Plan and market(s): a theoretical model of the Chinese grain economy

Dabai Chen
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Plan and market(s): A theoretical model of the Chinese grain economy

Chen, Dabai, Ph.D.
Iowa State University, 1990

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Plan and market(s): A theoretical model of the Chinese grain economy

by

Dabai Chen

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For the Graduate College

Iowa State University
Ames, Iowa
1990

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CHAPTER I. INTRODUCTION

We are facing changing economies in the world. Governments make policies to influence changes in the economies, or even changes in the nature of the economies. People are adjusting their behavior in response to the changes, especially to the structural changes in the economies. Economists are trying to keep pace with these changes by using economic principles to get pictures of the changing behavior and structure. To understand the changes in the economies, one should look at the past economic structure, at the current economic structure, and even forward to the structural changes that may occur over time as a consequence of the current policies or their alternatives. This is a very difficult and important task.

Statement of the Problem

A new structure has emerged in socialist economies during the last 25 years. The Soviet Union, the East European countries, and, finally, China have begun their own economic reforms. Although the manifestations of these reforms may vary from time to time and from country to country, the basic measures of the economic reforms can be seen in several ways: decentralizing management, reducing control of ownership, price reform, opening free markets, encouraging individual initiative, more concern for consumer welfare, etc. From the point of view of operational mechanisms

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1 The data used in this chapter are mainly from: Chinese State Statistics Bureau, 1987; Agricultural Yearbook Editing Committee, 1986, 1987; FAO, 1988a, b.
in the economy, the transition from a traditional planned economy to a mixed economy—which combines the dual features of planned and market economies—is the most meaningful and interesting structural change in the socialist economies.

The above situation leads to key questions for all economists. First, what is a planned economy and what is its operational mechanism? More specifically, what roles are played by the plan and the market, and what characterizes the behaviors of the government and the people in a planned economy? Second, what is a mixed economy and what is its operational mechanism? Particularly, what roles are played by the plan and the market(s), respectively; how do the plan and the market(s) relate and interact with each other; and what characterizes the behaviors of the government and the people in a mixed economy? Finally, what is the difference between basic features of planned and mixed economies; and what are the differences between the behaviors of the government and the people in a planned and a mixed economy?

In attempting to answer the above questions, the author will concentrate on the Chinese economy. This is because the Chinese economy is the second largest in socialist countries and because the Chinese government has applied a series of policies to promote economic reform. Although China has been engaged in its economic reform for only ten years, it has moved further toward a mixed economy than many of the other socialist countries and has achieved greater success in some respects.

Furthermore, this study will deal with only the grain economy in Chinese agriculture. One reason is that agriculture plays such an
important role in the Chinese economy. China has been predominantly an agrarian country throughout history. Feeding over one billion people, or 21.7% of the world's population, with only 6.9% of the world's arable land, China remains one of the largest agricultural economies in the world. In today's China, about 80% of the population lives in rural areas, about 50% of the labor force is engaged in the agricultural sector, and about 33% of the GNP is contributed by the agricultural sector. On the other hand, agriculture supplies much raw material to processing industries, provides considerable savings for sustained investment in a growing industrial sector, and, generally, earns some foreign currency for purchasing equipment in international markets. In this sense, the Chinese government has recognized agriculture as the base of the national economy.

In addition, agriculture plays an important role in economic reform in China. Agriculture, as an experimental sector, was chosen by the Chinese government for a major breakthrough in economic reform, and the success of agricultural reform led to economic reform in the industrial sector as well as in other sectors. The depth and breadth of economic reform in the agricultural sector are far beyond those in other sectors. The transition from a collective to an individual economy in agriculture has been almost completed throughout the whole country except for the ownership of land. Agricultural households can sell their products in reopened free markets after the government procurement contracts are satisfied. The improvement of agricultural efficiency and productivity provides a secure base for further growth of the rest of the economy and
a continuously rising standard of living—in short, for further economic reform.

Within agriculture, the grain economy is the most important. The grain sector was recognized as a treasure with which to manage the country in all past Chinese dynasties. At present, the Chinese government still thinks of grain as the base of the base (agriculture) of the national economy. To encourage grain production, to ensure grain supply, and to sustain an increasingly huge population seems to be a long-run goal for the Chinese government. The importance of the grain economy in China can be understood from the following three aspects:

1. Consumption aspect. Grain is the most important commodity in Chinese consumption. On the average, Chinese spend about 50% of their income on food consumption, and about 80% of their caloric intake and 30% of their protein intake comes from food grain. In addition, China has been the largest consumer of food grains in the world because it has had the largest population. For food grain only, the Chinese consumed 270 million metric tons, which was almost 15.3% of the total grain produced in the world in 1987.

2. Production aspect. Grain is the most important product in the agricultural sector of China. Generally, about 80% of the arable land in China is intensively used for annual grain production. In addition, China is one of the largest producers of grain in the world. For instance, in 1987, China was the
largest producer for rice (38.6%)\textsuperscript{2}, the largest producer for wheat (17.0%), the second-largest producer for corn (17.3%), the third-largest producer for soybeans (12.1%), and the largest producer for grain generally (20.1%) in the world.

3. Trade aspect. Grain is the most important good in China's agricultural trade. To import some grain can help the Chinese government balance domestic demand and supply, guarantee political security and improve the standard of living for the people. To export some grain can help the Chinese government obtain valuable foreign currency to pay for equipment needed for industrial modernization. In the international grain market, China has been very active and played an important role. For example, in 1987, China's grain import and export in trade volume were ranked third (9.9%)\textsuperscript{3} and eighth (2.5%) in the world, respectively. In the same year, China was the second-largest buyer for wheat (13.4%); the third-largest buyer (4.6%) and the fourth-largest seller (9.6%) for rice; the third-largest buyer (8.1%) and the fourth-largest seller for corn (6.1%); and the fifth-largest buyer (7.6%) and the third-largest seller (5.9%) for soybeans.

\textsuperscript{2}The numbers in the parentheses in this paragraph indicate the corresponding percentage of volume of the world product.

\textsuperscript{3}The numbers in the parentheses in this paragraph indicate the corresponding percentage of volume traded in the world market.
Besides, the grain economy represents well the features of planned and mixed economies. Due to the importance of the grain economy, the Chinese government applied a series of rigid policies to control grain consumption and production. From the late 1950s, only the state grain market had existed in the country. After more than 20 years' experience with that system, the government eventually shifted under the economic reform from a planned to a mixed grain economy and opened a free market for grain as a supplement to the state market. At present, dual markets for grain, i.e., the state market and the free market, coexist. Consequently, dual prices, i.e., a state market price and a free market price, coexist. The government even discussed the possibility of a completely open free market for grains, with only free market price as the signal to guide grain consumption and production. Thus, studying the Chinese grain economy can help people learn the features of planned and mixed economies and, in turn, learn more about the features of the Chinese grain economy itself.

To study the Chinese grain economy, a theoretical grain model for China will be developed. This work can be used by the Chinese government to appraise policy alternatives, and to make decisions on further economic reforms in the agricultural sector and, therefore, in the national economy.

Objective of the Study

The main objective of this study is to conduct theoretical analyses of the Chinese grain economy, more particularly, to develop a theoretical
framework for both planned and mixed economies. The associated specific objectives are:

1. To construct a theoretical grain model for China, and to evaluate the interaction between the domestic and international markets for both planned and mixed economy assumptions.

2. To build a rationing system into the grain modeling framework and to make theoretical analyses of the rationing effects on both the domestic and international markets in both planned and mixed economy scenarios.

3. To build both domestic prices, including state market and free market prices, and international prices into the grain modeling framework to make theoretical analyses of the effects of these prices on both domestic and international markets in both planned and mixed economy scenarios.

Organization of the Study

The study is organized as follows:

Chapter II reviews all of the relevant theoretical literature, discusses controversies and the limitations of the previous studies, and, finally, indicates the direction of this study.

Chapter III presents the theoretical framework of a Chinese grain model for a planned economy by both mathematical and graphical analysis. The behaviors of urban households (urban consumers) and rural production teams (rural consumers and producers) in the domestic market are treated separately. The behavior of the government in the international market
is described. The urban demand, the rural demand and supply, and the import demand (or export supply) for grain are derived. Comparative statics is used to provide analysis of alternative policies of key instruments. The roles of the state market and the international market in the planned economy are stressed and analyzed.

Chapter IV presents the theoretical framework of a grain model for a mixed economy by using mathematical and graphical analysis. Similarly, the behaviors of the urban households (urban consumers) and rural households (rural consumers and producers) in the domestic markets, including both the state market and the free market, are treated separately, and the behavior of government in the international market also is described. The urban demand, the rural demand and supply, and the import demand (or export supply) for grain are also derived. Comparative statics also are used to provide analysis of alternative policies. Last, the roles of the state market, free market and international market in the mixed economy are discussed and analyzed.

Chapter V constructs a complete grain model of China containing features of both a planned and a mixed economy. The domestic state market is controlled by state monopoly power and is assumed in disequilibrium, whereas the domestic free market is competitive and is assumed in equilibrium. The international markets are assumed to be competitive and/or imperfectly competitive.

Chapter VI includes a summary of the analyses, comparison of policy alternatives, conclusions, and suggestions for further research.
CHAPTER II. REVIEW OF LITERATURE

The review of literature in this study is classified into three sections: the urban household model, the agricultural household model, and the international trade model. The review is mainly focused on the theoretical contributions that are related to this study. Following that, the limitations of the previous studies for both the planned grain economy and mixed grain economy are discussed.

Urban Household Model

In agreement with early works done by several economists such as Davenant (1699), Gregory King (1696), and others, Adam Smith wrote in 1776, "Price varies directly as the quantity demanded, which depends on price."

The foundation of preference theory was built by Daniel Bernoulli in the 1730s. Later in the eighteenth century, Smith provided the proposition that the demand curve is downward sloping. Edgeworth (1881) defined a cardinal utility function and derived the indifference curve. After that, Fisher (1892) and Pareto (1906) established the modern theory on the assumption of ordinal utility.

An outstanding contribution of the period was made by Engel in 1857. He formulated the relationship between income and particular categories of expenditure. Walras (1874) presented a "theory of demand" and obtained demand as a function of all prices and income from a model of utility maximization by assuming cardinal utility. In the late nineteenth century, Marshall (1890) introduced the concept of the elasticity...
of demand.

Finally, Slutsky (1915) showed that the assumption of ordinal utility and other restrictions guarantee a utility maximum subject to a budget constraint. He also obtained a "Slutsky matrix" possessing symmetry and other characteristics. In 1934, Allen and Hicks rediscovered the Slutsky model independently.

Hicks gave an example of rationing demand in his famous *Value and Capital* in 1939. Several articles were published during World War II in British Journals. Scitovsky (1942) mentioned primary conditions for maximum utility under rationing. Nicholson (1942-43) used a two-dimensional geometric argument to describe the consumers' behavior under rationing. A general approach to demand under rationing was worked out by Samuelson (1947) and Graff (1948).

Malmquist (1948) investigated the aggregation problem of demand under rationing. He derived expressions for collective income, price, and ration elasticity of demand for a rationed good in terms of individual demand elasticities. Assuming random differences in individuals' tastes, Tobin (1952) revisited Malmquist's argument and studied the relation between the collective and the individual's demand.

Tobin and Houthakker (1951) provided a basic theory of consumer behavior for straight rationing by using the traditional Lagrangian method to maximize constrained consumer's utility and did comparative statics for changes in demand under rationing due to changes in some demand parameters including the ration itself. They discussed the effect of a change in income on demand with rationing and compared it with the
effect of a change in income on the same demand but without rationing.

Tobin (1952), in his survey paper, had summarized a series of theoretical and empirical findings for a rationing demand system from World War II. Clower (1965) and Barro and Grossman (1971) used general disequilibrium to approach the consumer's behavior under a rationing system. Malinvaud (1977) and Muellbauer and Portes (1978) also extended rationing theory to provide equilibria that depend on properties of the demand and supply system under rationing.

Pollak published his two papers in 1969 and 1971 and applied theory to the case in which the ration is just binding. Howard (1977) developed Tobin's rationing framework into a more general case by using the Kuhn-Tucker conditions and making some acceptable assumptions (such as strict equalities). He derived different demand functions for a representative consumer whose rations are not binding except for one and made some comparative statics. This is a theoretical improvement.

Deaton (1978) expressed a hope that the duality theory can be used to generate empirically estimable demand functions under both rationing and nonrationing in the same way. Neary and Roberts (1980) developed a new approach by minimizing a constrained expenditure function to get a compensated constrained demand function and then used partial derivatives based on duality theory to derive elasticities of demand under a rationing system. Hausman (1979) developed a solution procedure that is used for a unique optimum given the maximization of the function on a convex budget set. His general principle used in the analysis of piecewise nonlinear budget sets is to have the consumer choose the most preferred
consumption point on each budget segment, determine the corresponding utility of that consumption, and then consume at his maximum utility across all segments.

Deaton and Muellbauer (1980) even discussed a modified AIDS (Almost Ideal Demand System) to project a demand system under rationing. Later, Deaton also introduced rationing into a modified LES (Linear Expenditure System).

Not much theoretical work has been done on demand under a rationing system in a socialist country. In his early work, Market Control and Planning in Communist China, Perkins (1966) described the movement from a market economy to a planned economy in China during the 1950s. His conclusion for scope of market control is that the importance of markets declined after planned control. He discussed why the government used (formal) rationing for a number of key commodities such as grain and cotton textiles. He laid a foundation for further research on the topic, but he did not give any mathematical expression for the structural change in this classical book for the Chinese planned economy.

Making the analysis of the Chinese economy, Chow (1985) pointed out the differences between planned and mixed economies. He also tried to use demand rationing to explain the planned economy. After comparing working hours of workers under rationing with those under an alternative purchase tax system (without rationing), he concluded that the rationing system discourages the worker from working as much as before. He did not express the mixed economy mathematically.
Tong-eng Wang (1980) used a graphic analysis to explain the changes in demand under rationing in China but neglected the difference in the shape of the demand curves under planned and market economy assumptions.

Terry Siculur (1988) examined interactions between the markets and the plan in the context of the Chinese agricultural sector. She derived urban consumers' demand, which is a function of a free market price vector, wage income plus government transfer, or the benefit consumers obtain from the dual markets (state and free markets) and dual prices (state and free market prices). From there, she drew her main conclusion that the aggregate demand for an agricultural product in a mixed economy is directly affected only by free market prices and not by state market prices (planning). Overall, this work is a major theoretical improvement for the Chinese economy. She did permit urban consumers to have free choices in both the state market (up to the rationing level) and the free market in her urban demand model; but as the case stands, her results always lead to urban consumption in the state market being strictly equal to the ration. That means, she did force urban consumers to use up their coupons, by assuming the equilibrium solution. This may not well reflect the real situation in the urban areas in China during the 1980s.

Agricultural Household Model

The first usage of the word 'supply' as an economic term in English appeared in 1767. Stewart mentioned that supply is found to be in proportion to demand. Soon after that, Adam Smith (1776) wrote, "Price varies directly as the quantity supplied which also depends on price."
Walras (1874) was the first to express quantity supplied mathematically as a function of more than one price. All of his supply functions, however, were derived from a utility maximization. Marshall (1890) laid the foundation for a theory of cost and supply. His most important contribution to the theory of supply was his concept of the time period, especially the short run and the long run.

Hicks (1939) was the first to derive the output supply function from a profit maximization, in his *Value and Capital*. Later, Samuelson (1947), in his *Foundations of Economic Analysis*, also discussed the problem of a profit maximization subject to a production function.

In about the last twenty years, economists developed the so-called agricultural household model to describe the behavior of rural households that are both consumers and producers. The important issue is recognizing the owner's time as a scarce resource.

Becker (1965) provided a new approach with his theory of the allocation of time. He used utility maximizing subject to constraints of expenditure (including time spent in consuming) and a production function to combine both consumption and production activities into a household model. Although his main interest was focused on labor supply of the household, he founded a theoretical basis for the household model and, therefore, the agricultural household model.

After that, Lancaster (1966) studied consumer theory under an assumption that consumption is an activity in which goods, singly or in combination, are inputs and in which the output is a collection of characteristics. He found his model to be richer in explanatory and
predictive power than the traditional model. Muth (1966) considered the hypothesis that commodities purchased on the market by consumers are inputs into the production of goods within the household.

Hymer and Resnick (1969) concentrated on the necessary condition for using a household model. They assume that the leisure and labor demand are not choice variables. Then they found that there are two virtual prices for labor and goods, respectively. Thus, there are two equalities on supply and demand rather than one. They thought of relaxing the assumption, in other words, assuming that a labor market may be assumed to exist, or that the consumption and the production for the household are dependent on each other. After 1975, World Bank and Stanford University staffs have developed agricultural household models that combine the consumption and the production with the labor supply of the household.

Some theoretical works have been completed on the rural demand and supply under rationing and procurement in the Chinese economy. Without using any mathematical expression, Perkins (1968) also introduced the transition from the market economy to the planned economy in rural areas in China during the mid-1950s. He discussed the interaction of the market and plan in the late 1950s and early 1960s.

Walker (1984) studied grain procurement and consumption in China. He provided plenty of data that had been carefully collected from all possible publications including journals, news, and other sources. Where data was missing, his work was very helpful in furthering research on rural demand and supply, as well as on urban demand.
Chow (1985) illustrated the rationing system in China. He examined a demand system with rationing for the Chinese economy and discussed relationships between the rural consumption and production for agricultural households. Later he did distinguish between urban consumption and rural consumption and tried to explain urban-rural income redistribution in China. He used an aggregate production function to model Chinese agriculture.

Wang (1980) used a normal supply curve to graph China's situation and tried to explain the price policy of the government, but neglected the difference in the shape of supply curves in both market and planned economies.

Sicular (1983) first provided an agricultural household model for the Chinese planned economy. She assumed that the households' behaviors were represented by a utility maximization subject to a budget, the production function, and a quota constraint. On a theoretical basis, she conducted analysis for the team behavior with and without a quota. Although she tried to use alternative unrestricted utility maximization with the exact same first-order conditions to replace the restricted utility maximization, there was a minor mistake in her work so that the first-order conditions for both the utility maximization problems were not identical as she wished.

Using her early work, Sicular (1988) published a paper in the *Journal of Political Economy* in which the interaction between the markets and plan in the Chinese mixed economy was examined. At this
time, she applied a modern aggregate agricultural household model to represent the agricultural sector in the economy. According to her result, the rural demand function is a function of the free market price, income plus an extra term that is named a lump-sum transfer and equals the sum of the quota levels times the difference between the market and state prices. On the other hand, the rural supply function is identical to what would exist in the absence of planning, in other words, the market economy. As mentioned before in urban household models in this chapter, by this kind of aggregation and assumption for equilibrium condition, she could ensure the results derived. The general rural demand and supply functions may not be properly specified for the Chinese mixed economy without allowing for the differences in activities of agricultural households in the market(s).

International Trade Model

James Mill (1804) was the first to formulate the theory of comparative advantage and explain its practical meaning in terms of cost ratios. David Ricardo (1817), in his famous On the Principles of Political Economy and Taxation, gave an explicit formulation of the theory and attempted to spell out the implications of trade theory so that the theory became well known as the Ricardian theory.

Later in the nineteenth century, John S. Mill (1909; the first edition published in 1848), the son of James Mill, completed the theory by taking account of the role of reciprocal demands to explain how the exact international terms of trade would occur between the two closed
economy equilibrium price ratios of two commodities by means of reciprocal demand in each country. Mill described this situation as the equation of international demand.

To make an attempt to reduce a complex problem to a more manageable form by isolating one sector of the economy or one (or several) commodities in the economy, the partial equilibrium model was developed in the tradition of Alfred Marshall (1890). The partial equilibrium model also is often used for the international trade of agricultural commodities.

Not much theoretical work has been done on international agricultural trade in the Chinese economy. Perkins (1967), in his paper, "The International Impact on Chinese Central Planning," pointed out the basic strategy of international trade for Chinese planners. He also mentioned the limited size of trade due to the self-sufficiency of the Chinese economy and limited availability of foreign currency for the Chinese government.

Surls (1984) suggested several possible explanatory variables that affect China's grain trade levels. He identified the factors as the urban grain demand and supply balance, world grain market prices, the balance of payments situation, trade policy, and economic priorities. He even tried to use an econometric model to verify the determinants of grain trade for China.

The World Bank (1985a, 1985b) publication, Long-Term Development Issues and Options for China, used a material-balance table to simulate the trade behavior of the Chinese government. The basic consideration is that foreign trade, in particular sectors, is adjusted in each period to
fill the gap between the domestic demand and supply. Other projections, such as Noh (1983), or Carter and Zhong (1988), followed the same idea to form a balance sheet to predict grain trade trends for China.

Discussion of Limitations

As just mentioned, the theoretical derivations of grain models for the Chinese economy so far are unsatisfactory. For the current investigation on the demand and the supply functions for both planned and mixed economies, the theory is not well constructed. In particular, the roles of both state and market price mechanisms in either planned or mixed economies are not completely clear. Both the theoretical and empirical results are still open to question. Therefore, it is necessary to discuss the limitations of previous studies and hence to indicate further developments required for this study.

The planned grain economy

What is the proper modelling framework for the planned grain economy in China? More specifically, what is the role of the state price mechanism for the planned grain economy in China? These questions are still not resolved. The reasons can be understood from the discussion in this study.

Behaviors of the urban households and the urban grain demand

It is widely accepted that the domestic demand for food grain in the Chinese planned economy can be divided into two sections: urban demand and rural demand. The former is derived from urban households' behavior(s), whereas the latter is mainly derived from behavior(s) of production teams.
Still, some technical problems need to be solved. It has been obvious that the behaviors of urban households are classified into different types of groups by their situation relative to the ration in the state market. Allowing for the difference in types among the urban households, how to aggregate the individual demands into urban grain demand in the planned economy has not been discussed. Therefore, the correct specified functional form for the urban grain demand, which has to include all proper factors that influence or change the demand, has not yet been set up. Furthermore, the right shape of the urban grain demand function in the Chinese economy has not been traced out. Finally, the role of the state price of grain in the planned economy has not been clearly described.

To solve these problems, we suggest using the two-stage aggregation method in this study. First, we have to derive different individual demand functions by distinguishing different types of urban households' behaviors, then aggregate the individual demands into the group demand by assuming that all individuals within the group are identical, and, finally, into the total urban demand in the planned economy. Working in this way, the aggregate urban grain demand function contains all of the proper factors that reflect the real situation correctly and makes its shape and the role of the state price mechanism much clearer.

Behaviors of the state farms and the agricultural production teams and the rural grain demand and supply. It is known that there were two different kinds of rural enterprises in the Chinese planned economy: state farms, which were state-owned and production teams, which were
collective. The past theoretical studies on the rural grain demand and supply concentrated only on behavior of the production teams and not on the behavior of the state farms. Furthermore, the studies ignored the different behaviors of the production teams in the state market.

Some theoretical work needs to be done in this study. Fundamentally speaking, how do the state farms or production teams relate to the rural grain demand and supply in the Chinese planned economy? More directly, what are the behaviors of the state farms or the production teams in the state market? Is there any difference in relationships to the state market among the state farms or production teams? The basic questions are how to derive and aggregate both individual demands and supplies for the state farms or production teams, allowing for the differences in their situations under the procurement and rationing systems, respectively.

Using the two-stage aggregation method and following the similar assumption; step by step, we can work out the above problems and reach the correctly specified functional forms for both the rural grain demand and supply in the Chinese planned economy. These functions will contain all of the proper factors that influence the market activities and make the shapes of the rural demand and supply curves apparent. It is necessary to point out that the role of the state price will be clearly seen by means of the effective and ineffective components within the rural grain demand and/or supply function(s).

Behavior of the Chinese government and international grain trade
The behavior of the Chinese government in international (grain) trade in
a planned economy has been described in many previous studies. It is
generally acknowledged that the Chinese government has the monopoly power
to manage its foreign grain trade. For the roles played by both the
domestic and international price in the international market, the past
discussions were mainly qualitative rather than quantitative and empiri-
cal rather than theoretical. Up to now, there has been no theoretical
model that correctly reflects the behavior of the government in the
international grain markets.

The unsolved issues to be considered in this study are how to design
a theoretical model to describe the activities of the Chinese government
in the international grain markets and how to derive an import grain
demand or an export grain supply function for the planned economy. In
particular, are the grain import and the grain export independent of each
other or are there any trade-offs between grain and other agricultural
commodities, such as cotton, and trade-offs between different grains,
such as wheat and rice? What roles in international grain trade are
played by either the domestic or the international prices? The author
will try to create a new economic model to answer these questions.

Combining the domestic market and the international markets to-
gether, what is the equilibrium situation for the Chinese planned grain
economy, how is this equilibrium situation different from a usual
equilibrium condition for the free grain economy, and, finally, how do
the government's grain policy instruments (either domestic or interna-
tional) affect the equilibrium? These questions also will be answered by
making a proper theoretical analysis in this study.
The mixed grain economy

The proper modeling framework for the mixed grain economy in China, more specifically, the roles of the state and the free market price mechanisms in the Chinese mixed economy, are still questions in need of answers. The readers can recognize these issues from the next discussion.

Behaviors of the urban households and the urban grain demand As studied in the planned grain economy, the demand in the Chinese mixed grain economy is regularly divided into two sections: the urban demand and the rural demand. The urban grain demand is still derived from the behaviors of urban households.

After the shift from a planned economy to a mixed economy, the urban households have found themselves facing a new environment. The new model that reflects the change has been created by Byrd (1987) and Sicular (1988) without considering the difference of the households' situations related to the state market and the free market.

Still, some technical problems exist in the theoretical derivation. Given the differences in the individual households' situations in the state market in the planned economy, what is the change in their behaviors to adjust themselves to fit the new economy? More specifically, changes in the functional forms for the individual and aggregate urban grain demand(s) that allow for the existence of different types of urban households in the mixed economy have to be explored and stressed. The roles of both the state and the free market price in both the state and free market have to be illustrated separately and clearly. Last, the
shapes of urban demand functions in both the state and the free market have to be truly traced out.

**Behaviors of state farm (households) and agricultural households and rural grain demand and supply** Since the institutional change in the economic reform, the basic unit of rural consumption and production has become the individual or household, either the state farm household that belongs to the state farm, or the agricultural household which belonged to the production team. The previous theoretical studies on the rural grain demand and supply were focused mainly on the behavior of the agricultural households. Moreover, most of the studies ignored the difference in the behaviors among the agricultural households in both the state market and free market.

The same theoretical questions, that were presented for the planned rural economy need to be settled in the study for the mixed rural economy. They are: how the state farm (households) or agricultural households connect with the rural grain demand and supply in the Chinese mixed economy, in detail, how to derive and aggregate individual demands and supplies, allowing for different types of state farm (households) or agricultural households, respectively, and how to evaluate the price effects (both the state and the free market prices) on these aggregate rural grain demands and supplies.

**Behavior of the Chinese government and international trade** The behavior of the Chinese government in international (grain) trade in a mixed economy has been discussed in several studies. Although the
Chinese government has decentralized the management of foreign trade, it is still acknowledged that the central government has the authority to make the final decision on its foreign grain trade. Unfortunately, there has been no theoretical model that properly presents the changing behavior of the government in the international grain markets.

The unsolved problems are similar to those presented in the planned economy and it is unnecessary to repeat them here; but special attention will be paid to those changes brought by the structural reform, in other words, change in the model designing, change in the function form of import grain demand/export grain supply, change in market roles played by the domestic and the international market prices, and finally, change in interactions of government plan and domestic and international market.

Modeling the Chinese Grain Economy

After the theoretical analysis for both the planned and the mixed grain economy, the empirical analyses can be made on the basis of the theoretical models and careful comparisons for both the planned and the mixed grain economies.

It may be possible to build a theoretical model to include both the planned economy and the mixed economy by distinguishing different time periods. Until now, there has been no such econometric model to deal with projections for the Chinese grain economy and to evaluate policy alternatives for the structural changes within the economies.
CHAPTER III. THE CHINESE PLANNED GRAIN ECONOMY

Central planning is considered by the Chinese government as essential to the national economy and is set up on the basis of public ownership of production means including state and collective ownership. After organizing collectives in the mid-1950s, combining a planning system with a marketing system, the government centralized control of the grain economy. To ensure rapid growth of industries and meet basic needs of the population, the government became involved, through direct and indirect plans, in almost all agricultural economic activities, such as resource allocation, investment distribution, production decisions, and, especially, marketing decisions.

This chapter deals with a theoretical model for the Chinese planned grain economy. To begin with, a brief overview is presented on the planning and the marketing frameworks in the economy to help the readers understand the theoretical work. This is followed by three sections of theoretical discussion: the behavior of the urban households, the behaviors of state farms and agricultural production teams, and the behavior of the Ministry of Foreign Trade (MOFT). These sections develop an urban household model, a state farm model and an agricultural production team model, and a MOFT model, respectively.

General Framework of the Chinese Planned Grain Economy

Because the Chinese government managed its grain economy by using the forces of both the plans and the market, it is necessary to introduce
the Chinese planned grain economy from both aspects of planning and marketing processes. The material used in the planning framework is mainly borrowed from Tuan and Crook's (1983) excellent report, Planning and Statistical Systems in China's Agriculture.

Planning framework

In mainland China, the grain economy was planned and managed at two levels: central and local. The local governments included provincial, prefecture, and county levels. The top apparatus of the central government is the State Council. Managed by the Premier and several Vice Premiers, the State Council sets up several various commissions, ministries, and other agencies to perform its administrative functions. The responsibilities of the commissions are to help the State Council to coordinate its programs. Among these commissions, the State Planning Commission bears most of the responsibility for central plans, including agricultural plans. It works with the State Economic Commission, the State Capital Construction Commission, the State Statistical Bureau, the People's Bank of China, the economic ministries, and other agencies subordinate to the State Council to formulate national plans for different periods, such as five years or annually. The annual plans are actually carried out in practical economic activities. Plans for the coming year are formed in the fall before the new year so that the agricultural output for the current year can be known by the planners.

There were 29 provincial, municipal, or autonomous region governments, 220 prefectural and 2300 county governments in China in 1978. Zhongguo nongye nianjian bianji weiyuanhui, 1981.
The foundation of an annual plan is material-balance tables specified in physical units.

The provincial government is led by a governor (or head of Provin­
cial Revolution Committee during the Cultural Revolution) who is assisted by several vice governors. It also sets up various commissions and departments that correspond to the national commissions and ministries, respectively. These commissions and departments perform their duties mainly under the governor's administration and receive guidance from the corresponding organs at the national level.

The prefectural government is responsible for governing several counties within a province and delivering documents and reports between counties and provinces. The county government is controlled by a county chairperson (or head of County Revolution Committee during the Cultural Revolution). Both prefectures and counties use the same models as provincial and national. Different commissions and bureaus in both the prefectural and the county government also correspond to the commissions and departments at provincial levels, but on a smaller scale. The communes were basic governmental and administrative units in the country­side and were managed by heads of commune (or the head of the Commune Revolution Committee during the Cultural Revolution).

There were two kinds of rural enterprises in the Chinese planned economy: state farms^ and collective farms or the commune system which was composed of four parts--commune, production brigades, production

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^There were 2002 state farms in China in 1978. Zhongguo nongye nianjian bianji weiyuanhui, 1981.
teams, and agricultural households. The planning process was different for state farms and collective farms.

**State farm planning**  
State farms drew up their own preliminary plans according to national objectives and previous production, then the copies were sent to the provincial government. After summarizing and balancing these preliminary plans, the provincial governments sent a draft plan to the central government.

The Planning Commission and its subordinate ministry received the draft plans for state farms and integrated those into the comprehensive plan for the whole economy by holding an annual planning convention. After this plan was approved, the relevant local plans were passed down from the central government to the provincial level and to the state farm system. This is the direct planning process.

**Collective farm planning**  
For collective farms, the planning process was indirect or semidirect. At the beginning, the central planners estimated demand and supply for each important and centrally controlled good, including grain, to form a draft plan. After considering transfer between the provinces, and possible trade with other countries, the Planning Commission brought total quantity of grain supplied and demanded into balance for each province. The initial plans were sent down to province-level administrations. The corresponding Planning Commissions in the province broke plans down among their subordinate

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6 There were 52,781 communes in China in 1978; each had an average of 13 brigades and 91 teams and averaged 3279 households, 15,134 persons, and over 1,800 hectares. Zhongguo nongye nianjian bianji weiyuanhui, 1981.
prefectures and then their subordinate counties as well as their state farms. In turn, the county planning commissions would distribute plan targets for their subordinate communes. Finally, communes allotted goals to their brigades and then to production teams, the basic production and accounting units in the countryside.

At each local level, individual units received their target input allocations and output quantities. Managers, farmers, and accountants compared the targets with their own projections, then adjusted their goals and sent revised figures back up through the planning chain. The central ministries and commissions evaluated the revised targets, repeated the material balance procedure, and used the results as the final plan, which was then officially approved by the State Council.

Approved annual plans were formulated at the province level, then sent through the same process of disaggregation. They eventually became provincial annual plans. Finally, the provincial planner sent annual plans to the county, then to commune, brigade, and production team. Individual units would receive their output quotas and figures for their resource allocation and organize their production by seasons. The planning process was finished.

Figure 3.1 briefly describes the agricultural planning process just introduced. The arrows in the figure indicate the delivery of the plans either between the governments at different levels or between the governments and the enterprises.
Figure 3.1. Agricultural planning process in the Chinese planned economy

Marketing framework

The state market In the planned economy, the only grain market is managed by the government and, therefore, is referred to as the state grain market in this study. Figure 3.2 shows the circulation of grain in the Chinese planned economy. The sellers to the state market include most of the state farms and most of the production teams (Type I and Type II). The former sell almost all of their grain product except storage,
Figure 3.2. Circulation of grain in the Chinese planned economy

including seed and feed for next year's production, and the latter usually sell part of their grain product and keep the remaining part for their next year's production and internal distribution.

The buyers from the state market consist of all urban households, all households in the state farms, the military, some industries, and part of the production teams. The production teams, as the figure shows, include Type II and Type III production teams. They are classified by their activities in the state market. The Type III production teams are complete buyers or zero sellers for the state market. The Type II production teams are partial buyers or partial sellers because they also
are involved in the partial selling (buying) activities. Last, the remaining Type I production teams are zero buyers or complete sellers in the state market.

International trade and the state stock are both used by the government to adjust and maintain the balance of the demand and supply for grain in the state market. The functions of the international grain market will be discussed in more detail later in this chapter. The functions of the state stock can be summarized: 1) to ensure the needs between harvests, 2) to protect people against natural disasters and/or plan mistakes, 3) to have flexibility for fluctuation in the international market to avoid unnecessary loss, 4) to overcome time lag difficulties caused by both domestic and international transportation, and 5) to provide for emergency needs due to some abnormal situations, such as war or embargo.

For those economic and/or political reasons, by affording high storage costs, the Chinese government has accumulated a surprisingly large grain stock. It is estimated that the grain stock in China was at least 50% to 70% of the annual grain consumption. To simplify our theoretical model, we assume that the grain stock demand (or supply) equals zero, and will leave it to be handled in Chapter V.

Team-own market and team market: Obviously, a study of the state market excludes the distribution activities within the production teams. For convenience, let us call an assumed market in which this distribution happens the team-own market.

According to the roles set by the government, the team-own market of
grain does not open to anybody else except for their own household members. In this so-called team-own market, the only seller is the team itself, and the buyers are its own households. The distributions are generally made according to the work, ages, and sexes of the members within the households and at the state market prices (or less).

In addition, another assumed market in which all grain distribution activities in all of team-own markets are added together is called the team market. The sellers in this market are teams, and the buyers, of course, are still agricultural households.

Finally, we have a newly assumed combined market, which combines both the state market and the team market. Thus, in this discussion, demand and supply usually mean the demand and supply in this combined market, unless we mark out a specific market.

The dashed line in Figure 3.2 divides the circulation of grain in the Chinese planned economy into two parts. The right and upper part is the circulation of grain occurring in the state market and under direct control of the government plan. The left and lower part is the circulation of grain occurring within the team market and under indirect or semidirect control of the plan.

In general, the circulation of grain in the state market was a smaller portion of the total output in the planned economy and the circulation of grain in the team market occupied a larger portion. For instance, in 1975, the government purchased 21.4% of the total grain output and the production teams distributed 78.6% of the total grain output. In the same year, the urban population comprised 15.7% of the
total population whereas the rural population accounted for 84.3%. To a
certain extent, these facts provide a rough proof that the agricultural
households in the Chinese planned grain economy still possessed self-
sufficiency.

Theory of Urban Grain Demand
in the Planned Economy

Because the Chinese government had applied different measures to
distinguish and treat the basic food grain needs for urban and rural
people, it is essential to explore and derive both urban and rural demand
theories for the planned economy separately. To start with, this section
of the chapter focuses on the behavior of urban households in such an
economy.

For convenience, the general assumptions used in demand analysis are
followed through this study. More specifically, there exist utility
functions for households. These utility functions have the usual pro-
terties, in other words, completeness, reflexivity, transitivity, con-
tinuity, nonsatiation, and strict convexity. Finally, the objective of
the urban household is supposed to be to maximize its utility subject to
given constraints.

The discussion of urban grain demand in the planned economy is
divided into the following three parts: policy environment, urban house-
hold model, and aggregate grain demand of urban households. The policy

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The data used here are from the State Statistical Bureau, P.R.C.,
1987.
environment existing for urban households is introduced to make further theoretical analysis. Immediately after that, the individual urban household grain demands are derived from utility maximization and distinguished by different types of households' situations relative to the rationing system. The aggregate urban grain demand is finally formulated on the basis of the discussion of these different situations.

**Policy environment**

**Ownership of assets** In the Chinese planned economy, almost all schools, stores, and other enterprises are owned by either the public or by cooperatives.

**Labor** Urban labor is hired by the government, public, or cooperative enterprises. The government agents set the wage rates for the working people according to their education, working experience, and positions. The people usually work a fixed eight hours every day except Sunday and the public holidays. Labor cannot be mobile between different enterprises without permission and, in general, neither can they be fired by their employers.

**Plan control** To guarantee basic living needs of the urban households, the government issues ration coupons for food grain as well as other necessary goods such as vegetable oil, cotton, cloth, meat, and whatever else the government thinks necessary. The grain coupons are distributed to members of urban households according to their ages and occupations. In this way, the government can easily make a plan to supply comparatively sufficient food grain based on the amount of ration
coupons issued.

**Market control** Urban households can buy food grain only from state-owned grain retail stations with both money and the ration coupons in the amount indicated. Without coupons, people cannot use only money to buy any food grain; and, in theory, coupons cannot be bought, sold, or exchanged legally in the market or in private. The government also sets fixed retail prices for food grain in the state market, although it may adjust retail prices for some various reasons.

**Urban household model**

To some extent, any study is considered the continuation and the development of all previous relevant studies. Our study, of course, cannot be an exception. Noting that Howard (1977) has written a very fine article titled "Rationing, Quantity Constraint, and Consumption Theory," we are going to follow his approach in this part until the general urban household demand functions are derived. Working in his model we will, under the general circumstances in the planned economy, expand his derivation from a single ration binding to multiple ration binding and, therefore, deduce more general demand functions for both the rationed and the unrationed goods.

Under the environment mentioned early in this section, the urban household in the Chinese planned economy faces a budget line formed by wage income and expenditure on all goods, given state prices and some ration constraints for some of the specific goods including food grain.

Suppose an urban household has a utility function
u (X₁...X₈, X₉...X₉, X₁...X₈)

where X₁...X₈ and X₉...X₉ are rationed goods but X₁...X₈ are not.

Assuming that the rations for commodities X₁...X₈ are R₁...R₉ and the prices for commodities X₁...X₈, X₉...X₉ are P₁...P₉, P₉...P₉, respectively, and that a given household's wage income is M because of fixed working hours set by the government, we express the Lagrangian function as:

\[ L = U(X₁...X₈, X₉...X₉, X₁...X₈) + \lambda_1(M - \sum_{i=1}^{n} P_i X_i) \]

\[ + \sum_{i=1}^{k} \lambda_{2i}(\bar{R}_i - X_i) \]  \hspace{1cm} (3.1)

The Kuhn-Tucker conditions will be:

\[ \frac{\partial L}{\partial X_i} - \frac{\partial U}{\partial X_i} - \lambda_1 - \lambda_{2i} \leq 0 \]

\[ i = 1, \ldots, k \]  \hspace{1cm} (3.2)

\[ \frac{\partial L}{\partial X_i} - \frac{\partial U}{\partial X_i} - \lambda_1 P_i \leq 0 \]

\[ i = 1, \ldots, n \]  \hspace{1cm} (3.3)

\[ X_i \frac{\partial L}{\partial X_i} - X_i \left( \frac{\partial U}{\partial X_i} - \lambda_1 P_i - \lambda_{2i} \right) = 0 \]

\[ i = 1, \ldots, k \]  \hspace{1cm} (3.4)

\[ X_i \frac{\partial L}{\partial X_i} - X_i \left( \frac{\partial U}{\partial X_i} - \lambda_1 P_i \right) = 0 \]

\[ i = 1, \ldots, n \]  \hspace{1cm} (3.5)

\[ X_i \geq 0 \]

\[ i = 1, \ldots, n \]  \hspace{1cm} (3.6)

\[ \frac{\partial L}{\partial \lambda_1} - \sum_{i=1}^{n} P_i X_i \geq 0 \]

\[ i = 1, \ldots, n \]  \hspace{1cm} (3.7)

\[ \lambda_1 \frac{\partial L}{\partial \lambda_{2i}} - \lambda_1 (M - \sum_{i=1}^{n} P_i X_i) \geq 0 \]

\[ i = 1, \ldots, n \]  \hspace{1cm} (3.8)

\[ \lambda_{2i} \geq 0 \]  \hspace{1cm} (3.9)

\[ \frac{\partial L}{\partial \lambda_{2i}} - \bar{R}_i - X_i \geq 0 \]

\[ i = 1, \ldots, k \]  \hspace{1cm} (3.10)
To simplify the Kuhn-Tucker conditions, it is better to make some further assumptions:

1) Suppose that the household buys each kind of good, in other words, instead of (3.6):

\[ X_i > 0 \quad i = 1, \ldots, n \]  

then (3.2) and (3.3) can be replaced by

\[ -\lambda_i - 1 \leq \lambda_i - 1 - \lambda_i - 2 = 0 \quad i = 1, \ldots, k \]  

After that, both (3.4) and (3.5) can be dropped out because they are automatically satisfied.

2) Suppose that

\[ M - \sum_{i} P_i X_i = 0 \quad i = 1, \ldots, n \]  

or the urban household spends all of its income in today's market.

Combining (3.16) with (3.9), (3.8) can be automatically ignored.

3) Suppose that some ration constraints in (3.10), say from \( h \)-th to \( k \)-th, are not strictly binding for this household,

\[ \bar{R}_i - X_i > 0 \quad i = h, \ldots, k \]  

immediately according to (3.11), this means that

\[ \lambda_{2i} = 0 \quad i = h, \ldots, k \]  

Undoubtedly, (3.17) and (3.18) can both be used for parts of (3.10)
and (3.12), respectively.

4) On the other hand, suppose that the remaining ration constraints are just binding, or

\[ \bar{R}_i - X_i = 0 \quad i = 1, \ldots, g \] (3.19)

from (3.11), this implies that

\[ \lambda_{2i} \geq 0 \quad i = 1, \ldots, g \] (3.20)

substituting (3.18) and (3.20) into (3.14) separately, and noting that (3.15) represents (3.14) if (3.18) holds, we obtain (3.21) and (3.22).

\[ \frac{\partial L}{\partial X_i} = \frac{\partial U}{\partial X_i} - \lambda_i P_i = \lambda_{2i} = 0 \quad i = 1, \ldots, g \] (3.21)

\[ \frac{\partial L}{\partial X_i} = \frac{\partial U}{\partial X_i} - \lambda_i P_i \leq 0 \quad i = h, \ldots, n \] (3.22)

Thus, the simplified Kuhn-Tucker conditions consist of (3.21), (3.22), (3.16), (3.19), (3.13), (3.9), (3.20), and (3.18).

