted to perform the analysis. Parameters which were highly uncertain and difficult to estimate (Karkee and Steward, 2011) were selected for this study. The study helped explain the results of parameter estimation from experimental data that was conducted for the same set of model parameters. The following paragraphs will explain the methods used in this study.

Vehicle and Tire Model

As described by Walter and Pronzato (1997), physical systems are generally modeled in continuous time and described by a set of differential equations,

$$\dot{x}(t) = f(x(t), \theta, u(t)) \quad (1)$$

(2)

$$y_m(t) = h(x(t), \theta, u(t))$$

where $\mathbf{x}$ is the state vector, θ is the parameter vector, $\mathbf{u}$ is the vector of controlled inputs, $\mathbf{t}$ is time, and $\mathbf{y_m}$ is the vector of model outputs.

At a high level, several different model types can be considered, each having advantages and disadvantages in different applications (Walter and Pronzato, 1997; Ljung, 1999; Bohlin, 2006). The most common type is the "white box" model, which is guided by first principles such as conservation and balance to represent system phenomena. At the other end of the modeling spectrum is the "black box" model, in which an arbitrary mathematical structure is used to fit an input(s) to an output(s) recorded through some experiments. Black box model parameters, in general, do not contain any physical meaning. In between white box and black box models is the "grey box" modeling approach. A first principles model structure is used to explain most, if not all, of the system behavior. Some of the model parameter values may be known with greater certainty, but other parameter values may be unknown. The unknown parameters are then identified, or "estimated", from experimental data. The main advantage of a grey-box model is that its parameters retain physical meaning, yet it has been calibrated to match observed system behavior.

The subject of this work was a dynamic model of an agricultural tractor and a grain cart system, studied extensively by Karkee and Steward (2010a). The actual system being modeled was a John Deere 7930 MFWD (mechanical front wheel drive) tractor (Deere and Co., Moline, IL) and a single axle, 18 m^3 (500 bu.) grain cart (model 500, Alliance Product Group, Kalida, OH). Their research efforts included the modeling of vehicle and tire force dynamics and an examination of open and closed loop system characteristics. Among the different models studied, they found that a dynamic bicycle model with tire relaxation length dynamics represented the system most accurately, particularly for speeds above 4.5 m/s. This conclusion was based on a comparison of time and frequency responses. This model was used for sensitivity analysis (Karkee and Steward, 2010b) and parameter identification studies (Karkee and Steward, 2011) that followed.

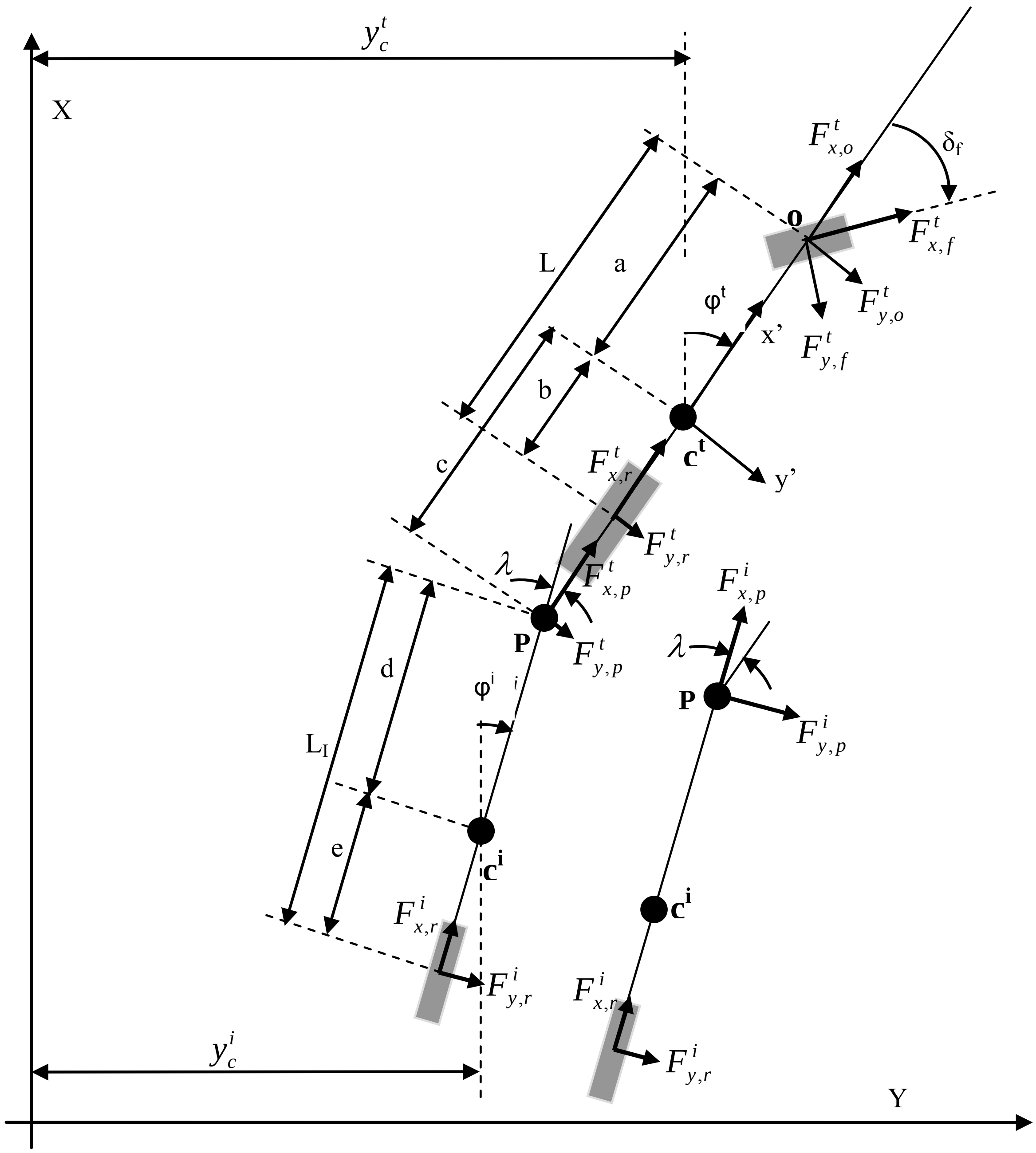
Tire lateral forces were represented by a linear model based on the tire lateral slip angle, α , and a tire cornering stiffness, $\mathbf{C_\alpha}$, by,

