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# Insuring customers of a unionized firm against loss of network benefits

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## **Keywords**

insurance, unions, crises, network externalities, network benefits, network effects, global games, optimal public policy, Detroit, automakers

## **Disciplines**

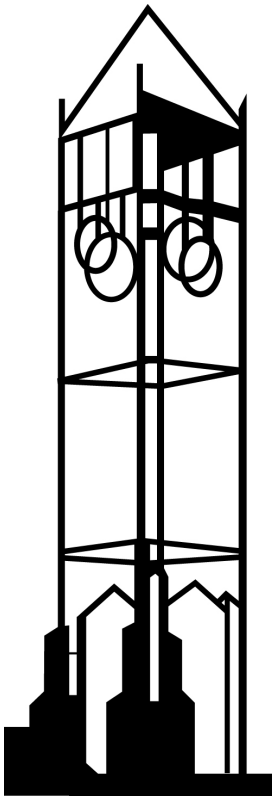
Economics

# **Insuring Customers of a Unionized Firm Against Loss of Network Benefits**

David M. Frankel

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# Insuring Customers of a Unionized Firm Against Loss of Network Benefits

David M. Frankel\*

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February 3, 2015

## Abstract

We study how optimally to insure customers of a unionized firm, such as an auto maker, against the loss of network benefits that occurs when other consumers abandon the firm. The union first announces a wage. A random demand shock is then realized. The firm then chooses its price and, finally, consumers decide whether or not to buy from the firm. Common knowledge of payoffs is perturbed slightly in order to obtain a unique outcome. In this outcome the union chooses an excessive wage, leading consumers to abandon the firm too often. The first best can be costlessly attained by providing consumers with countercyclical insurance.

JEL: C72; D42; D62; H21.

Keywords: Insurance, Unions, Crises, Network Externalities, Network Benefits, Network Effects, Global Games, Optimal Public Policy, Detroit, Automakers.

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If you buy a car from Chrysler or General Motors, you will be able to get your car serviced and repaired, just like always. Your warranty will be safe. In fact, it will be safer than it's ever been, because starting today, the United States government will stand behind your warranty. [*Remarks by the President on the American Automotive Industry*, March 30, 2009.]

## 1 Introduction

The auto industry plays a vital role in the United States economy. Cole *et al* [9] estimate that the disappearance of G.M., Ford, and Chrysler in 2009 would have lowered U.S. nonfarm employment by 2.2% in that year.<sup>1</sup> U.S. personal income would have declined by 1.2%, while the cost to governments would have amounted to 1.6% of tax revenues.<sup>2</sup> As a result of these risks, policymakers have been reluctant to let the industry die. Chrysler was saved from liquidation in 1980, while G.M. and Chrysler were rescued in 2009.

The most recent auto bailout included a new approach, in which new car warranties were guaranteed (U.S. Government Accountability Office [45]). A possible rationale is as follows. If a firm is liquidated, continued warranty coverage is unlikely.<sup>3</sup> Thus, worries about a firm's survival may lead consumers to buy elsewhere.<sup>4</sup> If enough do so, liquidation becomes inevitable. A public warranty guarantee might help calm consumers' fears, thus keeping

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<sup>1</sup>Cole *et al* [9] estimate losses of 239,341 jobs at the firms themselves, 973,369 indirect/supplier jobs, and 1.7 million spinoff (expenditure-induced) jobs. The total amounts to 2.2% of actual U.S. nonfarm employment in 2009, which was 130.9 million (U.S. Bureau of Labor Statistics, *Monthly Labor Review*, May 2010, Table 1, p. 62).

<sup>2</sup> Cole *et al* [9] estimate a loss of \$150.7 billion in personal income in 2009. This would have amounted to 1.2% of actual U.S. personal income in that year, which was \$12,087.5 billion (U.S. Bureau of Economic Analysis, *National Income and Product Accounts*, Table 2.1). They also estimate a a \$14.5 billion increase in transfer payments and a \$45.8 billion decline in tax revenues in 2009. The total, \$60.1 billion, would have amounted to 1.6% of actual governmental revenue (at all levels) in 2009, which was \$3,689 billion (U.S. Bureau of Economic Analysis, *National Income and Product Accounts*, Table 3.1).

<sup>3</sup>Warranty coverage is uncertain in bankruptcy because consumers are treated as unsecured creditors and, moreover, newly formed firms are not obligated to provide warranty coverage for models produced by the former, liquidated firm. See Hortaçsu *et al* [26, n. 2].

<sup>4</sup>In J.D. Power's 2009 Avoider Study, 18% of new car buyers who avoided a particular vehicle model cited concerns about the model's future as a reason.

them loyal to the firm. This, in turn, might help the firm survive a bad shock.

In fact, lost warranty coverage is not the only danger consumers face from a dwindling customer base. The resulting loss of economies of scale will jeopardize customers' access to cheap replacement parts and to the firm's network of repair shops. Consumers could be insured against these risks as well. Such comprehensive insurance might enhance welfare by heading off the negative externality that would be imposed on other consumers if new car buyers were to turn away from a firm *en masse*. It might also save the government money by making another bailout less likely.

Such insurance would also raise consumers' valuation of the firm's product, thus leading to higher sales revenue.<sup>5</sup> This creates an obvious vulnerability. Seeing a now larger pie, the firm's union(s) would have an incentive to demand higher wages and benefits. By leaving the firm more fragile, this response would tend to shrink - and might even reverse - the beneficial effects of the insurance.

Past experience makes such opportunism seem likely. The early UAW leader Walter Reuther summarized his bargaining philosophy as follows.

"We have talked in the past about this being a kind of golden goose and no matter how much fat we take off that goose at the bargaining table, it has the capability of coming back and the next time it is even fatter." [Victor Reuther [42, p. 305]]

By virtue of its aggressive approach, the UAW gradually won compensation that far exceeded that paid by foreign automakers (Cooney and Yacobucci [13, p. 1]). While part of this differential was eliminated as part of the 2009 bailouts (Leonhardt [33]), similar concessions to Chrysler during its 1980 bailout were reversed in subsequent years (Katz [28, pp. 60-61]). In one negotiation in 1995, the UAW urged Chrysler to dip into its \$7.5 billion cash reserve in order to "repay its 73,000 hourly and salaried workers for the concessions they made to

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<sup>5</sup>Hortaçsu *et al* [26, p. 3] find that a ten percentage point increase in the probability that an automaker will default on its bonds lowers the prices of its used cars by an average of 0.5%. The effect is stronger for cars that have more time left on their warranties, indicating that concerns over warranty coverage are reflected in prices.

help keep the automaker afloat through the 1980s.”<sup>6</sup> During the 2006 strike at Delphi (the auto parts supplier owned by G.M.), union workers reportedly were aware that G.M. had a cash balance of \$20 billion dollars and “hope[d] the threat of a strike [would] prompt GM’s management to dip into its cash reserves to compensate them for accepting lower pay and benefits.”<sup>7</sup>

We propose an insurance scheme that is immune to union rent extraction. Rather than being full, the insurance is partial and state-dependent. Full insurance is given only at the lowest state at which it is efficient for the firm to produce. As the state rises above this level, the insurance is gradually phased out. Under fairly mild conditions, this policy implements the first best at zero cost.

Our stylized model is as follows. There is a single union, a single firm, and a continuum of *ex ante* identical consumers. The union first announces a wage.<sup>8</sup> A demand shock - the "state" - is then realized. Finally, the firm announces a price and the consumers simultaneously decide whether or not to buy the firm’s product. There are network externalities: a consumer’s valuation is increasing in the proportion of other consumers who buy. It is also increasing in the state.

In order to find the union’s optimal wage, we must predict how the firm and consumers will respond to different wages. However, if the state is common knowledge, the subgame in which the consumers buy or not will generally have multiple equilibria. In order to obtain a unique prediction, we perturb the assumption of common knowledge by assuming that each consumer sees a slightly noisy private signal of the state. In the limit as the signal noise vanishes, a unique equilibrium emerges. In this equilibrium, the consumers’ reservation price is a deterministic, increasing function of the state.

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<sup>6</sup>Nichole M. Christian, “UAW Gets Tough With Cash-Rich Chrysler,” *Wall Street Journal*, January 11, 1996.

<sup>7</sup>*The Economist*, “Last Tango in Detroit? General Motors, Delphi and the Unions,” April 2006, p.70.

<sup>8</sup>A linear wage is assumed here for simplicity. In the model, nonlinearity is permitted. Union-firm bargaining is discussed in section 4.3.

We assume the union knows only the prior distribution of the state. This captures the idea, discussed below, that union wages tend not to respond to short-run fluctuations. As for the firm, we assume it observes a noisy public signal of the state, which the consumers also see. After we take the consumers' private signal errors to zero, we take the noise in this public signal to zero.<sup>9</sup> Hence the firm has precise information about the state but - because the private signal noise shrinks first - a unique equilibrium is preserved.

Giving the firm precise information about the state captures the idea that prices are flexible and thus can fall quickly in a downturn. For instance, when fuel costs rise, U.S. automakers tend to offer rebates and other discounts in order to prevent falling sales (Langer and Miller [32, pp. 1206-7]; McManus [34]). This price flexibility is mirrored in our model: the firm sells whenever consumers are willing to pay enough to cover its costs.

In the double limit we study, the firm can precisely estimate the state and thus the consumers' reservation price. It will charge the reservation price (and thus sell to all consumers) as long as this price exceeds its unit cost; else it will shut down. The firm's unit cost includes the union's wage. This presents the union with a tradeoff: a higher wage raises workers' income if the firm produces, but also makes a shutdown more likely. The union's optimal wage equates this cost and benefit. While the firm's greater fragility also harms the firm and consumers, the union considers only its own harm. As a result, it chooses a socially excessive wage, which leads consumers to abandon the firm too often.

We then consider an insurance scheme in which buyers are compensated for the network benefits they lose when others fail to buy. In the limit as consumers' private signal noise vanishes, either all buy or none do, so the insurance is costless.<sup>10</sup> As full insurance boosts consumer demand without correcting the monopoly distortion, it is not optimal. Rather, payments should be phased out as the state rises. This raises the consumers' reservation

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<sup>9</sup>Morris and Shin [40, pp. 170 ff.] use the same order of limits to study the effects of IMF lending on country moral hazard.

<sup>10</sup>Outside of this limit, there can be states at which some agents buy while others do not, so positive transfers are paid. This set of states shrinks to zero as the private signal noise shrinks.



price more in bad states, thus flattening the consumer demand curve.<sup>11</sup> Hence the union has an incentive to moderate its wage demands: if it lowers its wage until the firm's unit cost equals the height of the flattened demand curve, its members will suffer layoffs in far fewer states of the world.

Shrinking domestic car production in recent decades has made workers anxious about job security. Since the 1980s, the UAW has managed to negotiate plant closing moratoriums and limitations on layoffs (Katz, MacDuffie, and Pil [29, 30]). However, these agreements generally include liberal escape clauses:

[In committing to a plant closing moratorium] it is understood that conditions may arise that are beyond the control of the Company (i.e. market related volume decline, act of God) and could make compliance with this commitment impossible. ... Should it be necessary to close or idle a plant..., the Company will attempt to redeploy employees to other locations.... [2011 UAW-GM Contract Settlement Agreement, p. 280]

It thus seems likely that the UAW would be open to a scheme that enhances job security in return for modest wage concessions.

Our model assumes that the wage cannot be renegotiated after the public signal is revealed. This fits common practice. In unionized firms, management tends to favor multiyear contracts as they permit long-term planning, avoid strikes, and minimize costly negotiations (Jacoby and Mitchell [27]). As a result, wages are generally unresponsive to short run fluctuations:

... the U.S. auto industry endured sharp swings in output, employment, and profits over the postwar period. ... Yet wage setting was not sensitive to these short run employment fluctuations. [Katz [28, p. 18]]

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<sup>11</sup>More precisely, the insurance is zero at states that are so low that it is better for society that the firm not produce. At the lowest state where production is welfare enhancing, full insurance is given. As the state rises, the insurance is gradually phased out so as to yield a level demand curve. At some state, the insurance reaches zero. No insurance is given at higher states.

In hard times, UAW also resists lowering fringe benefits, which are generally the results of years of tough, incremental negotiations (Katz [28, p. 25]). In his autobiography, early UAW organizer Victor Reuther (Walter's younger brother) writes:

We knew that once the principles of the [Supplemental Unemployment Benefit] program were solidly established, subsequent negotiations would widen the range of benefits. As with pensions and health care, it became a matter of "building brick by brick," as Walter used to put it, fleshing out the bare bones of a basic fringe benefit. [Victor Reuther [42, p. 317]]

On the other hand, UAW contracts have supplemented fixed wages with some profit sharing since 1979 (Katz, MacDuffie, and Pil [30, p. 62]). In our model, profit sharing entices the union to lower its fixed wage in order to make the firm less fragile. Under our scheme, this benefit of a lower wage disappears, so the firm will not offer profit sharing. Hence, the scheme works even when profit sharing is permitted (section 4.4). Firm-union bargaining does not disrupt our results either (section 4.3). Finally, our scheme is shown to be superior to an efficient wage cap, as the latter requires union members to work for less than their reservation wage (section 4.1).

The subgame played by the consumers in our model is a global game. Global games were first studied by Carlsson and van Damme [8] in the context of 2-player, 2-action games with two pure Nash equilibria. They show that if, instead of the game's payoffs being common knowledge, each player receives a slightly noisy signal of these payoffs, there is a unique equilibrium.<sup>12</sup>

In a global game, small changes in fundamentals can yield large shifts in aggregate behavior. This makes global games useful for studying aggregate fluctuations and crises. Applications include bank runs and international contagion (Goldstein and Pauzner [20, 21]),

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<sup>12</sup>These results have been generalized to multiple players and actions, and more general information and payoff structures (e.g., Frankel, Morris and Pauzner [17], Morris and Shin [37]). Similar results are obtained in dynamic games with frictions and shocks under common knowledge of payoffs (Burdzy, Frankel, and Pauzner [7]; Frankel and Pauzner [18]). For limitations on the uniqueness result, see Angeletos, Hellwig, and Pavan [1], Angeletos and Werning [4], Chassang [12], Hellwig, Mukherji, and Tsyvinski [25], and Morris and Shin [39].

currency crises and debt pricing (Morris and Shin [36, 39]), search-driven business cycles (Burdzy and Frankel [6]), investment cycles (Chamley [11], Oyama [41]), and merger waves (Toxvaerd [44]).

Because of network effects, a consumer’s valuation of the good is higher if she believes that other consumers will buy it. Thus, for each price there is an interval of states for which both all-buy and none-buy are Nash equilibria when the state is common knowledge. The global games information structure partitions this interval of states into two subintervals, a higher one in which all consumers buy and a lower one in which they do not. At the boundary that separates them, a consumer is just willing to buy on the counterfactual belief that the proportion who buy will be uniform on the unit interval.<sup>13</sup> The insurance scheme expands the upper interval of states by raising a consumer’s willingness to pay under these counterfactual beliefs. This permits us to redraw the consumers’ demand curve in fairly arbitrary ways. In the limit as the consumers’ signals become precise, the scheme has a nonvanishing effect on the boundary but consumers perfectly coordinate so the event that triggers the insurance - some but not all consumers buying - never occurs: the insurance is costless.

Angeletos and Pavan [3] also study optimal policy in a setting with public and private signals. Unlike our model, their underlying complete information game has a unique equilibrium: there is no boundary to manipulate. Hence, positive taxes on some agents are needed in order to change the outcome in a revenue neutral way. Also, there are no large players in their model.

Our model also contributes to the literature on network externalities. When payoffs are common knowledge, the consumers’ purchase decision typically displays multiple equilibria (e.g., Farrell and Katz [15]). We instead assume that the consumers play a global game,

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<sup>13</sup>Kim [31] was the first to show that in a global game informational setting with two actions and multiple agents, an agent chooses the action that is a best response under the counterfactual belief that all proportions who choose that action are equally likely. An intuition appears in Morris and Shin [37, pp. 61-63]. In a more general informational setting, other actions may be selected (Weinstein and Yildiz [46]). However, experiments support Kim’s [31] prediction even in settings where payoffs are common knowledge (Heinemann, Nagel, and Ockenfels [24]).

so the equilibrium is unique. Similarly, Argenziano [5] uses global games techniques to study the interaction between two firms that sell competing goods which display network externalities, while Frankel [16] studies the monopoly case. Neither author considers optimal policy.

Finally, we significantly weaken the sufficient conditions for uniqueness in a global game setting. These studies assume that an agent's incentive to choose a given action is continuous and increasing in the state, controlling for the proportion of others who choose that action (e.g., Morris and Shin [37, Proposition 2.2, p. 67]). These properties are violated by our insurance scheme, so we prove uniqueness without them.

Throughout we focus on the case of small private signal errors. We introduce these errors as a perturbation to common knowledge that permits us to make unique predictions, and thus to study optimal policy. With private signal errors that are large, either absolutely or relative to the public signal error, multiple equilibria remain possible (Morris and Shin [38]). We leave these cases unstudied, in the hope that future techniques will be developed that allow unique predictions.

The rest of this paper is as follows. We present the model in section 2. We discuss our main results informally and with a minimum of technical details in section 3. Extensions are then studied: a wage cap in section 4.1, taxation of buyers in section 4.2, firm-union bargaining in section 4.3, profit sharing in section 4.4, firm-provided insurance in section 4.5, noise in the policymaker's information in section 4.6, and bargaining with dealers in section 4.7. Our formal results appear in section 5 and are proved in Appendix A. Concluding remarks appear in section 6.

## 2 The Model

There is a single monopoly union, a single firm, and a unit measure of *ex ante* identical consumers.<sup>14</sup> All participants are risk-neutral and fully rational. The firm produces a

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<sup>14</sup>The results are essentially the same with a finite number (at least two) of consumers.

quantity  $G(L)$  where  $L \geq 0$  is labor. The production function  $G$  is increasing and satisfies  $G(0) = 0$ . Let  $\bar{L} > 0$  be the amount of labor needed to produce one unit:  $G(\bar{L}) = 1$ . Consumers have unit demands, so the firm demands at most  $\bar{L}$  units of labor.

In return for  $L$  units of labor the firm pays the union  $w(L) + \underline{w}L$  where  $\underline{w} > 0$  is the workers' constant, exogenous reservation wage and  $w(L) \geq 0$  is an endogenous wage premium. For simplicity we will refer to  $w$  as the wage schedule, keeping in mind that it omits the reservation wage payment  $\underline{w}L$ . The union cannot force the firm to pay if no labor is demanded, so  $w(0) = 0$ .

The union first chooses a wage schedule  $w$  from the set  $\Omega$  of nonnegative functions  $w : [0, \bar{L}] \rightarrow \mathfrak{R}_+$  that satisfy  $w(0) = 0$ . A public signal  $Y = \theta + \tau v$  is then revealed, where  $\theta \sim \Phi$  is an exogenous, random state with bounded density  $\phi$ ,  $\tau$  is a positive scalar, and  $v \sim H$  is an exogenous noise term with bounded density  $h$ . Both the state  $\theta$  and the noise term  $v$  have support equal to the whole real line. The densities  $\phi$  and  $h$  of these random variables are Lipschitz continuous: there are constants  $k_\phi, k_h \in (0, \infty)$  such that for all  $z, z' \in \mathfrak{R}$ ,

$$|\phi(z) - \phi(z')| \leq k_\phi |z - z'| \text{ and } |h(z) - h(z')| \leq k_h |z - z'|. \quad (1)$$

In addition, the state and the public signal noise term have finite 4th moments:<sup>15</sup>

$$E[\theta^4] < \infty \text{ and } E[v^4] < \infty. \quad (2)$$

After the public signal  $Y$  is revealed, the firm announces a price  $p$ . Each consumer  $i \in [0, 1]$  then sees a private signal  $x_i = \theta + \sigma \varepsilon_i$  of the state  $\theta$ , where  $\sigma \in (0, \bar{\sigma}]$  is a scale factor and  $\bar{\sigma}$  is an arbitrary positive constant. The noise term  $\varepsilon_i$  has a fixed, continuous density  $f$ , with a corresponding distribution function  $F$  and connected support contained in  $[-1/2, 1/2]$ .<sup>16</sup> The state  $\theta$  and the noise terms  $\nu$  and  $\varepsilon_i$  (for all  $i$ ) are mutually independent.

