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Abstract
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Keywords
Nuclear data analysis, nuclear power, probability theory, signal generators, statistical model calculations

Disciplines
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A STATISTICAL MODEL FOR EDDY-CURRENT SIGNALS FROM STEAM GENERATOR TUBES

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ABSTRACT. We propose a model for characterizing amplitude and phase probability distributions of eddy-current signals. The squared amplitudes and phases of the potential defect signals are modeled as independent, identically distributed (i.i.d.) random variables following gamma and von Mises distributions, respectively. We derive a maximum likelihood (ML) method for estimating the amplitude and phase distribution parameters from measurements corrupted by additive complex white Gaussian noise. Newton-Raphson iteration is utilized to compute the ML estimates of the unknown parameters. The obtained estimates can be used for flaw detection as well as efficient feature extractors in a defect classification scheme. Finally, we apply the proposed method to analyze rotating-probe eddy-current data from tube inspection of a steam generator in a nuclear power plant.

INTRODUCTION

Eddy-current (EC) testing of steam generator tubes is performed extensively to detect and size flaws [1]. Rotating-probe EC testing has been proposed to improve the detection, interpretation and sizing of defects [1]. Rotating probes usually consist of three coils spaced $2\pi/3$ rad (120°) apart, see Figure 1. Each coil scans the inner surface of the tube by moving along a helical path. To extract meaningful information from the rotating-probe data, a preprocessing step is performed first [2]. The raw data is one-dimensional in nature and a synchronization step converts it to a 2-D image, where each column of the resulting image contains the data from one rotation. Figure 2 illustrates the result of this process. Figures 2(a) and 2(b) show the raw one-dimensional signal and the synchronized 2-D image, respectively. Figure 2(c) is a result of the calibration process where the potential defect signals show up. (The details of the calibration process are described in [2].) Further analysis of the potential defects is needed to discriminate between defects and nondefects, as well as between different kinds of defects. Here, we propose a statistical model for characterizing the amplitude and phase probability distributions of the potential defects and derive a maximum likelihood (ML) method for estimating the unknown amplitude and phase distribution parameters from noisy measurements. The proposed model is generally applicable to scenarios where signal amplitudes and phases have unimodal distributions.

This paper is organized as follows. We first introduce the signal and noise models. We then describe the ML method for estimating the unknown parameters and apply it to characterize the amplitude and phase distributions of several potential defects. Finally, we conclude by outlining suggestions for future work.

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SIGNAL AND NOISE MODELS

Characterizing the amplitude and phase probability distributions of eddy-current signals is important for flaw detection and classification. For example, after preprocessing and calibration of rotating-probe eddy-current data, the "true" defect signals should have sufficiently large amplitudes (compared with the noise level) and their phases should lie in the first and second quadrants of the impedance plane (i.e. between 0 and π rad), see [2]. The phase information is also essential for discriminating between inner diameter (ID) and outer diameter (OD) defects, see [2] and Figure 3. [Note that the defect signals in Figure 3 were collected from machined defects in a low-noise environment.] Below, we describe a statistical model for characterizing the amplitude and phase probability distributions of the potential defects.

Assume that we have collected K complex measurements $y_k,\ k = 1, 2, \ldots, K$ of an eddy-current signal from neighboring spatial locations. The measurements are modeled as follows:

$$y_k = \sqrt{\alpha_k} \cdot e^{j\beta_k} + e_k, \quad k = 1, 2, \ldots, K, \tag{1}$$

where

(i) α_k, $k = 1, 2, \ldots, K$ are independent, identically distributed (i.i.d.) squared signal amplitudes (powers) following a Gamma(a, b) distribution, described by the probability density function (pdf):

$$p_\alpha(\alpha_k; a, b) = \frac{b^a}{\Gamma(a)} \alpha_k^{a-1} \exp(-b\alpha_k), \quad \alpha \geq 0. \tag{2}$$

Figure 4 (left) illustrates of the Gamma distribution and its versatility. Interestingly, in the special case where $a = 1$, the amplitudes $\sqrt{\alpha_k}$ follow a Rayleigh distribution.