Assuming that the second-order conditions are satisfied, we can solve the first-order conditions for the above simplified constrained utility maximization, given a rationing system. Let \( \bar{R} = (\bar{R}_1, \ldots, \bar{R}_g) \), and \( P = (P_1, \ldots, P_n) \), the household demand functions for \( X_i \) (\( i = 1, \ldots, n \)) would be either

\[ X_i = f_i (\bar{R}_1, \ldots, \bar{R}_g, P_1, \ldots, P_n, M) \]

\[ = f_i (\bar{R}, P, M) \quad i = h, \ldots, n \] (3.23)

\[ \geq \bar{R}_i \quad i = h, \ldots, k \] (3.24)

or

\[ X_i = \bar{R}_i \quad i = 1, \ldots, g \] (3.25)

We have extended Howard's results into functional forms in which multiple rations are binding. From now on we are going to develop our
own way by using these results.

Especially for a particular rationed good $i$ ($i = 1, \ldots, g$), the demand function of a specific household may be either

$$X_i = f_i(\_R_1, \ldots, \_R_{i-1}, \_R_{i+1}, \ldots, \_R_g, P, \ldots, P_{n}, M)$$

or

$$X_i = f_i(\_R', P, M) \leq R_i$$

(3.26)

or

$$X_i = \_R_i$$

(3.27)

where $\_R' = (\_R_1, \ldots, \_R_{i-1}, \_R_{i+1}, \ldots, \_R_g)$ and $P$ was given before.

These results mean that the household may have either one of the above two types of demand functions for a particular rationing good. Its choice may (3.26) or may not (3.25) be directly affected by the ration. Conversely, a specific ration may or may not affect a household decision or demand function. In the former case, the household's demand would be the ration itself; whereas in the latter case, the household's demand would be a function of prices, income, and all other effective (binding) rations until it reaches the ration level. Please note the difference between the ration effect and the rationing system effect. It is true that a ration effect will lead to a rationing system effect, but it is not true that a rationing system leads to a specific ration effect. In fact, the rationing system has an effect on a household demand function if and only if there is at least one ration that is binding.

Obviously, there are two possible types of urban households if they face a rationed good, say food grain $X_j$. For the first type of household, the demand is not affected by the food grain ration but is by some other rations:
\[ X_j = f_i (\bar{R}^*, \bar{P}, M) \leq \bar{R}_j \]  

(3.28)

where \( \bar{R}^* \) is \( \bar{R}' \), but excluding \( \bar{R}_j \); and \( \bar{P} \) is the same as before.

For the second type of household, the demand is only affected by the official ration on food grain; therefore, it equals the ration,

\[ X_j = \bar{R}_j \]  

(3.29)

The formulas (3.27) and (3.28) imply the planned food grain demand of an urban household in the planned economy. Because the demand for food grain is not more than or equal to the official ration level for both the first and the second type of household, the task of a central planner is to make available the ration level of food grain to satisfy the restricted need of either the first or the second type of household. Thus, the planned demand for food grain is just the ration level \( \bar{R}_j \) to any urban household in the planned grain economy.

Comparative statics The comparative statics are made for two types of urban household.

1. The first type of urban household

The food grain demand function for the first type of household is

\[ X_j = f_i (\bar{R}^*, \bar{P}, M) \leq \bar{R}_j \]

The comparative statics can be obtained as the following:

\[ \frac{\partial X_i}{\partial R_j} < 0 \quad (i \neq j) \]  

for substitutes within the effective rationed goods

\[ \frac{\partial X_j}{\partial P_i} > 0 \quad (i \neq j) \]  

for complements within the effective rationed goods

\[ \frac{\partial X_i}{\partial R_j} = 0 \]  

for ineffective rationed goods

\[ \frac{\partial X_j}{\partial P_i} < 0 \]

\[ \frac{\partial X_j}{\partial P_i} > 0 \]
\[
\frac{\partial X_j}{\partial P_j} < 0 \quad \text{for ineffective rationed goods}
\]
\[
\frac{\partial X_j}{\partial M} > 0 \quad \text{for normal ineffective rationed goods}
\]
\[
\frac{\partial X_j}{\partial M} < 0 \quad \text{for inferior ineffective rationed goods}
\]

A brief interpretation of the results is that, for the first type of household, the grain demand decreases when grain price increases, it increases when household's income increases assuming grain as a normal good, and it does not change if the grain ration increases.

2. The second type of urban household

The food grain demand function for the second type of household is
\[
X_j = \bar{R}_j
\]

The comparative statics are shown as the following:
\[
\frac{\partial X_j}{\partial \bar{R}_i} = 0 \quad (i \neq j)
\]
\[
\frac{\partial X_j}{\partial \bar{R}_j} = 1
\]
\[
\frac{\partial X_j}{\partial P_j} = \frac{\partial X_j}{\partial P_i} = 0
\]
\[
\frac{\partial X_j}{\partial M} = 0
\]

It is easy to conclude that, for the second type of household, the grain demand is only influenced by the official ration level but not by other factors at all.

Graphic analysis The graphic analyses also are made for two...
1. The first type of urban household

As Figure 3.3a shows, $X_1$ and $X_2$ are commodities. $X_2$ is a rationed good—say food grain—but $X_1$ is not. $\bar{R}$ is the official ration of $X_2$. AB is a budget line for an urban household with rationing, and AC is a budget line for that household without rationing.

![Diagram](image)

Figure 3.3. The derivation of urban grain demand of the first type of urban household in the planned economy

For the first type of urban household, its indifference curve is tangent on the budget line AB, either at point D, which is between A and B, or at point B. Because the ration is not binding, the household need not constrain its grain consumption at all.

By assuming the normal negative sign for the price effect on the grain demand function of the first type of household, its demand curve is kinked as shown in Figure 3.3b. As usual, the demand curve is sloping downward and becomes vertical after it meets the ration line $\bar{R}$. That is, under the rationing system, the food grain consumption of the first type
of urban household increases as the state market price goes down until the ration becomes effective.

2. The second type of urban household

Facing the same state market price and official ration $\overline{R}$, the second type of urban household makes a different choice. Its indifference curve, as Figure 3.4a indicates, would be tangent to the budget line on $AC$ but beyond $B$. The ration in this case is effective, so the household is forced to reduce its grain consumption to the ration level $\overline{R}$. This means a lower indifference curve that would pass through the ration line at $B$.

![Figure 3.4. The derivation of urban grain demand of the second type of urban household in the planned economy](image)

The corresponding demand curve for food grain of the second type of urban household is a vertical line $\overline{R}$ in $(P_{2}^{S}, X_{2})$ space (see Figure 3.4b). Under the rationing system, the food grain consumption of the second type
of urban household is just the official ration level and is not affected by any change in the state market price for food grain.

**Aggregate grain demand of urban households**

According to economic theory, the aggregate urban food grain demand is the sum of all individual urban households' food grain demands:

$$D_{UP} = \sum_{j=1}^{n} X_j$$

(3.30)

where $D_{UP}$ = aggregated urban food grain demand in the planned economy and $X_j$ = individual urban food grain demand.

Considering that there are two types of urban households, the aggregate urban food grain demand $D_U$ is the sum of aggregate urban food grain demands for both the first type and the second type of urban households, or

$$D_{UP} = D_{UPI} + D_{UPII}$$

(3.31)

where $D_{UPI}$ = aggregated urban food grain demand for the first type of urban households in the planned economy

$D_{UPII}$ = aggregated urban food grain demand for the second type of urban households in the planned economy.

For the first type of urban households, whose ration coupons are not used up, we assume that there are $n$ households in this group and all of them are identical, even facing the same ration restriction, $\bar{R}_j$. The aggregate food grain demand, $D_{UPI}$, would be

$$D_{UPI} = \sum_{j=1}^{n_1} X_{j,UP}$$
For the second type of urban households, whose ration coupons are just used up, we assume that there are \( n_2 \) identical households in this group, and they face the same ration \( R_j \). Then, their aggregate food grain demand would be

\[
D_{UPII} = \sum_{j=1}^{n_2} X_j^{UP} - n_2 R_j
\]

Using the above results, \( D_{UP} \), the aggregate food grain demand in the urban area in the planned economy would be

\[
D_{UP} = D_{UPI} + D_{UPII} = n_1 X_j^{UP} (R'', P, M) + n_2 R_j \leq (n_1 + n_2) R
\]

Suppose the total number of urban households in China is \( n_u \) so that

\[
n_u = n_1 + n_2
\]

The total number of urban households is the sum of the numbers of the first and the second types of urban households. In addition let

\[
\frac{n_1}{n_u} = P_{u1}
\]

where \( P_{u1} \) is the relative frequency of the first type of urban households. Substituting (3.32) and (3.33) into (3.31) gives

\[
D_{UP} = n_1 X_j^{UP} (R'', P, M) + (n_u - n_1) R_j
\]
Thus, the aggregate urban food grain demand in the planned economy is theoretically specified as a function of the relative frequency of the first type of urban households, the total number of urban households, the official ration level for grain, the ration levels for other rationed goods, the state market price of food grain and other commodity prices, and the individual household's income. Furthermore, suppose both the first and the second type of household are the same in size and income. Then, the change in the total number of urban households is consistent with the change in the total population, and the change in the individual household's income is agreeable to the change in any representative household's income, regardless of whether the ration is binding, or in total households' income, regardless of the method for aggregation.

In analyzing formula (3.36) we notice that $D_{UP}$ consists of two components, $D_{UPI}$ and $D_{UPII}$. This conclusion is different from past studies. Checking the structure of both components, we find that they contain some different factors as well as some similar factors. This fact implies that each component sometimes plays its own role to affect the total urban grain demand, $D_{UP}$, because of the difference in the existing factors. On the other hand, both components sometimes play their roles together to affect the $D_{UP}$ because of the similarity in the existing factors.
Second, formula (3.37) indicates that the total urban grain demand is no more than the product of \( n_u \) and \( R_j \), or the planned aggregate urban grain demand. In order to estimate the urban grain demand roughly, the central planners need to find out the total number of urban households and to decide the official ration level for grain. Then they can obtain the result by performing simple multiplication.

Last, it is useful for the government to consider relaxing the ration restriction or even the ration system. By the Le Chatelier principle and observing formula (3.37), we know that the individual urban grain demand of the second type of urban household cannot be reduced by relaxing the ration constraint. In other words, the individual urban grain demand of the second type of urban household without ration constraint cannot be less than that with the ration constraint. Thus, the total urban grain demand of the second type of household cannot be reduced if the government decides to cancel the usage of ration coupons for grain. On the other hand, if the government relaxes other ration constraints except for grain, by the same principle and similar inference, we can conclude that the total urban grain demand cannot be reduced because the \( X_j \) and \( D_{UPI} \) cannot be reduced. Suppose the government relaxes all rationing systems; by the same principle, the total urban grain demand cannot be reduced because both components in \( D_{UP} \) cannot be reduced. If this is true, the total urban grain demand would be the same as the total urban grain demand in the market economy despite the planned state price policy.

**Comparative statics**  By using (3.37), the results of the
comparative statics can be calculated

$$\frac{\partial D_{UP}}{\partial u_I} = \frac{\partial D_{UPI}}{\partial u_I} + \frac{\partial D_{UPII}}{\partial u_I}$$

$$= n_u X_j - n_u R_j$$

$$= n_u (X_j - R_j) \leq 0$$

The total urban grain demand decreases or remains unchanged if the relative frequency of the first type of urban household increases.

$$\frac{\partial D_{UP}}{\partial n_u} = P_{uI} X_j + (1 - P_{uI}) R_j > 0$$

The total urban grain demand increases as the urban population increases.

$$\frac{\partial D_{UP}}{\partial R_j} = \frac{\partial D_{UPII}}{\partial R_j}$$

$$= (1 - P_{uI}) n_u > 0$$

The total urban grain demand increases if the government increases the official ration level. Please note that the ration effect on the urban grain demand becomes operative because of only one component, $D_{UPII}$ within $D_{UP}$, rather than all $D_{UP}$. In other words, the change in the official ration level affects only part of the urban households (the second type) but not all. In this sense, we name this kind of ration effect as the partial ration effect to emphasize the new discovery of the partial role of the official ration level in the planned grain economy.
The total urban grain demand is partly influenced by \( P_j \), the state price of food grain. If the sign of \( \frac{\partial X_i}{\partial P_j} \) is negative, as the state price goes up, the total urban grain demand of the first type of urban households goes down; therefore, the total urban grain demand goes down. Thus, the slope of the urban grain demand curve is determined by

\[
\frac{\partial D_{\text{UP}}}{\partial P_j} = \frac{\partial D_{\text{UPI}}}{\partial P_j}
\]

\[
= P_{uiu} \frac{\partial X_i}{\partial P_j} < 0
\]

This situation means a partial state price effect in the planned urban grain economy or the change in the government price policy for food grain can directly influence grain consumption of only part of the urban households (the first type). Differing from all previous studies, this is a new theoretical finding for the planned economies.

\[
\frac{\partial D_{\text{UP}}}{\partial P_i} = \frac{\partial D_{\text{UPII}}}{\partial P_i}
\]

\[
= P_{uiu} \frac{\partial X_i}{\partial P_i} < 0 \quad (i \neq j)
\]

\[
= P_{uiu} \frac{\partial X_i}{\partial P_i} > 0 \quad (i \neq j)
\]

Total urban grain demand decreases or increases if the relative prices of goods increase. Note that this also is the partial price effect.
The total urban grain demand increases when household income increases, provided that the food grain is a normal good for the household. This means that the government income policy for households has also a partial (income) effect on the grain consumption of a part of the households (the first type).

Graphic analysis The graphic analyses are made for aggregate urban grain demands.

Formula (3.32) gives the aggregate urban grain demand of the first type of urban household, \( D_{UPI} \). Figure 3.5a shows that the \( D_{UPI} \) is a kinked line that consists of two segments, a downward sloping line and a vertical line passing through \( Q_{UI} = P_{ul} u R_j \) on the \( Q^S \) axis in the \( (P^S, Q^S) \) space.

![Diagram](image)

(a) (b) (c)

Figure 3.5. The aggregation of the urban grain demands in the planned economy
From formula (3.33), we obtain the total urban grain demand of the second type of urban household, \( D_{UPII} \). As Figure 3.5b indicates, the \( D_{UPII} \) is a vertical line that crosses \( Q_{uII} = (1 - P_{uI})n_u R_j \) on the \( Q^S \) axis in the \((P^S, Q^S)\) space.

Finally, formula (3.34) indicates that the total urban grain demand, \( D_{UP} \), is the sum of the total urban grain demand of both the first and the second type of urban household. Figure 3.5c shows that the shape of the \( D_{UP} \) is similar to that of \( D_{UPI} \), except that (1) the aggregate constant is now \( Q_u = n_u R_j \) on the \( Q^S \) axis in the \((P^S, Q^S)\) space, and (2) the aggregate downward sloping segment in \( D_{UP} \) is steeper than that in \( D_{UPI} \) because of the effect of \( D_{UPII} \). In addition, by the Le Chatelier principle, we know that the slope in \( D_{UP} \) is much steeper than that in an assumed market economy because of the rationing system. That is why the elasticity of the urban grain demand in the planned economy may be very small but is still greater than zero.

Theory of Rural Grain Demand and Supply
of State Farms and Agricultural Production Teams
in the Planned Economy

As mentioned at the beginning of this chapter, two kinds of basic rural enterprises existed in the Chinese planned economy: state farms and agricultural production teams. The behaviors of the state farm and the production team were different from each other because of different ownerships of their properties and different policy environments faced by them. Thus, this section contains two subsections: one is studying the
behavior of the state farm in the planned grain economy; the other one is studying the behavior of the agricultural production team in the planned economy. Each subsection is divided into three parts: the policy environment, the state farm model or the agricultural production team model, and aggregate grain demand and supply of all state farms or all production teams. To begin with, the policy environment presents basic background materials for state farms or production teams. Assuming the state farm's physical output maximization or the production team's utility maximization, we build a state farm model or an agricultural production team model to derive an individual farm's or team's grain demand and supply. After that, the aggregate grain demand and supply of the state farms and the agricultural production teams are formulated from derived individual farm's or team's demands and supplies in each subsection, respectively. Finally, the state market analysis is added for exploring and stressing the roles of the government in the rural state market.

We now start with the first subsection: the theory of grain demand and supply of state farms in the planned economy.

Policy environment of state farms

The state farm sector consists of a small but significant portion of Chinese agriculture. According to statistics for 1978, state farms occupied about 4.5% of the total arable land in mainland China and produced about 2% of the grain output, which was about 10% of the total grain commodities purchased by the government.
**State-owned enterprises**  
As state-owned enterprises, state farms were completely run under the government's plans. State farms delivered their agricultural products to the government, and the government distributed equipment and inputs to the state farms. In case profit was generated, the state farms turned it over to the government, and the government gave the state farm financial assistance in case of a loss.

**Labor**  
All laborers employed by state farms were paid a fixed wage, and they were not transferable between farms without permission.

**Consumption of food grain**  
Laborers earned wage income and were free to use it to buy any commodities under the plan control. In the grain market, they were treated the same as the urban household by the government.

**State farm model**  
As discussed before, under the government's plan, the state farm manager made decisions including the labor supply for grain production, and the individual households within the farm determined their own food grain consumption. Thus, the state farm model in the planned economy can be subdivided to deal with consumption and production separately.

**Food grain demand of state farm household**  
As individual consumers facing fixed wage income and ration coupons for food grain consumption, state farm households did not differ from urban households in the state market. The urban household grain demand functions (3.28) and (3.29) can be used for the state farm household. Thus, we have

\[ X_{jS} = X_j(R', P, M) \]  
(3.38)
or

\[ X_{js} = \bar{R}_j \]  \hspace{1cm} (3.39)

For the same reasons, the formulas (3.36) and (3.37) represent the state farm households' different attitudes toward the ration coupons for food grain in the state market. The comparative statics and graphic analyses also are the same as before and, therefore, are omitted here.

**Food grain supply of state farm** Before the economic reform, the operation of the state farms was generally not profitable. Most of the state farms were established for pioneer settlement or land development in the border areas. There were several targets given by the state government to the farms. Among these targets, the gross output target might be the most important one for a state farm, whereas other targets might be met or neglected by the farm's manager. Assume that the state farm's objective had been to maximize its gross output, subject to the government's plan. Furthermore, assume that the production functions are all concave, homogeneous in a particular input or output, and twice continuously differentiable. The Lagrangian function is then

\[
L = \sum_{i=1}^{n} Q_i(X_{1}, \ldots, X_{k}) + \sum_{i=1}^{n} \lambda_{1i}(\bar{X}_h - \sum_{i=1}^{n} X_{h})
\]

\[
+ \sum_{i=1}^{n} \lambda_{2i}(Q_i - \bar{Q}_i) \]  \hspace{1cm} (3.40)

where \( Q_i \) = the \( i \)th output produced by the state farm

\( X_{h} \) = the \( h \)th input used by the state farm

\( \bar{X}_h \) = the ration on the \( h \)th input to the state farm
\( X_{hi} \) = the \( h^{th} \) input used for the \( i^{th} \) output

\( Q_{i} \) = the \( i^{th} \) output target set by the government

\( X_{h} = \sum_{i=1}^{n} X_{hi} \geq 0 \) is an input allocation constraint imposed by the government's plans. That is, the state farm can receive only \( X_{h} \) amount of the \( h^{th} \) input from the government. On the other hand, \( Q_{i} - Q_{i} \geq 0 \) is an output target set by the government. That means the state farm has to provide no less than \( Q_{i} \) amount of the \( i^{th} \) output.

The Kuhn-Tucker conditions would be:

\[
\frac{\partial L}{\partial X_{hi}} = \frac{\partial L}{\partial Q_{i}} \frac{\partial Q_{i}}{\partial X_{hi}} - \lambda_{1i} + \lambda_{2i} \frac{\partial Q_{i}}{\partial X_{hi}} \leq 0 \quad h = 1, \ldots, k \quad i = 1, \ldots, n \tag{3.41}
\]

\[
\frac{\partial L}{\partial X_{hi}} = 0 \quad h = 1, \ldots, k \quad i = 1, \ldots, n \tag{3.42}
\]

\[
\frac{\partial L}{\partial \lambda_{1i}} = X_{h} - \sum_{i=1}^{n} X_{hi} \geq 0 \quad i = 1, \ldots, n \tag{3.43}
\]

\[
\lambda_{1i} = 0 \quad i = 1, \ldots, n \tag{3.44}
\]

\[
\frac{\partial L}{\partial \lambda_{2i}} = Q_{i} - Q_{i} \geq 0 \quad i = 1, \ldots, n \tag{3.45}
\]

\[
\lambda_{2i} = 0 \quad i = 1, \ldots, n \tag{3.46}
\]

\[
X_{hi} \geq 0 \quad h = 1, \ldots, k \quad i = 1, \ldots, n \tag{3.47}
\]

\[
Q_{i} \geq 0 \quad i = 1, \ldots, n \tag{3.48}
\]

\[
\lambda_{1i} \lambda_{2i} \geq 0 \quad i = 1, \ldots, n \tag{3.49}
\]

Suppose that \( X_{hi}, Q_{i}, \) and \( \lambda_{2i} > 0 \ (h = 1, \ldots, k; \ i = 1, \ldots, n) \), then immediately we find out that (3.41) and (3.45) become
\[
\frac{\partial L}{\partial x_{hi}} = \frac{\partial L}{\partial q_i} \frac{\partial q_i}{\partial x_{hi}} - \lambda_{1i} - \lambda_{2i} \frac{\partial q_i}{\partial x_{hi}} = 0 \quad h = 1, \ldots, k \quad i = 1, \ldots, n (3.50)
\]

and
\[
\frac{\partial L}{\partial \lambda_{2i}} = q_i - \bar{q}_i = 0 \quad i = 1, \ldots, n (3.51)
\]

Furthermore, if
\[
\lambda_{1i} = 0 \quad i = 1, \ldots, n (3.52)
\]

then we have
\[
\frac{\partial L}{\partial \lambda_{1i}} = \bar{x}_i - \sum_{n=1}^{n} x_{hi} \geq 0 \quad i = 1, \ldots, n (3.53)
\]

in other words, the constants (3.53) are not strictly binding or if
\[
\lambda_{1i} > 0 \quad i = 1, \ldots, n (3.54)
\]

then
\[
\frac{\partial L}{\partial \lambda_{hi}} = \bar{x}_i - \sum_{n=1}^{k} x_{hi} = 0 \quad i = 1, \ldots, n (3.55)
\]

in other words, the constraints (3.54) are just binding.

In fact, the constraints (3.43) are generally binding because the state farm accepted planned allocations from the government, and the constraints (3.45) must be fulfilled because the output targets presented the management level for the managers of the farms and are relevant to their future official positions.

Note that if \(\lambda_{1i} = \lambda_{2i} = 0 \quad (i = 1, \ldots, n) (3.41)\) becomes
\[
\frac{\partial L}{\partial x_{hi}} = \frac{\partial L}{\partial q_i} \frac{\partial q_i}{\partial x_{hi}} = 0 \quad h = 1, \ldots, k \quad i = 1, \ldots, n (3.56)
\]
That is, there is no freedom for the state farm to choose the inputs and, therefore, outputs because every target is strictly given by the control plan. If this is true, solve the first-order conditions. Provided the second-order conditions are satisfied, we obtain the input demand $X_{hi}$'s, which would be functions of $Q_i$ and $X_h$; in other words, if both $Q_i$ and $X_h$ are just binding then

$$
X_{hi} = X_{hi}(Q_i, X_h) \quad h = 1, \ldots, k; \ i = 1, \ldots, n \quad (3.57)
$$

On the other hand, if $\lambda_{1i} = 0$ but $\lambda_{2i} > 0$, the input demand $X_{hi}$'s would be the functions of $X_h$ itself:

$$
X_{hi} = X_{hi}(X_h) \quad h = 1, \ldots, k; \ i = 1, \ldots, n \quad (3.58)
$$

Thus, the corresponding grain supply functions by the state farm would be either

$$
Q_i^F = Q_i[\bar{X}_1(Q_i, \bar{X}_h) \ldots \bar{X}_n(Q_i, \bar{X}_h)]
$$

$$
= Q_i(Q, \bar{X}_h) \quad i = 1, \ldots, n; \ h = 1, \ldots, k \quad (3.59)
$$

or

$$
Q_i^F = Q_i[\bar{X}_1(\bar{X}_h) \ldots \bar{X}_n(\bar{X}_h)]
$$

$$
= Q_i(\bar{X}_h) \quad i = 1, \ldots, n; \ h = 1, \ldots, k \quad (3.60)
$$

For the state farms, the grain supply functions are functions either of both the input allocation quotas and the output targets or of only the input quotas.
Comparative statics The comparative statics are shown only for the two different grain supplies by the state farms here.

1. $Q_i^F = Q_i(Q, X_h)$

$$\frac{\partial Q_i}{\partial Q} > 0 \quad i = 1, \ldots, n;$$

$$\frac{\partial Q_i}{\partial X_h} > 0 \quad i = 1, \ldots, n; \quad h = 1, \ldots, k$$

The state farm will probably increase its $i^{th}$ grain output if the government either improves its plan allocation for the $h^{th}$ input or raises its plan target for $i^{th}$ grain output to the farm.

2. $Q_i^F = Q_i(X_h)$

$$\frac{\partial Q_i}{\partial X_h} > 0$$

The state farm will probably increase its $i^{th}$ grain output to respond to an increase in the $h^{th}$ input supply from the government.

These results imply that the grain supply by the state farm, in both cases, does not change in response to either input or output prices, but to the relevant plans.

Graphic analysis For both supply functions, the supply curves are vertical lines in $(P^g, Q^g)$ space (Figure 3.6a or b). Figure 3.6a displays a situation in which the output target is binding; Figure 3.6b displays another situation in which the output target is not binding.
Aggregate grain demand of state farm households and supply of state farms

The food grain demand and supply by state farms has been derived. It is time now to derive the aggregate grain demand and supply for state farms. We first aggregate the grain demand of all state farmers, and then aggregate the grain supply:

Aggregate grain demand of state farm households

Because grain demands of households in the state farms are the same as those for urban consumers, it is easy to conclude that the aggregate grain demand of state farm households is the same as that of urban consumers. Thus, the aggregate grain demand for state farm households is:

\[ D^F = D^{FI} + D^{FII} \]  

\[ = P_{HFI} n_{HF} \bar{R}_j + (1 - P_{HFI}) n_{HF} X_j (R', P, M) \]

\[ = D(P_{HFI}, n_{HF}, \bar{R}_j, R', P, M) \]  

where
\( D_{FI} , D_{II} \) = aggregate grain demand for the first type or the second type of households in state farms

\( P_{HFI} \) = relative frequency of first type households in state farms

\( n_{HF} \) = total number of households in state farms

\( R_j \) = ration of food grain for households in state farms

\( R' \) = rations of other commodities except food grain for households in state farms

\( P \) = state prices of all commodities

\( M \) = wage income for households in state farms

**Aggregate grain supply of state farm households**

From the analysis for individual state farms, state farms have food grain supplies for either

\[ Q_1^F = Q_1^F(Q, \bar{X}_h) \]

or

\[ Q_1^F = Q_1^F(\bar{X}') \]

Assuming that all state farms of the same type are identical, that there are \( n_F \) state farms total, and that the relative frequency of the first type of state farms is \( P_{FFI} \), the aggregate grain supply for state farms is

\[ S^F = S^{FI} + S^{FII} \quad (3.63) \]

\[ = P_{FFI} n_F \bar{Q}_1^F(Q, \bar{X}_h) + (1 - P_{FFI}) n_F \bar{Q}_1^F(\bar{X}') \]

\[ = S(P_{FFI}, n_F, Q, \bar{X}_h, \bar{X}') \quad (3.64) \]
where $S_F^F$ = aggregate grain supply for state farms

$S_{FI}, S_{FII}$ = aggregate grain supply for the first or the second type state farms

$n_F$ = total number of state farms

$\bar{Q}$ = output target set by the government

$\bar{X}_h$ = input ration that is binding for the first type of state farm

$\bar{X}'$ = input rations that are binding for the second type of state farm

**Comparative statics** We are conducting comparative statics for only the aggregate grain supply of the state farms and dropping out comparative statics for the aggregate grain demand of the state farm households because of similarity between the individual grain demands for the state farm households and urban households. Therefore, the aggregate grain demand for the state farm households is similar to those for the urban households. The following are comparative statics for the aggregate grain supply for state farms.

By formulas (3.63) and (3.64)

$S_F^F = S_{FI} + S_{FII}$

$S_{FI} = P_{FPI}n_F\bar{Q}_i^{F}(\bar{Q}, \bar{X}_h)$

$\frac{\partial S_{FI}}{\partial P_{FF}} = n_F\bar{Q}_i^{F} > 0$

$\frac{\partial S_{FI}}{\partial n_F} = P_{FPI}\bar{Q}_i^{F} > 0$
That is, the aggregate grain supply by the first type of state farm increases if the relative frequency of the first type of state farm within all state farms, or the total number of state farms, or the output target set for the state farms increases. On the other hand, the aggregate grain supply by the first type of state farm may increase or decrease if the input ration increases.

\[
\frac{\partial s_{FI}}{\partial Q} = p_{FFI}n_F \frac{\partial q_i^F}{\partial Q} > 0
\]

\[
\frac{\partial s_{FI}}{\partial x_h} = p_{FFI}n_F \frac{\partial q_i^F}{\partial x_h} < 0 \quad \text{as} \quad \frac{\partial q_i^F}{\partial x_h} > 0
\]

Thus, we finally have

\[
\frac{\partial s_{FI}}{\partial p_{FFI}} = \frac{\partial s_{FI}}{\partial p_{FFI}} + \frac{\partial s_{FII}}{\partial p_{FFI}}
\]

\[
- n_F q_i^F(\bar{Q}, \bar{x}_h) - n_F q_i^F(\bar{x}') < 0
\]
That is, the aggregate grain supply by the state farms is increasing if the relative frequency of the first type of state farms decreases, or the total number of state farms or the output target set for the state farms increases. If the government raises an input ration, the result in the aggregate grain supply by the state farms is still unclear.

Note that the output target effect on the aggregate grain supply is partially through the aggregate grain supply by the first type of state farms but not the second type. This is a partial output target effect. In addition, the state grain market price has no effect on the aggregate grain supply by the state farms.

**Graphic analysis** The graphic analysis is subdivided into two parts:

1. Aggregate grain demand by state farm households

The shape of aggregate grain demand of the state farm households is the same (kinked) as that of urban households, but the intercept on the Q axis has been changed from \( n_{u_i} \) to \( n_{f_i} \) (Figure 3.7).
2. Aggregate grain supply by state farms

The aggregate grain supply by state farms is the sum of the aggregate grain supply by both the first and the second type of state farms. The former is drawn in Figure 3.8a, the latter is drawn in Figure 3.8b, and their sum—the aggregate grain supply by state farms—is given in Figure 3.8c. As Figure 3.8 shows, all of the three curves are vertical lines in the (Q, P) space and have respective intercepts:

$$Q_{FI} = P_{FFI} n_{FI} Q_{FI}^F, \quad Q_{FII} = (1-P_{FFI}) n_{FI} Q_{FI}^F, \quad Q_F = Q_{FI} + Q_{FII}.$$ 

We have finished discussion of the first subsection. We now move on to the second subsection: theory of grain demand and supply of agricultural production teams in the planned economy.

**Policy environment of agricultural production teams**

**Collective of agricultural households**  As a basic unit of production and distribution in rural areas, a production team made its own decisions to buy agricultural inputs from the state for use, to
produce agricultural commodities by allocating its resources, to pay the agricultural taxes to the government, to sell products to the state market, and, finally, to distribute part of its products and income to its household members and to keep the necessary stock for its further reproduction. Thus, given the plan and the market control, the households' incomes and food grain consumption were mainly determined by their production teams.

**Labor** The agricultural laborers were hired by the production team. Without approval from higher authority, the laborers could not be mobile even between teams. Usually, production teams set the minimum working days for laborers according to their ages, sex, and physical conditions.

The laborers were paid annually by accounting their accumulated working points, which were determined on the basis of their working assignments and times, working attitudes and experiences, ages, and sex, etc. The accumulated working points were used to finance the grain

![Diagram](image_url)  
**Figure 3.8.** The aggregation of grain supplies of the state farms in the planned economy
apportionment plus remaining cash income.

**Plan control**  Most of the economic activities of the production teams were restricted or guided by the government's plans directly or indirectly. On the production side, the government set some minimum area quotas for the main crops and rationed purchases of some inputs in short supply such as chemicals, fertilizer, diesel fuel. On the consumption side, the government set minimum delivery quotas or sales quotas for the main grain commodities as well as others produced by the team, and a floor quota as well as a ceiling quota for food grain distribution within the production team. Finally, the government set stock quotas for the production teams. All quotas just mentioned would be determined by local authorities and varied from place to place and from year to year.

**Market control**  The grain market for rural people was a state monopolized system in the planned economy. The government reserved for itself the exclusive right to purchase grain from the production teams by using a quota system. Moreover, the government decided the procurement prices of grain in the state market and imposed agricultural taxes that were normally fixed in grain upon the production teams.

The government procured grain mainly from the majority of production teams that were totally self-sufficient; they were able to meet their floor consumption quotas after satisfying the procurement quotas and were not allowed to buy any food grain from the state market. Thus, those production teams are called nonbuyers or complete sellers, and they are classified into the first type in this study.

There was a channel through which rural households could buy food
grain from the government. Two additional types, the second and third types of production teams, had the government's permission to do so.

The second type of production teams are called partial buyers or partial sellers in the state market. They produced grain and fulfilled the delivery quotas set by the government, but they could not meet the floor consumption quotas for their household members because of either predictable reasons such as normally poor production conditions or unpredictable reasons such as some accidental disasters. Hence, they were given the right to buy grain back from the government until their floor quotas were met.

The third type of production teams are called complete buyers or nonsellers in this study. According to the government's plan, they did not produce food grain, but produced other agricultural commodities such as cash crops, livestock, vegetables, fruits. The government set the ceiling consumption quotas for those production teams, and they bought all their food grain from the state market.

Agricultural production team model

For a production team, both production and consumption were managed by the team leader and guided directly or indirectly by the government's plans. Suppose the team's objective is to maximize its utility subject to its budget constraint, as well as several other constraints given by government's policies. Furthermore, suppose the team's utility function $U(X_1, \ldots, X_n)$ and production function $G(q_1, \ldots, q_n)$ have the usual
properties mentioned earlier in this chapter as well as in general microeconomic theory. To reflect the distribution activities happening within the production teams, we developed Siciliano's model (1983) by adding two more constraints. In our model, the Lagrangian function for the production team would be

\[
L = U(X_1, \ldots, X_n) + \lambda_1 \sum_{i=1}^{n} (p_i q_i - p_i X_i) + \lambda_2 G(q_1, \ldots, q_n) + \sum_{i=1}^{n} \lambda_3 (q_i - X_i - S_i) + \sum_{i=1}^{n} \lambda_4 (\bar{X}_{ic} - X_i) + \sum_{i=1}^{n} \lambda_5 (X_i - \bar{X}_i) \tag{3.65}
\]

where

- \(X_i\) - the \(i^{th}\) commodity consumed by the production team
- \(q_i\) - the \(i^{th}\) commodity produced by the production team
- \(S_i\) - sales quota for the \(i^{th}\) commodity
- \(\bar{X}_{ic}\) - ceiling consumption quota set for the \(i^{th}\) commodity
- \(\bar{X}_{if}\) - floor quota set for the \(i^{th}\) commodity

\(\sum_i (p_i q_i - p_i X_i) \geq 0\) is a budget constraint for the team. That is, the team's total income from its production cannot be less than its total expenditure.

\(\bar{X}_{ic} - X_i \geq 0\) or \(X_i - \bar{X}_{if} \geq 0\) is a constraint that represents a ceiling consumption quota or a floor consumption quota set by the government, in other words, the team's consumption of the \(i^{th}\) commodity cannot exceed the ceiling limit or cannot be below the floor limit. Last, \(q_i - X_i - S_i \geq 0\) is a fixed sales quota imposed by the government. That is, the
team's $i^{th}$ output has to be not less than the sum of the team's consumption and its sales quota.

The Kuhn-Tucker conditions for our model are now:

\[
\frac{\partial L}{\partial x_i} = \frac{\partial u}{\partial x_i} - \lambda_{1i} p_i - \lambda_{3i} - \lambda_{4i} - \lambda_{5i} \leq 0 \quad i = 1, \ldots, n \quad (3.66)
\]

\[
x_i \frac{\partial L}{\partial x_i} = 0 \quad i = 1, \ldots, n \quad (3.67)
\]

\[
\frac{\partial L}{\partial q_i} = \lambda_{1i} p_i - \lambda_{2i} \frac{\partial G}{\partial q_i} - \lambda_{3i} \leq 0 \quad i = 1, \ldots, n \quad (3.68)
\]

\[
q_i \frac{\partial L}{\partial q_i} = 0 \quad i = 1, \ldots, n \quad (3.69)
\]

\[
\frac{\partial L}{\partial \lambda_{1i}} = \sum_{i=1}^{n} (p_i q_i - p_i x_i) \geq 0 \quad (3.70)
\]

\[
\lambda_{1i} \frac{\partial L}{\partial \lambda_{1i}} = 0 \quad i = 1, \ldots, n \quad (3.71)
\]

\[
\frac{\partial L}{\partial \lambda_{2i}} = G(q_1, \ldots, q_n) = 0 \quad i = 1, \ldots, n \quad (3.72)
\]

\[
\frac{\partial L}{\partial \lambda_{3i}} = q_i - x_i - \bar{s}_i \geq 0 \quad i = 1, \ldots, n \quad (3.73)
\]

\[
\lambda_{3i} \frac{\partial L}{\partial \lambda_{3i}} = 0 \quad i = 1, \ldots, n \quad (3.74)
\]

\[
\frac{\partial L}{\partial \lambda_{4i}} = \bar{x}_{1c} - x_i \geq 0 \quad i = 1, \ldots, n \quad (3.75)
\]

\[
\lambda_{4i} \frac{\partial L}{\partial \lambda_{4i}} = 0 \quad i = 1, \ldots, n \quad (3.76)
\]

\[
\frac{\partial L}{\partial \lambda_{5i}} = x_i - \bar{x}_{1p} \geq 0 \quad i = 1, \ldots, n \quad (3.77)
\]

\[
\lambda_{5i} \frac{\partial L}{\partial \lambda_{5i}} = 0 \quad i = 1, \ldots, n \quad (3.78)
\]

Unlike a usual agricultural household model, our production team model has three additional constraints: (3.73), (3.75), and (3.77).
Thus the results in our model, or the grain demand and supply by the production team, would be influenced by the constraint(s) if at least one constraint is binding or not influenced if no constraint is binding.

The situations for constraint binding can be easily explained. For instance, if (3.75) or (3.77) is binding, the food grain demand of the team is fixed at the ceiling level or the floor level. In addition, as Sicular (1986) pointed out, if (3.73) is binding, that is, $q_i - X_i = \bar{s}_i$, given the sales quota $\bar{s}_i$, the grain demand and the grain supply by the team are dependent on each other, and separability of the model of the agricultural production team is eliminated by this binding constraint.

Using the same procedures as in the urban demand section; making the necessary assumptions, such as the positiveness of all demand $X_i$ and supply $q_i$, equality of budget constraint; discussing the combined situations for these additional constraints; then solving the first order conditions simultaneously, we obtain the food grain demand function $X_i$ and the food grain supply $q_i$ for three different cases:

<table>
<thead>
<tr>
<th>Demand Function</th>
<th>Supply Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td></td>
</tr>
<tr>
<td>(3.77) is binding</td>
<td></td>
</tr>
<tr>
<td>1.1. (3.73) is binding</td>
<td>$q_i - \bar{X}_i + \bar{s}_i$ (3.80)</td>
</tr>
<tr>
<td>1.2. (3.73) is not binding</td>
<td>$q_i = q_i(P)$ (3.81)</td>
</tr>
<tr>
<td>Case 2</td>
<td></td>
</tr>
<tr>
<td>both (3.77) and (3.73) are not binding</td>
<td></td>
</tr>
<tr>
<td>2.1. (3.75) is binding</td>
<td>$X_i = \bar{X}_i$ (3.82)</td>
</tr>
<tr>
<td>2.2. (3.75) is not binding</td>
<td>$q_i = q_i(P)$ (3.83)</td>
</tr>
</tbody>
</table>
Case 3

(3.77) is not binding but (3.73) is binding

3.1. (3.75) is binding

\[ X_i = \bar{X}_{ic} \]  \hspace{1cm} (3.85) \quad q_i = \bar{X}_{ic} + S_i \]  \hspace{1cm} (3.86)

3.2. (3.75) is not binding

\[ X_i = X_i(P, S_i) \]  \hspace{1cm} (3.87) \quad q_i = X_i(P, S_i) + S_i \]

These results indicate that, under certain circumstances, the grain demand function by the production team is either fixed \( \bar{X}_{ic} \) or \( \bar{X}_{ic} \) or varied \( X_i(P) \). In other words, the grain demand \( X_i \) is either the floor quota or the ceiling quota or a function of the consumer and the producer prices. On the other hand, the grain supply function of the production team is either fixed \( \bar{X}_{ic} + S_i \) or varied \( q_i(P) \) or \( q_i(P, S_i) \). In other words, the grain supply \( q_i \) is either the sum of the floor quota and the sales quota for grain or a function of the consumer and producer prices or of the prices and sales quotas. The corresponding relationship between the grain demand and the grain supply can be easily checked.

Case 3.2 is an extremely restricted one. By the formula (3.88)

\[ q_i(P, S_i) = X_i(P, S_i) + S_i \]

Thus, both the supply and the demand curves must have the same positive or negative slopes with respect to their own price change and must keep an equal horizontal distance \( S_i \) from each other in the \((Q, P)\) space. Assuming nonpositiveness of the slope of the demand curve and nonnegativeness of the slope of the supply curve, we drop out further discussion about the case in this study.

It is obvious that Case 3.1 and all the remaining cases (Case 1 and Case 2) can be included in the activities of the first type of production.
teams, which is nonbuyer and complete seller in the state market. In addition to this type, two more special types of production teams exist; and therefore, our basic model (3.65) has to be adjusted to fit their cases.

The second type of production team produced grain that was less than the sum of the floor consumption quota and sales quota. Thus, instead of (3.73) we have

\[ q_i - X_i - \bar{S}_i < 0 \quad i = 1, \ldots, n \quad (3.89) \]

for this case in our model. Noting that (3.75) is not binding, solving the new first order conditions we obtain

<table>
<thead>
<tr>
<th>Case 4</th>
<th>Demand Function</th>
<th>Supply Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.77) is binding</td>
<td>( X_i = X_{iF} )</td>
<td>( q_i = q_i(P) )</td>
</tr>
<tr>
<td>both (3.75) and (3.89)</td>
<td>( \bar{S}_i = 0 )</td>
<td>(3.90)</td>
</tr>
<tr>
<td>are not binding</td>
<td></td>
<td>(3.91)</td>
</tr>
</tbody>
</table>

Another case applies to the third type of production teams, which do not produce grain by permission, \( q_i = 0 \) then \( \bar{S}_i = 0 \), and their grain consumption up to the ceiling quota is supplied by the government. In this case, (3.75) is dropped out of our model. Because of a drain on their strength and relatively lower ceiling quota as well as price level, the production team would not buy grain below the ceiling level. Thus, we have

<table>
<thead>
<tr>
<th>Case 5</th>
<th>Demand Function</th>
<th>Supply Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.75) is binding</td>
<td>( X_i = X_{ic} )</td>
<td>( q_i = 0 )</td>
</tr>
<tr>
<td>and ( \bar{S}_i = 0 )</td>
<td></td>
<td>(3.92)</td>
</tr>
<tr>
<td>(3.77) is not binding</td>
<td></td>
<td>(3.93)</td>
</tr>
</tbody>
</table>
In analyzing all of the above realistic demand functions of production teams, note the roles played by the government's floor quotas and ceiling quotas. Indeed, all possible demand functions are not less than the floor quota and not more than the ceiling quota, that is,

\[
\bar{X}_{jF} \leq \bar{X}_j \leq \bar{X}_{jc} \tag{3.94}
\]

In other words, the floor quota \(\bar{X}_{jF}\) can be seen as the lower bound of grain demand by production teams and the ceiling quota \(\bar{X}_{jc}\) can be seen as the upper bound. The government can easily figure out both the minimum and the maximum grain demand by any production team. Thus, the government uses the floor quota and the ceiling quota as the policy instruments to plan rural grain demand for production teams.