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<sup>15</sup>The common assumption of finite variances implies (2). However, we do not need finite variances for our results.

<sup>16</sup>By idiosyncratic, we mean that each  $\varepsilon_i$  is independent of any of the other exogenous random variables

The consumers then simultaneously decide whether or not to buy the firm's product. As consumers have unit demands, the proportion  $\ell \in [0, 1]$  who buy also equals the quantity produced. We refer to  $\ell$  as the *purchase rate*. It and the state  $\theta$  are observable *ex post* but not verifiable.<sup>17</sup> Hence, the firm cannot make the price  $p$  contingent on  $\ell$  or on  $\theta$ , although the price it chooses can depend on the public signal  $Y$  and thus, indirectly, on  $\theta$ . Likewise, the union's wage schedule  $w$  cannot be made an explicit function of the state  $\theta$ , although it is a function of labor  $L$  and thus of the purchase rate  $\ell = G(L)$ . (Profit sharing - a type of state-dependent wage - is studied in section 4.4.)

The union's realized payoff is the wage premium  $w(L)$ . The firm's realized payoff is its revenue  $p\ell$  less the sum of its wage payment  $w(L) + \underline{w}L$  and its exogenous nonwage costs  $\zeta(\ell)$ , where  $\zeta$  is an arbitrary nonnegative function. Let  $c$  denote the *marginal cost* of supplying to all consumers at a marginal state  $\theta$ : the sum of the opportunity cost  $\underline{w}\bar{L}$  of the workers plus the nonwage cost  $\zeta(1)$  of the firm. If all consumers buy, the firm's profit is thus  $p - c - W$  where  $W$  denotes  $w(\bar{L})$ : the union's wage premium if all consumers buy. For brevity, we refer to  $W$  simply as the *wage*.

If a consumer  $i \in [0, 1]$  buys, her realized payoff is her valuation  $v_\theta^\ell \in \Re$  less the price  $p$ . If she does not buy, she receives an outside option  $o_\theta^\ell \in \Re$ . Let the *relative payoff*  $r_\theta^\ell = v_\theta^\ell - o_\theta^\ell$  denote an individual consumer's benefit from buying, gross of the price  $p$ , for a given purchase rate  $\ell$  and state  $\theta$ . Let the mean relative payoff  $R_\theta$  denote the mean  $\int_{\ell=0}^1 r_\theta^\ell d\ell$  of this relative payoff over all possible purchase rates  $\ell$ .

We assume the decision to buy displays strategic complementarities, which are bounded:

**AM. Action Monotonicity.** There is a constant  $k_1 \in (0, \infty)$  such that for any state  $\theta$  and purchase rates  $\ell' > \ell$ ,  $0 \leq r_\theta^{\ell'} - r_\theta^\ell \leq k_1$ .

For instance, customers may need to buy accessories and services in the future: repairs, spare parts, software, etc. If these are produced subject to economies of scale, then they

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in the model.

<sup>17</sup>The idea that some variables may have this property is due to Tirole [43].

are more likely to be provided by the firm or other firms if more consumers buy the firm's product to begin with. A consumer may also worry that if too few consumers buy, the firm will not survive to provide warranty service or pay compensation for defects (Hortaçsu *et al* [26, p. 1]). By requiring only that the relative payoff be nondecreasing rather than increasing, we allow threshold effects; for instance, it may be that the firm will survive if and only if at least 1/3 of the consumers buy. However, our insurance scheme does require some complementarities. In particular, the gap between the highest relative payoff  $r_\theta^1$  and the mean relative payoff  $R_\theta$  is bounded below by a positive constant:

**PMC. Positive Mean Complementarities.** There is a constant  $k_2 \in (0, \infty)$  such that for any state  $\theta$ ,  $r_\theta^1 - R_\theta > k_2$ .

Finally, a consumer's incentive to purchase is increasing in the state  $\theta$  at a bounded rate:<sup>18</sup>

**SM. State Monotonicity.** There are constants  $0 < k_3 < k_4 < \infty$  such that for every pair of states  $\theta' > \theta$  and each purchase rate  $\ell$ ,  $\frac{r_{\theta'}^\ell - r_\theta^\ell}{\theta' - \theta} \in (k_3, k_4)$ .

By SM, a consumer's relative payoff  $r_\theta^\ell$  is negative for low enough states. All we actually need is that for low enough states, it is strictly dominant not to buy. Since the firm will never price below its minimum average cost, it suffices that the relative payoff  $r_\theta^\ell$  be less than this minimum if the state is low enough.<sup>19</sup> Weakening SM in this way would significantly complicate our proofs and exposition, so we do not do so.

Let

$$s_\theta = v_\theta^1 - o_\theta^0 \tag{3}$$

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<sup>18</sup>Heinemann, Nagel, and Ockenfels [24] find that experimental subjects play in accordance with the predictions of global games even when the state (i.e., the game's payoffs) are common knowledge. Under common knowledge, State Monotonicity has no bite since it refers to states that are commonly known not to be the true state. Hence, our model may still have predictive power in settings in which State Monotonicity fails.

<sup>19</sup>More precisely, let  $z$  denote the firm's minimum average cost net of the wage premium:  $z = \inf_{\ell \in (0,1]} \frac{wG^{-1}(\ell) + \zeta(\ell)}{\ell} > 0$ . We can replace SM with the weaker condition that  $r_\theta^\ell$  is nondecreasing in  $\theta$  and there is some  $z' \in (0, z)$  and a threshold  $\underline{\theta}$  such that  $r_\theta^\ell < z'$  for any  $\theta < \underline{\theta}$  and  $\frac{r_{\theta'}^\ell - r_\theta^\ell}{\theta' - \theta} \in (k_3, k_4)$  for any  $\theta' > \theta > \underline{\theta}$ .

be the *social benefit* of all consumers buying: the benefit (gross of the price  $p$ ) that the consumers get if, as a group, they buy rather than getting their outside option. (Recall that  $o_\theta^0$  is a consumer's valuation of the outside option when all other consumers choose it: when  $\ell = 0$ .) We assume the maximum relative payoff exceeds this social benefit:

$$r_\theta^1 > s_\theta. \quad (4)$$

By definition of  $r^1$  and  $s$ , this is equivalent to  $o_\theta^0 > o_\theta^1$ : the outside option is more attractive when more choose it.<sup>20</sup> For instance, some of those who choose the outside option may end up buying from the same alternative firm, which may itself be subject to network externalities.

We also assume that

$$v_\theta^1 - \int_{\ell=0}^1 v_\theta^\ell d\ell > o_\theta^0 - \int_{\ell=0}^1 o_\theta^\ell d\ell. \quad (5)$$

This means that a consumer's absolute valuation of a given choice is more sensitive to the proportion of others who make that choice when the choice is buying the firm's product than when the choice is the outside option. Intuitively, network effects should be weaker from choosing the outside option, as consumers will generally have several alternatives from which to choose. While it simplifies and shortens our exposition, assumption (5) is not essential: our main results hold without it.<sup>21</sup>

We also assume the social benefit function  $s_\theta$  is increasing in the state  $\theta$ . E.g., if the firm specializes in fuel-inefficient vehicles and  $\theta$  is inversely related to fuel costs, this property means that the social benefit of the consumers' collectively buying from the firm is greater when gas is cheaper. We assume, moreover, that the social benefit curve lies below (above)

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<sup>20</sup>We need this plausible condition in order to implement the first best. Why? Suppose instead that for all  $\theta$ , the social benefit  $s_\theta$  exceeds the consumers' maximum willingness to pay  $r_\theta^1$ . Our scheme uses purchase insurance to raise the agents' willingness to pay at different states  $\theta$ . However, even full insurance raises this willingness only to its maximum possible value,  $r_\theta^1$ . If this is less than the social benefit  $s_\theta$ , there will generally exist states  $\theta$  at which  $s_\theta$  exceeds the firm's marginal cost  $c$ , so it is socially beneficial to produce, but the firm cannot sell because  $r_\theta^1$  is less than  $c$ . Thus, our scheme cannot attain the first best. A purchase subsidy might do so, but it would not be costless.

<sup>21</sup>Details are available from the author on request.



the marginal cost  $c$  for sufficiently low (high) states.

**ISB. Increasing Social Benefit.** The social benefit  $s_\theta$  is increasing and continuous in the state  $\theta$  and satisfies  $\lim_{\theta \rightarrow -\infty} s_\theta < c$  and  $\lim_{\theta \rightarrow \infty} s_\theta > c$ .

ISB ensures that the social benefit  $s$  equals the firm's marginal cost  $c$  at a unique, finite state  $\theta^*$ , which is the socially optimal production threshold for the firm.

For technical reasons, we assume the union's monopoly power is bounded: by paying an arbitrarily high but finite relocation expense  $\bar{C}$ , the firm can transfer its operations to an overseas location where it can hire any amount  $L \in [0, \bar{L}]$  of labor at a fixed linear wage  $\bar{w}$ . Hence the union's maximum payment  $w(\bar{L}) + \underline{w}\bar{L}$  cannot exceed the firm's maximum payment if it relocates,  $\bar{C} + \bar{w}\bar{L}$ . It follows that the wage  $W$  cannot exceed the arbitrarily high but finite bound  $\bar{W} = \bar{C} + (\bar{w} - \underline{w})\bar{L}$ .<sup>22</sup>

Similarly, the firm's market power is also bounded: a technology is available that permits an entrant to produce a perfect substitute for the firm's product at an arbitrarily high but finite marginal cost  $\bar{p}$  and zero fixed costs. The firm will thus never choose a price  $p > \bar{p}$  since an entrant could capture the market by offering a slightly lower price. Finally, we assume that at the maximum wage  $\bar{W}$ , the firm can sell profitably to all consumers at the price  $\bar{p}$ :

$$c + \bar{W} < \bar{p}. \tag{6}$$

This assumption is inessential but slightly shortens the proofs.

### 3 Main Results

We first solve the model informally, keeping technical details to a minimum. Formal statements of all results appear in section 5, which can be skipped by the casual reader.

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<sup>22</sup>Any wage schedule  $w$  that violates this constraint is dominated, for the union, by the wage schedule  $\hat{w}(L) = \min\{w(L), \bar{W}\}$ .

We focus on the limit as the private signal noise  $\sigma$  goes to zero for fixed public signal noise  $\tau$ , and then consider the limit of these limiting cases as  $\tau$  goes to zero. The purchase subgame played by the consumers is a global game with two actions: buy and not buy. Such a game has a unique equilibrium in the limit as  $\sigma$  shrinks: consumers buy when doing so is optimal under the counterfactual belief that the proportion who buy is uniform on the unit interval.<sup>23</sup> This holds if  $\int_{\ell=0}^1 (r_{\theta}^{\ell} - p) d\ell > 0$  or, equivalently, if the price  $p$  is less than the mean relative payoff  $R_{\theta}$ .

In the limit as  $\tau$  then goes to zero, the firm can precisely estimate the state  $\theta$  and thus the consumers' reservation price  $R_{\theta}$ . To maximize profits, the firm will produce and charge the reservation price  $R_{\theta}$  as long as  $R_{\theta}$  exceeds the firm's unit cost  $c + W$ . Let  $\theta_W$  satisfy

$$R_{\theta_W} = c + W. \tag{7}$$

That is,  $\theta_W$  is the lowest state at which the firm can profitably produce when the union's wage is  $W$ . The union's payoff is then

$$U(W) = W [1 - \Phi(\theta_W)] : \tag{8}$$

the wage  $W$  times the probability  $1 - \Phi(\theta_W)$  that the state exceeds the firm's production threshold  $\theta_W$ .

In the limit, a consumer's reservation price equals her mean relative payoff  $R_{\theta}$ . The consumers also perfectly coordinate: if one buys, all buy. Hence, when a consumer buys, her realized valuation of the good is actually her maximum relative payoff  $r_{\theta}^1$ . Accordingly, consumers receive rents equal to  $r_{\theta}^1 - R_{\theta}$ . These *strategic rents* are due to the effect of miscoordination costs on which equilibrium is selected in the purchase subgame (Frankel [16, section 2.1.2]). Our scheme works by shrinking these rents by different amounts at different states.

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<sup>23</sup>For an intuition, see Morris and Shin [37, pp. 61-63]. If instead the public signal were precise relative to the private ones, multiple equilibria might re-emerge, as first shown by Morris and Shin [37, pp. 77 ff.].

Since the firm's production threshold  $\theta_W$  is increasing in the wage  $W$ , we can think of the union as choosing the firm's production threshold directly. In order to implement a given threshold  $\theta$ , the union will choose the highest wage  $W$  for which the firm produces at all states above  $\theta$ . Since the consumers' willingness to pay  $R$  is increasing by SM but may be discontinuous, this is just the wage  $W = R_\theta^+ - c$  where  $R_\theta^+$  denotes  $\lim_{\theta' \downarrow \theta} R_{\theta'}$ : the right limit of  $R$  at  $\theta$ . Substituting this formula for  $W$  into (8) and writing  $\theta$  in place of  $\theta_W$ , the union's payoff from choosing the production threshold  $\theta$  becomes

$$V(\theta) = (R_\theta^+ - c) [1 - \Phi(\theta)]. \quad (9)$$

This leads to a further useful equivalence. As the distribution  $\Phi$  of the state is increasing, choosing a production threshold  $\theta$  is equivalent to choosing the firm's probability of sale,  $1 - \Phi(\theta)$ . Since there is a unit measure of consumers with unit demands,  $1 - \Phi(\theta)$  also equals the expected quantity sold. In this way, we can interpret the union as a quantity-setting monopolist - except that it is setting the firm's expected quantity, not its own.

We now develop this analogy further. Assume henceforth that the primitive mean relative payoff function  $R$  is differentiable. The change in the wage  $W = R_\theta - c$  from increasing the probability  $1 - \Phi(\theta)$  of sale by one infinitesimal unit is then  $\frac{d(R_\theta - c)}{d[1 - \Phi(\theta)]} = \frac{d(R_\theta - c)/d\theta}{d[1 - \Phi(\theta)]/d\theta} = -\frac{R'_\theta}{\phi(\theta)}$ . So by (9), the union's net marginal benefit  $\mu_\theta = \frac{dV(\theta)}{d[1 - \Phi(\theta)]}$  from this change increase equals  $m_\theta - c$  where

$$m_\theta = R_\theta - [1 - \Phi(\theta)] \frac{R'_\theta}{\phi(\theta)} \quad (10)$$

is the standard marginal revenue function corresponding to the demand curve  $R$ . The derivative  $R'_\theta$  is positive by SM, so

$$m_\theta < R_\theta. \quad (11)$$

That is, the firm's marginal revenue curve lies below its demand curve.

The union's first order condition for a threshold  $\theta$  that maximizes its payoff  $V(\theta)$  is simply  $m_\theta = c$ : marginal revenue equals marginal cost. In the standard monopoly problem, the second order condition for profit maximization holds if marginal revenue is decreasing.

Likewise, in our model the second order condition holds if  $m$  is decreasing in the probability  $1 - \Phi(\theta)$  of sale - or, equivalently, increasing in the threshold  $\theta$ . The following assumption combines this property with the differentiability of the primitive mean relative payoff function  $R$ , assumed above, as well as a limit property that (combined with (11) and SM) ensures that  $m$  crosses the marginal cost line  $c$  at a unique finite state  $\hat{\theta}$ , which must be the threshold in the laissez-faire equilibrium.

**IMR. Increasing Marginal Revenue.** The primitive mean relative payoff function  $R$  is differentiable in the state  $\theta$ . The corresponding marginal revenue function  $m$  is increasing and continuous in  $\theta$  and satisfies  $\lim_{\theta \rightarrow \infty} m_\theta > c$ .

We can now compute social welfare. Suppose the firm sells if and only if the state exceeds some threshold  $\tilde{\theta}$ . For instance,  $\tilde{\theta}$  may be the laissez-faire equilibrium threshold  $\hat{\theta}$  or the socially optimal threshold  $\theta^*$ . At states  $\theta < \tilde{\theta}$ , the consumers get their outside option  $o_\theta^0$  while the union and firm get nothing. At states  $\theta > \tilde{\theta}$ , the consumers get their valuation  $v_\theta^1$  less the price  $R_\theta$ , the firm gets its price  $R_\theta$  less its cost  $W + c$ , and the union gets its wage  $W$ . Hence, consumer welfare is  $CW = CW_F + \int_{\theta=\tilde{\theta}}^\infty [s_\theta - R_\theta] d\Phi(\theta)$  where  $CW_F$  is the fixed amount  $\int_{\theta=-\infty}^\infty o_\theta^0 d\Phi(\theta)$ , firm profits are  $\Pi = \int_{\theta=\tilde{\theta}}^\infty (R_\theta - W - c) d\Phi(\theta)$ , and union revenue is  $W [1 - \Phi(\tilde{\theta})]$ . Social welfare is the sum of consumer welfare, firm profits, and union revenue:  $SW = CW_F + \int_{\theta=\tilde{\theta}}^\infty [s_\theta - c] d\Phi(\theta)$ . Intuitively, at each state  $\theta$  at which the consumers purchase, the surplus created is just the social benefit  $s_\theta$  of the consumers purchasing rather than obtaining their outside option, less the marginal cost  $c$  of providing the good. The other quantities that change hands (the price  $R_\theta$  and the wage  $W$ ) are transfers that do not affect social welfare.

In order to depict the limiting economy graphically, we need to determine the relative heights of the curves  $r^1$ ,  $R$ ,  $m$ , and  $s$ . By PMC and (11),  $r^1 > R > m$ . By (4),  $r^1 > s$ . Finally, equation (5) can be rearranged to yield  $s_\theta > R_\theta$ : the social benefit of all consumers buying the firm's product exceeds their willingness to pay for the product.

