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Income and Price Effects in an Alternate Exposition of the Theory of Clubs

Ron D. Adams
Iowa State University

Jeffrey S. Royer
Iowa State University

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Income and Price Effects in an Alternate Exposition of the Theory of Clubs

Abstract
As Buchanan noted in his original paper, the economic theory of clubs “allows us to move one step forward in closing the awesome Samuelson gap between the purely private and the purely public good.” [3, p.1]. Although the theory applies only to goods from which nonpayers can be excluded, the step forward his analysis permits is large. An alternate exposition of the theory is presented in Richard and Peggy Musgrave’s textbook [6, pp. 615-622]; and a graphic condensation of their exposition was developed by Allen, Amacher, and Tollison [1, pp. 386-91].

Disciplines
Economic Theory | Finance and Financial Management | Income Distribution | Statistical Models
Income and Price Effects in an Alternate Exposition of the Theory of Clubs

by

Roy D. Adams
and
Jeffrey S. Royer

No. 37
Summary: Income and Price Effects in an Alternate Exposition of the Theory of Clubs. This paper presents a concise, but useful graphic exposition of the theory of clubs, clarifies the role of comraderie and congestion, and investigates income and price effects upon club equilibrium. Price changes have the expected effect upon consumption of club good services, and an unpredictable effect on equilibrium club membership and club purchases of club goods. Income effects explain both the segregation of individuals in different clubs, and income-induced movements between clubs. It is found that high income clubs will not necessarily have small memberships, because there is a strong incentive to add members to high income-high dues clubs.
INCOME AND PRICE EFFECTS IN AN ALTERNATE EXPOSITION
OF THE THEORY OF CLUBS

Introduction

As Buchanan noted in his original paper, the economic theory of clubs "allows us to move one step forward in closing the awesome Samuelson gap between the purely private and the purely public good." [3, p.1]. Although the theory applies only to goods from which non-payers can be excluded, the step forward his analysis permits is large. An alternate exposition of the theory is presented in Richard and Peggy Musgrave's textbook [6, pp. 615-622]; and a graphic condensation of their exposition was developed by Allen, Amacher, and Tollison [1, pp. 386-91]. Although Ng [7, 8] has claimed that Buchanan's equilibrium conditions are not Pareto optimal, Berglas [2] has vindicated the Pareto efficiency rules derived by Buchanan and has shown that firms supplying club services for user fees may attain the efficiency conditions for clubs. A paper by Polinsky [9] explores some possible extensions and applications of the theory of clubs to local public finance issues. One suspects that the theory of clubs will contribute significantly to further advances in the theory of public finance.

This paper considers income and price effects upon club equilibrium by using a concise graphical and mathematical exposition. This exposition is also useful as a pedagogical improvement since it efficiently illustrates the essential relations of the model and the conditions under which clubs with finite membership may be expected to exist. In our exposition we clarify two points inadequately treated by Buchanan:
(a) the role of comraderie in the model, and (b) the differences between purely public goods (in the Samuelson sense), and goods which are not purely public, but which are sufficiently public to result in equilibrium club membership of infinity.

In our analysis of income effects we verify Buchanan's argument that higher income leads to reduced club membership when the quantity of the club good is held constant. However, we also show that increased income may either increase or decrease equilibrium club membership if the level of club goods is allowed to vary. This ambiguity results from a previously neglected effect, namely that the economic incentive to expand membership is stronger in high income clubs than in low income clubs. We present an economic explanation for the tendency of clubs and communities to be segregated by income levels, for individuals to change clubs when their income changes, and an explanation for the rich remaining in sharing arrangements. We find that club good price changes have the expected effect on consumption of club good services, but that the new service level may be provided via a club which is either larger or smaller in terms of membership and purchases of club goods. The income and substitution effects of a price change upon club good purchases and club membership are separated in "Slutsky equations."

The Model

To facilitate our graphic exposition while retaining the essential relations of Buchanan's model we reduce the utility function he used,

\[ U^i = U^i[(X_1^i, N_1^i), (X_2^i, N_2^i), \ldots, (X_{n+m}^i, N_{n+m}^i)] \]
to:

(1) \( u^i = u^i [(x^i_r, 1), (x^i_j, n^i_j)] \).

