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Scattering of Ultrasound by Ellipsoidal Cavities

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Abstract

Before I begin I would like to emphasize that the work reported here is the result of a team effort, and was carried out in close collaboration with Neil Paton of the Science Center in the sample fabrication, Ken Lakin of USC in the characterization of the transducers, Dick Cohen of the Science Center in the theoretical calculations from "exact theory," John Richardson and Dick Elsley of the Science Center in the Fourier analysis of the ultrasonic pulses and the synthesis of the calculations over the band width of the transducers. We thank Jim Krumhansl and his group at Cornell University for the Born approximation results and Lazlo Adler of the University of Tennessee for the Keller theory results.

Disciplines

Materials Science and Engineering

SCATTERING OF ULTRASOUND BY ELLIPSOIDAL CAVITIES

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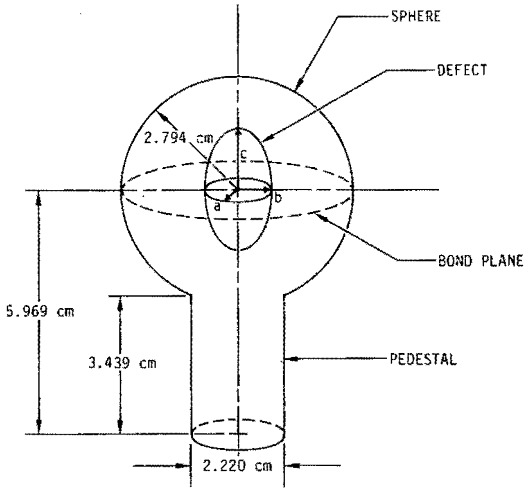
Before I begin I would like to emphasize that the work reported here is the result of a team effort, and was carried out in close collaboration with Neil Paton of the Science Center in the sample fabrication, Ken Lakin of USC in the characterization of the transducers, Dick Cohen of the Science Center in the theoretical calculations from "exact theory," John Richardson and Dick Elsley of the Science Center in the Fourier analysis of the ultrasonic pulses and the synthesis of the calculations over the band width of the transducers. We thank Jim Krumhansl and his group at Cornell University for the Born approximation results and Lazlo Adler of the University of Tennessee for the Keller theory results.

Objectives in this program are two-fold: firstly, to conduct those experiments that will explore and define the scattering of elastic waves from defects; in particular, to determine experimentally the scattering cross-sections of ellipsoidal defects in solids and to provide a critical data base for testing the regimes of validity of various approximate scattering theories; secondly, to explore and define the role of the scattering studies in the failure prediction processes where fracture is controlled by the slow growth of a single flaw; i.e.: to determine key failure prediction parameters, such as size, shape, and orientation of the flaw.

Now, you might ask why we are studying these simple ellipsoidal shapes when we are really interested in cracks. There are two reasons for this. First, we need to build up a data base for the simple shapes so that after understanding those we can launch into more complex geometries. Second, there are a lot of "real-world" defects that have the simpler geometries such that the results of our present studies would become useful immediately.

Figure 1 shows schematically the configurations of the samples which are made by the diffusion bonding process. The bond plane goes across the middle of the spherical dome with the defect in this case an exaggerated prolate ellipsoid.

The samples range from prolate spheroids to spheres to oblate spheroids and circular disks. For example, one of the disk shaped defects has a thickness of about 200μm and a diameter of 1200μm. The ultrasonic wave length is roughly 10 to 30 times the thickness of the disk.



DESCRIPTION	a (μm)	b (μm)	c (μm)	STAMP NO.
PROLATE SPHEROID	200	200	800	40
PROLATE SPHEROID	400	400	800	41
SPHERE	200	200	200	35
SPHERE	400	400	400	36
SPHERE	600	600	600	37
OBLATE SPHEROID	400	400	200	39
OBLATE SPHEROID	400	400	100	38
CIRCULAR DISC	600	600	100	62
ELLIPTICAL DISC	2500	600	250	61

Figure 1. Sample configuration.