$$F_y = -C_\alpha \alpha \tag{3}$$

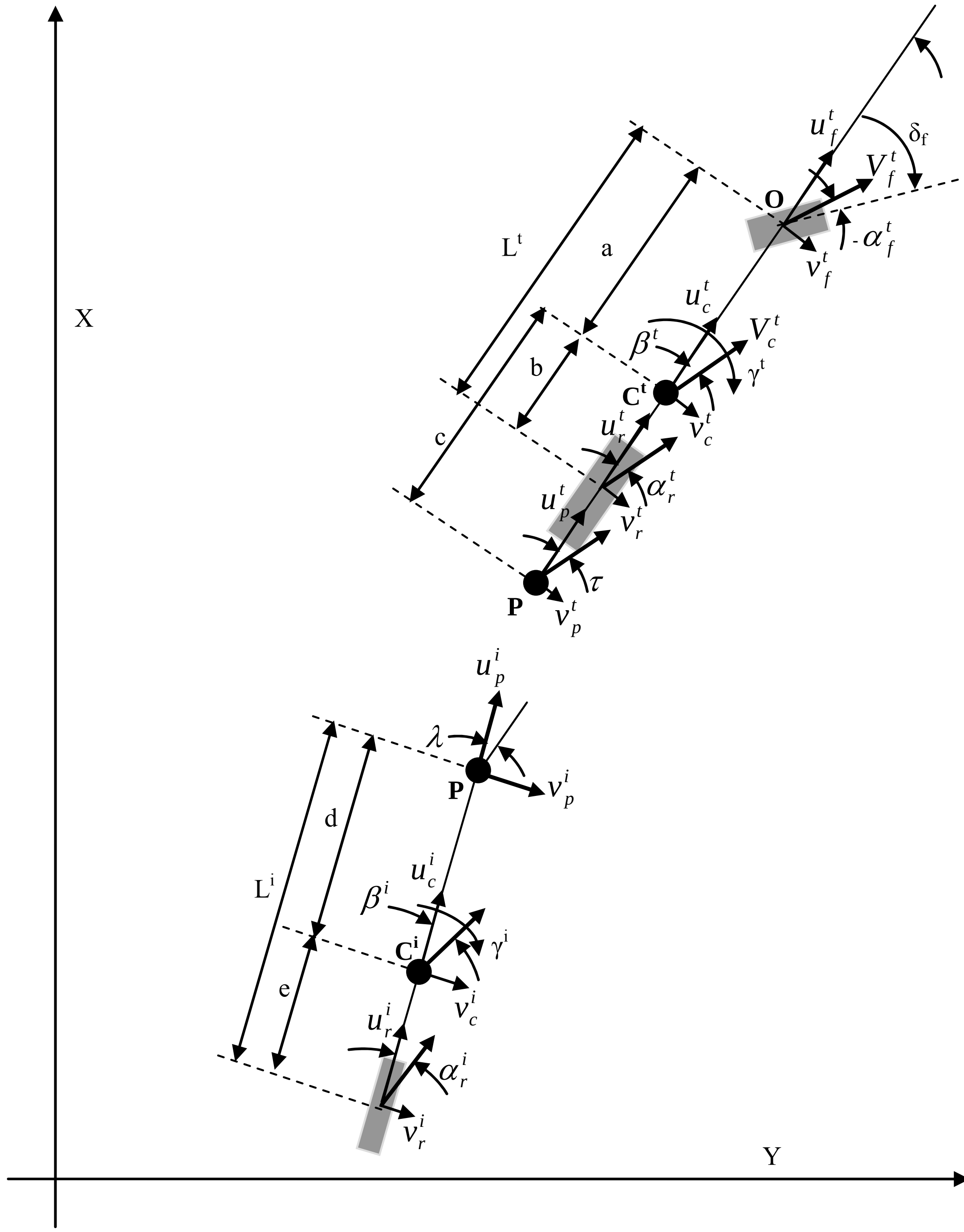
The development of each tire slip angle was modeled as a first-order delay and parameterized by a relaxation length, σ , so that,

$$\dot{\alpha} = \frac{u}{\sigma}(\alpha_0 - \alpha) \tag{4}$$

where α_0 is the steady state slip angle (Bevly et al., 2002). The overall vehicle model (Figure 1) is described by Eqs. 5-12:



a)



b)

Figure 1 - Dynamic bicycle model of a tractor and single axle towed implement system (Karkee and Steward, 2010a); a) forces on the system, and b) velocities at different locations of the system.

$$(m^t + m^i)\dot{v}_c^t - m^i c\dot{\gamma}^t - m^i d\dot{\gamma}^i = -(m^t + m^i)u_c^t\gamma^t - C_{\alpha,f}^t\alpha_f^t - C_{\alpha,r}^t\alpha_r^t - C_{\alpha,r}^i\alpha_r^i \quad (5)$$

$$(I_z^t + m^i c^2)\dot{\gamma}^t - m^i c\dot{v}_c^t + m^i c d\dot{\gamma}^i = m^i c u_c^t\gamma^t - aC_{\alpha,f}^t\alpha_f^t + bC_{\alpha,r}^t\alpha_r^t + cC_{\alpha,r}^i\alpha_r^i \quad (6)$$

$$\left(I_z^i + m^i d^2\right)\dot{\gamma}^i - m^i d\dot{v}_c^t + m^i cd\dot{\gamma}^t = m^i du_c^t\gamma^t + (d+e)C_{\alpha,r}^i\alpha_r^i \tag{7}$$

$$\dot{\alpha}_f^t = \frac{v_c^t}{\sigma_f^t} + \frac{a\gamma^t}{\sigma_f^t} - \frac{u_c^t}{\sigma_f^t}\delta - \frac{u_c^t}{\sigma_f^t}\alpha_f^t \tag{8}$$

$$\dot{\alpha}_r^t = \frac{v_c^t}{\sigma_r^t} - \frac{b\gamma^t}{\sigma_r^t} - \frac{u_c^t}{\sigma_r^t}\alpha_r^t \tag{9}$$