\( x^i_r \) is the quantity of a composite purely-private good consumed by individual \( i \) in a club of optimum size equal to one. \( x^i_j \) is the quantity of the impurely public good which the individual consumes in a club of \( n^i_j \) members. We assume that the marginal utility of both \( x^i_r \) and \( x^i_j \) is positive and for reasons explained below we assume that the marginal utility of \( n^i_j \) is negative for all \( n^i_j > 1 \). Figure 1 illustrates an indifference surface for \( x^i_r, x^i_j, n^i_j \) plotted on a graph in which the origin corresponds to \( (x^i_r, x^i_j, n^i_j) = (0, 0, 1) \). For any value of \( n^i_j \), the indifference

[Insert Figure 1 about here]

surface displays the usual property of a diminishing marginal rate of substitution (MRS) between \( x^i_j \) and \( x^i_r \).

Considering the slope of the indifference surface in the \( n^i_j \) direction points to an apparent inconsistency in Buchanan's exposition. He asserts in a footnote [3, p. 2, note 1] that the economic theory of clubs deals only with economic motivations for joining clubs and that "in so far as individuals join clubs for comraderie, as such, the theory does not apply." However, in his figure one, which displays cost and benefit levels in relation to the number of users of a fixed size facility, the benefit curve initially rises before declining. This construction implies that the individual would seek fellow club members to share in the ownership and consumption of \( x^i_j \) even if doing so did not reduce his costs. In our view this would involve joining
a club solely for comraderie and is therefore inconsistent with the contents of Buchanan's footnote.

To be consistent with Buchanan's benefit curve construction our indifference curves would decline in the $N_j$ direction over a limited range then begin to rise when the value of additional comraderie is offset by congestion. However, if comraderie as a motive to join clubs is excluded à la Buchanan's footnote, then the indifference surface has positive slope in the $N_j$ direction for all $N_j > 1$. We have opted for the latter construction for the following reason. Since $N_j$ is the number of persons who share ownership-consumption rights, it is the number who share control over use of the facility. Having a private facility with $N_j = 1$ does not preclude comraderie in its use, but it does give the owner exclusive control over use of the facility by others. Therefore it seems that $N_j = 1$ will be preferred to $N_j > 1$ by all persons whose desires for companionship can be satisfied by invited friends joining him in consumption of his privately owned facility. Buchanan's construction applies only to those persons whose desire for companionship cannot be met by invited friends and who find clubs appealing because of the guaranteed companionship they provide. Whether the indifference surfaces decline before rising or rise monotonically in the $N_j$ direction is of limited significance, however, because the club member's budget constraint is such that the equilibrium always occurs at a point where the indifference surface has positive slope with respect to $N_j$.

We reduce Buchanan's general form of the "cost or production function as this confronts the individual,"
The equilibrium conditions derived by Buchanan for the case of \( n + m \) goods are retained in this condensed form of the model. The Lagrangean expression for utility maximization subject to the constraints of the production function is:

(3) \( L = U^i[(X^i, 1), (X^i_j, N^i_j)] + \lambda \{ F[(X^i, 1), (X^i_j, N^i_j)] \} \)

Eliminating \( \lambda \) from the necessary first order conditions produces the equivalent of Buchanan's equation 7:

(4) \( \frac{U^i}{U^r} = \frac{f^i}{f^r} = \frac{U^i}{NJ} \)

Rearranging the first two terms gives Buchanan's equation 5,

(5) \( \frac{U^i}{U^r} = \frac{f^i}{f^r} \)

and rearranging the second and third terms gives Buchanan's equation 6,

(6) \( \frac{U^i}{NJ} = \frac{f^i}{f^r} \)

A third aspect of the equilibrium, which is implicit in condition 4 but was not explicitly considered by Buchanan, is obtained by rearranging the first and third terms:

(7) \( \frac{U^i}{NJ} = \frac{f^i}{f^r} \)
This condition is explicit in Polinsky's exposition of the model [9, p. 172]. The exact form of the production function confronting the individual depends upon both the cost conditions of $X_J$ production and the cost sharing arrangements of the club. If the unit cost of $X_J$ is independent of the quantity purchased, if costs are equally shared by all club members, and if the individual can costlessly find others to join him in a club, the individual's budget constraint is:

$$y = X_r + P_J X_j / N_j$$

where $y$ represents the individual's income, $P_J$ is the annual service cost per unit of $X_J$, and the price of $X_r$ equals one. When this constraint is used in the Lagrangean of 3, the equilibrium conditions 4, 5, 6, and 7 take the following forms:

$$\frac{U^i_J}{P_J / N_J} = \frac{U^i_r}{1} = \frac{U^i_N}{1} = \frac{U^i}{1}$$

$$\frac{U^i_J}{N_J} = \frac{P_J}{1}$$

$$\frac{U^i_N}{U^i_r} = -\frac{X_J P_J}{N_J^2}$$

, and
Although an equilibrium can be defined by any two of conditions 5, 6, and 7, each condition describes a separate aspect of full equilibrium. Condition 5 states that the MRS between the composite private good $X_f$ and the club good $X_J$ will be equated to the cost per member of an additional unit of the club good; with the budget constraint of 8, this price is $P_J/N_J$. Condition 6 states that in equilibrium, the MRS between the composite private good and additional club members, will be equated to the price of additional members, which with 8, is equal to $-X_JP_J/N_J$. The negative sign reflects the fact that spreading the cost of a fixed amount of club goods over more members reduces dues per member. Finally condition 7 states that in equilibrium the MRS between additional club goods and additional members will be equated to the (negative of the) increment to the club's goods provided by the addition of each member to the club. This quantity is the ratio of the "price" of additional members and the per capita price of additional units of the club good.

Several graphic descriptions of these equilibrium conditions are possible. Buchanan's $Q^{opt}$ and $N^{opt}$ schedules are plots in $N_J, X_J$ space which satisfy respectively, conditions 5 and 6, and the intersection of the two schedules defines the point where condition 4 holds. Since condition 7 is implicit in 4, it also holds at this point. Our figure 2 contains both the club budget constraint and an indifference
surface for persons with incomes of \( y \), and identical preferences; it shows, in one figure, the conditions which hold in club equilibrium.

[Insert Figure 2 about here]

In this diagram the inverse of the slope of the budget constraint in \( X_r, X_j \) space is \( \frac{P_j}{N_j} \); at point A, the MRS between \( X_r \) and \( X_j \) is equated to the per capita price of additional club goods for a club of given membership. The slope of the budget constraint in \( X_j, N_j \) space is \( \frac{X_j}{N_j} \); at point A, this value is equated to the MRS between additional club members and additional club goods. Condition 6 may also be described in this diagram; it is considered below in our discussion of income effects.

Condition 7 provides interesting insights into the differences among private, public, and club goods, particularly the conditions under which clubs with unique and finite memberships of more than one will emerge. This discussion is facilitated by expressing this aspect of the equilibrium in terms of the elasticity of \( X_j \) with respect to \( N_j \) on the budget constraint and on the indifference surfaces. Solving the budget constraint of 8 for \( X_j \) produces

\[
X_j = \frac{(y - X_r)}{P_j}. \quad \text{Thus,}
\]

\[
\frac{\partial X_j}{\partial N_j} = \frac{y - X_r}{P_j} = \frac{X_j}{N_j},
\]

and the elasticity of club goods, \( X_j \), with respect to additional dues-paying members, \( N_j \), which is defined as \( \frac{\partial X_j}{\partial N_j} \cdot \frac{N_j}{X_j} \), is equal to one along the budget constraint for any level of club dues per member.
Given this, the elasticity of $X_j$ with respect to $N_j$ ($\varepsilon$) on the indifference surface determines the equilibrium club size. With the budget constraint assumed here, a unique and finite equilibrium $N_j$ greater than one exists only if this elasticity is less than one at $N_j = 1$, is equal to one at a unique $N_j$, and is greater than one for all larger values of $N_j$. Such a case was illustrated in Figure 2.

