5-2001

The Impact of Surface Errors on the Location and Orientation of a Cylindrical Workpiece in a Fixture

Supapan Sangnui
Iowa State University

Frank E. Peters
Iowa State University, fpeters@iastate.edu

The Impact of Surface Errors on the Location and Orientation of a Cylindrical Workpiece in a Fixture

Abstract

Surface error at the contact region is one factor that contributes to the displacement of the workpiece in a fixture. This work employs the Newton-Raphson technique to develop a mathematical model that predicts the impact of surface errors on the location and orientation of a cylindrical workpiece. Model validation and measurement analysis were conducted to verify the proposed mathematical model. An experimental fixture and workpiece were developed to simulate the effect of surface errors on the placement of a workpiece. The study showed that the numerical model is reliable in predicting the behavior of the workpiece.

Keywords

Newton-Raphson technique

Disciplines

Industrial Engineering | Systems Engineering

Comments

This article is from *Journal of Manufacturing Science and Engineering, Transactions of the ASME* 123 (2001): 325, doi: 10.1115/1.1367269. Posted with permission.

Supapan Sangnui
Graduate Research Assistant

Frank Peters[1]
Assistant Professor

Industrial and Manufacturing
Systems Engineering,
Iowa State University,
2019 Black Engineering Building,
Ames, IA 50011-2164

The Impact of Surface Errors on the Location and Orientation of a Cylindrical Workpiece in a Fixture

Surface error at the contact region is one factor that contributes to the displacement of the workpiece in a fixture. This work employs the Newton-Raphson technique to develop a mathematical model that predicts the impact of surface errors on the location and orientation of a cylindrical workpiece. Model validation and measurement analysis were conducted to verify the proposed mathematical model. An experimental fixture and workpiece were developed to simulate the effect of surface errors on the placement of a workpiece. The study showed that the numerical model is reliable in predicting the behavior of the workpiece. [DOI: 10.1115/1.1367269]

1 Introduction

Proper design of fixtures to hold workpiece is essential to produce components that satisfy part specifications. The devices should locate the workpiece accurately, maintain its location precisely during the operation, and support the part, if needed. For machining, the objective of the fixture is to locate the workpiece relative to the cutter to enable the creation of dimensional accurate components. When fixturing castings, forgings, and rough machined parts, the surface variability may cause the component to be deviated from the desired position. Accommodating this error should be considered to achieve better fixture designs, efficient process plans, and help designers choose appropriate tolerances.

The displacement of the workpiece from the desired (nominal) position could occur when it is being located on a fixture or during machining as shown in Fig. 1. Prior to machining, the workpiece could be deviated from the theoretically nominal position by the inaccuracy of locator positions, the workpiece irregularities, clamp force and gravitational force [1–11]. Previous researchers have studied workpiece displacement, but little consideration has been given to the effect of the workpiece surface errors.

In this study, the term *surface error* refers to any irregularities on the surface of the workpiece. The severity of the surface error typically depends on the manufacturing processes. This paper studies the effect of surface errors, where the workpiece contacts the five spherical locators, on the location and orientation of a cylindrical workpiece. The Newton-Raphson method was employed to formulate a numerical model that was created to predict the impact of surface errors on location and orientation. The model was implemented with Matlab programming. A coordinate measuring machine and a special fixture, which simulated variable surface errors, were used to validate the model.

Dimensional accuracy of a workpiece largely depends upon its relative position to the cutting tool. The relationship between the coordinate system of the workpiece and that of the tool must be carefully defined. Datum targets are locations on the workpiece selected to contact the fixture. The datum targets will form a datum which is used as a reference to locate a feature on a product during manufacturing or inspection. The number of datum targets required varies by the locating method, and the workpiece configuration [12].

Most research conducted in fixturing focuses on developing a method to secure a workpiece in a fixture. King and Hutter [8] proposed a tool to generate optimal fixturing locations for prismatic workpieces in automated assembly. Objective functions for rotational and translational slippage, expressed in terms of fixture element stiffness, are introduced and analyzed. Trappey and Matrubhutam [11] presented algorithms to determine the fixturing locations of a nonprismatic part. Orientation and geometry of a workpiece, including magnitudes and directions of cutting forces are considered. The application of projective geometry introduced in this paper simplifies the fixture configuration problem. Optimal sheet metal fixturing layout design algorithm is developed by Cai et al. [1] to obtain minimum deformation of the deformable sheet metal. An N-2-1 locating principle is introduced and verified as compared to 3-2-1 principle.