The samples are mounted in a measurement fixture, shown in Fig. 2, which allows control of the transducer location in both elevation and azimuth in a coordinate system as shown in Fig. 3.

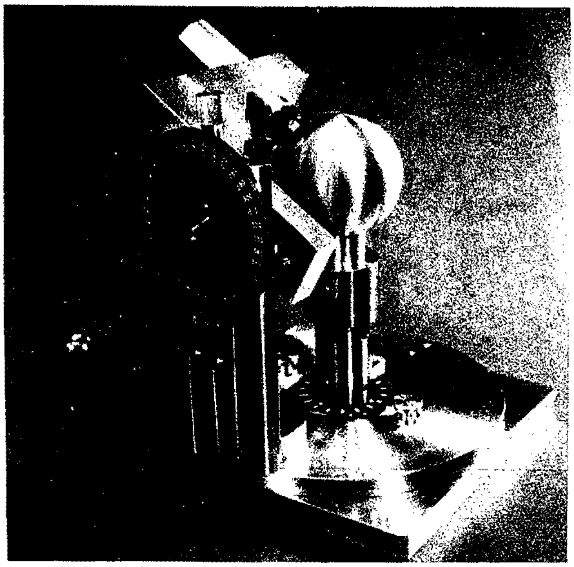


Figure 2. Photo of measurement fixture.

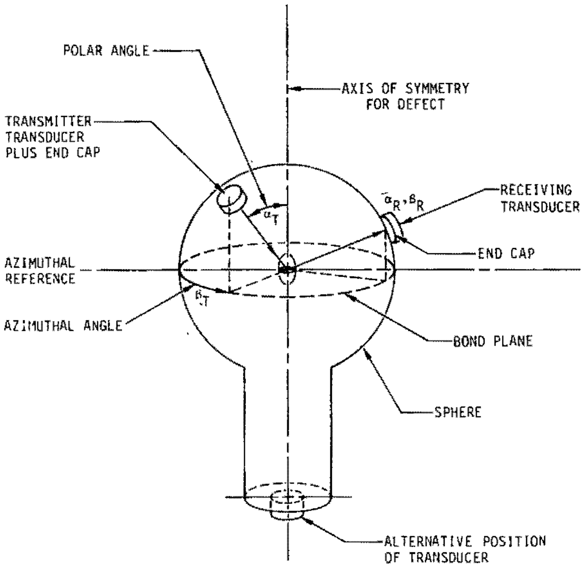


Figure 3. Measurement coordinate system.

Figure 4 summarizes results obtained with a single transducer by the pulse echo method. The graph plots the power scattered from several different defects as a function of polar angle. The polar angle is defined as the angle which the transducer makes with the axis of rotational symmetry of the defect. The data are normalized at zero polar angle. The results of Fig. 4 show that for the four different classes of defects, we get very characteristic defect signatures. As you might expect, the sphere gives us a flat response with changes in the polar angle. The prolate spheroids fall above this flat line; the oblate spheroids below that line, and if we go to the limit of the very thin disk, we get a rapid fall off to very low amplitudes. So, these results, therefore, suggest that by making a few measurements at small polar angles, we can readily distinguish the shapes of these four principal classes of objects, even though they are approximately the same size.

The second point to be made about Fig. 4 is that the main features discussed above are in good qualitative agreement with the theoretical calculations obtained from the Born approximation. In fact, the solid line is the theoretically predicted curve from the Born approximation and is shown here for a quantitative comparison. As you can see, good agreement is observed for this defect, an oblate spheroid, at a frequency of about 5 MHz. The dimensions of the spheroid are $a=b=400\mu m$ and $C=100\mu m$. The results for the oblate spheroids and the disks are also in good qualitative agreement with calculations from the Keller theory.

