A RELIABILITY-CONFIDENCE METHODOLOGY FOR COMPLEX SYSTEMS

R.M. Bevensee

Lawrence Livermore National Laboratory
Livermore, CA 94550

INTRODUCTION

An important problem of many complex systems is that of assessing the reliability in the minimum probability of survival, from the reliabilities of the components. If system performance can be represented by a network flow diagram, the system probability of failure can be expressed in terms of path probabilities of failure, each of which is a function of component failure probabilities.

Measurement errors and other uncertainties place an interval of ambiguity around the mean value of system probability of failure and each path and component failure probability. Associated with a specified reliability R that a failure probability will lie within an interval is a confidence C in that reliability. The statement that the system probability of failure lies within a specified interval with a declared reliability R implies a derivable confidence in that statement.

A formalism for analyzing the reliability R of an interval for system probability of failure and its confidence C from the component R-C intervals is outlined as follows:

1. Obtain the network representation of the system, in which the (dependent) path failure probabilities are indicated relative to their component failure probabilities.

2. Derive by an algorithm such as "STOP"[1] a representation of the system by disjoint paths and their state probabilities. The system probability of failure is a sum of these.

3. Declare an R-reliable interval for each component probability of failure and express a confidence in that interval.

4. For a declared R-reliable interval for system probability of failure derive the confidence in that interval.

The general formalism will now be developed with reference to aircraft engine component retirement-for-cause.[2]

THE GENERAL CASE, APPLIED TO AIRCRAFT ENGINE FAILURE

An aircraft engine can fail when one or more rotor components fail; we assume for simplification no other types of failure. Each rotor component can fail because of one or more flaws, such as corner flaws in the balance flange scallop or disk line rim. Therefore the network diagram consists of paths for the various rotor components, each path passing through nodes representing the flaws peculiar to that rotor component.

These network paths are not independent if they share common flaws. In such a case algorithms can be utilized to derive a set of disjoint paths whose state probabilities depend on the various flaws F1, F2,..., as indicated in Table 1. The table shows, for example, that the state probability of path 1 for rotor component 1 is the product of the following probabilities: F1 being in state 1 (non-failure), F4 being in state 0 (failure),...

A table such as Table 1 yields the probability of system failure, P_F, as a sum of the disjoint path state probabilities,

$$P_F = \sum_{n=1}^{N} P_n \quad , \tag{1}$$

Table 1. Disjoint Paths and Their State Probabilities as Indicated by the States of Their Flaws.

	Flaws[a,b]				
Paths n	F1	F2	F3	F4	...
1	1	--	--	0	
2	0	1	--	--	
3	0	0	--	--	...
.			.		
.			.		
.			.		

[a] 0(1) indicates the flaw has (has not) caused failure.
[b] a dash indicates a flaw irrelevant to the probability of failure for that path.

each P_n being a product of probabilities of the flaw states in that path (i.e., rotor component).

To avoid confusion it is convenient to subtract from (1) the expected values of the probabilities, $\langle P_F \rangle$ and $\langle P_n \rangle$ and write

$$P_F - \langle P_F \rangle = \sum_n (P_n - \langle P_n \rangle$$

in the form

$$X = \sum_n x_n \quad . \tag{2}$$

X can be regarded as a sum of steps x_n with random lengths, and one can study the statistics for an R-C interval for X (i.e., system probability of failure, in terms of the statistics for R-C intervals of the x_n, i.e., flaw state probabilities).

For aircraft engine retirement-for-cause we ask the question: given intervals for the path state probabilities in which they are expected to lie at least 100R% of the time, with associated confidences, what is the confidence C_X that the engine probability-of-survival will lie within a given interval at least 100R% of the time? The R-C figures for an interval of engine probability-of-survival measure the errors and uncertainties involved in the assessment of engine performance.

R-C Intervals for the Path State Probabilities

Figure 1 introduces the probability $\tau(\bar{q})$ of stress $\bar{q}$ as a vector, $\bar{q} = (q^1,\ldots,q^s)$, with components indicated by superscripts to distinguish them from paths and flaws. τ determines the statistics of path state probability $P_n(\bar{q})$ and system probability of survival.