Turning to analyze the corresponding supply functions of production teams, we find out the role played by the government sales quotas. All supply functions are not less than the sales quota except for that of the second and the third type of production teams, that is

\[
\bar{X}_{jF} + \bar{S}_j \leq q_j \tag{3.95}
\]

In other words, the sum of the floor quota and the sales quota \(\bar{X}_{jF} + \bar{S}_j\) is the lower bound of grain supply by the first type of production teams, which is the majority within production teams. The government can easily figure out the minimum grain supply by a usual production team. Thus, the government uses the floor quota or sales quota as a policy instrument to guarantee planned grain supply.

**Comparative statics** The comparative statics are reported for all three types of production teams as follows:
1. The first type of production teams in the state market

There are three cases for the first type of production teams:

Case 1

1.1. \( X_1 = X_{1F}, \quad q_1 = \bar{x}_{1F} + s_1 \)

\[
\frac{\partial X_1}{\partial X_{1F}} = 1
\]

\[
\frac{\partial X_1}{\partial P_1} = \frac{\partial X_1}{\partial S_1} = 0
\]

\[
\frac{\partial q_1}{\partial S_1} = \frac{\partial q_1}{\partial X_{1F}} = 0
\]

\[
\frac{\partial q_1}{\partial P_1} = 0
\]

Both the grain demand and supply by the production team are increasing as the government raises its floor consumption quota. Only the grain demand, not the grain supply, is increasing as the government raises its sales quota. Finally, both the grain demand and supply keep unchanged when the government changes the grain price.

1.2. \( X_1 = \bar{x}_{1F}, \quad q_1 = q_1(P) \)

\[
\frac{\partial X_1}{\partial P_1}, \quad \frac{\partial X_1}{\partial S_1}, \quad \frac{\partial X_1}{\partial X_{1F}} \text{ are the same as that in case 1.1.}
\]

\[
\frac{\partial q_1}{\partial P_1} > 0
\]

\[
\frac{\partial q_1}{\partial S_1} = \frac{\partial q_1}{\partial X_1} = 0
\]
The grain demand increases, but the grain supply does not change, if the government raises the floor consumption quota. On the other hand, the grain demand does not change, but the grain supply increases, if the government raises its grain price.

Case 2

2.1. \( X_i = \bar{X}_{ic}, \ q_i = q_i(P) \)

These functions are the same as those in Case 1.2 except the ceiling quota \( \bar{X}_{ic} \) is used here instead of the floor quota \( \bar{X}_{IF} \). Thus, we omit comparative statics for Case 2.1.

2.2. \( X_i = X_i(P), \ q_i = q_i(P) \)

\[
\frac{\partial X_i}{\partial P_i} < 0
\]

\[
\frac{\partial X_i}{\partial S_i} = \frac{\partial X_i}{\partial \bar{X}_{ic}} = 0
\]

\[
\frac{\partial q_i}{\partial P_i} = 0
\]

\[
\frac{\partial q_i}{\partial S_i} = \frac{\partial q_i}{\partial \bar{X}_{ic}} = 0
\]

These are the same as the standard demand and supply functions in a market economy except that the price here is the state market price.

Case 3

3.1. \( X_i = \bar{X}_{ic}, \ q_i = \bar{X}_{ic} + \bar{s}_i \)

These functions are the same as those in case 1.1 except that the ceiling quota \( \bar{X}_{ic} \) here replaces the floor quota \( \bar{X}_{IF} \). Thus, we omit comparative statics for Case 3.1.
2. The second type of production teams in the state market

Case 4 \( X_i = \bar{x}_{1F}, q_i = q_i(P) \)

\[
\frac{\partial X_i}{\partial x_{1F}} = 1 \\
\frac{\partial X_i}{\partial p_i} = \frac{\partial X_i}{\partial s_i} = 0 \\
\frac{\partial q_i}{\partial p_i} > 0 \\
\frac{\partial q_i}{\partial x_{1F}} = \frac{\partial q_i}{\partial s_i} = 0
\]

The grain demand is fixed at the floor consumption quota and is increasing as the floor quota increases. The grain supply is increasing as the grain price increases.

3. The third type of production team in the state market

Case 5 \( X_i = \bar{x}_{1C}, q_i = 0 \)

\[
\frac{\partial X_i}{\partial x_{1C}} = 1 \\
\frac{\partial X_i}{\partial s_i} = \frac{\partial X_i}{\partial p_i} = 0 \\
\frac{\partial q_i}{\partial s_i} = \frac{\partial q_i}{\partial p_i} = \frac{\partial q_i}{\partial x_{1F}} = 0
\]

The grain demand is increasing if the ceiling consumption quota increases. The grain supply is always zero.

**Graphic analysis** The graphic analysis is made in the combined market with the state price for all three types of production teams.
1. The first type of production teams

Case 1

For Case 1.1, both the grain demand and supply curves are vertical lines in (P, Q) space (Figure 3.9a). While the grain demand is passing through $X_{1F}$ on Q axis, the grain supply $q_i$ is passing through $X_{1F} + S_i$ on the Q axis. Thus, the horizontal distance between the two curves is $S_i$.

Figure 3.9. The demand and the supply curves in Case 1

For Case 1.2, the demand curve is still vertical but the supply curve is upward sloping, and the horizontal distance between the demand and the supply is greater than $S_i$ because $S_i$ is not binding (Figure 3.9b).

Case 2

For Case 2.1, the demand curve is a vertical line and the supply curve is upward sloping, and the horizontal distance between these two curves is always greater than $S_i$ (Figure 3.10a).
For Case 2.2, the demand slopes downward and the supply slopes upward (Figure 3.10b), and the horizontal distance between the demand and the supply curve is greater than \( S_1 \) because \( X_1 \) has to be more than \( \bar{X}_{1F} \), the floor consumption quota, and less than \( \bar{X}_{1c} \), the ceiling consumption quota. The demand curve becomes vertical at \( X_1 = \bar{X}_{1F} \) and \( X_1 = \bar{X}_{1c} \).

Case 3

The graph for Case 3.1 is the same as that for case 1.1 (Figure 3.9a) except that the intercept \( \bar{X}_{1F} \) is replaced by \( \bar{X}_{1c} \).

2. The second type of production teams

Case 4

As Figure 3.11a shows, the grain demand \( X_1 \) is a vertical line passing through \( \bar{X}_{1F} \) on \( Q_1 \) axis, and the grain supply \( q_1 \) is upward sloping and on the left of a vertical line that passes through \( \bar{X}_{1F} + S_1 \) on \( Q_1 \) axis. The supply curve \( q_1 \) crosses the demand curve \( X_1 \). Given price \( P \), if \( q_1 \) is on the right of \( X_1 \), the team is a net seller in the combined market. On the other hand, if \( q_1 \) is on the left of \( X_1 \), the team becomes
a net buyer in the combined market.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3_11.png}
\caption{The demand and supply curves in Case 4 and Case 5}
\end{figure}

3. The third type of production teams

As Figure 3.11b shows, the demand curve \( X_1 \) for Case 5 is a vertical line that passes through \( X_{1c} \) on the \( Q_1 \) axis.

**Aggregate grain demand and supply of production teams**

**Aggregate grain demand of production teams** There are three types of production teams, according to their activities in the state market, or, alternatively, their situations relative to the ceiling quota or the floor quota and the sales quota. Situations are complex for the first type of production teams. Within the first type of teams, there are three subtypes that may contain even two different cases. Start with the first subtype of production teams. Assuming that the total number of all production teams is \( n_T \), the relative frequency of the first type and the first subtype (within the first type) of production teams are \( P_{T_1} \) and
are $P_{TI}$ and $P_{TI1}$, respectively. Furthermore, assuming that all teams of the first subtype are identical, we can obtain $D^{TI1}$, the aggregate grain demand by the first subtype of production teams, as follows:

$$D^{TI1} = P_{TI} P_{TI1} n_T \bar{X}_{1F}$$

$$= D^{TI1} (P_{TI}, P_{TI1}, n_T, \bar{X}_{1F}) \quad (3.96)$$

By using the same method, we can derive $D^{TI2}$ and $D^{TI3}$, the aggregate grain demand of the second and the third subtype of production teams. The only difference for $D^{TI2}$ is that it contains two components rather than one to reflect Cases 2.1 and 2.2 within the subtype. Thus,

$$D^{TI2} = D^{TI21} + D^{TI22}$$

$$= P_{TI} P_{TI2} P_{TI21} n_T \bar{X}_{1c} + P_{TI} P_{TI2} (1 - P_{TI21}) n_T \bar{X}_{1c}(P)$$

$$= P_{TI} P_{TI2} [P_{TI21} n_T \bar{X}_{1c} + (1 - P_{TI21}) n_T \bar{X}_{1c}(P)]$$

$$= D^{TI2} (P_{TI}, P_{TI2}, P_{TI21}, n_T, \bar{X}_{1c}, P) \quad (3.97)$$

and

$$D^{TI3} = P_{TI} (1 - P_{TI1} - P_{TI2}) n_T \bar{X}_{1c}$$

$$= D^{TI3} (P_{TI}, P_{TI1}, P_{TI2}, n_T, \bar{X}_{1c}) \quad (3.98)$$

Adding (3.96), (3.97) and (3.98) together, we obtain $D^{TI}$, aggregate grain demand by the first type of production teams.

$$D^{TI} = D^{TI1} + D^{TI2} + D^{TI3}$$
That is, the aggregate grain demand by the first type of production teams is a function of the relative frequencies \( P_{TI} \), \( P_{TI1} \), \( P_{TI2} \), and \( P_{TI21} \), the total number of production teams, the floor consumption quotas, the ceiling consumption quotas, and commodity prices.

Obviously, we can derive \( D_{TII} \) and \( D_{TIII} \), aggregate grain demand by the second and the third type of production teams in the same way:

\[
D_{TII} = P_{TI} n_T \bar{X}_{1F}
\]

\[
D_{TII} = D_{TII} (P_{TI}, n_T, \bar{X}_{1F})
\] (3.100)

and

\[
D_{TIII} = (1 - P_{TI} - P_{TIII}) n_T \bar{X}_{1c}
\]

\[
D_{TIII} = D_{TIII} (P_{TI}, P_{TIII}, n_T, \bar{X}_{1c})
\] (3.101)

Thus, the aggregate grain demand by the second and the third type of production teams are a function of the relative frequency \( P_{TIII} \), the total number of production teams, and the floor quotas, and a function of the relative frequencies \( P_{TI} \) and \( P_{TIII} \), the total number of teams, and the ceiling quotas.
Finally, $D^T$, the aggregate grain demand of all production teams can be formulated by summing (3.99), (3.100) and (3.101).

$$D^T = D^{TI} + D^{TIII}$$

$$= P_{TI} \left[ P_{TI1} n_T \bar{X}_{1F} + P_{TI2} \left[ P_{TI21} n_T \bar{X}_{1c} ight. \right.$$

$$+ (1 - P_{TI21}) n_T \bar{X}_1 (P)) + (1 - P_{TI1} - P_{TI2}) n_T \bar{X}_{1c} \right]$$

$$+ P_{TIII} n_T \bar{X}_{1F} + (1 - P_{TI} - P_{TIII}) n_T \bar{X}_{1c}$$

$$= D^{TI} \left( P_{TI}, P_{TIII}, P_{TI1}, P_{TI2}, P_{TI21}, n_T, \bar{X}_{1F}, \bar{X}_{1c}, P \right)$$ (3.103)

where

$D^T = \text{aggregate grain demand by production teams}$

$D^{TI}, D^{TII}, D^{TIII} = \text{aggregate grain demand by the first, second, and third type of production teams}$

$P_{TI}, P_{TIII} = \text{the relative frequency of the first and the second type of production teams}$

$P_{TI1}, P_{TI2} = \text{the relative frequency of the first or the second subtype within the first type of production teams}$

$P_{TI21} = \text{the relative frequency of case 2.1 of the first subtype of production teams}$

$n_T = \text{the total number of production teams}$

$\bar{X}_{1F} = \text{the floor quota for production teams}$

$\bar{X}_{1c} = \text{the ceiling quota for production teams}$

$P = \text{prices of commodities}$

Formula (3.115) indicates that aggregate grain demand of production teams is a function of the relative frequencies $P_{TI}, P_{TIII}, P_{TI1}, P_{TI2}$.
and \( P_{TI21} \), total number of production teams \( n_T \), the floor quota and the ceiling quota, and commodity prices.

In addition, observing (3.100) and (3.101) we note that

\[
P_{TI} n_T \bar{X}_{IF} < D^{TI} - P_{TI} n_T \bar{X}_{1c} \tag{3.104}
\]

\[
(1 - P_{TII} - P_{TI}) n_T \bar{X}_{IF} = D^{TI} < (1 - P_{TII} - P_{TI2}) n_T \bar{X}_{1c} \tag{3.105}
\]

That is, the aggregate grain demand by the second type of production teams is staying on the upper bound \( P_{TI} n_T \bar{X}_{1c} \) under the government policy, whereas the aggregate grain demand by the third type of production teams is staying on the lower bound \( (1 - P_{TII} - P_{TI}) n_T \bar{X}_{IF} \) restricted by the government. Furthermore, looking at (3.96), (3.97) and (3.98) carefully, we find that

\[
P_{TII} P_{TII1} n_T \bar{X}_{IF} - D^{TII} < P_{TII} P_{TII1} n_T \bar{X}_{1c} \tag{3.106}
\]

\[
P_{TII} P_{TII2} n_T \bar{X}_{IF} - D^{TII} < P_{TII} P_{TII2} n_T \bar{X}_{1c} \tag{3.107}
\]

\[
P_{TII}(1-P_{TII} - P_{TI2}) n_T \bar{X}_{IF} < D^{TII} - P_{TII}(1-P_{TII} - P_{TI2}) n_T \bar{X}_{1c} \tag{3.108}
\]

The second inequality (3.107) is obtained by adding the following two inequalities:

\[
P_{TII} P_{TII2} P_{TI21} n_T \bar{X}_{IF} < D^{TII} - P_{TII} P_{TII2} P_{TI21} n_T \bar{X}_{1c} \tag{3.109}
\]

\[
P_{TII} P_{TII2}(1-P_{TI21}) n_T \bar{X}_{IF} < D^{TII} - P_{TII} P_{TII2}(1-P_{TI21}) n_T \bar{X}_{1c} \tag{3.110}
\]

Now, we add (3.106), (3.107) and (3.108) into a new inequality:

\[
P_{TII} n_T \bar{X}_{IF} < D^{TII} < P_{TII} n_T \bar{X}_{1c} \tag{3.111}
\]
That is, $D_{\text{TI}}$, aggregate grain demand by the first type of production teams is more than $P_{\text{TI}} n_T \bar{X}_{iF}$ and less than $P_{\text{TI}} n_T \bar{X}_{1c}$. Thus, the former is its lower bound and the latter is its upper bound. Finally, summing (3.104), (3.105) and (3.111), we immediately have

$$n_T \bar{X}_{iF} < D^T < n_T \bar{X}_{1c} \tag{3.112}$$

That means aggregate grain demand by production teams in the planned economy is between the product of total number of production teams and the floor quota and the product of total number of production teams and the ceiling point. In other words, $n_T \bar{X}_{iF}$ and $n_T \bar{X}_{1c}$ are the lower bound and the upper bound for the aggregate grain demand by production teams, respectively. In this case, the government can easily plan and manage the grain demand for production teams.

**Aggregate grain supply of production teams**

By using the same aggregation method as in the analysis of aggregate grain demand by production teams, and making appropriate assumptions such as the total number of production teams, the relative frequencies, and identicalness of production teams within a certain type or subtype, we can derive

$$S_{\text{TI}} = S_{\text{TI1}} + S_{\text{TI2}} + S_{\text{TI3}} \tag{3.113}$$

where

$$S_{\text{TI1}} = S_{\text{TI11}} + S_{\text{TI12}}$$

$$= P_{\text{TI}} P_{\text{TII1}} n_T (\bar{X}_{iF} + \bar{S}_1) + P_{\text{TI}} P_{\text{TII1}} (1-P_{\text{TII1}}) n_T q_1(P)$$

$$= P_{\text{TI}} P_{\text{TII1}} [P_{\text{TII1}} n_T (\bar{X}_{iF} + \bar{S}_1) + (1-P_{\text{TII1}}) n_T q_1(P)]$$

$$= S_{\text{TI}} (P_{\text{TI}}, P_{\text{TII1}}, P_{\text{TII1}}, n_T, \bar{X}_{iF}, \bar{S}_1, P) \tag{3.114}$$
Substituting (3.114), (3.115), and (3.116) into (3.113) we obtain

\[ S_{TI}^{II} = P_{TI} P_{TI1} n_T q_1(P) \]
\[ = S_{TI}^{II} (P_{TI}, P_{TI2}, n_T, P) \]  

(3.115)

and

\[ S_{TI}^{III} = P_{TI} (1 - P_{TI1} - P_{TI21}) (\bar{X}_{1c} + \bar{S}_i) \]
\[ = S_{TI}^{III} (P_{TI}, P_{TI1}, P_{TI2}, \bar{X}_{1c}, \bar{S}_i) \]  

(3.116)

Substituting (3.114), (3.115), and (3.116) into (3.113) we obtain

\[ S_{TI}^{II} = P_{TI} P_{TI1} \left( P_{TI11} n_T (\bar{X}_{1F} + \bar{S}_i) + (1-P_{TI11}) n_T q_1(P) \right) \]
\[ + P_{TI} P_{TI2} n_T q_1(P) + P_{TI} (1 - P_{TI1} - P_{TI2}) (\bar{X}_{1c} + \bar{S}_i) \]
\[ - P_{TI} (P_{TI1} n_T (\bar{X}_{1F} + \bar{S}_i) + (1-P_{TI1}) n_T q_1(P)) \]
\[ + P_{TI2} n_T q_1(P) + (1 - P_{TI1} - P_{TI2}) (\bar{X}_{1c} + \bar{S}_i) \)
\[ = S_{TI}^{II} (P_{TI}, P_{TI1}, P_{TI2}, P_{TI11}, n_T, \bar{X}_{1F}, \bar{X}_{1c}, \bar{S}_i, P) \]  

(3.117)

By using the same procedures, \( S_{TI}^{III} \) and \( S_{TI}^{III} \) can be derived.

\[ S_{TI}^{II} = P_{TI} n_T q_1(P) \]
\[ = S_{TI}^{II} (P_{TI}, n_T, P) \]  

(3.118)

and

\[ S_{TI}^{III} = 0 \]  

(3.119)

Formulas (3.117), (3.118) and (3.119) specify aggregate grain supply functions by the first, the second, and the third type of production teams, respectively. Adding (3.117), (3.118) and (3.119) together, we
finally get

\[ S_T = S_{T_1} + S_{T_2} + S_{T_3} \]  \hspace{1cm} (3.120)

\[ = P_{T_1} \left( P_{T_{11}} n_T (\bar{X}_{1F} + \bar{S}_1) + (1-P_{T_{11}}) n_T q_i(P) \right) \]

\[ + P_{T_{12}} n_T q_i(P) + (1 - P_{T_{11}} - P_{T_{12}})(\bar{X}_{1C} + \bar{S}_1) \]  \hspace{1cm} (3.121)

\[ + P_{T_1} n_T q_i(P) \]

where

- \( S_T \) = aggregate grain supply of production teams
- \( S_{T_1}, S_{T_2}, S_{T_3} \) = aggregate grain supply of the first, second and third type of production teams, respectively
- \( S_{T_{11}}, S_{T_{12}}, S_{T_{13}} \) = aggregate grain supply of the first, second, and third subtype within the first type of production teams
- \( \bar{S}_i \) = the sales quota for production teams
- \( P_{T_{11}} \) = the relative frequency of case 1.1 of the first subtype of production teams

Other factors such as \( P_{T_1}, P_{T_{11}}, P_{T_{12}}, n_T, \bar{X}_{1F}, \bar{X}_{1C} \), and \( P \) are defined as the same as those used in the aggregate grain demand function.

Formula (3.121) indicates that aggregate grain supply by production teams is a function of the relative frequencies \( P_{T_1}, P_{T_{11}}, P_{T_{12}}, \) the total number of production teams, the floor quota, the ceiling quota, and sales quota, and, finally, commodity prices. Making the bounds analysis for aggregate grain supply by the third type of production teams, we can figure out the following inequalities by
observing (3.114), (3.115) and (3.116).

\[
P_{TI} P_{TI1} n_T (\bar{X}_{1F} + \bar{S}_1) < S^{TI1} = P_{TI} P_{TI1} n_T (\bar{X}_{1F} + \bar{S}_1)
+ (1 - P_{TI1}) n_T \bar{q}_1(P)
\]

\[
P_{TI} P_{TI1} n_T (\bar{X}_{1F} + \bar{S}_1) < S^{TI2} = P_{TI} P_{TI2} n_T \bar{q}_1(P)
\]

\[
P_{TI} (1 - P_{TI1} - P_{TI2}) \bar{X}_{1c} < S^{TI3}
\]

Thus, summing up (3.122), (3.123) and (3.124) immediately we obtain

\[
P_{TI} n_T (\bar{X}_{1F} + \bar{S}_1) < S^{TI}
\]

That means aggregate grain supply by the third type of production teams, or the majority of production teams, is more than \(P_{TI} n_T (\bar{X}_{1F} + \bar{S}_1)\). In other words, the product of the relative frequency of the first type of production teams, the total number of teams, and the sum of the floor quota and the sales quota is the lower bound for aggregate grain supply by the third type of production team. If this is true, the government can easily guarantee the minimum grain supply plan for most production teams.

**Comparative statics**

The comparative statics are made for both aggregate grain demand and supply of production teams:

1. Aggregate grain demand of production teams

From (3.102) and other relevant formulas we know that

\[
D^T = D^{TI} + D^{TII} + D^{TIII}
\]
The aggregate grain demand by the first type of production teams decreases as the relative frequency \( P_{TI1} \) or \( P_{TI2} \) or its own price increases, and it increases as the relative frequency \( P_{TI} \) or \( P_{TI21} \), the total number of productions teams, the floor quota, or the ceiling quota increases.

\[
\text{a. } D_{TI}^{TII} = D_{TI1}^{TII} + D_{TI2}^{TII} + D_{TI3}^{TII}
\]

\[
\frac{\partial D_{TI}^{TII}}{\partial P_{TI}} = \frac{\partial D_{TI1}^{TII}}{\partial P_{TI}} + \frac{\partial D_{TI2}^{TII}}{\partial P_{TI}} + \frac{\partial D_{TI3}^{TII}}{\partial P_{TI}} > 0
\]

\[
\frac{\partial D_{TI}^{TII}}{\partial P_{TI1}} = \frac{\partial D_{TI1}^{TII}}{\partial P_{TI1}} + \frac{\partial D_{TI3}^{TII}}{\partial P_{TI1}} < 0
\]

\[
\frac{\partial D_{TI}^{TII}}{\partial P_{TI2}} = \frac{\partial D_{TI2}^{TII}}{\partial P_{TI2}} + \frac{\partial D_{TI3}^{TII}}{\partial P_{TI2}} < 0
\]

\[
\frac{\partial D_{TI}^{TII}}{\partial P_{TI21}} = \frac{\partial D_{TI2}^{TII}}{\partial P_{TI21}} > 0
\]

\[
\frac{\partial D_{TI}^{TII}}{\partial n_T} = \frac{\partial D_{TI1}^{TII}}{\partial n_T} + \frac{\partial D_{TI2}^{TII}}{\partial n_T} + \frac{\partial D_{TI3}^{TII}}{\partial n_T} > 0
\]

\[
\frac{\partial D_{TI}^{TII}}{\partial x_1F} = \frac{\partial D_{TI1}^{TII}}{\partial x_1F} + \frac{\partial D_{TI3}^{TII}}{\partial x_1F} > 0
\]

\[
\frac{\partial D_{TI}^{TII}}{\partial x_{ic}} = \frac{\partial D_{TI2}^{TII}}{\partial x_{ic}} + \frac{\partial D_{TI3}^{TII}}{\partial x_{ic}} < 0
\]

\[
\text{b. } D_{TII}^{TII} = P_{TII} n_T \bar{x}_{1F}
\]

\[
\frac{\partial D_{TII}^{TII}}{\partial P_{TII}} = n_T \bar{x}_{1F} > 0
\]

\[
\frac{\partial D_{TII}^{TII}}{\partial n_T} = P_{TII} \bar{x}_{1F} > 0
\]
The more the relative frequency of the second type of production teams, or the total number of production teams, or the floor quota is, the more the aggregate grain demand by the second type of production teams is.

c. \( D_{\text{III}} = (1 - P_{\text{TII}}) - P_{\text{III}} n - X_{\text{IC}} \)

\[
\frac{\partial D_{\text{III}}}{\partial P_{\text{III}}} = \frac{\partial D_{\text{III}}}{\partial P_{\text{TII}}} = -n - X_{\text{IC}} < 0
\]

\[
\frac{\partial D_{\text{III}}}{\partial n} = (1 - P_{\text{TII}}) X_{\text{IC}} > 0
\]

\[
\frac{\partial D_{\text{III}}}{\partial X_{\text{IC}}} = (1 - P_{\text{TII}}) n - > 0
\]

The aggregate grain demand by the third type of production teams is increasing if the relative frequency \( P_{\text{TII}} \) or \( P_{\text{III}} \) decreases, the total number of production teams, or the ceiling quota increases. Thus,

\[
\frac{\partial D_{\text{T}}}{\partial P_{\text{TII}}} = \frac{\partial D_{\text{TI}}}{\partial P_{\text{TII}}} + \frac{\partial D_{\text{III}}}{\partial P_{\text{TII}}} < 0
\]

\[
\frac{\partial D_{\text{T}}}{\partial P_{\text{TII}}} = \frac{\partial D_{\text{TI}}}{\partial P_{\text{TII}}} + \frac{\partial D_{\text{III}}}{\partial P_{\text{TII}}} < 0
\]

\[
\frac{\partial D_{\text{T}}}{\partial P_{\text{TII}}} = \frac{\partial D_{\text{TI}}}{\partial P_{\text{TII}}} < 0
\]

\[
\frac{\partial D_{\text{T}}}{\partial P_{\text{TII}}} = \frac{\partial D_{\text{TI}}}{\partial P_{\text{TII}}} > 0
\]
The aggregate grain demand of production teams is increasing if the relative frequency $P_{TI21}$, the total number of production teams, the floor quota, or the ceiling quota increases. On the other hand, the aggregate grain demand is decreasing if the relative frequency $P_{TI}$, $P_{TI1}$, or $P_{TI2}$ or its own price increases.

Note that every factor (including $P_1$) except $n_T$ has only a partial effect on the aggregate grain demand through only a certain component or only a certain group of components within the aggregate demand function. By the previously detailed statics, the government can know exactly how a policy instrument is going to affect the aggregate grain demand and how a certain type of production team is affected by such a policy instrument. The government also can pay attention to a change in relative frequency of a certain type or subtype of production teams to estimate its effect on the aggregate grain demand.

2. Aggregate grain supply by production teams

From (3.132) and other relevant formulas we have
The more the relative frequency $p_{T1}$ is, the less the aggregate grain supply of the first type of production team is. On the other hand, the more the total number of production teams, the floor quota, the ceiling quota, the sales quota, or its own price is, the more the aggregate grain supply of the first type of production team is.
b. $S^{TII} = P^{TII} n_T q_1(P)$

$$\frac{\partial S^{TII}}{\partial P^{TII}} = n_T q_1(P) > 0$$

$$\frac{\partial S^{TII}}{\partial n_T} = P_T q_1(P) > 0$$

$$\frac{\partial S^{TII}}{\partial P^1_T} = P^{TII} n_T \frac{\partial q_1(P)}{\partial P} > 0$$

The aggregate grain supply by the second type of production teams increases if the relative frequency of the second type of teams, the total number of production teams, or its own price increases. Thus,

$$\frac{\partial S^T}{\partial P^{TI}} = \frac{\partial S^{TII}}{\partial P^{TII}} > 0$$

$$\frac{\partial S^T}{\partial P_T^{TI}} = \frac{\partial S^{TII}}{\partial P_T^{TII}} > 0$$

$$\frac{\partial S^T}{\partial P^{TII}} = \frac{\partial S^{TI}}{\partial P^{TII}} \geq 0$$

$$\frac{\partial S^T}{\partial P^{TII2}} = \frac{\partial S^{TI}}{\partial P^{TII2}} < 0$$

$$\frac{\partial S^T}{\partial P^{TI11}} = \frac{\partial S^{TI}}{\partial P^{TI11}} \geq 0$$

$$\frac{\partial S^T}{\partial n_T} = \frac{\partial S^{TI}}{\partial n_T} + \frac{\partial S^{TII}}{\partial n_T} > 0$$

$$\frac{\partial S^T}{\partial X_{1F}} = \frac{\partial S^{TI}}{\partial X_{1F}} > 0$$

$$\frac{\partial S^T}{\partial X_{1c}} = \frac{\partial S^{TI}}{\partial X_{1c}} > 0$$
The aggregate grain supply by production teams increases if the relative frequency \( P_{TI} \) or \( P_{TII} \), the total number of production teams, the floor quota, the ceiling quota, the sales quota, or its own price increases. The aggregate grain supply is uncertain if the relative frequency \( P_{TII}, P_{TII}, \) or \( P_{TII} \) increases.

Finally, we also need to point out that the effect of any factor (including own price) within the aggregate grain supply function is a partial one because it affects the grain supply through only a certain component or a certain group of components rather than all components within the aggregate supply function. This knowledge is useful for the government to use as a policy instrument to affect part of the aggregate supply and to predict the different responses from the different types of production teams.

**Graphic analysis** The graphic analyses of aggregate grain demands and supplies of three types of production teams are presented below:

1. The first type of production teams
   a. The first subtype of production teams

   For the first subtype of the first type of production teams, aggregate demand curve is a vertical line whose intercept on the \( Q_i \) axis is
   \[
   Q_{TIII} = \frac{P_{TI} P_{TII} n_T K_{IF}}{T} \quad \text{(Figure 3.12a).}
   \]
   On the other hand, the
aggregate supply curve of the first subtype of teams is formed by the sum of a vertical line that passes through $Q_{DI1b} = P_{DI} P_{DI1} n_T (X_{IF} + S_1)$ and an upward-sloping line (Figure 3.12a). Note that the horizontal distance between $D_{DI1}$ and $S_{DI1}$ is always not less than $S_{DI1} = P_{DI} P_{DI1} n_T S_1$.

b. The second subtype of production teams

For the second subtype of production teams, the aggregate grain demand curve $D_{DI2}$ is the sum of a vertical line whose intercept on the $Q_1$ axis is $Q_{DI2a} = P_{DI} P_{DI2} P_{DI21} n_T X_{ic}$ and a downward-sloping line between $Q_{DI2b} = P_{DI} P_{DI2} P_{DI21} n_T X_{IF}$ and $Q_{DI2b}$ (Figure 3.12b). On the other hand, aggregate supply curve $S_{DI2}$ is an upward-sloping line on the right of $D_{DI2}$, and their horizontal distance is more than $S_{DI2} = P_{DI} P_{DI2} n_T S_1$ (Figure 3.12b).

![Figure 3.12](image)

Figure 3.12. Aggregate grain demands and supplies of the first, the second, and the third subtype within the first type of production teams

c. The third subtype of production teams

Figure 3.12c shows that $D_{DI3}$, aggregate grain demand by the third subtype of the first type of production teams, is a vertical line whose
intercept on the $Q_i$ axis is $Q_{TII3a} = P_{TI} (1 - P_{TI1} - P_{TI2}) n_T \bar{X}_{ic}$; and its aggregate grain supply $S_{TII3}$ also is a vertical line whose intercept on the $Q_i$ axis is $Q_{TII3b} = P_{TI} (1 - P_{TI1} - P_{TI2}) n_T \bar{X}_{IF} + \bar{S}_1$. Thus, the horizontal distance between $D_{TII3}$ and $S_{TII3}$ is $S_{TII3} = P_{TI} (1 - P_{TI1} - P_{TI2}) n_T (\bar{X}_{IF} + \bar{S}_1 - \bar{X}_{ic})$. Adding $D_{TII}$, $D_{TII2}$, and $D_{TII3}$ together we obtain $D_{TII}$, the aggregate grain demand by the first type of production teams, which contains two segments: one is a vertical line whose intercept is $Q_{TIIa} = Q_{TIIa} + Q_{TII2a} + Q_{TII3a}$, another one is a downward-sloping line (Figure 3.12a). Similarly, we draw $S_{TII}$, aggregate grain supply by the first type of production teams in Figure 3.12c, which also contains two segments: one is a vertical line whose intercept is $Q_{TIIb} = Q_{TIIb} + Q_{TII2b} + S_{TII2} + Q_{TII3b}$, another one is an upward-sloping line (Figure 3.13a).

Figure 3.13. Aggregate grain demands and supplies of the first, the second, and the third type of production teams

2. The second type of production teams

As Figure 3.13b shows, aggregate grain demand by the second type of production teams is a vertical line passing through $Q_{TIIa} = P_{TII} n_T \bar{X}_{IF}$ on the $Q_i$ axis, and the aggregate grain supply by the second type of
production team is an upward slope in the \((P_1, Q_1)\) space.

3. The third type of production teams

For the third type of production team, aggregate grain demand is a vertical line that passes \(Q_{TIIA} = P_{TI} n_T \bar{x}_{1c}\) on the \(Q_1\) axis (Figure 3.13c).

Now we can complete graphic analysis for the aggregate grain demand and supply of all types of production teams. By summing \(D_{II}, D_{III},\) and \(D_{TIII}\) in Figure 3.12 we obtain \(D_T\), the aggregate grain demand by production teams, which consists of two segments. One is a vertical piece whose intercept is \(Q_{Ta} = Q_{TIA} + Q_{TIIA} + Q_{TIIIa}\); another one is a downward-sloping line (Figure 3.14). Similarly, the aggregate grain supply curve is obtained, and it is an upward sloping line in the same figure. In general, \(S_T\) is on the right of \(D_T\) because the team surplus is provided for urban households as well as other uses.

Figure 3.14. Aggregate grain demand and supply of all production teams

The state market analysis

In the view of central planners, the main interest in food grain is the balance between the demand and the supply occurring in the state.
considering that the role played by a state farm is a nonbuyer or complete seller in the state market, and the state farm household behaves in the same way as the urban household does in the state market, there is no need to repeat the state market analysis for the state farms, but there is a need to split both the grain demand and the grain supply of production teams in the combined market into a state market analysis and a team market analysis.

**Derivation of team’s grain demand and supply in the state market**

At the beginning of the second subsection, we already classified the production teams into three types according to their activities in the state market. In order to make the state market analysis, we are formulating the grain demand and the grain supply functions corresponding to the roles played by these three types of production teams in the state market.

1. **Nonbuyer**

As a nonbuyer (complete seller), the production team provides sufficient grain to the state market as well as its own households by the government standard. It is not permitted to buy any grain from the state market; therefore, its grain demand in the state market is

\[ X_{jSI} = 0 \] (3.126)

Its grain supply in the state market, \( q_{jSI} \), is the difference between its supply and its demand in the combined market, or

\[ q_{jSI} = q_j - X_j \] (3.127)

Substitute \( X_j \) and \( q_j \) into (3.127) according to (3.79) through (3.84) individually, and we have the results for three subtypes of nonbuyer:
a. the first subtype of nonbuyer

\[ q_{j\text{SI11}} = \bar{s}_j \] (3.128)

\[ q_{j\text{SI12}} = q_j(P) - \bar{X}_jP \]

\[ = q_{j\text{SI12}} (\bar{X}_jP, P) \] (3.129)

b. the second subtype of nonbuyer

\[ q_{j\text{SI21}} = q_j(P) - \bar{X}_jC \]

\[ = q_j (P, \bar{X}_jC) \] (3.130)

\[ q_{j\text{SI22}} = q_j(P) - X_j(P) \]

\[ = q_{j\text{SI22}} (P) \] (3.131)

c. the third subtype of nonbuyer

\[ q_{j\text{SI3}} = q_j(P) - \bar{X}_jC \]

\[ = q_{j\text{SI3}} (\bar{X}_jC, P) \] (3.132)

That is, the grain supply by the first type of production team is

the sales quota, or a function of the floor quota and commodity prices,

or a function of the ceiling quota and commodity prices, or a function of

commodity prices, or, finally, a function of the ceiling quota and

commodity prices.

2. Partial buyer

A partial buyer (partial seller) sells its grain to both the state

market and its households and has the right to buy some grain back up to

the floor quota. Obviously, its grain supply in the state market is

\[ q_{j\text{II}} = \bar{s}_j \] (3.133)
Its grain demand in the state market $X_{jsII}$ is the difference between its floor quota and the remainder left after subtracting its state market supply, $q_{jsII}$, from $q_j$, its total grain supply in the combined market, or

$$X_{jsII} = X_j - (q_j - q_{jsII})$$

(3.134)

Substituting (3.90), (3.91) and (3.132) into (3.133) we obtain

$$X_{jsII} = \bar{X}_{jF} - \{q_j(P) - S_j\}$$

$$= \bar{X}_{jF} + S_j - q_j(P)$$

$$= X_{jsII} \left( \bar{X}_{jF}, S_j, P \right)$$

(3.135)

Thus, the grain demand by the second type of production team is a function of the floor quota, the sales quota, and both consumer and producer price of grain; and its grain supply is just the sales quota.

3. Complete buyer

For a complete buyer (nonseller) in the state market, the grain demand function $X_{jsIII}$ and the supply function $q_{jsIII}$ are the same as (3.92) and (3.93),

$$X_{jsIII} = \bar{X}_{jc}$$

(3.136)

and

$$q_{jsIII} = 0$$

(3.137)

Comparative statics of teams' grain demand and supply Comparative statics are calculated for all three types of production teams in the state market:

1. Nonbuyer

For the nonbuyer or complete seller in the state market, the grain
demand $X_{jsI} = 0$, but there are three subtypes of nonbuyer in analyzing its grain supply:

a. The first subtype of the nonbuyer

1) $q_{jsI11}^* = S_j$

$$\frac{\partial q_{jsI11}}{\partial S_i} = 1$$

$$\frac{\partial q_{jsI11}}{\partial P_j} = 0$$

The grain supply in the state market is increasing if the sales quota increases.

2) $q_{jsI12} = q_j(X_{jF}, P)$

$$\frac{\partial q_{jsI12}}{\partial P_i} = \frac{\partial q_i(P)}{\lambda P_i} > 0$$

$$\frac{\partial q_{jsI12}}{\partial X_{jF}} = -1$$

The grain supply in the state market increases as the grain price increases or the floor quota decreases.

b. The second subtype of nonbuyer

1) $q_{jsI21} = q_j(P, X_{jc})$

$$\frac{\partial q_{jsI21}}{\partial P_j} = \frac{\partial q_i}{\partial P_j} > 0$$
The grain supply in the state market increases if the grain price increases; it decreases if the ceiling quota increases.

2) \( q_{jsI2} = q_{jsI2}(P) \)

\[
\frac{\partial q_{jsI22}}{\partial P_j} = \frac{\partial q_j(P)}{\partial P_j} - \frac{\partial X_j(P)}{\partial P_j} > 0
\]

The grain supply in the state market increases if the grain price increases.

c. the third subtype of nonbuyer

\( q_{jsI3} = q_{jsI3}(\bar{X}_{jc}, P) \)

\[
\frac{\partial q_{jsI3}}{\partial P_j} = \frac{\partial q_j(P)}{\partial P_j} > 0
\]

\[
\frac{\partial q_{jsI3}}{\partial X_{jc}} = -1
\]

The grain supply is increasing if the grain price increases or the ceiling quota decreases.

2. Partial buyer

\( X_{jsII} = X_{jsII}(\bar{X}_{jF}, \bar{S}_j, P), q_{jsII} = \bar{S}_j \)

\[
\frac{\partial X_{jsII}}{\partial X_{jF}} = 1
\]

\[
\frac{\partial X_{jsII}}{\partial S_j} = 1
\]
For the partial buyer in the state market, the grain demand is increasing if the ceiling quota or sales quota increases, or grain price decreases, and its grain supply is increasing if the sales quota increases. Thus, the state price has an effect on its grain demand but not its grain supply in the state market.

3. Complete buyer

\[
\frac{\partial X_{jsIII}}{\partial P_j} = \frac{\partial q_{js}}{\partial P_j} < 0
\]

\[
\frac{\partial q_{jsIII}}{\partial S_j} = 1
\]

\[
\frac{\partial q_{jsIII}}{\partial P_j} = 0
\]

For the partial buyer in the state market, the grain demand is increasing if the ceiling quota or sales quota increases, or grain price decreases, and its grain supply is increasing if the sales quota increases. Thus, the state price has an effect on its grain demand but not its grain supply in the state market.

3. Complete buyer

\[X_{jsIII} = \bar{X}_{jc}, \ q_{jsIII} = 0\]

\[
\frac{\partial X_{jsIII}}{\partial X_{ic}} = 1
\]

\[
\frac{\partial X_{jsIII}}{\partial P_j} = 0
\]

For the complete buyer in the state market, the higher the ceiling quota is, the higher the grain demand is. The state price of grain has no effect on either the grain demand or the grain supply.

Looking back at all of the above comparative statics, we can conclude that all the factors included in both the demand and the supply function only have an effect on only a partial group of production teams. For instance, the change in grain price has an effect only on the demand
of the partial buyer and the demand and the supply of a part of the nonbuyers. This finding is important for policy makers to recognize the roles played by such policy parameters.

Graphic analysis of team's demand and supply

Graphic analyses also are made for all these three types of production teams in the state markets:

1. Nonbuyer

For nonbuyer or complete seller, the grain supply in the state market is either a vertical line (Figure 3.15a) or an upward-sloping line (Figure 3.15b). The former reflects the first case of the first subtype of nonbuyer, whereas the latter describes the remaining situations.

2. Partial buyer

Figure 3.16a shows that the grain demand in the state market for partial buyers is a downward sloping line and becomes vertical at both \( X_{jsII} = \bar{X}_jF \) and \( X_{jsII} = \bar{X}_jC \) because of restriction of both the floor
quota and the ceiling quota. The grain supply for this partial buyer is a vertical line that passes through $\bar{S}_j$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.16.png}
\caption{The grain demand and supply of the second and the third type of production team in the state market}
\end{figure}

3. Complete buyer

For the complete buyer, the grain demand in the state market is a vertical line passing through $\bar{X}_{jc}$ on the Q axis (Figure 3.16b).

**Aggregate grain demand of production teams in the state market**

In the state market, $D^{TS}$, aggregate grain demand by all production teams, is the sum of aggregate grain demands of all three types of production teams, that is,

$$D^{TS} = D^{TSI} + D^{TSII} + D^{TSIII}$$

(3.138)

Making the similar assumptions and following the similar procedures in aggregation of grain demand in the combined market using formula (3.126), (3.135) and (3.136), we can derive aggregate grain demand by the first, the second, and the third type of production teams, $D^{TSI}, D^{TSII}$,
and $D^{TSIII}$, separately.

1. Nonbuyers

$$D^{TSI} = P_{TI} n_T \overline{X}_{js1}$$

$$= 0$$

(3.139)

2. Partial buyers

$$D^{TSII} = P_{TII} n_T \overline{X}_{js2}$$

$$= P_{TII} n_T [\overline{X}_{jF} + \overline{S}_j - q_i(P)]$$

$$= D^{TSII} (P_{TII}, n_T, \overline{X}_{jF}, \overline{S}_j, P)$$

(3.140)

In the state market, aggregate grain demand by the second type of production teams is a function of the relative frequency $P_{TII}$, the total number of production teams, the floor quota $\overline{X}_{jF}$, the sales quota $\overline{S}_j$, and commodity prices $P$.

3. Complete buyers

$$D^{TSIII} = (1 - P_{TI} - P_{TII}) n_T \overline{X}_{js3}$$

$$= (1 - P_{TI} - P_{TIII}) n_T \overline{X}_{jc}$$

$$= D^{TSIII} (P_{TI}, P_{TIII}, n_T, \overline{X}_{jc})$$

(3.141)

Aggregate grain demand of the third type of production teams in the state market is a function of the relative frequencies $P_{TI}$ and $P_{TIII}$, the total number of production teams $n_T$, and the ceiling quotas $\overline{X}_{jc}$.