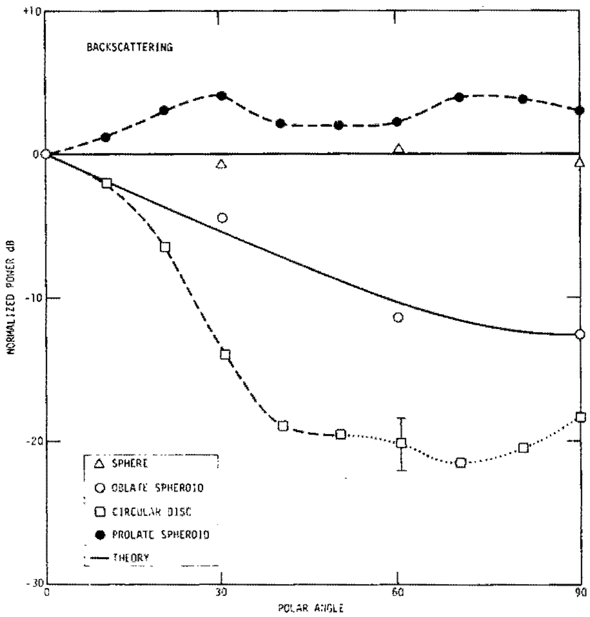


Figure 4. Angular dependence of pulse-echo intensity. (Data normalized).

If we lift the restriction of normalizing at zero polar angle, we find that the curves of Fig. 4 are displaced vertically, approximately in proportion to the product of the radii of curvature. As shown in Fig. 5, if we look at the 400μm radius and the 600μm spheres, we find the two corresponding experimental curves are separated by about 4 db. This is in agreement with physical intuition and in good quantitative agreement with geometric optics, which scales the intensity by p^2, where p is the radius of curvature - in this case p= a - and predicts a difference of 3.5 dB. This trend is qualitatively also borne out by the data on the oblate spheroid with a=b=400μm, c=200μm and by the circular disk with a=b=600μm, which gave higher back-scattered power levels at a=0 than the sphere with a=b=c=400μm, and the sphere with a=b=c=600μm respectively.

We also find that if we look at the edge of the disk, the separation in the curves of Fig. 6 is in direct proportion to the change in the cross sectional area. In this regime of angles, the curves are dotted to indicate that the main beam splits into two beams. This effect goes along with the notion that when the transducer is illuminating a disk-shaped defect at an oblique angle in the pulse echo mode, and because of specular reflection, the transducer does not see the flat portion of the disk, but only the near and far edges. If one calculates what one might expect for the splitting of the two beams on the basis of the difference in travel paths, then the calculated time delays are very close to those observed.

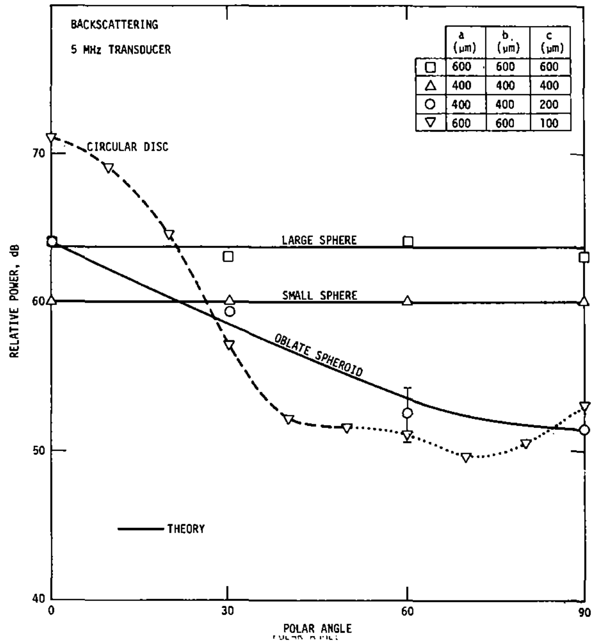


Figure 5. Angular dependence of pulse-echo intensity.