(ii) β_k, $k = 1, 2, \ldots, K$ are i.i.d. signal phases, independent of the amplitudes, which follow a von Mises distribution, described by the pdf:

$$p_\beta(\beta_k; c, d) = \frac{1}{2\pi I_0(d)} \exp[d \cos(\beta_k - c)], \quad 0 < \beta_k \leq 2\pi, \tag{3}$$

where $I_0(\cdot)$ denotes the modified Bessel function of the first kind and order zero. The von Mises distribution is one of the most used distributions for modeling random phase and is analogous to the normal distributions on the real line. It is also known as the Tikhonov distribution in the communications literature, see e.g. [3, eq. (3.37)] and [4, eq. (6.1)]. Von Mises distributions with different values of c and d are shown in Figure 4 (right).

(iii) e_k, $k = 1, 2, \ldots, K$ are i.i.d. zero-mean complex Gaussian noise samples independent of the signal amplitudes and phases, having *known* variance σ^2. [The noise variance σ^2 can be estimated from the neighboring measurement locations that contain only noise.]

Our goal is to find the ML estimates of the unknown parameters a, b, c, and d using the observations $y_k,\ k = 1, 2, \ldots, K$. Define the unknown parameter vector

$$\boldsymbol{\lambda} = [a, b, c, d]^T \tag{4}$$

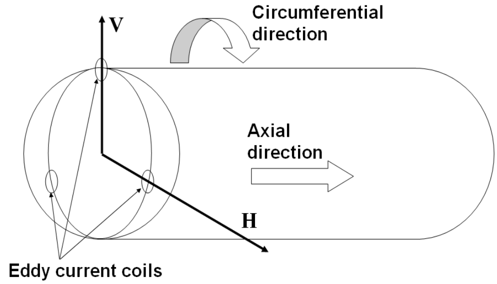

FIGURE 1. Rotating-probe eddy-current system.

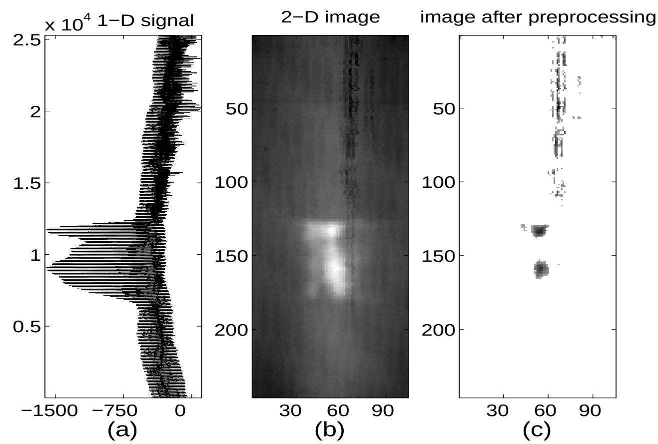

FIGURE 2. Signal preprocessing: (a) 1-D raw data, (b) 2-D image and, (c) signals after preprocessing.

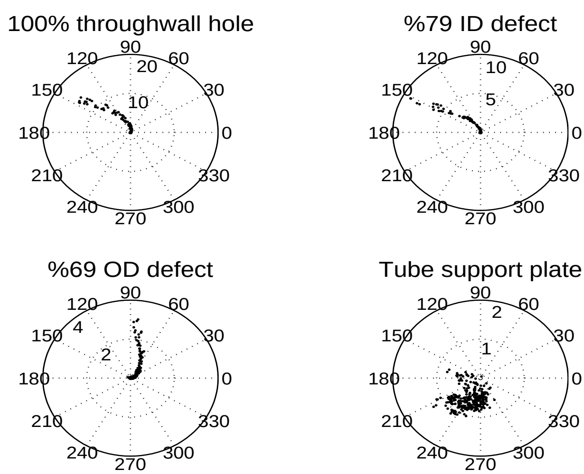

FIGURE 3. Signals from different discontinuities in impedance plane.