The existence of a range wherein $\varepsilon < 1$ is quite plausible for goods containing some publicness, as is a point where the publicness is exhausted and $\varepsilon = 1$. However, an explanation for the existence of a range wherein $\varepsilon > 1$ is more elusive. $\varepsilon > 1$ requires that club members be made worse off by equal proportionate increases in club facilities and membership. If congestion is measured by density of people relative to facilities, then congestion does not explain $\varepsilon > 1$, because equal percentage changes of $X_j$ and $N_j$ do not increase density. The difficulty of identifying an explanation for $\varepsilon > 1$ for goods having a range of publicness ($\varepsilon < 1$) suggests that in some cases the elasticity may equal one over a large range of $N_j$. If this is so, there may be no unique equilibrium membership level for clubs which provide these goods, given the budget constraint assumed here.

There may, however, be reasons for assuming a budget constraint along which the elasticity is not always equal to one. With such a budget constraint, there may be a unique equilibrium membership level, even if the elasticity on the indifference surface is equal to one along a range of $N_j$. A budget constraint along which the elasticity is not always equal to one may result from decreasing returns to scale in the production of the club good; from transactions costs involved
in the act of sharing; or from maintenance costs on shared durable goods. Maintenance costs may increase more than in proportion to use when the goods are moved from private to joint ownership because shared ownership reduces the individual's share of the costs resulting from carelessness or misuse.

Purely Private and Purely Public Goods

The \( X_J, N_J \) sub-space of figure 2 illustrates condition 7a and may be used to show the special cases of purely private and purely public goods. According to the widely accepted private good definition used by Samuelson [10], a good is private if consumption of it is rival. Buchanan prefers to avoid such a priori definitions and would presumably classify as private, those goods for which the optimum club size is one. As he states, "A good for which the equilibrium value of \( N_J \) is large can be classified as containing much 'publicness.' By contrast, a good for which the equilibrium value of \( N_J \) is small can be classified as largely private" [3, p.6]. For goods which are private in the Samuelson sense, the elasticity of substitution between the good and more sharers of it along an indifference surface is at least equal to one. If \( \varepsilon = 1 \) a club provides no economic advantages; in \( X_J, N_J \) space the indifference surfaces and budget constraints coincide. If \( \varepsilon > 1 \), as it will be if the act of sharing per se is disliked, the optimum club size is one; this case is illustrated in figure 3 which contains the relevant portions of two indifference surfaces and budget constraints.

[Insert Figure 3 about here]

The special case of the purely public good, characterized by completely non-rival consumption, is represented by indifference surfaces which are flat in the \( N_J \) direction indicating that expansion of the sharing
group does not diminish the benefits received by other users of the existing $X_j$. Plotting this relation along with the relevant portion of the budget constraint provides a graphic demonstration of the well-known fact that any finite sharing group for purely public goods is too small because the dues of additional members will increase the quantity of $X_j$ available to all club members without diminishing the benefits received from it by existing members. As long as incremental $X_j$ is positively valued, there is reason to expand the club's dues paying membership. This case is illustrated in figure 4.

[Insert Figure 4 about here]

A problem with the Buchanan notion of a public good is that it fails to distinguish between the type of good which is purely public à la Samuelson and goods which are not purely public, but which are sufficiently public to result in an equilibrium club size of infinity; the latter case is illustrated in figure 5. Pure publicness is a sufficient, but not a necessary condition, for an equilibrium size of infinity; a necessary condition is that the good be such that the $\epsilon$ of $X_j$ with respect to $N_j$ along the indifference surface be less than one for all values of $N_j$. While such goods contain much publicness, they are not purely public à la Samuelson.

[Insert Figure 5 about here]

**Income Effects and Equilibrium Club Membership**

Previous expositions of the theory of clubs have assumed that all individuals have equal income and identical tastes and preferences. We retain the assumption of identical preferences but relax the assumption regarding income by assuming that different incomes exist and that
there are enough people at each income level to form the equilibrium club for that income group. This framework implies two related results. One is that segregation by income levels is a result of full equilibrium; the corollary is that changes in income will generally motivate one to change clubs. If the income elasticity of demand for club services is different from zero, then there will be different equilibrium clubs for each different income group. 3

A. Quantity of Club Good Fixed

Buchanan notes [3, p. 12, note 1] that his analysis "suggests clearly that the optimal club size, for any quantity of good, will tend to become smaller as the real income of an individual is increased." An extracted two-dimensional portion of our graph depicting condition (6a) provides a demonstration that this is generally true, given the constraint that the quantity of the club good is fixed. However, as will be shown in the next section, equilibrium club membership does not necessarily decrease as income rises, if the quantity of the club good is unconstrained.