The role of forces at contact regions between the fixture and the workpiece has been considered in several papers. De Meter [3] analyzed the placement of the workpiece in terms of friction. Workpiece restraints can be theoretically classified into two types, form closed restraint and force closed restraint. The workpiece is force closed when it adopts the aid of friction to counteract the wrenches of the contact region. Such friction is caused from the clamping forces which act as preloads on the workpiece surface. De Meter [5] formulated the model to predict the impact of locator and clamp placement on the workpiece displacement during machining. The author adopted the scheme of coulombic friction, which assumed that the workpiece and the fixture are rigid bodies. They also proposed techniques used to determine workpiece displacement during clamping [6]. Hockenberger and De Meter [7]

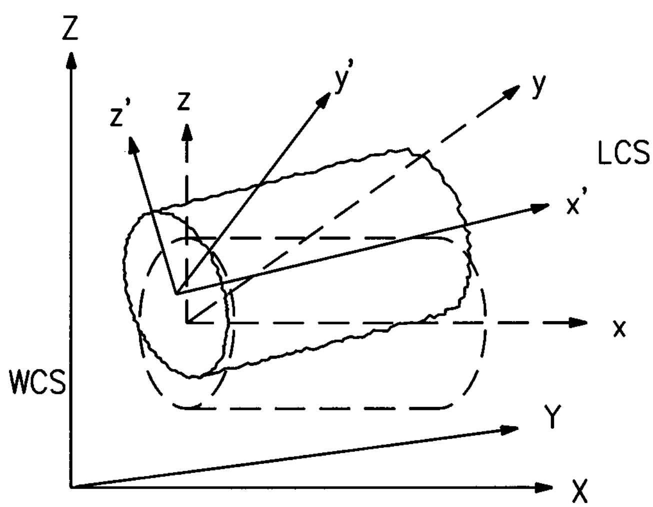

Fig. 1 The deviation of the workpiece from the nominal position

[1]Corresponding author.

Contributed by the Manufacturing Engineering Division for publication in the JOURNAL OF MANUFACTURING SCIENCE AND ENGINEERING. Manuscript received Oct. 1999; revised Aug. 2000. Associate Editor: E. C. DeMeter.

applied quasi-static analysis and meta modeling to predict the impact of fixture design parameters on prismatic workpiece displacement.

Rong and Bai [9] analyzed the dependent relationship of operational dimensions to estimate machining errors in terms of linear and angular dimensions. Machining errors are divided, based on an analysis of machining processes, into deterministic and random components. The deterministic machining errors are caused by locating errors of the fixture, position error of the fixture, locating component and datum variation of the workpiece. Random errors are determined by clamping deformation, cutting force and thermal deformation.

Cai et al. [2] developed a variational method to conduct robust fixture design to minimize the workpiece positional errors caused by workpiece surface and fixture set-up errors. They showed that when the rank of the Jacobian of the constraint equations equals to the degrees of freedom of the workpiece, the deterministic locating condition will be achieved. Salisbury and Peters [10] proposed a model to predict the worst case final workpiece location and orientation errors, which are based on the maximum possible surface errors at each datum locator. This model calculates the displacement in step-wise approach for 3-2-1 fixturing method for prismatic parts.

2 Fixturing Model

A numerical model was constructed to predict the displacement of a workpiece from its nominal position due to surface errors where the locators contact the workpiece. Before machining, all twelve degrees of freedom must be restrained through a combination of locators and clamps. In this study, five locators were used to restrict eight degrees of freedom as shown in Fig. 2. Four of the locators restrict translation along $-z$, y and $-y$ directions and rotations about the z and y axes. The fifth locator, at the face end of the workpiece, prohibits a translation along $-x$ axis. The remaining degrees of freedom need to be eliminated by clamps. Throughout this paper, the locators are referred as P_1-P_5. P_5 is on the face end while P_1 and P_3 are on the end nearest P_5. P_1 and P_2 are on the same side of the cylinder.

