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Thermal conductivity of metals at high temperatures

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Thermal conductivity of metals at high temperatures

Abstract
A new method of measuring thermal diffusivity and hence thermal conductivity of metals is suggested. Like previously reported dynamic methods, this method uses a heat source, whose temperature varies sinusoidally located at one end of an effectively infinite rod. Unlike these methods only one period of the heat wave is required to eliminate the unknown coefficient determining the heat lost by radiation since both velocity and amplitude decrement of the heat wave are measured. The new method is faster in taking data and simpler in computation. The thermoelectric potentials from two thermojunctions are amplified and plotted on a Brown "Electronic" recorder in order to obtain a permanent record of all necessary data for computing the thermal diffusivity. Results for copper over the temperature range 0-560°C and for thorium over the temperature range 0-430°C are given.

Keywords
Ames Laboratory

Disciplines
Ceramic Materials | Engineering | Engineering Physics | Materials Science and Engineering | Metallurgy | Physics

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THERMAL CONDUCTIVITY OF METALS AT HIGH TEMPERATURES

By
Paul H. Sidles
G. C. Danielson

December 1951

Ames Laboratory
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>ABSTRACT</td>
<td>4</td>
</tr>
<tr>
<td>II.</td>
<td>INTRODUCTION</td>
<td>4</td>
</tr>
<tr>
<td>III.</td>
<td>THEORY</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>A. Basic Differential Equation</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>B. Theory of Velocity Method</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>C. Theory of Amplitude Decrement Method</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>D. Theory of New Method</td>
<td>8</td>
</tr>
<tr>
<td>IV.</td>
<td>APPARATUS</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>A. Samples</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>B. Sinusoidal Heat Source</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>C. Bucking Potential and Thermocouple Switching</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>D. Amplifier and Recorder</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>E. Furnace</td>
<td>11</td>
</tr>
<tr>
<td>V.</td>
<td>PROCEDURE</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>A. Calibration</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>B. Measurement</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>C. Calculation</td>
<td>19</td>
</tr>
<tr>
<td>VI.</td>
<td>RESULTS</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>A. Copper</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>B. Thorium</td>
<td>21</td>
</tr>
<tr>
<td>VII.</td>
<td>DISCUSSION</td>
<td>23</td>
</tr>
<tr>
<td>VIII.</td>
<td>REFERENCES</td>
<td>24</td>
</tr>
</tbody>
</table>
THERMAL CONDUCTIVITY OF METALS AT HIGH TEMPERATURES*

by

Paul H. Sidles and G. C. Danielson

I. ABSTRACT

A new method of measuring thermal diffusivity and hence thermal conductivity of metals is suggested. Like previously reported dynamic methods, this method uses a heat source, whose temperature varies sinusoidally, located at one end of an effectively infinite rod. Unlike these methods only one period of the heat wave is required to eliminate the unknown coefficient determining the heat lost by radiation since both velocity and amplitude decrement of the heat wave are measured. The new method is faster in taking data and simpler in computation. The thermoelectric potentials from two thermojunctions are amplified and plotted on a Brown "Electronic" recorder in order to obtain a permanent record of all necessary data for computing the thermal diffusivity. Results for copper over the temperature range 0-560°C and for thorium over the temperature range 0-430°C are given.

II. INTRODUCTION

Many methods have been suggested for measuring the thermal conductivities of metals at high temperatures. These methods may be classified into two basic categories, static and dynamic. The earliest measurements at high temperatures were made by Forbes (1) who used a static method. The Forbes method, though modified considerably by Baillie (2), Lees (3), Bidwell (4) and others, consists essentially of heating one end of a bar, cooling the other end, and measuring the steady state temperature distribution along the bar. In order to calculate the thermal conductivity the heat lost from the surface of the bar must be either measured experimentally or reduced to insignificance. Reduction of the surface heat loss can be accomplished by thermal lagging or by enclosing the sample in a vacuum. However, the problem of experimentally eliminating or measuring the surface heat loss limits the temperature range over which the Forbes method is useful. At low temperatures radiation losses are negligible and the metal specimen can be enclosed in a vacuum to prevent conduction and convection losses.