Figure 6 describes the relations between $X_r$ and $N_J$ for a fixed level of the club good. It is extracted from figure 2, with $X_r$ fixed. The slope of this portion of the budget constraint, $rac{\partial X_r}{\partial N_J} = \frac{X_r P_J}{N_J}$, illustrates the (diminishing) reduction of dues per member permitted by expansion of membership. The slope of this portion of the indifference surface reflects the reduction of dues which the membership requires to willingly accept another user of their fixed-size facility.
If the equilibrium at point A is disturbed by an increase in income to \( y' \), the individual finds himself at point A'. At point A' the slope of the budget constraint \( \frac{X_jP_j}{N_j} \) is the same as at point A, but if \( X_r \) is subject to diminishing marginal utility, the marginal rate of substitution of \( N_j \) for \( X_r \) will be more at A' than at A, and a higher indifference surface can be attained by reducing membership to attain point B.

B. Income Effect in General

Next we consider the effect of a change in income without the above constraint on the amount of the club good. This analysis is facilitated by using the concept of sharing technologies \([4, pp. 3-4]\) and the variable \( X_s \), defined as the level of club services provided by combinations of \( X_j \) and \( N_j \),

\[
(9) \quad X_s = X_s(X_j, N_j).
\]

This permits reduction of the utility function of equation (1)

\[
U^i = U^i[(X_r^i, 1), (X_j^i, N_j^i)]
\]

to

\[
(10) \quad U^i = U^i(X_r^i, X_s^i).
\]

The sharing technology of equation 9 determines the shape of \( X_s \) isoquants. The plot of these "isoquants" in \( X_j, N_j \) space is equivalent to the previously discussed indifference curves between \( X_j \) and \( N_j \) for fixed levels of \( X_r \). Both show combinations of \( X_j \) and \( N_j \) between which the individual is indifferent. One set of such isoquants is depicted.
in Figure 7a, which also contains budget constraints for clubs with different levels of dues per member. Since the origin is \((X_J, N_J) = (0, 0)\), all constraints pass through the origin. The slope of each constraint is \(\frac{D^i}{P_J}\), the additional \(X_J\) provided by the dues \((D^i)\) of each added member. The fact that this slope increases with the level of club dues illustrates the previously neglected fact that the incentive to expand membership increases with the level of dues.

The tangencies between the \(X_s\) isoquants and the budget constraints identify the most efficient \(X_J, N_J\) combinations for the provision of each \(X_s\) level. The corresponding dues are the cost per member of each service level. The locus of such tangencies determines the function:

\[
D^i = D^i(X_s)
\]

The individual who may choose among clubs efficiently providing various service levels has the budget constraint

\[
y = X_J + D^i(X_s)
\]

illustrated in figure 7b. The (negative of the) slope of the constraint is the marginal cost, \(\frac{\partial D^i}{\partial X_s}\), of moving to a club with a higher service level.

[Insert Figure 7 about here]

Equilibrium service level is determined in figure 7b by the tangency between the individual's budget constraint and the highest attainable indifference curve. The optimal combination of \(X_J\) and \(N_J\) for the provision of that service level appears in figure 7a.  

An increase of income moves the individual from the prior equilibrium at A to A' in figure 7b. If \(X_r\) and \(X_s\) are both normal goods, point A' is inferior to point B, which is attainable by spending
part of the income increment on the higher annual dues of a club providing the service level $X_s^{''}$ corresponding to point B. Clubs providing this service level most efficiently have membership and facilities corresponding to point F in figure 7a.

Whether the high income clubs providing higher service levels will in equilibrium have smaller or larger memberships than lower service level clubs depends on the characteristics of the sharing technology applicable to the good $X_j$. The expected desire of the rich to reduce the number with whom they share $X_j$ is at least partially offset by the high contributions of additional members in high dues clubs. The result depends on the elasticity of substitution between $X_j$ and $N_j$ along the $X_s$ isoquants. Since the club budget constraints of figure 7a have constant unitary elasticity, the equilibrium membership for each service level occurs at the point of unitary elasticity on the isoquant.