$$\dot{\alpha}_r^i = \frac{v_c^t}{\sigma_r^i} - \frac{c\gamma^t}{\sigma_r^i} - \frac{(d+e)\gamma^i}{\sigma_r^i} + \frac{u_c^t}{\sigma_r^i}\varphi^t - \frac{u_c^t}{\sigma_r^i}\varphi^i \quad \frac{u_c^t}{\sigma_r^i}\alpha_r^i \tag{10}$$

$$\dot{\varphi}^t = \gamma^t \tag{11}$$

$$\dot{\varphi}^i = \gamma^i \tag{12}$$

Full development of this model was documented by Karkee (2009). These equations can be represented in matrix differential equation representation as,

$$M\dot{X} = NX + PU \tag{13}$$

where the state vector is $X = \begin{bmatrix} v_c^t & \gamma^t & \gamma^i & \alpha_f^t & \alpha_r^t & \alpha_r^i & \varphi^t & \varphi^i \end{bmatrix}^T$, and the input is $U = [\delta]$. Matrices M, N, and P are given by,

$$M = \begin{bmatrix} m^t + m^i & -m^i c & -m^i d & 0 & 0 & 0 & 0 & 0 \\ -m^i c & I_z^t + m^i c^2 & m^i cd & 0 & 0 & 0 & 0 & 0 \\ -m^i d & m^i cd & I_z^i + m^i d^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$N = [■(0\& -(m^{\uparrow}t + m^{\uparrow}i)\ u_{\downarrow}c^{\uparrow}t\&0\& -C_{\downarrow}(\alpha,f)^{\uparrow}t\& -C_{\downarrow}(\alpha,r)^{\uparrow}t\& -C_{\downarrow}(\alpha,r)^{\uparrow}i\&0\&0@0\&m^{\uparrow}i\ cu_{\downarrow}c^{\uparrow}$
(15)

$$P = \begin{bmatrix} 0 & 0 & 0 & -\frac{u_c^t}{\sigma_f^t} & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{16}$$

In the context of a field experiment for data collection, there are a limited number of system-level, tractor-implement outputs that can be reasonably measured with common, commercially-available sensors and data acquisition equipment and that have meaning with respect to the level of fidelity of the model being used. For the linear bicycle model considered here, these measurements could potentially include tractor and implement positions, heading angles, yaw rates, velocities, and accelerations. This study considered position and yaw rate measurements.

Vehicle positions are commonly measured using Global Positioning System (GPS) receivers, which may incorporate real-time kinematic (RTK) technology for increased accuracy. Although the GPS receiver will generally not be mounted directly over the tractor center of gravity (CG) in an experiment, it is assumed that this placement has been made possible for the purpose of this analysis. Based on the parameters and model states, the trajectory of the tractor CG was calculated as,

$$\dot{x}_c^t = u_c^t\cos(\varphi^t) - v_c^t\sin(\varphi^t) \tag{17}$$

(18)

$$\dot{y}_c^t = u_c^t \sin(\varphi^t) + v_c^t \cos(\varphi^t)$$

Similarly, the trajectory of the implement CG was calculated as follows based on the position of the tractor CG and the kinematics of the tractor and towed implement.

(19)

$$x_c^i = x_c^t - c\ cos(\varphi^t) - d \cos(\varphi^i)$$

(20)

$$y_c^i = y_c^t - c\ sin(\varphi^t) - d \sin(\varphi^i)$$

Yaw rate measurements are commonly obtained using gyroscopic sensors. These sensors can be mounted at any point on the object of interest as long as the measurement axis is oriented properly (i.e., parallel to an object's vertical axis). The tractor yaw rate, $\gamma^{\mathbf{t}}$, and implement yaw rate, $\gamma^{\mathbf{i}}$, were already calculated as states of the model. The PottersWheel (Maiwald and Timmer, 2008) toolbox was used in this study, which required mathematical models to be entered in a specific format compatible with its functions. In particular, since the model was required to be entered as a set of ordinary differential equations, it was necessary to convert the tractor-implement model from matrix differential equation representation, Eq. (13), to state-space representation to obtain the state equation,

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{B}\,\mathbf{U} \qquad (21)$$

where $\mathbf{A} = \mathbf{M}^{-1}\mathbf{N}$ and $\mathbf{B} = \mathbf{M}^{-1}\mathbf{P}$. The MATLAB Symbolic Math Toolbox was used to perform the conversion. From this representation, the eight state equations were extracted. The only model input, the front wheel steer angle, δ , was specified using a driving input function with predefined input types.

Parameter Identification

A typical method for identifying parameters in a given model structure is to find the set of values for which the simulated model output most closely represents the physical system experimental output for a given input (Figure 2; Walter and Pronzato, 1997). Closeness of representation may be determined by comparing the time history of one or more sensed outputs of the system with the time history of the same outputs of the model. For a common input vector, $\mathbf{u}$, the error vector, $\mathbf{e_y}$, between the system output vector, $\mathbf{y}$, and the corresponding model output vector, $\mathbf{y_m}$, is calculated as,

$$\mathbf{e_y}(t,\theta) = \mathbf{y}(t) - \mathbf{y_m}(t,\theta) \tag{22}$$

To obtain the best estimate of parameter values, $\bar{\theta}$, of the system model, an objective function was formulated that calculates a scalar value as a function (e.g., the sum of squares) of the output error, $\mathbf{e_y}$. The purpose of the objective function is to quantify the suitability of the model with a particular set of parameter values. The prediction error minimization (PEM) algorithm was used to search the parameter space for the minimum value of the objective function. Detailed explanation of the methods used is available in Karkee and Steward (2011).