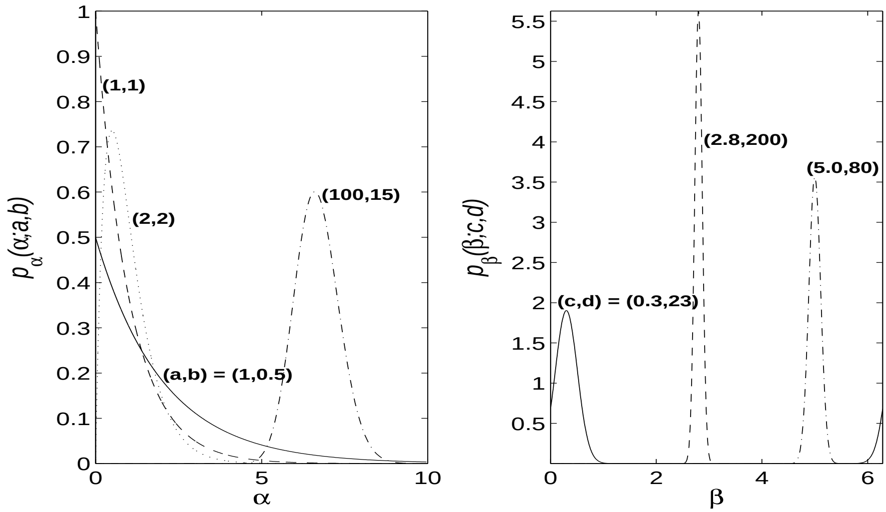

FIGURE 4. Gamma distribution pdf (left) von Mises distribution pdf (right).

and the vectors of signal amplitudes and phases

$$\boldsymbol{\theta}_k = [\alpha_k, \beta_k]^T, \quad k = 1, 2, \ldots, K, \tag{5}$$

where "T" denotes a transpose. From the assumptions (i)–(iii) and equations (1)–(3) it follows that the conditional pdf of y_k given $\boldsymbol{\theta}_k$ is complex Gaussian:

$$p_{y|\theta}(y_k|\boldsymbol{\theta}_k) = \frac{1}{\pi\sigma^2} \exp\Big[- \frac{|y_k - \sqrt{\alpha_k} \cdot e^{j\beta_k}|^2}{\sigma^2} \Big], \quad k = 1, 2, \ldots, K, \tag{6}$$

and the pdf of $\boldsymbol{\theta}_k$ is

$$p_\theta(\boldsymbol{\theta}_k; \boldsymbol{\lambda}) = p_\alpha(\alpha_k; a, b) \cdot p_\beta(\beta_k; c, d), \quad k = 1, 2, \ldots, K. \tag{7}$$

The marginal distribution of y_k is then

$$\begin{aligned} p_y(y_k; \boldsymbol{\lambda}) &= \int_{\boldsymbol{\Theta}} p_{y|\theta}(y_k|\boldsymbol{\theta}) p_\theta(\boldsymbol{\theta}; \boldsymbol{\lambda}) d\boldsymbol{\theta} \\ &= \frac{1}{\pi\sigma^2} \int_0^{2\pi} d\beta \int_0^\infty \exp\Big[- \frac{|y_k - \sqrt{\alpha} e^{j\beta}|^2}{\sigma^2} \Big] p_\alpha(\alpha; a, b) p_\beta(\beta; c, d)\, d\alpha, \end{aligned} \tag{8}$$

where $\boldsymbol{\Theta} = \{(\alpha, \beta) : 0 < \alpha,\ 0 < \beta \leq 2\pi\}$. The ML estimate of $\boldsymbol{\lambda}$ is obtained by maximizing the log-likelihood function of all available measurements $\boldsymbol{y} = [y_1, y_2, \ldots, y_K]^T$:

$$L(\boldsymbol{y}; \boldsymbol{\lambda}) = \sum_{k=1}^{K} \ln p_y(y_k; \boldsymbol{\lambda}). \tag{9}$$

The difficulty in estimating the unknown parameters in the above model arises due to the integral form of the density function (8). In the following, we present the Newton-Raphson method for finding the ML estimates of $\boldsymbol{\lambda}$.