A given $\bar{q}$ causes specific probabilities of failure via the various flaws and a corresponding value of state probability $P_n(\bar{q})$ for path n. It is the task of fracture mechanics and/or NDE to evaluate these flaw failure probabilities and the $P_n(\bar{q})$. A probability distribution $\tau(\bar{q})$ for the $\bar{q}$ will impose a distribution on the $P_n(\bar{q})$ of which

$$\langle P_n(\bar{q}) \rangle = \int P_n(\bar{q})\ \tau(\bar{q})\ d\bar{q} \tag{3}$$

is the average. Defining $x_n(\bar{q}) = P_n - \langle P_n \rangle$, the probability that $x_n(\bar{q})$ lies in a range $[-e_n, e_n]$ is

$$\int_{\bar{q}_1(-e_n)}^{\bar{q}_2(e_n)} \tau(\bar{q})\ d\bar{q} \quad . \tag{4}$$

The statement that the path state probability lies in this range 100R% of the time means

$$\int_{\bar{q}_1}^{\bar{q}_2} \tau(\bar{q})\, d\bar{q} = R \quad , \tag{5}$$

and, because the τ-distribution is completely specified, the confidence in this R-reliable range is 100 %.

In practice the τ-distribution is not completely known because of errors, uncertainties and incomplete information, so we designate it as $\tau(\bar{q}|\bar{\nu})$, $\bar{\nu} = (\nu_1,\ldots,\nu_M)$ being a vector of imperfectly known parameters. Now the statement that $[-e_n, e_n]$ is R-reliable for $x_n(q)$ is

$$\int_{\bar{q}_1}^{\bar{q}_2} \tau(\bar{q}_1|\bar{\nu})\, d\bar{q} \leq R \quad . \tag{6}$$

Equation (6) defines a region $\bar{\nu}_{nR}$ of parameter space. The confidence C_n that $[-e_n, e_R]$ is indeed R-reliable is the probability of $\bar{\nu}$ falling within the $\bar{\nu}_{nR}$ region,

$$\int_{\bar{\nu}_{nR}} g(\bar{\nu}|\text{measurements})\, d\bar{\nu} = C_n. \tag{7}$$

g is an ad hoc probability function which may very well involve an unknown "prior" distribution. This could be chosen to represent maximum ignorance.

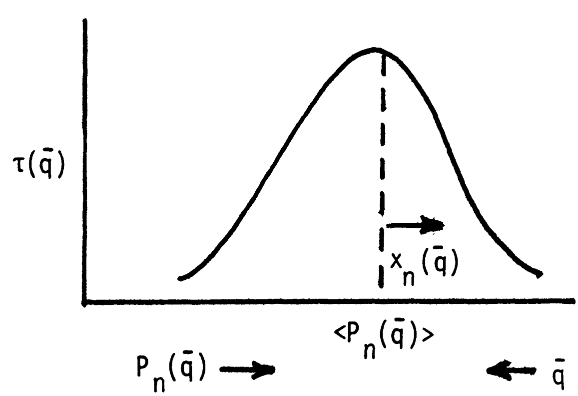

Fig. 1. The probability of stress $\bar{q}$, $\tau(\bar{q})$ and the state probability $P_n(\bar{q})$ of path n.

The R-C Interval for the System Probability of Failure

According to (2) the engine failure probability about its mean value is, for a given stress vector $\bar{q}$,

$$X(\bar{q}) = \sum_{n=1}^{N} x_n(\bar{q}) \tag{8}$$

Designate $\tau_X(|\bar{\nu})$ as the probability of X for a given $\bar{\nu}$, averaged over $\tau(\bar{q})$. The assertion that the interval [-E, E] is R-reliable for X is

$$\int_{-E}^{E} \tau_X\,(X|\bar{\nu})\; dX \geq R \quad , \tag{9}$$

and this actually defines a region $\bar{\nu}_{xR}$ of parameter space $\bar{\nu}$.