To determine the position of the workpiece relative to the fixture and cutting tool, the relationship between the coordinate system of the workpiece and that of the fixture must be established. In this model, a coordinate system of the fixture is defined as a world coordinate system (*WCS*), while a local coordinate system (*LCS*) belongs to the coordinate system of the workpiece. By utilizing geometric transformations, these two coordinate systems can be transformed to each other and objects in one system can be transformed to a different coordinate system. The transformation applied in this model is point-based, since the basic element of all rigid bodies is a point and the distances between these points remain constant during the transformations.

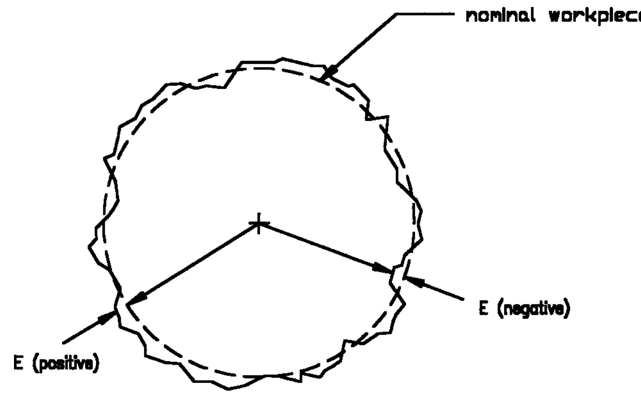

Fig. 3 Examples of surface errors

The location and orientation of a cylindrical part in space will be defined by its origin and the direction cosine of its axis. The origin of the workpiece is the center point of the cylinder's face end. The workpiece's origin and axis are coincidental with the origin and x axis of the *LCS*, respectively. The surface model depicted in Fig. 3 is employed to formulate the boundary equations. The surface error is measured radially from the nominal surface, except for the fifth locator. The error is assumed to be positive when the actual surface is located beyond the nominal surface. Similarly, a negative error exists where the workpiece is undersized as shown in Fig. 3. The assumptions are described in Eq. (1).

$$r_i = r + E_i \tag{1}$$

where $i = 1,2,..4$. The goal of this model is to predict the displacement of the workpiece caused by the surface errors during the initial placement of the part. Therefore, liftoff during clamping is not considered, however it could be included if the values were known.

When the workpiece makes contact with all five locators, the shortest distance from $x'_{ci}, y'_{ci}, z'_{ci}$ to the workpiece is equal to $R + E_i$.