*This report is based on a master's thesis by Paul H. Sidles, submitted December, 1951.
Therefore, no appreciable heat loss will occur and the Forbes method can be used with little difficulty. At higher temperatures, especially at temperatures above a few hundred degrees centigrade, radiation losses become large and additional precautions are necessary. Guard rings maintained at the average temperature of the sample are often used to reduce this radiation loss. However, guard rings are difficult to maintain at a constant temperature at high temperatures and are not completely effective at any temperature since the bar possesses a temperature gradient not possessed by the guard ring.

Another static method which has been employed over a large range of temperatures is that proposed by Kohlrausch (5) and used with various modifications by Jaeger and Diesellohorst (6), Angell (7), Meissner (8) and others. Basically this method consists of measuring either the radial or the longitudinal temperature distribution in a cylindrical sample which is heated by an electric current and cooled by surface radiation or electrode conduction. This method also requires that elaborate precautions be taken to eliminate heat loss corrections.

A dynamic method first used by King (9) and later modified by Starr (10) is the method with which this investigation is concerned. This method will be discussed in detail. In this case the surface heat loss need not be either measured or reduced in magnitude in order to calculate the thermal conductivity.

Experimentally all dynamic methods are similar. A heat source whose temperature varies sinusoidally impresses a sinusoidal heat wave at one end of a long rod. By sampling this heat wave as it moves down the rod the necessary information for determining the thermal conductivity of the rod may be obtained.

III. THEORY

A. Basic Differential Equation

The differential equation for heat flow in an infinite rod is

$$ k \frac{\partial^2 \Theta}{\partial x^2} = \frac{\partial \Theta}{\partial t} $$ (1)

where $\Theta$ is the temperature, $t$ is the time, and $x$ is the distance measured along the rod. By definition, the thermal diffusivity $k$ is

$$ k = \frac{K}{cd} $$ (2)

where $K$ is the thermal conductivity, $c$ is the specific heat and $d$ is
the density of the material. If the rod is radiating to its surroundings and the temperature difference between the rod and its surroundings is small enough for the heat loss to be a linear function of temperature, the differential equation will include the heat loss term \( \mu \theta \)

\[
k \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial \theta}{\partial t} + \mu \theta
\]  

(3)

where \( \mu \) is the coefficient of surface heat loss. This coefficient is given by the equation

\[
\mu = \frac{Ep}{Ac}\d
\]  

(4)

where \( E \) is the emissivity of the surface of the rod, \( p \) is the perimeter, \( A \) is the cross-sectional area and \( c \) is the specific heat of the rod.

The solution of this differential equation under the boundary conditions

\[
\begin{align*}
  &x = 0, \ \theta = \theta_1 + \theta_2 \cos \omega t \\
  &x = \infty, \ \theta = 0
\end{align*}
\]  

(5)

is

\[
\theta = \theta_1 e^{-\sigma x} + \theta_2 e^{-\alpha x} \cos (\omega t - \beta x)
\]  

(6)

where

\[
\sigma = \sqrt{\frac{\mu}{k}}
\]  

(7)

\[
\alpha = \sqrt{\frac{1}{2k} \left( \sqrt{\mu^2 + \omega^2} + \mu \right)}
\]  

(8)

\[
\beta = \sqrt{\frac{1}{2k} \left( \sqrt{\mu^2 + \omega^2} - \mu \right)}
\]  

(9)

An examination of either \( \alpha \) or \( \beta \) shows that with the exception of all of the quantities necessary to calculate \( k \) can be measured experimentally without difficulty. The quantity \( \mu \), however, becomes increasingly difficult to measure accurately or to reduce to a negligible
value as the temperature is increased. An alternative to measuring $\mu$ is to measure other quantities necessary to obtain two expression in $\mu$ and $k$ and thus eliminate $\mu$. Three ways of accomplishing this will now be discussed.