Markoff's Method[3] yields explicit formulas by which this $\bar{\nu}_{xR}$ region can be found:

$$\tau_x(X|\bar{\nu} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\rho X}\, A_N(\rho)\; d\rho, \tag{10}$$

where $A_N(\rho)$ is given completely generally as

$$A_N(\rho) = \int dq^1 \ldots \int dq^s \exp[i\rho \sum_{n=1}^{N} x_n\,(\bar{q})]\; \tau(\bar{q}|\bar{\nu}). \tag{11}$$

The confidence $\bar{C}_X$ that [-E, E] is indeed R-reliable for X is the probability of ν falling within this ν_{xR} region:

$$\int_{\nu_{xR}} g(\bar{\nu}\,|\text{measurements})\; d\bar{\nu} = C_X \tag{12}$$

A SIMPLIFIED CASE FOR GAUSSIAN POPULATIONS

The foregoing very general formalism simplifies considerably if the following conditions are valid:

1. The path state probabilities x_n in (2) are independent. This would be so if the flaws determining one path (rotor component) state probability did not affect any other path probability. Then $\tau(\bar{q}|\nu)$ would become a product of $\tau_n(\bar{q}_n|\bar{\nu}_n)$ for the individual paths.

2. Each path probability τ is Gaussian with unknown variance ν_n, with stress q relative to the value at $x_n = 0$ measured directly by x_n. Then $\tau_n = \tau_n(x_n|\nu_n)$.

3. n_n measurements are made to estimate P_n by the sample variance s_n.

Under these conditions (6) would evaluate to the form

$$R = R(e_n/\sqrt{\nu_{nR}}) \tag{13}$$

for the nth path, ν_{nR} being a boundary point separating ν_n space into R-reliable and non-R-reliable portions. Eq. (7) would read

$$C_n = \int_{\nu_n \le \nu_{nR}} g(\nu_n | s_n)\, d\nu_n. \tag{14}$$

$g(\nu_n|s_n)$ can be written with the aid of Bayes' Theorem as

$$g(\nu_n|s_n) = \frac{f(s_n|\nu_n)g(\nu_n)}{\int_0^\infty f(s_n|\nu_n)\, g(\nu_n)\, d\nu_n}\ , \tag{15}$$

where $f(s_n|\nu_n)$ is the chi-squared distribution with n_n degrees of freedom. If the "prior" $g(\nu_n)$ is taken to be $g_n \exp(-\varepsilon\nu_n)$ over $0 \le \nu_n \le \infty$ and ε is sufficiently large to keep $g(\nu_n|s_n)$ from having an impractically large variance (because $\langle P_n \rangle + x_n$ must not extend appreciably outside [0,1]) then (14) is of the form

$$C_n = C_n\ (\nu_{nR}|s_n) \tag{16}$$

We have a simplification in the X-population since it too must be Gaussian. If ν_x is its variance, $\nu_x = \Sigma\nu_n$ and (9) becomes

$$R = R(E/\nu_{xR}). \tag{17}$$

Eq. (12) is of the form

$$C_x = \int_{\nu_x \le \nu_{xR}} g(\nu_x|s_1,\ldots,s_N)\, d\nu_x \tag{18}$$

and this may be evaluated from the Markoff Eq. (10) and (11), the latter allowing analytic solution if the n_n are odd and ≥ 3.

The final solution for C_x involves sums and products of a number of integrals which can be quickly evaluated in the complex plane by an adaptive integration algorithm.

The details of the solution will not be presented because of the special nature of the conditions assumed.

REFERENCES

1. G.C. Corynen, "STOP: A Fast Procedure for the Exact Computation of the Performance of Complex Probabilistic Systems," UCRL-5320, Lawrence Livermore National Laboratory, January 1982.
2. C.G. Annis, M.C. VanWanderham and J.A. Harris, "Engine Component Retirement-for-Cause: A Nondestructive Evaluation (NDE) and Fracture Mechanics Based Maintenance Concept," Proceedings of the DARPA/AFWAL Review of Progress in Quantitataive NDE, AFWAL-TR-81-4080, Rockwell International Science Center, January 1981.
3. S. Chandrasekhar, "Stochastic Problems in Physics and Astronomy," Rev. Mod. Phys., 15:8, 1943. Reprinted in Noise and Stochastic Processes, N. Wax (ed.), Dover, New York, 1954.

ACKNOWLEDGEMENT

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract No. W-7405-Eng-48.