$$x'_{c5} + R + E_5 = 0 \tag{2}$$

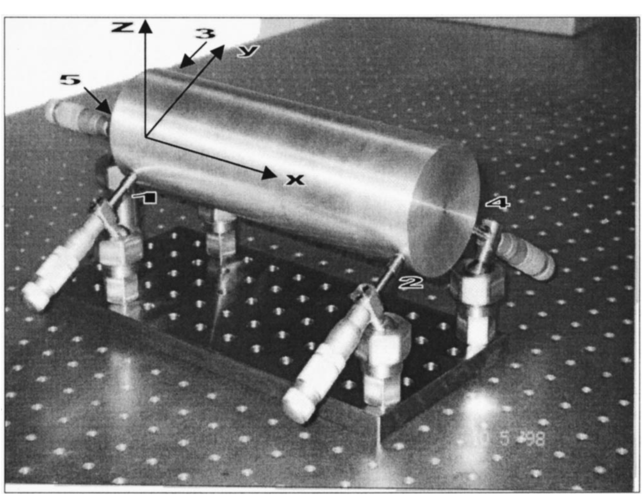

Fig. 2 The positions of the datum targets on the cylinder

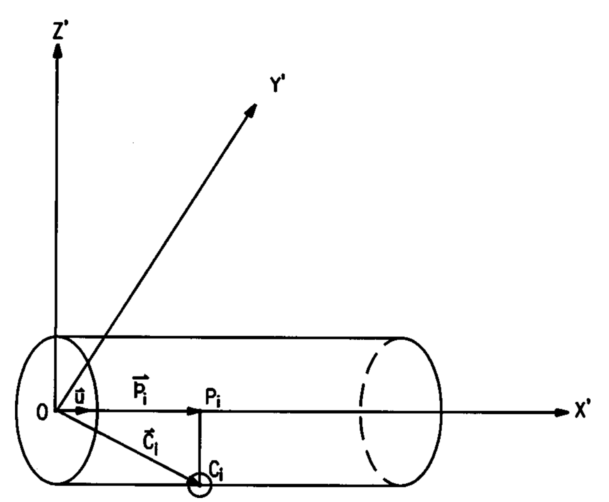

Fig. 4 The geometric relationship between the cylinder and the locator

$$\sqrt{((x'_{ci}-x'_{pi})^2+(y'_{ci}-y'_{pi})^2+(z'_{ci}-z'_{pi})^2)}-(R+r_i)=0 \quad (3)$$

where $i=1,2,..4$

The point x'_{pi},y'_{pi},z'_{pi} is on the cylinder's axis closest to the corresponding x'_{ci},y'_{ci},z'_{ci}. The components of Eq. (3) are related to each other as shown in Fig. 4 and Eq. (4).

$$\vec{p}_i=\left(\frac{\vec{u}\cdot\vec{c}_i}{|\vec{u}|}\right)\vec{u} \quad (4)$$

3 Geometric Analysis

Surface variability of the workpiece causes the workpiece position to deviate from its nominal position. The objective solution of this problem is the location and orientation of the cylinder after it is displaced by the surface errors at the contact points. Geometric transformations are employed to achieve the mapping between the *LCS* (workpiece) and the *WCS* (fixture). The inputs to this analysis are the radius of the nominal workpiece (r), the radius of the locators (R), the coordinates of the locators' centers (x_{ci},y_{ci},z_{ci}), and the surface errors at each contact point (E_i).

The impact of the surface errors on the workpiece's position can be represented by a translation and two rotations (about the y and z axes of the *LCS*). A third coordinate system, WCS', is created such that its origin is coincident with the workpiece coordinate system, *LCS*, and its axes are parallel to *WCS*. The variables used to define the location and orientation of the workpiece are the origin of the workpiece in *WCS* (x_0,y_0,z_0), the angle that the workpiece rotates about the y axis of WCS' (Φ), and the angle that the workpiece rotates about the z axis of WCS' (θ). These transformations are shown in Fig. 5. It is not necessary to rotate the workpiece about the x axis because the axis of the cylinder is defined to be coincident with the x axis. The Newton-Raphson method is applied to solve five boundary equations with five variables.

Algorithm:

1 The known inputs are r, R, x_{ci}, y_{ci}, z_{ci}, and E_i.

2 The first solution that is composed of the origin of the workpiece in WCS (x_0,y_0,z_0) and the angles that the workpiece rotates about y and z axes of WCS' (Φ and θ) is established.

3 x_{ci},y_{ci},z_{ci} are transformed to *LCS* by parameters of the first solution.

4 The boundary equations are evaluated by substituting the solution into the procedures of Newton-Raphson.

5 If the solution satisfies the boundary equations, it will be accredited as the final solution and the calculation is accomplished.

6 If the solution does not satisfy the boundary equations, the next iteration is created and the procedures are repeated.

Note that since the number of equations is equal to the number of unknowns, there is a unique solution to the problem. The new position of the workpiece is determined by the translation and rotation transformations relative to *WCS*. The output of the simulation using the Newton-Raphson formulation provides the new origin of the workpiece in *WCS* and the angles that the workpiece rotates about y and z axes of WCS' to approach the contacts with all spherical locators. The coordinate of the cylinder's origin represents the location of the cylinder in *WCS*. The dislocation of the cylinder is denoted by $\vec{\mathbf{DL}}$ as shown in Fig. 6.