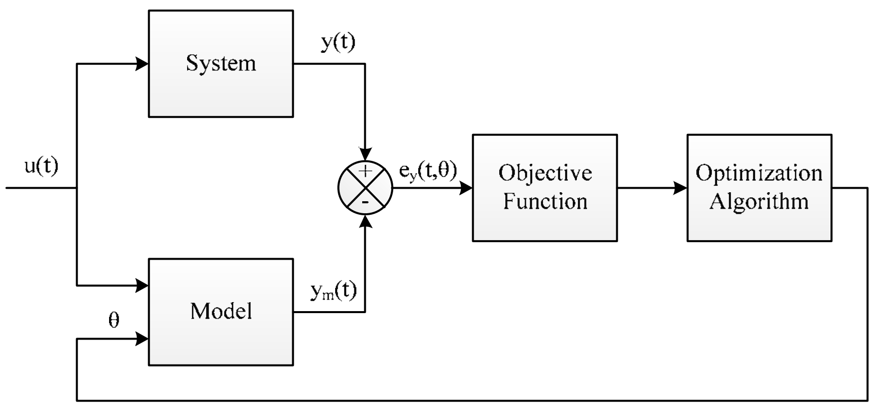


Figure 2 - The general parameter identification process (adapted from similar figures given by Walter and Pronzato (1997))

Identifiability Analysis

Identifiability refers to the uniqueness of a parameter vector $\hat{\boldsymbol{\theta}}$ as an estimate of the true parameter vector $\boldsymbol{\theta}^*$ in a model representing a physical system (Walter and Pronzato, 1997). Structural identifiability is considered independent of any data properties and considers a model to exactly represent the system of interest. Accordingly to Walter and Pronzato (1997) and Ljung (1999), "identical input-output behavior" of two identical model structures implies that the estimated parameter set $\hat{\boldsymbol{\theta}}$ is unique and corresponds to the true parameter set $\boldsymbol{\theta}^*$. A model structure is considered structurally unidentifiable if one or more of its parameters is unidentifiable. A number of methods are available for testing the structural identifiability of mathematical models (Walter and Pronzato, 1997). However, it is often impractical to test global identifiability for even simple models, and the scope will be limited to local identifiability (Serban and Freeman, 2001).

Practical identifiability, however, considers model identifiability in light of the characteristics of the experimental data used for parameter identification (Balsa-Canto and Banga, 2010), such as the quality (presence of error), quantity (sampling rate), and richness (frequency content) of the data. Therefore, it is possible for a structurally identifiable parameter to be practically unidentifiable once experimental data is introduced.

Raue et al. (2009) and (2011) described a numerical approach to local structural and practical identifiability based on the profile likelihood of the model parameters. For the optimization problem, an objective function which is the weighted sum of squared residuals is considered

$$\chi^2(\theta) = \sum_{k=1}^{m}\sum_{l=1}^{d}\left(\frac{y_{kl}^D - y_k(\theta, t_l)}{\sigma_{kl}^D}\right)^2 \quad (23)$$

where $\mathbf{k}$ is the index of $\mathbf{m}$ outputs measured, $\mathbf{l}$ is the index of $\mathbf{d}$ data points collected, $\mathbf{y_{kl}^D}$ is an experimental data point, $\mathbf{y_k}$ is a model output, and $\sigma_{\mathbf{kl}}^{\mathbf{D}}$ is the corresponding measurement error of a data point. Assuming that the noise on the measurements is normally distributed, $\epsilon \sim \mathbf{N}(0, \sigma^2)$, minimization of this objective function yields maximum likelihood estimates of the parameter set, $\boldsymbol{\theta}$. Although asymptotic confidence intervals for the parameters can be obtained based on a quadratic approximation of the likelihood at the estimated parameter values if the model "sufficiently describes the experimental data", Raue et al. (2009) acknowledged that this approximation may not hold as well for cases with data of lower quality and/or quantity. For those cases, confidence intervals based on a "threshold" in the likelihood were recommended, defined by

$$\left\{\theta \middle| \chi^2(\theta) - \chi^2\left(\hat{\theta}\right) < \Delta_\alpha\right\} \quad (24)$$

$$\Delta_\alpha = Q\left(\chi_{df}^{2,1} - \alpha\right) \quad (25)$$

where Δ_α is the $1 - \alpha$ quantile of the χ^2-distribution with $\mathbf{df}$ degrees of freedom.

In this work, the profile likelihood approach was used to study the parameter identifiability of a tractor-implement model. The profile likelihood, a plot of how the likelihood estimate changes with variation in each individual parameter, was selected to analyze if parameters could be identified from specific output data. In this analysis, each parameter is individually incremented in increasing and decreasing directions around its estimate, reoptimizing all of the other parameters to the data and recording the χ^2 (objective function) value at each step. Therefore, the approach is able to capture the effects of parameter sensitivity as well as parameter interaction on the identification of model parameters. The computation produces a profile likelihood plot for each parameter, showing how its likelihood changes with respect to the parameter values. Based on Eqs. (24) and (25), upper and lower confidence bounds for a parameter are determined by the locations at which the likelihood crosses a certain χ^2 threshold. A parameter is identifiable if it has finite confidence bounds, i.e., a profile likelihood that reaches a specific χ^2 threshold. A completely flat profile likelihood with no minimum indicates a functional relation between parameters such that a change in one parameter value can be compensated by a change in at least one other parameter with no increase in the objective function. Such a parameter is structurally unidentifiable (Raue et al., 2009).

The profile likelihood approach was implemented into the third-party PottersWheel mathematical modeling toolbox (Maiwald and Timmer, 2008) for MATLAB (The MathWorks, Inc., Natick, MA). Although the toolbox is tailored specifically toward the systems biology community, it has the capability to handle general mathematical models defined as a set of ordinary differential equations as well. In addition, the toolbox has many other functionalities that are useful in mathematical modeling, parameter identification, and model analysis. The PottersWheel built-in

CVODES solver for ordinary differential equations (Hindmarsh et al., 2005), with "methods for stiff and nonstiff systems", was used for integration.

The profile likelihood approach was first performed on simulated data for the tractor and single axle towed implement model described in Eqs. (5) – (12). Simulated data was analyzed to study the model's identifiability free from any model characterization errors or unknown experimental error and to have complete control over the addition of error to the output data. These results represented a best-case scenario for parameter identification, upon which actual experimental data would not be likely to improve. Of primary interest in the simulated data analysis were the effects of measurement noise, data sampling rate, and input signal type on the identifiability of the tractor-implement tire-soil parameters, which were considered to be the most uncertain and most difficult to measure (Karkee and Steward, 2011). Although it is acknowledged that the values of the other parameters (masses, yaw moments of inertia, and geometric dimensions) have a degree of uncertainty associated with them as well, this assumption provided a narrowing of scope for the analysis. The values of these "fixed" parameters were measured or estimated by Karkee (2009; Table 1).