It is plausible that the slope of the isoquants above any given $N_j$ will increase with the service level, because generally the $X_j$ increment required to maintain a high service level while enlarging membership exceeds the required increment to maintain a lower service level. Were it not for the greater slope of high-dues clubs' budget constraints the increasing steepness of the isoquants would lead to smaller equilibrium memberships in high dues-high income clubs. However, the increasing slope at least partially offsets this effect and may result in larger memberships in high-dues clubs than in low-dues clubs. We find no a priori basis for predicting the net effect. However, this relationship probably explains why many high income persons join clubs instead of privately providing all their consumed goods.
Notice that the preceding analysis provides an economic explanation for the tendency of persons with increased income to leave their old clubs and join clubs comprised of higher income individuals. The person who stays in his original club after an increase in his income is at point A' in figure 7b. This is an equilibrium position only if his income elasticity of demand for club services is zero. If higher club services are desired at the higher income level, there is a sound economic reason to move to a club comprised of higher income persons paying higher dues. 5

Price Effects

The effect of club good price changes upon equilibrium club membership is similarly indeterminant. The effect of a reduction in $P_J$ is illustrated in figure 8.

[Insert Figure 8 about here]

The price reduction decreases both the cost of attaining each $X_s$ level and the marginal cost of moving to a higher level. The resulting shift of the individual budget constraint is illustrated in figure 8b. There is both an income and a pure price effect in the direction of increased consumption of club services, $X_s$. As in the case of an increase in income, the change in equilibrium $N_J$ is unpredictable because again the point of unitary elasticity on the higher $X_s$ isoquant may lie to the left or right of the point of unitary elasticity on the lower $X_s$ isoquant which was the equilibrium prior to the price change.

Notice that the effect of a price change (or a change in income) upon the equilibrium club good level is also indeterminant. While
increased income or a decrease in the price of the club good will generally result in increased equilibrium consumption of club services, the optimal combination of club facilities and membership for the provision of this increased service level relative to the prior equilibrium is indeterminate. The difference between the two equilibrium states depends on the sharing technology applicable to the good and cannot be predicted in general.

As in the standard two-private goods case, the effect of the change in the price of the club good upon the club's purchases of it can be separated into a substitution and an income effect. In the standard two-private goods model, these effects can be examined and the Slutsky equation may be derived by differentiation of the first order conditions for utility maximization subject to a budget constraint [5, pp. 31-32]. As our appendix shows, applying this procedure to the previously discussed first-order conditions for club equilibrium, produces the following "Slutsky equation", in which the effect of a price change on club good purchases is separated into a substitution and income effect.

\[
(13) \frac{\partial X_j}{\partial P_j} = \left( \frac{\partial X_j}{\partial P_j} \right) U = \text{constant} - \frac{X_j}{N_j} \left( \frac{\partial X_j}{\partial y} \right) \text{prices = constant.}
\]

This expression differs from the standard Slutsky equation in that the coefficient of the income effect in (13) is \( \frac{X_j}{N_j} \) rather than \( X_j \). Notice that this generalization of the Slutsky equation encompasses the special case of two-private goods, in which \( N_j = 1 \). The explanation for the \( \frac{X_j}{N_j} \) coefficient of the income effect lies in the budget constraint of club members. This constraint assumes that club costs are equally shared, so
that each member pays only $1/N_j$ of the cost of club goods. It follows that the income effect of a change in the club good's price is only $1/N_j$ of what it would be if each individual paid the full cost of consumed club goods.

As was previously illustrated, a decrease in the price of $X_j$ will generally result in an increased level of club services consumption, but the movement to the higher service level generally involves an adjustment in both $X_j$ and $N_j$. Equation (13) separated the adjustment of $X_j$ into a substitution and an income effect. Analogously the adjustment of $N_j$ may be written as

$$
\frac{\partial N_j}{\partial P_j} = \left( \frac{\partial N_j}{\partial P_j} \right) \frac{X_j}{N_j} \left( \frac{\partial N_j}{\partial y} \right) \text{ prices = constant.}
$$

Again it is noteworthy that the coefficient of the income effect, $X_j/N_j$, reflects the assumption that the cost of $X_j$ is shared equally among the $N_j$ members of the consuming group.