Substituting (3.139), (3.140), and (3.141) into (3.138), we obtain

$$D^{TS} = P_{TIII} n_T [\overline{X}_{jF} + \overline{S}_j - q_i(P)] + (1 - P_{TI} - P_{TIII}) n_T \overline{X}_{jc}$$
In the state market, aggregate grain demand by production teams is a function of the relative frequency $P_{T_1}$ and $P_{T_3}$, the total number of production teams $n_T$, the floor quotas $\bar{X}_{jF}$, the ceiling quotas $\bar{X}_{jc}$, the sales quotas $\bar{S}_j$, and commodity prices $P$. In addition, observing (3.140) carefully we can figure out the following inequality:

$$0 < D_{T_{SI_1}} < P_{T_1}(\bar{X}_{jF} + \bar{S}_j)$$ (3.143)

That is, the aggregate grain demand by partial buyers in the state market is more than zero and less than $D_{T_{SI_1}} = P_{T_1}(\bar{X}_{jF} + \bar{S}_j)$. Thus, zero is the lower bound, and $D_{T_{SI_1}}$ is the upper bound for the aggregate grain demand in the state market, or the government needs to provide at most $D_{T_{SI_1}}$ of grain to those partial buyers.

Since formula (3.141) indicates that $D_{T_{SII}} = (1 - P_{T_1} - P_{T_3})n_T\bar{X}_{jc}$, the government needs to set a plan to sell exactly $D_{T_{SII}}$ amount of grain to those complete buyers. Adding (3.139), (3.141) and (3.143) together we obtain

$$(1 - P_{T_1} - P_{T_3})n_T\bar{X}_{jc} < D_{T_{SS}} < P_{T_1}(\bar{X}_{jF} + \bar{S}_j) + (1 - P_{T_1} - P_{T_3})n_T\bar{X}_{jc}$$ (3.144)

Formula (3.144) indicates that aggregate grain demand of production teams in the state market is between $D_{T_{SS}} = (1 - P_{T_1} - P_{T_3})n_T\bar{X}_{jc}$ and $D_{T_{SS}} = P_{T_1}(\bar{X}_{jF} + \bar{S}_j) + (1 - P_{T_1} - P_{T_3})n_T\bar{X}_{jc}$. The former is the lower bound and the latter is the upper bound for the total grain demand in the state market. To satisfy the basic need of food grain to team
households, the government has to provide at least $D_{\text{min}}^{TS}$ but at most $D_{\text{max}}^{TS}$ of grain.

**Aggregate grain supply of production teams in the state market**

$S^{TS}$, aggregate grain supply of production teams in the state market, is the sum of $S^{TSI}$, $S^{TSII}$, and $S^{TSIII}$, aggregate grain supply of the first, the second, and the third types of production teams in the state market. Algebraically,

\[ S^{TS} = S^{TSI} + S^{TSII} + S^{TSIII} \]  

(3.145)

Using the similar assumptions and methods in the analysis of the aggregate grain supply of production teams in the combined market and applying formulas (3.127), (3.128), (3.129), (3.130), (3.131), (3.132), (3.133) and (3.137), we can induce $S^{TSI}$, $S^{TSII}$, and $S^{TSIII}$ individually.

1. **Nonbuyers**

There are three subtypes of nonbuyers; therefore, $S^{TSI}$ is the aggregate grain demand of the first, the second, and the third subtypes of nonbuyers in the state markets,

\[ S^{TSI} = S^{TSI1} + S^{TSI2} + S^{TSI3} \]  

(3.146)

where

\[
S^{TSI1} = S^{TSI11} + S^{TSI12} \\
S^{TSI2} = P_{TI1} P_{TI11} n_T q_{jSI11} + P_{TI1} P_{TI11} (1-P_{TI11}) n_T q_{jSI12} \\
S^{TSI3} = P_{TI1} P_{TI11} n_T S_j + P_{TI1} P_{TI11} (1-P_{TI11}) n_T [q_j(P) - X_{jF}] \\
S^{TSI4} = P_{TI1} P_{TI11} n_T S_j + (1 - P_{TI11}) n_T [q_j(P) - X_{jF}] \\
S^{TSI5} = P_{TI1} P_{TI11} n_T (q_j(P) - X_{jF})
\]
\[ S_{TSI}^{TI1} = (P_{TI}, P_{TI1}, P_{TI11}, n_T, \overline{x}_{IF}, \overline{S}_j, P) \] (3.147)

\[ S_{TSI}^{TI2} = S_{TSI}^{TI21} + S_{TSI}^{TI22} \]

\[ S_{TSI}^{TI21} = P_{TI} P_{TI2} P_{TI21} n_T q_{JSI21} + P_{TI} P_{TI2} (1 - P_{TI21}) n_T q_{JSI22} \]

\[ S_{TSI}^{TI22} = \overline{P_{TI} P_{TI2} P_{TI21} n_T [q_j(P) - \overline{x}_{jc}]} + \overline{P_{TI} P_{TI2} (1 - P_{TI21}) n_T [q_j(P) - X_j(P)]} \]

\[ S_{TSI}^{TI2} = (P_{TI}, P_{TI2}, P_{TI21}, \overline{x}_{jc}, n_T, P) \] (3.148)

\[ S_{TSI}^{TI3} = P_{TI} (1 - P_{TI1} - P_{TI2}) q_{JSI3} \]

\[ S_{TSI}^{TI3} = P_{TI} (1 - P_{TI1} - P_{TI2}) [q_j(P) - \overline{x}_{jc}] \]

\[ S_{TSI}^{TI} = (P_{TI}, P_{TI2}, \overline{x}_{jc}, P) \] (3.149)

Substitute (3.147), (3.148) and (3.149) into (3.146):

\[ S_{TSI} = P_{TI} P_{TI1} (P_{TI11} n_T \overline{S}_j + (1 - P_{TI11}) n_T [q_j(P) - \overline{x}_{IF}]) \]

\[ + P_{TI} P_{TI2} P_{TI21} n_T [q_j(P) - \overline{x}_{jc}] \]

\[ + P_{TI} P_{TI2} (1 - P_{TI21}) n_T [q_j(P) - X_j(P)] \]

\[ + P_{TI} (1 - P_{TI1} - P_{TI2}) [q_j(P) - \overline{x}_{jc}] \]

\[ S_{TSI} = (P_{TI}, P_{TI1}, P_{TI2}, P_{TI11}, n_T, \overline{x}_{IF}, \overline{x}_{jc}, \overline{S}_j, P) \] (3.150)

That is, in the state market, the aggregate grain supply by the nonbuyers is a function of relative frequencies \( P_{TI}, P_{TI1}, P_{TI2}, P_{TI11}, \) and \( P_{TI21}, \) the total number of production teams, the floor quotas, the
ceiling quotas, the sales quotas, and commodity prices.

2. Partial buyers

For partial buyers, their aggregate grain supply in the state market $S^{TSII}$ is given as the following:

$$S^{TSII} = P_{TII} n_T q_{JS2}$$

$$= P_{TII} n_T S_j$$

$$= S^{TSII} (P_{TII} n_T S_j)$$

(3.151)

In the state market, the aggregate grain supply by partial buyers is a function of relative frequency of $P_{TII}$, the total number of production teams, and the sales quotas.

3. Complete buyers

For complete buyers, their aggregate grain supply in the state market $S^{TSIII}$ is zero, or

$$S^{TSIII} = (1 - P_{TII1} - P_{TII}) n_T q_{JS3}$$

$$= 0$$

(3.152)

Finally, adding (3.150), (3.151) and (3.152) together we obtain

$$S^{TS} = P_{TII} P_{TII1} (P_{TII1} n_T S_j + (1 - P_{TII1}) n_T [q_j(P) - X_j])$$

$$+ P_{TII} P_{TII2} P_{TII21} n_T [q_j(P) - X_j]$$

$$+ P_{TII} P_{TII2} (1 - P_{TII21}) n_T [q_j(P) - X_j(P)]$$

$$+ P_{TII} (1 - P_{TII1} - P_{TII2}) [q_j(P) - X_j] + P_{TII} n_T S_j$$
Thus, in the state market, the aggregate grain supply by production teams is a function of the relative frequencies $P_{TI'$, $P_{TI''}$, $P_{TI1}$, $P_{TI2}$, $P_{TI11}$, and $P_{TI21}$, the total number of production teams, the floor quotas, the ceiling quotas, the sales quotas, and commodity prices.

Additionally, observing (3.147), (3.148), and (3.149), we note that

$$-TQ_{TI} < S_{j} < S$$ (3.154)

$$-TQ_{TI2} < S_{j} < S$$ (3.155)

$$P_{TI} (1 - P_{TI1} - P_{TI2}) n_{T} S_{j} < S$$ (3.156)

Summing (3.154), (3.155), and (3.156) we get

$$P_{TI} n_{T} S_{j} < S$$ (3.157)

That is, in the state market, the aggregate grain supply by non-buyers or most of the production teams is always guaranteed to be more than $S_{j} = P_{TI} n_{T} S_{j}$, which is the lower bound for the aggregate grain supply by nonbuyers.

Formula (3.151) indicates that $S_{TI}$ is a constant $P_{TI} n_{T} S_{j}$, or the government can keep a fixed procurement of grain from partial buyers. Combining (3.157) with (3.151) we obtain a new inequality:

$$(P_{TI} + P_{TI1}) n_{T} S_{j} < S$$ (3.158)

Thus, the government has a guarantee to procure $S_{min} = (P_{TI} + P_{TI1}) n_{T} S_{j}$ amount of grain from production teams. In other words, $S_{min}$ is the
minimum planned grain supply by production teams.

Lastly, comparing aggregate grain demand with aggregate grain supply for nonbuyers, partial buyers, and complete buyers in the state market, respectively. We can figure out the balances between the demand and supply by these different buyers. Subtracting (3.139) from (3.150) and noting (3.154) we can get

\[ P_{TI} P_{TII} n_T S_j < S^TSI - S^TSI - D^TSI \] (3.159)

Similarly,

\[ S^TSII - D^TSII = P_{TII} n_T [q_i(P) - \bar{x}_{jF}] > 0 \] (3.160)

\[ S^TSIII - D^TSIII = -(1 - P_{TI} - P_{TII}) n_T \bar{x}_{jF} < 0 \] (3.161)

In general, \( S^TSII - D^TSII \) might be negative, that is, the government provides a net amount of grain to partial buyers. Assuming \( S^TSII - D^TSII < 0 \) and adding (3.159), (3.160) and (3.161) together we get

\[ P_{TI} P_{TII} n_T S_j - P_{TII} n_T [\bar{x}_{jF} - q_i(P)] - (1 - P_{TI} - P_{TII}) n_T \bar{x}_{jF} < S^TS - D^TS \] (3.162)

Let \((S^TS - D^TS)'\), equal the left of the inequality (3.162). In the case of \( S^TSII - D^TSII > 0 \), \((S^TS - D^TS)'\) becomes even bigger, but is still less than \( S^TS - D^TS \). Thus, regardless of positiveness or negativeness of \( S^TS - D^TS \), the government can have more than \((S^TS - D^TS)'\) amount of grain for use, excluding that resold to production teams or used by the rural people themselves. This surplus provides a base for the government to
make further plans to consider other uses, which include urban people's consumption and industrial uses as well as the needs of international grain trade.

Comparative statics of aggregate grain demand and supply by teams

Comparative statics is divided into two parts.

1. Aggregate grain demand of production teams in the state market

From (3.138) and other relevant formulas

\[ D_{TSI} = D_{TSI}^0 + D_{TSII} + D_{TSIII} \]

a. Nonbuyers

\[ D_{TSI}^0 = 0 \]

b. Partial buyers

\[ D_{TSII} = P_{TIII} n_T [\bar{X}_{jF} + S_j - q_1(P)] \]

\[ \frac{\partial D_{TSII}}{\partial P_{TIII}} = n_T [\bar{X}_{jF} + S_j - q_1(P)] > 0 \]

\[ \frac{\partial D_{TSII}}{\partial n_T} = P_{TIII} [\bar{X}_{jF} + S_j - q_1(P)] > 0 \]

\[ \frac{\partial D_{TSII}}{\partial \bar{X}_{jF}} = \frac{\partial D_{TSII}}{\partial S_j} = P_{TIII} n_T > 0 \]

\[ \frac{\partial D_{TSII}}{\partial P_j} = - P_{TIII} n_T \frac{\partial q_1(P)}{\partial P_j} < 0 \]

In the state market, the aggregate grain demand by partial buyers is increasing if the relative frequency of the total number of production teams, or the floor quotas increase, or its own price decreases.
c. Complete buyers

\[
D_{TSIII} = (1 - P_{TI} - P_{TII}) n_T \bar{x}_{jc}
\]

\[
\frac{\partial D_{TSIII}}{\partial P_{TI}} = \frac{\partial D_{TSIII}}{\partial P_{TII}} - n_T \bar{x}_{jc} < 0
\]

\[
\frac{\partial D_{TSIII}}{\partial n_T} = (1 - P_{TI} - P_{TII}) \bar{x}_{jc} > 0
\]

\[
\frac{\partial D_{TSIII}}{\partial x_{jc}} = (1 - P_{TI} - P_{TII}) n_T > 0
\]

The aggregate grain demand of complete buyers in the state market increases as the total number of production teams or the ceiling quotas increases, or as the relative frequency \( P_{TI} \) decreases. Then

\[
\frac{\partial D_{TS}}{\partial P_{TI}} = \frac{\partial D_{TSIII}}{\partial P_{TI}} < 0
\]

\[
\frac{\partial D_{TS}}{\partial P_{TII}} = \frac{\partial D_{TSIII}}{\partial P_{TII}} + \frac{\partial D_{TSIII}}{\partial P_{TII}} > 0
\]

\[
\frac{\partial D_{TS}}{\partial n_T} = \frac{\partial D_{TSIII}}{\partial n_T} + \frac{\partial D_{TSIII}}{\partial n_T} > 0
\]

\[
\frac{\partial D_{TS}}{\partial x_{jF}} = \frac{\partial D_{TSIII}}{\partial x_{jF}} > 0
\]

\[
\frac{\partial D_{TS}}{\partial x_{jc}} = \frac{\partial D_{TSIII}}{\partial x_{jc}} > 0
\]

\[
\frac{\partial D_{TS}}{\partial s_j} = \frac{\partial D_{TSIII}}{\partial s_j} > 0
\]
Thus, in the same market, the aggregate grain demand by production teams increases when the total number of production teams, floor quotas, ceiling quotas, or sales quotas increase. On the other hand, it decreases when the relative frequency $P_{TI}$ or its own price increases. In addition, each of these factors affects the aggregate grain demand in a partial way through a component or a group of components in the aggregate demand function; therefore, these factor effects are called partial effects in this study.

2. Aggregate grain supply of production teams in the state market

From (3.146) and other relevant formulas

$$S_{TS} = S_{TSI} + S_{TSII} + S_{TSIII}$$

a. Nonbuyers

$$S_{TSI} = S_{TSI1} + S_{TSI2} + S_{TSI3}$$

where

$$S_{TSI1} = P_{TI} P_{TI11} n_T S_j + (1 - P_{TI11}) n_T [q_j(P) - X_j(P)]$$

$$S_{TSI2} = P_{TI} P_{TI2} P_{TI21} n_T [q_j(P) - X_j(P)]$$

$$+ P_{TI} P_{TI2} (1 - P_{TI21}) n_T [q_j(P) - X_j(P)]$$

$$S_{TSI3} = P_{TI} (1 - P_{TI1} - P_{TI2}) [q_j(P) - X_j(P)]$$

Thus,
\[
\begin{align*}
\frac{\partial S_{TSI}}{\partial P_{TI}} &= \frac{\partial S_{TSI1}}{\partial P_{TI}} + \frac{\partial S_{TSI2}}{\partial P_{TI}} + \frac{\partial S_{TSI3}}{\partial P_{TI}} > 0 \\
\frac{\partial S_{TSI}}{\partial P_{TII}} &= \frac{\partial S_{TSI1}}{\partial P_{TII}} + \frac{\partial S_{TSI3}}{\partial P_{TII}} < 0 \\
\frac{\partial S_{TSI}}{\partial P_{TII1}} &= \frac{\partial S_{TSI2}}{\partial P_{TII2}} + \frac{\partial S_{TSI3}}{\partial P_{TII2}} < 0 \\
\frac{\partial S_{TII1}}{\partial P_{TII1}} &= \frac{\partial S_{TII1}}{\partial P_{TII1}} < 0 \\
\frac{\partial S_{TII1}}{\partial P_{TII21}} &= \frac{\partial S_{TII1}}{\partial P_{TII21}} < 0 \\
\frac{\partial S_{TII1}}{\partial \pi_T} &= \frac{\partial S_{TII1}}{\partial \pi_T} + \frac{\partial S_{TII2}}{\partial \pi_T} + \frac{\partial S_{TII3}}{\partial \pi_T} > 0 \\
\frac{\partial S_{TSI}}{\partial X_{JF}} &= \frac{\partial S_{TSI1}}{\partial X_{JF}} < 0 \\
\frac{\partial S_{TSI}}{\partial X_{JF}} &= \frac{\partial S_{TSI2}}{\partial X_{JF}} + \frac{\partial S_{TSI3}}{\partial X_{JF}} < 0 \\
\frac{\partial S_{TSI}}{\partial X_{Jc}} &= \frac{\partial S_{TSI1}}{\partial X_{Jc}} < 0 \\
\frac{\partial S_{J}}{\partial \pi_T} &= \frac{\partial S_{J}}{\partial \pi_T} > 0 \\
\frac{\partial S_{TSI}}{\partial P_{j}} &= \frac{\partial S_{TSI2}}{\partial P_{j}} + \frac{\partial S_{TSI3}}{\partial P_{j}} > 0
\end{align*}
\]

In the state market, the aggregate grain supply by nonbuyers is increasing if the relative frequency $P_{TI}$, the sales quotas $S_j$, or its own price $P_j$ increases. The aggregate grain supply is uncertain if the relative frequencies $P_{TII1}$, $P_{TII2}$, $P_{TII3}$, and $P_{TII21}$ or the ceiling quotas increases. Lastly, the aggregate grain supply is decreasing if the floor quotas increase.
b. Partial buyers

\[ S^{TSII} = P_{TII} n_T S_j \]

\[ \frac{\partial S^{TSII}}{\partial P_{TII}} = n_T S_j > 0 \]

\[ \frac{\partial S^{TSII}}{\partial n_T} = P_{TII} S_j > 0 \]

\[ \frac{\partial S^{TSI}}{\partial S_j} = P_{TII} n_T > 0 \]

For partial buyers, the aggregate grain supply in the state market is increasing if the relative frequency \( P_{TII} \), the total number of production teams, or the sales quotas increases.

c. Complete buyers

\[ S^{TSIII} = 0 \]

Thus, we finally have the results:

\[ \frac{\partial S^{TS}}{\partial P_{TII}} = \frac{\partial S^{TSI}}{\partial P_{TII}} > 0 \]

\[ \frac{\partial S^{TS}}{\partial P_{TII}} = \frac{\partial S^{TSII}}{\partial P_{TII}} > 0 \]

\[ \frac{\partial S^{TS}}{\partial P_{TI1}} = \frac{\partial S^{TSI}}{\partial P_{TI1}} < 0 \]

\[ \frac{\partial S^{TS}}{\partial P_{TI2}} = \frac{\partial S^{TSI}}{\partial P_{TI2}} < 0 \]

\[ \frac{\partial S^{TS}}{\partial P_{TI11}} = \frac{\partial S^{TSI}}{\partial P_{TI11}} < 0 \]

\[ \frac{\partial S^{TS}}{\partial P_{TI21}} = \frac{\partial S^{TSI}}{\partial P_{TI21}} < 0 \]
For production teams, the aggregate grain supply in the state market increases as the relative frequency $P_{Ti}$ or $P_{TII}$, the total number of production teams, or the sales quotas increases. The aggregate grain supply is uncertain as the relative frequency $P_{TII}$, $P_{TII}$, or $P_{TII}$ or ceiling quotas increases, and it decreases as the floor quotas, or its own price, increases.

**Graphic analysis of aggregate grain demand and supply in the state market** Based on these mathematical analyses, the graphs of the aggregate grain demand and supply by production teams can be drawn in three subparts.

1. Nonbuyers
   a. The first subtype of nonbuyers
   
   The aggregate grain supply by the first subtype of nonbuyers is the sum of a vertical line passing through $Q_{TSI1b} = P_{TII}P_{TII}P_{TII}n_{T}^{-}\bar{S}_{j}$ on the $Q_{j}$ axis and an upward sloping line (Figure 3.17a).
Figure 3.17. Aggregation of the grain supplies of the first and the second subtype within the first type of production teams in the state market

b. The second and the third subtypes of nonbuyers

For both the second and the third subtypes of nonbuyers, their aggregate grain supplies in the state market have the same shape (Figure 3.17b and c), although they may have a slight change in their slopes. Thus, we can figure out the shape of the aggregate grain supply of nonbuyers in the state market. That is an upward sloping line shown in Figure 3.18a.

2. Partial buyers

For partial buyers in the state market, the aggregate grain demand is a downward sloping line on the left of $Q_{TSIIIa} = P_{TII} n_T \bar{x}_{jc}$, and its aggregate grain supply is a vertical line that passes through $Q_{TSIIb} = P_{TII} n_T \bar{s}_j$ (Figure 3.18a).

3. Complete buyers

For complete buyers in the state market, the aggregate grain demand is a vertical line that passes through $Q_{TSIIIa} = (1 - P_{TI} - P_{TII}) n_T \bar{x}_{jc}$ and its aggregate grain supply is zero (Figure 3.18c). Thus, adding
Figure 3.18. Aggregate grain demands and supplies of the first, the second and the third type of production teams in the state market.

Figure 3.18a, 3.18b and 3.18c together, we obtain the aggregate grain demand and aggregate grain supply of production teams in the state market. The former has two segments: one is a downward-sloping line and the other one is a vertical line passing through \( Q_{TSa} = Q_{TSIIa} + Q_{TSIIIa} \) on the \( Q_j \) axis. The latter also has two segments: one is an upward-sloping line and the other one is a vertical line passing through \( Q_{TSb} = Q_{TSIb} + Q_{TSIIb} \) on the \( Q_j \) axis (Figure 3.19). The minimum horizontal distance between \( D^{TS} \) and \( S^{TS} \) is \( Q_{TS} = Q_{TSb} - Q_{TSa} \), which is the minimum net grain supply of production teams in the state market.

Figure 3.19. Aggregate grain demand and supply of all production teams in the state market.
Theory of International Grain Trade
in the Planned Economy

During the period of the Chinese planned economy, the government built up a state trading system to manage international trade. Thus, the government had significant power to use trade to achieve not only economic but also political objectives. For simplicity, this study focuses on the economic issues rather than the political issues.

The discussion of international grain trade in the Chinese planned economy contains four major parts: policy environment, the Ministry of Foreign Trade (MOFT) model, aggregate grain import demand or export supply, and interactions of plans and markets. Under policy environment, the existing background for the decision-making process in the MOFT is introduced so that theoretical work can be pursued. Next, the individual grain import demands or export supplies are derived from the MOFT model under the assumption that the MOFT maximizes its utility, subject to its budget constraint and other constraints due to the planning control. After that, total import grain demand or export grain supply is aggregated from all individual import demands or export supplies. In the last part, plan impacts on both the domestic grain market and the international grain market are examined and analyzed. The roles of plans and markets in the planned economy are illustrated and described.

Policy environment

State monopoly of foreign trade After collectivization of the agricultural sector and the nationalization of the industrial sector in
China in the mid-1950s, the state government had the sole responsibility for the management of foreign trade, including the foreign grain trade. This situation was unchanged until 1978.

In practice, under the guidance of the State Planning Commission (SPC), the Ministry of Foreign Trade (MOFT) made annual, as well as medium-term, foreign trade plans. The plans were carried out by operations of several different Foreign Trade Corporations (FTCs) through their subordinated local Foreign Trade Bureaus (FTBs), and then exchange activities in the international markets. Among the FTCs, the China National Cereals, Oils and Foodstuffs Import and Export Corporation (CNCOFIEC), and its branches in different provinces and cities are specialized in the trade of grain, as well as in other foodstuffs.

**Objective of foreign trade** The Chinese government usually thinks of foreign trade as the engine of socialist development. The main task of foreign trade is to import relatively advanced plants and equipment that cannot be produced or cannot be produced in sufficient quantities inside China. In this way, the new technology would be absorbed and the modernization of industry would be facilitated and accelerated. The problem lies in that China's import capacity mainly depends on its export capacity. To fulfill the main task, the government has to make a great effort to export goods (such as raw material, agricultural commodities, and products of light industry) to earn as much hard currency as possible. On the other hand, the government wants, to the maximum feasible extent, to rely on China's own production for those basic needs of its people, such as food and cloth.
In the international markets, the MOFT may behave as a special consumer with a utility function within imported commodity space and may face a budget constraint, as well as other constraints due to the planning control. The assumed objective for the MOFT is to maximize its utility from purchasing commodities in the international markets.

Plan control The MOFT makes its foreign trade plan, which states its revenues and expenditures, imposed import and export quotas, and expected targets in quantities of goods to be imported and exported.

For the MOFT, one main concern is the external balance, which is the budget in foreign exchange. To pay for its imports, the Chinese government has to collect foreign currency from its export earnings and/or its foreign exchange reserves and foreign loans because the Chinese currency is not an internationally convertible currency. In other words, the MOFT must extend its exports to guarantee its ability to purchase the desired foreign goods. Comparatively speaking, the internal balance is not so important for the MOFT in the planned economy because its profits have to be turned over to the State Treasurer, and its losses can be covered by the State Treasurer.

Another concern for the MOFT is the balance of domestic demand and supply for some important goods. According to needs in the Chinese economy, the government identifies all important goods that are needed to keep the balance. For those types of equipment necessary for industrial development, the MOFT must fulfill plan requirements. For those types of goods necessary for the people's livelihood, such as grain, the MOFT must strictly maintain the planned balance (rather than the real
balance) in the domestic state market (rather than the combined market).

Market operation Because the main interest in this section is focused on the foreign grain trade managed by the MOFT, for simplicity, we assume that the MOFT is a price taker in all international markets except the international grain market. The behavior of the MOFT in the foreign grain markets changes as its position changes. When the MOFT is a buyer in the foreign grain market, it behaves as either perfectly competitive or imperfectly competitive. In the former case, it is a price taker; and in the latter case, it is a price follower because of its limited export capacity.

Bilateral trade agreement Political factors as well as economic factors affect the negotiation of package deals for other traded commodities, and the MOFT makes several bilateral agreements with different foreign governments. In these agreements, the Chinese government usually guarantees to buy a minimum amount of a specific commodity such as wheat from the other country in the following several years. Therefore, during the period, the MOFT's total purchases are at least the sum of the amounts stated in all agreements with these countries.

Model of the Ministry of Foreign Trade

In the Chinese planned economy, the MOFT is assumed to maximize its utility in the import commodity space subject to given constraints. Suppose that the MOFT has a utility function $U(X_{i1}, \ldots, X_{j1}, X_{k1}, \ldots, X_{n1})$ where $X_{i1}$ $(i = 1, \ldots, n)$ is the $i^{th}$ imported good. Furthermore, to guarantee utility maximization, suppose that $U(X_{i1})$ has all proper
properties such as completeness, reflexivity, transivity, continuity, nonsatiation, and strict convexity. The Lagrangian function is formulated as the following:

\[ L = U (X_{1I}, \ldots, X_{nI}, X_{kI}, \ldots, X_{nI}) \]

\[ + \lambda_1 (\sum_{i=1}^{n} p_i^I x_{iE} + \kappa FR + \eta FL - \sum_{i=1}^{n} p_i^I x_{iI}) \]

\[ + \lambda_2 \left[ \sum_{i=1}^{j} (\bar{x}_{iE} - x_{iI}) - (S_{G}^{PS} - D_{G}^{PS}) \right] \]

\[ + \sum_{i=k+1}^{n} \lambda_{3i} \left[ (\bar{x}_{iE} - x_{iI}) - (S_{i}^{PS} - D_{i}^{PS}) \right] \]

\[ + \sum_{i=1}^{n} \lambda_{4i} (\bar{x}_{iI} - x_{iI}) \]  (3.163)

where

\[ X_{iI} \] is the \( i \)-th imported commodity \( (i = 1, \ldots, n) \)

\[ \bar{x}_{iE} \] is the export quota for the \( i \)-th commodity \( (i = 1, \ldots, n) \)

in case there is no potential to export, \( \bar{x}_{iE} = 0 \)

\[ \bar{x}_{iI} \] is the import quota for the \( i \)-th commodity \( (i = 1, \ldots, n) \)

in case there is no desire to import, \( \bar{x}_{iI} = 0 \)

\[ p_i^I \] is international price for the \( i \)-th commodity \( (i = 1, \ldots, n) \)

\( FR \) is the foreign currency reserves

\( FL \) is the foreign loans

\( S_{G}^{PS} \) planned domestic grain supply in the state market

\( D_{G}^{PS} \) planned domestic grain demand in the state market
$S_{i}^{PS} = \text{planned domestic supply of the } i^{th} \text{ commodity in the state market} \quad (i = k, \ldots, n)$

$D_{i}^{PS} = \text{planned domestic demand of the } i^{th} \text{ commodity in the state market} \quad (i = k, \ldots, n)$

$\kappa, \eta = \text{fixed proportional coefficient } 0 \leq \kappa \leq 1, 0 \leq \eta \leq 1$

The constraints in the Lagrangian function can be classified into three categories:

1) external balance.

$$\sum_{i=1}^{n} P_{i}^{I} X_{iE} + \kappa \text{ FR} + \eta \text{ FL} - \sum_{i=1}^{n} P_{i}^{I} X_{iI}$$

is an external balance constraint. It says that $\sum_{i=1}^{n} P_{i}^{I} X_{iE}$, total expenditure for importing goods, must be covered by $\sum_{i=1}^{n} P_{i}^{I} X_{iI}$ total earnings from possibly exporting goods plus part of reserves FR and loans FL in foreign currency.

2) balance of the planned domestic demand and supply.

$(X_{iE} - X_{iI})$ is a net export for good $i$ if $X_{iE} > X_{iI}$ or a net import for good $i$ if $X_{iE} < X_{iI}$, and $(S_{i}^{PS} - D_{i}^{PS})$ is a planned surplus for good $i$ if $S_{i}^{PS} > D_{i}^{PS}$ or a planned shortage for good $i$ if $S_{i}^{PS} < D_{i}^{PS}$. The constraint $(X_{iE} - X_{iI}) - (S_{i}^{PS} - D_{i}^{PS}) \geq 0$ means that, for an important good $i$, the net export or net import has to be covered by the planned domestic surplus or has to cover the planned domestic shortage. Grain, as a composite good, is considered in the balance table by the central planners, and, therefore, is an exception. An individual variety of grain is not considered to be a single good, but more importantly, is
considered to be part of a composite good. Suppose that the first $j^{th}$
commodities belong to the grain category, then

$$\sum_{i=1}^{j} (X_{iE} - X_{iI}) - (S_{G}^{PS} - D_{G}^{PS})$$
is a constraint that guarantees the planned balance of
domestic grain demand and grain supply in only the state market by using
international grain markets.

3) Import quota imposed for some commodities.

Due to the central plan, the government may impose import quotas for
some specific commodities. $X_{iI} - \bar{X}_{iI}$ is the import quota constraint,
that is, import of commodity $i$ must be no more than the import quota set
by the government.

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial X_{iI}} = \frac{\partial U}{\partial X_{iI}} - \lambda_{I} P_{I} - \lambda_{2} - \lambda_{4i} \leq 0$$
for $\bar{X}_{iI} \neq 0$, $i = 1, \ldots, j$ (3.164)

$$\frac{\partial L}{\partial \lambda_{i}} = \frac{\partial U}{\partial \lambda_{i}} - \lambda_{1} P_{1} - \lambda_{2} \leq 0$$
for $\bar{X}_{iI} = 0$, $i = 1, \ldots, j$ (3.165)

$$\frac{\partial L}{\partial X_{iI}} = \frac{\partial U}{\partial X_{iI}} - \lambda_{1} P_{1} - \lambda_{3i} - \lambda_{4i} \leq 0$$
for $\bar{X}_{iI} \neq 0$, $i = k, \ldots, n$ (3.166)

$$\frac{\partial L}{\partial \lambda_{i}} = \frac{\partial U}{\partial \lambda_{i}} - \lambda_{1} P_{1} - \lambda_{3i} \leq 0$$
for $\bar{X}_{iI} = 0$, $i = k, \ldots, n$ (3.167)

$$\bar{X}_{iI} \frac{\partial L}{\partial \lambda_{i}} = 0$$
for $i = 1, \ldots, n$ (3.168)

$$\frac{\partial L}{\partial \lambda_{i}} = \sum_{i=1}^{n} P_{i} \bar{X}_{iE} + \kappa FR + \eta FL - \sum_{i=1}^{n} P_{i} \bar{X}_{iI} \geq 0$$
(3.169)

$$\lambda_{i} \frac{\partial L}{\partial \lambda_{i}} = 0$$
for $i = 1, \ldots, n$ (3.170)
Suppose that the MOFT strictly keeps its budget in foreign currency at the balance level,

\[
\frac{\partial L}{\partial \lambda_2} = \sum_{i=1}^{n} (X_{1E} - X_{1I}) - (S^*_{GS} - D^*_{GS}) \geq 0 \tag{3.171}
\]

\[
\frac{\partial L}{\partial \lambda_2} = 0 \tag{3.172}
\]

\[
\frac{\partial L}{\partial \lambda_{31}} = (X_{1E} - X_{1I}) - (S^*_{1} - D^*_{1}) \geq 0 \quad i = k, \ldots, n \tag{3.173}
\]

\[
\frac{\partial L}{\partial \lambda_{31}} = 0 \tag{3.174}
\]

\[
\frac{\partial L}{\partial \lambda_{41}} = X_{1I} - X_{1I} \geq 0 \quad \text{for } X_{1I} \neq 0 \quad i = 1, \ldots, n \tag{3.175}
\]

\[
\frac{\partial L}{\partial \lambda_{41}} = 0 \quad \text{for } X_{1I} \neq 0 \quad i = k, \ldots, n \tag{3.176}
\]

\[
\lambda_{1}, \lambda_{2}, \lambda_{41} \geq 0 \tag{3.177}
\]

\[
\lambda_{31} \geq 0 \tag{3.178}
\]

Then both (3.169) and (3.170) can be omitted. Similarly, suppose that the MOFT keeps grain and other commodities at the planned balance levels, then

\[
\frac{\partial L}{\partial \lambda_1} = \sum_{i=1}^{n} P_{1}^{-1} X_{1E} + \kappa FR + \eta FL - \sum_{i=1}^{n} P_{1}^{-1} X_{1I} = 0 \tag{3.179}
\]

and

\[
\frac{\partial L}{\partial \lambda_2} = \sum_{i=1}^{j} (X_{1E} - X_{1I}) - (S^*_{GS} - D^*_{GS}) = 0 \tag{3.180}
\]
After that, (3.171), (3.172), (3.173), and (3.174) can be dropped from the model.

Furthermore, suppose that the second-order conditions are satisfied; solving the first-order conditions, we find three possible situations for $X_{II}$:

1. $X_{II} = 0$ (3.182)

2. $X_{II} = X_{II} (P^I, FR, FL, X_{E}^I, X_{I}^I, S_{G}^{PS} - D_{G}^{PS}, S_{1}^{PS} - D_{1}^{PS})$ (3.183)

and

3. $X_{II} = X_{II}$ (3.184)

The first formula (3.182) implies a corner solution. That is, the MOFT does not import any amount of good i. Formula (3.183) says that import demand for good i is a function of international commodity prices, foreign reserves in the country, foreign loans borrowed by the government, the export quotas, the planned balance of domestic grain supply and demand in the state market, and the planned balances of domestic supplies and demands of other commodities in the state market. The last formula (3.184) gives imported good i as just its quota.

If $X_{II}$ is determined, the corresponding quantities of either net export supply $X_{NE}$ or net import demand $X_{NI}$ are immediately determined.

We now discuss the three situations of $X_{II}$:

1. $X_{II} = 0$
There are two possible solutions for \( x_{1E} \) if \( x_{1I} = 0 \).

a. \( \bar{x}_{1E} = 0 \)
\[
x_{1NI} = x_{1NE} = 0 \tag{3.185}
\]
The MOFT is not buying and selling good \( i \) in international markets. This case will be dropped out of this study because it is meaningless.

b. \( \bar{x}_{1E} > 0 \)
\[
x_{1NE} = x_{1E} \tag{3.186}
\]
The MOFT exports good \( i \) at the quota level only and, therefore, is called a command exporter for good \( i \).

2. \( x_{1I} = x_{1I} (P^I, FR, FL, \bar{x}_E, \bar{x}_I, \bar{s}_G^{PS} - s_G^{PS}, s_P^{PS} - d_P^{PS}) \)

a. \( \bar{x}_{1E} = 0 \), then
\[
x_{1NI} - x_{1I} (P^I, FR, FL, \bar{x}_E, \bar{x}_I, \bar{s}_G^{PS} - d_G^{PS} - s_P^{PS} - d_P^{PS}) \tag{3.187}
\]
Net import demand for good \( i \) is a function of international prices of goods, the foreign reserves, the foreign loans, the export quotas, the import quotas, the planned balance of domestic grain supply and demand in the state market, and the planned balances of domestic supply and demand for other goods in the state market. Therefore, the MOFT is a flexible importer for good \( i \).

b. \( \bar{x}_{1E} > 0 \), then we have

1) if \( \bar{x}_{1E} > x_{1I} \)
\[
x_{1NE} = \bar{x}_{1E} - x_{1I} (P^I, FR, FL, \bar{x}_E, \bar{x}_I, \bar{s}_G^{PS} - d_G^{PS} - s_P^{PS} - d_P^{PS})
\quad - x_{1NE} (P^I, FR, FL, \bar{x}_E, \bar{x}_I, \bar{s}_G^{PS} - d_G^{PS} - s_P^{PS} - d_P^{PS}) \tag{3.188}
\]
2) if $\bar{X}_{IE} < X_{II}$

$$X_{INI} = X_{II} - \bar{X}_{IE}$$

$$= X_{II} (P^I, FR, FL, \bar{X}_E, \bar{X}_I, S^PS - D^PS, S^PS - D^PS) - \bar{X}_E$$

$$= X_{INI} (P^I, FR, FL, \bar{X}_E, \bar{X}_I, S^PS - D^PS, S^PS - D^PS)$$

(3.189)

These are either net export supply or net import demand of good $i$, which is a function of international prices of goods, the foreign reserves, the foreign loans, the export quotas, the import quotas, the planned balance of domestic grain supply and demand in the state market, the planned balances of domestic supply and demand for other goods in the state market. Because export quota is effective, the MOFT is called either a restricted flexible exporter or importer for good $i$ in case (2) a and b.

3) if $\bar{X}_{IE} = X_{II}$

$$X_{INI} - X_{INE} = 0$$

(3.190)

either net export supply or net import demand for good $i$ is zero. The MOFT is a nonexporter and nonimporter. This case is not discussed further.

3. $X_{II} = \bar{X}_I$

a. $\bar{X}_{IE} = 0$

thus

$$X_{INI} = \bar{X}_I$$

(3.191)

The MOFT imports good $i$ at the quota level only and is called a command importer for good $i$. 
b. $\overline{X}_{1E} > 0$

1) if $\overline{X}_{II} > \overline{X}_{1E}$

$$X_{INI} = \overline{X}_{II} - \overline{X}_{1E} \quad (3.192)$$

2) if $\overline{X}_{II} < \overline{X}_{1E}$

$$X_{INI} = \overline{X}_{1E} - \overline{X}_{II} \quad (3.193)$$

3) if $\overline{X}_{II} = \overline{X}_{1E}$

$$X_{INI} = \overline{X}_{INE} = 0 \quad (3.194)$$

Because (3.194) does not make sense, it is dropped from the discussion. Thus, if $\overline{X}_{1E} > 0$, either net import demand or net export supply for good $i$ is fixed at the difference of the import quota and the export quota by the MOFT. The MOFT is both a dual command importer and exporter in these cases because it is restricted by both the import and the export quota at the same time.

All of the above results indicate that the MOFT tries to decide all net imports and net exports for all goods together, although it may pay prior attention to export quotas in order to collect the foreign currency needed to purchase goods in the international market.

**Comparative statics**  
From the above discussion, we can classify the MOFT's behavior by its importing and exporting behaviors.

1. Importer
   a. Dual command importer
For dual command importer, the net import demand increases or decreases as the import or the export quota increases.

b. Command importer

\[ x_{INI1} = x_{1I} - x_{1E} \]

\[ \frac{\partial x_{INI1}}{\partial x_{1I}} = 1 \]

\[ \frac{\partial x_{INI1}}{\partial x_{1E}} = -1 \]

Net import demand for the \( i \)th good is not affected by any factor except for the import quota.

c. Flexible importer

\[ x_{INI3} = x_{1I} (P^I, FR, FL, X_E, \bar{X}_I, S^P - D^P, S^P - D^P) \]

\[ \frac{\partial x_{INI3}}{\partial P^I} = - \frac{\partial x_{1I}}{\partial P^I} < 0 \]

\[ \frac{\partial x_{INI3}}{\partial FR} = \frac{\partial x_{1I}}{\partial FR} > 0 \]

\[ \frac{\partial x_{INI3}}{\partial FL} = \frac{\partial x_{1I}}{\partial FL} > 0 \]

\[ \frac{\partial x_{INI3}}{\partial x_{1I}} = - \frac{\partial x_{1I}}{\partial x_{1I}} < 0 \]

\[ \frac{\partial x_{INI3}}{\partial x_{1E}} = \frac{\partial x_{1I}}{\partial x_{1E}} > 0 \]

\[ \frac{\partial x_{INI3}}{\partial x_{1I}} = \frac{\partial x_{1I}}{\partial x_{1I}} > 0 \]
Net import demand for the $i^{th}$ good is increasing if the foreign reserves, the foreign loans, or the import quotas increase. On the other hand, net import demand for the $i^{th}$ good is decreasing if its own price, the export quota, or its planned balance of domestic supply and demand in the state market increases.

Especially, for net import demand of a kind of grain $X_{INI} (i = 1, \ldots, j)$ and net import demand of total grain $X_{GNI}$, where

$$X_{GNI} = \sum_{n=1}^{j} (X_{iE} - X_{i})$$

then

$$\frac{\partial X_{GNI}}{\partial (S_{PS} - D_{PS})} > 0$$

but

$$\frac{\partial X_{GNI}}{\partial (S_{G} - D_{G})} < 0$$

That is, when the balance of domestic grain supply and demand in the state market decreases, net import demand of total grain increases, but net import of a specific kind of grain $X_{INI}$ may increase or decrease depending on the situations. The reason this happens is explained as follows. As the balance of domestic grain supply and demand in the state market decreases, the government must increase the net grain import to
satisfy the planned domestic needs because grain is so important to people's lives as well as political stability in the country. But the increase in net grain import is constrained by the budget balance in foreign currency. To increase net grain import and keep other imports and exports unchanged, the government must have additional money to keep its budget balanced. If the MOFT cannot get more foreign currency from the State Treasury, it must adjust its net import and/or net export to compensate its additional expenses from increasing net grain import. There are two ways for the MOFT to deal with this matter. First, it can reduce imports and expand exports, and, therefore, reduce net imports or expand net exports for other goods rather than grain. Alternatively, it can make adjustments only within the grain trade to avoid financial difficulty. For example, it can expand net exports for a specific kind of grain, say rice, whose price is relatively higher in the international market, and expand net imports for another specific kind of grain, say wheat, whose price is relatively lower in the international market. Working in this way, making a trade-off between rice exporting and wheat importing by utilizing their price difference, the MOFT can easily have its net grain import $X_{\text{GNI}}$ expanded without any further financial request in foreign currency from the State Treasury. In addition, without changing the aggregate grain demand and supply in the domestic state market, the government can easily use its play to change or restrict the proportion of different varieties of grain demanded or supplied in the state market to guarantee the trade-off in the international market.
d. Restricted flexible importer

\[ X_{1 I 4} = X_{1 I} \left( P^I, FR, FL, X_{E}, X_{I}, S^G - D^P, S^P - D^P \right) + X_{1 E} \]

Because \( X_{1 E} \) is fixed, the partial derivatives for restricted flexible importer are exactly the same as those for the flexible importer.