From Fig. 6, it can be seen that

$$\vec{\mathbf{AL}}=\vec{\mathbf{NL}}+\vec{\mathbf{DL}} \quad (5)$$

The direction cosine of the cylinder's axis is used to define its orientation.

$$\vec{\mathbf{AO}}=\vec{\mathbf{NO}}+\vec{\mathbf{DO}} \quad (6)$$

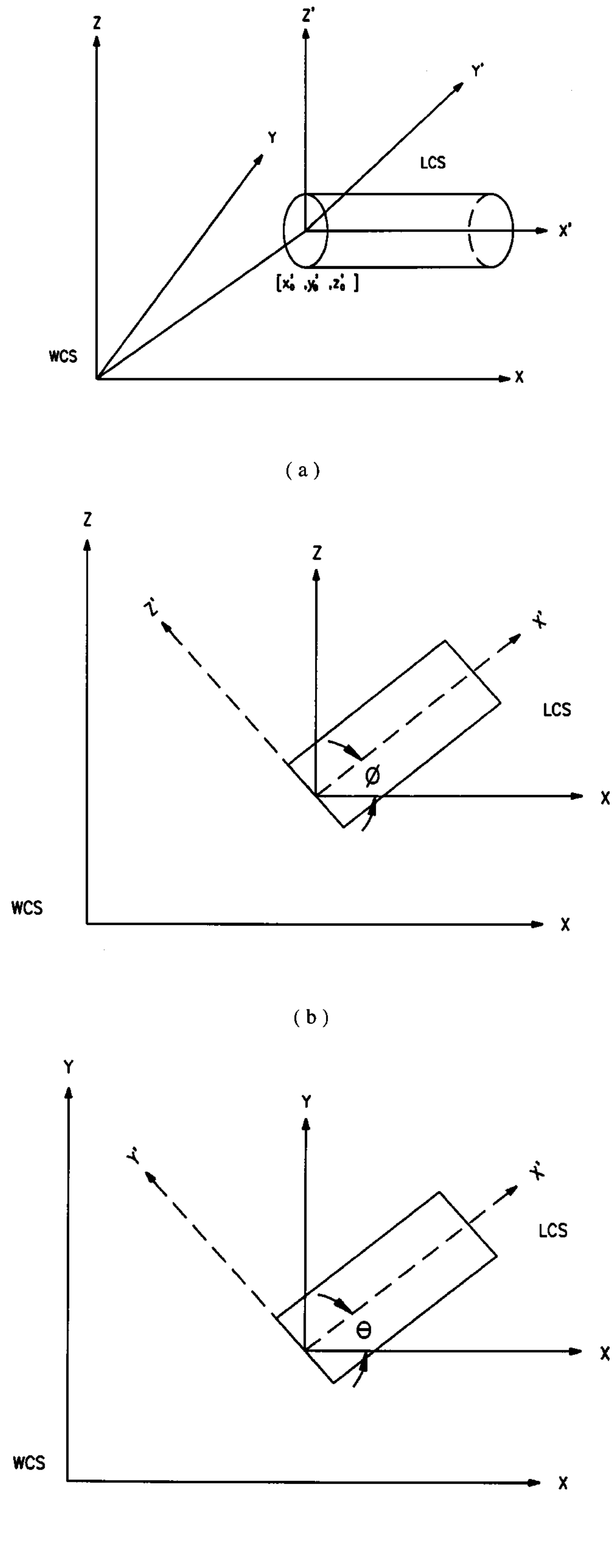

Fig. 5 The transformation of the cylinder (a) translation to x'_0,y'_0,z'_0 (b) rotation about y axis by Φ degree (c) rotation about z axis by θ degree