Gathering inequalities, we obtain  $r^1 > s > R > m$ . These curves are depicted in Figure 1. The purchase probability  $1 - \Phi(\theta)$  appears on the horizontal axis, so the threshold  $\theta$  falls

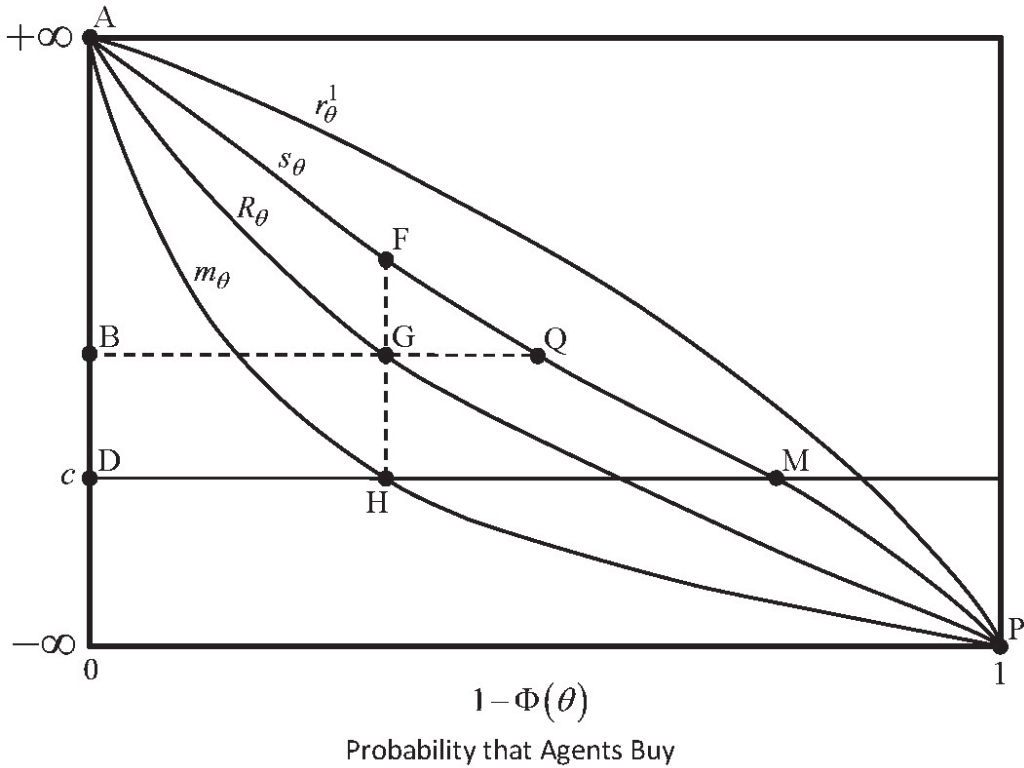


Figure 1: Laissez-Faire Outcome. Vertical axis is progressively compressed (more so at bottom) so as to contain whole real line. Horizontal axis depicts purchase probability  $1 - \Phi(\theta)$ : state  $\theta$  falls gradually from  $\infty$  on left axis to  $-\infty$  on right axis. Social optimum is point M. Laissez-faire equilibrium is point H: wage  $W$  is  $BD$ , union revenue is  $BGHD$ , firm profits are  $AGB$ , consumer welfare is  $AFG$ , and social welfare is  $AFHD$ . (Consumer and social welfare omit fixed term  $CW_F$ , defined in the text.)

gradually from  $\infty$  on the left vertical axis to  $-\infty$  on the right axis. Hence the curves  $r^1$ ,  $s$ ,  $R$ , and  $m$ , which are increasing in  $\theta$ , are downwards sloping in the figure. The vertical axis is compressed (more so near the bottom) so as to contain the whole real line. In the laissez-faire outcome, the union picks the point H, where  $m_\theta = c$ . The union's wage  $W$  is  $BD$ , union revenue is  $BGHD$ , firm profits are  $AGB$ , consumer welfare is  $AFG$ , and social welfare is  $AFHD$ .

The social optimum is at point M, where  $s_\theta = c$ .<sup>24</sup> As point H lies to the left of M, the firm

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<sup>24</sup>This notion of first best is that of Angeletos and Pavan [2, p. 1105]: "... the best a society could do if its agents were to internalize their payoff interdependencies and appropriately adjust their use of

is too fragile in equilibrium: it produces in too few states. There are two reasons for this. The first is the usual monopoly distortion: the union chooses the production threshold  $\theta$  at which marginal cost  $c$  equals marginal revenue  $m_\theta$  rather than consumer willingness to pay  $R_\theta$ . Since  $R_\theta$  exceeds  $m_\theta$ , this leads to suboptimal production in equilibrium. Intuitively, the union ignores the negative externality that is created when, by raising its wage  $W$ , it makes the firm more fragile.

The second source of inefficiency is that the consumers' willingness to pay  $R$  for the firm's product is less than the social benefit  $s$  the consumers get if they choose *en masse* to buy it. Hence, for states between G and Q, there is no price that the consumers are willing to pay that covers the firm's unit cost (since  $R < c + W$ ) even though there are potential gains from trade between the firm and consumers (since  $s > c + W$ ). This inefficiency increases the distance between points H and M, thus making the firm even more fragile relative to the social optimum.

We now consider a set of insurance schemes  $t$  in which, if a consumer buys at the state  $\theta$  and the purchase rate is  $\ell$ , the government pays her  $t_\theta^\ell$ . (Nonbuyers are not paid under the scheme.) For now, we restrict to the following class of schemes, which are defined more formally in section 5.

**Definition 1** (*Informal.*) A Predictable Costless Subsidy Scheme (PCSS) is a real-valued function  $t$  with the following three properties. (a) The augmented relative payoff function  $\tilde{r} = r + t$  satisfies sufficient conditions for a unique equilibrium of the purchase subgame. (b) For all  $\theta \in \mathfrak{R}$ ,  $t_\theta^1 \leq 0$ . (c) For all  $\ell \in [0, 1]$  and  $\theta \in \mathfrak{R}$ ,  $t_\theta^\ell \geq 0$ .

Property (b) states that no subsidy is given to buyers if all consumers buy, while property (c) says that no buyers are ever taxed. Together, they imply that  $t_\theta^1 = 0$ : if all buy, the

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available information without communicating with one another." Point M is attained while respecting the informational constraints as follows: the union sets  $W = 0$ , the firm sets  $p = c$ , and each agent  $i$  buys if and only if the price  $p$  is less than  $s_{x_i}$ : the social benefit of all agents buying, evaluated at the agent's private signal of the state. Each signal  $x_i$  differs from the true state  $\theta$  by at most  $\sigma/2$ . Hence, in the limit as  $\sigma$  goes to zero, the agents buy if and only if  $s_\theta > p = c$ : point M is chosen.

payment to buyers is identically zero. Thus, in the limit as the private signal errors vanish, the scheme is revenue-neutral: either all consumers buy and thus all are paid zero, or no consumers buy and thus there are no buyers. We discuss weakening these assumptions in section 3.1.3.

Let  $\Gamma(\theta, \theta')$  equal the union's payoff  $(r_\theta^1 - c)[1 - \Phi(\theta)]$  from the threshold  $\theta$  if it faces the demand curve  $r^1$ , less its payoff  $V(\theta')$  from the threshold  $\theta'$  if it faces the laissez-faire demand curve  $R$ . Our main result can now be stated informally as follows.

**Theorem 1** (*Informal.*) *Assume the relative payoff function  $r$  satisfies AM, SM, and PMC. If  $\Gamma(\theta^*, \hat{\theta}) > (<) 0$ , then there is (not) a PCSS  $t$  that induces the union to choose a wage schedule that implements the first best purchase threshold  $\theta^*$ .*

Figure 2 illustrates a PCSS  $t$  that implements the first best outcome if  $\Gamma(\theta^*, \hat{\theta}) > 0$ . A PCSS  $t$  augments the consumers' reservation price at the state  $\theta$  from  $R_\theta$  to  $\tilde{R}_\theta = R_\theta + T_\theta$ , where  $T_\theta$  denotes the mean transfer  $\int_{\ell=0}^1 t_\theta^\ell d\ell$ . At states  $\theta < \theta^*$  (those to the right of the socially optimal point M), no transfers are given ( $t_\theta^\ell = 0$ ) so the augmented demand curve  $\tilde{R}$  coincides with segment NP of the original demand curve  $R$ . At state  $\theta^*$  (point M), full insurance is given ( $t_{\theta^*}^\ell = r_{\theta^*}^1 - r_{\theta^*}^\ell$ ), so  $\tilde{R}_{\theta^*}$  jumps up to equal the maximum relative payoff  $r_{\theta^*}^1$ . Here the augmented demand curve rises vertically from point N to point L.

At states  $\theta > \theta^*$ , which lie to the left of point M, the insurance is gradually phased out so that the augmented demand function  $\tilde{R}$  rises at some small positive rate  $k'_3$ . (That is,  $\tilde{R}_\theta$  equals  $r_{\theta^*}^1 + k'_3(\theta - \theta^*)$ .) By taking  $k'_3$  to be arbitrarily small, we can guarantee that the augmented demand curve is nearly horizontal: it is depicted as segment IL in Figure 2. The mean transfer  $T_\theta$  reaches zero at the state  $\theta = \omega(k'_3)$  at which  $\tilde{R}_\theta = R_\theta$ . This is just point I in Figure 2. No insurance is given at states above  $\omega(k'_3)$ , so the augmented demand curve to the left of point I coincides with segment AI of the original demand curve  $R$ .

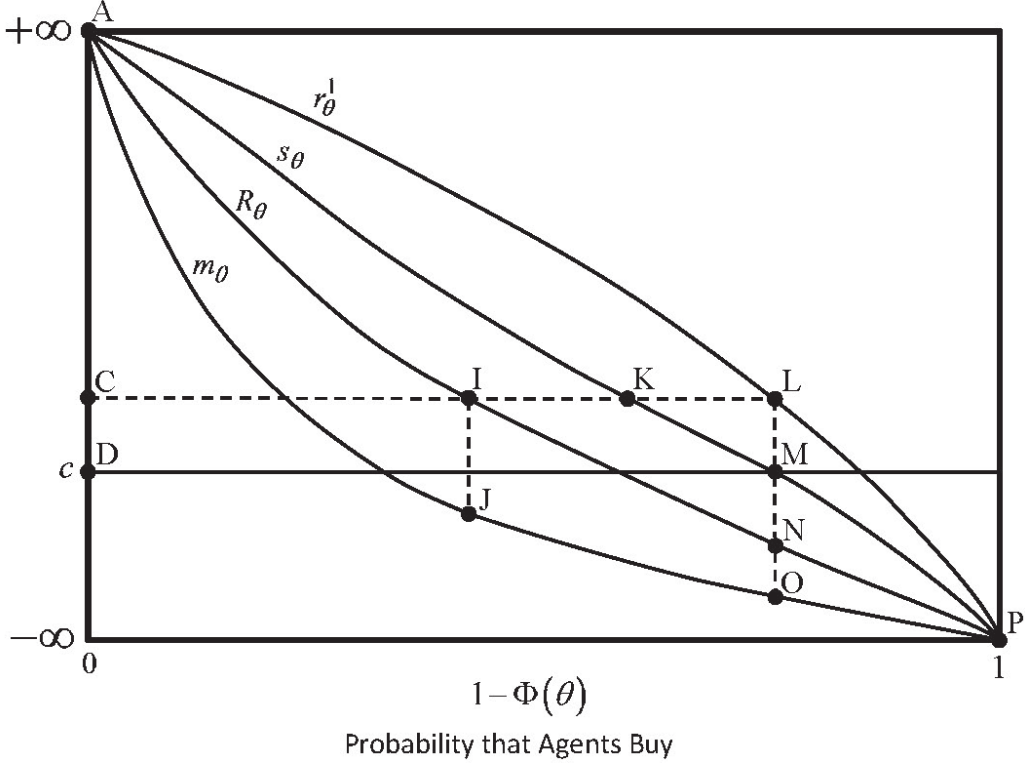


Figure 2: Optimal Insurance Scheme. Under scheme, demand curve is AILNP and marginal revenue curve is AJILOP. Union picks socially optimal point M. Wage  $W$  is CD. Union revenue is CLMD, firm profits are AIC, consumer welfare is AIK minus KLM, and social welfare is AMD. (Consumer and social welfare omit fixed term  $AW_F$ .)

Written more compactly, the mean transfer function is

$$T_\theta = \begin{cases} 0 & \text{if } \theta < \theta^* \\ r_{\theta^*}^1 - R_\theta + k'_3(\theta - \theta^*) & \text{if } \theta^* \leq \theta \leq \omega(k'_3) \\ 0 & \text{if } \theta > \omega(k'_3) \end{cases} \quad (12)$$

The PCSS itself is given by

$$t_\theta^\ell = \frac{r_\theta^1 - r_\theta^\ell}{r_\theta^1 - R_\theta} T_\theta, \quad (13)$$

which integrates (over purchase rates  $\ell$ ) to  $T_\theta$ . As the definition of PCSS requires,  $t_\theta^1$  is identically zero: no payments occur if all consumers buy. Thus, as noted, the scheme is costless in the limit as the private signal noise vanishes.



With this scheme, the new demand curve  $\tilde{R}$  is denoted AILNP. The corresponding marginal revenue curve is AJILOP. The union will choose point M if its payoff from doing so - area CLMD - exceeds its payoff from the laissez-faire outcome (area BGHD in Figure 1). This is just the condition  $\Gamma(\theta^*, \hat{\theta}) > 0$  in Theorem 1. In this case the firm gets AIC and social welfare rises to AMD. Consumer welfare is AIK minus KLM. (Again, consumer and social welfare omit the fixed term  $CW_F$ .)

### 3.1 Discussion

#### 3.1.1 The Role of Complementarities

In order for the scheme to work, there must be strategic complementarities among buyers. Without complementarities, the buyers' reservation price  $R_\theta$  already equals their valuation  $r_\theta^1$ . Insurance against the risk that others will not buy thus has no effect on the consumer reservation price or on the equilibrium outcome, which remains inefficient because of the union monopoly distortion.

#### 3.1.2 The Union's Tradeoff

The scheme rewards union wage concessions with greater job security. In the past, UAW leaders have successfully sold such a tradeoff to their members (Katz [28, pp. 57-58]). Moreover, wage concessions may not be needed. Let us mentally raise the curve  $r^1$  until segment CD in Figure 2 is longer than segment BD in Figure 1. In this case, the insurance scheme will lead both to greater job security *and* a higher wage.

#### 3.1.3 Weakening PCSS

By weakening property (b) or (c) of PCSS, can we enlarge the set of parameters for which the first best can be costlessly attained? The answer is "no" for property (b), while for (c) it is "yes". Why? In the social optimum, no consumers buy in the limit if the state  $\theta$  is less than the socially optimal threshold  $\theta^*$ . Since  $\theta^*$  lies below the laissez-faire equilibrium

threshold  $\widehat{\theta}$ , there is no social benefit to subsidizing or taxing buyers at states  $\theta < \theta^*$ : a tax is not needed to deter the union from choosing a threshold below  $\theta^*$  and a subsidy, which raises consumers' willingness to pay at such states, will only encourage the union to make such an undesirable choice. Hence, setting  $t_\theta^\ell = 0$  for all  $\theta < \theta^*$  does *not* restrict the set of parameters for which we can costlessly attain the first best. So it would not help to weaken (b) or (c) at such states.

What of states  $\theta$  above the socially optimal threshold  $\theta^*$ ? Since all consumers buy at such states, for the policy to be costless  $t_\theta^1$  must be nonpositive. Hence, property (b) *cannot* be weakened at such states without sacrificing zero cost. As for property (c), taxes on buyers at states  $\theta > \theta^*$  and participation rates  $\ell < 1$  *can* help attain the social optimum. We consider such a policy in section 4.2.

## 4 Extensions

We now consider a number of extensions.

### 4.1 A Wage Cap

We first compare the insurance scheme to an efficient wage cap. In Figure 1, the primitive demand curve  $R$  lies below the marginal cost curve  $c$  at the socially optimal threshold  $\theta^*$ . An efficient wage cap would thus have to hold the union to the negative wage  $W = -(c - R_{\theta^*}) = -DE$ , as depicted in Figure 3. As workers will not work for less than their reservation wage, a cap cannot implement the first best outcome.

### 4.2 Allowing Taxation

We now consider a broader class of schemes, in which buyers may be taxed when not all consumers buy. Informally, we relax the restrictions in PCSS as follows. (A formal version appears in section 5.)

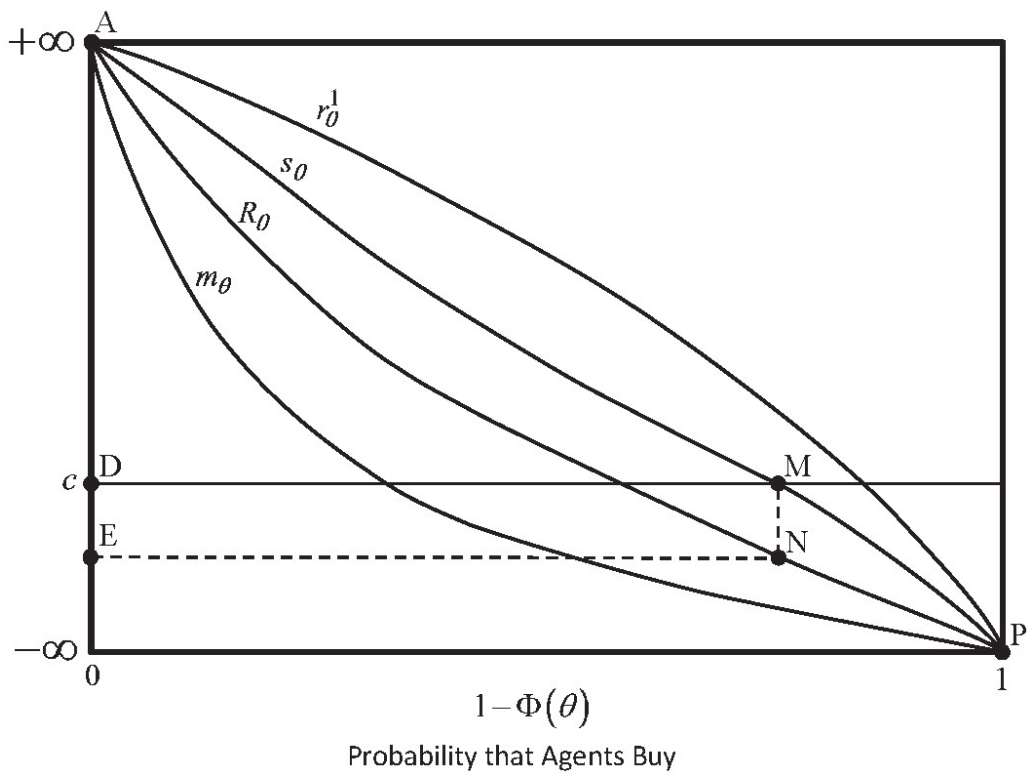


Figure 3: Wage Cap. In order to induce union to choose point M, wage  $W$  is capped at negative DE. The policy infeasibly requires workers to work for less than the reservation wage.

**Definition 2** (*Informal*) A Predictable Revenue-Neutral Scheme (PRNS) is a real-valued function  $\hat{t}$  with the following two properties. (a) The augmented relative payoff function  $\hat{r} = r + \hat{t}$  satisfies sufficient conditions for a unique equilibrium of the purchase subgame. (b) For all  $\theta \in \mathfrak{R}$ ,  $\hat{t}_\theta^1 = 0$ .

Like a PCSS, a PRNS is revenue-neutral: by property (b') no taxes or subsidies are paid in the limit, where the consumers perfectly coordinate. But unlike a PCSS,  $\hat{t}_\theta^\ell$  can be negative when  $\ell < 1$ : the policymaker may tax buyers when not all consumers buy. With this restriction lifted, we can now implement the first best without the condition  $\Gamma(\theta^*, \hat{\theta}) > 0$  of Theorem 1:

**Theorem 2** (*Informal.*) Assume the primitive relative payoff function  $r$  satisfies AM, SM, and PMC. There is a PRNS  $\hat{t}$  that induces the union to choose a wage schedule that implements the first best purchase threshold  $\theta^*$ .

For an intuition, consider Figure 2. For states to the right of point I in each Figure, the PRNS  $\hat{t}$  coincides with the PCSS  $t$ : the demand curve is still given by ILNP. For states to the left of point I, we use the ability to tax to keep the demand curve horizontal at the level of point I. In this way, the full demand curve that results from the PRNS  $\hat{t}$  is ACLNP. If the wage  $W$  exceeds CD, the firm fails for sure, so the union gets zero. This implies that the union will choose CD: the PRNS implements the first-best.

### 4.3 Union-Firm Bargaining

In our model, the union has all the bargaining power. We can relax this assumption using the bargaining framework of Hart and Moore [23, p. 10]. Assume bargaining occurs before the public signal  $Y$  is revealed. In stage 1, the firm offers the union a wage schedule  $w$ . If this offer is rejected, then in stage 2 one party is chosen at random (the union with probability  $\beta \in (0, 1)$ , the firm with probability  $1 - \beta$ ) to make a take-it-or-leave-it offer to the other.