2. Exporter

a. Dual command exporter

\[ X_{1 N E 1} = X_{1 E} - X_{1 I} \]

For the dual command exporter, comparative statics is the same as that for the dual command importer but with opposite signs.

b. Command exporter

\[ X_{1 N E 2} = X_{1 E} \]

\[ \frac{\partial X_{1 N E 2}}{\partial X_{1 E}} = 1 \]

Net export supply for the \( i^{th} \) good is not affected by any factor except for the export quota.

c. Restricted flexible exporter

\[ X_{1 N E 3} = X_{1 E} - X_{1 I} \left( P^I, FR, FL, X_{E}, X_{I}, S^G - D^P, S^P - D^P \right) \]

Obviously, any partial derivative for \( X_{1 N E 3} \) is the same as that for a negative \( X_{1 I} \) or a negative \( X_{1 N I 3} \) (see flexible importer). Thus, it is not necessary to continue the discussion of comparative statics here.

Graphic analysis The graphic analysis also is divided into two parts by importing and exporting behaviors of the MOFT.

1. Importer

a. Dual command importer
For a dual command importer, its net import demand for a grain commodity $i$, $X_{\text{INI}1}$, is a vertical line that passes through $X_{\text{INI}1} = X_{11} - X_{1E}$ on the $X_{11}$ axis in the $(X_{11}, P_{11}^I)$ space (Figure 3.20a).

\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure_a}
\caption{Figure 3.20a. Net import demands of the MOFT}
\end{subfigure}
\hspace{0.5cm}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure_b}
\caption{Figure 3.20b}
\end{subfigure}
\end{figure}

b. Command importer

For a command importer, its net import demand for a grain commodity $i$, $X_{\text{INI}2}$, is a vertical line that passes through $X_{\text{INI}2} = X_{12}$ on the $X_{12}$ axis in the $(X_{12}, P_{12}^I)$ space, and therefore, can be represented by Figure 3.20a.

c. Flexible importer

As Figure 3.20b shows, for a flexible importer, its net import demand is a downward-sloping line in the $(X_{13}, P_{13}^I)$ space.

d. Restricted flexible importer

For a restricted flexible importer, its net import demand also is downward-sloping, and, therefore, is represented by Figure 3.20b.
2. Exporter

a. Dual command exporter

For a dual command exporter, its net export supply $X_{jNE1}$ is a vertical line that passes $X_{jNE1} = X_{JE} - X_{JI}$ on the $(X_{j1}, P_{j1})$ space (Figure 3.21a).

b. Command exporter

$X_{jNE2}$, net export supply of a grain commodity $j$ for a command exporter, is the same as that for a dual command exporter, except that its intercept on the $X_{j2}$ axis is $X_{jNE2} = X_{JE}$, rather than $X_{jNE1}$ (Figure 3.21a).

c. Restricted flexible exporter

As Figure 3.21b shows, when the MOFT behaves as a restricted flexible exporter, $X_{jNE3}$, its net export supply for a grain commodity $j$, is an upward-sloping line in the $(X_{j3}, P_{j3})$ space.
Aggregate import grain demand and export grain supply

Within the first $j^{th}$ grain commodities, suppose that the first $e^{th}$ are net importing grain commodities and the remaining are net exporting grain commodities. In the planned economy, the aggregate import grain demand $D^M$ is then

$$D^M = \sum_{i=1}^{n} x_{i1} + \sum_{i=1}^{n} x_{i2} + \sum_{i=1}^{n} x_{i3} + \sum_{i=1}^{n} x_{i4}$$

$$= \sum_{i=1}^{n} (x_{i1} - x_{i1E1}) + \sum_{i=1}^{n} x_{i2}$$

$$+ \sum_{i=1}^{n} (P^I, FR, FL, xE, xi, sPS - D^P - S^P - D^P)$$

That is, as long as the MOFT behaves as a flexible or a restricted flexible importer for at least one grain commodity, $D^M$, the aggregate import grain demand, is a function of international prices of commodities, the foreign reserves, the foreign loans, the export quotas, the import quotas, the planned balance of domestic grain supply and demand in the state market, and the planned balances of domestic supplies and demands of other goods in the state market. On the other hand, $S^X$, the aggregate export grain supply in the planned economy is as follows:

$$S^X = \sum_{i=1}^{n} x_{i1E1} + \sum_{i=1}^{n} x_{i1E2} + \sum_{i=1}^{n} x_{i1E3}$$

$$= \sum_{i=1}^{n} (x_{i1E1} - x_{i1E1}) + \sum_{i=1}^{n} x_{i1E2}$$
As long as the MOFT behaves as a restricted flexible exporter for at least one grain commodity, the aggregate export grain supply is similar to the aggregate import grain demand as a function of international prices of commodities, foreign reserves, foreign loans, export quotas, import quotas, the planned balance of domestic grain supply and demand in the state market, and the planned balances of domestic demands of other goods in the state market.

Two important things should be pointed out. First, both the aggregate import grain demand and the aggregate export grain supply by the MOFT do contain international price vectors, especially the international grain price, but do not contain domestic price vectors, especially the domestic grain price. This fact indicates that the MOFT's behavior in the international grain market is affected by international prices but not by domestic prices. Second, whether China becomes a net importing grain country or a net exporting grain country depends on the difference between the aggregate import grain demand, $D^I$, and the aggregate export grain supply, $S^X$. If $D^I > S^X$, the MOFT is a net grain importer; otherwise, it is a net grain exporter. Looking back at the history of Chinese grain trade after the 1950s, we found out that China was a net importing grain country most of the time, especially in the planned economy. That means, in general, the aggregate import grain
demand $D^{MP}$ is greater than the aggregate supply of grain $S^{XP}$ in China. If this is true, the net grain import demand, $D^{NMP}$, is as follows:

$$D^{NMP} = D^{MP} - S^{XP} \quad (3.200)$$

Undoubtedly, $D^{NMP}$ is a function that contains all the factors $D^{MP}$ and/or $S^{XP}$ have/has.

**Comparative statics** The comparative statics are calculated for both aggregate import grain demand and export grain supply.

1. Aggregate import grain demand

$$\frac{\partial D^{MP}}{\partial X_{I11}} = \Sigma \frac{\partial X_{I11}}{1 - e^{-X_{I11}}} + \Sigma \frac{\partial X_{I11}}{1 - e^{-X_{I11}}} + \Sigma \frac{\partial X_{I11}}{1 - e^{-X_{I11}}} > 0$$

$$\frac{\partial D^{MP}}{\partial X_{I12}} = \Sigma \frac{\partial X_{I12}}{1 - e^{-X_{I12}}} + \Sigma \frac{\partial X_{I12}}{1 - e^{-X_{I12}}} + \Sigma \frac{\partial X_{I12}}{1 - e^{-X_{I12}}} > 0$$

As the import quota or the export quota faced by the dual command importer or the command importer increases, the total effect on the aggregate grain demand is unclear because the effects on the last two terms, aggregate import grain demand by the flexible importers and restricted flexible importers are unclear.

Based on similar reasoning, we have

$$\frac{\partial D^{MP}}{\partial X_{I13}} = \Sigma \frac{\partial X_{I13}}{1 - e^{-X_{I13}}} + \Sigma \frac{\partial X_{I13}}{1 - e^{-X_{I13}}} > 0$$

$$\frac{\partial D^{MP}}{\partial X_{I14}} = \Sigma \frac{\partial X_{I14}}{1 - e^{-X_{I14}}} + \Sigma \frac{\partial X_{I14}}{1 - e^{-X_{I14}}} < 0$$
The remaining comparative statics are focused on the behaviors of flexible and restricted importers.

\[
\frac{\partial D_{MP}}{\partial x^{ij}} = \sum_{i=1}^{j} \frac{\partial X_{i}^{ij}}{\partial x^{ij}} + \sum_{i=1}^{j} \frac{\partial X_{i}^{ij}}{\partial x^{ij}} > 0
\]

\[
\frac{\partial D_{MP}}{\partial x^{ij}} = \sum_{i=1}^{j} \frac{\partial X_{i}^{ij}}{\partial x^{ij}} + \sum_{i=1}^{j} \frac{\partial X_{i}^{ij}}{\partial x^{ij}} < 0
\]

\[
\frac{\partial D_{MP}}{\partial x^{ij}} = \sum_{i=1}^{j} \frac{\partial X_{i}^{ij}}{\partial x^{ij}} + \sum_{i=1}^{j} \frac{\partial X_{i}^{ij}}{\partial x^{ij}} > 0
\]

The aggregate import grain demand increases when the foreign reserves or the foreign loans increase. It decreases when its international price, \( P^{I}_{G} \), average grain price, or the balance of domestic grain supply and demand in the state market increases. Finally, it becomes uncertain when the international price of one grain commodity increases.

2. Aggregate export grain supply

Under the changes of the import quotas or the export quotas, the final results of comparative statics for aggregate export grain supply are the same as those for the aggregate import grain demand. Otherwise,
the results are given as follows:

\[ \frac{\partial S_{XP}}{\partial P_1} = \sum_{f-j} \frac{\partial x_{IE2}}{\partial P_1} > 0 \]

\[ \frac{\partial S_{XP}}{\partial P_G} = \sum_{f-j} \frac{\partial x_{IE2}}{\partial P_G} < 0 \]

\[ \frac{\partial S_{XP}}{\partial FR} = \sum_{f-j} \frac{\partial x_{IE2}}{\partial FR} < 0 \]

\[ \frac{\partial S_{XP}}{\partial FL} = \sum_{f-j} \frac{\partial x_{IE2}}{\partial FL} < 0 \]

\[ \frac{\partial S_{MP}}{\partial (S^P_G - D^P_G)} = \sum_{f-j} \frac{\partial x_{IE2}}{\partial (S^P_G - D^P_G)} > 0 \]

These results indicate that, given the above change(s) in the factor(s), aggregate export grain supply is changing in the opposite direction of changes in the aggregate grain demand.

**Graphic analysis**

Graphic analysis also is presented for both aggregate grain demand and export grain supply.

1. Aggregate import grain demand

Given the average international grain price, we can aggregate the import grain demand by graph. As Figure 3.22a shows, \( D^{MPI} \), the sum of aggregate import grain demands by dual commanders and commanders, is a vertical line that intercepts on the \( Q^I_G \) axis is \( Q^I_G = \sum (\bar{X}_{II1} - \bar{X}_{IE1}) \) + \( \sum \bar{X}_{II2} \) in the \((Q^I_G, P^I_G)\) space.
In Figure 3.22b, $D^\text{MPI}$, aggregate import grain demand by flexible and restricted flexible importers, is given as a downward-sloping line in the $(Q^I_G, P^I_G)$ space. Finally, after summing $D^\text{MPI}$ and $D^\text{MPII}$ we obtain $D^\text{MP}$, aggregate import grain demand by the MOFT. As Figure 3.22c shows, $D^\text{MP}$ is still a downward-sloping line, but it is much steeper than $D^\text{MPI}$ in the $(Q^I_G, P^I_G)$ space because of the effect of $D^\text{MPI}$.

![Diagram](a) (b) (c)

Figure 3.22. Aggregation of import grain demands in the planned economy

2. Aggregate export grain supply

Similarly, we can figure out aggregate export grain supply in the $(Q^I_G, P^I_G)$ space. In Figure 3.23a, $S^\text{XPI}$, the sum of the aggregate export grain supplies by dual command exporters and command exporters is a vertical line that passes through $Q^G_{GE} = \sum_{f-j} \bar{X}^{1E1}_{1I1} + \sum_{f-j} \bar{X}^{1E2}_{1I1}$ on $Q^I_G$ axis in the $(Q^I_G, P^I_G)$ space.

Lastly, $S^\text{XP}$, the aggregate export grain supply by the MOFT is obtained from the sum of $S^\text{XPI}$ and $S^\text{MPII}$, which is the aggregate grain
supply by restricted flexible exporters (Figure 3.23b). It is still an upward-sloping line, but it is steeper than $X_{PPI}$ because of the effect of $X_{PI}$ (Figure 3.23c).

![Figure 3.23. Aggregation of export grain supplies in the planned economy](image)

**Interactions of plans and markets**

In the Chinese grain economy, the markets, especially the domestic state markets, are different from markets in the usual sense. Determined by the government authority and not by the interaction of market forces, the state market price lacks the mechanism to adjust the marketing process automatically and instantaneously. Moreover, through the planning process, the government almost controls grain consumption and production. In the state market, the government limits the maximum grain demand by using ration coupons in urban areas and floor quotas in rural areas and guarantees the minimum grain supply by applying the output targets to state farms and the sales quotas and the ceiling quotas to production teams. That the government has monopoly power in the state
market is not only because it has the authority to decide the prices but also because it has authority, to a great extent, to decide the quantities exchanged. Hence, the state market becomes a buyer market when the government procures grain from state farms and production teams and becomes a seller market when the government retails grain to the urban households and sells or resells grain to those production teams permitted. Under such circumstances, in a certain sense, the state grain market can be seen as an extension of government policies.

In the international grain markets, the Chinese government has no power to decide prices, but still has full monopoly power to decide the variety and quantity of grain imported and exported. Thus, the international grain markets can be partly seen as extensions of government policies. The next is the analysis of the policy impacts on the equilibrium situations of the domestic as well as the international market.

Market equilibrium situations

Generally, one particularly noticeable feature of the state grain market is its disequilibrium. Indeed, disequilibrium is an internal part of the Chinese grain economy. Without perfect information and knowledge, exogenous prices cannot be the equilibrium prices in the state market. In addition, it is believed by both the Chinese officials and professional authorities that the grain price in the state market is well below its equilibrium price. One reason is that the government wants to set lower prices for agricultural products but higher prices for industrial products to create a difference between prices of agricultural and industrial products and to collect
funds from the agricultural sector. After economic reform, this hypothe­sis was partly proved by the much higher free-market prices, whose average might be considered as the equilibrium price. Obviously, this too-low price would create an excess grain demand in the state market. Moreover, usage of the ration coupons imply an existing shortage problem in quantities of grain in the planned economy. Even under such a rationing system, restricted excess grain demand still existed throughout the state market because the Chinese government had adjusted its grain price within only a narrow range, and China had been a net grain importing country from 1961. Hence, the equilibrium quantities of grain were reached by means of international trade in the planned economy.

In the international grain markets, there exist equilibrium prices and quantities. Although the equilibrium prices are mainly determined by excess grain suppliers of the rest of the world, the equilibrium quantities are mainly determined by the planned excess grain demand, rather than the real excess grain demand of the MOFT.

**Domestic policy instruments** In the domestic grain market, as already analyzed, the policy instruments include ration coupons, sales quotas, ceiling quotas, floor quotas, and the state market prices. Among these policy instruments, the ration coupons, the sales quotas and the state market prices are more important for the central planners because these three mainly determine a disequilibrium in the state grain market. The first two of the three policy instruments, needless to say, are quantity parameters to adjust either the grain demand or the grain supply in the state market. The last one is regarded as a policy instrument
because the state market price is centrally decided and generally is uniform throughout the country (although there are some local differentials). It is weakly (partly) related to the grain demand and supply in the state market. Next are comparative studies for these three policy instruments:

1. Ration coupons

Ration coupons are issued to the urban households plus the households of state farms. With the rationing system the government can plan the maximum grain demand in the state market. Suppose the government raised the amount of ration coupons to the urban people in order to improve their standard of living. The market impacts of this policy are illustrated in Figure 3.24.

![Figure 3.24](image-url)

Figure 3.24. The effects of a change in ration level in the planned economy

On the left, $D^P$ and $S^P$ separately represent the aggregate grain demand and supply in the domestic state market in the planned economy.
At the given state market price $P^s$, the grain demand is $Q^D_1$ and the grain supply is $Q^S_1$. The planned grain demand and supply are $Q^{PD}_{1}$ and $Q^{PS}_{1}$, which are not affected by the state market price $P^s$. Obviously, $Q^{PD}_{1} > Q^D_1$ and $Q^{PS}_{1} < Q^S_1$ because the government wants to make sure that the plans would fully coordinate the balance of real demand and supply in the state grain market. On the right, the international grain market is partly separated from the state market because the state market price is irrelevant to the planned excess demand, but it is partly related to the state market because the balance of the planned demand and supply is relevant to the planned excess demand. Suppose that China is an importing grain country. We find out that the planned excess demand is $ED$, which is downward-sloping, and the excess supply of the rest of the world is $ES(R)$, which is a horizontal line because China is assumed to be either a price taker or a price follower in the international market in spite of the fact that it is a larger country. The import grain demand is determined by a cross point $Q^M_{1}$ under a presumed international grain price $P^I_{1}$. All of these descriptions about the beginning situations of the markets are fitted to discussion of any other policy instruments, and, therefore, are not repeated in this part.

Because the government increases the amount of ration coupons distributed, the domestic grain demand in the state market is expanding from $Q^D_1$ to $Q^D_{1,2}$. At the same time, the planned grain demand also is expanding from $Q^{PD}_{1}$ to $Q^{PD}_{1,2}$. Neither the domestic grain supply nor the planned grain supply is changing. Thus, the balance of total grain
demand and supply as well as the balance of total planned grain demand and supply in the state market is increasing. The change in the latter balance rather than the former balance would cause a rightward shift of the planned excess demand curve $ED$ in the international market. The import grain demand then increases to $Q^M_2$.

2. Sales quotas

Sales quotas are set to production teams. Using sales quotas, the government can plan the minimum grain supply in the state market. Figure 3.25 shows the impacts on both the domestic and the international market when the government increases sales quotas.

The beginning situations have already been discussed. As the government decides to increase the sales quota $S^1$ to $S'^1$ in the state market,

![Diagram](image)

**Figure 3.25.** The effects of a change in the sales quota in the planned economy

the total grain supply line $S^P$ is shifted to the right, and results in an increase of both total grain supply from $Q^S_1$ to $Q^S_2$ and planned grain supply in the state market from $Q^S_1$ to $Q^S_2$. The government can plan the minimum grain supply in the state market by increasing the sales quota.
supply from $Q_{1}^{PS}$ to $Q_{2}^{PS}$. Thus, both the balance of total grain demand and supply and the balance of total planned grain demand and supply decrease. Because of a decrease in the balance of total planned grain demand and supply, the planned excess grain demand line $ED$ is shifted back to $ED'$. Immediately, the import grain demand decreases from $Q_{1}^{M}$ to $Q_{2}^{M}$.

3. The state market price

The state grain markets are set for both domestic consumers and producers. By adjusting the state market prices, the government can partly influence the behaviors of urban households and rural production teams. Figure 3.26 indicates the effects of a change in the state market price on both the state and the international markets.

The beginning situations are the same as before. When the government decides to raise the state market price—say from $P_{1}^{S}$ to $P_{2}^{S}$—to

![Diagram](image)

Figure 3.26. The effects of a change in the state market price in the planned economy
Improve the rationality of the price, immediately in the state market, the total grain demand decreases and the total grain supply increases, but both total planned grain demand and supply are unchanged. Hence, the balance of total grain demand and supply decreases but the balance of total planned grain demand and supply does not change. There is no impact on the international market because all factors specified in ED stay the same as before. Here, we can draw a conclusion that a change in the state grain market price has an impact only on the domestic market but not on the international market.

**International policy instruments**  In the international grain markets, the main policy instruments used by the Chinese government in the planned economy are import quotas and export quotas. Comparative studies of these two policy instruments follow.

1. Import quota

The import quotas are used to restrict import grain demand. Theoretically, the impacts on an individual import grain demand due to a change in grain import quotas are not certain. Suppose the import quotas for all grains increase simultaneously, or suppose the import quota for total import grain demand increases.

As Figure 3.27 shows, when the import grain quota $Q_G$ increases, the planned excess demand in the international market shifts to the right, the import grain demand increased from $Q^M_1$ to $Q^M_2$. There is no change in the domestic market.
2. Export quota

Export quotas are used to limit the export grain supply. The impacts on an individual import grain demand due to an increase in the grain export quota also are not certain. Suppose the export quotas for all grains increase together, or suppose the export quota for total export grain supply increases, then the result is in the opposite direction to the result of an increase in the import grain quota (Figure 3.27b).
CHAPTER IV. THE CHINESE MIXED GRAIN ECONOMY

A major aspect of the Chinese mixed economy is the integration of central planning with the markets' mechanism in spite of the fact that central planning is still considered as fundamental by the government. Even though the state market still plays the dominant role, and the free market is merely an auxiliary means of regulating the economy, current economic reform has eventually provided a new approach for combining the state market with the free market to distribute and circulate goods, especially important goods such as grain. In this sense, the Chinese grain economy has been transformed in the last decade from being planned to being mixed. After the collective disappeared earlier in the economic reforms and the free market reemerged later, the powerful influence of the government in the grain economy declined significantly.

This chapter deals with a theoretical model for the Chinese mixed grain economy. The expositional process is similar to that in Chapter III. At the beginning, a brief overview of the planning and the marketing framework in the mixed economy is presented. Following this are three sections of theoretical discussions: the behavior of urban households, the behavior of state farm households and agricultural households, and the behavior of the MOFT in the mixed grain economy. These sections generate an urban household model, a state farm (household) model and an agricultural household model, and, finally, a MOFT

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For convenience, we call the households that belonged to the state farms state farm households, and the households that belonged to the production teams agricultural households.
General Framework of the Chinese Mixed Grain Economy

Because we have introduced the general framework of the Chinese planned grain economy, this section is mainly focused on the distinctions, rather than the similarities, between the frameworks of the mixed and the planned grain economy.

Planning framework

Since the economic reform, the macro aspects of the grain economy have still been planned and managed by the central and local governments, but institutional changes have occurred on the micro aspects of the economy. Among these institutional changes, the most significant and important for the planning framework is that the rural households, including both state farm households and agricultural households, have replaced state farms and production teams, respectively, and become the fundamental units of management and production in the agricultural sector with state or collective ownership of land and major fixed assets. The Production Responsibility System (PRS) consists of contracts that define the rights and responsibilities of owners (including state, collective, and private) and managers of assets. The most widely accepted contractual form is the Bao Gao Dao Hu (BGDH). Under the BGDH, the households have the right to manage collectively owned land according to their size or labor force. The households are obligated to pay agricultural taxes, make contributions to social welfare funds, and provide their share in
satisfying state procurement requirements. After that, all remaining output may be kept and disposed of by the households.

In addition to the replacement of state farms and production teams, the other institutions in the countryside have been restructured. The communes have been transformed into the townships, and the brigades have been renamed as the villages. All these institutional changes have brought about change in the planning framework. At the central government level, there is no big change at all. At the local government level, however, the channel from county to basic units—agricultural households through the townships then the villages—has been established to replace the old channel. There are now two kinds of new rural enterprises in the Chinese mixed economy: the state farm households and the agricultural households. The planning processes for state farm households and agricultural households are almost the same as that for state farms and production teams before the economic reform, except that the plan targets and sales quotas that were set for the state farm and the production team have to be split into the corresponding small shares to fit the households' sizes and production situations.

Figure 4.1 briefly describes the planning process in the mixed grain economy. The arrows in the graph indicate the transition of the plans between the central government and the local governments and between the local governments and the state farm households and the agricultural households.
Figure 4.1. Agricultural planning process in the Chinese mixed economy

Marketing framework

In the mixed economy, there are dual grain markets. The state market is continuously controlled by the government, while the free market is managed by the government but is opened for private exchange. Figure 4.2 shows the circulation of grain in the Chinese mixed economy.
Figure 4.2. Circulation of grain in the Chinese mixed economy

The arrows in the graph indicate the flow directions of grain within the economy. We will discuss the situations in these markets separately.

The state market Sellers in the state grain market include state farm households and a majority of the agricultural households (Type I and Type II). The former sell almost all their product to the state market, and the latter sell part of their product to the state market. Among the latter, Type I agricultural households are complete sellers or nonbuyers and Type II are partial sellers in the state markets. On the other hand,
buyers in the state grain market include urban households, the military, industry, state farm households, and a minority of agricultural households (Type II and Type III). Obviously, the first four buyers purchase most of their grain from the state market by either ration coupons or the government's permission. For the last buyers, Type III agricultural households are complete buyers and Type II are partial buyers in the state market.

Under the guidance of the government's plans, both the international market and the state stock still are used to adjust and maintain the balance of the demand and supply for grain in the state market. Indeed, the government now may even purchase or sell some grain in the free market to change the level of the state stock. For simplicity, we continue to assume that the stock grain demand/supply equals zero and to leave it to be dealt with in Chapter V. The relationship between the international market and the state market in the mixed grain economy is discussed in the last section of this chapter.

The free market Everybody in the mixed economy can buy goods in the free market. As the figure shows, buyers in the free market consist of urban households, industry, state farm households, and a minority of agricultural households (Type II and Type III). The first three buyers have the right to buy grain from the state market at the lower prices, so they may purchase additional amounts of grain from the free market at generally higher prices to satisfy themselves. The seller in the free market is usually a Type I agricultural household, that is called a complete seller in the free market.
**Household-own market and household market** The previous analyses for the state market and the free market clearly eliminate the part of agricultural households' own grain consumption (Type I and Type II). Among these households, Type I is a complete self-supplier whereas Type II is only a partial self-supplier.

For convenience, we call an assumed market in which such a household's own consumption occurs as the household-own market. Clearly, in this household-own market, the only seller and buyer is the household itself, and the trading price can be seen as the free market price. In addition, we call another assumed market in which, for all involved agricultural households, all the own grain consumptions without circulation within either the state market or the free market are combined together the household market. Sellers and buyers in this market are the households themselves.

Finally, we assume an additional united market that combines both the free market and the household market in the mixed grain economy. In these discussions, we mention grain demand and supply with a specification for the market.

The dashed line in Figure 4.2 divides the circulation of grain in the Chinese mixed economy into four parts. The central part is the circulation of grain occurring in the state market and under direct control of the government plans. The other three parts, except a rectangle on the left corner, compose the circulation of grain occurring in the free market and under weakened indirect plan control. The rectangle represents the circulation of grain within the so-called household market.
and is almost out of government control.

Generally, the circulation of grain in both the state market and the free market hold only a smaller share, whereas the own grain consumption by most of the agricultural households still holds a larger share of total grain consumption in the Chinese mixed economy. It is estimated that the government retailed about 34% of the total grain consumed in China and the agricultural households sold about 5% of the total grain consumed to the free market in 1985. Thus, in the household market, the own grain consumption by the agricultural households was roughly figured out to be about 61% of total grain consumed in the same year. These figures illustrate that the Chinese mixed grain economy, to a certain degree, is still self-sufficient but not commercialized.

Theory of Urban Grain Demand in the Mixed Economy

Even though the free market is now open to everyone without any restriction from the government, the urban and the rural people still receive different treatment in the state grain market. Thus, it is still necessary to explore and develop urban and rural grain demand theories separately for the mixed economy. Again, behavior of the urban households in the mixed grain economy will be discussed first.

To insure the correct derivation of the urban grain demand function, the general assumptions mentioned in Chapter III are consciously maintained here. Like the corresponding section in Chapter III, the

9 The data used here are from State Statistical Bureau, 1987.
study for urban grain demand in the mixed economy contains three similar parts: policy environment, urban household model, and aggregate grain demand of urban households. Unlike the corresponding section in Chapter III, this section has two additional parts: the state market analysis and the free market analysis. Under the policy environment, the policy changes due to the economic reform are especially stressed. The individual urban households still are classified into different types, and they maximize their utilities subject to new constraints based on the policy changes. Then, the aggregate urban grain demand is derived from the aggregation of grain demands by all different types of individual urban households. Finally, the exchange activities in both the state market and the free market are analyzed separately.

Policy environment

Ownership of assets In the Chinese mixed economy, most schools, stores, and other enterprises still are owned by either the public or the cooperatives, but some stores and enterprises have emerged under private or foreign ownership.

Labor Urban laborers who are hired by either public or private businesses are paid by wages plus bonuses. Those working for the public cannot change jobs without their employer's permission, but they may work in other places in their spare time. Those working for private businesses are free to take offers from the job market.

Plan control The government still issues ration coupons for food grain for all urban households. The issuing and distributing method is
Market control There is a dual market system. Urban households still need coupons to buy grain from the state market. Prices in the state grain market are determined by the government and are usually fixed for comparatively long time periods. In spite of existence of the state market, the free market is encouraged by the government and outlets are scattered in urban areas. The urban households can freely buy grain from the free market. The prices in the free grain market are determined by market forces and change from time to time. In general, for food grains, the free market prices are much higher than the state market prices.

Urban household model

Concentrating on the commodity demand, we simply assume that each urban household earns a fixed wage income, M, plus a fixed bonus, B, even though members may work in their spare time. Thus, the urban household in the mixed economy now faces a new budget, dual markets and dual prices. Suppose the urban household is buying a good, say food grain $X_1$, and let

$$X_1 = X_{1S} + X_{1F}$$

(4.1)

That is, total food grain bought is the sum of $X_{1S}$ grain purchased from the state market and $X_{1F}$ grain purchased from the free market. Obviously, $X_1$, $X_{1S}$, and $X_{1F} \geq 0$.

Furthermore, suppose the price for the state market grain $X_{1S}$ is $P_{1S}$, and the price for the free market grain $X_{1F}$ is $P_{1F}$. Comparing the
state market grain price with the free market grain price, we find all three possible situations: (1) $P_{iS} > P_{iF}$, (2) $P_{iS} = P_{iF}$, and (3) $P_{iS} < P_{iF}$.

Sicular (1988) carefully examined these three situations in her paper. Apparently, the first situation may occur if a surplus of grain in the free market exists. In the case of $P_{iS} > P_{iF}$, the urban households would buy all of their food grain from the free market at a lower price; therefore, an increasing demand for food grain would push the free market price up quickly. On the other hand, the rural households would try to sell as much food grain as possible to the state market at a higher price. Some of them might even attempt to buy food grain from the free market and then resell it to the state market to make a profit. Thus, both the higher demand and the lower supply in the free market would cause a quick increase of the free market price, then a reduction in the price difference between the markets until an end of this situation.

The second situation may happen only by coincidence. Obviously, if $P_{iS} = P_{iF}$, there is no price difference between the state market and the free market, and the ration coupons are no longer meaningful for the urban households because they could buy as much as desired in either market at the same price. With this case, there is no real reason for keeping such a dual market system, or the dual markets for those goods without coupon limits can be treated as a single market. As a result, the problem has returned to the usual utility maximization problem for the urban household.
The third situation occurs when a shortage of food grain in the free market exists. Looking at the long-run process, the case of $P_{1S} < P_{1F}$ is the most likely because the problem of food grain shortage in China has not yet been solved and might become more serious in the future. In this case, if we assume all food grain products are homogeneous, the urban households would buy food grain $X_{1S}$ in the state market first until their coupons are used up and then consider further purchasing $X_{1F}$ in the free market.

In theory, the third situation exists continuously, because China usually faces constant grain shortages in both the state market and the free market. In practice, according to statistics during the period 1979 to 1986, the state market prices had been much lower than the corresponding free market prices. Similar analyses can be used for the other goods. Thus, this study is mainly concentrating on the third situation with an effective rationing coupon system.

Focusing on the third situation, Byrd (1987) created a model to deal with the (urban) demand from industry in the mixed economy. After that, Sicular (1988) developed her model to handle the urban demand for agricultural goods in the mixed economy. First of all, perhaps for simplicity, she assumed that the urban producer also is a consumer of agricultural commodities. This is true for those urban industries that use agricultural goods as their inputs or for those urban households which open private enterprises; but it is not true for the majority of urban households which earn fixed wages in the public or cooperative enterprises and consume agricultural goods in their own home. Thus, her model
may not fit the mixed grain economy very well. Second, perhaps by accident, she ignored the dependence between the total urban grain demand (total urban demand for agricultural goods)\(^1\) and the total urban grain demand in only the state market. In detail, she made the mistake of choosing both \(X_1 (X_{12})\) and \(X_{1S} (\gamma_{12})\) as the decision variables in the Lagrangian function for the urban household, but \(X_{1S}\) is either just \(X_1\) itself if \(X_{1S} \leq \bar{R}_1 (X_{12} - \gamma_{12} \leq \bar{\gamma}_{12})\) or is included in \(X_1\) if \(X_{1S} = \bar{R}_1 (\gamma_{12} = \bar{\gamma}_{12})\) and \(X_{1F} > 0 (X_{12} \geq \bar{\gamma}_{12})\). Fortunately, by assuming the equilibrium conditions and such an aggregate demand function, she had grain consumption in the state market equal the ration level and avoided the technical trouble caused by variable choices in her model, but she totally omitted the possibility of \(X_1 = X_{1S} (X_{12} = \gamma_{12} \leq \gamma_{12})\). Finally, based on such derivation, Sicolar drew her main conclusion that the state market price has no direct effect on grain demand (the demand for agricultural goods) but only the free market price does.

In fact, this result may not well represent the situation in today's China. It is known that most of the urban households in China cannot use up their food grain coupons; they accumulate the coupons at home instead. Some of them trade illegally with the rural people for agricultural goods such as eggs, chickens or even for money. In this case, urban households purchase food grain from the free market not because the total quantity of grain in the state market is limited but because the qualities and

\(^1\)Unless otherwise noted, from now on the parentheses in this paragraph indicate the similar item or symbol used by Sicolar (1988) in her model.
varieties of grain available in the state market are limited. For instance, in Northern China, sometimes the state market limits the purchase of some grains such as rice or wheat flour within the ration level for total grains provided.

Noting Sicilari's mistake, considering a multiple-good case instead of her three-good case, and assuming the situation in which the state market prices are less than the free market prices, we extend our urban household model in the planned economy to fit the changes in the mixed grain economy. Regardless of the rationing system, our decision variables would be \( X_i \) (\( i = 1, \ldots, n \)) and all relevant Lagrangian multipliers. The Lagrangian function for an urban household in the mixed economy is as follows:

\[
L = U[X_1, \ldots, (X_{1S} + X_{1P}), \ldots, X_k, X_1, \ldots, X_n] \\
+ \lambda_1 [M + B - \sum_{i=1}^{k} (P_{iS}X_{1S} + P_{iF}X_{1F}) - \sum_{i=1}^{n} P_i X_i] \\
+ \sum_{i=1}^{k} \lambda_{2i} (R_i - X_{1S}) \tag{4.2}
\]

\[
= U[X_1, \ldots, (X_{1S} + X_{1P}), \ldots, X_k, X_1, \ldots, X_n] \\
+ \lambda_1 [M + B - \sum_{i=1}^{k} P_{iF} (X_{1S} + X_{1F}) - \sum_{i=1}^{n} P_i X_i - \sum_{i=1}^{k} (P_{iF} - P_{iS}) X_{1S}] \\
+ \sum_{i=1}^{k} \lambda_{2i} (R_i - X_{1S}) \tag{4.3}
\]

where the first \( k \) goods are rationed and the remaining \( n-k \) goods are unrationed. \( M + B - \sum (P_{iS}X_{1S} + P_{iF}X_{1F}) - \sum P_i X_i \) is a new budget constraint faced by the urban household in the mixed economy. It implies
that the total income of the household, $M + B$, has to cover the total expenditure, including the expenses on the rationed goods in both the state market and the free market and the unrationed goods. Another constraint, $R_i - X_{iS}$, means the ration for purchasing goods in the state market.

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial X_1} - \frac{\partial L}{\partial (X_{iS} + X_{iF})} \leq 0 \quad i = 1, \ldots, k \quad (4.4)$$

$$\frac{\partial L}{\partial X_1} - \frac{\partial L}{\partial X_1} - \lambda_1 P_{i1} \leq 0 \quad i = 1, \ldots, n \quad (4.5)$$

$$X_i \frac{\partial L}{\partial X_i} = 0 \quad i = 1, \ldots, n \quad (4.6)$$

$$\frac{\partial L}{\partial \lambda_1} = M + B - \sum_{i=1}^{k} (P_{iS}X_{iS} + P_{iF}X_{iF}) \quad \sum_{i=1}^{n} P_i X_i \geq 0 \quad \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0 \quad \lambda_1, \lambda_{2i} \geq 0 \quad i = 1, \ldots, k \quad (4.7)$$

$$\frac{\partial L}{\partial \lambda_{2i}} = -R_i - X_{iS} \geq 0 \quad i = 1, \ldots, k \quad (4.8)$$

$$\lambda_{2i} \frac{\partial L}{\partial \lambda_{2i}} = 0 \quad i = 1, \ldots, k \quad (4.9)$$

$$X_{iS}, X_{iF} \geq 0 \quad i = 1, \ldots, k \quad (4.10)$$

$$X_i \geq 0 \quad i = 1, \ldots, n \quad (4.11)$$

$$\lambda_1, \lambda_{2i} \geq 0 \quad i = 1, \ldots, k \quad (4.12)$$

Let us make some assumptions to simplify the Kuhn-Tucker conditions:

1. Suppose

$$M + B - \sum_{i=1}^{k} (P_{iS}X_{iS} + P_{iF}X_{iF}) - \sum_{i=1}^{n} P_i X_i = 0 \quad (4.14)$$

that is, the urban household spends all its money in the market(s). Then
(4.7) and (4.8) can be dropped out.

(2) Suppose

$$X_1 > 0 \quad i = 1, \ldots, n \quad (4.15)$$

the household buys every good in the market(s). Hence, (4.5) equals zero
and becomes (4.5)', and (4.6) can be eliminated.

Looking at the new Kuhn-Tucker conditions carefully, we have trouble
with (4.4), which is expressed as an implicit condition. To obtain an
explicit expression, we need a technique to develop a mathematical
transformation for \( \frac{\partial L}{\partial X_1} \) \((i = 1, \ldots, k)\):

$$\frac{\partial L}{\partial X_1} = \frac{\partial L}{\partial (X_{1s} + X_{1F})}$$

$$= - \frac{1}{\frac{\partial (X_{1s} + X_{1F})}{\partial L}}$$

$$= - \frac{1}{\frac{\partial X_{1s}}{\partial L} + \frac{\partial X_{1F}}{\partial L}}$$

$$= - \frac{1}{\frac{\partial L}{\partial X_{1s}} + \frac{\partial L}{\partial X_{1F}}} \quad (i = 1, \ldots, k) \quad (4.16)$$

Because \( X_1 > 0 \) \((i = 1, \ldots, k)\) by (4.15), immediately we have

$$\frac{\partial L}{\partial X_1} = 0 \quad (4.17)$$

Substituting (4.17) into (4.16) we have
\[
\frac{\partial L}{\partial x_{1s}} \quad \frac{\partial L}{\partial x_{1f}} = 0 \quad (4.18)
\]

Consequently, it is necessary to have either
\[
\frac{\partial L}{\partial x_{1s}} = 0 \quad \text{but} \quad \frac{\partial L}{\partial x_{1f}} \neq 0 \quad (4.19)
\]
or
\[
\frac{\partial L}{\partial x_{1f}} = 0 \quad \text{but} \quad \frac{\partial L}{\partial x_{1s}} \neq 0 \quad (4.20)
\]

Moreover, in order to avoid the situation of multiple solutions for \(x_1\), we need to reduce the ranges for \(\frac{\partial L}{\partial x_{1s}} \neq 0\) in (4.19) and \(\frac{\partial L}{\partial x_{1f}} \neq 0\) in (4.20). Clearly only two cases in which a unique solution for \(x_1\) is insured can exist and are discussed below:

**Case 1.** \(\frac{\partial L}{\partial x_{1s}} = 0\) and \(\frac{\partial L}{\partial x_{1f}} < 0\) \( (4.21)\)

This is because \(\frac{\partial L}{\partial x_{1f}} < 0\) guarantees a corner solution \(x_{1f} = 0\); therefore, the situation of multiple solutions for \(x_1\) under concave constraint sets can be avoided.

The above conditions imply that, to maximize the utility, the urban household is buying food grain only in the state market; hence,
\[x_{1s} = x_{1s} (\ldots) \leq \bar{R}_1 \quad (4.22)\]

and
\[x_{1f} = 0 \quad (4.23)\]

Substituting (4.22) and (4.23) into (4.1), we obtain
\[x_1 = x_{1s} (\ldots) \leq \bar{R}_1 \quad (4.24)\]

where \(x_{1s} (\ldots)\) is an implicit function for \(x_{1s}\) at this time.
These results indicate that the ration constraint is not binding. The urban household seems to satisfy its initial needs in the state grain market at a lower price and does not need the free market for further purchases at a higher price. Indeed, if the household’s preference has been unchanged, this case fits well the first type of urban household in the planned economy. Thus, we conclude that for the first type of urban households, their preference and, therefore, grain demand function are not affected by the existence of the free market in the mixed economy.

Case 2. \( \frac{\partial L}{\partial x_{is}} > 0 \) and \( \frac{\partial L}{\partial x_{if}} = 0 \) \hspace{1cm} (4.25)

This is because \( \frac{\partial L}{\partial x_{is}} > 0 \) guarantees a corner solution \( x_{is} = \bar{r}_i \); therefore, it guarantees continuity and a unique solution of \( x_i \) under nonlinear budget constraints.

These conditions imply that, to maximize utility, the urban household decides to use both the state market and the free market to purchase its food grain. Thus,

\[
x_{is} = \bar{r}_i \tag{4.26}
\]

and

\[
x_{if} = x_{if} ( ) \tag{4.27}
\]

Substituting (4.26) and (4.27) into (4.1), we get

\[
x_i = \bar{r}_i + x_{if} ( ) > \bar{r}_i \tag{4.28}
\]

where \( x_{if} ( ) \) is an implicit function for \( x_{if} \) at this time.

The results indicate that the ration constraint in the state market is just binding. The urban household seems not to satisfy its initial purchase in the state grain market and is willing to pay a higher price.
for an additional purchase in the free market. Assuming the household's preference has been unchanged, this household in the mixed economy exactly belonged to the second type of urban households in the planned economy because their attitudes toward the grain coupons in the state market are exactly the same. The only difference is that the household in the mixed economy needs the free market to purchase additional grain to make itself better off, whereas the household in the planned economy had the same need for further purchase but had no way to go (to the free market).

Now we continue to discuss the Kuhn-Tucker conditions by using the above results. Within the first k rationed goods, supposing the first g goods belong to Case 1 and the remaining k-g goods belong to Case 2; we could obtain a set of explicit conditions for replacing undeveloped (4.4):

$$\frac{\partial L}{\partial x_1} - \frac{\partial L}{\partial x_{iS}}$$

\[= \frac{\partial u}{\partial x_1} \frac{\partial (x_{iS} + x_{iF})}{\partial x_{iS}} - \lambda_i p_{iS} - \lambda_{2i} \]

\[= \frac{\partial u}{\partial x_1} - \lambda_i p_{iS} - \lambda_{2i} \leq 0 \quad i = 1, \ldots, g \quad (4.29)\]

$$x_1 \frac{\partial L}{\partial x_1} - x_{iS} \frac{\partial L}{\partial x_{iS}} = 0 \quad i = 1, \ldots, g \quad (4.30)$$

$$\frac{\partial L}{\partial x_1} - \frac{\partial L}{\partial (x_{iS} + x_{iF})}$$

\[= \frac{\partial l}{\partial (R_i + x_{iF})} \]
In addition, according to conditions that appeared in Case 1 and Case 2, we have

\[ \frac{\partial L}{\partial \lambda_{2i}} - R_i - X_{iS} = 0 \quad i = h, \ldots, k \]  

and \[ \frac{\partial L}{\partial \lambda_{2i}} = R_i - X_{iS} > 0 \quad (i = 1, \ldots, k) \] can be omitted from our model.

We already assumed all \( X_i > 0 \) \( (i = 1, \ldots, n) \), so that (4.29) and (4.31) become the equalities (4.29)' and (4.31)', and (4.30) and (4.32) can be dropped out. Thus, a complete new set of Kuhn-Tucker conditions consists of (4.29)', (4.31)', (4.5)', (4.14), (4.33), and a set of positive constraints (4.11), (4.12) and (4.13).

Solving these first-order conditions simultaneously, provided that the second-order conditions are satisfied, additionally, let

\[ P'_{is} = P_{is} \quad (i = 1, \ldots, g), \]

\[ P'_{IF} = P_{IF} \quad (i = h, \ldots, k), \]

\[ P'_i = P_i \quad (i = 1, \ldots, n), \]

\[ R'_i = R_i \quad (i = h, \ldots, k), \]

and \( M' = M + B \).

The implicit demand functions for the urban household in the mixed
The following discussion for urban household grain demand in the mixed economy is similar to that in the planned economy, and, therefore, will be given briefly. Facing rationed grain in the state market in the mixed economy, an urban household can have two different choices, hence two possibly different grain demand functions. All the urban households in the mixed economy still can be classified into two types by their choices. Suppose that the preferences of these two types of urban households have not changed and that their real income and the real price of grain also have not changed. Immediately, we obtain individual grain demand functions for those two types of urban households and note that those two types to which the urban households belong have not changed at all since the period of the planned economy.