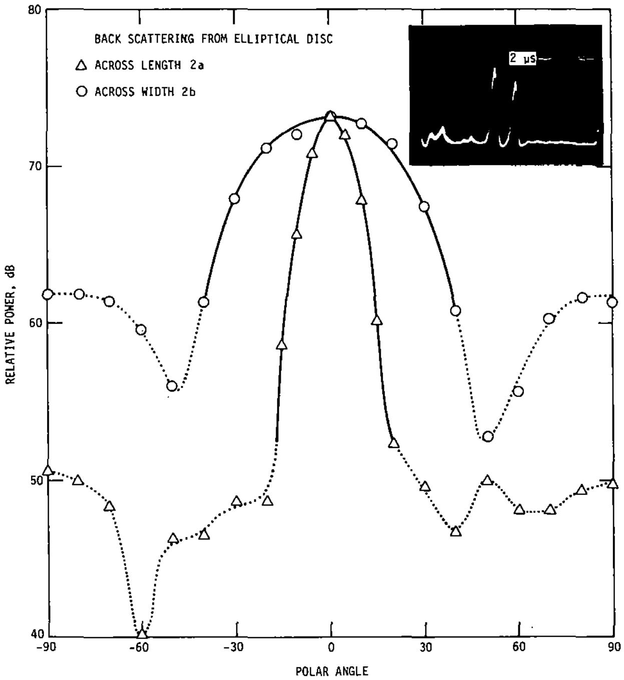


Figure 6. Angular dependence of pulse-echo intensity for disk at 2.25 MHz. (ka = 5.625) The dashed lines indicate regimes where double pulse (see insert) appears.

Figure 6 presents an interesting result because it was performed on a quasi-unknown defect by virtue of the fact that the sample was mislabeled. By making measurements as a function of the polar angle we could ascertain what the principal axes of symmetry were, and that we were having an encounter with an elliptical disk. In Fig. 6, we also see that across the width (the small dimension) the fall off in power is much less rapid in angle than for the length (the large dimension) with the notion that phase cancellation would occur at smaller angles for traversals across the large dimensions.

We have also conducted measurements with two transducers in the pitch-catch mode in which one transducer is placed at the axis of symmetry for the defect, and the other one at right angles. The result of this measurement is shown in Fig. 7 (see proceedings article by J. Krumhansl), which plots the ratio of the back scattered to side scattered power as a function of the aspect ratio. The solid line is the prediction of the Born approximation and is in semi-quantitative agreement with the experimental results shown as open circles.

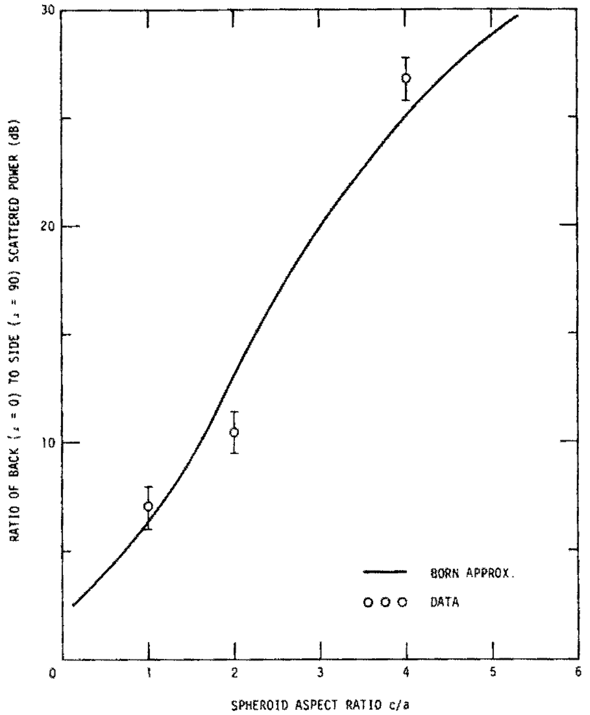


Figure 7. Ratio of back-scattered (pulse-echo) to side-scattered (pitch-catch) power.