B. Theory of Velocity Method

In the method proposed by King (9) a sinusoidal heat source is placed at one end of a long rod and the velocity $v$ of the heat wave is measured in passing between two fixed points on the rod for two different periods $T$ of the sinusoidal boundary condition. This determines the quantities $\beta_1$ and $\beta_2$ corresponding to $\omega_1$ and $\omega_2$. By eliminating $\mu$ between these expressions a value for the thermal diffusivity $k$ is obtained.

If $\lambda$ is the wavelength, equation (6) shows that

$$\beta x = \frac{x}{\lambda} \ 2\pi \ \text{whence} \ v = \frac{\lambda}{T} = \frac{2\pi}{T} \beta \ \text{or} \ \beta = \frac{2\pi}{vT}. \ \ \ (10)$$

Using equation (10) to replace $\beta_1$ and $\beta_2$ by $v_1$ and $v_2$, after eliminating $\mu$,

$$k = \frac{1}{4\pi} \left[ \frac{T_1^2 T_2^2 v_1^2 v_2^2 (v_1^2 - v_2^2)}{T_2^2 v_2^2 - T_1^2 v_1^2} \right]^{\frac{1}{2}}. \ \ \ (11)$$

The thermal conductivity is then found by multiplying $k$ by the product of the specific heat and the density of the rod.

$$K = k \rho c \ \ \ (12)$$
C. Theory of Amplitude Decrement Method

Starr (10) measured the amplitude decrement $q$ of the heat wave between two points on the rod for two different periods $T_1$ and $T_2$ of the sinusoidal boundary condition. The two resulting expressions for $\alpha$ and $\omega$ were used to eliminate the radiation constant $\mu$. By definition,

$$q = \frac{e^{-\alpha x_1}}{e^{-\alpha x_2}}$$

whence

$$\ln q = \alpha(x_2-x_1) = \alpha l$$

or

$$\alpha = \frac{\ln q}{l} \quad (13)$$

Using equation (13) to replace $\alpha_1$ and $\alpha_2$ by $q_1$ and $q_2$, after eliminating $\mu$,

$$k = \frac{\pi L^2}{T_1 \ln q_1 \ln q_2} \left[ \frac{a^2-b^2}{b^2-1} \right]^{\frac{1}{2}} \quad (14)$$

where

$$b = \frac{\ln q_2}{\ln q_1} \quad \text{and} \quad a = \frac{T_1}{T_2} \quad (15)$$

The thermal Conductivity is given by equation (12) as in King's method.

D. Theory of New Method

Both of the above methods, in addition to requiring lengthy calculations, also require that measurements be made over a considerable period of time. After the necessary data for period $T_1$ has been taken, the period of the sinusoidal boundary condition must be changed and the rod allowed to reach an equilibrium condition for the new period $T_2$ before the remaining data can be taken. During this comparatively long period of time required to reach a second equilibrium and take a second
set of data, errors may enter into the experiment due to surface reactions altering the radiation loss. Also, the efficiency of heat transfer from the sinusoidal heater to the sample may change.

To reduce this possibility of errors arising from the variation of experimental conditions with time and to allow measurements to be made more rapidly, the following method was used. Instead of either the velocity or the amplitude decrement being measured for two periods of the heat wave, both the velocity and the amplitude decrement were measured for a single period.

The amplitude decrement \( q \), measured between two points on the rod separated by a distance \( L \), determine the quantity \( \alpha \) by equation (13). The velocity \( v \) and the period \( T \) determines the quantity \( \beta \) by equation (10). Eliminating the unknown radiation constant \( \mu \) by equating the product of \( \alpha \) and \( \beta \) from equations (8) and (9) to the product of \( \alpha \) and \( \beta \) from equations (10) and (13)

\[
\alpha \beta = \frac{\omega}{2k} = \left(\frac{2\pi}{vT}\right) \left(\frac{\ln q}{L}\right).
\]  

or replacing \( \omega \) by \( \frac{2\pi}{T} \),

\[
k = \frac{Lv}{2 \ln q}.
\]  

With this value of \( k \) the thermal conductivity is determined from equation (12) as before. The expression for \( k \) using this new method is much simpler in form than for either of the preceding methods and has the added advantage that it is independent of the period of the sinusoidal boundary condition. The number of quantities which must be measured for a determination of \( k \) is reduced, from the four quantities required by either King's method or Starr's method, to the two quantities \( q \) and \( v \). This new method, which uses only one period of the heat wave, is therefore not only superior in reliability but also simpler in taking data and simpler in computation.