Suppose the policymaker still offers the insurance scheme of Figure 2. What will happen? If the firm is chosen in bargaining stage 2, it will offer the wage  $W^F = 0$ . If the union is

chosen, it will choose a wage  $W^U$  equal to CD. Hence the firm's optimal stage 1 offer is  $\beta * CD$ , which the union will accept. In Figure 2, the outcome is efficient as before: the scheme still implements the first best.

## 4.4 Profit Sharing

Modern UAW contracts with the Detroit 3 feature some profit sharing. From 1983 to 2012, workers received an average annual profit sharing payment of \$2,945 at Ford, \$1,318 at G.M., and \$2,444 at Chrysler (2012 dollars).<sup>25</sup> These mean payments are less than ten percent of full-time, full-year autoworkers' earnings in 2012.<sup>26</sup>

We can allow profit sharing as follows. The firm first announces a profit-sharing proportion  $\alpha \in [0, 1)$ . The game then proceeds as before: the union announces a wage schedule  $w$ , the public signal  $Y$  is revealed, and so on. If the firm buys  $L$  units of labor and sells to  $\ell = G(L)$  consumers at the price  $p$ , it pays the union its wage  $w(L)$  plus a proportion  $\alpha$  of its profits.

Without intervention, the firm may propose some profit sharing so as to lower the union's wage  $W$ .<sup>27</sup> However, the insurance scheme eliminates this incentive. Why? When  $\alpha = 0$ , the union's optimal wage equals CD (Figure 2). Offering  $\alpha \in (0, 1)$  can benefit the firm only if the union lowers its wage. However, by lowering its wage to  $W \in [0, CD)$  the union lowers its expected payoff by  $(1 - \alpha)(CD - W)[1 - \Phi(\theta^*)] > 0$ : it will not do so. Hence, the firm will not offer profit sharing. It follows that our results are robust to the possibility

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<sup>25</sup>Author's calculations using nominal payments from Katz, MacDuffie, and Pil [30, Table 5] and CPI-U from Table B-60 in *2013 Economic Report of the President*.

<sup>26</sup>These earnings ranged from \$30,742 for an entry level worker at G.M. to \$60,549 for a tier-2 worker at Chrysler (Center for Automotive Research [10, p. 14]).

<sup>27</sup>At each state  $\theta > \theta_W$  the union gets its wage  $W$  plus the profit-sharing payment  $\alpha(R_\theta - c - W)$ , so its expected revenue is  $\widehat{U}(W, \alpha) = (1 - \alpha)U(W) + \alpha \int_{\theta=\theta_W}^{\infty} (R_\theta - c) d\Phi(\theta)$ . By IMR and (7),  $\theta_W$  is differentiable, so  $\widehat{U}_W(W, \alpha) = (1 - \alpha)U'(W) - \alpha W\phi(\theta_W)\theta'_W$ . Now say the wage  $W$  satisfies the union's first order condition without profit sharing:  $0 = U'(W) = 1 - \Phi(\theta_W) - W\phi(\theta_W)\theta'_W$ . Then if  $\alpha > 0$ ,  $\widehat{U}_W(W, \alpha) = -\alpha W\phi(\theta_W)\theta'_W = -\alpha[1 - \Phi(\theta_W)] < 0$ : profit sharing induces the union to lower its wage. This may create an incentive for the firm to offer some profit sharing in the laissez-faire case.

of profit sharing.

## 4.5 Firm-Provided Insurance

We focus on publicly provided insurance. Why can't the firm itself insure consumers against the loss of network benefits? In practice, we are not aware of any such scheme. One reason may be that the event that triggers payments under the scheme is an exodus of buyers that threatens the firm's very existence. As consumers are treated as unsecured creditors in bankruptcy proceedings, their claims under such a scheme would be unlikely to be honored. Thus the scheme would lack credibility.

That being said, we can ask whether the firm would choose the efficient scheme if credibility were not an issue. In fact, it would not. To show this, it suffices to consider schemes that have the same form as the efficient one but with a possibly different threshold. That is, the firm offers no insurance at states below some threshold  $\theta''$  and full insurance at  $\theta''$ . At states  $\theta > \theta''$ , it phases the insurance out gradually so as to yield a horizontal demand curve as in the efficient scheme. We now show that the firm prefers a threshold  $\theta''$  that lies below the efficient threshold  $\theta^*$ . This benefits the firm by leading to a lower wage  $W$ . It also leads the firm to produce inefficiently often.

In order for the firm to produce at all states above the threshold  $\theta''$ , the union must choose the wage  $W = r_{\theta''}^1 - c$ , which is increasing in  $\theta''$ . The firm's profits

$$\int_{\theta=-\infty}^{\infty} \max \{0, R_{\theta} - (c + W)\} dG(\theta)$$

are decreasing in  $W$  and thus in  $\theta''$ . Thus, the firm will choose the lowest production threshold  $\theta''$  that induces the union to choose the wage  $W = r_{\theta''}^1 - c$  rather than its laissez-faire wage. It follows that  $\Gamma(\theta'', \hat{\theta}) = 0$ : the union's payoff from choosing  $W = r_{\theta''}^1 - c$  equals its laissez-faire payoff. Assuming  $\Gamma(\theta^*, \hat{\theta}) > 0$ , which is the condition in Theorem 7 for the first best to be implementable by a PCSS, the firm's optimal threshold  $\theta''$  must be less than the efficient threshold  $\theta^*$ : the wage  $W$  is lower than under the efficient scheme and

the firm produces too often.

## 4.6 Noise in the Policymaker's Information

Our model assumes the policymaker observes the state  $\theta$  and purchase rate  $\ell$  without noise. We now weaken each assumption in turn. First, we show that the policymaker can implement the optimal policy with knowledge of  $\ell$  alone. We make the additional mild assumption that consumers always play a threshold equilibrium in the purchase subgame.<sup>28</sup>

**Claim 1** *Assume the primitive relative payoff function  $r$  satisfies AM, SM, and PMC and that  $\Gamma(\theta^*, \hat{\theta}) > 0$ . Let  $t$  be a PCSS that induces the union to choose a wage schedule that implements the first-best purchase threshold  $\theta^*$ . Assume that, for any price  $p$ , the consumers' purchase decisions are given by a threshold equilibrium: each consumer  $i$  buys if and only if her signal  $x_i$  exceeds some threshold  $\kappa(p)$ . Then the policymaker can implement  $t$  without directly observing  $\theta$  or any signal of  $\theta$ .*

**Proof.** For each price  $p$ , let  $\kappa(p)$  denote the consumers' purchase threshold. Each consumer  $i$  buys whenever  $x_i = \theta + \sigma\varepsilon_i > \kappa(p)$  or, equivalently, if  $\varepsilon_i > \frac{\kappa(p)-\theta}{\sigma}$ , which holds with probability  $1 - F\left(\frac{\kappa(p)-\theta}{\sigma}\right)$ . This probability must then equal the proportion  $\ell$  who purchase by the law of large numbers. Hence  $F\left(\frac{\kappa(p)-\theta}{\sigma}\right) = 1 - \ell$ . If  $\ell$  is either zero or one, no payments are called for under  $t$ : the policymaker does nothing. Now suppose instead that  $\ell$ , and thus  $F\left(\frac{\kappa(p)-\theta}{\sigma}\right)$ , lies in  $(0, 1)$ . As the support of  $F$  is connected,  $F$  is strictly increasing at  $\frac{\kappa(p)-\theta}{\sigma}$ . But then the policymaker can perfectly infer  $\frac{\kappa(p)-\theta}{\sigma} = F^{-1}(1 - \ell)$ . From this and her knowledge of  $\kappa(p)$ , she can also infer the state  $\theta$  and thus any required payment under the scheme  $t$ . Hence, she can always compute her required payment under  $t$ . ■

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<sup>28</sup>A threshold equilibrium always exists: as the purchase subgame is supermodular, the lowest and highest strategy profiles of this subgame that survive iterated strict dominance are both threshold equilibria (Milgrom and Roberts [35, Theorem 5, p. 1265]). If there are multiple threshold equilibria, we assume the policymaker knows which threshold equilibria is selected for each price  $p$ . This is just the standard assumption that the policymaker knows which equilibrium of the full game is being played.

Now suppose the policymaker sees the state  $\theta$  but not the purchase rate  $\ell$ . If applying for an insurance payment is costless, all  $\ell$  buyers will apply: the government will be able to infer the proportion  $\ell$  by observing these applications. If for some reason it does not wish to use this information, it can still implement the PCSS  $t$  without precise knowledge of  $\ell$  as long as it knows both the state  $\theta$  and the realized miscoordination loss  $r_\theta^1 - r_\theta^\ell$ . The latter quantity might be estimable using hedonic regressions and the occurrence of public events that affect buyers, such as closure of the firm’s repair network, loss of warranty coverage, denial of class actions lawsuits against the firm, and so on.

Finally, one may wonder what happens if the policymaker sees both  $\theta$  and  $\ell$  with noise and *cannot* infer  $\ell$  from the measure of applications for payments. In this case, it seems likely that the policymaker cannot implement the first best. We leave this question for future research.

## 4.7 Bargaining with Dealers

Our motivating example is the new car market. In this market, a manufacturer sets an invoice price that becomes a dealer’s unit cost. The dealer then bargains with a customer over a car’s price.

This is captured by the following extension of our model. Add a new class of players - “dealers” - to the model. The number of dealers equals the number of consumers, and each dealer has an exclusive relationship with a single consumer. On seeing the union wage schedule  $w$  and public signal  $Y$ , the firm chooses an “invoice” price  $p$ . The consumers then see their signals as before. Each consumer’s dealer also sees the given consumer’s signal: there is symmetric information between a consumer and her dealer. Each consumer  $i$  then bargains with her dealer over a price  $\rho_i$ . The expected payoffs of consumer  $i$  and her dealer from trading are  $E(r_\theta^\ell | x_i) - \rho_i$  and  $\rho_i - p$ , respectively. Hence, trade will occur if and only if the expected relative payoff conditional on the signal of consumer  $i$ ,  $E(r_\theta^\ell | x_i)$ , exceeds the firm’s price  $p$ . But this is exactly the condition for consumer  $i$  to buy in the original model:



the models are strategically equivalent.<sup>29</sup> Accordingly, the insurance scheme will work here as well.

## 5 Formal Results

In prior sections, we minimize technical details so as to focus on intuitions. We now discuss those details and present our results formally. This section can be skipped by the casual reader.

As noted in section 2, the primitive relative payoff function  $r_\theta^\ell$  is assumed to satisfy AM, PMC, and SM. However, under the optimal PCSS  $t$  of section 3, the relative payoff function is not  $r_\theta^\ell$  but rather the augmented relative payoff function  $r_\theta^\ell + t_\theta^\ell$  where  $t_\theta^\ell$  is the insurance payment a buyer gets at state  $\theta$  when the purchase rate is  $\ell$ . Unfortunately, this augmented relative payoff function violates both PMC and SM. Luckily, it satisfies a set of weaker properties that are implied by SM and are sufficient for uniqueness.<sup>30</sup> These weaker properties are as follows.<sup>31</sup> First, SM holds when restricted to the maximal purchase rate  $\ell = 1$ :

**XSM. Maximum State Monotonicity.** For all  $\theta' > \theta$ ,  $\frac{r_{\theta'}^1 - r_\theta^1}{\theta' - \theta} \in (k_3, k_4)$ .

Second, the rate of *decrease* of  $r$  in the state  $\theta$  is bounded:

**OSL. One Sided Lipschitz Continuity.** There is a constant  $k_5 \in (0, \infty)$  such that for all purchase rates  $\ell$  and states  $\theta' > \theta$ ,  $\frac{r_{\theta'}^\ell - r_\theta^\ell}{\theta' - \theta} > -k_5$ .

Finally, the lower bound in SM applies to the *mean* relative payoff function  $R$ . As for the upper bound, it applies where  $R$  is continuous, which it is at all but a finite number

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<sup>29</sup>The only difference is that an agent's payoff in the original model is now shared with the agent's dealer.

<sup>30</sup>These statements also hold for the optimal PRNS  $\hat{t}$  of section 4.2.

<sup>31</sup>Why then is PMC needed? As noted, our uniqueness proof does not rely on it. However, it does rely on the upper bound in AM. The proof that  $\tilde{r}$  satisfies this bound relies on the assumption that  $r$  satisfies PMC.

(which may be zero) of states. Let  $R_\theta^- = \lim_{\theta' \uparrow \theta} R_\theta$  and  $R_\theta^+ = \lim_{\theta' \downarrow \theta} R_\theta$  be the left- and right-continuous versions of  $R$ .

**MSM. Mean State Monotonicity.** For any states  $\theta'$  and  $\theta$  such that  $\theta' > \theta$ ,  $\frac{R_{\theta'} - R_\theta}{\theta' - \theta} > k_3$ .

Moreover,  $R$  has a finite number (possibly zero) of points of discontinuity. If  $R$  is continuous throughout the interval  $(\theta, \theta')$ , then  $\frac{R_{\theta'}^- - R_\theta^+}{\theta' - \theta} < k_4$ .

**Claim 2** *SM implies MSM, OSL, and XSM. Proof: Trivial.*

Our assumptions also imply the usual dominance regions property assumed in global games:

**Claim 3** *Assume AM and XSM. Then for any price  $p$ , there are finite thresholds  $\underline{\theta}_p \leq \bar{\theta}_p$  such that if a consumer knows the state  $\theta$ , it is strictly dominant for her (not) to buy if  $\theta > \bar{\theta}_p$  ( $\theta < \underline{\theta}_p$ ): for any purchase rate  $\ell \in [0, 1]$ ,  $r_\theta^\ell > p$  ( $r_\theta^\ell < p$ ).*

Our equilibrium concept is Perfect Bayesian Equilibrium (PBE).<sup>32</sup> In analyzing the purchase subgame, we do not require the relative payoff function  $r_\theta^\ell$  to be continuous in the state  $\theta$  or to be nondecreasing in  $\theta$  for all purchase rates  $\ell$ . As standard uniqueness results do assume these properties (e.g., Morris and Shin [37, Proposition 2.2, p. 67]), the following result is proved from first principles.

**Theorem 3 (Agent Subgame)** *Assume  $r$  satisfies AM, XSM, OSL, and MSM. For each public noise scale factor  $\tau > 0$  there is a constant  $\gamma_\tau \in (0, \infty)$  such that all consumers buy if  $p \leq R_{\theta - \gamma_\tau \sigma}$  and no consumers buy if  $p \geq R_{\theta + \gamma_\tau \sigma}$  for any private noise scale factor  $\sigma \in (0, \bar{\sigma})$ , wage schedule  $w$ , public signal  $Y$ , and price  $p$ .*

We now consider the firm's problem. While the result may look complex, its essence is simple: in the limit as the private noise  $\sigma$  and then the public noise  $\tau$  goes to zero, the firm sells to all consumers at its best estimate  $R_Y$  of their reservation price, as long as this estimate exceeds the firm's cost  $c + W$ ; else it does not sell.

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<sup>32</sup>See Fudenberg and Tirole [19].

**Theorem 4 (Firm Behavior)** *Assume  $r$  satisfies AM, XSM, OSL, and MSM. For any  $a \in (0, 1)$  and any  $\varepsilon > 0$  there is a  $\tau^* > 0$  and, for each wage schedule  $w$  with associated wage  $W = w(\bar{L})$ , a set  $S \subset \mathfrak{R}$ , such that for all  $\tau \in (0, \tau^*)$ :*

1. *the ex ante probability that  $Y$  lies in  $S$  is at least  $1 - \varepsilon$ ;*
2. *there is a  $\sigma^*$  (which depends on  $\tau$ ) such that for all  $\sigma \in (0, \sigma^*)$  and for all  $Y$  in  $S$ ,*
  - (a) *if  $R_Y - c - W$  is positive, then the firm chooses a price  $p$  in  $[R_{Y-\varepsilon}, R_{Y+\varepsilon}]$  and its probability of selling to all of the consumers is at least  $1 - \varepsilon^a$ ; and*
  - (b) *if  $R_Y - c - W$  is negative, then the firm chooses a price  $p \geq R_{Y-\varepsilon}$  and its probability of selling to none of the consumers is at least  $1 - \varepsilon^a$ .*

Stepping back again, we solve the union's problem. For any wage  $W$ , let

$$\theta_W = \sup \{ \theta : R_\theta < c + W \} = \inf \{ \theta : R_\theta > c + W \} \quad (14)$$

be the boundary between states  $\theta$  at which the firm can and cannot profitably sell to all the consumers at the price  $R_\theta$ .<sup>33</sup> In the limit as the private noise  $\sigma$  and then the public noise  $\tau$  goes to zero, the public signal  $Y$  converges to the state  $\theta$  so by Theorem 4 the firm sells to all (no) consumers if  $R_\theta > (<) c + W$ , which by (14) holds only if  $\theta \geq (\leq) \theta_W$ : the union's payoff converges to  $U(W)$ , which is defined above in (8).

To state this result precisely, we first fix a PBE  $E_\sigma^\tau$  for each pair  $(\sigma, \tau)$  of noise scale factors. For any wage schedule  $w$ , let  $u_\sigma^\tau(w)$  denote the union's payoff from  $w$  if the firm and consumers play according to  $E_\sigma^\tau$ . In the limit as  $\sigma$  and then  $\tau$  goes to zero, this payoff converges to the function  $U$ , defined in (8), evaluated at the wage  $w(\bar{L})$ , uniformly in the wage schedule  $w$ :

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<sup>33</sup>By MSM,  $R$  is increasing in  $\theta$ , so the supremum and infimum in (14) are equal.

**Theorem 5 (Union Payoff)** *Assume  $r$  satisfies AM, XSM, OSL, and MSM. For any  $\varepsilon > 0$  there is a  $\tau^* > 0$  such that for all  $\tau \in (0, \tau^*)$  there is a  $\sigma^* \in (0, \bar{\sigma}]$  such that for all  $\sigma \in (0, \sigma^*)$  and every wage schedule  $w$  in  $\Omega$ ,  $|u_\sigma^\tau(w) - U(w(\bar{L}))| < \varepsilon$ .*

For any scalar  $\mu \in \mathfrak{R}$  and any real valued, two dimensional sequence  $(\mu_\sigma^\tau)$ , let " $\mu_\sigma^\tau \rightrightarrows \mu$ " (with negation " $\neg(\mu_\sigma^\tau \rightrightarrows \mu)$ ") mean that  $\mu_\sigma^\tau$  converges to  $\mu \in \mathfrak{R}$  in the limit as  $\sigma$  and then  $\tau$  goes to zero: for any  $\varepsilon > 0$  there is a  $\tau^* > 0$  such that for all  $\tau \in (0, \tau^*)$  there is a  $\sigma^* \in (0, \bar{\sigma}]$  such that for all  $\sigma \in (0, \sigma^*)$ ,  $|\mu_\sigma^\tau - \mu| < \varepsilon$ .