We may conclude that both techniques, the single transducer or pulse echo technique and the two transducer or pitch-catch method, are powerful techniques for deducing the shape and orientation of geometrically shaped scattering objects. This discussion represents a brief survey of what we have been trying to do in characterizing the scattering from ellipsoids of revolution. We plan to carry out detailed measurements of the angular and frequency dependence of the scattered power and try to obtain both amplitude and phase information. We have done some very careful experiments with one category of the defects, namely, the sphere, and I will show some of those results briefly, with emphasis on the angular dependence. Comparisons will be shown between experiment and "exact theory" as developed by Lamb[1], Pao and Mow[2], Ying and Truell[3], and more recently Tittmann, Cohen and Richardson[4]. To remind you, the differential scattering cross section falls into two components for a longitudinal incident wave, i.e., the directly scattered longitudinal wave and the mode converted shear wave. The independent variable used is the scattering angle which is defined as the angle with respect to the forward direction.

Figure 8 shows theoretical and experimental results for a spherical cavity plotted in the form of a polar diagram for the case of an incident longitudinal wave. In the top portion of the graph are shown the results of "exact" theory, experimental observations as well as the Born approximation. On the bottom of the graph, similar results are presented for the mode converted shear waves, and you see that for this case where the product of the wave vector and the radius ka≈1, the results are in quite reasonable agreement.

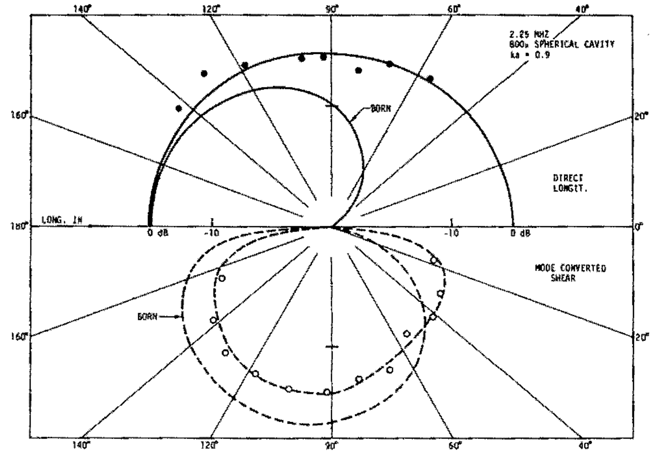


Figure 8. Scattered radiation patterns for spherical void at 2.25 MHz.

Figure 9 shows results for ka≈2 and we see deviations start to develop between the predictions of the Born approximation on one hand and "exact theory" and experiment on the other. This disagreement is expected and becomes worse as ka is increased.

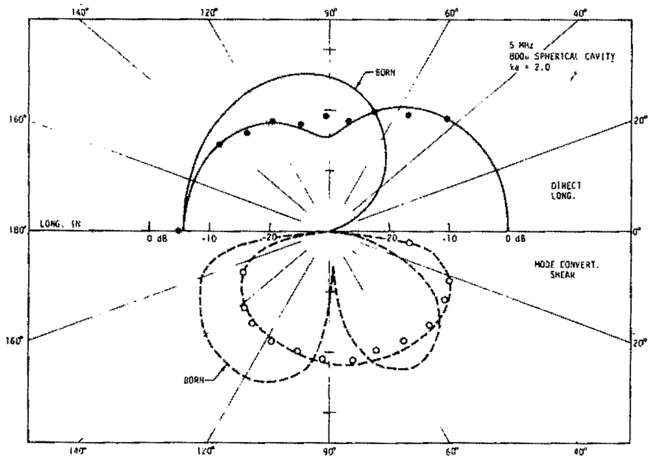


Figure 9. Scattered radiation pattern for spherical void at 5.0 MHz incident longitudinal waves.