The nominal values of the tire model parameters were set at or near the values initially selected by Karkee and Steward (2010a) based on their review of the literature, but the upper and lower bounds, used to determine the range that the parameters were varied in the analysis and given in Table 2, were defined relatively wide around those nominal values, as if their values were unknown. These parameters are physically limited to real values greater than zero, so the lower bound selection was straightforward. However, the upper bounds were set more arbitrarily because there was no additional information available to guide their definition. The outputs for a given model were determined by the particular sensor configuration being simulated and were calculated based on the model states and parameters according to the development in the previous two sub-sections. As part of the format for defining outputs, an error model with noise could be specified as well. Simulated data collection times were specified using a vector with start and stop times and intermediate times determined by a fixed collection frequency (e.g., 5 Hz).

Table 1 - Dynamic bicycle model parameters for the JD 7930 tractor and Parker 500 grain cart system (Karkee, 2009).

Tractor			Implement (Grain Cart)		
Parameter	Nominal Value	Units	Parameter	Nominal Value	Units
a	1.7	m	d	3.62	m
b	1.2	m	e	0.1	m
c	2.1	m			
m^t	9391	kg	m^i	2127	kg
I_z^t	35709	kg-m^2	I_z^i	6402	kg-m^2

Table 2 - Upper and lower bounds as well as nominal values for tire model parameters during the optimization.

Parameter	Units	Lower Bound	Nominal	Upper Bound
$C_{\alpha,tf}$	N/rad	10000	220000	700000
$C_{\alpha,tr}$	N/rad	10000	486000	700000
$C_{\alpha,ir}$	N/rad	10000	167000	700000

σ_{tf}	m	0.1	0.5	2.0
σ_{tr}	m	0.1	1.0	2.0
σ_{lr}	m	0.1	0.5	2.0

After creating the simulated data, it was necessary to reoptimize the six free parameters to the data to ensure that the optimum set of parameter values was reached; even though the parameter values used to create the data were known, a slightly different set of values will generally fit the simulated data with a lower objective function value. PottersWheel was used for this parameter identification process. A "trust region" optimization algorithm was selected for this process, starting from the known parameter values used to create the data. A global optimization technique would generally be chosen for the initial optimization step of a complex, multi-dimensional identification problem, but it was assumed that the optimal parameter set for the simulated data could be reached with a local technique since it started at the known, true values. Optimization was conducted in logarithmic parameter space since the normal values of the parameters extended more than one order of magnitude and can only have positive values.

From the identified parameter values, the profile likelihood approach was run. A trust region optimization algorithm was used to fit parameters in logarithmic space within the predefined bounds (Table 2). The χ^2 "threshold" for identifiability was calculated based on a simultaneous confidence level of 68% for which all parameter confidence intervals hold jointly. Simultaneous confidence intervals consider the joint effects of parameter uncertainty on model validity. The computation time required was typically between five and ten minutes per parameter for conservative settings on a 2.8 GHz workstation with 8 GB of RAM. The profile likelihood was computed in relatively small steps to ensure that it would be smooth; this required a greater number of function calls.

The analysis in this work considered the influence of measurement noise on the identifiability of the six tire model parameters. The tractor forward velocity was held constant at 4.5 m/s, and a rate-limited step input from 0 to 10 degrees at the front wheels was applied over 0.5 seconds. The model outputs were the tractor yaw rate and the implement yaw rate, each sampled at 5 Hz for a period of 10 seconds, a length of time that provided measurements that were composed of approximately half transient response and half steady-state response. The 5 Hz sampling frequency was selected based on the specifications of a GPS receiver with yaw rate sensing capabilities that is commonly used in agricultural applications. For the purpose of this investigation and to maintain the validity of the identifiability approach, Gaussian noise was added to the simulated data. The signal-to-noise ratio (SNR) of data has been expressed as the reciprocal of the coefficient of variation (Meeker and Escobar, 1998), in which

$$SNR = \frac{E(T)}{SD(T)} \tag{26}$$

In this equation, $\mathbf{E(T)}$ and $\mathbf{SD(T)}$ represent the expected value and standard deviation, respectively, of a continuous random variable $\mathbf{T}$. In this study, the numerator term of the SNR equation was specifically defined as the maximum amplitude of each tractor yaw rate signal, $\max(\gamma^t)$, in the maneuver, such that

$$SNR = \frac{\max(\gamma^t)}{\sigma} \tag{27}$$

The denominator term, σ , was the standard deviation of the specific noise model applied to the output. Therefore, the signal-to-noise ratio was varied from 1000, a nearly undistorted signal, to 25, a signal for which the transient response was more difficult to detect visually.

Results and Discussion

An earlier parameter estimation study conducted for six tire-soil interaction parameters showed that variance of the cornering stiffness parameter estimates was smaller than that of the relaxation length parameters (Table 3). The standard deviations of the cornering stiffness parameters among three different experimental trials varied from 6.7% to 12.7% of estimated parameter values whereas the same for the relaxation length parameters varied from 19% to 80% of the estimated parameter values (Karkee, 2009; Karkee and Steward, 2011). Even though all parameters of interest were identified using the PEM parameter estimation method (Karkee and Steward, 2011), standard deviations associated with the relaxation length parameters, which also can be viewed as the uncertainty in the estimated parameters, were generally very high. This result raised some concerns about those estimates and hence encouraged further identifiability analysis.

Identifiability analysis of the simulated data with a known noise model provided a means by which the model structure and certainty of the parameter estimates could be evaluated. The impact of noise became apparent as they were varied, and the overall trends agreed with expectations. The nearly noise-free dataset with signal-to-noise ratio of 1000 represented an ideal situation for parameter identification (Figure 3). The profile likelihood of each parameter was nearly parabolic (Figure 4), approaching the quadratic approximation for asymptotic confidence intervals. Confidence intervals with finite upper and lower bounds indicated that the tractor-implement model, with six uncertain tire model parameters, was practically identifiable from the tractor yaw rate and implement yaw rate data and, therefore, structurally identifiable (Table 4). Even though it was identifiable, the implement tire relaxation length was estimated least accurately, and its true value was narrowly outside of the likelihood-based confidence region.