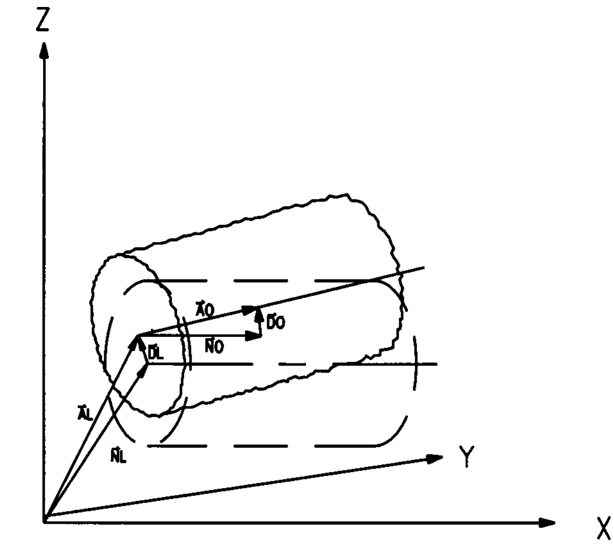

Fig. 6 The displacement vector of the cylinder's location and orientation

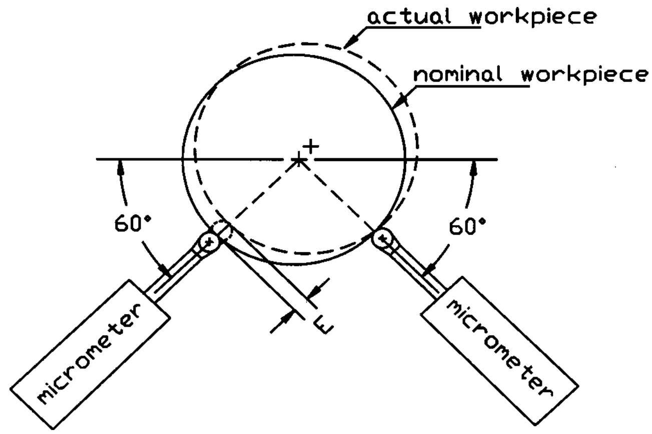

Fig. 7 The deviation of the cylinder due to surface error at the contact point

4 Model Validation

In an actual workpiece fixture system, the locators are in defined locations and the workpiece varies due to the surface error. This was imitated by using a workpiece with negligible surface errors and locators attached to micrometers, so they were adjustable. The locators supporting the cylinder on its cylindrical surface were installed at a 60 deg angle relative to the fixture base as shown in Fig. 7. The last locator is applied at the end of the cylinder to position the workpiece in the *x* direction. The locators had spherical tips (8 mm diameter) and sphericity of 0.02 mm. The workpiece used in this experiment was an aluminum cylinder of diameter 76.2 mm and length of 254 mm. The cylinder had cylindricity of 0.015 mm; therefore this error will be not considered.

In this experiment, the cylinder is moved from its nominal position by adjusting each micrometer by the amount of surface errors at that contact point. For example, if the surface error at one contact point is $+1$ mm, the micrometer supporting the cylinder at the corresponding point will be moved toward the workpiece by that amount to push the cylinder away. This simulated a workpiece with excess material at this locator location. The surface error is positive when it is radially directed outward from the surface of the cylinder. Negative error exists when the workpiece is undersized. The nominal position is derived when the nominal cylinder is initially set on the fixture as illustrated in Fig. 7. This figure also shows the impact of a positive surface error on the workpiece location.

After the simulated errors were added to each of the locators, the position and orientation of the workpiece was measured using a Brown & Sharpe MicroVal CMM. The actual position of the locator was determined by measuring ten points around its surface. The cylinder was measured by randomly selecting twenty points along its upper half. To validate the numerical model, the following parameters were collected.

1 The coordinates of the centers (x_{ci}, y_{ci}, z_{ci}) of the locators
2 The direction cosine of the cylinder
3 The origin of the cylinder (x_0, y_0, z_0)

The radius of the locator (R) and the radius of the cylinder (r) were also measured. The origin of the cylinder was established by the intersection of the cylinder's axis and the plane of the face end. To minimize measurement error, all measurements were repeated three times and the average value was used. It was assumed that the workpiece remained in contact with the same positions with the spherical locators. In reality, when the workpiece is pushed away or pulled back by one locator, the contact regions on the workpiece surface and the locator surfaces would shift to other locations. However, this dislocation is negligible.

5 Results

The results for five sets of surface errors are presented here. The sets of surface errors are presented in Table 1. Table 2 contains the theoretical and experimental location of the cylinder's origin for each experiment. Table 2 also contains the orientation

Table 1 The surface errors (mm)

Data set	P_1	P_2	P_3	P_4	P_5
0 (nominal)	0	0	0	0	0
1	+0.635	+0.635	+0.635	+0.635	+0.635
2	+0.635	+0.635	+0.635	+0.635	-0.635
3	-0.635	+0.635	+0.635	+0.635	+0.635
4	-0.635	+0.635	+0.635	+0.635	-0.635
5	+0.635	-0.635	+0.635	+0.635	+0.635

Table 2 The location and orientation of the cylinder after applying surface errors