Figure 10 is an example of our measurements with shear wave incidence and compares the data with the predictions of "exact theory." We see that considerable structure develops in the angular dependence and that the data points coincide reasonably well with theory.

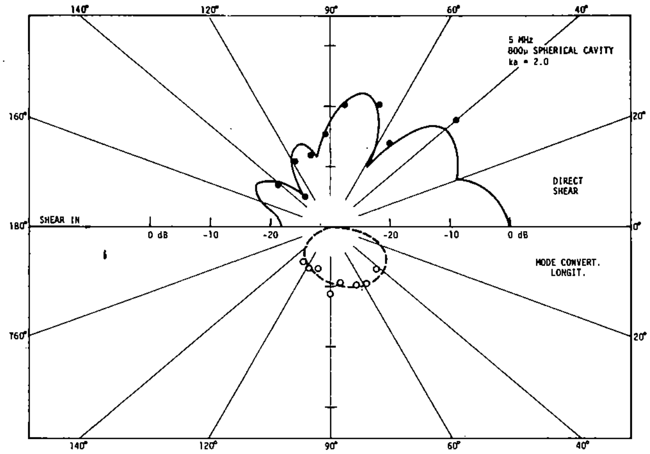


Figure 10. Scattered radiation pattern for spherical void at 5 MHz for incident shear waves.

In these studies, we come across an interesting result, namely, that the process of mode conversion is reciprocal; that is to say, if one has an incident longitudinal wave and looks at the mode converted shear wave, one gets the same angular dependence as when one sends in a shear wave and looks at the mode converted longitudinal wave. This result, shown in Fig. 11, is very useful and was originally unexpected but has now been verified theoretically, both from the Born approximation and "exact theory."

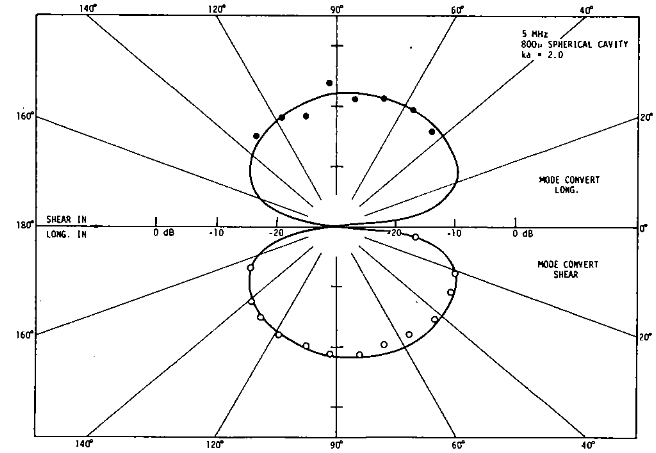


Figure 11. Reciprocity observed for spherical void.

For the Born approximation the displacement of the scattered wave in the far field can be written as[5]

$$u_j^{(s)}(\underline{r}) = |K_s|^2 \frac{e^{i\underline{K}_s\cdot\underline{r}}}{4\pi r} G_{jm} u_m^{(0)} \times$$

$$\int e^{i(\underline{K}_s - K_o)\cdot r'} \, d^3 r' \qquad (1)$$

where u^0 is the amplitude of the displacement of the incident wave with wave vector $\underline{K}_0$, $u_j^{(s)}(\underline{r})$ is the far field amplitude of the scattered wave with wave vector $\underline{K}_s$, and the matrix G_{jm} depends only on the angle of scattering and the properties of the scatterer. The term containing the integral is essentially the Fourier transform of the shape of the scatterer. From the expression it is clear that if the roles of K_s and K_0 are reversed, i.e., $K_s \rightarrow -K_0$ and $-K_0 \rightarrow K_s$ the expression is unchanged. The result of reciprocity is significant from several points of view, one of which is just in reducing the number of measurements and calculations in the study of mode conversion.