Our conclusion regarding price effects is that decreased club good prices result in greater consumption of club good services, but one cannot predict a priori whether club good price changes will lead to increases or decreases in equilibrium club membership and club purchases of club goods.

Conclusion

The theory of clubs has filled a significant gap in economic theory. Berglas's demonstration that the market may efficiently provide the services of some club goods is interesting, but it does not negate the relevance of the theory of clubs to public finance theory. Although there are examples of market provision of club services, there are also many examples of club
good provision by clubs per se and by various levels of government.

Our exposition of the theory clarifies some ambiguities in the original formulation regarding comraderie, congestion, and the difference between purely public goods and club goods. Although the effects of club good price changes are not very surprising, our consideration of income effects has provided several significant results. We have shown that, even if all individuals have identical preferences, they will segregate themselves by income groups in communities providing different levels of club good services. We have also shown that increased income does not necessarily result in clubs with fewer members, and that high income individuals will not necessarily remove themselves from sharing arrangements, because the economic incentive to add members to a sharing arrangement is actually stronger in a high-dues club than in a low-dues club.
Appendix

Derivation of the Slutsky equation describing the effect of a change in the price of the club good on the club's purchases of the good begins with the Lagrangean expression for utility maximization subject to a budget constraint:

\[ L = U^1[(X_r, 1), (X_j, N_j)] + \lambda[y - X_r - P_j X_j N_j^{-1}] \]

A rational individual will plan his expenditures so that the first-order conditions for utility maximization subject to the budget constraint are satisfied:

\[ \frac{\partial L}{\partial X_j} = U_j - \lambda P_j N_j^{-1} = 0 \]

\[ \frac{\partial L}{\partial X_r} = U_r - \lambda = 0 \]

\[ \frac{\partial L}{\partial N_j} = U_N + \lambda P_j X_j N_j^{-2} = 0 \]

\[ \frac{\partial L}{\partial \lambda} = y - X_r - P_j X_j N_j^{-1} = 0 \]

The effect of price and income changes on the individual's expenditure decisions are determined by allowing all variables to vary simultaneously, accomplished here by total differentiation of the first-order conditions:

\[ U_{jj} dX_j + U_{jr} dX_r + (U_{jn} + \lambda P_j N_j^{-2}) dN_j - P_j N_j^{-1} d\lambda = \lambda N_j^{-1} dP_j \]

\[ U_{rr} dX_r + U_{rn} dX_r + U_{rn} dN_j - d\lambda = 0 \]
Solution of these four equations for the four unknowns, \(dX_j, dX_r, dN_j,\) and \(d\lambda,\) can be determined by treating the terms on the right hand sides as constants. Derivation of the solution is facilitated by using the matrix expression:

\[
\begin{bmatrix}
U_{JJ} & U_{Jr} & U_{JN} + \lambda P J N J^{-2} & -P J N J^{-1} \\
U_{rJ} & U_{rr} & U_{rN} & -1 \\
U_{NJ} + \lambda P J N J^{-2} & U_{Nr} & U_{NN} - 2\lambda P J X J N J^{-3} & P J X J N J^{-2} \\
-P J N J^{-1} & -1 & P J X J N J^{-2} & 0
\end{bmatrix}
\begin{bmatrix}
dX_j \\
dX_r \\
dN_j \\
d\lambda
\end{bmatrix}
= \begin{bmatrix}
\lambda N J^{-1} dP J \\
0 \\
-\lambda X J N J^{-2} dP J \\
-dy + X J N J^{-1} dP J
\end{bmatrix}
\]

The matrix on the left is the bordered Hessian matrix. The second order conditions for utility maximization subject to the budget constraint are satisfied if the determinant of this matrix is negative definite. The borders of the bordered Hessian are composed of the negatives of the implicit prices of \(X_j, X_r, N_j,\) and \(\lambda\) respectively.