Data Set	Location of Cylinder Origin (mm)		Direction Cosines of Cylinder Axis	
	Theoretical	Experimental	Theoretical	Experimental
0	-36.263, –29.277, –48.282	-36.287, –29.269, –48.316	0.999903 -0.013141, –0.004401	0.999903, –0.013145, –0.004404
1	-35.626, –29.276, –47.061	-36.926, –29.262, –47.061	0.999905, –0.013181, –0.004030	0.999905, –0.013164, –0.004031
2	-36.895, –29.259, –47.056	-35.651, –29.279, –47.059	0.999905, –0.013181, –0.004030	0.999905, –0.013184, –0.004021
3	-35.654, –28.502, –48.377	-35.692, –28.504, –48.374	0.999865, –0.016363, 0.001386	0.999865, –0.016365, 0.001472
4	-36.924, –28.481, –48.379	-36.952, –28.484, –48.382	0.999865, –0.016363, 0.001386	0.999865, –0.016360, 0.001376
5	-35.579, –29.322, –46.977	-35.609, –29.325, –46.979	0.999901, –0.010003, –0.009795	0.999901, –0.010053, –0.009836

Table 3 Displacement of cylinder, measured relative to nominal cylinder location and orientation. The change in orientation is measured as the difference of a unit vector in the nominal orientation and the displaced orientation.

Data set	Location (mm)						Orientation (10^{-3} mm)					
	Theoretical			Experimental			Theoretical			Experimental		
	x	y	z	x	y	z	x	y	z	x	y	z
1	0.637	0.001	1.221	0.639	0.007	1.255	0.002	-0.040	0.371	0.002	-0.019	0.373
2	-0.632	0.018	1.226	-0.636	0.010	1.257	0.002	-0.040	0.371	0.002	-0.039	0.383
3	0.608	0.775	-0.095	0.595	0.765	-0.058	-0.038	-3.222	5.787	-0.038	-3.220	5.876
4	-0.661	0.796	-0.097	-0.665	0.785	-0.066	-0.038	-3.222	5.787	-0.038	3.215	5.780
5	0.684	-0.045	1.305	0.678	-0.056	1.337	-0.002	3.138	-5.394	-0.002	3.092	-5.432

Table 4 The deviation between the theoretical and experimental displacement vectors

Data set	Location (mm)			Orientation (10^{-3} mm)		
	Theoretical	Experimental	Percent Deviation	Theoretical	Experimental	Percent Deviation
1	1.377	1.408	2.20%	0.373	0.373	0
2	1.379	1.409	-2.12%	0.373	0.384	-2.16%
3	0.990	0.970	2.06%	6.623	6.700	-1.15%
4	1.039	1.030	0.87%	6.623	6.614	0.13%
5	1.474	1.500	-1.73%	6.240	6.250	-0.16%

of cylindrical workpiece's axis. Table 3 contains the theoretical and experimental displacement vectors for the location and orientation of the workpiece. The deviation between the theoretical and experimental results are presented in Table 4.

6 Conclusion

The goal of this work was to develop a tool to predict the displacement of a workpiece based on the surface errors at the locators. Such a model could be used to determine the tolerance required of the preceding operation. This could also be used to help design the fixtures and the location of the locators. The coordinate systems of the workpiece and the fixture are separately defined and the workpiece is transformed to different coordinate systems by using geometric transformations. Based on the surface error model conditions, the boundary or objective equations are established. The model acquires the solution that satisfies all boundary equations by employing Newton-Raphson formulation. The final solution is the displacement vectors of location and orientation that the workpiece deviates from the nominal position. An experimental fixturing system, which simulated the surface errors on the workpiece, was fabricated to verify the numerical model. The difference between the resultant simulation and the resultant measurement falls within the range of ± 2.5 percent.