If  $U$  has a strict global maximizer  $W_0$ , then the wage of any optimal wage schedule must converge to  $W_0$ :

**Theorem 6** *Assume  $r$  satisfies AM, XSM, OSL, and MSM. For each pair  $(\sigma, \tau)$  of noise scale factors, let the wage schedule  $w_\sigma^\tau \in \Omega$  be a best response for the union in the PBE  $E_\sigma^\tau$  chosen above. If  $U$  has a strict global maximizer  $W_0 \in (0, \infty)$ , then  $w_\sigma^\tau(\bar{L}) \rightrightarrows W_0$ .*

In the limit, the union's payoff depends only on its wage  $W$  (Theorem 5), which must equal the global maximizer of the union's limiting payoff function  $U$  if this maximizer exists (Theorem 6). Consider any wage  $W$ . If the mean relative payoff function  $R$  is discontinuous at the limiting purchase threshold  $\theta_W$ , so that  $R_{\theta_W}^- < R_{\theta_W}^+$ , then by MSM and (14) any wage  $W'$  in  $[R_{\theta_W}^- - c, R_{\theta_W}^+ - c]$  yields the same threshold:  $\theta_{W'} = \theta_W$ . But then by (8), the upper endpoint of this interval is the wage in this interval that yields the highest value of  $U$ . Hence if  $W$  maximizes  $U$ , it must equal  $R_{\theta_W}^+ - c$ . This also holds if  $R$  is continuous at  $\theta_W$  since then, by MSM and (14),  $W$  equals  $R_{\theta_W} - c$  which equals  $R_{\theta_W}^+ - c$ . Combining the two cases, the limiting purchase threshold  $\theta = \theta_W$  that corresponds to a maximizer  $W$  of  $U$  must itself maximize  $V(\theta)$  which is defined above in (9). This suggests the following equivalence, which is proved in the appendix:

**Claim 4** *If the wage  $W$  is a maximizer of  $U$ , then  $W$  equals  $R_\theta^+ - c$  where the purchase threshold  $\theta = \theta_W$  maximizes  $V$ . Conversely, if the purchase threshold  $\theta$  maximizes  $V$ , then the wage  $W = R_\theta^+ - c$  maximizes  $U$ .*

Moreover, the union's optimal purchase threshold cannot converge to a state that does not maximize  $V$ ; and if a state uniquely maximizes  $V$ , then the union's optimal purchase threshold must converge to this state:

**Claim 5** *Assume  $r$  satisfies AM, XSM, OSL, and MSM.*

1. *For any state  $\theta'$ , if  $V(\theta'') > V(\theta')$  for some other state  $\theta''$  then  $\neg(\theta_{w_\sigma^\tau(\bar{L})} \rightrightarrows \theta')$ .*
2. *If  $V$  has a strict global maximizer  $\hat{\theta} \in \mathfrak{R}$ , then  $\theta_{w_\sigma^\tau(\bar{L})} \rightrightarrows \hat{\theta}$ .*

We can now formally define our insurance schemes. These are the formal versions of definitions 1 and 2.

**Definition 3** *A Predictable Costless Subsidy Scheme (PCSS) is a real-valued function  $t$  with the following three properties. (a) The augmented relative payoff function  $\tilde{r} = r + t$  satisfies AM, XSM, OSL, and MSM.<sup>34</sup> (b) For all  $\theta \in \mathfrak{R}$ ,  $t_\theta^1 \leq 0$ . (c) For all  $\ell \in [0, 1]$  and  $\theta \in \mathfrak{R}$ ,  $t_\theta^\ell \geq 0$ .*

**Definition 4** *A Predictable Revenue-Neutral Scheme (PRNS) is a real-valued function  $\hat{t}$  with the following two properties. (a) The augmented relative payoff function  $\hat{r} = r + \hat{t}$  satisfies AM, XSM, OSL, and MSM.<sup>35</sup> (b) For all  $\theta \in \mathfrak{R}$ ,  $\hat{t}_\theta^1 = 0$ .*

Finally, we give precise versions of Theorems 1 and 2.

**Theorem 7** *Assume the primitive relative payoff function  $r$  satisfies AM, SM, and PMC. If  $\Gamma(\theta^*, \hat{\theta}) > (<) 0$ , then there is (not) a PCSS  $t$  that induces the union to choose a wage schedule  $w_\sigma^\tau$  for which  $\theta_{w_\sigma^\tau(\bar{L})} \rightrightarrows \theta^*$ .*

**Theorem 8** *Assume the primitive relative payoff function  $r$  satisfies AM, SM, and PMC. There is a PRNS  $\hat{t}$  that induces the union to choose a wage schedule  $w_\sigma^\tau$  for which  $\theta_{w_\sigma^\tau(\bar{L})} \rightrightarrows \theta^*$ .*

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<sup>34</sup>The function  $\tilde{r}$  is allowed to satisfy these assumptions using a different set of constants  $k_1, k_2, k_3, k_4$ , and  $k_5$  than the function  $r$ .

<sup>35</sup>The comment in footnote 34 applies also to  $\hat{r}$ .

## 6 Conclusions

The recent bailouts of GM and Chrysler included explicit measures to protect new car buyers against loss of warranty coverage. We explore the potential of such insurance to fix two inefficiencies. First, union monopoly power leads to a socially excessive wage. Second, each consumer ignores the benefit to other buyers of her decision to purchase. Both effects make a firm too fragile.

Under certain conditions, these inefficiencies can be corrected by an insurance scheme that pays consumers who buy when others do not. In the limit as consumers become well informed about fundamentals, the insurance is costless: either all buy or none do. However, the insurance raises the reservation price of the consumers by reducing the strategic risk of buying.

The optimal insurance scheme is countercyclical: as the state falls, more insurance is given. This flattens the consumers' demand curve. If the union then lowers its price so that the firm's cost lies slightly below the height of the new demand curve, the firm will be able to sell in many more states. We give a necessary and sufficient condition for the union to do so, and thus for our costless scheme to implement the first best.

## A Proofs

We begin with some preliminary results.

**Lemma 1** 1. For  $n = 0, 1, 2$ , the limits  $\lim_{z \rightarrow -\infty} [z^n \phi(z)]$  and  $\lim_{z \rightarrow \infty} [z^n \phi(z)]$  exist and are zero.

2. For  $n = 0, 1, 2$  and any  $a > 0$ ,  $z^n \phi(z)$  is bounded on  $z \in [-a, \infty)$ .

3. The limits  $\lim_{z \rightarrow -\infty} z \Phi(z)$  and  $\lim_{z \rightarrow \infty} z [1 - \Phi(z)]$  exist and equal zero.

Moreover, these properties all hold if  $\Phi$  and  $\phi$  are replaced by  $H$  and  $h$ , respectively.

**Proof.** We prove the theorem for  $\phi$  and  $\Phi$ ; the proof for  $h$  and  $H$  is identical.

Part 1. Suppose otherwise: for some fixed  $j = \{0, 1\}$  and for any  $\iota > 0$ , there is a sequence  $(z_m)_{m=1}^\infty$  such that for each  $m$ ,  $(-1)^j z_m > m$  and  $\left[(-1)^j z_m\right]^n \phi(z_m) > \iota$ . By (1),  $|\phi(z_m) - \phi(z)| \leq k_\phi |z_m - z|$  so  $\phi(z) \geq \phi(z_m) - k_\phi |z_m - z| > 0$  for all  $z \in I_m = \left[z_m - \frac{\phi(z_m)}{k_\phi}, z_m + \frac{\phi(z_m)}{k_\phi}\right]$ . Dropping  $z_m$ 's and renumbering as needed, we can assume that  $(-1)^j (z_m - z_{m-1}) \geq 2\bar{\phi}/k_\phi$  for all  $m$ , whence the intervals  $I_m$  do not overlap. By (2) and Jensen's Inequality,  $E[\theta^4]$  and  $E[\theta^2] \leq \sqrt{E[\theta^4]}$  are finite. From this we derive a contradiction:  $\infty > E[\theta^{2n}] \geq \sum_{m=1}^\infty \int_{z=z_m}^{z_m + \frac{\phi(z_m)}{k_\phi}} z_m^{2n} [\phi(z_m) - k_\phi(z - z_m)] dz = \frac{1}{2} \sum_{m=1}^\infty \frac{z_m^{2n} [\phi(z_m)]^2}{k_\phi} \geq \frac{1}{2k_\phi} \sum_{m=1}^\infty \iota^2 = \infty$ .

Part 2. Suppose otherwise: for all  $m = 1, 2, \dots$ , there is a  $z_m \in [-a, \infty)$  such that  $z_m^n \phi(z_m) > m$ . Then  $|z_m| > (m/\bar{\phi})^{1/n}$ . Thus, as  $z_m$  is bounded away from  $-\infty$ , it must go to infinity as  $m$  does. We have produced a sequence  $(z_m)_{m=1}^\infty$  that goes to infinity such that for all  $m$ ,  $z_m^n \phi(z_m) > \iota$  for any  $\iota \in (0, 1)$ . This contradicts part 1.

Part 3.  $\lim_{z \rightarrow \infty} z[1 - \Phi(z)] = \lim_{z \rightarrow \infty} \frac{1 - \Phi(z)}{z^{-1}} = \lim_{z \rightarrow \infty} \frac{-\phi(z)}{-z^{-2}} = \lim_{z \rightarrow \infty} [z^2 \phi(z)] = 0$  by L'Hôpital's rule and part 1. The proof that  $\lim_{z \rightarrow -\infty} z\Phi(z) = 0$  is the same, *mutatis mutandis*. ■

Let  $\Psi$  and  $\psi$  denote the posterior distribution and density of the state  $\theta$  given the public noise scale factor  $\tau$  and a fixed realization  $y$  of the public signal. By Bayes's Rule,

$$\psi(\theta) = k_y^\tau h\left(\frac{y - \theta}{\tau}\right) \phi(\theta) \quad (15)$$

where

$$k_y^\tau = \left[ \int_{\theta'=-\infty}^{\infty} h\left(\frac{y - \theta'}{\tau}\right) \phi(\theta') d\theta' \right]^{-1} > 0. \quad (16)$$

The posterior density  $\psi$  is positive since  $h$  and  $\phi$  have support equal to the whole real line. Moreover,  $\psi$  is bounded above by the finite constant

$$\bar{\psi} = k_y^\tau \bar{h} \bar{\phi} \in (0, \infty). \quad (17)$$

Finally,  $\psi$  is Lipschitz: by the triangle inequality, for  $\theta, \theta' \in \mathfrak{R}$ ,

$$|\psi(\theta') - \psi(\theta)| \leq k_\psi |\theta' - \theta| \quad (18)$$

where, by (1),

$$k_\psi = k_y^\tau \left[ \bar{h} k_\phi + \bar{\phi} \frac{k_h}{\tau} \right] \in (0, \infty). \quad (19)$$

As  $Y = \theta + \tau v$ ,  $\Pr(Y \leq y | \theta = \theta_0) = H\left(\frac{y - \theta_0}{\tau}\right)$  and the associated conditional density is  $\tau^{-1} h\left(\frac{y - \theta_0}{\tau}\right)$ . So the unconditional density  $\lambda$  of  $Y$  at  $y$  is

$$\lambda(y) = \tau^{-1} \int_{\theta=-\infty}^{\infty} \phi(\theta) h\left(\frac{y - \theta}{\tau}\right) d\theta = \int_{\theta=-\infty}^{\infty} \phi(\theta) d\left[1 - H\left(\frac{y - \theta}{\tau}\right)\right] \leq \bar{\phi}. \quad (20)$$

Let

$$\begin{aligned} c_3 &= \int_{v=-\infty}^0 H(v) dv + \int_{v=0}^{\infty} [1 - H(v)] dv = \int_{v=-\infty}^{\infty} |v| dH(v) \\ &\leq \int_{v \in [-1, 1]} dH(v) + \int_{v \in \mathfrak{R} \setminus [-1, 1]} v^2 dH(v) \leq 1 + E[v^2] < \infty, \end{aligned} \quad (21)$$

by part 3 of Lemma 1 and equation (2). Let  $\Lambda$  be the unconditional distribution of  $Y$ .

**Lemma 2** For any  $\tau > 0$  and  $y_0 \in \mathfrak{R}$ ,  $|\Lambda(y_0) - \Phi(y_0)| < c_3 \bar{\phi} \tau$ .

**Proof.** Let  $\kappa(x)$  be an indicator function for the event  $x > 0$ . By (20),

$$\begin{aligned} \Lambda(y_0) &= \int_{y=-\infty}^{y_0} \lambda(y) dy = \tau^{-1} \int_{\theta=-\infty}^{\infty} \phi(\theta) \int_{y=-\infty}^{y_0} h\left(\frac{y - \theta}{\tau}\right) dy d\theta \\ &= \int_{\theta=-\infty}^{\infty} \phi(\theta) \left[ \int_{y=-\infty}^{y_0} dH\left(\frac{y - \theta}{\tau}\right) \right] d\theta = \int_{\theta=-\infty}^{\infty} \phi(\theta) H\left(\frac{y_0 - \theta}{\tau}\right) d\theta \end{aligned}$$

while  $\Phi(y_0) = \int_{\theta=-\infty}^{\infty} \phi(\theta) \kappa(y_0 - \theta) d\theta$ , so using the change of variables  $v = \frac{y_0 - \theta}{\tau}$ , the absolute gap  $|\Lambda(y_0) - \Phi(y_0)|$  is at most  $\int_{\theta=-\infty}^{\infty} \phi(\theta) |H\left(\frac{y_0 - \theta}{\tau}\right) - \kappa(y_0 - \theta)| d\theta$ , which equals  $\tau \int_{v=-\infty}^{\infty} \phi(y_0 - \tau v) |H(v) - \kappa(v)| dv \leq \tau \bar{\phi} \int_{v=-\infty}^{\infty} |H(v) - \kappa(v)| dv = \tau \bar{\phi} c_3$ . ■



**Proof of Claim 3:** By XSM, for all  $p$  there exists a  $\underline{\theta}_p$  such that for all  $\theta \leq \underline{\theta}_p$ ,  $r_\theta^1 \leq p$  whence, by AM,  $r_\theta^\ell \leq p$  for all  $\ell$ . Also by XSM there exists a  $\bar{\theta}_p$  such that for all  $\theta \geq \bar{\theta}_p$ ,  $r_\theta^1 \geq p + k_1$  whence, by AM,  $r_\theta^\ell \geq p$  for all  $\ell$ . Q.E.D.<sub>Claim 3</sub>

**Proof of Theorem 3:** Suppose a consumer  $i$  sees private signal  $x_i = x$  and believes that each other consumer will buy if and only if his signal exceeds some threshold  $k \in \mathfrak{R}$ . Then her expected relative payoff, gross of the firm's price  $p$ , is  $\pi_\sigma(x, k) = \int_{\theta=x-\sigma/2}^{x+\sigma/2} w_\sigma(x, \theta) r_\theta^{1-F(\frac{k-\theta}{\sigma})} d\theta$  where  $w_\sigma(x, \theta) = f\left(\frac{x-\theta}{\sigma}\right) \psi(\theta) \left[ \int_{\theta'=x-\sigma/2}^{x+\sigma/2} f\left(\frac{x-\theta'}{\sigma}\right) \psi(\theta') d\theta' \right]^{-1}$ .

**Lemma 3** *The function  $\pi_\sigma(x, k)$  is continuous in  $x \in \mathfrak{R}$ .*

**Proof.** Fix a constant  $c_1 > 0$  and an  $x' \in \mathfrak{R}$ . We will show that for any  $\varepsilon \in (0, c_1)$ , there is a  $\delta \in (0, c_1)$  such that for any  $x'' \in [x' - \delta, x' + \delta]$ ,  $|\pi_\sigma(x', k) - \pi_\sigma(x'', k)| \leq \varepsilon$ ; thus, the function  $\pi_\sigma(x, k)$  is continuous at  $x = x'$ . By the Cauchy-Schwarz inequality,

$$\begin{aligned} |\pi_\sigma(x', k) - \pi_\sigma(x'', k)| &= \left| \int_{\theta=x'-\delta-\sigma/2}^{x'+\delta+\sigma/2} [w_\sigma(x', \theta) - w_\sigma(x'', \theta)] r_\theta^{1-F(\frac{k-\theta}{\sigma})} d\theta \right| \\ &\leq \sqrt{\int_{\theta=x'-\delta-\sigma/2}^{x'+\delta+\sigma/2} [w_\sigma(x', \theta) - w_\sigma(x'', \theta)]^2 d\theta} \sqrt{\int_{\theta=x'-\delta-\sigma/2}^{x'+\delta+\sigma/2} \left[ r_\theta^{1-F(\frac{k-\theta}{\sigma})} \right]^2 d\theta}. \end{aligned}$$

The second square root is no greater than  $c_2 \sqrt{2c_1 + \sigma}$  where  $c_2 \in \mathfrak{R}_{++}$  is the supremum of  $|r_\theta^\ell|$  over pairs  $(\theta, \ell)$  in the set  $[x' - c_1 - \sigma, x' + c_1 + \sigma] \times [0, 1]$ . By the triangle inequality,

$$\begin{aligned} |w_\sigma(x', \theta) - w_\sigma(x'', \theta)| &\leq \frac{|f\left(\frac{x'-\theta}{\sigma}\right) - f\left(\frac{x''-\theta}{\sigma}\right)| \psi(\theta)}{\int_{\theta'=x'-\delta-\sigma/2}^{x'+\delta+\sigma/2} f\left(\frac{x'-\theta'}{\sigma}\right) \psi(\theta') d\theta'} \\ &+ f\left(\frac{x''-\theta}{\sigma}\right) \psi(\theta) \left( \frac{\int_{\theta'=x'-\delta-\sigma/2}^{x'+\delta+\sigma/2} |f\left(\frac{x''-\theta'}{\sigma}\right) - f\left(\frac{x'-\theta'}{\sigma}\right)| \psi(\theta') d\theta'}{\left[ \int_{\theta'=x'-\delta-\sigma/2}^{x'+\delta+\sigma/2} f\left(\frac{x'-\theta'}{\sigma}\right) \psi(\theta') d\theta' \right] \left[ \int_{\theta'=x'-\delta-\sigma/2}^{x'+\delta+\sigma/2} f\left(\frac{x''-\theta'}{\sigma}\right) \psi(\theta') d\theta' \right]} \right) \end{aligned}$$

Let  $I = [x' - c_1 - \sigma/2, x' + c_1 + \sigma/2]$ ,  $\bar{\psi}_I = \max_{\theta \in I} \psi(\theta) < \infty$ ,  $\underline{\psi}_I = \min_{\theta \in I} \psi(\theta) > 0$ , and  $\bar{f} = \max_{z \in [-1/2, 1/2]} f(z)$ . For  $x = x', x''$ ,  $\int_{\theta=x'-\delta-\sigma/2}^{x'+\delta+\sigma/2} f\left(\frac{x-\theta}{\sigma}\right) d\theta = \sigma$ , so the integral  $\int_{\theta=x'-\delta-\sigma/2}^{x'+\delta+\sigma/2} f\left(\frac{x-\theta}{\sigma}\right) \psi(\theta') d\theta'$  lies in  $[\sigma \underline{\psi}_I, \sigma \bar{\psi}_I]$ . Since  $f$  is continuous, it is uniformly continuous on any compact set by the Heine-Cantor theorem. Hence, for any  $\varepsilon' > 0$  there

exists a  $\delta' > 0$  such that if  $|z' - z''| \leq \delta'$  and  $z', z'' \in I$  then  $|f(z') - f(z'')| \leq \varepsilon'$ . Now let  $I = [-1/2, 1/2]$ ,  $z' = \frac{x' - \theta}{\sigma}$ , and  $z'' = \frac{x'' - \theta}{\sigma}$ . Let

$$\varepsilon' = \varepsilon (c_2)^{-1} \left[ \frac{\bar{\psi}_I}{\sigma \underline{\psi}_I} + \left( \frac{\bar{\psi}_I}{\sigma \underline{\psi}_I} \right)^2 \bar{f} (2c_1 + \sigma) \right]^{-1} (2c_1 + \sigma)^{-1},$$

and let  $\delta'$  be the corresponding constant. Let  $\delta = \min \{c_1, \delta' \sigma\}$ , whence  $|x'' - x'| \leq \delta' \sigma$ , which implies  $|f(\frac{x' - \theta}{\sigma}) - f(\frac{x'' - \theta}{\sigma})| \leq \varepsilon'$ . Hence, the absolute gap  $|w_\sigma(x', \theta) - w_\sigma(x'', \theta)|$  is at most  $\varepsilon' \left[ \frac{\bar{\psi}_I}{\sigma \underline{\psi}_I} + \left( \frac{\bar{\psi}_I}{\sigma \underline{\psi}_I} \right)^2 \bar{f} (2c_1 + \sigma) \right]$ , whence the absolute difference  $|\pi_\sigma(x', k) - \pi_\sigma(x'', k)|$  is at most  $\varepsilon' \left[ \frac{\bar{\psi}_I}{\sigma \underline{\psi}_I} + \left( \frac{\bar{\psi}_I}{\sigma \underline{\psi}_I} \right)^2 \bar{f} (2c_1 + \sigma) \right] c_2 (2c_1 + \sigma) = \varepsilon$ . ■

**Lemma 4** *The function  $\pi_\sigma(x, k)$  is nonincreasing in  $k$ .*

**Proof.** For  $k' > k$ ,  $\pi_\sigma(x, k') - \pi_\sigma(x, k) = \frac{\int_{\theta=-\infty}^{\infty} f(\frac{x-\theta}{\sigma}) \left[ r_\theta^{1-F(\frac{k'-\theta}{\sigma})} - r_\theta^{1-F(\frac{k-\theta}{\sigma})} \right] d\Psi(\theta)}{\int_{\theta=-\infty}^{\infty} f(\frac{x-\theta}{\sigma}) d\Psi(\theta)}$  which is nonpositive by AM. ■

By Lemma 4,  $\bar{\beta}_p(k) = \sup \{x : \pi_\sigma(x, k) \leq p\}$  and  $\underline{\beta}_p(k) = \inf \{x : \pi_\sigma(x, k) \geq p\}$  are each nondecreasing in  $k$ .<sup>36</sup> And for all  $x > \bar{\beta}_p(k)$ ,  $\pi_\sigma(x, k) > p$ , and for all  $x < \underline{\beta}_p(k)$ ,  $\pi_\sigma(x, k) < p$ . So by AM, if all others are known (not) to buy when their signals exceed (are less than)  $k$ , then it is optimal for a given consumer (not) to buy when her signal exceeds  $\bar{\beta}_p(k)$  (is less than  $\underline{\beta}_p(k)$ ).