In conclusion, we have measured the scattering of elastic waves from a variety of defects ranging from spherical cavities to ellipsoidal cavities and to very thin elliptical disks. We have explored the validity of various theories, such as the Born approximation and the Keller theory, and have tested their regimes of validity. We have observed features which aid in the identification of size, shape, and orientation of geometrically shaped defects, which are three of the parameters important in failure prediction.

What we now need is to determine defect dimensions in an absolute sense in the regime of flaw criticality. Experimentally, we need to evaluate the use of the pulse echo versus the two transducer or pitch-catch method and pursue vigorously the obtainment of amplitude and phase information from the scattering signature with frequency and angular dependence as parameters, so that we can start to tackle the inverse problem. Finally, we should also direct our attention to crack-like defects.

Acknowledgements

The author is grateful to E. Domany, E.R. Cohen and L. Adler for making available the results of their calculations and to H. Nadler and L. Ahlberg for their assistance in the experiments.

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5. E. R. Cohen, private communication.

DISCUSSION

DR. PAPADAKIS: Questions?

MR. ROY SHARPE (Harwell Labs): I still don't know how you use all this information in practice. I think it came up this morning that "real" defects are not in nice spheres. You can normally only look at them from one direction. I just don't see how you're going to tackle the inverse problem.

DR. TITTMANN: As I mentioned earlier, we have to first develop a theoretical and experimental data base with simple geometries from which we can then launch into complex shapes, which may always be approximated by combinations of the simpler shapes. I think the results for the very thin elliptical disk are a good start in the direction towards "real" cracks. We have demonstrated the ease with which you can identify the axes of symmetry, the changes in radii of curvature, and the aspect ratios. We are therefore confident that many of these features can be obtained by interrogation over a limited range of angles and frequencies, as would be required by "real world" inspections of parts.

DR. EYTAN DOMANY (Cornell University*): One of the slides that Prof. Krumhansl showed displayed results for cracks in the regime of low ka, so that we can now make available calculations for comparison with experiment in this regime.

DR. TITTMANN: Thank you. This would be very useful when well characterized samples with cracks become available.

DR. PAPADAKIS: I wanted to ask you whether you have gotten an integrated cross section to--

DR. TITTMANN: You mean the power averaging over all angles?

DR. PAPADAKIS: Yes, power averaging over all angles. Does that agree with what has been published concerning the grain scattering contribution of longitudinal to shear conversion and so on?

DR. TITTMANN: That's difficult to do experimentally because of the need to collect very thoroughly all the angular information. One of the difficulties is that in the forward scattering direction the direct beam of the transducer completely masks the scattered radiation.

DR. PAPADAKIS: How about in theory?

DR. TITTMANN: The theoretical work has been done. I think Ying and Truell[3] calculate total scattering cross sections and make these comparisons.

DR. PAPADAKIS: The same approximations that go into your theory and Dr. Krumhansl's theory?

DR. TITTMANN: I don't know about that. Would you care to comment on that, Jim?

PROF. KRUMHANSL(Cornell University): We know that in some regimes for strong scattering from a cavity, the Born approximation will not give quite the right total because it simply won't. On the other hand, the Born approximation is only a "plug-in" at a second stage in the general equation. We do have some general expressions for scattering cross sections which would provide very good approximations to the Ying-Truell calculations[3] for the sphere.

DR. DICK COHEN (Rockwell International): I think the main problem with trying to do experimentally the evaluation of the power split between longitudinally scattered and shear scattered is trying to calibrate your transducers absolutely. You have a longitudinal receiver and a shear wave receiver and then to make sure that they're really calibrated to the same standard is quite difficult.

DR. PAPADAKIS: Yes.

DR. COHEN: You can make relative measurements quite easily, of course, and get the angular distribution, but to get those cross sections evaluated on an absolute basis is very difficult.

DR. PAPADAKIS: Okay. I didn't want to belabor the point, because I know you were actually aiming at different things, that is, characterizing the shape and orientation of the flaw.

* Now at the University of Washington.