The first type of urban households, whose grain demand was not affected by the grain ration in the planned economy, are continuously satisfying their initial purchase in the state grain market. Its demand function for food grain in the mixed economy is

\[ X_{i} = X_{iS} \]
\[ = X_{iS} (P_{iS}, P_{iF}, P_{i}, R_{i}, M') \leq R_{i} \quad i = 1, \ldots, g \] (4.34)

\[ X_{i} = X_{iS} + X_{iF} \]
\[ = R_{i} + X_{iF} (P_{iS}, P_{iF}, P_{i}, R_{i}, M') > R_{i} \quad i = h, \ldots, k \] (4.35)

\[ X_{i} = X_{i} (P_{iS}, P_{iF}, P_{i}, R_{i}, M') \quad i = 1, \ldots, n \] (4.36)

\[ X_{j} = X_{jS} \]
\[ = X_{jS} (P_{jS}, P_{jF}, P_{j}, R_{j}, M') \leq R_{j} \quad (4.37) \]
Note that $P_i^S$ includes $P_j^S$ but $P_i^F$ and $R_j^*$ exclude $P_j^F$ and $R_j^*$, respectively. Thus, formula (4.37) implies that the grain demand of the first type of household in the mixed economy is a function of the state market price vector $P_i^S$, including grain price $P_j^S$; the free market price vector $P_i^F$, excluding grain price $P_j^F$, the price vector of unrationed goods, the ration vector $R_i^*$, excluding grain ration $R_j^*$; and, finally, its wage $M$ plus bonus $B$.

On the other hand, the second type of urban household, whose grain demand was the ration itself in the planned economy, is getting better off by an additional purchase in the free market. Its demand function for food grain in the mixed economy becomes:

$$X_j^{II} = X_j^{II} + X_j^{FII}$$

$$= R_j + X_j^{FII} (P_i^S, P_i^F, P_i^*, R_i^*, M^*) > R_j$$  \[4.38\]

$$= X_j^{FII} (P_i^S, P_i^F, P_i^*, R_i^*, M^*)$$  \[4.39\]

Note that $P_i^S$ excludes $P_j^S$, but $P_i^F$ and $R_i^*$ include $P_j^F$ and $R_j^*$, respectively. Moreover, $M^*$ is the difference between $M'$ and expenditure on $X_j^S$, or

$$M^* = M' - P_j^S R_j$$  \[4.40\]

Hence, formula (4.39) implies that the grain demand by the second type of urban household in the mixed economy is a function of the state market price vector $P_i^S$, excluding grain price $P_j^S$; the free market price $P_i^F$, including grain price $P_j^F$; the price vector of unrationed goods, the ration vector $R_i^*$, including grain ration $R_j^*$; and its wage and bonus income.
Comparative statics

The comparative statics are given for the above two types of urban households. To save space and time, we concentrate on only own price effects, own ration effect, and the income effect as well as the bonus effect.

1. The first type of urban household

From formula (4.37), the food grain demand of the first type of urban household in the mixed economy is

\[ X_{j_1} = X_{j_1} \left( P_{i_1}^{'}, P_{i_F}^{'}, P_1^{'}, R_1^{'}, M^{'}, F^{'}, R_{j} \right) \]

The comparative statics can be calculated as the following:

\[ \frac{\partial X_{j_1}}{\partial P_{jS}} = \frac{\partial X_{j_1}}{\partial P_{jS}} < 0 \]

\[ \frac{\partial X_{j_1}}{\partial P_{jF}} = \frac{\partial X_{j_1}}{\partial P_{jF}} = 0 \]

\[ \frac{\partial X_{j_1}}{\partial R_j} = \frac{\partial X_{j_1}}{\partial R_j} = 0 \]

\[ \frac{\partial X_{j_1}}{\partial M} = \frac{\partial X_{j_1}}{\partial M} > 0 \text{ for normal goods} \]

\[ \frac{\partial X_{j_1}}{\partial B} = \frac{\partial X_{j_1}}{\partial B} > 0 \text{ for normal goods} \]

The results indicate that, for the first type of urban household in the mixed economy, its food grain demand is driven by the own state market price in the opposite direction and by its wage and bonus income in the same direction, but is not driven by the own free market price and the official food grain ration level. Noting that both \( P_{i_1}^{'}, \) and \( P_{i_F}^{'}, \) vectors appeared in the function form, and checking our results, we can conclude that the food grain demand by the first type of urban household
in the mixed economy is affected by the dual price system but not by the dual prices of grain.

2. The second type of urban household

By formula (4.38), the food grain demand of the second type of urban household in the mixed economy is

$$X_{jII} = \bar{R}_j + X_{jFII}(P_{IS}^n, P_{IF'}^n, P_{I}'^n, \bar{R}_I^n, M^n)$$

The comparative statics can be calculated as the following:

$$\frac{\partial X_{jII}}{\partial P_{JS}} = \frac{\partial X_{jFII}}{\partial M^n} < 0 \quad \text{if} \quad \frac{\partial X_{jFII}}{\partial M^n} > 0$$

$$\frac{\partial X_{jII}}{\partial P_{IF'}} = \frac{\partial X_{jFII}}{\partial F} < 0$$

$$\frac{\partial X_{jII}}{\partial R} = \frac{\partial X_{jFII}}{\partial M^n} \frac{\partial M^n}{\partial R} > 0 \quad \text{if} \quad \frac{\partial X_{jFII}}{\partial M^n} \frac{\partial M^n}{\partial R} > -1$$

$$\frac{\partial X_{jII}}{\partial M} = \frac{\partial X_{jFII}}{\partial M} > 0 \quad \text{for normal goods}$$

$$\frac{\partial X_{jII}}{\partial B} = \frac{\partial X_{jFII}}{\partial B} > 0 \quad \text{for normal goods}$$

These results imply that, for the second type of urban household in the mixed economy, its food grain demand is driven by the own free market price directly through the price effect, by the own state market price indirectly through the income effect in the opposite direction, and by its wage and bonus income in the same direction. The ration effect consists of two components: one directly impacts on $X_{jSII}$ and the other one indirectly impacts on $X_{jFII}$ through the income effect. As the official grain ration level increases, $X_{jII}$ might increase. Similarly,
both $P^S_1$ and $P^F_1$ vectors are contained in the function form of $X_{jii}$.

Combining this with the previous results, we can draw a conclusion that the food grain demand by the second type of urban household in the mixed economy is directly influenced by the dual price system but not by grain own dual prices.

**Graphic analysis** In order to make a graphic analysis of the mixed grain economy, it is necessary to introduce a dual market graph in this part. As Figure 4.3 shows, the usual $(Q, P)$ space can be divided into two subspaces by a vertical line of the free market price $P^F$. On the left of $P^F$ is the $(Q^S, P^S)$ subspace, where $Q^S$ is the quantity demanded or supplied in the state market and $P^S$ is the state market price. All activities occurring in the state market are represented in $(Q^S, P^S)$ subspace. On the right of $P^F$ is the $(Q^F, P^F)$ subspace, where $Q^F$ is quantity demanded or supplied in the free market and $P^F$ is the free market price. All activities happening in the free market are reflected in $(Q^F, P^F)$ subspace. To learn the quantity demanded or supplied in both the state and the free market in the mixed economy, one should first read $Q^S$ in $(Q^S, P^S)$ subspace according to the given state market price $P^S$, then read $Q^F$ in $(Q^F, P^F)$ subspace according to the given free market price $P^F$, and, finally, add $Q^S$ and $Q^F$ together to get total quantity demanded or supplied in the combined market.

This graph is especially useful for a rationed good such as food grain in the mixed economy. Under rationing, there is a vertical limit for the rationed good in $(Q^S, P^S)$ subspace. For example, $Q^S_j = R_j$ is an
individual urban grain demand, and $Q^S = \sum R_j$ is an aggregate urban grain demand by a certain type of urban household. Thus, we can move the $P^F$ axis to this vertical limit, such as $R_j$ or $\Sigma R_j$, to combine both the state market and the free market together (Figure 4.3).

Moreover, under the assumption that $P^S < P^F$, or the state market price is less than the free market price, given the ration level $R_j$ or whatever it is, and the state market price $P^S$, as Figure 4.3 shows, the exchange activity occurring in the state market would be located in the south of $(Q^S, P^S)$ subspace, with the right boundary $R_j$ and the upper boundary $P^S$. On the other hand, the exchange activity happening in the free market would be located in the north of $(Q^F, P^F)$ subspace, with the left boundary $R_j$ and the lower boundary $P^S$. Indeed, $R_j$ and $P^S$ limit the state market $(Q^S, P^S)$ subspace, as well as the free market $(Q^F, P^F)$ subspace. In other words, the government directly controls the state market by means of two policy instruments: ration coupons and the state market price. Any change in the ration coupons or the state market price
means a change in the state market space, and, therefore, a change in the free market space. Because of shortages, an expansion or a reduction of the state market space would cause a reduction or an expansion of the free market space. In this sense, the free market space can be seen as a residual space and is indirectly controlled by these government policy instruments. The following are graphic analyses for two types of urban households in the mixed economy.

1. The first type of urban household

In Figure 4.4a, both $X_1$ and $X_2$ are goods. $X_2$ is a rationed good—say food grain—but $X_1$ is not. $\bar{R}$ is the official ration level for $X_2$. ABC is a kinked budget line for the first type of urban household in the mixed economy. Assume the income of the household is fixed, and AB is part of the budget line that reflects the state market price $P_S$ and the ration $\bar{R}$, whereas BC, another part of the budget line, reflects the free market price $P_F$. Under the assumption that $P_S < P_F$, BC is steeper than AB.

![Diagram](image)

Figure 4.4. The derivation of grain demand of the first type of urban household in the mixed economy
For the first type of urban household in the mixed economy, its indifference curve would be tangent to the budget line AB because the official ration is not binding. The shape of the food grain demand is the same as that in the planned economy. Figure 4.4b gives a kinked demand in \((Q^S, P^S)\) space for the first type of urban household. The ration \(\overline{R}\) plus the dotted line separate \((Q, P)\) space into the state market space \((Q^S, P^S)\) and the free market space \((Q^F, P^F)\). Of course, the exchange activity participated in by the first type of household is limited in the state market space only.

2. The second type of urban household

Figure 4.5a shows that another situation exists for the second type of urban household. Its indifference curve would be tangent on the budget line BC. Suppose the real prices of goods \(X_1\) and \(X_2\) in the state market and the real income of the household have been unchanged since the period of the planned economy. Consequently, the household's utility in the mixed economy (at point D) obviously would be higher than that in the planned economy (at point B) because of opening the free market.

In Figure 4.5b, two pieces of demand line for the second type of urban household are given. The first piece is a vertical line \(X_{1S} = \overline{R}\), which is located in \((Q^S, P^S)\) subspace. Given the state market price \(P^S\), a corresponding point on \(X_{1S} = \overline{R}\) should be read as quantity demanded in the state market. There is no change in this demand at all if the state market price changes. The second piece is a downward-sloping curve that is located in \((Q^F, P^F)\) subspace. Because the official ration \(\overline{R}\) is given by the government and \(P^S < P^F\) is provided by the assumption, this piece
Figure 4.5. The derivation of grain demand of the second type of urban household in the mixed economy

would be on the right of $\bar{R}$ and on the north of $P^S$. Given the free market price, a corresponding point on $X_F$ should be read as quantity demanded in the free market only. Thus, in general, for dual markets, there are dual demand lines. Given dual prices in the dual markets, there are always corresponding dual points on the dual demand lines, respectively.

Finally, because the urban household is always buying its initial food grain in the state market, the demand in the free market can be called the residual demand, to correspond to the total grain demand by the urban household.

Aggregate grain demand of urban households

The aggregate urban food grain demand is the sum of all individual urban household's food grain demands. In the mixed economy we have

$$D_{UM} = \sum_{j=1}^{n} X_j$$  \hspace{1cm} (4.41)
where $D^{UM}$ = aggregated urban food grain demand in the mixed economy

$X_j$ = individual urban household food grain demand in the mixed economy

Clearly, the aggregate urban food grain demand $D^{UM}$ can be the sum of aggregate urban households' food grain demands of both the first and the second types of households. That is,

$$D^{UM} = D^{UMI} + D^{UMII} \quad (4.42)$$

where $D^{UMI}$ = aggregate urban food grain demand of the first type of households in the mixed economy

$D^{UMII}$ = aggregate urban food grain demand of the second type of households in the mixed economy

For the first type of urban households, who purchase food grain from only the state market, we assume that the total number of households in this group is $n_{UI}$ and all of them are identical and face the same official ration $R_j$. Then their aggregate food grain demand $D^{UMI}$ would be

$$D^{UMI} = \sum_{j=1}^{n_{UI}} X_{j} 
= \sum_{j=1}^{n_{UI}} X_{jSI} (P_{IS}', P_{IF}', P_i', \bar{R}_i', M_i') 
= n_{UI} X_{jSI} (P_{IS}', P_{IF}', P_i', \bar{R}_i', M_i') \leq n_{UI} \bar{R}_j \quad (4.43)$$

For the second type of urban households, who purchase food grain from both the state market and the free market, we assume that there are $n_{UII}$ identical households in this group and that they are facing the same ration $\bar{R}_j$. Hence, their aggregate food grain demand $D^{UMII}$ would be
Substituting (4.43) and (4.44) into (4.42), we obtain

\[ D_{U} = \sum_{j=1}^{n_{UI}} X_{jII} \]

\[ = \sum_{j=1}^{n_{UII}} [R_{j} + X_{jIII}(P_{iS}', P_{IF}', P_{i}', \overline{R}_{i}', M_{i}')] \]

\[ = n_{UII} \overline{R}_{j} + n_{UII}X_{jIII}(P_{iS}', P_{IF}', P_{i}', \overline{R}_{i}', M_{i}')] \geq n_{UII} \overline{R}_{j} \quad (4.44) \]

Substituting (4.43) and (4.44) into (4.42), we obtain

\[ D_{UM} = n_{UI} X_{jSI}(P_{iS}', P_{IF}', P_{i}', \overline{R}_{i}', M_{i}') \]

\[ + n_{UII} \overline{R}_{j} + n_{UII}X_{jIII}(P_{iS}', P_{IF}', P_{i}', \overline{R}_{i}', M_{i}')] \quad (4.45) \]

Let the total number of urban households in China be \( n_{U} \), so that

\[ n_{U} = n_{UI} + n_{UII} \quad (4.46) \]

In addition, let

\[ \frac{n_{UI}}{n_{U}} = P_{UI} \quad (4.47) \]

where \( P_{UI} \) is the relative frequency of the first type of urban households. Substituting (4.46) and (4.47) into (4.45) gives

\[ D_{UM} = n_{UI} X_{jSI}(P_{iS}', P_{IF}', P_{i}', \overline{R}_{i}', M_{i}') \]

\[ + (n_{U} - n_{UI}) \overline{R}_{j} + (n_{U} - n_{UI}) X_{jIII}(P_{iS}', P_{IF}', P_{i}', \overline{R}_{i}', M_{i}') \]

\[ = P_{UI} n_{UI} X_{jSI}(P_{iS}', P_{IF}', P_{i}', \overline{R}_{i}', M_{i}') \]

\[ + (1-P_{UI})n_{U} \overline{R}_{j} + (1-P_{UI})n_{U}X_{jIII}(P_{iS}', P_{IF}', P_{i}', \overline{R}_{i}', M_{i}') \]

\[ = D_{UM}(P_{UI}, n_{U}, P_{iS}', P_{IF}', P_{i}', \overline{R}_{i}', M_{i}') \quad (4.48) \]

Hence, the aggregate urban food grain demand in the mixed economy is theoretically specified as a function of the relative frequency of the first type of urban households, the total number of urban households, the
state market prices including the state grain price; the free market prices including the grain price; the prices of unrationed goods; the ration levels including the grain ration; and the individual household's wage plus bonus income.

Analyzing formula (4.48), we find out that $D_{UM}$ contains two components: $D_{UMI}$ and $D_{UMII}$. This conclusion is different from the previous studies. Looking at both $D_{UMI}$ and $D_{UMII}$, we note that they have some different factors and some similar factors. This fact implies that each component sometimes plays its own role to affect the total urban grain demand $D_{UM}$ because of the difference in the existing factors, and sometimes both components play their roles together to affect $D_{UM}$ because of the similarity in the existing factors.

Second, comparing (4.48), aggregate urban grain demand in the mixed economy, with (3.34), aggregate urban grain demand in the planned economy, we cannot draw any conclusion here because of too many uncertain factors. However, by assuming $P_{UI}$ and $R_j$ are fixed for both the mixed economy and the planned economy, and neglecting the differences between $X_{JSI}(P'_S, P'_F, P'_1, R'_1, M'_1)$ in the mixed economy and $X_{JS}(P, R'_j, M')$ in the planned economy, the sum of the first two terms in $D_{UM}$ is consistent with $D_{UP}$; consequently, we can have

$$D_{UM} = D_{UP} + (1 - P_{UI})n X_{JFI} (P''_S, P''_F, P'_1, R'_1, M''_1) > D_{UP}$$  \hspace{1cm} (4.49)

That is, given the assumed conditions above, the aggregate urban grain demand in the mixed economy would be higher than that in the planned economy. If this is true, we can conclude that the urban households are benefited in food grain consumption by the economic
Comparative statics As we did in the study for individual types of households, the comparative statics in this part are still focused on the effects of own prices, own ration, the income, and the bonus. Using (4.48), the comparative statics of the aggregate urban grain demand in the mixed economy can be calculated:

\[ \frac{\partial D_{UM}}{\partial p_{UI}} = \frac{\partial D_{UMI}}{\partial p_{UI}} + \frac{\partial D_{UMII}}{\partial p_{UI}} \]

\[ = n_U x_{JSI} + n_U r_j + n_U x_{JFI} < 0 \]

The total urban grain demand decreases if the relative frequency of the first type of urban household increases.

\[ \frac{\partial D_{UM}}{\partial n_U} = \frac{\partial D_{UMI}}{\partial n_U} + \frac{\partial D_{UMII}}{\partial n_U} \]

\[ = P_{UI} x_{JSI} + (1 - P_{UI}) r_j + (1 - P_{UI}) x_{JFI} > 0 \]

The total urban grain demand increases as the total number of urban households increases.

\[ \frac{\partial D_{UM}}{\partial p_{JS}} = \frac{\partial D_{UMI}}{\partial p_{JS}} + \frac{\partial D_{UMII}}{\partial p_{JS}} \]

\[ = P_{UI} n_U \frac{\partial x_{JSI}}{\partial p_{JS}} + (1 - P_{UI}) n_U \frac{\partial x_{JSII}}{\partial p_{JS}} < 0 \]

\[ \frac{\partial D_{UMII}}{\partial p_{JF}} = \frac{\partial D_{UMII}}{\partial p_{JF}} \]

\[ = (1 - P_{UI}) n_U \frac{\partial x_{JFI}}{\partial p_{JF}} < 0 \]
An increase in either the state grain market price or the free grain market price can drive the total urban grain demand down. Note that both the state market price and the free market price have only a partially direct price effect on the total urban grain demand through corresponding components \( (D_{\text{UMI}} \text{ in } D_{\text{UM}} \text{ for } P_j^S) \) and \( (D_{\text{UMII}} \text{ in } D_{\text{UM}} \text{ for } P_j^F) \) individually. This conclusion is different from the previous one (Sicular, 1988) that the aggregate demand of agricultural goods in the mixed economy is only directly affected by the free market price but not by the state market price.

\[
\frac{\partial D_{\text{UM}}}{\partial R_j} = \frac{\partial D_{\text{UMI}}}{\partial R_j} + \frac{\partial D_{\text{UMII}}}{\partial R_j} \\
= (1 - P_{\text{UI}}) n_u [1 + \frac{\partial x_{\text{FII}}}{\partial m} \frac{\partial m}{\partial R_j}] > 0
\]

When the official ration level is increased, the total urban grain demand also increases, but only the second type of urban households want more food grain. This also is a partial ration effect in the mixed economy.

\[
\frac{\partial D_{\text{UM}}}{\partial m} = \frac{\partial D_{\text{UMI}}}{\partial m} + \frac{\partial D_{\text{UMII}}}{\partial m} \\
= P_{\text{UI}} n_u \frac{\partial x_{\text{FII}}}{\partial m} + (1 - P_{\text{UI}}) n_u \frac{\partial x_{\text{FII}}}{\partial m} > 0
\]

\[
\frac{\partial D_{\text{UM}}}{\partial B} = \frac{\partial D_{\text{UMI}}}{\partial B} + \frac{\partial D_{\text{UMII}}}{\partial B} \\
= P_{\text{UI}} n_u \frac{\partial x_{\text{FII}}}{\partial B} + (1 - P_{\text{UI}}) n_u \frac{\partial x_{\text{FII}}}{\partial B} > 0
\]

When the government increases the wage or gives an additional bonus
to the urban households, the total urban grain demand increases, provided that grain is a normal good.

**Graphic analysis**  The graphic analysis is made for two types of urban households separately.

1. Aggregate grain demand by the first type of urban households

From (4.48) and (4.43) we know that

\[ D_{UMI} = p_{UI} n_{U} x\sum_{j=1}^{n} (p_{IS}^{'} p_{IF}^{'} p_{1}^{'} R_{j}^{'} M_{i}^{n}) \leq p_{UI} n_{U} R_{j} \quad (4.50) \]

Figure 4.6a indicates that the aggregate food grain demand by the first type of urban households in the mixed economy would be a kinked line in \((Q^S, P^S)\) subspace, and nothing in the \((Q^F, P^F)\) subspace. The main piece of \(D_{UMI}\) is downward sloping with \(Q_{UI} = p_{UI} n_{U} R_{j}\) as its right boundary and \(\bar{P}_S\) as its upper boundary.

2. Aggregate grain demand by the second type of urban households

By formulas (4.48) and (4.44) we can get

\[ D_{UMII} = (1 - p_{UI}) n_{U} R_{j} + (1 - p_{UI}) n_{U} x\sum_{j=1}^{n} (p_{IS}^{''} p_{IF}^{''} p_{1}^{''} R_{j}^{''} M_{i}^{n}) \geq (1 - p_{UI}) n_{U} R_{j} \quad (4.51) \]

In Figure 4.6b, the shape of \(D_{UMII}\) is the same as that of the graph for individual urban households within the second type (Figure 4.5b), except that the right bound line for \((Q^S, P^S)\) space has become \(Q_{UI} = (1 - p_{UI}) n_{U} R_{j}\).

Based on formula (4.48), referring to Figure 4.6a and 4.6b, we can draw a graph of the aggregate urban grain demand in the mixed economy (Figure 4.6c). The shape of \(D_{UM}\) is similar to that of \(D_{UMII}\), except
To close this section we present the market analysis, which includes the state market analysis and the free market analysis. These two parts can help us to recognize fully the roles of both the state market and the free market for urban households in the mixed economy.

**The state market analysis**

Although dual markets exist in the mixed economy, the government still is paying more attention to the state market. Thus, it is necessary to conduct the state market analysis to explore whether or not there is a change in the government plans regarding urban grain demand in the state market.

**Grain demand by urban household in the state market**

As we know, there are two types of urban households in the state market, and their
behavior in the mixed economy is similar to that in the planned economy.

1. The first type of urban household

\[ X_{jS1} = X_{jS1} (P'_{1S}, P'_{1F}, P'_1, \bar{R}'_1, M') \leq \bar{R}_j \]  (4.52)

For the first type of urban household, its grain demand in the state market is a function of the state market price vector \( P'_{1S} \), the free market price vector \( P'_{1F} \), the prices of other goods, the state ration levels, and the money income.

2. The second type of urban household

\[ X_{jSII} = \bar{R}_j \]  (4.53)

For the second type of urban household, its grain demand in the state market is just the amount of the ration coupons received.

Comparative statics for the individual grain demands

Comparative statics are made for individual grain demands of the two types of urban household in the state market.

\[
\begin{align*}
\frac{\partial X_{jS1}}{\partial P_{jS}} &< 0 \\
\frac{\partial X_{jS1}}{\partial P_{jF}} &< 0 \\
\frac{\partial X_{jS1}}{\partial R_j} &< 0 \\
\frac{\partial X_{jS1}}{\partial M'} &> 0 \\
\frac{\partial X_{jSII}}{\partial P_{jS}} &= 0 \\
\frac{\partial X_{jSII}}{\partial P_{jF}} &= 0 \\
\frac{\partial X_{jSII}}{\partial R_j} &= 1 \\
\frac{\partial X_{jSII}}{\partial M'} &= 0 \\
\end{align*}
\]

These results indicate that, for urban households in the state market,
market, (1) grain demand by the first type decreases, but demand by the second type does not change when the state grain market price increases; (2) grain demands by both the first and the second type do not change when the free grain market price increases; (3) grain demand by the first type does not change, but demand by the second type increases when the government increases the official ration level; and (4) grain demand by the first type increases, but demand by the second type does not change when the government raises the income of urban households.

Aggregate grain demand by urban households in the state market

Now we formulate $D^{UMS}$, the aggregate urban grain demand in the state market of the mixed economy, as follows:

$$D^{UMS} = D^{UMSI} + D^{UMSII}$$  \hspace{1cm} (4.54)

where $D^{UMSI}$ and $D^{UMSII}$ are aggregate urban grain demands by the first and the second type of urban households in the state market of the mixed economy. From formulas (4.43), (4.44) and (4.48), we know that

$$D^{UMSI} = P_{UI} \sum_{u} X_{JSI} (P'_{1S}, P'_{1F}, P'_{i}, \bar{R}'_{i}, M') \leq P_{UI} \sum_{u} \bar{R}_{j}$$  \hspace{1cm} (4.55)

$$D^{UMSII} = (1 - P_{UI}) \sum_{u} X_{JSII}$$

$$= (1 - P_{UI}) \sum_{u} \bar{R}_{j}$$  \hspace{1cm} (4.56)

and

$$D^{UMS} = P_{UI} \sum_{u} X_{JSI} (P'_{1S}, P'_{1F}, P'_{i}, \bar{R}'_{i}, M') + (1 - P_{UI} \sum_{u} \bar{R}_{j})$$

$$= D^{UMS} (P_{UI}, \sum_{u} P'_{1S}, P'_{1F}, P'_{i}, \bar{R}'_{i}, M') \leq \sum_{u} \bar{R}_{j}$$  \hspace{1cm} (4.57)

That is, total grain demand by urban households in the state market is a function of the relative frequency $P_{UI}$, the total number of urban households, the state market price vector, the free market price vector,
the prices of other goods, the state ration levels, and the money income.

In addition, formula (4.57) implies that the total urban grain demand in the state market of the mixed economy is not more than the product of the total number of urban households and the official ration level, or $D^\text{UMP}$, the planned urban grain demand in the mixed economy. In other words, the government still is using the same method to plan the aggregate grain demand in the urban area, even though it permits reopening of the free market. Without any further assumptions, we cannot compare $D^\text{UMP}$ with $D^\text{UPP}$, the planned urban grain demand in the planned economy. If we suppose $\bar{R}_j$ is fixed, however, $D^\text{UMP}$ is certainly greater than $D^\text{UPP}$ because $n_U$, the total number of urban households, has certainly increased in the urban area in mainland China.

Comparative statics for aggregate grain demand The comparative statics are shown for aggregate grain demands by two types of urban households in the state market separately.

1. The first type of urban households

$$D^\text{UMSI} = \frac{P_i}{P_{iF}} n_U \bar{R}_j (P'_{iL}, P'_{iF}, P'_{i}, \bar{R}_i, M')$$

Because $D^\text{UMSI}$ is exactly the same as $D^\text{UMI}$, it is not necessary to recalculate the comparative statics for $D^\text{UMSI}$ here.

2. The second type of urban households

$$D^\text{UMSII} = (1 - P_{UI}) n_U \bar{R}_j$$

$$\frac{\partial D^\text{UMSII}}{\partial P_{UI}} = - n_U \bar{R}_j < 0$$

$$\frac{\partial D^\text{UMSII}}{\partial n_U} = (1 - P_{UI}) \bar{R}_j > 0$$
These results illustrate that, for the second type of urban households, the aggregate grain demand in the state market (1) decreases when the relative frequency $P_{UI}$ increases, (2) does not change when the state market price or the free market price increases, and (3) increases when the total number of urban households or the official grain ration increases.

Now we perform comparative statics for aggregate grain demand by all urban households in the state market.

\[
\frac{\partial D_{UMS}}{\partial P_{UI}} = \frac{\partial D_{UMS}}{\partial P_{UI}} + \frac{\partial D_{UMSII}}{\partial P_{UI}} < 0
\]

\[
\frac{\partial D_{UMS}}{\partial P_{JS}} = \frac{\partial D_{UMS}}{\partial P_{JS}} < 0
\]

\[
\frac{\partial D_{UMS}}{\partial P_{JF}} = 0
\]

\[
\frac{\partial D_{UMS}}{\partial R_j} = \frac{\partial D_{UMSII}}{\partial R_j} > 0
\]

\[
\frac{\partial D_{UMS}}{\partial M'} = \frac{\partial D_{UMSII}}{\partial M'} > 0
\]

For urban households in the mixed economy, the aggregate grain demand in the state market (1) is uncertain if the relative frequency $P_{UI}$
increases, (2) decreases if the state grain market price increases, (3) does not change if the free grain market price increases, and (4) increases if the state grain ration level or the money income increases.

Based on these results, we can conclude that both the state grain market price and the state ration level have partial (not total) direct (not indirect) effects on the aggregate grain demand in the state market through $D^{UMSI}$ and $D^{UMSII}$, respectively. Besides, the free grain market price has no effect on $D^{UMSI}$ and $D^{UMSII}$, and hence on $D^{UMS}$ at all.

**Graphic analysis** The graphic analyses for either individual or aggregate grain demands by either individual type or aggregate urban households in the state market have been completed; readers interested can refer to the $(Q^{S}, P^{S})$ subspace in Figures 4.4b, 4.5b, and 4.6.

**The free market analysis**

The existence of the free grain market provides a channel through which urban households can purchase additional grain to satisfy themselves. In particular, the free market is not a unique market within the mixed economy but coexists with the state market so that it is somewhat different from the free market in a free economy in the common sense. Thus, it is necessary to make the free market analysis to explore its operational mechanism in such an environment.

**Grain demand by urban households in the free market** There are two types of urban household in the free market.

1. The first type of urban household
For the first type of urban household, the grain demand in the free market is zero.

2. The second type of urban household

\[ X_{jFII} = X_{jFII} (P_i^{IS} , P_i^{IF} , P'_i , R'_i , M) \]  

For the second type of urban household, its grain demand in the free market is a function of the state market price vector \( P_i^{IS} \), the free market price vector \( P_i^{IF} \), the prices of other goods, the state ration levels, and the household's income minus its expenditure in the state grain market.

**Comparative statics for the individual grain demand**

Comparative statics are made for individual grain demand by two types of urban household in the free market.

1. The first type of urban household

\[ X_{jFI} = 0 \]

For the first type of urban household, the grain demand in the free market is always zero and is unaffected by any factor.

2. The second type of urban household

\[ X_{jFII} = X_{jFII} (P_i^{IS} , P_i^{IF} , P'_i , R'_i , M) \]

\[ \frac{\partial X_{jFII}}{\partial P_{jS}} = \frac{\partial X_{jFII}}{\partial M} \frac{\partial M}{\partial P_{jS}} < 0 \text{ if } \frac{\partial X_{jFII}}{\partial M} > 0 \]

\[ \frac{\partial X_{jFII}}{\partial P_{jF}} < 0 \]

\[ \frac{\partial X_{jFII}}{\partial R_j} = \frac{\partial X_{jFII}}{\partial M} \frac{\partial M}{\partial R_j} > 0 \text{ if } \frac{\partial X_{jFII}}{\partial M} < 0 \]
\[ \frac{\partial x_{jFII}}{\partial m'} > 0 \quad \text{for normal goods} \]

Thus, for the second type of urban household, the grain demand in the free market (1) decreases if the state grain market price or the free grain market price increases, (2) is uncertain but possibly increases if the state ration level increases, and (3) increases if its income increases.

**Aggregate grain demand of urban households in the free market**

Now we derive \( D_{UMF} \), the aggregate urban grain demand in the free market of the mixed economy as follows:

\[
D_{UMF} = D_{UMFI} + D_{UMFII} \quad (4.60)
\]

where \( D_{UMFI} \) and \( D_{UMFII} \) are the aggregate grain demands by the first and the second type of urban households in the free market of the mixed economy. From formulas (4.43), (4.44) and (4.48), we can have

\[
D_{UMFI} = 0 \quad (4.61)
\]

\[
D_{UMFII} = (1 - P_{UI}) n_U x_{jFII} \]

\[= (1 - P_{UI}) n_U x_{jFII} (P''_u, P''_f, P''_i, \bar{R}'_i, M') \]

\[= D_{UMFII} (P_{UI}, n_U, P''_u, P''_f, P''_i, \bar{R}'_i, M') \quad (4.62)\]

Thus, substituting (4.61) and (4.62) into (4.60), we obtain

\[
D_{UMF} = D_{UMFII} \]

\[= (1 - P_{UI}) n_U x_{jFII} (P''_u, P''_f, P''_i, \bar{R}'_i, M') \]

\[= D_{UMF} (P_{UI}, n_U, P''_u, P''_f, P''_i, \bar{R}'_i, M') \quad (4.63)\]

Consistent with the aggregate grain demand of the second type of urban households in the free market, \( D_{UMF} \) is a function of the relative
frequency $P_{UI}$, the total number of urban households, the state market
price vector $P_{iS}$, the free market price vector $P_{IF}$, the prices of other
goods, the state ration levels, and the household’s income minus the
expenditure on $X_{JIIS}$.

**Comparative statics for aggregate grain demand**

The comparative statics are made for the aggregate grain demand(s) of two types of urban
households as well as of all urban households in the free market.

1. The first type of urban households

$$D_{UMFI} = 0$$

There is nothing that can change the aggregate grain demand of the
first type of households in the free market.

2. The second type of urban households

$$D_{UMFII} = (1 - P_{UI}) n_U X_{JIJ} (P_{iS}, P_{IF}, P_i, R_i, M')$$

$$\frac{\partial D_{UMFII}}{\partial P_{UI}} = - n_U X_{JIJ} < 0$$

$$\frac{\partial D_{UMFII}}{\partial n_U} = (1 - P_{UI}) X_{JIJ} > 0$$

$$\frac{\partial D_{UMFII}}{\partial P_{JS}} = (1 - P_{UI}) n_U \frac{\partial X_{JIJ}}{\partial P_{JS}} < 0$$

$$\frac{\partial D_{UMFII}}{\partial P_{IF}} = (1 - P_{UI}) n_U \frac{\partial X_{JIJ}}{\partial P_{IF}} < 0$$

$$\frac{\partial D_{UMFII}}{\partial R_i} = (1 - P_{UI}) n_U \frac{\partial X_{JIJ}}{\partial R_i} > 0$$
For the second type of urban households, the aggregate grain demand in the free market (1) decreases if the relative frequency $P_{UI}$, the state grain market price, or the free grain market price increases, (2) is uncertain but possibly increases if the grain ration level increases, and (3) increases if the income of urban households increases.

We have $D^{UMF} = D^{UMFI}$ from formula (4.63), so that the comparative statics for both $D^{UMF}$ and $D^{UMFI}$ are exactly the same. Thus, we can conclude that for the second type, as well as all types of urban households, both the state grain market price and the state ration level have indirect effects, but the free market price has a direct effect on the aggregate grain demand in the free market through the income effect.

**Graphic analysis** Similarly, the graphic analyses for either individual or aggregate grain demands of either the individual type or the aggregate of urban households in the free market have been completed in the $(Q^F, P^F)$ subspace in Figure 4.4b, 4.5b and 4.6 and are not reprinted here.

Theory of Rural Grain Demand and Supply of State Farm (Households) and Agricultural Households in the Mixed Economy

After the institutional changes in the rural area in the recent economic reforms, there have been two kinds of basic rural enterprises in the Chinese mixed economy: state farm households and agricultural households. The state farm households became the basic production units
much later than agricultural households did. There are differences be­
tween the behaviors of state farm households and agricultural households
because they face different environments, especially the policy about
land ownership utilized by them. Consequently, the first two subsec­
tions in this section are formed separately. One subsection studies the
behavior of the state farm (households) and the other one studies the
behavior of the agricultural households. Each subsection consists of
three parts: the policy environment, the state farm (household) model or
agricultural household model, and the aggregate grain demand and supply
of all state farm (households) or all agricultural households. The pol­
icy environment introduces the basic background materials during the eco­
nomic reform for the state farm (households) or the agricultural house­
holds. The state farm model is developed based on an assumption of its
profit maximization whereas the agricultural household model is built
based on an assumption of its utility maximization. After that, the
aggregate grain demand and supply of the state farm (households) and the
agricultural households are given from the derived state farm (house­
hold’s) or agricultural household’s grain demand and supply in each
subsection.

To close this section, an additional subsection deals with the
market analysis. This subsection contains two parts: the state market
analysis and the free market analysis. Within these two parts, the
market situations are described, the policy impacts are discussed, and,
finally, the interactions among the state market, the free market, and
the household market in the mixed grain economy are explored and studied.
The topic of the first subsection is the theory of rural grain demand and supply of state farm (households) in the mixed economy. From the early 1980s, managers in the state farms had changed their behavior to catch up with the policy changes. Their goal of management had shifted away from the previous output maximization. The adjustment brought changes to the state farms. The initiative of state farm households had been partly stimulated by the bonuses attached to their wage income. In the mid-1980s, state farms applied the production responsibility system and turned their households' members into basic production units. After that, there seems to be no big difference between the state farm households and the agricultural households except for their production equipment and explanation of land ownership.

Policy environment of state farm (households)

This part deals with the earlier stage of state farms in the mixed economy, before the production responsibility system.

State ownership Before the mid-1980s, the state farm was still run under the government's plans. All properties, including land, livestock, houses, and equipment, belonged to the state. In the mixed economy, the state farm delivered all its agricultural products to the government. The government distributed equipment and material inputs to the state farm. Although the state compensated the losses of the state farm, the government permitted the state farm to keep profits for its own reinvestment and partial benefits of its own households. Thus, under the state plans, to a great degree, the managers of the state farms made an
effort to make as much profit as possible.

**Labor** All laborers of the state farm household still were not transferable. Just like the past, they were paid by a fixed wage. In order to encourage their working initiative, additional bonuses were provided by the farm manager.

**Consumption of food grain** The wage income earned by the state farm households could be spent in the market(s). These households were given ration coupons and permitted to purchase grain from the state market. They were treated as the urban households.

**Market control** Almost all economic activities of the state farms were limited to the state market. To motivate the state farm to develop agricultural production, the government set the minimum sales quotas for each state farm. On the other hand, the government increased the procurement prices for agricultural products including grain. It also set an above-quota price that was used for the grain delivered above the quota and was clearly higher than the procurement price. In addition, the state farm could buy some material inputs from the free market.

**State farm (household) model**

For a state farm, its grain economy possesses dual features. Its grain production decision was made by the farm manager, but its food grain consumption decisions were made by its households. Hence, the state farm model in the mixed economy can be subdivided to handle grain consumption and production.
Food grain demand of state farm household

As an individual consumption unit, earning fixed wages plus bonuses, receiving grain coupons from the government, and facing dual grain markets in the mixed economy, the state farm household does not differ from the urban household. Thus, the urban household grain demand functions can be used for the state farm household:

\[ X_{jsH} = X_{js} (P_{is}', P_{if}', P_{i}', R_{i}', M') \leq \bar{R}_j \]  \hspace{1cm} (4.64)

or

\[ X_{jsH} = \bar{R}_j + X_{jf} (P_{is}'', P_{if}'', P_{i}'', R_{i}'', M'') > \bar{R}_j \]  \hspace{1cm} (4.65)

Formulas (4.64) and (4.65) represent different attitudes toward the ration coupons for grain and the dual markets, especially the free market. The comparative statics and the graphic analysis are also the same as before and, therefore, are deleted here.

Grain supply of the state farm

Since the economic reform, the state farm had shifted its interest from concentrating on output to seeking profits for its own needs. Although the output target set by the government remained the same, the profit goal had become more important for the managers of the state farms. Assuming that the objective of the state farm is transformed into maximizing profit, subject to the government plans, and that the production functions for the state farm have all the usual properties, as mentioned in Chapter III, the Lagrangian function would be

\[ L = \sum_{i=1}^{k} (P_{is} Q_{is} + P_{iSA} Q_{iSA}) + \sum_{i=1}^{n} P_{i} Q_{i} - \sum_{j=1}^{m} (\gamma_{js} X_{js} + \gamma_{jf} X_{jf}) - \sum_{j=1}^{m} \gamma_{j} X_{j} \]
\begin{align}
+ \lambda_1 \left[ \sum_{i=1}^{k} x_{iS} - x_{iS} \right] + \sum_{j=1}^{k} \lambda_{2i} (Q_{iS} - \bar{Q}_{iS}) + \sum_{j=1}^{k} \lambda_{3i} (Q_{iS} - \bar{Q}_{iS}) \right)
\end{align}

where

\begin{align}
Q_{iS} &= \text{the } i^{\text{th}} \text{ output sold at the quota price } (i = 1, \ldots, k) \\
Q_{iSA} &= \text{the } i^{\text{th}} \text{ output sold at the above quota price } (i = 1, \ldots, k) \\
Q_i &= \text{the } i^{\text{th}} \text{ output without quota } (i = 1, \ldots, n) \\
\bar{Q}_{iS} &= \text{the sales quota for the } i^{\text{th}} \text{ output of the state farm } (i = 1, \ldots, k) \\
x_{jS} &= \text{the } j^{\text{th}} \text{ input bought at the state market price } (j = 1, \ldots, k) \\
x_{jF} &= \text{the } j^{\text{th}} \text{ input bought at the free market price } (j = 1, \ldots, k) \\
x_j &= \text{the } j^{\text{th}} \text{ input without ration } (j = 1, \ldots, m) \\
\bar{x}_{jS} &= \text{the purchase ration for the } j^{\text{th}} \text{ input of the state farm } (j = 1, \ldots, k) \\
P_{iS} &= \text{the quota price for the } i^{\text{th}} \text{ output } (i = 1, \ldots, k) \\
P_{iSA} &= \text{the above quota price for the } i^{\text{th}} \text{ output } (i = 1, \ldots, k) \\
P_i &= \text{the price for the } i^{\text{th}} \text{ output } (i = 1, \ldots, n) \\
\gamma_{jS} &= \text{the state market price for the } j^{\text{th}} \text{ input } (j = 1, \ldots, k) \\
\gamma_{jF} &= \text{the free market price for the } j^{\text{th}} \text{ input } (j = 1, \ldots, k) \\
\gamma_j &= \text{the price for the } j^{\text{th}} \text{ input } (j = 1, \ldots, m)
\end{align}

Because the sales quota \( \bar{Q}_{iS} \) set by the government must be fulfilled, we have

\begin{align}
Q_{iS} = \bar{Q}_{iS}
\end{align}

Substitute (4.67) into (4.66), and the Lagrangian function becomes
The Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial x_j} \leq 0 \quad j = 1, \ldots, k \tag{4.69}
\]

\[
\frac{\partial L}{\partial x_j} = \sum_{i=1}^{n} p_i \frac{\partial q_i}{\partial x_j} + \gamma_j + \lambda_1 \sum_{i=1}^{n} \frac{\partial q_i}{\partial x_j} \leq 0 \quad j = 1, \ldots, m \tag{4.70}
\]

\[
x_j \frac{\partial L}{\partial x_j} = 0 \quad j = 1, \ldots, m \tag{4.71}
\]

\[
\frac{\partial L}{\partial \lambda_1} = G [Q_1, \ldots, (Q_{iS} + Q_{iSA}), \ldots, Q_k, Q_1, \ldots, Q_n] \geq 0 \tag{4.72}
\]

\[
\lambda \frac{\partial L}{\partial \lambda_1} = 0 \tag{4.73}
\]

\[
\frac{\partial L}{\partial \lambda_{21}} = x_{js} - x_{js} \geq 0 \quad j = 1, \ldots, k \tag{4.74}
\]

\[
\lambda_{21} \frac{\partial L}{\partial \lambda_{21}} = 0 \quad j = 1, \ldots, k \tag{4.75}
\]

\[
x_j, \lambda_1 \geq 0 \quad j = 1, \ldots, k \tag{4.76}
\]

\[
\lambda_{21} \geq 0 \quad i = 1, \ldots, k \tag{4.77}
\]

Assume that \( x_j > 0 \), or that the state farm is purchasing each product in the market(s). Obviously, this assumption implies that (4.69) and (4.70) equal zero and (4.71) is dropped out. By the same reasons given in the last section, we let \( \gamma_{jS} < \gamma_{jF} \) for \( j = 1, \ldots, m \), the
state market price for the \( j \)th input is less than the free market price, and ignore the situations of \( \gamma_{js} \geq \gamma_{jf} \). Thus, the state farm always makes its initial purchase of \( X_{js} \) in the state market until \( \bar{X}_{js} \) is fulfilled. Under the condition of \( \gamma_{js} < \gamma_{jf} \), we now consider the transformation of (4.69). By the mathematical transformation of (4.16), we understand that \( \frac{\partial L}{\partial X_j} = 0 \) is equivalent to these two cases:

Case 1. \( \frac{\partial L}{\partial X_{js}} = 0 \), \( \frac{\partial L}{\partial X_{jf}} < 0 \).