If the determinant of the bordered Hessian is denoted by \(D\) and the cofactor of the element in the \(g\)th row and \(h\)th column of the matrix by \(D_{gh},\) the solution for \(dX_j\) by Cramer's rule can be expressed as:

\[
dX_j = \frac{D_{11}(\lambda N J^{-1} dP J) + D_{31}(-\lambda X J N J^{-2} dP J) + D_{41}(-dy + X J N J^{-1} dP J)}{D}
\]

If the individual's income and the price of the numeraire good are assumed to remain unchanged, \(dy = 0,\) and the overall effect of a change in the price of the club good on the club's purchases of the club good is determined by dividing both sides of this expression by \(dP J:\)
\[
\frac{\partial X_j}{\partial P_j} = \frac{D_{11}(\lambda N_j^{-1}) + D_{31}(\lambda X_jN_j^{-2}) + D_{41}(X_jN_j^{-1})}{D}.
\]

The change in \( X_j \) which results from a change in the individual's income while prices are held constant is determined by setting \( dP_j = 0 \) and dividing the above expression for \( dX_j \) by \( dy \):

\[
\left( \frac{\partial X_j}{\partial y} \right) \text{prices = constant} = \frac{D_{41}(-1)}{D}.
\]

The change in \( X_j \) which results from a change in the price of the club good while utility is held constant is more difficult to determine. If utility is held constant, \( dU = 0 \), and taking the total differential of the utility function produces:

\[
dU = U_J dX_J + U_r dX_r + U_N dN_J = 0.
\]

From the first-order conditions, the following relationships can be derived:

\[
\lambda = \frac{U_J}{P_J N_j^{-1}} = \frac{U_r}{1} = \frac{U_N}{P_J X_j N_j^{-2}}.
\]

By solving each of these for the respective partial derivative, substituting these into the total differential of the utility function, and dividing the resulting expression by \(-\lambda\), it can be shown that:

\[
-P_J N_j^{-1} dX_J - dX_r + \lambda P_J X_j N_j^{-2} dN_J = 0.
\]

If this is so, then from the fourth of the total differential equations resulting from the first-order conditions:

\[-dy + X_j N_j^{-1} dP_j = 0.\]
Using this information, the change in $X_j$ which results from a change in the price of the club good while utility is held constant is determined by again setting $dy = 0$ and dividing the expression for $dX_j$ by $dP_j$:

\[
\left( \frac{\partial X_j}{\partial P_j} \right) U = constant \quad = \quad \frac{D_{11}(\lambda N_j^{-1}) + D_{31}(\lambda X_j N_j^{-2})}{D}
\]

From this and the expressions for $(\partial X_j/\partial P_j)$ and $(\partial X_j/\partial y)$ prices = constant, the Slutsky equation can be written and $(\partial X_j/\partial P_j)$ can be broken down into a substitution and income effect:

\[
\frac{\partial X_j}{\partial P_j} = \left( \frac{\partial X_j}{\partial P_j} \right) U = constant - \frac{X_j}{N_j} \left( \frac{\partial X_j}{\partial y} \right) \text{ prices} = \text{constant}.
\]

The analogous expression for the effect of a change in the price of the club good on the equilibrium club membership can similarly be derived from the same set of total differential equations.
NOTES

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1. This form of the budget constraint, and the utility function in our equation 2, are essentially equivalent to those used by Polinsky [9 p. 172] in his consideration of possible extensions of the theory of clubs.

2. Mathematically the \( N_{opt} \) curve can be derived by differentiating equation 3 with respect to \( X_i, N_j, \) and \( \lambda \), while treating \( X_i \) as a parameter, and solving the first order conditions for \( N_j \) in terms of \( X_i \). Likewise, the \( Q_{opt} \) curve can be derived by differentiating 3 with respect to \( X_j, X_i, \) and \( \lambda \), while treating \( N_j \) as a parameter, and solving the first order conditions for \( X_i \) in terms of \( N_j \). The intersection of the two curves, found by equating them, corresponds to the equilibrium, where both equations 5 and 6, as well as equation 7, are satisfied.

3. Thus our results are similar to the "Tiebout effect" [11]. The difference is that here only incomes differ; in Tiebout's model preferences differ.

4. The constraints shown in 7b assume that higher service levels (which are measured ordinally) are attained under increasing cost; the special case of constant costs would be shown with straight-line budget constraints. Our results hold in both situations.

5. The alternative of gratuitously supplementing the services of his old club with the increment to his income is less appealing than joining another club in which his contribution would be matched by the dues of other members. Using his entire increment to supplement club A's facilities will put the club members at a point to the right of A, but short of budget line \( y_1 \).
REFERENCES