MAXIMUM LIKELIHOOD ESTIMATION

We derive the Newton-Raphson algorithm for maximizing (9). The first- and second-order derivatives of $L(\boldsymbol{\lambda})$ with respect to $\boldsymbol{\lambda}$ are

$$\frac{\partial L(\boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} = \sum_{k=1}^{K} \frac{\partial \ln p_y(y_k;\boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}}, \qquad \frac{\partial^2 L(\boldsymbol{\lambda})}{\partial \boldsymbol{\lambda} \partial \boldsymbol{\lambda}^T} = \sum_{k=1}^{K} \frac{\partial^2 \ln p_y(y_k;\boldsymbol{\lambda})}{\partial \boldsymbol{\lambda} \partial \boldsymbol{\lambda}^T}, \tag{10}$$

where the terms in the above summations have been computed using the following formulas:

$$\frac{\partial}{\partial \lambda_i}\{\ln p_y(y_k;\boldsymbol{\lambda})\} = \frac{1}{p_y(y_k;\boldsymbol{\lambda})} \int_{\boldsymbol{\Theta}} p_{y|\theta}(y_k|\boldsymbol{\theta}) \frac{\partial p_\theta(\boldsymbol{\theta};\boldsymbol{\lambda})}{\partial \lambda_i} d\boldsymbol{\theta} \tag{11a}$$

$$\begin{aligned}\frac{\partial^2}{\partial \lambda_i \partial \lambda_m}\{\ln p_y(y_k;\boldsymbol{\lambda})\} &= \frac{1}{p_y(y_k;\boldsymbol{\lambda})} \int_{\boldsymbol{\Theta}} p_{y|\theta}(y_k|\boldsymbol{\theta}) \frac{\partial^2 p_\theta(\boldsymbol{\theta};\boldsymbol{\lambda})}{\partial \lambda_i \partial \lambda_m}\, d\boldsymbol{\theta} \\ &- \frac{1}{p_y^2(y_k;\boldsymbol{\lambda})} \cdot \int_{\boldsymbol{\Theta}} p_{y|\theta}(y_k|\boldsymbol{\theta}) \frac{\partial p_\theta(\boldsymbol{\theta};\boldsymbol{\lambda})}{\partial \lambda_i}\, d\boldsymbol{\theta} \cdot \int_{\boldsymbol{\Theta}} p_{y|\theta}(y_k|\boldsymbol{\theta}) \frac{\partial p_\theta(\boldsymbol{\theta};\boldsymbol{\lambda})}{\partial \lambda_m}\, d\boldsymbol{\theta}\end{aligned} \tag{11b}$$

for $i, m = 1, 2, 3, 4$ and $k = 1, 2, \ldots, K$. After applying the change-of-variable transformation

$$u = b\alpha, \tag{12}$$

the above integral expressions can be easily computed using Gauss quadratures, see [6, Ch. 5.3]. We applied the Gauss-Chebyshev and generalized Gauss-Laguerre quadratures (of orders N_{C} and N_{L}) to approximate integrals over β and u (respectively), yielding

$$\int_0^{2\pi} d\beta \int_0^{\infty} f(u,\beta) u^{a-1} \exp(-u)\, du \approx \frac{2\pi}{N_{\mathrm{C}}} \sum_{n=1}^{N_{\mathrm{C}}} \sum_{i=1}^{N_{\mathrm{L}}} w_i(a-1) f(u_i(a-1), \beta_n), \tag{13}$$

where $f(u,\beta)$ is an arbitrary real function, $u_i(a-1)$ and $w_i(a-1)$ are the abscissas and weights of the generalized Gauss-Laguerre quadrature of order N_{L} with parameter $a-1$, and

$$\beta_n = \frac{(2n-1)\pi}{N_{\mathrm{C}}}, \quad n = 1, 2, \ldots, N_{\mathrm{C}} \tag{14}$$

are the abscissas of the Gauss-Chebyshev quadrature. For example, applying (12) and (13) to (8) yields