Thus,

\[
\frac{\partial L}{\partial X_{js}} = \sum_{iSA} \frac{\partial q_i}{\partial X_j} \frac{\partial (X_{js} + X_{jf})}{\partial X_{js}} - \gamma_{js} + \lambda_1 \sum \frac{\partial g_i}{\partial q_i} \frac{\partial q_i}{\partial X_{js}} - \lambda_{21} = 0 \quad j = 1, \ldots, k \quad (4.78)
\]

Case 2. \( \frac{\partial L}{\partial X_{js}} > 0 \), \( \frac{\partial L}{\partial X_{jf}} = 0 \).

Thus,

\[
\frac{\partial L}{\partial X_{jf}} = \sum_{iSA} \frac{\partial q_i}{\partial X_j} \frac{\partial (X_{js} + X_{jf})}{\partial X_{jf}} - \gamma_{jf} + \lambda_1 \sum \frac{\partial g_i}{\partial q_i} \frac{\partial q_i}{\partial X_{jf}} = 0 \quad j = 1, \ldots, k \quad (4.79)
\]

The Kuhn-Tucker conditions now consist of (4.78), (4.79), (4.72), (4.73), and (4.74). Furthermore, suppose that \( \lambda_1 > 0 \), and that the first \( h \)th inputs belong to case 1 and the \( i \)th to the \( k \)th inputs belong to case 2. Solving the equations simultaneously, provided the second-order conditions are satisfied, let

\[ p'_{iSA} = p_{iSA} (i = 1, \ldots, k) \]
We obtain the implicit input demand functions of the state farm in the mixed economy as follows:

\[ X_j = X_{JS} (P_{iSA}', P_i', \gamma_{JS}', \gamma_{jF}', \gamma_j', X'_{iS}) \quad j = 1, \ldots, h \quad (4.80) \]

\[ X_j = X_{JS} + X_{jF} \]

\[ = X_{JS} + X_{jF} (P_{iSA}', P_i', \gamma_{JS}', \gamma_{jF}', \gamma_j', X'_{iS}) \quad j = 1, \ldots, k \quad (4.81) \]

\[ X_j = X_j (P_{iSA}', P_i', \gamma_j', \gamma_{jF}', \gamma_j', X'_{iS}) \quad j = 1, \ldots, m \quad (4.82) \]

Substituting (4.80), (4.81) and (4.82) into the grain supply function of the state farm, we get

\[ Q_i = Q_i (X_1, \ldots, X_h, X_1, \ldots, X_k, X_1, \ldots, X_m) \]

\[ = Q_i (P_{iSA}', P_i', \gamma_{JS}', \gamma_{jF}', \gamma_j', X'_{iS}) \geq Q_{iS} \quad (4.83) \]

That is, the grain supply of the state farm is a function of the above quota prices, the prices of other outputs, input prices in the state market and the free market, prices of other inputs, and, finally, input rations in the state market. It needs to be pointed out that the state market grain quota price is excluded from the grain supply function, or it does not affect the grain supply. Thus, we can conclude that when two prices exist in the same state market, the grain supply of the
state farm is guided by only the higher price, the above-quota price, and not by the lower price, the quota price. In our case, after the state farm fulfills the government's quota requirement, the additional grain supply depends on only the above-quota price. The case can easily be extended to where more than two prices exist.

**Comparative statics** The comparative statics for the grain demand by state farm households are the same as those by urban households, and are, hence, not necessary to recalculate here. The comparative statics for the grain supply by the state farm are focused on the grain own prices.

Using formula (4.83), we have

\[
\frac{\partial Q_i}{\partial P_{iS}} = 0
\]

\[
\frac{\partial Q_i}{\partial P_{iSA}} > 0
\]

The above results imply that the grain supply is increasing if the above quota price increases, but it remains unchanged if the quota price increases.

**Graphic analysis** Like comparative statics, the graphic analysis also is made for only grain supply by the state farms. Because the grain supply function does not contain the quota price but the above-quota price, the supply curve is kinked in Figure 4.7. The lower piece is a vertical line whose intercept on the \(q_i^S\) axis is \(Q_{1S}\), and the upper piece is upward-sloping and is connected with \(Q_{1S}\) at the quota price \(P_{1S}\).
The individual grain demand and supply of the state farm (household) have been derived. In this part, aggregate grain demand of state farm households and aggregate grain supply of state farms are derived separately.

**Aggregate grain demand of state farm households**

Individual grain demand of the state farm household is similar to that of urban households, so the aggregate grain demand of the state farm households is no exception.

Using formulas (4.42) and (4.48), we have

\[
D_{FM} = D_{FMI} + D_{FMII}
\]

\[
= P_{1F} n_F X_{JS} (P_{1IS}', P_{1IF}', P_{1}'', R_{1}', M')
+ (1 - P_{1F}) n_F [R_{1} + X_{jF} (P_{1IS}'', P_{1IF}'', P_{1}'', R_{1}'', M)]
= D_{FM} (P_{1F}', n_F, P_{1IS}'', P_{1IF}'', P_{1}'', R_{1}'', M')
\]

where

\[D_{FM} = \text{the aggregate grain demand of state farm households}\]
D^{FMI}, D^{FMII} = aggregate grain demand of the first type and the second type of state farm households

n_F = the total number of state farm households

P_{FI} = the relative frequency of the first type of state farm households

Other symbols used here are the same as those in (4.48).

Formula (4.85) indicates that the aggregate grain demand is a function of the relative frequency P_{FI}, the total number of state farm households, the state market prices, the free market prices, the prices of other unrationed goods, and the household’s disposable income.

Aggregate grain supply of state farms Suppose that all the state farms are identical and there are n_S state farms total. From formula (4.83) we can derive S^F, aggregate grain supply of state farms.

S^{FM} = n_F Q_i (P_i^{ISA}, P_i, \gamma_j^S, \gamma_j^{IF}, \bar{x_j})

S^{FM} = n_F Q_i (P_i^{ISA}, P_i, \gamma_j^S, \gamma_j^{IF}, \bar{x_j}) \geq n_F Q_i (4.86)

Comparative statics This subpart is subdivided into two parts.

1. Aggregate grain demand of state farms

Aggregate grain demand of state farm households is the same as that of urban households, so the comparative statics are the same for both state farm households and urban households.

2. Aggregate grain supply of state farms

Comparative statics for aggregate grain supply of state farms concentrate mainly on the grain quota price and the above-quota price.

S^{FM} = n_F Q_i (P_i^{ISA}, P_i, \gamma_j^S, \gamma_j^{IF}, \bar{x_j})
The higher the above quota price is, the higher the aggregate grain supply by the state farms is.

**Graphic analysis** The graphic analysis for aggregate grain demand is presented in Figure 4.6. The graphic analysis here is limited to only the aggregate grain supply of the state farms. The shape of the aggregate grain supply of the state farms is the same as that of an individual state farm, but the intercept on the \( Q^S \) axis has been changed to \( n_F Q^S_{iS} \) (Figure 4.8).

![Figure 4.8. Aggregate grain supply of state farms in the mixed economy](image)

We have finished our study of the first subsection, and we are starting our work with the second subsection, the topic of which is the theory of rural grain demand and supply of agricultural households in the mixed economy.
Policy environment of agricultural households

Private agricultural household  As a basic unit of production and consumption in rural areas in the mixed economy, the agricultural household makes its own decisions in its economic activities. It owns almost all of the means of production except for the land used. According to contract, it is assigned a specific plot of land to plant within a specified period.

Labor  Agricultural laborers are transferable in the mixed economy. The agricultural household can hire labor to help its operation or can have its labor hired by someone else. In either case, the hired laborer is paid wages.

Plan control  The government has loosened its plan control in the rural areas. First, it reduced the grain quota and agricultural tax in grain, and tried to use price as a signal to guide the grain delivery to the state market. Second, in early 1985, the government introduced a contract system to replace the quota system for grain. According to the contract, the government negotiates its purchase with agricultural households before the sowing season. Thus, the households have their own right to decide whether or not to sign the contract with the government. In theory, the contracts are voluntary, but in practice, the contracts become semi-mandatory to a great extent. In a certain sense, this contract system can be thought of as a new quota system.

Market operation  There are dual grain markets in the mixed economy. The state market is considered to be dominated by the government whereas the free market is regarded as a supplement to state
commerce. The government made an effort to improve the situation in the state grain market. It raised the grain quota price about 30% to 50% and set a higher above quota purchase price for grain delivered above the quota. The government even tried to use a negotiated purchase price in response to the free market price. In general, the above quota price is less than the negotiated purchase price, which may be very close to and even exceed the free market price. Since 1985, the government abolished multiple prices in the state market. Instead, it set a uniform contract price, 30 percent of the quota price plus 70 percent of the above-quota price. Thus, this contract price seems to be more than the quota price but less than the above-quota price and, of course, less than the free market price. Besides, the government maintains the negotiated price, which may approximate the free market price.

To be consistent with the analysis for the planned economy, we still classify the agricultural households into three types by their activities in the state market rather than in the dual markets. The first type is called nonbuyer or complete seller in the state market because it has to sign contracts and fulfill their obligations. Among the first type of agricultural households, a majority generally produce grain to supply themselves and the state market, even the free market. The minority may produce other cash crops to make a profit and buy grain from the free market for their own use and resale to the government. Hence, the nonbuyer in the state market may be either a nonbuyer or a complete buyer in the free market.

The second type is a partial buyer or a partial seller in the state
market because it has to fulfill its obligation by the contract and can buy some grain back with the government's permission because of unexpected natural disasters. According to their situations in the free market the second type of agricultural households can be subdivided into two: one is a nonbuyer who does not need to buy grain from the free market, the other one is a complete buyer who buys part of the consumed grain from the free market to satisfy itself fully.

The last type of agricultural household is called a complete buyer or nonseller in the state market because it does not need to produce and sell grain by the government's plan. This type also contains two subtypes: the nonbuyer who does not buy any grain in the free market, and the complete buyer who buys but does not sell grain in the free market.

The basic situations are the same in the input markets. In brief, there are also dual markets and dual (multiple) prices for the inputs used by the agricultural households. Generally, the households purchase their initial needs up to the official ration level in the state market, then their additional needs at the higher prices in the free market.

**Agricultural household model**

For an agricultural household in the mixed economy, both its production and consumption are managed by itself and guided by the government's plan and information from both the state market and the free market. Suppose that the household's objective is to maximize its utility, subject to its budget, production function, and other constraints given by the plan control and that its utility function and production function
have the usual properties mentioned in Chapter III as well as in general microeconomic theory.

To build a general agricultural household model in the mixed economy, we have to take into further consideration the changes in the price policy of the state government. It is known that there were multiple prices and there are dual prices in the state grain market. In fact, any multiple (more than two) price system can be seen as an extension of a dual price system in which a quota price $P_S$ and an above-quota price $P_{SA}$ exist and, of course, $P_{SA} > P_S$. Considering the relationship between $P_{SA}$ and the free market price $P_F$, we ignore the cases $P_{SA} > P_F$ or $P_{SA} = P_F$. Readers can refer to the reasons from the discussion about the relationship between $P_S$ ($P_{SA}$ here now) and $P_F$ in the last section. Now we discuss the case $P_{SA} < P_F$.

The reasons for an agricultural household to sell its product to the state market at the lower $P_{SA}$ rather than to the free market at the higher price $P_F$ may be briefly summarized as follows: 1) lack of the free market information; 2) consideration of additional transportation cost; 3) receipt of the government's benefits, for example, receiving lower cost inputs such as chemical fertilizer or an awarded right to buy a good in shortage such as a name brand bicycle; or 4) influence of local authorities. From the point of view of economics, the first reason can be considered as the value of market information and the second reason can be treated as sales cost. In fact, both of the reasons are deducting some value from the free market price $P_F$. The third reason, actually, is adding some value to the above-quota price, $P_{SA}$. All three reasons are
reductions in the difference between $P_{SA}$ and $P_F$, even a change in sign in case $P_{SA} < P_F$. For simplicity, we ignore these three situations in our theoretical model. The last reason is apparently noneconomic and, therefore, is not discussed in this study.

Overall, there is no real economic reason for an agricultural household facing a free market to take such a voluntary action unless the government asks it to do so. Furthermore, the negotiated price can be seen as the approximate free market price and can be ignored in this study because it plays a role to transfer part of supply in the free market to the state market. Under these circumstances, we assume a unique and uniform price $P_S$ that can be considered as either the above-quota price (in the earlier stage of the mixed economy) or the contract price (in the later stage of the mixed economy)--the new quota price--in our model.

Under all of these assumptions, we extend Sicular's (1988) model from a three-good case to a multiple-good case and change her decision variables from unreasonable $X_i$ and $\gamma_i$ to reasonable $X_i$. Our Lagrangian function for the agricultural household in the Chinese mixed economy is then as follows:

$$L = U \left[ X_1, \ldots, (X_{iS} + X_{iF}), \ldots, X_k, X_1, \ldots, X_n \right]$$

$$+ \lambda_1 \left( M + \sum_{i=1}^{k} P_{iS} Q_{iS} + \sum_{i=1}^{n} P_{iF} Q_{iF} + \sum_{i=1}^{k} \sum_{j=1}^{k} P_{i} X_{iS} - \sum_{i=1}^{k} \sum_{j=1}^{k} P_{iF} X_{iF} - \sum_{i=1}^{k} P_{i} X_{i} \right)$$

$$+ \lambda_2 \left( Q_1, \ldots, (Q_{iS} + Q_{iF}), \ldots, Q_k, Q_1, \ldots, Q_n \right)$$

$$+ \sum_{i=1}^{k} \lambda_{3i} (X_{iS} - X_{iS}) + \sum_{j=1}^{k} \lambda_{4i} (Q_{iS} - Q_{iS})$$

(4.87)

where
$X_i$ - the $i^{th}$ commodity consumed by the household ($i = 1, \ldots, n$)

$X_{iS}$ - the $i^{th}$ commodity consumed by the household in the state market ($i = 1, \ldots, k$)

$X_{iF}$ - the $i^{th}$ commodity consumed by the household in the free market ($i = 1, \ldots, k$)

$X_{iS}$ - the state market ration for the $i^{th}$ commodity ($i = 1, \ldots, k$)

$Q_i$ - the $i^{th}$ commodity produced by the household ($i = 1, \ldots, n$)

$Q_{iS}$ - the $i^{th}$ commodity sold to the state market ($i = 1, \ldots, k$)

$Q_{iF}$ - the $i^{th}$ commodity accounted at the free market price ($i = 1, \ldots, k$)

$Q_{iS}$ - the sales quota to the state market for the $i^{th}$ commodity ($i = 1, \ldots, k$)

$P_{iS}$ - the state market price for the $i^{th}$ commodity ($i = 1, \ldots, k$)

$P_{iF}$ - the free market price for the $i^{th}$ commodity ($i = 1, \ldots, k$)

$P_i$ - the price for the $i^{th}$ commodity ($i = 1, \ldots, n$)

$M$ - other income received by the household

For the household, $X_{iS} \geq X_{iS}$ is a rationing constraint and $Q_{iS} \geq Q_{iS}$ is a sales quota constraint in the state market.

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial X_i} \leq 0 \quad i = 1, \ldots, k \quad (4.88)$$

$$\frac{\partial L}{\partial X_i} = \frac{\partial U}{\partial X_i} - \lambda_1 P_i \leq 0 \quad i = 1, \ldots, n \quad (4.89)$$

$$X_i \frac{\partial L}{\partial X_i} = 0 \quad i = 1, \ldots, n \quad (4.90)$$

$$\frac{\partial L}{\partial Q_i} \leq 0 \quad i = 1, \ldots, k \quad (4.91)$$
\[ \frac{\partial L}{\partial q_i} = \frac{\partial G}{\partial q_i} + \lambda_1 q_i \leq 0 \quad i = 1, \ldots, n \quad (4.92) \]

\[ Q_1 \frac{\partial L}{\partial q_1} = 0 \quad i = 1, \ldots, n \quad (4.93) \]

\[ \frac{\partial L}{\partial \lambda_1} = m \sum_{i=1}^{k} p_i s_{iS} + \sum_{i=1}^{k} p_i f_{iF} + \sum_{i=1}^{k} p_i q_{iQ} - \sum_{i=1}^{n} x_{iS} \sum_{i=1}^{n} x_{iF} - \sum_{i=1}^{n} p_i x_i \geq 0 \quad (4.94) \]

\[ \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0 \quad (4.95) \]

\[ \frac{\partial L}{\partial \lambda_2} = G \left[ q_{iS} + q_{iF} \right] \quad (4.96) \]

\[ \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0 \quad (4.97) \]

\[ \frac{\partial L}{\partial \lambda_3} = x_{iS} - x_{iS} \geq 0 \quad i = 1, \ldots, k \quad (4.98) \]

\[ \lambda_3 \frac{\partial L}{\partial \lambda_3} = 0 \quad i = 1, \ldots, k \quad (4.99) \]

\[ \frac{\partial L}{\partial \lambda_4} = q_{iS} - q_{iS} \geq 0 \quad i = 1, \ldots, k \quad (4.100) \]

\[ \lambda_4 \frac{\partial L}{\partial \lambda_4} = 0 \quad i = 1, \ldots, k \quad (4.101) \]

\[ x_{iS}, x_{iF}, q_{iS}, q_{iF} \geq 0 \quad i = 1, \ldots, n \quad (4.102) \]

\[ \lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0 \quad i = 1, \ldots, k \quad (4.104) \]

**The second type of agricultural household**

Suppose that \( \lambda_1 > 0 \),

then (4.94) becomes

\[ \frac{\partial L}{\partial \lambda_1} = m \sum_{i=1}^{k} p_i s_{iS} + \sum_{i=1}^{k} p_i f_{iF} + \sum_{i=1}^{k} p_i q_{iQ} - \sum_{i=1}^{n} x_{iS} \sum_{i=1}^{n} x_{iF} - \sum_{i=1}^{n} p_i x_i = 0 \quad (4.105) \]

so that (4.95) can be dropped out. Moreover, suppose \( \lambda_2 > 0 \), then we have
\[
\frac{\partial L}{\partial \lambda_2} = G \left[ Q_1, \ldots, (Q_{1S} + Q_{1F}), \ldots, Q_k, Q_1, \ldots, Q_n \right] = 0 \quad (4.106)
\]

Hence, (4.97) can be omitted. Now we work with (4.88) and (4.91).

Suppose all \( X_i \) and \( Q_i \) (\( i = 1, \ldots, n \)) are positive, that is, the household consumes and produces each commodity. Then

\[
\frac{\partial L}{\partial x_i} = 0 \quad i = 1, \ldots, n \quad (4.107)
\]

\[
\frac{\partial L}{\partial q_i} = 0 \quad i = 1, \ldots, n \quad (4.108)
\]

Immediately applying the mathematical transformation (4.16), assuming the household makes both its initial purchase and sale from and to the state market rather than the free market, both \( x_{1S} \) and \( q_{1S} > 0 \), and considering the possible situations in the mixed economy, both (4.107) and (4.108) imply only three cases:

**Case 1:**

\[
\frac{\partial L}{\partial x_{1S}} = 0, \quad \frac{\partial L}{\partial x_{1F}} < 0, \quad \frac{\partial L}{\partial q_{1S}} = 0, \quad \frac{\partial L}{\partial q_{1F}} < 0
\]  

**Case 2:**

\[
\frac{\partial L}{\partial x_{1S}} > 0, \quad \frac{\partial L}{\partial x_{1F}} = 0, \quad \frac{\partial L}{\partial q_{1S}} = 0, \quad \frac{\partial L}{\partial q_{1F}} < 0
\]  

**Case 3:**

\[
\frac{\partial L}{\partial x_{1S}} > 0, \quad \frac{\partial L}{\partial x_{1F}} = 0, \quad \frac{\partial L}{\partial q_{1S}} > 0, \quad \frac{\partial L}{\partial q_{1F}} = 0
\]  

Furthermore, suppose that for the agricultural household, the first \( d \)th within the first \( k \)th commodities belong to case 1, the \( e \)th through the \( g \)th commodities belong to case 2, and the remaining \( h \)th through the \( k \)th commodities belong to case 3. We then have

\[
\frac{\partial L}{\partial x_{1S}} = \frac{\partial u}{\partial x_{1S}} - \lambda_{1P1S} - \lambda_{31} = 0 \quad i = 1, \ldots, d \quad (4.112)
\]
Because of a shortage of consumed goods and the price difference between the state market and the free market, rural households usually use up their purchase rations. On the other hand, by the contract, the sales quotas must be fulfilled. Thus, we simply assume that all $\lambda_{3i}$ and $\lambda_{4i} > 0$ ($i = 1, \ldots, k$), immediately:

$$\frac{\partial L}{\partial x_{1s}} - \frac{\partial \bar{L}}{\partial x_{1s}} - \lambda_{1p_{1s}} - \lambda_{3i} > 0 \quad i = e, \ldots, k \quad (4.113)$$

$$\frac{\partial L}{\partial x_{1f}} - \frac{\partial \bar{L}}{\partial x_{1f}} - \lambda_{1p_{1f}} < 0 \quad i = 1, \ldots, d \quad (4.114)$$

$$\frac{\partial L}{\partial x_{1f}} - \frac{\partial \bar{L}}{\partial x_{1f}} - \lambda_{1p_{1f}} = 0 \quad i = e, \ldots, k \quad (4.115)$$

$$\frac{\partial L}{\partial q_{1s}} - \frac{\partial \bar{L}}{\partial q_{1s}} - \lambda_{1p_{1s}} + \lambda_{4i} = 0 \quad i = 1, \ldots, g \quad (4.116)$$

$$\frac{\partial L}{\partial q_{1s}} - \frac{\partial \bar{L}}{\partial q_{1s}} - \lambda_{1p_{1s}} + \lambda_{4i} > 0 \quad i = h, \ldots, k \quad (4.117)$$

$$\frac{\partial L}{\partial q_{1f}} - \frac{\partial \bar{L}}{\partial q_{1f}} - \lambda_{1p_{1f}} < 0 \quad i = 1, \ldots, g \quad (4.118)$$

$$\frac{\partial L}{\partial q_{1f}} - \frac{\partial \bar{L}}{\partial q_{1f}} - \lambda_{1p_{1f}} = 0 \quad i = h, \ldots, k \quad (4.119)$$

Finally, consider situations of both constraints (4.98) and (4.100).
no free choices for the agricultural household in the state market. After fulfilling the sales quotas and using up purchase rations, the agricultural household can have free choices in the market. Thus, we let

\[ M' = M + \sum_{i=1}^{k} P_{iS} (Q_{iS} - X_{iS}) \quad (4.122) \]

\(M')\) represents the real disposable income of the agricultural household before its free choices both in the state market without any restriction and in the free market.

Provided that the second-order conditions are satisfied, considering final Kuhn-Tucker conditions simultaneously, and letting

\[ P'_{iF} = P_{iF} (i = h, \ldots, k) \text{ and } P'_{i} = P_{i} (i = 1, \ldots, n), \]

the implicit (if any) grain demand and supply functions for the agricultural household would be

\[ X_{i} = \bar{X}_{iS} \quad i = 1, \ldots, d \quad (4.123) \]

\[ X_{i} = \bar{X}_{iS} + X_{iF} \]

\[ = \bar{X}_{iS} + X_{iF} (P'_{iF}, P'_{i}, M') > \bar{X}_{iS} \quad i = e, \ldots, k \quad (4.124) \]

\[ X_{i} = X_{i} (P'_{iF}, P'_{i}, M') \quad i = 1, \ldots, n \quad (4.125) \]

\[ Q_{i} = \bar{Q}_{iS} \quad i = 1, \ldots, g \quad (4.126) \]

\[ Q_{i} = \bar{Q}_{iS} + Q_{iF} \]

\[ = \bar{Q}_{iS} + Q_{iF} (P'_{iF}, P'_{i}, M') > \bar{Q}_{iS} \quad i = h, \ldots, k \quad (4.127) \]

\[ Q_{i} = Q_{i} (P'_{iF}, P'_{i}, M') \quad i = 1, \ldots, n \quad (4.128) \]

Because we derive the above results by assuming both \(X_{iS}\) and \(Q_{iS} > 0\), these results fit the case of the partial buyer or partial seller, the second type of agricultural household. Thus, we can conclude that
the possible combination of grain demand and grain supply functions for any of the second type of households might be (4.123) and (4.126), or (4.124) and (4.126), or (4.124) and (4.127). In other words, we have

\[
X_{jIII} = X_{jJS}
\]  
(4.129)

\[
X_{jII2} = X_{jII3} - X_{jJS} + X_{jF} (P'_{IF}, P'_1, M') > X_{jJS}
\]  
(4.130)

and

\[
Q_{jIII} = Q_{jII2} = Q_{jJS}
\]  
(4.131)

\[
Q_{jII3} = Q_{jJS} + Q_{jF} (P'_{IF}, P'_1, M') > Q_{jJS}
\]  
(4.132)

These results indicate that, for the second type of agricultural household, the grain demand is either the grain ration (the first subtype) or a function of the free market price vector, the prices of other goods, and its disposable income (the second and the third subtype), whereas its grain supply is either the sales quota (the first and the second subtype) or a function of the same factors used in its demand functions plus its grain sales quotas (the third subtype).

To obtain the grain demand and supply functions for the first and the third types of agricultural households, we can use our basic model but adjust part of our assumptions to fit their situations.

The first type of agricultural household

For the first type of household, the nonbuyer in the state market, the only thing we need to do is note that assumption \( X_{IS} = 0 \) for \( i = 1, \ldots, b \) has replaced part of the assumption of \( X_{IS} > 0 \) for \( i = 1, \ldots, k \). Consequently, the change in the Kuhn-Tucker conditions are that part of (4.114), (4.120) and (4.121) for \( i = 1, \ldots, b \) drop out, and part of (4.117) for \( i = 1, \ldots, b \)
is added in the model used for the second type of agricultural household. In this case, the results of this model become

\[ X_i = X_{iF} (P''_i, P'_i, M') \]

\[ X_i = \bar{X}_{iS} \]

\[ X_i = X_i (P''_i, P'_i, M') \]

\[ X_i = X_{iS} + X_{jF} (P''_i, P'_i, M') \]

\[ X_i = X_i (P''_i, P'_i, M') \]

where

\[ P''_i = P'_i (i = 1, \ldots, b, h, \ldots, k) \]

and other results are unchanged except that \( P''_i \) has replaced \( P'_i \) in all new functional forms, which are numbered (4.137) through (4.139) corresponding to (4.126) through (4.128) and are not listed here. All possible solutions of grain demand and supply by the first type of agricultural household are then

\[ X_{j11} = X_{j12} = X_{jF} (P''_i, P'_i, M') \]

and

\[ Q_{j11} = \bar{Q}_{jS} \]

\[ Q_{j12} = \bar{Q}_{jS} + Q_{jF} (P''_i, P'_i, M') \]

That is, for the first type of agricultural household, its grain demand is a function of the free market price vector, the other price vector, and its disposable income, whereas its grain supply is either its sales quota or a function of the same factors used in its grain demand function plus its grain sales quota.
The third type of agricultural household

Similarly, for the third type of households, complete buyers or nonsellers in the state market, we need to use \( Q_1 = 0 \); therefore, \( Q_{1S} = 0 \) for \( i = 1, \ldots, b \). This replaces part of the assumption \( Q_1 > 0 \) and \( Q_{1S} > 0 \) for \( i = 1, \ldots, n \).

Furthermore, we cancel part of (4.100) for \( i = 1, \ldots, b \) and add part of (4.117) for \( i = 1, \ldots, b \) to the model of the second type of household.

All possible solutions of grain demand and supply of the third type of agricultural household are

\[
X_{jIII1} = \overline{X}_{jS} \tag{4.143}
\]

\[
X_{jIII2} = \overline{X}_{jS} + X_{jF} (P'_{iF}, P'_1, M') \tag{4.144}
\]

and

\[
Q_{jIII} = 0 \tag{4.145}
\]

These results mean that, for the third type of agricultural household, its grain demand is either the official ration (the first subtype) or a function of the free market price vector, the other price vector, and its disposable income (the second subtype), whereas its grain supply is always zero.

Comparative statics

The comparative statics are shown for the grain demands and grain supplies of all three types of agricultural household. To save time and space, we calculate only grain price, grain ration, and grain quota effects.

1. The first type of agricultural household
   a. the first subtype

\[
X_{jI1} = X_{jF} (P''_{1F}, P'_1, M') \quad Q_{jI1} = \overline{Q}_{jS}
\]
Thus,
\[ \frac{\partial X_{I2}}{\partial P_{JS}} = \frac{\partial X_{IF}}{\partial P_{JS}} \frac{\partial M'}{\partial P_{JS}} < 0 \quad \text{if} \quad \frac{\partial X_{IF}}{\partial P_{JS}} > 0 \]
\[ \frac{\partial Q_{I2}}{\partial P_{JS}} = 0 \]
\[ \frac{\partial X_{I1}}{\partial P_{JF}} = \frac{\partial X_{IF}}{\partial P_{JF}} < 0 \]
\[ \frac{\partial Q_{I1}}{\partial P_{JF}} = 0 \]
\[ \frac{\partial X_{I1}}{\partial Q_{JS}} = \frac{\partial X_{IF}}{\partial Q_{JS}} \frac{\partial M'}{\partial Q_{JS}} < 0 \quad \text{if} \quad \frac{\partial X_{IF}}{\partial Q_{JS}} > 0 \]
\[ \frac{\partial Q_{I1}}{\partial Q_{JS}} = 1 \]

These results imply that, for the first subtype within the first type of agricultural households, (1) grain demand decreases but grain supply does not change if the state market price or the free market price increases; and (2) grain demand decreases but grain supply increases if the sales quota increases.

b. the second subtype

\[ X_{J12} = X_{JF} (P''_F, P'_I, M') \]
\[ Q_{J12} = Q_{JS} + Q_{JF} (P''_F, P'_I, M') \]

Because \( X_{J12} = X_{J1l} \), we do not need to discuss the partial derivatives for \( X_{J12} \) here. For \( Q_{J12} \) we have
\[ \frac{\partial Q_{J12}}{\partial P_{JS}} = \frac{\partial Q_{IF}}{\partial P_{JS}} \frac{\partial M'}{\partial P_{JS}} > 0 \quad \text{if} \quad \frac{\partial Q_{IF}}{\partial P_{JS}} > 0 \]
\[ \frac{\partial Q_{J12}}{\partial P_{JF}} = 0 \]
\[ \frac{\partial Q_{J12}}{\partial Q_{JS}} = 1 + \frac{\partial Q_{IF}}{\partial M'} \frac{\partial M'}{\partial Q_{JS}} > 0 \quad \text{if} \quad \frac{\partial Q_{IF}}{\partial M'} \frac{\partial M'}{\partial Q_{JS}} > -1 \]

The results illustrate that, for the second subtype within the first
type of agricultural household, grain supply increases when the state market price, the free market price, or the sales quota increases.

2. The second type of agricultural household

a. the first subtype

\[ X_{\text{III}} = X_{\text{JS}}, \quad Q_{\text{III}} = Q_{\text{JS}} \]

\[ \frac{\partial X_{\text{III}}}{\partial P_{\text{JS}}} = \frac{\partial X_{\text{III}}}{\partial P_{\text{IF}}} = 0 \quad \frac{\partial Q_{\text{III}}}{\partial P_{\text{JS}}} = \frac{\partial Q_{\text{III}}}{\partial P_{\text{IF}}} = 0 \]

\[ \frac{\partial X_{\text{III}}}{\partial X_{\text{JS}}} = 1 \quad \frac{\partial Q_{\text{III}}}{\partial X_{\text{JS}}} = 0 \]

\[ \frac{\partial X_{\text{III}}}{\partial Q_{\text{JS}}} = 0 \quad \frac{\partial Q_{\text{III}}}{\partial Q_{\text{JS}}} = 1 \]

The results illustrate that, for the first subtype within the second type of agricultural household, (1) grain demand and supply do not change when either the state market price or the free market price increases; (2) grain demand increases but grain supply does not change when the grain ration increases; and (3) grain demand does not change but grain supply increases when the sales quota increases.

b. the second subtype

\[ X_{\text{II}2} = X_{\text{JS}} + X_{\text{IF}} (P_{\text{IF}}, P_{i}', M'), \quad Q_{\text{II}2} = Q_{\text{JS}} \]

Because \( Q_{\text{II}2} = Q_{\text{III}1} \), all partial derivatives for \( Q_{\text{II}2} \) are the same as those for \( Q_{\text{III}1} \) and, hence, do not need to be repeated and rediscussed here. For \( X_{\text{II}2} \) we have

\[ \frac{\partial X_{\text{II}2}}{\partial P_{\text{JS}}} = \frac{\partial X_{\text{IF}}}{\partial M'} < 0 \quad \text{if} \quad \frac{\partial X_{\text{IF}}}{\partial M'} > 0 \]
The results mean that, for the second subtype within the second type of agricultural household, the grain demand is decreasing if the state market price, the free market price, or the sales quota increases; and it increases if the ration level increases.

c. the third subtype

\[ X_{jII3} = X_{jJS} + X_{jIF} (P_{1F}', P_1', M') \]

\[ Q_{jIII2} = Q_{jJS} + Q_{jIF} (P_{1F}', P_1', M') \]

Because \( X_{jII3} = X_{jII2} \), we do not repeat calculations and discussion for the partial derivatives of \( X_{jII3} \) here. For \( Q_{jIII3} \), we have

\[ \frac{\partial Q_{jIII3}}{\partial P_{jJS}} = \frac{\partial Q_{jIF}}{\partial P_{jJS}} \frac{\partial M'}{\partial M'} > 0 \quad \text{if} \quad \frac{\partial Q_{jIF}}{\partial M'} > 0 \]

\[ \frac{\partial Q_{jIII3}}{\partial P_{jIF}} = \frac{\partial Q_{jIF}}{\partial P_{jIF}} > 0 \]

\[ \frac{\partial Q_{jIII3}}{\partial X_{jJS}} = \frac{\partial Q_{jIF}}{\partial X_{jJS}} \frac{\partial M'}{\partial X_{jJS}} < 0 \quad \text{if} \quad \frac{\partial Q_{jIF}}{\partial M'} > 0 \]

\[ \frac{\partial Q_{jIII3}}{\partial Q_{jJS}} = 1 + \frac{\partial Q_{jIF}}{\partial Q_{jJS}} \frac{\partial M'}{\partial Q_{jJS}} > 0 \quad \text{if} \quad \frac{\partial Q_{jIF}}{\partial M'} > -1 \]
For the third subtype within the second type of agricultural household, the grain supply increases as the state market price, the free market price, or the sales quota increase; and it decreases as the ration level increases.

3. The third type of agricultural household
   a. the first subtype

   Using formula (4.143) and (4.145) we have

   \[ X_{jIII1} = X_{jS} \]
   \[ Q_{jIII1} = 0 \]

   thus,

   \[ \frac{\partial X_{jIII1}}{\partial P_{jS}} = \frac{\partial X_{jIII1}}{\partial P_{jF}} = 0 \]
   \[ \frac{\partial X_{jIII1}}{\partial X_{jS}} = 1 \]

   For the first subtype within the third type of agricultural household, its grain demand increases if the official ration increases, and its grain supply is always zero.

   b. the second subtype

   From formula (4.144) and (4.145)

   \[ X_{jIII2} = X_{jS} + X_{jF} (P'_{1F}, P'_{1}, M') \]
   \[ Q_{jIII2} = 0 \]

   hence

   \[ \frac{\partial X_{jIII2}}{\partial P_{jS}} = \frac{\partial X_{jF}}{\partial M'} \frac{\partial M'}{\partial P_{jS}} < 0 \quad \text{if} \quad \frac{\partial X_{jF}}{\partial M'} > 0 \]
We can conclude that, for the second subtype within the third type of agricultural household, the grain demand (1) decreases if the state market price or the free market price increases, (2) increases if the official ration increases, and (3) does not change if the sales quota increases. On the other hand, its grain supply always remains fixed at zero.

**Graphic analysis** The graphic analyses are presented for all three types of agricultural households in both the state market and the united market that combines the free market with the household-own market. Thus, we need to change our previous dual market graph (graph for the state market and the free market) into a new dual market graph (graph for the state market and the united market) but use the same procedures used before.

1. The first type of agricultural household
   a. the first subtype

   For the first subtype of nonbuyer in the state market, its demand in the \((Q^u, P^u = P^F)\) subspace is a downward-sloping line, and its supply in the \((Q^S, P^S)\) subspace is a vertical line (Figure 4.9a).
Figure 4.9. Grain demands and supplies of the first type of agricultural household

b. the second subtype

For the second subtype of nonbuyer, its only demand in the united market is a downward sloping line, and its supply consists of one vertical line in the \((Q^S, P^S)\) subspace and one upward-sloping line in the \((Q^U, P^U)\) subspace (Figure 4.9b).

2. The second type of agricultural household

a. the first subtype

For the first subtype of partial buyer in the state market, both grain demand and supply are vertical lines in the \((Q^S, P^S)\) subspace (Figure 4.10a).

b. the second subtype

For the second subtype of partial buyer in the state market, grain demand consists of two segments: one is a vertical line in the \((Q^S, P^S)\) subspace, and the other one is a downward sloping line in the \((Q^U, P^U)\) subspace. Grain supply is a vertical line in the \((Q^S, P^S)\) subspace.
As Figure 4.10c shows, for the third subtype of partial buyers in the state market, both demand and supply are composed of two segments. For the former, one is a vertical line in the \((Q^S, P^S)\) subspace, and the other one is a downward-sloping line in the \((Q^U, P^U)\) subspace. For the latter, one is a vertical line in the \((Q^S, P^S)\) subspace, and the other one is an upward-sloping line in the \((Q^U, P^U)\) space.

3. The third type of agricultural household
   a. the first subtype

For the first subtype of complete buyer in the state market the demand curve is only a vertical line in the \((Q^S, P^S)\) subspace (Figure 4.11a).

b. the second subtype

As Figure 4.11b indicates, for the second subtype of complete buyer...
in the state market, its demand curve is the same as that in Figure 4.10b.

Figure 4.11. Grain demands and supplies of the third type of agricultural household

Aggregate grain demand and supply of agricultural households

Aggregate grain demand of agricultural households  As we pointed out, agricultural households in the mixed economy can be classified into three types by their behavior in the state market. To aggregate grain demand of agricultural households, we start with aggregation of grain demand of the first type of household. Assuming that the total numbers of all agricultural households is \( n_H \), that the relative frequency of the first type and the first subtype within the first type of agricultural households is \( p_{HI} \) and \( p_{HII} \), respectively, and that all households of the first type are identical except for their behavior in the market(s), we can draw \( D^{HI} \), the aggregate grain demand of the first type of agricultural household, as the following:

\[
D^{HI} = D^{HII} + D^{HII2}
\]  

(4.146)
where

\[ D_{HI1} = P_{HI} P_{HI1} n_H X_{JI1} \]
\[ = P_{HI} P_{HI1} n_H X_{JF} (P''_{IF}, P'_i, M') \]
\[ = D_{HI1} (P_{HI}, P_{HI1}, n_H, P''_{IF}, P'_i, M') \] (4.147)

\[ D_{HI2} = P_{HI} (1 - P_{HI1}) n_H X_{JI2} \]
\[ = P_{HI} (1 - P_{HI1}) n_H X_{JF} (P''_{IF}, P'_i, M') \]
\[ = D_{HI2} (P_{HI}, P_{HI1}, n_H, P''_{IF}, P'_i, M') \] (4.148)

Consequently,

\[ D_{HI} = P_{HI} n_H X_{JF} (P''_{IF}, P'_i, M'') \]
\[ = D_{HI} (P_{HI}, n_H, P''_{IF}, P'_i, M'') \] (4.149)

That is, aggregate grain demand of the first type of agricultural households is a function of the relative frequency \( P_{HI} \), the total number of agricultural households, the free market price vector \( P''_{IF} \), the other prices \( P'_i \), and the disposable income \( M' \).

In the same way, we can get \( D_{HII} \) and \( D_{HIII} \), aggregate grain demand of the second and the third type of agricultural households. Algebraically,

\[ D_{HII} = D_{HII1} + D_{HII2} + D_{HII3} \] (4.150)

where

\[ D_{HII1} = P_{HII} P_{HII1} n_H X_{JI1} \]
\[ = P_{HII} P_{HII1} n_H X_{JS} \]
\[ = D_{HII1} (P_{HII}, P_{HII1}, n_H, X_{JS}) \] (4.151)
\[ D^{HII2} = P_{HII} P_{HII2} n_H X_{JII2} \]
\[ = P_{HII} P_{HII2} n_H [\bar{X}_{JS} + X_{JF} (P'_1, P'_1, M')] \]
\[ = D^{HII2} (P_{HII}, P_{HII2}, n_H, \bar{X}_{JS}, P'_1, P'_1, M') \] (4.152)

\[ D^{HII3} = P_{HII} (1 - P_{HII2}) n_H X_{JI13} \]
\[ = P_{HII} (1 - P_{HII1} - P_{HII2}) n_H [\bar{X}_{JS} + X_{JF} (P'_1, P'_1, M')] \]
\[ = D^{HII3} (P_{HII}, P_{HII1}, P_{HII2}, n_H, \bar{X}_{JS}, P'_1, P'_1, M') \] (4.153)

Hence
\[ D^{HII} = P_{HII} P_{HII1} n_H X_{JS} + P_{HII} P_{HII2} n_H [\bar{X}_{JS} + X_{JF} (P'_1, P'_1, M')] \]
\[ + P_{HII} (1 - P_{HII1} - P_{HII2}) n_H [\bar{X}_{JS} + X_{JF} (P'_1, P'_1, M')] \]
\[ = P_{HII} n_H \bar{X}_{JS} + P_{HII} (1 - P_{HII1}) n_H X_{JF} (P''_1, P''_1, M') \]
\[ = D^{HII} (P_{HII}, P_{HII1}, n_H, \bar{X}_{JS}, P''_1, P''_1, M') \] (4.154)

Similarly,
\[ D^{HIII} = D^{HIII1} + D^{HIII2} \] (4.155)

where
\[ D^{HIII1} = (1 - P_{HI} - P_{HII}) P_{HIII} n_H X_{JIII1} \]
\[ = (1 - P_{HI} - P_{HII}) P_{HIII} n_H \bar{X}_{JS} \]
\[ = D^{HIII1} (P_{HI}, P_{HII}, P_{HIII}, n_H, \bar{X}_{JS}) \] (4.156)

\[ D^{HIII2} = (1 - P_{HI} - P_{HII}) (1 - P_{HIII}) n_H X_{JIII2} \]
\[ = (1 - P_{HI} - P_{HII}) (1 - P_{HIII}) n_H [\bar{X}_{JS} + X_{JF} (P'_1, P'_1, M')] \]
\[ = D^{HIII2} (P_{HI}, P_{HII}, P_{HIII}, n_H, \bar{X}_{JS}, P'_1, P'_1, M') \] (4.157)
Thus,

\[ D_{\text{HI}} - (1 - P_{HI} - P_{HII}) P_{HIII} n_H X_{JS} \]

\[ + (1 - P_{HI} - P_{HII})(1 - P_{HIII}) n_H [\bar{X}_{JS} + X_{JF}(P_{1I}, P_1', M')] \]

\[ = (1 - P_{HI} - P_{HII}) n_H X_{JS} \]

\[ + (1 - P_{HI} - P_{HII}) (1 - P_{HIII}) n_H X_{JF}(P_{1I}', P_1', M') \]

\[ = D_{\text{HIII}} (P_{HI}, P_{HII}, P_{HIII}, n_H, X_{JS}, P_{1I}', P_1', M') \]  (4.158)

These results imply that the aggregate grain demand of the second and third type of agricultural households are functions of the relative frequencies \(P_{HI}, P_{HII},\) and \(P_{HIII},\) the total number of agricultural households, the free market prices, the prices of other commodities, the ration limits, and the disposable income \(M',\) and a function of the relative frequencies \(P_{HI}, P_{HII}\) and \(P_{HIII},\) the total number of agricultural households, the free market prices, the prices of other commodities, the ration limits, and the disposable income \(M'.\)

Finally, adding (4.149), (4.154) and (4.158) together, we obtain \(D^H,\) the aggregate grain demand of all agricultural households:

\[ D^H = D_{HI} + D_{HII} + D_{HIII} \]  (4.159)

\[ = P_{HI} n_H X_{JF}(P_{1I}', P_1', M') \]

\[ + P_{HII} n_H X_{JS} + P_{HIII} (1 - P_{HIII}) n_H X_{JF}(P_{1I}', P_1', M') \]

\[ + (1 - P_{HI} - P_{HII}) n_H X_{JS} \]

\[ + (1 - P_{HI} - P_{HII}) (1 - P_{HIII}) n_H X_{JF}(P_{1I}', P_1', M') \]
where
\[ D^H = \text{aggregate grain demand of agricultural households} \]
\[ D_{HI}, D_{HII}, D_{HIII} = \text{aggregate grain demand of the first, second, or third type of agricultural households} \]
\[ P_{HI}, P_{HII} = \text{the relative frequency of the first or the second type of agricultural households} \]
\[ P_{HIII}, P_{HIII} = \text{the relative frequency of the first subtype within the second or third type of agricultural households} \]
\[ n_H = \text{the total number of agricultural households} \]

Other symbols used here, such as \( \bar{x}_{JS} \), \( P_{IF}' \), and \( M'' \), are the same as those mentioned earlier in this section.