Table 3: Three different estimations of six model-parameters of a tractor and gain cart system. Three sets of parameters were estimated from three different experimental step input trajectories collected in a field at 4.5 m/s forward velocity.

Parameter	Estimated Parameter Values		
	First Trial	Second Trial	Third Trial
$C_{\alpha f}^{t}$ (KN/rad)	090±06* (±6.7%)	100±10 (±10.0%)	090±07 (±7.8%)
$C_{\alpha r}^{t}$ (KN/rad)	600±40 (±6.7%)	550±70 (±12.7%)	560±40 (±7.1%)
$C_{\alpha r}^{i}$ (KN/rad)	070±05 (±7.1%)	090±10 (±11.1%)	060±05 (±8.3%)
σ_f^t (m)	1.5±0.4 (±26.7%)	1.0±0.8 (±80.0%)	2.1±.1.0 (±47.6%)
σ_r^t (m)	2.1±0.4 (±19.0%)	2.6±0.7 (±26.9%)	1.3±0.5 (±38.5%)
σ_r^i (m)	1.3±0.9 (±69.2%)	1.7±1.0 (±58.8%)	0.8±0.6 (±75.0%)

**Standard deviation of the estimates.*

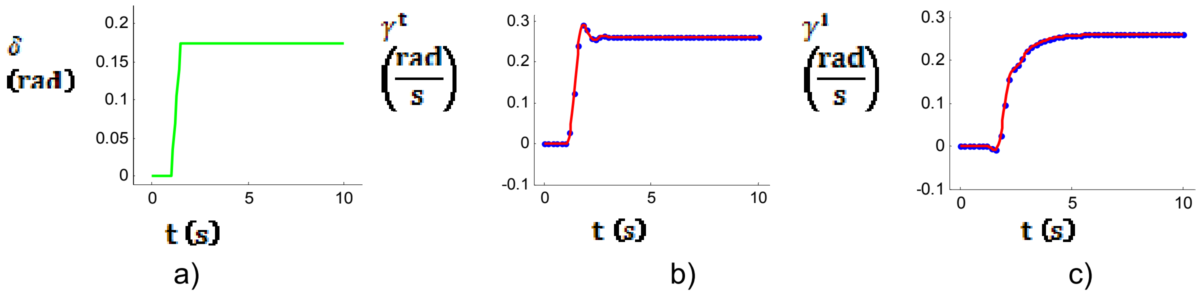


Figure 3 - Time histories of a) front wheel steering input (rad) and simulated data with signal-to-noise ratio of 1000 for b) tractor yaw rate (rad/s), and c) implement yaw rate (rad/s).

However, as the signal-to-noise ratio decreased from 1000, practical unidentifiabilities became apparent based on widening, sometimes infinite, confidence intervals and less accurate identification of the true parameter values. For the dataset with signal-to-noise ratio of 25 (Figure 5), practical unidentifiabilities became apparent based on widening confidence intervals (Figure 6). In fact, only the tractor's front tire cornering stiffness had finite upper and lower confidence bounds.

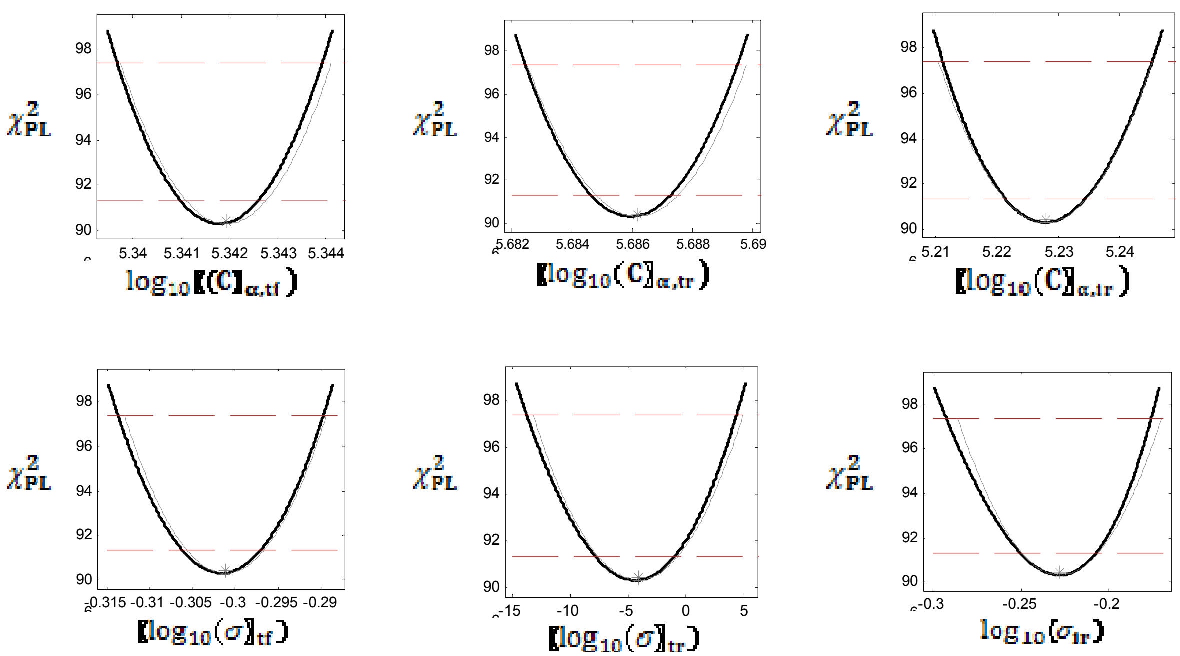


Figure 4 - Profile likelihoods for each of the six tire model parameters, plotted in logarithmic parameter space, for the simulated data shown in Figure 3. Black lines represent the profile likelihood; gray parabolas represent the quadratic approximation for asymptotic intervals. Gray asterisks at the valley of each curve indicate the estimated values of the parameters. The upper red dashed line of each plot represents the threshold for 68% simultaneous confidence intervals. The lower red dashed line represents the threshold for 68% pointwise confidence intervals.

Table 4 - True values of each parameter, as well as estimated values and 68% simultaneous

likelihood-based confidence intervals (all in normal parameter space) for the simulated data shown in Figure 4.