IV. APPARATUS

A. Samples

A block diagram of the apparatus for measuring thermal conductivities using this new method is shown in Figure 1. The samples that were measured were approximately 1/8" in diameter and at least 50 cm in length.
SINUSOIDAL BUCKING POTENTIAL
HEAT SOURCE

VACUUM PUMPS AND GAUGES

FURNACE POWER SUPPLY

BUCKING POTENTIAL AND THERMOCOUPLE SWITCHING
D.C. AMPLIFIER
RECORDER

APPARATUS FOR MEASURING THERMAL CONDUCTIVITY

Fig. 1
Two American wire gauge 28 chromel-alumel thermocouples were peened into holes drilled in the sample with a No. 70 drill. The thermocouples were separated a distance \( L \) whose magnitude was chosen according to the thermal conductivity of the sample. For metals with high thermal conductivity the heat wave travels with a higher velocity than for those with low thermal conductivity. Therefore the thermocouple separation \( L \) was selected so that the time of travel and hence the velocity \( v \) could be most conveniently measured. The maximum separation was 16.8 cm in the case of copper and the minimum was 5.32 cm in the case of thorium.

A distance of several centimeters separated the sinusoidal heater at one end of the rod and the nearest thermocouple. Thus any irregularity in the cross-sectional distribution of the heat wave in the vicinity of the heater would be smoothed out by the time it reached the first thermocouple. The remainder of the sample was coiled into a helix as shown in Figure 1 so that it could be accommodated in a small furnace. The sample was enclosed in a container which was evacuated to a pressure of \( 5 \times 10^{-4} \) mm Hg.

B. Sinusoidal Heat Source

Before the theory which has been developed can be applied to the measurement of thermal conductivity a means must be devised to cause the temperature at one end of a long rod to vary with time according to the expression \( \theta = \theta_1 + \theta_2 \cos \omega t \). Ideally a sample heater whose temperature varies as a cosine function should be used. However, it is not possible to generate this function using electric heaters since such heaters can supply heat but cannot extract heat from the sample. The sinusoidal heat source used in this investigation is shown in Figure 2. The displacement of a point on a disk which is rotating with constant angular velocity is a sinusoidal function. Using the apparatus detailed in Figure 2 this displacement was used to rotate a Model 116U Powerstat variable transformer so that its output voltage varied as \((1-\cos \omega t)\). This potential was applied to a resistance heater, also detailed in Figure 2, so that the heat developed in the heater varied as \((1-\cos \omega t)^2\). With this heat function applied at one end of the rod the wave form of the resulting temperature at the nearest thermocouple was examined. This wave form was, to a good approximation, sinusoidal in form and the required boundary condition was therefore satisfied.
SINUSOIDAL HEAT SOURCE

Fig. 2
As a further check on the justification of this procedure, an examination was made of the effect of different periods and different heat inputs on the measured values of thermal conductivity. This examination showed that the effect of varying these quantities over a wide range had very little effect on the measured value of thermal conductivity. However, it was found that, when the period of the heat function was greater than 3 minutes, large power inputs caused an increase in the value of k. For this reason most of the measurements of k were made using periods shorter than 2 minutes and power inputs less than one-half that required to cause an increase in k. A typical operating condition would be a period of 110 seconds and a power input of 0.30 watts. These conditions would result in a sinusoidal temperature variation at the thermocouple nearest the heater of 0.5°C.