By Claim 3, for each price  $p$  there exist finite thresholds  $\underline{\theta}_p$  and  $\bar{\theta}_p$  such that it is strictly dominant for a consumer to buy (not to buy) if her signal is at least  $\bar{\theta}_p + \sigma/2$  (at most  $\underline{\theta}_p - \sigma/2$ ). So  $\pi_\sigma(x, \underline{\theta}_p - \sigma/2) < p$  for all  $x \leq \underline{\theta}_p - \sigma/2$ . By Lemma 3, then,  $\underline{\beta}_p(\underline{\theta}_p - \sigma/2) > \underline{\theta}_p - \sigma/2$ . Let  $\underline{k}_p^0 = \underline{\theta}_p - \sigma/2$  and, for  $n > 0$ , let  $\underline{k}_p^n = \underline{\beta}_p(\underline{k}_p^{n-1})$ . Since  $\underline{\beta}_p(k)$  is a nondecreasing function of  $k$ , the sequence  $(\underline{k}_p^n)_{n=0}^\infty$  is nondecreasing and bounded above by  $\bar{\theta}_p + \sigma/2$ , so it converges to a limit  $\underline{k}_p$  that satisfies  $\underline{\beta}_p(\underline{k}_p) = \underline{k}_p$ . No consumers buy if their signals are below

<sup>36</sup>If a set  $S \subseteq \Re$  is empty, let  $\inf S = \infty$  and  $\sup S = -\infty$ .

$\underline{k}_p$ . Since  $\underline{k}_p = \underline{\beta}_p(\underline{k}_p)$  is the infimum of  $\{x : \pi_\sigma(x, \underline{k}_p) \geq p\}$  which is nonempty by Claim 3,  $\pi_\sigma(\underline{k}_p, \underline{k}_p) = p$  by Lemma 3. We can construct an analogous nonincreasing sequence  $\bar{k}_p^0 = \bar{\theta}_p + \sigma/2$  and  $\bar{k}_p^n = \bar{\beta}_p(\bar{k}_p^{n-1})$  that converges to a limit  $\bar{k}_p$  such that  $\bar{\beta}_p(\bar{k}_p) = \bar{k}_p$  (and thus  $\pi_\sigma(\bar{k}_p, \bar{k}_p) = p$ ) and all consumers buy if their signals exceed  $\bar{k}_p$ . Clearly,  $\bar{k}_p \geq \underline{k}_p$ .

Let  $k_y^\sigma(p)$  denote either  $\underline{k}_p$  or  $\bar{k}_p$ . Using the change of variables  $\ell = 1 - F\left(\frac{k_y^\sigma(p) - \theta}{\sigma}\right)$ ,

$$\begin{aligned} p &= \pi_\sigma(k_y^\sigma(p), k_y^\sigma(p)) = \frac{\int_{\theta=k_y^\sigma(p)-\sigma/2}^{k_y^\sigma(p)+\sigma/2} f\left(\frac{k_y^\sigma(p)-\theta}{\sigma}\right) r_\theta^{1-F\left(\frac{k_y^\sigma(p)-\theta}{\sigma}\right)} \psi(\theta) d\theta}{\int_{\theta=k_y^\sigma(p)-\sigma/2}^{k_y^\sigma(p)+\sigma/2} f\left(\frac{k_y^\sigma(p)-\theta}{\sigma}\right) \psi(\theta) d\theta} \\ &= \int_{\ell=0}^1 \alpha(\ell, k_y^\sigma(p), \sigma) r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^\ell d\ell, \end{aligned} \quad (22)$$

where  $\alpha(\ell, k_y^\sigma(p), \sigma) = \frac{\psi(k_y^\sigma(p)-\sigma F^{-1}(1-\ell))}{\int_{\ell'=0}^1 \psi(k_y^\sigma(p)-\sigma F^{-1}(1-\ell')) d\ell'}$ . The terms  $\alpha(\ell, k_y^\sigma(p), \sigma)$  are positive and integrate to one over  $\ell \in [0, 1]$ , so  $p$  is a weighted average of the terms  $r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^\ell$  over  $\ell \in [0, 1]$  and hence lies in the interval

$$I = \left[ \inf_{\ell \in [0,1]} r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^\ell, \sup_{\ell \in [0,1]} r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^\ell \right]$$

which, by AM, is contained in  $I' = \left[ \inf_{\ell \in [0,1]} r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^0 - k_1, \sup_{\ell \in [0,1]} r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^1 \right]$ . The support of  $F$  is  $[-1/2, 1/2]$ , so  $F^{-1}(1-\ell) \in [-1/2, 1/2]$  for any  $\ell \in [0, 1]$ . So by XSM,

$$r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^1 \leq r_{k_y^\sigma(p)+\sigma/2}^1. \quad (23)$$

Moreover, by OSL,  $r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^0 \geq r_{k_y^\sigma(p)-\sigma/2}^0 - k_5\sigma$ , so the length of  $I'$  is at most

$$r_{k_y^\sigma(p)+\sigma/2}^1 - r_{k_y^\sigma(p)-\sigma/2}^1 + r_{k_y^\sigma(p)-\sigma/2}^1 - r_{k_y^\sigma(p)-\sigma/2}^0 + k_5\sigma \leq k_4\sigma + k_1 + k_5\sigma \quad (24)$$

by (23), XSM, and AM. Finally, by (18) and the triangle inequality,

$$|1 - \alpha(\ell, k_y^\sigma(p), \sigma)| \leq \frac{\int_{\ell'=0}^1 \left| \begin{array}{c} \psi(k_y^\sigma(p) - \sigma F^{-1}(1 - \ell')) \\ -\psi(k_y^\sigma(p) - \sigma F^{-1}(1 - \ell)) \end{array} \right| d\ell'}{\int_{\ell'=0}^1 \psi(k_y^\sigma(p) - \sigma F^{-1}(1 - \ell')) d\ell'} \leq c_\tau \sigma \quad (25)$$

where, by (19) and (17),  $c_\tau = \frac{k_\psi}{\psi} = \frac{k_\phi}{\phi} + \frac{k_h}{h\tau}$ .

For any  $\tau > 0$ , let  $\gamma_\tau = 2\hat{c}_\tau/k_3$  where  $\hat{c}_\tau = k_3 + k_5 + c_\tau[\bar{p} + k_1 + \bar{\sigma}(k_4 + k_5)]$ . Assume  $R_{\theta-\gamma_\tau\sigma} \geq p$ . Then all consumers must buy. Why? By (22),

$$0 \leq R_{\theta-\gamma\sigma} - \int_{\ell=0}^1 \alpha(\ell, k_y^\sigma(p), \sigma) r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^\ell d\ell = R_{\theta-\gamma\sigma} - R_{k_y^\sigma(p)-\gamma\sigma+\sigma/2} + A$$

where  $A = R_{k_y^\sigma(p)-\gamma\sigma+\sigma/2} - \int_{\ell=0}^1 \alpha(\ell, k_y^\sigma(p), \sigma) r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^\ell d\ell$ . Now  $A = B + C + D$  where

$$B = R_{k_y^\sigma(p)-\gamma\sigma+\sigma/2} - R_{k_y^\sigma(p)-\sigma/2} \leq -k_3[\gamma\sigma - \sigma] \text{ by MSM,} \quad (26)$$

$$C = \int_{\ell=0}^1 \left[ r_{k_y^\sigma(p)-\sigma/2}^\ell - r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^\ell \right] d\ell \leq k_5\sigma \text{ by OSL, and} \quad (27)$$

$$D = \int_{\ell=0}^1 r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^\ell [1 - \alpha(\ell, k_y^\sigma(p), \sigma)] d\ell. \quad (28)$$

By Holder's inequality,

$$|D| \leq \left( \int_{\ell=0}^1 |1 - \alpha(\ell, k_y^\sigma(p), \sigma)| d\ell \right) \sup_{\ell \in [0,1]} \left| r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^\ell \right| \quad (29)$$

Since  $p > 0$  lies in  $I$  and is less than  $\bar{p}$ , no element of  $I$  can exceed, in absolute value, the sum of  $\bar{p}$  and the width of  $I'$ : by (24),

$$\sup_{\ell \in [0,1]} \left| r_{k_y^\sigma(p)-\sigma F^{-1}(1-\ell)}^\ell \right| \leq \bar{p} + k_1 + (k_4 + k_5)\sigma. \quad (30)$$

Combining this with (25) and using (29) and  $\sigma \leq \bar{\sigma}$  we obtain

$$|D| \leq c_\tau \sigma [\bar{p} + k_1 + \bar{\sigma} (k_4 + k_5)]. \quad (31)$$

Hence, by (26) and (27),  $A \leq \sigma (-k_3 \gamma_\tau + \hat{c}_\tau) < 0$ , so  $R_{\theta - \gamma\sigma} > R_{k_y^\sigma(p) - \gamma\sigma + \sigma/2}$  and hence, by MSM,  $\theta > k_y^\sigma(p) + \sigma/2$ , whence all consumers' signals exceed  $k_y^\sigma(p)$ : all will buy.

Assume now  $R_{\theta + \gamma\tau\sigma} \leq p$ . By (22) and MSM,

$$0 \geq R_{\theta + \gamma\sigma} - R_{k_y^\sigma(p) + \gamma\sigma - \sigma/2} + A' \quad (32)$$

where  $A' = R_{k_y^\sigma(p) + \gamma\sigma - \sigma/2} - \int_{\ell=0}^1 \alpha(\ell, k_y^\sigma(p), \sigma) r_{k_y^\sigma(p) - \sigma F^{-1}(1-\ell)}^\ell d\ell = B' + C' + D$  where

$$B' = R_{k_y^\sigma(p) + \gamma\sigma - \sigma/2} - R_{k_y^\sigma(p) + \sigma/2} \geq k_3 [\gamma\sigma - \sigma] \text{ by MSM,} \quad (33)$$

$$C' = \int_{\ell=0}^1 \left[ r_{k_y^\sigma(p) + \sigma/2}^\ell - r_{k_y^\sigma(p) - \sigma F^{-1}(1-\ell)}^\ell \right] d\ell \geq -k_5 \sigma \text{ by OSL,} \quad (34)$$

and  $D$  is given by (28). By (31), (33), and (34),  $A' \geq B' + C' - |D| \geq \sigma [k_3 \gamma_\tau - \hat{c}_\tau] > 0$ , so  $\theta < k_y^\sigma(p) - \sigma/2$  by (32) and MSM, whence all signals are less than  $k_y^\sigma(p)$ : no consumer buys. Q.E.D. Theorem 3

**Proof of Theorem 4:** Let the "closed ball with radius  $b \in \mathfrak{R}_+$  centered at  $a \in \mathfrak{R}$ " denote the interval  $[a - b, a + b]$  and let  $c_4$  denote  $4n\bar{\phi}^{-1}$  where  $n$  is one plus the number of discontinuities of  $R$ .

**Lemma 5** *For any finite, positive constants  $\varepsilon$ ,  $\iota$ , and  $m$ , there is a number  $\tau^* > 0$  and, for each wage  $W$ , a compact set  $S_W^\varepsilon$  (which does not depend on  $\iota$  or  $m$ ) such that:*

1. *at all states  $\theta$  that lie in the closed ball with radius  $c_4\varepsilon$  centered at any element of  $S_W^\varepsilon$ , the mean relative payoff function  $R$  is continuous and not equal to  $W + c$ ;*
2. *for each public noise scale factor  $\tau < \tau^*$ ,*

(a) *the unconditional probability that  $Y$  lies in  $S_W^\varepsilon$  is at least  $1 - \varepsilon$  and*

(b) if  $Y$  is in  $S_W^\varepsilon$ , then the probability is at least  $1 - \varepsilon^m$  that the state  $\theta$  lies in a closed ball with radius  $\iota$  centered at  $Y$ .

**Proof.** Given  $\varepsilon > 0$ , let  $y_0 = \max\{\Phi^{-1}(\varepsilon/8), \Phi^{-1}(1 - \varepsilon/8)\}$  whence the maximum of  $\Phi(-y_0)$  and  $1 - \Phi(y_0)$  is  $\varepsilon/8$ . Let  $\tau' = \frac{\varepsilon}{8c_3\bar{\phi}}$ . By Lemma 2 and the triangle inequality, for all  $\tau \leq \tau'$ ,

$$\Pr(|Y| > y_0) = \Lambda(-y_0) + 1 - \Lambda(y_0) \leq 2c_3\bar{\phi}\tau^* + \Phi(-y_0) + 1 - \Phi(y_0) \leq \varepsilon/2. \quad (35)$$

In addition, by MSM,  $R$  has a finite number  $n - 1 \geq 0$  of points of discontinuity,  $(\theta_i)_{i=1}^{n-1}$ . Let  $\theta_n = \sup\{\theta : R_\theta \leq W + c\}$  and  $B = \bigcup_{i=1}^n (\theta_i - c_4\varepsilon, \theta_i + c_4\varepsilon)$ . The measure of  $B$  is at most  $2c_4\varepsilon n$ , so by (20),

$$\Pr(Y \in B) \leq 2c_4\varepsilon n\bar{\phi} = \varepsilon/2. \quad (36)$$

Let  $S_W^\varepsilon$  be the intersection of (a) the complement of the set  $B$  and (b) the set  $[-y_0, y_0]$ . Property 1 then holds by construction. (By MSM,  $R_\theta$  equals  $W + c$ , if anywhere, only at  $\theta = \theta_n$ .) Property 2a holds for all  $\tau \leq \tau'$  by (35) and (36). By (15) and (16), on seeing  $Y = y \in S_W^\varepsilon$  the firm's posterior distribution of  $\theta$  is given by

$$\Psi(\theta') = \frac{\int_{\theta''=-\infty}^{\theta'} \phi(\theta'') d \left[ 1 - H\left(\frac{y-\theta''}{\tau}\right) \right]}{\int_{\theta''=-\infty}^{\infty} \phi(\theta'') d \left[ 1 - H\left(\frac{y-\theta''}{\tau}\right) \right]} \quad (37)$$

since  $d \left[ 1 - H\left(\frac{y-\theta''}{\tau}\right) \right] = \tau^{-1} h\left(\frac{y-\theta''}{\tau}\right) d\theta''$ . By (1) and the change of variables  $v = \frac{y-\theta''}{\tau}$ ,

$$\begin{aligned} & \left| \phi(y) - \int_{\theta''=-\infty}^{\infty} \phi(\theta'') d \left[ 1 - H\left(\frac{y-\theta''}{\tau}\right) \right] \right| \\ & \leq \left| \int_{\theta''=-\infty}^{\infty} [\phi(y) - \phi(\theta'')] dH\left(\frac{y-\theta''}{\tau}\right) \right| \\ & \leq k_\phi \int_{\theta''=-\infty}^{\infty} |y - \theta''| d \left[ 1 - H\left(\frac{y-\theta''}{\tau}\right) \right] = c_3 k_\phi \tau, \end{aligned}$$

where  $c_3$  is defined in (21). Hence, the denominator in (37) differs from  $\phi(y)$  by at most  $c_3 k_\phi \tau$ . So for  $\tau < \tau'' = \frac{\phi}{2c_3 k_\phi}$ ,

$$\int_{\theta''=-\infty}^{\infty} \phi(\theta'') d \left[ 1 - H \left( \frac{y - \theta''}{\tau} \right) \right] \geq \frac{\phi}{2} \quad (38)$$

where  $\underline{\phi} = \inf_{y' \in S_W^\varepsilon} \phi(y')$  is positive as  $S_W^\varepsilon$  is compact. The numerator in (37) is at most  $\bar{\phi} \left[ 1 - H \left( \frac{y - \theta'}{\tau} \right) \right]$ . Analogously, if we write  $1 - \Psi(\theta')$  with the same denominator as in (37), the numerator will be at most  $\bar{\phi} H \left( \frac{y - \theta'}{\tau} \right)$ . Thus, for  $\tau < \tau''$  we have  $\Psi(\theta') \leq \frac{2\bar{\phi}}{\underline{\phi}} \left[ 1 - H \left( \frac{y - \theta'}{\tau} \right) \right]$  and  $1 - \Psi(\theta') \leq \frac{2\bar{\phi}}{\underline{\phi}} H \left( \frac{y - \theta'}{\tau} \right)$  by (38). So for any realization  $y \in S_W^\varepsilon$  of  $Y$ , the posterior probability that  $|\theta - y| > \iota$  is  $\Psi(y - \iota) + [1 - \Psi(y + \iota)]$ , which is at most  $\frac{2\bar{\phi}}{\underline{\phi}} \left[ 1 - H \left( \frac{\iota}{\tau} \right) + H \left( -\frac{\iota}{\tau} \right) \right]$ , which goes monotonically to zero as  $\tau$  does as  $\iota > 0$ . Let  $\tau'''$  be the unique  $\tau$  that solves  $\frac{2\bar{\phi}}{\underline{\phi}} \left[ 1 - H \left( \frac{\iota}{\tau} \right) + H \left( -\frac{\iota}{\tau} \right) \right] = \varepsilon^m$ . Property 2b holds for all  $\tau \leq \min \{\tau'', \tau'''\}$ : the result holds for  $\tau^* = \min \{\tau', \tau'', \tau'''\}$ . ■

The firm's realized profits lie in  $[-c - W, p]$  and so are bounded above in absolute value by  $\bar{p}$  by (6). Fix any  $a \in (0, 1)$ ; let  $m = 2$  and

$$\iota = \frac{c_4 \varepsilon^m}{4}. \quad (39)$$

If Theorem 4 holds for some  $\varepsilon$ , then it must hold for all  $\varepsilon' \geq \varepsilon$ , so we may assume w.l.o.g.

$$0 < \varepsilon < \varepsilon^* = \min \left\{ \frac{1}{4}, \left( \frac{1}{2k_4} \right)^{\frac{1}{m-1}}, \left( \frac{c_4}{4\bar{p}} \right)^{\frac{1}{m-1}}, \left( \frac{3c_4}{(4+c_4)\bar{p}+c_4 \max\{c_4, 4k_4+1\}} \right)^{\frac{1}{m-a-1}} \right\}. \quad (40)$$

Let  $\tau < \tau^*$  where  $\tau^*$  depends on  $\varepsilon$ ,  $\iota$ , and  $m$  as specified in Lemma 5. Assume

$$\sigma \leq \sigma^* = \iota (\gamma_\tau \max \{2\bar{\psi}, 1\})^{-1} \quad (41)$$

where  $\gamma_\tau$  is defined in Theorem 3. By Theorem 3, some but not all consumers buy only if  $R_{\theta - \sigma\gamma_\tau} < p < R_{\theta + \sigma\gamma_\tau}$  or, equivalently, only if  $\theta \in [\theta_0 - \sigma\gamma_\tau, \theta_0 + \sigma\gamma_\tau]$  where

$\theta_0 = \sup \{\theta' : R_{\theta'} < p\} = \inf \{\theta' : R_{\theta'} > p\}$  by MSM. Hence by (17) and (41),

$$\Pr(\text{some but not all consumers buy} \mid Y) \leq (2\sigma\gamma_\tau)\bar{\psi} \leq \iota. \quad (42)$$

By Lemma 5,  $\Pr(Y \in S_W^\varepsilon) \geq 1 - \varepsilon$ . Assume  $Y \in S_W^\varepsilon$ . There are two cases; the firm does not know which holds.