$$\begin{aligned} p_y(y_k;\boldsymbol{\lambda}) &= \frac{1}{2\pi^2\sigma^2\Gamma(a)I_0(d)} \int_0^{2\pi} \exp[d\cos(\beta - c)]\, d\beta \\ &\quad \cdot \int_0^{\infty} \exp\Big[-\frac{|y_k - \sqrt{u/b} \cdot e^{j\beta}|^2}{\sigma^2} \Big] u^{a-1} \exp(-u)\, du \\ &\approx \frac{1}{\pi\sigma^2\Gamma(a)N_{\mathrm{C}}I_0(d)} \sum_{n=1}^{N_{\mathrm{C}}} \exp[d\cos(\beta_n - c)] \\ &\quad \cdot \sum_{i=1}^{N_{\mathrm{L}}} w_{\mathrm{L},i}(a-1) \exp\Big[-\frac{|y_k - \sqrt{u_{\mathrm{L},i}(a-1)/b} \cdot e^{j\beta_n}|^2}{\sigma^2} \Big], \end{aligned} \tag{15}$$

To compute the derivatives in (11), we also utilized the following formulas (see [5, eqs. (A.7) and (A.9)]):

$$\frac{dI_0(d)}{dd} = I_1(d), \tag{16a}$$

$$\frac{d^2 I_0(d)}{dd^2} = I_0(d) - \frac{I_1(d)}{d}. \tag{16b}$$

The (damped) Newton-Raphson algorithm updates the estimates of $\boldsymbol{\lambda}$ as follows (see e.g. [6], [7, eq. (13.25)], [8, Ch. 9.7], [9], and [10, Ch. 9.5]):

$$\boldsymbol{\lambda}^{(i+1)} = \boldsymbol{\lambda}^{(i)} - \delta^{(i)} \cdot \Big[\frac{\partial^2 L(\boldsymbol{\lambda}^{(i)})}{\partial \boldsymbol{\lambda}\, \partial \boldsymbol{\lambda}^T}\Big]^{-1} \frac{\partial L(\boldsymbol{\lambda}^{(i)})}{\partial \boldsymbol{\lambda}}, \tag{17}$$

where the damping factor $0 < \delta^{(i)} \leq 1$ is chosen (at every step i) to ensure that the log-likelihood function (9) increases and the parameter estimates remain in the allowable parameter space $(a, b, c, d > 0)$. Initialization of the above iteration is discussed in the Appendix.

EXPERIMENTAL RESULTS

We present two experimental examples from real defect to demonstrate the performance of the proposed ML estimation method. The K measurements y_k, $k = 1, 2, \ldots, K$ were selected from potential defect regions, and the noise variance σ^2 was estimated from the neighboring regions that contain only noise. The quadrature order of the Gauss-Chebyshev and generalized Gauss-Laguerre approximation were $N_{\mathrm{C}} = 120$ and $N_{\mathrm{L}} = 100$, respectively. The proposed algorithms converged within 30 iterations. In Figure 5, we show the estimated pdfs of the signal amplitudes $\sqrt{\alpha_k}$ and phases β_k. Here, the amplitudes $\rho_k = \sqrt{\alpha_k}$ follow the Nakagami-m pdf (see [3, eq. (2.20)]):

$$p_\rho(\rho_k; a, b) = \frac{2b^a}{\Gamma(a)} \rho_k^{2a-1} \exp(-b\rho_k^2), \quad \rho_k \geq 0. \tag{18}$$

CONCLUSIONS

We developed a statistical model for characterizing the amplitude and phase probability distributions of potential defects in rotating-probe eddy-current systems and derived a maximum likelihood method for estimating the unknown parameters from noisy measurements. Further research will include:

- computing Cramér-Rao bounds (CRBs) for the unknown parameters, and
- using the estimated distribution parameters as feature extractors in a defect classification scheme.

APPENDIX

We propose a method for initializing the Newton-Raphson iteration in (17). First, the initial values of the gamma distribution parameters a and b can be obtained using simple moment estimators:

$$a^{(0)} = \frac{(\widehat{\mathrm{E}}\,[\alpha])^2}{\widehat{\mathrm{var}}(\alpha)}, \quad b^{(0)} = \frac{\widehat{\mathrm{E}}\,[\alpha]}{\widehat{\mathrm{var}}(\alpha)},$$

where

$$\widehat{\mathrm{E}}\,[\alpha] = \frac{1}{K}\sum_{k=1}^{K} |y_k|, \quad \widehat{\mathrm{var}}(\alpha) = \frac{1}{K}\Big[\sum_{k=1}^{K} |y_k|^2\Big] - (\widehat{\mathrm{E}}\,[\alpha])^2. \tag{A.1}$$