Parameter	Units	θ_i^*	$\hat{\theta}_i$	$CI_i^{-,PL}$	$CI_i^{+,PL}$
$C_{\alpha,tf}$	N/rad	220000	219700	218600	220800
$C_{\alpha,tr}$	N/rad	486000	485400	481400	489200
$C_{\alpha,ir}$	N/rad	167000	169000	162700	175900
σ_{tf}	m	0.5	0.500	0.486	0.513
σ_{tr}	m	1.0	0.990	0.969	1.010
σ_{ir}	m	0.5	0.591	0.510	0.667

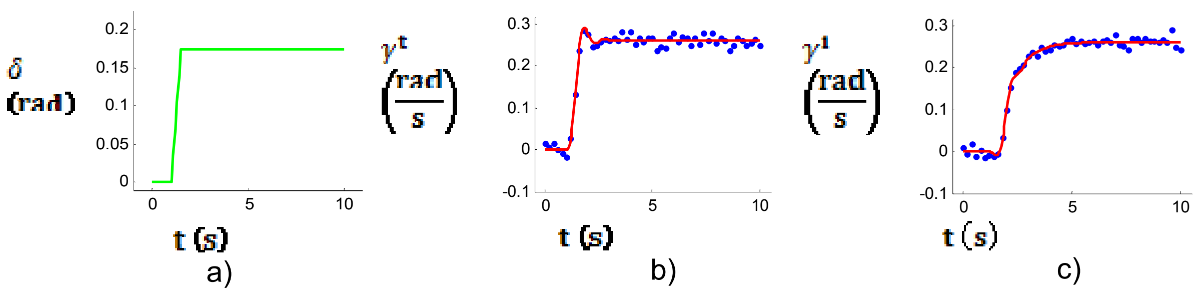


Figure 5 - Time histories of a) front wheel steering input (rad) and simulated data with signal-to-noise ratio of 25 for b) tractor yaw rate (rad/s), and c) implement yaw rate (rad/s).

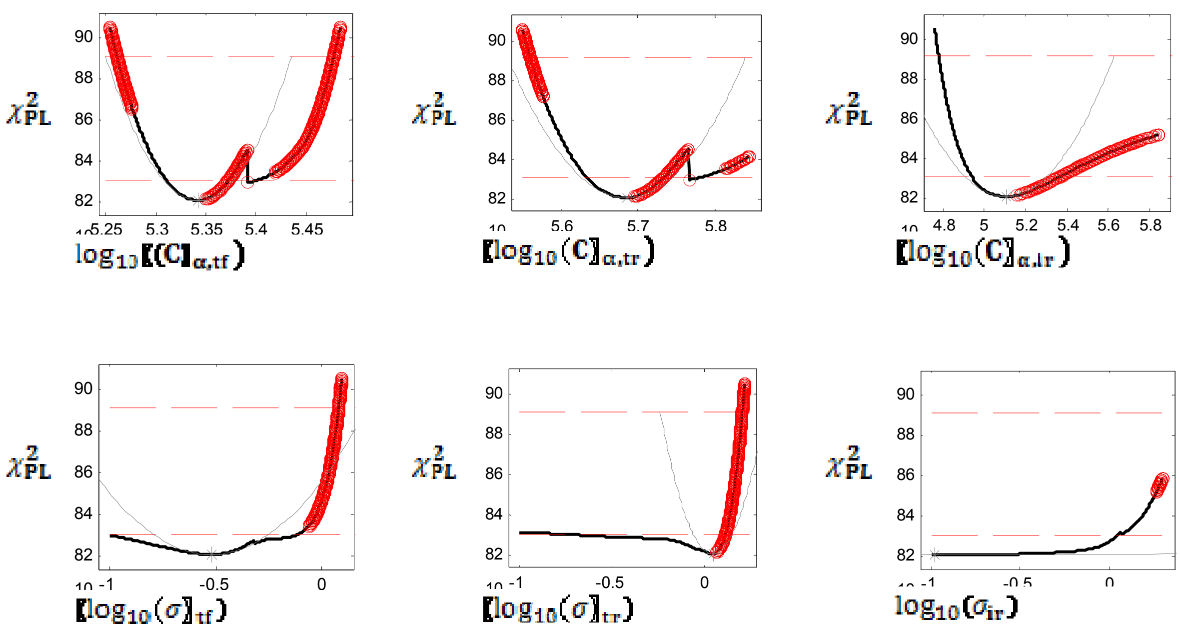


Figure 6 - Profile likelihoods for each of the six tire model parameters, plotted in logarithmic parameter space. Black lines represent the profile likelihood; gray parabolas represent the

quadratic approximation for asymptotic intervals. Gray asterisks at the valley of each curve indicate the estimated values of the parameters. The upper red dashed line of each plot represents the threshold for 68% simultaneous confidence intervals. The lower red dashed line represents the threshold for 68% point-wise confidence intervals. Red dots on a profile likelihood plot indicate simulation points where at least one of the other five parameters was fit to one of its bounds.

The identifiability analysis here showed that at lower signal to noise ratios these parameters may not be easily identifiable. Parameter identifiability analysis, thus, provides further insights into the uncertainty of tire-soil parameters and difficulties in estimating those parameters using experimental data.

Table 5 - True values of each parameter, as well as estimated values and 68% simultaneous likelihood-based confidence intervals (all in normal parameter space) for the simulated data shown in Figure 5.

Parameter	Units	θ_i^*	$\hat{\theta}_i$	$CI_i^{-,PL}$	$CI_i^{+,PL}$
$C_{\alpha,tf}$	N/rad	220000	220300	182500	300700
$C_{\alpha,tr}$	N/rad	486000	485100	363600	+∞
$C_{\alpha,ir}$	N/rad	167000	127300	60100	+∞
σ_{tf}	m	0.5	0.300	0	1.196
σ_{tr}	m	1.0	1.108	0	1.602
σ_{ir}	m	0.5	0.104	0	+∞

For this particular dataset, the tractor's front and rear tire cornering stiffness values were actually identified very close to their "true" values. The remaining four parameters were identified less accurately. The relative identifiabilities of the six parameters tend to follow the results of the sensitivity analysis (Karkee and Steward, 2010b); that is, the parameters to which the system dynamics are most sensitive are also the ones that can be estimated most confidently from the output data. The profile likelihood plot of each parameter shows how the likelihood changes as the parameter value is varied in increasing and decreasing directions around the parameter estimate (Figure 6).