1. With probability at least  $1 - \varepsilon^m$ ,  $|\theta - Y| \leq \iota$ . By Theorem 3 and (41) all consumers buy if  $p \leq R_{\theta-\iota}$  and none buy if  $p \geq R_{\theta+\iota}$ . The firm can ensure that all buy by charging  $p = R_{Y-2\iota}$ , which contributes at least  $(1 - \varepsilon^m)(R_{Y-2\iota} - c - W)$  to the firm's payoff if  $R_{Y-2\iota} > c + W$  and at most this amount otherwise. The firm can ensure that no consumers buy by charging  $p = R_{Y+2\iota}$ , which contributes zero to the firm's payoff.
2. With probability at most  $\varepsilon^m$ ,  $|\theta - Y| > \iota$ . What happens in this case raises or lowers the firm's payoff by at most  $\bar{p}\varepsilon^m$ .

We can also partition the event  $Y \in S_W^\varepsilon$  into two cases that the firm *can* distinguish, as follows.

- A. Suppose  $R_Y - c - W > c_4\varepsilon$ . By (40),  $\varepsilon < 1$ , so  $\varepsilon^m < \varepsilon$ . Hence, as  $Y \in S_W^\varepsilon$ ,  $R$  is continuous in  $[Y - c_4\varepsilon^m, Y + c_4\varepsilon^m]$  and thus in  $[Y - 2\iota, Y + 2\iota]$  by (39). So by MSM both  $R_Y - R_{Y-2\iota}$  and  $R_{Y+2\iota} - R_Y$  are at most  $k_4c_4\varepsilon^m/2$  which is less than  $c_4\varepsilon/4$  as  $\varepsilon < (2k_4)^{-\frac{1}{m-1}}$  by (40). Accordingly,

$$R_{Y-2\iota} - c - W > (3/4)c_4\varepsilon. \quad (43)$$

Referring to cases 1 and 2, the firm's payoff  $\Pi_1^A$  in case A from the price  $p = R_{Y-2\iota}$  satisfies

$$\Pi_1^A \geq (1 - \varepsilon^m)(R_{Y-2\iota} - c - W) - \varepsilon^m\bar{p} > (1 - \varepsilon^m)(3/4)c_4\varepsilon - \varepsilon^m\bar{p}. \quad (44)$$



The firm's payoff  $\Pi_3^A$  from a price  $p \in [R_{Y+2\iota}, \bar{p}]$  is at most  $\varepsilon^m \bar{p}$ . Since  $\varepsilon < 1/4$  by (40) and  $m > 1$ ,  $3(1 - \varepsilon^m) > 2$ , so  $\frac{\Pi_1^A - \Pi_3^A}{6(1 - \varepsilon^m)\varepsilon \bar{p}}$  is at least  $\frac{c_4}{8\bar{p}} - \frac{\varepsilon^{m-1}}{3(1 - \varepsilon^m)}$  which is positive as  $\varepsilon < \left(\frac{c_4}{4\bar{p}}\right)^{\frac{1}{m-1}}$  by (40):  $\Pi_1^A > \Pi_3^A$ . If the firm chooses a price in  $(R_{Y-2\iota}, R_{Y+2\iota})$ , let  $\pi$  be the probability that all consumers buy. As shown,  $R_{Y+2\iota} - R_{Y-2\iota} \leq k_4 c_4 \varepsilon^m$ , so the firm's payoff is at most  $\Pi_2^A = \pi (R_{Y-2\iota} - c - W + k_4 c_4 \varepsilon^m) + \bar{p} \iota$  by (42). The firm will choose a price in  $(R_{Y-2\iota}, R_{Y+2\iota})$  only if  $\Pi_2^A \geq \Pi_1^A$  which implies

$$\begin{aligned}
\varepsilon &< \left( \frac{3c_4}{(4 + c_4)\bar{p} + (4k_4 + 1)c_4} \right)^{\frac{1}{m-a-1}} \text{ by (40)} \\
\implies \varepsilon^{a+1-m} &> \varepsilon + \frac{(4 + c_4)\bar{p} + 4k_4 c_4}{3c_4} \text{ as } \varepsilon < 1/4 \text{ by (40)} \\
\implies \varepsilon^m [(4 + c_4)\bar{p} + (1 - \varepsilon^a)4k_4 c_4] &< 3c_4 \varepsilon (1 - \varepsilon^m) - 3c_4 \varepsilon (1 - \varepsilon^a) \\
\implies 1 - \varepsilon^a &< \frac{3c_4 \varepsilon (1 - \varepsilon^m) - \varepsilon^m (4 + c_4)\bar{p}}{3c_4 \varepsilon + 4k_4 c_4 \varepsilon^m} < \pi \text{ by (44), (39), and (43)}.
\end{aligned}$$

So  $\pi \geq 1 - \varepsilon^a$ : for all  $a \in (0, 1)$  and  $\varepsilon < \varepsilon^*$  there is a  $\tau^*$  (given in Lemma 5) such that for all  $\tau < \tau^*$  there is a  $\sigma^*$  (given in (41)) such that for all  $\sigma < \sigma^*$  and all  $Y$  in the subset of  $S_W^\varepsilon$  (defined in Lemma 5) satisfying  $R_Y - c - W > 0$ , the probability of selling to all consumers is at least  $1 - \varepsilon^a$  and the firm chooses a price  $p$  in  $[R_{Y-c_4\varepsilon^m}, R_{Y+c_4\varepsilon^m}]$  which, by the fifth entry in the min in (40), is contained in  $[R_{Y-\varepsilon}, R_{Y+\varepsilon}]$ .

B. Suppose  $R_Y - c - W < -c_4\varepsilon$ , whence  $R_{Y-2\iota} - c - W < -c_4\varepsilon$  by MSM. Referring to cases 1 and 2 above, the firm's payoff  $\Pi_1^B$  in case B from a price  $p \leq R_{Y-2\iota}$  satisfies

$$\Pi_1^B \leq (1 - \varepsilon^m)(R_{Y-2\iota} - c - W) + \varepsilon^m \bar{p} < -(1 - \varepsilon^m)c_4\varepsilon + \varepsilon^m \bar{p}. \quad (45)$$

If the firm chooses a price  $p \in [R_{Y+2\iota}, \bar{p}]$ , it will not sell to anyone unless  $|\theta - Y| > \iota$ , so its payoff  $\Pi_3^B$  is at least  $-\varepsilon^m \bar{p}$ . But  $\Pi_3^B - \Pi_1^B > (1 - \varepsilon^m)c_4\varepsilon/2 - 2\bar{p}\varepsilon^m > 0$  since, by (40),  $\varepsilon < \left(\frac{c_4}{4\bar{p}}\right)^{\frac{1}{m-1}}$ : the firm will not choose a price  $p \leq R_{Y-2\iota}$ . If the firm chooses a price in  $(R_{Y-2\iota}, R_{Y+2\iota})$ , let  $\Pi_2^B$  be the firm's payoff and let  $\pi$  be the probability that some or all consumers buy. All consumers buy with probability at least  $\pi - \iota$  by

(42). As shown in case A,  $R_Y - R_{Y-2\iota}$  and  $R_{Y+2\iota} - R_Y$  are each less than  $c_4\varepsilon/4$ , so  $R_{Y+2\iota} - c - W < -(3/4)c_4\varepsilon$ :  $\Pi_2^B \leq -(3/4)c_4\varepsilon(\pi - \iota) + \bar{p}\iota$  by (42). In order for the firm to choose a price in  $(R_{Y-2\iota}, R_{Y+2\iota})$ , it must be that  $\Pi_2^B \geq \Pi_3^B$  which by (39) and the fact that  $\varepsilon < 1/4$  implies that  $\pi$  is less than  $\varepsilon^{m-1} \frac{(4+c_4)\bar{p}+(c_4)^2}{3c_4}$  which is less than  $\varepsilon^a$  since  $\varepsilon < \left(\frac{3c_4}{(4+c_4)\bar{p}+(c_4)^2}\right)^{\frac{1}{m-a-1}}$  by (40). We have shown that for all  $a \in (0, 1)$  and  $\varepsilon < \varepsilon^*$  there is a  $\tau^*$  such that for all  $\tau < \tau^*$  there is a  $\sigma^*$  such that for all  $\sigma < \sigma^*$  and all  $Y$  in the subset of  $S_W^\varepsilon$  satisfying  $R_Y - c - W < 0$ , the probability of selling to no consumers is at least  $1 - \varepsilon^a$ . Moreover, the firm chooses a price  $p$  in  $[R_{Y-c_4\varepsilon^2}, \bar{p}]$  which, by the fifth entry in the min in (40), is contained in  $[R_{Y-\varepsilon}, \bar{p}]$ . Q.E.D.<sub>Theorem 4</sub>

**Proof of Theorem 5:** Fix an  $a \in (0, 1)$  and  $\varepsilon > 0$ , and let  $\hat{\varepsilon}$  be implicitly given by  $\bar{W}(2\hat{\varepsilon} + \hat{\varepsilon}^a) = \varepsilon$ . By Theorem 4, there is a  $\tau^* \in \left(0, \hat{\varepsilon}(c_3\bar{\phi})^{-1}\right)$  such that for all  $\tau \in (0, \tau^*)$  there is a  $\sigma^* \in (0, \bar{\sigma}]$  such that for all  $\sigma \in (0, \sigma^*)$ , the union's expected payoff  $U(w)$  from any wage schedule  $w$  with wage  $W$  lies in  $[\underline{U}(W), \bar{U}(W)]$  where  $\bar{U}(W) = \hat{\varepsilon}\bar{W} + W\{\hat{\varepsilon}^a + (1 - \hat{\varepsilon}^a)[1 - \Lambda(\theta_W)]\}$  and  $\underline{U}(W) = (1 - \hat{\varepsilon})(1 - \hat{\varepsilon}^a)W[1 - \Lambda(\theta_W)]$ .<sup>37</sup> Hence by Lemma 2,  $|\bar{U}(W) - W[1 - \Lambda(\theta_W)]| < \bar{W}(\hat{\varepsilon} + \hat{\varepsilon}^a)$ ,  $|W[1 - \Lambda(\theta_W)] - \underline{U}(W)| < \bar{W}(\hat{\varepsilon} + \hat{\varepsilon}^a)$ , and  $|W[1 - \Lambda(\theta_W)] - W[1 - \Phi(\theta_W)]| \leq \bar{W}c_3\bar{\phi}\tau < \bar{W}\hat{\varepsilon}$ , so  $|u_\sigma^\tau(w) - W[1 - \Phi(\theta_W)]| < \bar{W}(2\hat{\varepsilon} + \hat{\varepsilon}^a) = \varepsilon$  as claimed. Q.E.D.<sub>Theorem 5</sub>

**Claim 6** *The function  $U$  is continuous, strictly positive on  $W \in (0, \infty)$ , and satisfies  $U(0) = \lim_{W \rightarrow \infty} U(W) = 0$ .*

**Proof of Claim 6:** The products and compositions of continuous functions are continuous. Hence  $U$  is continuous by the following lemma.

**Lemma 6**  *$\theta_W$  is a Lipschitz-continuous function of  $W$  with Lipschitz constant  $1/k_3$ .*

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<sup>37</sup>Why?  $\Pr(Y \notin S) \in [0, \hat{\varepsilon}]$  and the union's payoff if  $Y \notin S$  is in  $[0, \bar{W}]$ . Moreover,  $\Pr(Y \in S) \in [1 - \hat{\varepsilon}, 1]$  and if  $Y \in S$ , the best outcome for the union is that the firm sells to all agents with probability one if  $\theta > \theta_W$  and with probability  $\hat{\varepsilon}^a$  otherwise. The union's worst outcome is that the firm sells to all agents with probability  $1 - \hat{\varepsilon}^a$  if  $\theta > \theta_W$  and with probability zero otherwise. The equations for  $\underline{U}$  and  $\bar{U}$  follow.

**Proof.** For any  $W'' > W'$ , let  $\theta' = \theta_{W'}$  and  $\theta'' = \theta_{W''}$ . For any  $\varepsilon > 0$ ,  $R_{\theta'+\varepsilon} \geq c + W'$  as  $\theta' = \sup \{\theta : R_\theta < c + W'\}$  and  $R_{\theta''-\varepsilon} < c + W''$  as  $\theta'' = \inf \{\theta : R_\theta > c + W''\}$ . Hence, by MSM,  $W'' - W' \geq R_{\theta''-\varepsilon} - R_{\theta'+\varepsilon} \geq k_3 (\theta'' - \theta' + 2\varepsilon)$ . Since this holds for all  $\varepsilon > 0$ , it must also hold in the limit as  $\varepsilon \rightarrow 0$  as  $W'$ ,  $W''$ ,  $\theta'$ , and  $\theta''$  do not depend on  $\varepsilon$ :  $\theta'' - \theta' \leq (1/k_3)(W'' - W')$  as claimed. ■

**Lemma 7** *There are finite constants  $W_0 \geq 0$  and  $c_5$  such that for  $W > W_0$ ,  $\theta_W \geq c_5 + W/k_4$ .*

**Proof.** By MSM,  $R$  has a finite number of discontinuities. Let  $\bar{\theta}$  be the highest one; if  $R$  has no discontinuities, let  $\bar{\theta} = -\infty$ . Since by MSM  $R$  is increasing at at least the rate  $k_3$  and, for  $\theta > \bar{\theta}$ , is Lipschitz with constant  $k_4$ ,  $R_\theta - c$  has a continuous inverse on  $\theta \in (\bar{\theta}, \infty)$  and this inverse is strictly increasing at at least the rate  $1/k_4$ . But if  $R$  is continuous at  $\theta = \theta_W$ , then  $R_{\theta_W} - c = W$  by (14): the inverse of  $R_\theta - c$  on  $\theta \in (\bar{\theta}, \infty)$  is  $\theta_W$ . Let  $W_0$  equal the maximum of  $R_{\bar{\theta}+1} - c$  and zero; if  $\bar{\theta} = -\infty$ , let  $W_0 = 0$ . For  $W \geq W_0$ ,  $\theta_W \geq \theta_{W_0} + (W - W_0)/k_4$ . Finally, let  $c_5 = \theta_{W_0} - W_0/k_4$ . ■

By Lemma 7, letting  $W' = c_5 + W/k_4$ , the limit  $\lim_{W \rightarrow \infty} U(W)$  is at most

$$\lim_{W \rightarrow \infty} \{W [1 - \Phi(c_5 + W/k_4)]\} = k_4 \lim_{W' \rightarrow \infty} \{W' [1 - \Phi(W')]\} - c_5 \lim_{W \rightarrow \infty} [1 - \Phi(c_5 + W/k_4)],$$

which equals zero by part 3 of Lemma 1. Trivially,  $U(0) = 0$  as well. Q.E.D.<sub>Claim 6</sub>

**Proof of Theorem 6:** Fix  $\varepsilon > 0$ . Since  $W_0$  strictly maximizes  $U$  and, by Claim 6,  $U$  is continuous and  $\lim_{W \rightarrow \infty} U(W) = 0$ , it follows that there is a  $W_1 > W_0 + 2\delta$  such that  $U(W) < U(W_0 + 2\delta)$  for all  $W > W_1$ . Let  $S_0$  be the set of wages  $W \in \mathfrak{R}_+$  satisfying  $|W - W_0| \geq \varepsilon$  and let  $S_1 = S_0 \cap [0, W_1]$ . As  $S_1$  is compact,  $U$  attains a maximum on  $S_1$  at some  $W_2 \in S_1$ . By definition of  $S_1$ ,  $W_2$  also maximizes  $U$  on  $S_0$ . Let  $\iota = [U(W_0) - U(W_2)]/2 > 0$ : for all  $W$  in  $S_0$ ,  $U(W) < U(W_0) - \iota$ . Let  $w_0$  be any wage schedule with a wage of  $W_0$ . By Theorem 5, there is a  $\tau^* > 0$  such that for all  $\tau \in (0, \tau^*)$  there is a  $\sigma^* \in (0, \bar{\sigma}]$  such that for all  $\sigma \in (0, \sigma^*)$ ,  $|u_\sigma^\tau(w_\sigma^\tau) - U(w_\sigma^\tau(\bar{L}))|$  and  $|u_\sigma^\tau(w_0) - U(W_0)|$  are both less than  $\iota/2$ ; if  $w_\sigma^\tau(\bar{L})$  is in  $S_0$  then  $U(w_\sigma^\tau(\bar{L})) < U(W_0) - \iota$

so  $u_\sigma^\tau(w_\sigma^\tau) < U(w_\sigma^\tau(\bar{L})) + \iota/2 < U(W_0) - \iota/2 < u_\sigma^\tau(w_0)$ : the union prefers  $w_0$  to  $w_\sigma^\tau$ , which is absurd. It follows that for any such  $\sigma$  and  $\tau$ ,  $w_\sigma^\tau(\bar{L}) \notin S_0$  as claimed. Q.E.D.<sub>Theorem 6</sub>

**Proof of Claim 4:** Fix a wage  $W$  and let  $\theta = \theta_W$  be the resulting purchase threshold. Fix the state  $\theta$  and let  $W$  vary within the interval  $[R_\theta^- - c, R_\theta^+ - c]$ . (If  $R$  is continuous at  $\theta$ , this interval consists of a single point.) By MSM and (14),  $\theta_W$  equals the fixed state  $\theta$  for all such wages  $W$ . So by (9), the union's payoff  $U$  is strictly increasing in  $W \in [R_\theta^- - c, R_\theta^+ - c]$ : if the wage  $W$  that maximizes  $U$  lies in this interval, it must equal  $R_\theta^+ - c$ . Any wage  $W$  that maximizes  $U$  must then be of the form  $R_\theta^+ - c$  where  $\theta$  is the purchase threshold  $\theta_W$ . In this case,  $\theta$  must also maximize  $V$ . For suppose not. Then there is a  $\theta'$  such that  $V(\theta') > V(\theta)$ . Let  $W'$  be the wage  $R_{\theta'}^+ - c$ , which satisfies  $\theta_{W'} = \theta'$ . Thus,  $U(W') = V(\theta') > V(\theta) = U(W)$ , a contradiction. Now suppose that  $\theta$  maximizes  $V$ , but the wage  $W = R_\theta^+ - c$  does not maximize  $U$ : there is another wage  $W'$  that does. By the preceding reasoning,  $W'$  must be of the form  $R_{\theta'}^+ - c$  where  $\theta' = \theta_{W'}$ , whence  $V(\theta') = U(W') > U(W) = V(\theta)$ , a contradiction. Q.E.D.<sub>Claim 4</sub>

**Proof of Claim 5:**

1. Let  $W' = R_{\theta'}^+ - c$  and  $W'' = R_{\theta''}^+ - c$ . By MSM and (14),  $\theta_{W'} = \theta'$  and  $\theta_{W''} = \theta''$ . By assumption and (8) and (9),  $0 < k \stackrel{d}{=} V(\theta'') - V(\theta') = U(W'') - U(W')$ . By MSM and (9),  $V$  is right-continuous and cannot jump downwards. Thus, there is a  $\delta > 0$  such that for any  $\theta$  in  $(\theta' - \delta, \theta' + \delta)$ ,  $V(\theta) \leq V(\theta') + k/2$ . Let  $w''$  be any wage schedule in  $\Omega$  satisfying  $w''(\bar{L}) = W''$ . By the optimality of  $w_\sigma^\tau$  in the PBE  $E_\sigma^\tau$ ,

$$\begin{aligned} 0 &\geq u_\sigma^\tau(w'') - u_\sigma^\tau(w_\sigma^\tau) \\ &= k + [u_\sigma^\tau(w'') - U(W'')] + [U(W') - U(w_\sigma^\tau(\bar{L}))] + [U(w_\sigma^\tau(\bar{L})) - u_\sigma^\tau(w_\sigma^\tau)]. \end{aligned}$$

By Theorem 6,  $u_\sigma^\tau(w'') - U(W'') \rightrightarrows 0$  and  $U(w_\sigma^\tau(\bar{L})) - u_\sigma^\tau(w_\sigma^\tau) \rightrightarrows 0$ . Hence for any  $\varepsilon > 0$  there is a  $\tau^* > 0$  such that for all  $\tau \in (0, \tau^*)$  there is a  $\sigma^* \in (0, \bar{\sigma}]$  such that for all  $\sigma \in (0, \sigma^*)$ ,  $U(w_\sigma^\tau(\bar{L})) - U(W') > k/2$  and thus  $V(\theta_{w_\sigma^\tau(\bar{L})}) - V(\theta') > k/2$

as  $U(w_\sigma^\tau(\bar{L})) \leq V(\theta_{w_\sigma^\tau(\bar{L})})$  and  $U(W') = V(\theta')$ , whence  $|\theta_{w_\sigma^\tau(\bar{L})} - \theta'| > \delta$ ; thus,  $\neg(\theta_{w_\sigma^\tau(\bar{L})} \rightrightarrows \theta')$ .