In the early stages of this investigation heaters were either wound tightly on a smooth sample or wound in threads cut on the outside of the sample. These heaters were insulated from the sample by the oxides which were formed on the heaters when they were heated in air. These oxides appeared to become unstable when the rod was heated in vacuum to high temperatures and caused the heater to short to the sample. For this reason this type of heater was abandoned. Heaters, which were separated from the sample by a small distance, were also tried. These heaters performed satisfactorily when making measurements near room temperature in an inert atmosphere. However, under vacuum conditions where radiative heat transfer predominates, this type of heater was unsatisfactory due to inefficient heat transfer to the sample. Since the solid angle subtended by that portion of the sample directly beneath the heater was less than $2\pi$, more than half of the heat supplied to the heater was lost to the sample container, to the remainder of the sample, and to the thermocouple leads.

To overcome these undesirable conditions the type of heater shown in detail in Figure 2 was developed. These heaters were wound with 4 mil tungsten wire using a No. 70 drill as a mandrel. After winding, one end of the wire was pulled through the coil and insulated from it by a fine Pyrex glass capillary. This assembly was placed inside a Pyrex tube 3 mm in inside diameter. This tube was then heated to the softening point and pulled down around the heater in order to have a thin layer of Pyrex glass on the outside. This layer of glass served to support the windings and to insulate them from each other and from the sample. The completed heater was dropped into a hole drilled axially with a No. 50 drill into the end of the sample. This type of heater has performed satisfactorily in vacuum up to a temperature of 650°C, the maximum temperature attained in this investigation. No material was placed between the heater and the sample to improve the thermal contact. The solid angle subtended by the sample with this type of heater is nearly $4\pi$, so that nearly all of the energy supplied to the heater is transferred to the sample.
C. Bucking Potential and Thermocouple Switching

The outputs of the thermocouples were fed into the circuit shown schematically in Figure 3. Here the thermocouple outputs were opposed by dc potentials which corresponded to the ambient temperature at which the measurements were being made. Only the sinusoidal component of the thermocouple output was left to be amplified and recorded. This circuit was contained in a constant temperature box to minimize thermal strays and to provide a constant cold junction temperature. Matched copper and low thermal solder were used in the connection from this unit to the dc amplifier. By using a 10 K Helipot in each bucking potential circuit a convenient means was provided for independently positioning the outputs of the thermocouples on the recorder chart.

D. Amplifier and Recorder

A Liston-Folb Model 10 dc amplifier and a high speed 0-12 mv Brown Electronik strip chart recorder were used to make a record of the data necessary for calculating the thermal conductivity. The Liston-Folb amplifier had a maximum gain of 80 db and a frequency response sufficient to follow a one cycle per second signal. The Brown recorder had a full scale pen travel time of 2 seconds which was more than ample for it to follow the signals used in this investigation.

Built-in calibration circuits in the Liston-Folb amplifier were used to calibrate the amplifier-recorder system. Since the output voltage of this amplifier is greater than 12 mv a voltage divider was required between the amplifier and the recorder.

E. Furnace

The furnace for heating the sample was a conventional resistance furnace. The temperature of the furnace was controlled by the apparatus shown schematically in Figure 4. With this apparatus the temperature of the sample could be held sufficiently constant during the time interval required to take the necessary data.

V. PROCEDURE

A. Calibration

The temperature of the furnace containing the sample was set at the temperature at which the measurement was to be made and the control system was set to hold the furnace at this temperature. When an equilibrium condition had been reached the following calibration procedure was
Some circuit as shown for Thermocouple I.

Thermocouple Cold Junctions

Some circuit as shown for Thermocouple I.

Special Cable to D.C. Amplifier To Galvanometer

BUCKING POTENTIAL AND THERMOCOUPLE SWITCHING.

Fig. 3
FURNACE POWER SUPPLY

Fig. 4
carried out. The gain selector of the amplifier was set at the desired value. The output of one of the thermocouples was switched to the amplifier input and the position control for that thermocouple used to position the recorder pen at the lower end of the recorder chart. A known calibrating potential was then inserted in the thermocouple circuit using the built-in calibrating equipment of the Liston-Folb amplifier. This caused the recorder pen to be deflected up-scale. The calibrating potential was then removed and the recorder allowed to return to its original position. This procedure was then followed for other gain settings of the amplifier and for the other thermocouple circuit. When all necessary amplifier gain-settings were calibrated the chart was removed from the recorder and the pen displacements were measured as accurately as possible. Dividing each calibration voltage by the pen displacement caused by that voltage gave the sensitivity of the amplifier-recorder system for that particular gain setting of the amplifier.