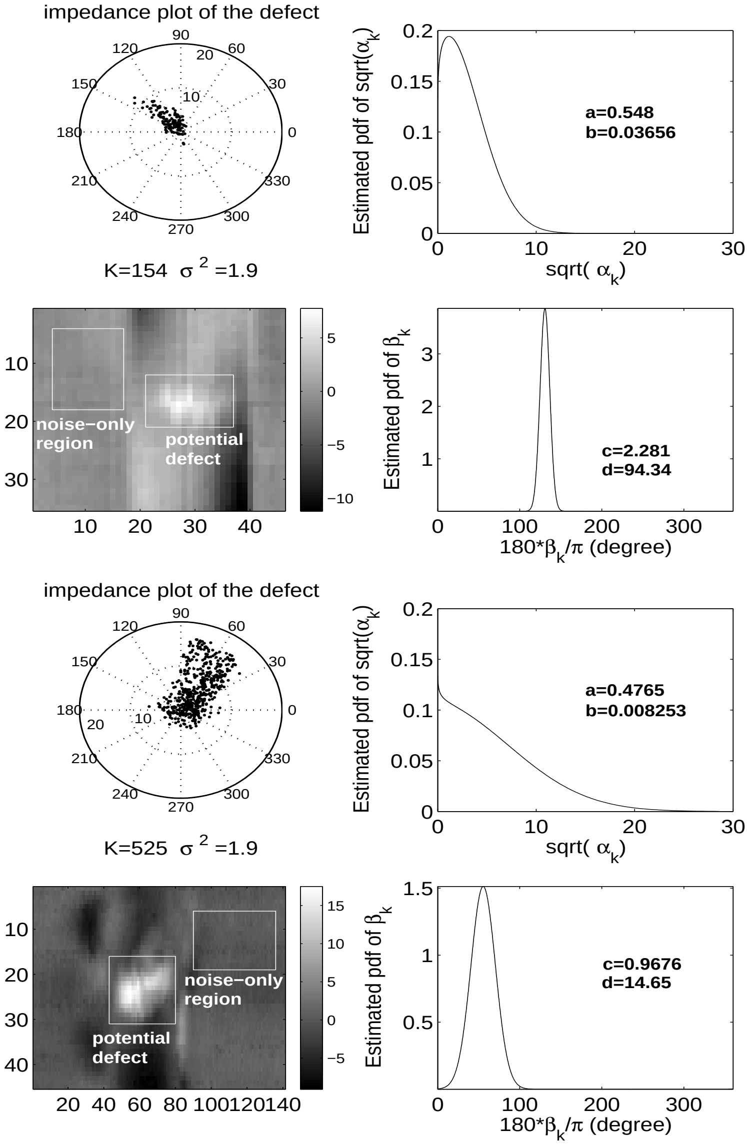

FIGURE 5. Impedance and amplitude plots (left) and estimated amplitude and phase distributions (right) of two potential defects.

We now discuss computing the initial estimates of c and d. Define

$$\overline{C} = \frac{1}{K}\sum_{k=1}^{K}\cos(\angle y_k), \quad \text{(A.2a)}$$

$$\overline{S} = \frac{1}{K}\sum_{k=1}^{K}\sin(\angle y_k). \quad \text{(A.2b)}$$

Then, the initial values of c and d can be obtained as (see [5, eqs. (2.2.4) and (5.3.11)]):

$$c^{(0)} = \begin{cases} \tan^{-1}(\overline{S}/\overline{C}), & \overline{C} \geq 0 \\ \tan^{-1}(\overline{S}/\overline{C}) + \pi, & \overline{C} < 0 \end{cases}, \quad \text{(A.3a)}$$

$$d^{(0)} = (1.28 - 0.53 \cdot \overline{R}^2) \cdot \tan(\pi\overline{R}/2), \quad \text{(A.3b)}$$

where

$$\overline{R} = (\overline{C}^2 + \overline{S}^2)^{1/2}. \quad \text{(A.4)}$$

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