2. Since  $\hat{\theta}$  uniquely maximizes  $V$ , the wage  $W_0 = R_\theta^+ - c$  uniquely maximizes  $U$  by Claim

4. Thus, by Theorem 6,  $w_\sigma^\tau(\bar{L}) \rightrightarrows W_0$ , so by Lemma 6,  $\theta_{w_\sigma^\tau(\bar{L})} \rightrightarrows \hat{\theta}$ . Q.E.D. Claim 5

**Proof of Theorem 7:** Since  $k'_3$  is arbitrarily small, assume w.l.o.g. that it lies in  $(0, k_3)$ .

Let us write  $t_\theta^\ell = \alpha_\theta (r_\theta^1 - r_\theta^\ell)$  where  $\alpha_\theta = \frac{T_\theta}{r_\theta^1 - R_\theta}$ .

**Lemma 8** *Assume  $r$  satisfies PMC and SM. For  $\theta$  in  $[\theta^*, \omega(k'_3)]$ , the weight  $\alpha_\theta$  lies in  $[0, 1]$ .*

**Proof.** It is nonnegative as  $T_\theta \geq 0$  and, by PMC,  $r_\theta^1 > R_\theta$ . By SM,  $r_\theta^1 \geq r_{\theta^*}^1 + k_3(\theta - \theta^*)$  for  $\theta > \theta^*$ , so since  $k'_3 < k_3$ ,  $T_\theta + R_\theta = r_{\theta^*}^1 + k'_3(\theta - \theta^*) \leq r_\theta^1$ , whence  $\alpha_\theta \leq 1$ . ■

**Lemma 9** *Assume  $r$  satisfies AM, SM, and PMC. Then  $\tilde{r} = r + t$  satisfies AM, XSM, OSL, and the version of MSM in which  $k_3$  is replaced by  $k'_3$ .*

**Proof.** Let  $\tilde{R}_\theta = R_\theta + T_\theta = \int_{\ell=0}^1 \tilde{r}_\theta^\ell d\ell$ . The axioms hold for  $\theta$  not in  $[\theta^*, \omega(k'_3)]$  by Claim 2. By Lemma 8,  $\tilde{r}_\theta^\ell = r_\theta^\ell + t_\theta^\ell$  is nondecreasing in the purchase rate  $\ell$  as  $r_\theta^\ell$  is. Moreover,  $\tilde{r}_\theta^1 - \tilde{r}_\theta^\ell = (1 - \alpha_\theta)(r_\theta^1 - r_\theta^\ell)$ , so as  $r$  satisfies AM,  $\tilde{r}_\theta^1 - \tilde{r}_\theta^0 = (1 - \alpha_\theta)(r_\theta^1 - r_\theta^0) \leq k_1 \min\{1 - \alpha_\theta, \tilde{r}_\theta^1 - \tilde{R}_\theta\}$ :  $\tilde{r}$  satisfies AM. XSM holds as  $r^1 = \tilde{r}^1$ . As  $t^\ell$  and thus  $\tilde{r}^\ell$  jumps upwards at  $\theta = \theta^*$ , it suffices to check OSL for  $\theta^* \leq \theta < \theta' \leq \omega(k'_3)$ . We have  $T_\theta \leq T_{\theta^*} = r_{\theta^*}^1 - R_{\theta^*} \leq k_1$  by AM and, by SM and (12),  $|T_{\theta'} - T_\theta| \leq (k'_3 + k_4)|\theta' - \theta|$ , so by SM and PMC,

$$\begin{aligned} |\alpha_{\theta'} - \alpha_\theta| &\leq \left| \frac{T_{\theta'} - T_\theta}{r_{\theta'}^1 - R_{\theta'}} \right| + T_\theta \left| \frac{r_\theta^1 - R_\theta - (r_{\theta'}^1 - R_{\theta'})}{(r_{\theta'}^1 - R_{\theta'})(r_\theta^1 - R_\theta)} \right| \\ &\leq \left( \frac{k'_3 + k_4}{k_2} + k_1 \frac{k_4 - k_3}{k_2^2} \right) (\theta' - \theta) \end{aligned}$$

whence by the triangle inequality and SM,

$$|\tilde{r}_{\theta'}^\ell - \tilde{r}_\theta^\ell| \leq |\alpha_{\theta'} - \alpha_\theta| (r_{\theta'}^1 - r_{\theta'}^\ell) + \alpha_\theta |(r_{\theta'}^1 - r_{\theta'}^\ell) - (r_\theta^1 - r_\theta^\ell)| + |r_{\theta'}^\ell - r_\theta^\ell|$$

which is at most  $k_5(\theta' - \theta)$  where  $k_5 = \frac{k'_3 + k_4}{k_2}k_1 + (k_4 - k_3)\left(\frac{k_1^2}{k_2^2} + 1\right) + k_4$ :  $\tilde{r}$  satisfies OSL. As for MSM, by construction  $\tilde{R}$  rises at a rate between  $k_3$  and  $k_4$  at states below  $\theta^*$  and above  $\omega(k'_3)$ ; jumps upwards at  $\theta^*$ ; and rises at the rate  $k'_3 \in (0, k_3)$  at states in  $[\theta^*, \omega(k'_3)]$ . Since it has only one point of discontinuity, it satisfies the version of MSM in which  $k_3$  is replaced by  $k'_3$ . ■

By Lemma 9, Theorems 3-6 and Claims 4-5 remain valid in the presence of our insurance scheme, where  $R$  in those results is replaced by the augmented mean relative payoff function  $\tilde{R}_\theta = \int_{\ell=0}^1 r_\theta^\ell d\ell$ . Hence, by Claim 5, in the limit as  $\sigma$  and then  $\tau$  goes to zero the union chooses a purchase threshold  $\theta$  that maximizes the augmented limiting payoff function

$$\tilde{V}(\theta) = \left(\tilde{R}_\theta^+ - c\right) [1 - \Phi(\theta)] \quad (46)$$

if there is a unique such maximizer. Moreover, the union cannot choose a threshold that is not a weak maximizer of  $\tilde{V}$ .

First suppose  $\Gamma(\theta^*, \hat{\theta}) > 0$ . Fix some  $k'_3 \in (0, k_3)$ . By (12) and (46),  $\tilde{V}(\theta^*)$  is  $(r_{\theta^*}^1 - c)[1 - \Phi(\theta^*)]$ , which exceeds  $(R_{\hat{\theta}} - c)[1 - \Phi(\hat{\theta})]$ , which equals  $V(\hat{\theta})$  by (9). Hence, for all  $\theta$  in the set  $\mathfrak{R} \setminus [\theta^*, \omega(k'_3)]$  of states at which no transfers are given,  $\tilde{V}(\theta^*) > V(\hat{\theta}) \geq V(\theta) = \tilde{V}(\theta)$  as  $\hat{\theta}$  strictly maximizes  $V$ . By (12) and (46), for  $\theta$  in  $(\theta^*, \omega(k'_3))$ ,

$$\begin{aligned} \tilde{V}(\theta^*) - \tilde{V}(\theta) &= (r_{\theta^*}^1 - c)[1 - \Phi(\theta^*)] - (r_{\theta^*}^1 + k'_3(\theta - \theta^*) - c)[1 - \Phi(\theta)] \\ &= (r_{\theta^*}^1 - c)[\Phi(\theta) - \Phi(\theta^*)] - k'_3(\theta - \theta^*)[1 - \Phi(\theta)]. \end{aligned}$$

The state  $\theta = \omega(k'_3)$  is defined by  $R_\theta - k'_3(\theta - \theta^*) = r_{\theta^*}^1$ . The left hand side is decreasing in  $k'_3$  (as  $\omega(k'_3) > \theta^*$ ) and increasing in  $\theta$  (by SM, as  $k'_3 < k_3$ ). Thus,  $\omega(k'_3)$  is increasing in  $k'_3$ , so  $\omega(k'_3) < \omega(k_3)$ . Let  $\underline{\phi}$  be a positive lower bound on  $\phi(\theta)$  over  $\theta \in [\theta^*, \omega(k_3)]$ , whence  $\Phi(\theta) - \Phi(\theta^*) \geq (\theta - \theta^*)\underline{\phi}$ . Thus, as  $1 - \Phi(\theta) < 1 - \Phi(\theta^*)$ , a sufficient condition for  $\tilde{V}(\theta^*) > \tilde{V}(\theta)$  for all  $\theta$  in  $(\theta^*, \omega(k'_3))$  is that  $k'_3 \leq \underline{\phi} \frac{r_{\theta^*}^1 - c}{1 - \Phi(\theta^*)}$  which lies in  $(0, \infty)$  by (4). Hence, for the constant  $k'_3 = \min\left\{\frac{k_3}{2}, \underline{\phi} \frac{r_{\theta^*}^1 - c}{1 - \Phi(\theta^*)}\right\} \in (0, k_3)$ , the augmented limiting payoff function  $\tilde{V}$  that results from PCSS  $t$  is uniquely maximized at the socially optimal threshold

$\theta^*$ .

Now suppose  $\Gamma(\theta^*, \hat{\theta}) < 0$ . Since  $t_\theta^1 = 0$ ,  $\tilde{r}_\theta^1 = r_\theta^1$ , so  $\tilde{r}_\theta^\ell \leq r_\theta^1$  by AM. And since transfers are nonnegative,  $\tilde{r}_\theta^\ell \geq r_\theta^\ell$ . It follows that  $\tilde{R}_\theta$  lies in  $[R_\theta, r_\theta^1]$ . Thus,  $\tilde{V}(\theta^*) \leq (r_\theta^1 - c)[1 - \Phi(\theta)]$  and  $\tilde{V}(\hat{\theta}) \geq (R_\theta^+ - c)[1 - \Phi(\theta)]$ . It follows that  $\tilde{V}(\theta^*) - \tilde{V}(\hat{\theta}) \leq \Gamma(\theta^*, \hat{\theta}) < 0$ , so by Lemma 9 and part 1 of Claim 5,  $\neg(\theta_{w\bar{z}(\bar{L})} \Rightarrow \theta^*)$ . Q.E.D.<sub>Theorem 7</sub>

**Proof of Theorem 8.** For any  $k'_3 \in (0, k_3)$  and  $\theta_0 \geq \omega(k'_3)$ , we will construct a PRNS  $\hat{t}$  with mean transfer  $\hat{T}_\theta$  equal to  $r_{\theta^*}^1 - R_\theta + k'_3(\theta - \theta^*)$  if  $\theta \in [\theta^*, \theta_0]$  and zero otherwise. Hence  $\hat{T}_\theta$  coincides with the mean subsidy  $T_\theta$  defined in equation (12) except on  $\theta \in (\omega(k'_3), \theta_0]$ , where  $T_\theta$  is zero but  $\hat{T}_\theta$  is negative. Let the scheme  $\hat{t}_\theta^\ell$  coincide with the PCSS  $t_\theta^\ell$  defined in (13) except on  $\theta \in (\omega(k'_3), \theta_0]$  where  $t_\theta^\ell$  is zero but  $\hat{t}_\theta^\ell$  is the negative amount  $2(1 - \ell)T_\theta$  (a tax). Clearly,  $\hat{T}_\theta = \int_{\ell=0}^1 \hat{t}_\theta^\ell d\ell$  and property (b') of PRNS holds. It remains to verify property (a):

**Lemma 10** *Assume  $r$  satisfies AM, SM, and PMC. Then  $\hat{r} = r + \hat{t}$  satisfies AM (with  $k_1$  replaced by  $k'_1 = k_1 + 2(r_{\theta^*}^1 - R_{\theta_0})$ ), XSM, OSL, and MSM (with  $k_3$  replaced by  $k'_3$ ).*

**Proof.** By Lemma 9, it suffices to check the axioms for  $\theta \in [\omega(k'_3), \theta_0 + \varepsilon)$  for some small  $\varepsilon > 0$ . For such  $\theta$ ,  $\hat{T}_\theta \leq 0$ , so  $\hat{r}_\theta^\ell = r_\theta^\ell + 2(1 - \ell)\hat{T}_\theta$  is nondecreasing in  $\ell$  as  $r_\theta^\ell$  is. Also for such  $\theta$ ,  $\hat{r}_\theta^1 - \hat{r}_\theta^0 = r_\theta^1 - r_\theta^0 - 2\hat{T}_\theta \leq k'_1$ :  $\hat{r}$  satisfies AM with  $k'_1$  replacing  $k_1$ . XSM holds as  $r^1 = \hat{r}^1$ . As  $\hat{t}_\theta^\ell$  and thus  $\hat{r}_\theta^\ell$  jumps upwards at  $\theta = \theta_0$ , it suffices to check OSL for  $\omega(k'_3) \leq \theta < \theta' \leq \theta_0$ , where  $|\hat{r}_{\theta'}^\ell - \hat{r}_\theta^\ell| \leq |r_{\theta'}^\ell - r_\theta^\ell| + 2(1 - \ell)|\hat{T}_{\theta'} - \hat{T}_\theta| \leq (3k_4 + 2k'_3)(\theta' - \theta)$  by the triangle inequality and SM:  $\hat{r}$  satisfies OSL with  $k_5 = 3k_4 + 2k'_3$ . As for MSM, by construction  $\hat{R}$  rises at a rate between  $k_3$  and  $k_4$  at states below  $\theta^*$  and above  $\theta_0$ ; jumps upwards at  $\theta^*$  and at  $\theta_0$ ; and rises at the rate  $k'_3 \in (0, k_3)$  at states in  $(\theta^*, \theta_0)$ . Since it has only two points of discontinuity, it satisfies the version of MSM in which  $k_3$  is replaced by  $k'_3$ . ■

Suppose the policymaker commits to the PRNS  $\hat{t}$ . By Theorem 3, the consumers' reservation price at the state  $\theta$  is now given by the augmented mean relative payoff function

$\widehat{R}_\theta = R_\theta + \widehat{T}_\theta$ . By Lemma 10, Theorems 3-6 and Claims 4-5 remain valid in the presence of  $\widehat{t}$ , where  $R$  in those results is replaced by the augmented mean relative payoff function  $\widehat{R}$ . Hence, by Claim 5, in the limit as  $\sigma$  and then  $\tau$  goes to zero the union chooses a purchase threshold  $\theta$  that maximizes the augmented limiting payoff function  $\widehat{V}(\theta) = (\widehat{R}_\theta^+ - c)[1 - \Phi(\theta)]$  if there is a unique such maximizer.

We now produce a  $k'_3 \in (0, k_3)$  and  $\theta_0 \geq \omega(k'_3)$  for which  $\widehat{V}$  is uniquely maximized at the socially optimal threshold  $\theta^*$ . Fix some  $k'_3 \in (0, k_3)$ . By (4),  $(r_{\theta^*}^1 - c)[1 - \Phi(\theta^*)] > 0$ . By SM,  $R_\theta \leq R_0 + k_4\theta$ , so by (9),  $V(\theta) \leq (R_0 - c)[1 - \Phi(\theta)] + k_4\theta[1 - \Phi(\theta)]$  which goes to zero as  $\theta \rightarrow \infty$  by part 3 of Lemma 1. Thus there is a  $\theta_0 \geq \omega(k_3)$  (which exceeds  $\omega(k'_3)$  as  $t(\cdot)$  is increasing) such that, for all  $\theta \geq \theta_0$ ,  $\Gamma(\theta^*, \theta) > 0$ . By definition of  $\widehat{T}_\theta$ ,  $\widehat{V}(\theta^*) = (r_{\theta^*}^1 - c)[1 - \Phi(\theta^*)]$  so, for all  $\theta > \theta_0$ ,  $\widehat{V}(\theta^*) > (R_\theta - c)[1 - \Phi(\theta)] = \widehat{V}(\theta)$ . As  $\theta^* < \widehat{\theta}$ ,  $m_\theta < c$  for all  $\theta \leq \theta^*$  by IMR and hence  $\widehat{V}(\theta) < \widehat{V}(\theta^*)$  for all  $\theta \leq \theta^*$ . By definition of  $\widehat{T}$ , for  $\theta$  in  $(\theta^*, \theta_0)$ ,  $\widehat{V}(\theta^*) - \widehat{V}(\theta) = (r_{\theta^*}^1 - c)[\Phi(\theta) - \Phi(\theta^*)] - k'_3(\theta - \theta^*)[1 - \Phi(\theta)]$ . Let  $\underline{\phi}$  be a positive lower bound on  $\phi(\theta)$  over  $\theta \in [\theta^*, \theta_0]$ , whence  $\Phi(\theta) - \Phi(\theta^*) \geq (\theta - \theta^*)\underline{\phi}$ . Thus, as  $1 - \Phi(\theta) < 1 - \Phi(\theta^*)$ , a sufficient condition for  $\widehat{V}(\theta^*) > \widehat{V}(\theta)$  for all  $\theta$  in  $(\theta^*, \theta_0)$  is that  $k'_3 \leq \underline{\phi} \frac{r_{\theta^*}^1 - c}{1 - \Phi(\theta^*)}$  which lies in  $(0, \infty)$  by (4). Hence we can let  $k'_3 = \min \left\{ \frac{k_3}{2}, \underline{\phi} \frac{r_{\theta^*}^1 - c}{1 - \Phi(\theta^*)} \right\}$ .

Q.E.D. Theorem 8

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