B. Measurement

After the system had been calibrated the sinusoidal boundary condition was applied and the sample allowed to come to equilibrium. When equilibrium had been reached and the outputs of both thermocouples had been positioned on the recorder chart the output of the thermocouple nearest the heater was recorded for several cycles. Immediately after either a maximum or a minimum was recorded the amplifier gain was changed and the recorder switched to the other thermocouple in order to record several cycles of its output. A typical record is shown in Figure 5. The amplitudes of every cycle were measured and the average amplitude for each thermocouple output was calculated. The calibration data previously obtained allowed conversion of these distance amplitudes to voltage amplitudes. The ratio of the amplitude for the thermocouple nearest the heater to the amplitude for the more distant thermocouple is the amplitude decrement \( q \). This is one of the two quantities which must be measured in order to determine the thermal diffusivity \( k \).

The other quantity which must be measured is the velocity \( v \) which was found in the following manner. The one-quarter, half and three-quarter points were geometrically determined on the chart for the nearest cycle on each side of the point at which the thermocouples were switched. The six points thus determined for the output of the thermocouple nearest the heater were then advanced along the chart a distance corresponding to one cycle. The distance \( m \) along the chart separating each of these points from the corresponding point on the output of the farthest thermocouple is a measure of the velocity of the heat wave in passing between the two thermocouples. This distance, together with the chart speed and the thermocouple separation, determines the velocity.
C. Calculation

Since the velocity \( v = \frac{Ls}{m} \) where \( L \) is the thermocouple separation, \( s \) is the chart speed, and \( m \) is the linear separation of corresponding points, we can write

\[
k = \frac{Lv}{2 \ln q} = \frac{L^2 s}{2 m \ln q} = \frac{L^2 s}{2} \left[ \frac{1}{m \ln q} \right]. \tag{18}
\]

The thermal diffusivity \( k \) was then multiplied by the specific heat and the density of the sample, using the best available data, to obtain the thermal conductivity \( K \).

VI. RESULTS

a. Copper

The results of this new method for measuring the thermal conductivity of copper are shown in tabular form in Table 1 and in graphical form in Figure 6. The points at \( 36^\circ \text{C} \) and \( 136^\circ \text{C} \) on the thermal diffusivity curve are the result of three independent measurements while the remaining points are the result of five independent determinations. The thermal conductivity of copper was calculated by multiplying these values for the thermal diffusivity by the product of the specific heat and the density of the sample. The International Critical Tables (11) give \( c_p = 24.33 + 6.03 \times 10^{-3} \text{ J mol}^{-1} \text{ K}^{-1} \) as the best value deduced from all available information for the specific heat of copper over the temperature range from 0 to \( 500^\circ \text{C} \). While no accuracy was given for this value it was stated that specific heat accuracies for metals are rarely better than one per cent and uncertainties of several per cent are not unusual. A value of \( 3.92 \text{ g cm}^{-3} \) at \( 20^\circ \text{C} \) is given by the International Critical Tables as the density of copper. These values for specific heat and density were those used in calculating the thermal conductivity \( K \) from the thermal diffusivity \( k \). The results of this calculation, after small corrections have been made for the change of density \( d \) and thermocouple separation \( L \) with temperature, are given in the right hand column of Table 1 and in the lower portion of Figure 6. The correction for changes of density and thermocouple separation with temperature were made using a value for the coefficient of thermal expansion of \( 1.07 \times 10^{-6} \) per degree centigrade as given by the Handbook of Chemistry and Physics (12) for the temperature range \( 0-625^\circ \text{C} \).