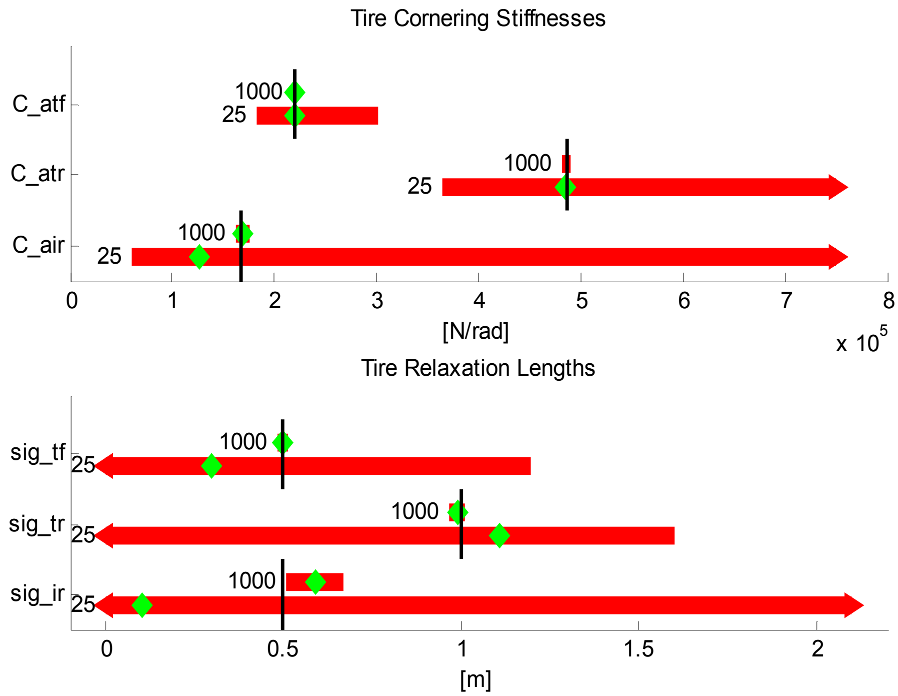


Figure 7. Likelihood-based, 68% simultaneous confidence intervals (red bars) for each of the six tire model parameters from simulated data analysis (5 Hz collection frequency of the tractor yaw rate and implement yaw rate for a rate-limited step steer input of 10 degrees). The number to the left of each bar indicates the signal-to-noise ratio of the data it pertains to. An arrow on the upper and/or lower end of a bar indicates a practical unidentifiability due to a confidence bound extending to +/- infinity (in logarithmic space). The black line in each cluster indicates the true value of the parameter which was used to create the simulated data. Each green diamond in a red bar indicates the estimated value of the parameter from the data.

Conclusions

The profile likelihood approach to identifiability analysis proposed by Raue et al. (2009) was used to study the identifiability of various tire-soil parameters of a tractor and towed implement model. A set of simulated data was generated from a dynamic model with constant tire model parameters, and different levels of Gaussian noise were added to study the parameter identifiability at different signal to noise ratios. The results showed that the tractor-implement model was structurally identifiable (at least locally), but there were issues with the practical identifiability due to the properties of the data. Although the tractor and implement yaw rates had non-zero sensitivities to each of the six tire model parameters (Karkee and Steward, 2010b), several of the parameters, especially the relaxation lengths and the implement cornering stiffness, were more difficult to identify accurately as measurement noise increased and were often practically unidentifiable in terms of their likelihood-based confidence intervals.

It can be concluded that:

- Even structurally identifiable parameters may become practically unidentifiable due to the presence of noise and other properties related to data collection.
- Parameters estimated from experimental data may be misleading if used without proper identifiability analysis because sometimes a parameter estimation method may generate results with reasonable certainty, even when the parameter is unidentifiable. In such a case, the estimated parameters should be used cautiously.

- The identifiability analysis methods used in this work are general in nature and can be applied to a variety of agricultural systems.

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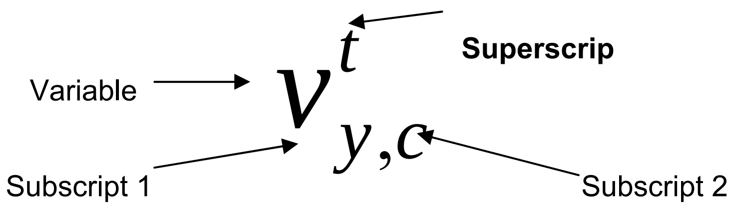
Nomenclature



Variable: The variable itself.
Superscript: Denotes whether the variable is related to tractor or implement.
t – tractor, i – implement
Subscript 1: Specifies the co-ordinate axis the variable corresponds to.
x – x axis, y – y axis, z – z axis
Subscript 2: Specifies the location the variable corresponds to.
f – front tire axle, r – rear tire axle, c – center of gravity, p – toe pin (hitch point)

List of variables

Tractor-implement model

α	tire lateral slip angle
α_0	steady-state tire lateral slip angle
γ	yaw rate
δ	wheel steer angle
σ	tire relaxation length
ψ	heading angle
a	distance between front axle and CG of tractor
b	distance between rear axle and CG of tractor
c	distance between hitch point and CG of tractor
C_α	tire cornering stiffness
d	distance between hitch point and CG of implement
e	distance between rear axle and CG of implement
F	force
I	yaw moment of inertia
m	mass
u	longitudinal velocity
v	lateral velocity
x	position of a CG in the x-axis of the world coordinate system
y	position of a CG in the y-axis of the world coordinate system

Identification

$\boldsymbol{\theta}$	vector of model parameters to be estimated
θ_i	parameter of index i in θ
$\boldsymbol{\theta}^*$	true value of θ
$\hat{\boldsymbol{\theta}}$	estimate of θ
σ	measurement error
χ2	chi-square distribution
CI	confidence interval bound
df	degrees of freedom
$\mathbf{e}_y$	output error
PL	profile likelihood abbreviation

SNR signal-to-noise ratio
t time
u vector of controlled inputs to a model or system
x model state vector
y system output vector
$\mathbf{y}_m$ model output vector