Nomenclature

x, y, z = coordinates of an object in *WCS*
x', y', z' = coordinates of an object in *LCS*
x_0, y_0, z_0 = coordinates of the origin of the cylinder in *WCS*
x'_0, y'_0, z'_0 = coordinates of the origin of the cylinder in *LCS*
P_i = spherical-tipped locators
x_{ci}, y_{ci}, z_{ci} = coordinates of the center of the ith locator in *WCS*
$x'_{ci}, y'_{ci}, z'_{ci}$ = coordinates of the center of the ith locator in *LCS*
x_{pi}, y_{pi}, z_{pi} = corresponding point of the center of the ith locator on the axis of the workpiece in *WCS*
$x'_{pi}, y'_{pi}, z'_{pi}$ = corresponding point of the center of the ith locator on the axis of the workpiece in *LCS*
r = radius of the nominal workpiece
r_i = assumed radius of the workpiece at ith contact point
R = radius of the supporting locator
E_i = amount of surface error at ith contact point
ϕ = angle that the workpiece rotates about y axis
θ = angle that the workpiece rotates about z axis
$\vec{\mathbf{u}}$ = unit vector of the workpiece's axis in *LCS*
$\vec{\mathbf{c}}_i$ = vector of the center of the spherical locator in *LCS*
$\vec{\mathbf{o}}$ = vector of the origin of the workpiece in *LCS*
$\vec{\mathbf{p}}_\mathbf{i}$ = vector of the target point on the axis of the workpiece in *LCS*
p_i = corresponding point of the center of the locator on the axis of the workpiece
$\overrightarrow{\mathbf{NL}}$ = nominal workpiece's origin vector in *WCS*
$\overrightarrow{\mathbf{AL}}$ = actual workpiece's origin vector in *WCS*
$\overrightarrow{\mathbf{DL}}$ = displacement vector of the workpiece's location in *WCS*
$\overrightarrow{\mathbf{NO}}$ = nominal unit vector of the cylinder's axis in *WCS*
$\overrightarrow{\mathbf{AO}}$ = actual unit vector of the cylinder's axis in *WCS*
$\overrightarrow{\mathbf{DO}}$ = displacement vector of the cylinder's orientation in *WCS*

References

[1] Cai, W., Hu, S. J., and Yuan, J. X., 1996, ''Deformable Sheet Metal Fixturing: Principles, Algorithms, and Simulations,'' ASME J. Manuf. Sci. Eng., **118**, pp. 318–324.
[2] Cai, W., Hu, S. J., and Yuan, J. X., 1997, ''A Variational Method of Robust Fixture Configuration Design for 3-D Workpieces,'' ASME J. Manuf. Sci. Eng., **119**, pp. 593–601.

[3] DeMeter, E. C., 1994, ''Restraint Analysis of Fixtures Which Rely on Surface Contact,'' J. Eng. Ind., **116**, pp. 207–215.
[4] DeMeter, E. C., 1994, ''The Min-max Load Criteria as a Measure of Machining Fixture Performance,'' J. Eng. Ind., **116**, pp. 500–507.
[5] DeMeter, E. C., 1995, ''Min-max Load Model for Optimizing Machining Fixture Performance,'' J. Eng. Ind., **117**, pp. 186–193.
[6] DeMeter, E. C., and Hockenberger, M. J., 1997, ''The Application of Tool Path Compensation for the Reduction of Clamping Induced Geometric Errors,'' Int. J. Prod. Res., **35**, No. 12, pp. 3415–3432.
[7] Hockenberger, M. J., and DeMeter, E. C., 1996, ''The Application of Meta Functions to the Quasi-static Analysis of Workpiece Displacement Within a Machining Fixture,'' ASME J. Manuf. Sci. Eng., **118**, pp. 325–333.
[8] King, S. L., and Hutter, I., 1997, ''Theoretical Approach for Generating Optimal Fixturing Locations for Prismatic Workparts in Automated Assembly,'' J. Manuf. Syst., **12**, No. 5, pp. 409–416.
[9] Rong, Y., and Bai, Y., 1996, ''Machining Accuracy Analysis for Computer-Aided Fixture Design Verification,'' ASME J. Manuf. Sci. Eng., **118**, pp. 289–300.
[10] Salisbury, E. J., and Peters, F. E., 1998, ''The Impact of Surface Errors on Fixtured Workpiece Location and Orientation,'' NAMRC Transactions, **26**, pp. 323–328.
[11] Trappey, A., and Matrubhutam, S., 1997, ''Fixture Configuration Using Projective Geometry,'' J. Manuf. Syst., **12**, No. 6, pp. 486–495.
[12] Foster, L. W., 1994, *Geo-metrics III: The Application of Geometric Tolerancing Techniques*, Addison-Wesley Publishing Co., Reading, MI