A literature survey showed that very few measurements of the thermal conductivity of copper as a function of temperature have been reported. For purposes of comparison the values for the thermal conductivity of copper as taken from Smithells' Metals Reference Book (13) are also shown.
THERMAL DIFFUSIVITY OF COPPER (a)

1.20
1.10
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00

TEMPERATURE, (°C)

THERMAL CONDUCTIVITY OF COPPER (b)

3.90
3.80
3.70
3.60
3.50

THERMAL CONDUCTIVITY (K, W/mK)

0 100 200 300 400 500 600

TEMPERATURE, (°C)

Fig. 6

- CALCULATED FROM K=kd
- SMITHULLS - METALS REFERENCE BOOK (1949)
- WILKINS & BUNN - COPPER AND COPPER BASE ALLOYS (1943)
in the lower portion of Figure 6. These values were taken from a table of thermal conductivities of metals which had been compiled from data taken from International Critical Tables and subsequent papers. The single point shown at 20°C was also taken from the Metals Reference Book and was credited to Wilkins' and Bunn's book Copper and Copper Base Alloys (14). Since small variations in the amount of impurity in a metal can appreciably change its physical properties the results obtained for copper are in good agreement with those of other workers.

B. Thorium

The results of measuring the thermal conductivity of high purity thorium are shown in Figure 7. The room temperature point on this curve is the result of a series of 22 independent measurements. Considerably fewer independent determinations were made for the other points on this curve. The large root-mean-square deviation for these points is at least partially due to the fact that less refined techniques and apparatus were used for these measurements than were used for copper. The smaller thermal conductivity of thorium is probably not responsible for these deviations.

The points on the thermal conductivity curve shown in the lower portion of Figure 7 were obtained from those on the thermal diffusivity curve using the relation $K = kcd$. The specific heat of thorium as determined by C. F. Miller (15) of the Ames Laboratory of the Atomic
THERMAL DIFFUSIVITY AND CONDUCTIVITY OF THORIUM

**Thermal Diffusivity (cm²/sec)**

- 0.300 ± 0.001
- 0.303 ± 0.032
- 0.312 ± 0.034
- 0.306 ± 0.036

**Thermal Conductivity (Watts/cm°C)**

- 0.412 ± 0.001
- 0.416 ± 0.044
- 0.428 ± 0.047
- 0.420 ± 0.049

**Temperature (°C)**

0, 100, 200, 300, 400, 500

**Fig. 7**
Energy Commission is 0.1188 joules/gm°C in the temperature range 0°C to 200°C. The density of thorium was measured as 11.558 gm/cm³. Since specific heat and density data were not available for elevated temperatures, these quantities were assumed to be constant in calculating the thermal conductivity. No thermal conductivity measurements of thorium by other observers were available for comparison with these results.

VII. DISCUSSION

The 22 independent determinations for thorium at room temperature were done under different conditions of the sinusoidal boundary conditions. Several different periods and heat inputs were used. By using the information obtained with the same heat input but for two different periods of the boundary condition, calculations were made for the thermal diffusivity of thorium using both the velocity and the amplitude decrement methods. The results of these calculations using the velocity method gave a value for k of 0.293 ± 0.025 cm²/sec°C for 16 independent calculations. Using the amplitude decrement method a value of 0.281 ± 0.025 cm²/sec°C from 16 independent calculations was obtained. These values for k agree with the value obtained using the new method within their root-mean-square deviations. A comparison of the root-mean-square deviations for the velocity and amplitude decrement methods with that of the method used in this investigation indicates that the new method is inherently capable of greater accuracy under the same experimental conditions. Further investigations of this type should be made to confirm this point.

Inadequate investigation was made of the effect of changing the waveform of the periodic boundary conditions on the measured value of thermal conductivity. If the nearest thermocouple was separated from the sample heater by only a few centimeters it was found that the waveform of the heat wave applied at the end of the rod did not have to be sinusoidal for the temperature at the nearest thermocouple to be sinusoidal in form. Further investigation of the effect on k of changing the waveform of the input to the sample heater is suggested.
VIII. REFERENCES

15. C. F. Miller, private communication.

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