Formula (4.160) indicates that the aggregate grain demand is a function of the relative frequencies \( P_{HI}, P_{HII}, P_{HIII}, \) and \( P_{HIII} \), the total number of agricultural households, the ration limits, the free market prices, the prices of other goods, and the disposable income \( M'' \).

**Aggregate grain supply by agricultural households** By using the same aggregation method as that used in the analysis of the aggregate
grain demand of agricultural households, and making proper assumptions about the total number of agricultural households and the relative frequencies and identicalness of agricultural households within a certain type or subtype, we can obtain

\[ S_{HI} = S_{HII} + S_{HI2} \]  

(4.161)

where

\[ S_{HII} = P_{HI} \bar{P}_{HII} n_H \bar{Q}_{JI} \]

\[ = P_{HI} \bar{P}_{HII} n_H \bar{Q}_{JS} \]

\[ = S_{HII} (P_{HI}, P_{HII}, n_H, \bar{Q}_{JS}) \]  

(4.162)

\[ S_{HI2} = P_{HI} (1 - P_{HII}) n_H \bar{Q}_{JI2} \]

\[ = P_{HI} (1 - P_{HII}) n_H [\bar{Q}_{JS} + Q_{IF} (P_{IF}, P_{I}, M')] \]

\[ = S_{HI2} (P_{HI}, P_{HII}, n_H, \bar{Q}_{JS}, P_{IF}, P_{I}, M') \]  

(4.163)

Then,

\[ S_{HI} = P_{HI} n_H \bar{Q}_{JS} + P_{HI} (1 - P_{HII}) n_H Q_{IF} (P_{IF}, P_{I}, M') \]

\[ = S_{HI} (P_{HI}, P_{HII}, n_H, Q_{JS}, P_{IF}, P_{I}, M') \]  

(4.164)

Similarly,

\[ S_{HII} = S_{HII1} + S_{HII2} + S_{HII3} \]  

(4.165)

where

\[ S_{HII1} = P_{HII} \bar{P}_{HII1} n_H Q_{JII1} \]

\[ = P_{HII} \bar{P}_{HII1} n_H \bar{Q}_{JS} \]

\[ = S_{HII1} (P_{HII}, P_{HII1}, n_H, \bar{Q}_{JS}) \]  

(4.166)
\[ S_{\text{HII2}} = P_{\text{HII}} P_{\text{HII2}} n^H Q_{\text{JII2}} \]
\[ = P_{\text{HII}} P_{\text{HII2}} n^H \bar{Q}_{JS} \]
\[ = S_{\text{HII2}} \left( P_{\text{HII}}, P_{\text{HII2}}, n^H, \bar{Q}_{JS} \right) \]  
\[ (4.167) \]
\[ S_{\text{HII3}} = P_{\text{HII}} \left( 1 - P_{\text{HII1}} - P_{\text{HII2}} \right) n^H Q_{\text{JII3}} \]
\[ = P_{\text{HII}} \left( 1 - P_{\text{HII1}} - P_{\text{HII2}} \right) n^H \left( \bar{Q}_{JS} + Q_{JF}(P_{1F}, P_i, M') \right) \]
\[ = S_{\text{HII3}} \left( P_{\text{HII}}, P_{\text{HII1}}, P_{\text{HII2}}, n^H, \bar{Q}_{JS}, P_{1F}, P_i, M' \right) \]  
\[ (4.168) \]

Thus, substituting (4.166) through (4.168) into (4.165) we obtain
\[ S_{\text{HII}} = P_{\text{HII}} n^H \bar{Q}_{JS} + P_{\text{HII}} \left( 1 - P_{\text{HII1}} - P_{\text{HII2}} \right) n^H Q_{JF}(P_{1F}, P_i, \bar{Q}_{JS}, M') \]
\[ = S_{\text{HII}} \left( P_{\text{HII}}, P_{\text{HII1}}, P_{\text{HII2}}, n^H, \bar{Q}_{JS}, P_{1F}, P_i, M' \right) \]  
\[ (4.169) \]

Finally,
\[ S_{\text{HIII}} = S_{\text{HII1}} + S_{\text{HII2}} \]  
\[ (4.170) \]

where
\[ S_{\text{HII1}} = \left( 1 - P_{\text{HII}} - P_{\text{HII}} \right) P_{\text{HII1}} n^H Q_{\text{JII11}} \]
\[ = 0 \]  
\[ (4.171) \]
\[ S_{\text{HII2}} = \left( 1 - P_{\text{HII}} - P_{\text{HII}} \right) \left( 1 - P_{\text{HII1}} \right) n^H Q_{\text{JII2}} \]
\[ = 0 \]  
\[ (4.172) \]

Substituting (4.171) and (4.172) into (4.170) we have
\[ S_{\text{HIII}} = 0 \]  
\[ (4.173) \]

Adding (4.164), (4.169), and (4.173) together; we have \( S^H \), the aggregate grain supply of all agricultural households:
\[ S^H = S_{HI} + S_{\text{HII}} + S_{\text{HIII}} \]  
\[ (4.174) \]
where

\[ S^H = \text{aggregate grain supply of agricultural households} \]

\[ S_{HI}, S_{HII}, S_{HIII} = \text{aggregate grain supply of the first, the second, or the third type of agricultural households} \]

Other symbols, such as \( P_{HI}, P_{HII}, P_{HIII}, n_H, P'_{1F}, P'_{1}, Q_{JS}, M' \), are the same as before.

Formula (4.175) shows that the aggregate grain supply of all agricultural households in the mixed economy is a function of the relative frequencies \( P_{HI}, P_{HII}, P_{HIII} \) and \( P_{HII2} \), the total number of agricultural households, the sales quotas, the free market prices, the prices of other goods, and the disposable income \( M' \), respectively.

Comparative statics Comparative statics are calculated for both aggregate grain demand and supply by agricultural households.

1. Aggregate grain demand of agricultural households

From (4.159) and other relevant formulas we have

\[ D^H = D^HI + D^HII + D^HIII \]

a. \[ D^HI = D^HII + D^HII2 \]
The aggregate grain demand of the first type of household increases if the relative frequency $P_{HI}$, or the total number of agricultural households increases; and it decreases if the sales quota, the state market price, or the free market price increases.
The results imply that aggregate grain demand of the second type of agricultural household increases as the relative frequency $P_{HII}$, the total number of agricultural households, or the ration level, increases; and it decreases as the sales quota, the state market price, or the free market price increases.

c. $D_{HII} = D_{HII1} + D_{HII2}$

$$
\frac{\partial D_{HII}}{\partial P_{HI}} = \frac{\partial D_{HII1}}{\partial P_{HI}} + \frac{\partial D_{HII2}}{\partial P_{HI}} < 0
$$

$$
\frac{\partial D_{HII}}{\partial P_{HII}} = \frac{\partial D_{HII1}}{\partial P_{HII}} + \frac{\partial D_{HII2}}{\partial P_{HII}} < 0
$$

$$
\frac{\partial D_{HII1}}{\partial P_{HII11}} = \frac{\partial D_{HII1}}{\partial P_{HII11}} + \frac{\partial D_{HII2}}{\partial P_{HII11}} < 0
$$

$$
\frac{\partial D_{HII1}}{\partial P_{HII11}} = \frac{\partial D_{HII1}}{\partial P_{HII11}} + \frac{\partial D_{HII2}}{\partial P_{HII11}} > 0
$$

$$
\frac{\partial D_{HII}}{\partial n_{H}} = \frac{\partial D_{HII1}}{\partial n_{H}} + \frac{\partial D_{HII2}}{\partial n_{H}} > 0
$$

$$
\frac{\partial D_{HII}}{\partial x_{JS}} = \frac{\partial D_{HII1}}{\partial x_{JS}} + \frac{\partial D_{HII2}}{\partial x_{JS}} > 0
$$
For the third type of agricultural household, the aggregate grain demand is increasing when the total number of agricultural households or the ration level increase; and its grain demand is decreasing when the relative frequency $P_{HI}$ or $P_{HII}$, the state market price, or the free market price increases.

Finally, we have

\[
\frac{\partial D_{HII}}{\partial P_{JS}} = \frac{\partial D_{HII}}{\partial P_{JS}} < 0
\]

\[
\frac{\partial D_{HII}}{\partial P_{JF}} = \frac{\partial D_{HII}}{\partial P_{JF}} < 0
\]
Hence, the aggregate grain demand of agricultural households increases if the relative frequency $P_{HII1}$ or $P_{HIII1}$, the total number of agricultural households, or the state ration increases; and it decreases if the state market price, $P_j$, or the free market price, $P_{jF}$, increases.

2. Aggregate grain supply of agricultural households

From (4.174) and other relevant formulas we have

\[
\frac{\partial S^H}{\partial P_{jF}} = \frac{\partial S^{H1}}{\partial P_{jF}} + \frac{\partial S^{HII}}{\partial P_{jF}} + \frac{\partial S^{HIII}}{\partial P_{jF}} < 0
\]

The aggregate grain supply of the first type of agricultural household increases if the relative frequency $P_{H11}$, the total number of agricultural households, the sales quota, the state market price, or the
free market price increases.

b. \( S_{\text{HII}} = S_{\text{HII1}} + S_{\text{HII2}} + S_{\text{HII3}} \)

\[
\frac{\partial S_{\text{HII}}}{\partial P_{\text{HII}}} = \frac{\partial S_{\text{HII1}}}{\partial P_{\text{HII}}} + \frac{\partial S_{\text{HII2}}}{\partial P_{\text{HII}}} + \frac{\partial S_{\text{HII3}}}{\partial P_{\text{HII}}} > 0
\]

\[
\frac{\partial S_{\text{HII}}}{\partial P_{\text{HII1}}} = \frac{\partial S_{\text{HII1}}}{\partial P_{\text{HII1}}} + \frac{\partial S_{\text{HII3}}}{\partial P_{\text{HII1}}} > 0
\]

\[
\frac{\partial S_{\text{HII}}}{\partial P_{\text{HII2}}} = \frac{\partial S_{\text{HII2}}}{\partial P_{\text{HII2}}} + \frac{\partial S_{\text{HII3}}}{\partial P_{\text{HII2}}} < 0
\]

\[
\frac{\partial S_{\text{HII}}}{\partial n_H} = \frac{\partial S_{\text{HII1}}}{\partial n_H} + \frac{\partial S_{\text{HII2}}}{\partial n_H} + \frac{\partial S_{\text{HII3}}}{\partial n_H} > 0
\]

\[
\frac{\partial S_{\text{HII}}}{\partial X_{\text{JS}}} = \frac{\partial S_{\text{HII3}}}{\partial X_{\text{JS}}} < 0
\]

\[
\frac{\partial S_{\text{HII}}}{\partial Q_{\text{JS}}} = \frac{\partial S_{\text{HII1}}}{\partial Q_{\text{JS}}} + \frac{\partial S_{\text{HII2}}}{\partial Q_{\text{JS}}} + \frac{\partial S_{\text{HII3}}}{\partial Q_{\text{JS}}} > 0
\]

\[
\frac{\partial S_{\text{HII}}}{\partial P_{\text{JS}}} > 0
\]

\[
\frac{\partial S_{\text{HII}}}{\partial P_{\text{JF}}} > 0
\]

For the second type of agricultural household, the aggregate grain supply increases as the relative frequency \( P_{\text{HII}} \), the total number of agricultural households, the sales quota \( Q_{\text{JS}} \), the state market price, or the free market price increases; it is uncertain if the relative frequency \( P_{\text{HII1}} \) or \( P_{\text{HII2}} \) increases or decreases as the ration level increases.
c. $s_{HI}^{III} = 0$

Thus, we have

\[ \frac{\partial s}{\partial P_{HI}} \frac{\partial s}{\partial P_{HI}} \frac{\partial s}{\partial P_{HI}} > 0 \]

\[ \frac{\partial s_{HI}}{\partial P_{HI}} \frac{\partial s_{HI}}{\partial P_{HI}} > 0 \]

\[ \frac{\partial s_{HI}}{\partial P_{HI}} \frac{\partial s_{HI}}{\partial P_{HI}} > 0 \]

\[ \frac{\partial s_{HI}}{\partial P_{HI}} \frac{\partial s_{HI}}{\partial P_{HI}} > 0 \]

\[ \frac{\partial s_{HI}}{\partial P_{HI}} \frac{\partial s_{HI}}{\partial P_{HI}} > 0 \]

\[ \frac{\partial s_{HI}}{\partial P_{HI}} \frac{\partial s_{HI}}{\partial P_{HI}} > 0 \]

\[ \frac{\partial s_{HI}}{\partial P_{HI}} \frac{\partial s_{HI}}{\partial P_{HI}} > 0 \]

These results indicate that the aggregate grain supply is increasing if the relative frequency $P_{HI}$ or $P_{HII}$, the total number of agricultural households, the sales quota, the state market price, or the free market price increases. It decreases if the ration level increases.

Graphic analysis Graphic analyses are performed for the aggregate grain demand and supply of all three types of agricultural households individually.

1. The first type of agricultural household
There are two subtypes within the first type of agricultural households. Aggregate grain demands and supplies of the first subtype (D_{HI1} and S_{HI1}) and the second subtype (D_{HI2} and S_{HI2}) of agricultural households are similar in their curves' shape to those individual agricultural households within the first subtype and the second subtype. For the former, the aggregate grain demand is a downward-sloping line in the (Q^U, P^U) subspace, and the aggregate supply is a vertical line in the (Q^S, P^S) subspace with an intercept Q_{HI1b} = P_{HI1} P_{HI1} n_H Q_{JS} (Figure 4.12a). For the latter, the aggregate grain demand is a downward-sloping line in the (Q^U, P^U) subspace, and the aggregate supply is composed of two segments: one is a vertical line with an intercept Q_{HI2b} = P_{HI2} (1 - P_{HI1}) n_H Q_{JS} on the Q^S axis in the (Q^S, P^S) subspace. The other is an upward-sloping line in the (Q^U, P^U) subspace (Figure 4.12b).

Figure 4.12. Aggregation of grain demands and supplies of two subtypes within the first type of agricultural households

Figure 4.12c shows the aggregate grain demand D_{HI} and supply S_{HI} of the first type of agricultural households. D_{HI} is a downward-sloping...
line in the \((Q^u, P^u)\) subspace, and \(S^H\) is similar to \(S^{HII2}\) in shape, but its intercept on the \(Q^S\) axis is now \(Q^{HIIb} = P_n^H \bar{Q}^{JS}\). At \(F\), the equilibrium price of the free market, there is a surplus in the united market so that the second subtype of the first type of agricultural households are main suppliers in the free market.

2. The second type of agricultural household

There are three subtypes within the second type of agricultural households. The aggregate grain demands and supplies for the first \((D^{HII1} \text{ and } S^{HII1})\), the second \((D^{HII2} \text{ and } S^{HII2})\), and the third \((D^{HII3} \text{ and } S^{HII3})\) subtype are illustrated in Figure 4.13a, b, and c. Both \(D^{HII1}\) and \(S^{HII1}\) are vertical lines with intercepts on the \(Q^S\) axis: \(Q^{HII1a} = P_n^H \bar{Q}^{JS}\). \(D^{HII2}\) is composed of two pieces: one is a vertical line with an intercept \(Q^{HII2a} = P_n^H \bar{Q}^{JS}\) in the \((Q^S, P^S)\) subspace, the other one is a downward-sloping line in \((Q^u, P^u)\) subspace. \(S^{HII2}\) is only a vertical line that has an intercept \(Q^{HII2b} = P_n^H \bar{Q}^{JS}\) in the \((Q^S, P^S)\) space. Both \(D^{HII3}\) and \(S^{HII3}\) consist of two segments. The former has one vertical segment in

![Figure 4.13](image)

Figure 4.13. Aggregate grain demands and supplies of three subtypes within the second type of agricultural households
the \((Q^S, P^S)\) subspace, with an intercept \(Q_{HII3a} = P_{HII}(1-P_{HII1}-P_{HII2})\) \(n_{H}^X\) on the \(Q^S\) axis and one downward-sloping segment in the \((Q^U, P^U)\) subspace. The latter has one vertical segment in the \((Q^S, P^S)\) subspace with an intercept \(Q_{HII3b} = P_{HII}(1-P_{HII1}-P_{HII2}n_{H}^O\) on the \(Q^S\) axis, and one downward-sloping segment in the \((Q^U, P^U)\) subspace.

By summing Figure 4.13a, b, and c, we obtain the aggregate grain demand \(D^{HII}\) and supply \(S^{HII}\) of the second type of agricultural household in Figure 4.14. Both \(D^{HII}\) and \(S^{HII}\) contain two segments and are similar in shape to Figure 4.13c, but the intercept on the \(Q^S\) axis for \(D^{HII}\) is \(Q_{HIIa} = Q_{HII1a} + Q_{HII2a} + Q_{HII3a}\) and that for \(S^{HII}\) is \(Q_{HIIb} = Q_{HII1b} + Q_{HII2b} + Q_{HII3b}\).

![Figure 4.14. Aggregate grain demand and supply of the second type of agricultural households](image)

3. The third type of agricultural household

There also are two subtypes within the third type of agricultural household. As Figures 4.15a and b show, the shapes of the aggregate grain demands and supplies for both the first subtype \(D^{HIII1}\) and \(S^{HIII1}\) and the second subtype \(D^{HIII2}\) and \(S^{HIII2}\) are similar to those in
Figures 4.11a and b. The intercepts for $D^{HII11}$ and $D^{HII12}$ now are

$$Q_{HII11a} = (1 - P_{HI} - P_{HII}) P_{HII11} \bar{x}_J S$$

and

$$Q_{HII12a} = (1 - P_{HI} - P_{HII}) (1 - P_{HII11}) \bar{x}_J S.$$

![Diagram of aggregate grain demands and supplies of the third type of agricultural households](image)

Figure 4.15. Aggregation of grain demands and supplies of the third type of agricultural households

The aggregate grain demand ($D^{HIII}$) and supply ($S^{HIII}$) of the third type of agricultural household are drawn in Figure 4.15c. They look like those in Figure 4.15b, but the intercept of $D^{HIII}$ on the $Q^S$ axis is

$$Q_{HII11a} = Q_{HII11a} + Q_{HII12a}.$$

Finally, adding Figure 4.12c, 4.14, and 4.15c together, we obtain the aggregate grain demand, $D^H$, and supply, $S^H$, of all agricultural households in Figure 4.16. Both $D^H$ and $S^H$ contain two segments. $D^H$ consists of one vertical line in the $(Q^S, P^S)$ subspace, with an intercept

$$Q_{Ha} = Q_{H1a} + Q_{HIIa} + Q_{HIIIa}$$
on the $Q^S$ axis and a downward-sloping line in the $(Q^U, P^U)$ subspace. $S^H$, on the other hand, consists of one vertical line in the $(Q^S, P^S)$ subspace, with an intercept

$$Q_{HB} = Q_{H1b} + Q_{HIIb} + Q_{HIIIb}$$
on the $Q^S$ axis and an upward-sloping line in the $(Q^U, P^U)$ subspace. Given $P^F$, obviously $S^H > D^H$ so that agricultural households,
especially the first type, are main suppliers in the free market.

Figure 4.16. Aggregate grain demand and supply of all types of agricultural households

The state market analysis

Because grain demand of the state farm households is the same as that of the urban households and grain supply of the state farms occurs in the state market only, it is not necessary to repeat the state market analysis for the state farm (households) in this part. Instead, the state market analysis concentrates on grain demand and supply of the agricultural households in the state market. This part is subdivided into two parts: one analyzes individual behavior of agricultural households; the other analyzes aggregate behavior of agricultural households. Now we work on the first subpart.

Grain demand and supply by agricultural household in the state market

There are three types of agricultural household in the state market.

1. Nonbuyer

\[ X_{jSI} = 0 \]  

(4.176)
Q_{JSI} = \bar{Q}_{JS} \quad (4.177)

The grain demand of nonbuyer in the state market is zero and the grain supply of nonbuyer in the state market is fixed at \( Q_{JS} \).

2. Partial buyer

\[ X_{JSII} = \bar{X}_{JS} \quad (4.178) \]
\[ Q_{JSII} = \bar{Q}_{JS} \quad (4.179) \]

For a partial buyer, both grain demand and supply in the state market are fixed at \( \bar{X}_{JS} \) and \( \bar{Q}_{JS} \). If \( X_{JS} < \bar{Q}_{JS} \), this partial buyer is a net seller in the state market. Otherwise, he is a net buyer in the state market.

3. Complete buyer

\[ X_{JSIII} = \bar{X}_{JS} \quad (4.180) \]
\[ Q_{JSIII} = 0 \quad (4.181) \]

For a complete buyer, the grain demand is fixed at \( \bar{X}_{JS} \) at the state market and the grain supply is zero.

**Comparative statics for the individual grain demands and supplies**

Comparative statics are made for individual grain demands and supplies of these three types of agricultural household.

1. Nonbuyer

\[ X_{JSI} = 0 \]
\[ Q_{JSI} = \bar{Q}_{JS} \]
\[ \frac{\partial Q_{JSI}}{\partial \bar{P}_{JS}} = \frac{\partial Q_{JSI}}{\partial \bar{P}_{JS}} = 0 \]
\[ \frac{\partial Q_{JSI}}{\partial Q_{JS}} = 1 \]

For a nonbuyer in the state market, the grain demand is zero and
unaffected by any policy instruments, and its grain supply is only affected by the sales quota in the same direction.

2. Partial buyer

\[ X_{jsII} = X_{js} \]
\[ \frac{\partial X_{jsII}}{\partial p_{js}} = \frac{\partial X_{jsII}}{\partial p_{jF}} = 0 \]
\[ \frac{\partial X_{jsII}}{\partial X_{js}} = 1 \]
\[ \frac{\partial X_{jsII}}{\partial Q_{js}} = 0 \]
\[ Q_{jsII} = Q_{js} \]
\[ \frac{\partial Q_{jsII}}{\partial p_{js}} = \frac{\partial Q_{jsII}}{\partial p_{jF}} = 0 \]
\[ \frac{\partial Q_{jsII}}{\partial X_{js}} = 0 \]
\[ \frac{\partial Q_{jsII}}{\partial Q_{js}} = 1 \]

For a partial buyer in the state market, grain demand increases if the ration level increases, and grain supply increases if the sales quota increases.

3. Complete buyer

\[ X_{jsIII} = X_{js} \]
\[ Q_{jsIII} = Q_{js} \]

The comparative statics for \( X_{jsIII} \) are the same as that for \( X_{jsII} \), whereas the comparative statics for \( Q_{jsIII} \) are always zero.

Next are analyses of the second subpart.

**Aggregate grain demand of agricultural households in the state market.** First, we aggregate \( D^{HIS} \), \( D^{HIIS} \) and \( D^{HIIS} \), grain demands of the first, the second, and the third type of agricultural households, then add them together to obtain \( D^{HS} \), aggregate grain demand of all agricultural households,

\[ D^{HS} = D^{HIS} + D^{HIIS} + D^{HIIS} \]  

(4.182)
From formulas (4.149), (4.154), and (4.158), we easily conclude that

$$D_{HIS} = 0 \quad (4.183)$$

In the state market, aggregate grain demand of nonbuyers is zero.

$$D_{HII} = P_{HII} n_H \bar{X}_{JS}$$

$$D_{HII} = D_{HII} \left( P_{HI}, n_{HI}, \bar{X}_{JS} \right) \quad (4.184)$$

Aggregate grain demand of partial buyers in the state market is a function of the relative frequency $P_{HII}$, the total number of agricultural households, and the ration level.

$$D_{HII} = (1 - P_H - P_{HII}) n_H \bar{X}_{JS}$$

$$D_{HII} = D_{HII} \left( P_{HI}, P_{HII}, n_H, \bar{X}_{JS} \right) \quad (4.185)$$

For complete buyers, aggregate grain demand in the state market is a function of the relative frequencies $P_{HI}$ and $P_{HII}$, the total number of all agricultural households, and the ration level.

Substituting (4.183), (4.184), and (4.185) into (4.182), we have

$$D_{HS} = P_{HII} n_H \bar{X}_{JS} + (1 - P_H - P_{HII}) n_H \bar{X}_{JS}$$

$$D_{HS} = (1 - P_H) n_H \bar{X}_{JS}$$

$$D_{HS} = D_{HS} \left( P_{HI}, n_{HI}, \bar{X}_{JS} \right) \quad (4.186)$$

Thus, aggregate grain demand of agricultural households in the state market is a function of the relative frequency $P_{HI}$, the total number of agricultural households, and the ration level $\bar{X}_{JS}$.

**Aggregate grain supply of agricultural households in the state market**

Similarly, we have the following formula for $S_{HS}^H$, aggregate grain supply of agricultural households in the state market:
where \( s^{\text{HIS}} \), \( s^{\text{HIIS}} \), and \( s^{\text{HIIIS}} \) are aggregate grain supply for the first, second, and third types of agricultural households and can be obtained from formulas (4.164), (4.169), and (4.173):

\[
s^{\text{HIS}} = P_{\text{HI}} n_{\text{H}} Q_{JS} - s^{\text{HIS}} (P_{\text{HI}}, n_{\text{H}}, Q_{JS})
\]

(4.188)

\[
s^{\text{HIIS}} = P_{\text{HII}} n_{\text{H}} Q_{JS} - s^{\text{HIIS}} (P_{\text{HII}}, n_{\text{H}}, Q_{JS})
\]

(4.189)

In the state market, grain supplies of both nonbuyers and partial buyers are functions of the relative frequency, the total number of agricultural households, and the sales quota.

\[
s^{\text{HIIIS}} = 0
\]

(4.190)

For complete buyers in the state market, the grain supply is zero.

Adding (4.188), (4.189) and (4.190) we have

\[
s^{\text{HS}} = P_{\text{HI}} n_{\text{H}} Q_{JS} + P_{\text{HII}} n_{\text{H}} Q_{JS}
\]

\[
= (P_{\text{HI}} + P_{\text{HII}}) n_{\text{H}} Q_{JS} - s^{\text{HS}} (P_{\text{HI}}, P_{\text{HII}}, n_{\text{H}}, Q_{JS})
\]

(4.191)

In the state market, aggregate grain supply of all agricultural households is a function of the relative frequency \( P_{\text{HI}} \) and \( P_{\text{HII}} \), the total number of agricultural households, and the sales quotas.

**Comparative statics for the aggregate grain demands and supplies**

Comparative statics are assessed for the aggregate grain demands and supplies of three types of agricultural households, then for the
aggregate grain demand and supply by all agricultural households.

1. Nonbuyers

\[ D_{HIS} = 0 \]

\[ S_{HIS} = P_{HI} n_H Q_{JS} \]

\[ \frac{\partial S_{HIS}}{\partial P_{HI}} = n_H Q_{JS} > 0 \]

\[ \frac{\partial S_{HIS}}{\partial n_H} = P_{HI} Q_{JS} > 0 \]

\[ \frac{\partial S_{HIS}}{\partial Q_{JS}} = P_{HI} n_H > 0 \]

For the first type of agricultural households in the state market, aggregate grain demand is always restricted at zero, and aggregate grain supply increases if the relative frequency \( P_{HI} \), the total number of agricultural households, or the sales quotas increase.

2. Partial buyers

\[ D_{HIIIS} = P_{HII} n_H x_{JS} \]

\[ S_{HIIIS} = P_{HII} n_H Q_{JS} \]

\[ \frac{\partial S_{HIIIS}}{\partial P_{HII}} = n_H x_{JS} > 0 \]

\[ \frac{\partial S_{HIIIS}}{\partial n_H} = P_{HII} Q_{JS} > 0 \]

\[ \frac{\partial S_{HIIIS}}{\partial x_{JS}} = P_{HII} n_H > 0 \]

For the second type of agricultural households in the state market, (1) both aggregate grain demand and supply increase as the relative frequency \( P_{HII} \), or the total number of agricultural households increases;
(2) aggregate grain demand increases, but aggregate grain supply does not change, as the ration level increases; and (3) aggregate grain demand does not change, but aggregate grain supply increases, as the sales quota increases.

3. Complete buyers

\[ D_{HIIIS} = (1 - P_{HI} - P_{HII}) n_{H} \bar{x}_{JS} \quad S_{HIIIS} = 0 \]

\[ \frac{\partial D_{HIIIS}}{\partial P_{HI}} = \frac{\partial D_{HIIIS}}{\partial P_{HII}} = - n_{H} \bar{x}_{JS} < 0 \]

\[ \frac{\partial D_{HIIIS}}{\partial n_{H}} = (1 - P_{HI} - P_{HII}) \bar{x}_{JS} > 0 \]

\[ \frac{\partial D_{HIIIS}}{\partial \bar{x}_{JS}} = (1 - P_{HI} - P_{HII}) n_{H} > 0 \]

For the third type of agricultural household in the state market, aggregate grain demand decreases if the relative frequency \( P_{HI} \) or \( P_{HII} \) increases, or it increases if the total number of agricultural households or the ration level increases; and the aggregate grain supply is always zero.

Lastly, we have comparative statics for \( D_{HS} \) and \( S_{HS} \), where

\[ D_{HS} = D_{HIIS} + D_{HIIIS} \]

\[ S_{HS} = S_{HIS} + S_{HIIS} \]

Thus,

\[ \frac{\partial D_{HS}}{\partial P_{HI}} = \frac{\partial D_{HIIS}}{\partial P_{HI}} > 0 \]

\[ \frac{\partial S_{HS}}{\partial P_{HI}} = \frac{\partial S_{HIS}}{\partial P_{HI}} > 0 \]

\[ \frac{\partial D_{HS}}{\partial P_{HI}} = \frac{\partial D_{HIIS}}{\partial P_{HI}} + \frac{\partial D_{HIIIS}}{\partial P_{HI}} > 0 \]

\[ \frac{\partial S_{HS}}{\partial P_{HI}} = \frac{\partial S_{HIS}}{\partial P_{HI}} + \frac{\partial S_{HIIS}}{\partial P_{HI}} > 0 \]
For all agricultural households in the state market, (1) both aggregate grain demand and supply increase if the relative frequency $P_{HI}$ or the total number of agricultural households increases; (2) aggregate grain demand does not change, but aggregate grain supply increases, if the relative frequency $P_{HII}$ or the sales quota increases; and (3) aggregate grain demand increases, but aggregate grain supply does not change, if the ration level increases.

**Graphic analysis**   The graphic analyses for either individual or aggregate grain demands and supplies of either individual type or aggregate of agricultural households in the state market are available in the $(Q^S, P^S)$ subspace in the dual market graphs. Readers can refer to Figures 4.9 through 4.11 and Figures 4.12 through 4.16.

**The free market analysis**

For state farm households, their activities in the free market are similar to the activities of urban households and hence are deleted here. We are only studying the behavior of agricultural households in the free market in this part. Like the state market analysis, this part also is subdivided into two parts. The first one studies individual behavior of
agricultural households. The second one studies aggregate behavior of agricultural households. We begin with the first subpart.

**Grain demand and supply of individual agricultural household in the free market**

When making the free market analysis, we depend on the results obtained from the united market analysis. Generally, grain demand or grain supply in the free market can be derived easily from the difference between the grain demand and the grain supply in the united market. In detail, both the grain demand and the grain supply in the free market are just the excess grain demand and the excess grain supply in the united market. To be consistent with the previous studies, we still insist on the previous classification of the three agricultural households by their behavior in the state market.

1. **Nonbuyer**

There are two subtypes of nonbuyer in the state market. The grain demand or grain supply in the free market for these two subtype nonbuyers can be derived from formulas (4.140) and (4.141) or (4.140) and (4.142).

\[
X_{jF1} = X_{jUI1} - Q_{jUI1} - X_{jF} (P_{iF}', P_i', M') \quad (4.192)
\]

For the first subtype of nonbuyer in the state market, the grain demand in the free market is a function of the free market price vector \(P_{iF}'\), the other goods' prices, and disposable income.

\[
X_{jF12} = X_{jUI2} - Q_{jUI2} \quad \text{if } X_{jUI2} > Q_{jUI2} - X_{jF} (P_{iF}', P_i', M') - Q_{jF} (P_{iF}', P_i', M') \quad (4.193)
\]

\[
= X_{jF12} (P_{iF}', P_i', M')
\]
or \( X_{jF_1} - Q_{jF_1} = 0 \) if \( X_{jUI_2} = Q_{jUI_2} \)
or \( Q_{jF_1} - X_{jUI_2} = 0 \) if \( X_{jUI_2} < Q_{jUI_2} \)

\[
- Q_{jF} (P_{1F}, P_i', M') - X_{jF} (P_i', P_i', M') \\
- Q_{jF_1} (P_{1F}', P_i', M')
\]

(4.194)

For the second subtype of nonbuyer in the state market, the grain demand (supply) in the free market is a function of the free market price vector, the prices of other goods, and the disposable income; or both its grain demand and supply in the free market are zero. In fact, the most frequent case for the nonbuyers in the state market is that they become nonbuyers in the free market, too. The main grain suppliers in the free market mainly come from this subtype.

2. Partial buyer

There are three subtypes of partial buyers in the state market. Their grain demands or supplies in the free market can be calculated from formulas (4.129) and (4.131), (4.130) and (4.131), and (4.130) and (4.132).

\[
X_{jFII_1} = Q_{jFII_1} = 0
\]

(4.195)

There is no grain demand or supply in the free market for the first subtype of partial buyer.

\[
X_{jFII_2} = X_{jUII_2} - Q_{jUII_2} \\
- X_{jF} (P_i', P_i', M') \\
- X_{jFII_2} (P_{1F}', P_i', M')
\]

(4.196)

For the second subtype of partial buyer in the state market, the grain demand in the free market is a function of the free market price
vector, the prices of other goods, and the disposable income.

\[
X_{jFI1} = X_{jUI1} - Q_{jUI1} \quad \text{if } X_{jUI1} > Q_{jUI1}
\]
\[
= X_F (P_{iF}', P_i', M') - Q_F (P_{iF}', P_i', M')
\]
\[
= X_{jFI1} (P_{iF}', P_i', M') \quad (4.197)
\]

or \[X_{jFI1} = Q_{jFI1} = 0 \quad \text{if } X_{jUI1} = Q_{jUI1} \]

or \[Q_{jFI1} = X_{jUI1} \quad \text{if } X_{jUI1} < Q_{jUI1}
\]
\[
= Q_F (P_{iF}', P_i', M') - X_F (P_{iF}', P_i', M')
\]
\[
= Q_{jFI1} (P_{iF}', P_i', M') \quad (4.199)
\]

For the third subtype of partial buyer in the state market, the grain demand or supply in the free market is a function of the free market price vector, the prices of other goods, and disposable income; or, both of them are zero.

3. Complete buyer

There are two subtypes of complete buyer in the state market. Their grain demands and supply in the free market can be figured out by using formulas (4.143) and (4.145) or (4.144) and (4.145).

\[
X_{jFI1I} = Q_{jFI1I} = 0 \quad (4.200)
\]
\[
X_{jFI1II} = X_{jUI1II} - Q_{jUI1II}
\]
\[
= X_F (P_{iF}', P_i', M')
\]
\[
= X_{jFI1II} (P_{iF}', P_i', M') \quad (4.201)
\]

For the first subtype of complete buyer in the state market, both
grain demand and supply in the free market are zero. For the second subtype of complete buyer in the state market, the grain demand in the free market is a function of free market prices, the prices of other goods, and disposable income.

**Comparative statics for individual grain demands and supplies**

Comparative statics are assessed for individual grain demands and supplies of all these three types of agricultural households.

1. Nonbuyer

\[ X_{jFII} = X_jF (P'_{iF}, P'_{i}, M') \]

Comparative statics for \( X_{jFII} \) are the same as those for \( X_{jII} \) and are omitted here.

By formula (4.193), if \( X_{jUI2} > Q_{jUI2} \)

\[ X_{jFII2} = X_{jF} (P''_{iF}, P'_{i}, M') - Q_jF (P''_{iF}, P'_{i}, M') \]

\[ \frac{\partial X_{jFII2}}{\partial P_{jS}} = \frac{\partial X_{jF}}{\partial M'} - \frac{\partial Q_jF}{\partial P_{jS}} < 0 \]

\[ \frac{\partial X_{jFII2}}{\partial P_{jF}} - \frac{\partial X_{jF}}{\partial P_{jF}} < 0 \]

\[ \frac{\partial X_{jFII2}}{\partial Q_{jS}} - \frac{\partial X_{jF}}{\partial Q_{jS}} < 0 \]

In case \( X_{jUI2} > Q_{jUI2} \), \( X_{jFII2} \) is decreasing if the state price or the free market price increases. If \( X_{jUI2} < Q_{jUI2} \), clearly, we have the opposite results for \( Q_{jFII2} \).

2. Partial buyer

\[ X_{jFIII} = Q_{jFIII} = 0 \]
There are no changes for both $X_{jFII1}$ and $Q_{jFII1}$ when any factor changes.

$$X_{jFII2} = X_{jF} (P_i', P_i', M')$$

$$\frac{\partial X_{jFII2}}{\partial P_jS} = \frac{\partial X_{jF}}{\partial M'} \frac{\partial M'}{\partial P_jS} > 0$$

$$\frac{\partial X_{jFII2}}{\partial P_jF} = \frac{\partial X_{jF}}{\partial P_jF} < 0$$

$$\frac{\partial X_{jFII2}}{\partial X_jS} = \frac{\partial X_{jF}}{\partial X_jS} \frac{\partial X_jS}{\partial M'} > 0$$

$$\frac{\partial X_{jFII2}}{\partial Q_jS} = \frac{\partial X_{jF}}{\partial Q_jS} \frac{\partial Q_jS}{\partial M'} < 0$$

For the second subtype of partial buyer in the state market, its grain demand in the free market decreases as the state market prices, the sales quota, or the free market price increases, and increases as the state ration level increases.

In case $X_{jUII3} > Q_{jUII3}$

$$X_{jFII3} = X_{jF} (P_i', P_i', M') - Q_{jF} (P_i', P_i', M')$$

$$\frac{\partial X_{jFII3}}{\partial P_jS} = \frac{\partial X_{jF}}{\partial M'} \frac{\partial Q_{jF}}{\partial P_jS} - \frac{\partial Q_{jF}}{\partial M'} \frac{\partial P_jS}{\partial P_jS} < 0$$

$$\frac{\partial X_{jFII3}}{\partial P_jF} = \frac{\partial X_{jF}}{\partial P_jF} - \frac{\partial Q_{jF}}{\partial P_jF} < 0$$

$$\frac{\partial X_{jFII3}}{\partial X_jS} = \frac{\partial X_{jF}}{\partial X_jS} \frac{\partial X_jS}{\partial M'} - \frac{\partial Q_{jF}}{\partial X_jS} \frac{\partial X_jS}{\partial M'} > 0$$
For the third subtype of partial buyer in the state market, in case $X_{JUII3} > Q_{JUII3}$, the grain demand in the free market decreases as the state market price, the free market price, or the sales quota increases, and increases as the state ration level increases. In case $X_{JUII3} < Q_{JUII3}$, we have opposite results for $Q_{JUII3}$. Thus, there is no need to calculate comparative statics for this case.

3. Complete buyer

$X_{JFIIII1} = 0$

$X_{JFIIII2} = X_{JF}(P'_1, P'_1, M')$

$\frac{\partial X_{JFIIII2}}{\partial P_{JS}} = \frac{\partial X_{JF}}{\partial P_{JS}} < 0$

$\frac{\partial X_{JFIIII2}}{\partial P_{JF}} < 0$

$\frac{\partial X_{JFIIII2}}{\partial P_{JS}} = \frac{\partial X_{JF}}{\partial P_{JS}} > 0$

Thus, for the first subtype of complete buyer in the state market, the grain demand in the free market is unaffected by any factor's change. For the second subtype of complete buyer, the grain demand in the free market decreases if the free market price or the state market price increases, and it increases if the state ration level increases.

The following is the second subpart of the free market analysis.
Aggregate demand of agricultural households in the free market

First, we aggregate $D^{\text{HIF}}$, $D^{\text{HIIF}}$, and $D^{\text{HIIF3}}$, grain demand of the first, the second, and the third type of agricultural households in the free market, then add them into $D^{\text{HF}}$, aggregate grain demand of all agricultural households in the free market:

$$D^{\text{HF}} = D^{\text{HIF}} + D^{\text{HIIF}} + D^{\text{HIIF3}}$$

Using formulas (4.201) through (4.201) and referring to formulas (4.149), (4.154), (4.158), (4.164), (4.169) and (4.173), we can derive $D^{\text{HIF}}$, $D^{\text{HIIF}}$, and $D^{\text{HIIF3}}$ individually.

$$D^{\text{HIF}} = D^{\text{HIF1}} + D^{\text{HIF2}}$$

$$D^{\text{HIIF}} = D^{\text{HIIF1}} + D^{\text{HIIF2}} + D^{\text{HIIF3}}$$

The aggregate grain demand in the free market for the first type of agricultural households is a function of the relative frequency $P_{\text{H1}}$, $P_{\text{H11}}$, the total number of agricultural households, the free market prices $P_{\text{IF}}$, the prices of other goods, and disposable income.
For the second type of agricultural households, the aggregate grain demand in the free market is a function of the relative frequencies $P_{HI}$, $P_{HII}$, $P_{HII2}$, $P_{HII3}$, total number of agricultural households, the free market price $P_i^F$, the prices of other goods, and disposable income.

$$D_{HIIF} = D_{HIIF1} + D_{HIIF2}$$

$$= 0 + (1 - P_{HI} - P_{HII}) (1 - P_{HII1}) n_H X_{IFIII2}$$
$$- (1 - P_{HI} - P_{HII}) (1 - P_{HII1}) n_H X_{IF} (P_i^F, P_i', M')$$

$$= D_{HIIF} (P_{HI}, P_{HII}, P_{HII1}, n_H, P_i^F, P_i', M')$$

That is, for the third type of agricultural household, the aggregate grain demand in the free market is a function of the relative frequencies $P_{HI}$, $P_{HII}$, and $P_{HII1}$, the total number of agricultural households, the free market price $P_i^F$, the prices of other goods, and disposable income.

Lastly, substituting (4.204), (4.206) and (4.208) into (4.202), we get

$$D^F = P_{HI} P_{HII1} n_H X_{IF} (P_i^F, P_i', M') + P_{HI} (1 - P_{HII1}) P_{HII1} n_H$$

$$[X_{IF} (P_i^F, P_i', M') - Q_{IF} (P_i^F, P_i', M') ]$$

$$+ P_{HII2} P_{HII1} n_H X_{IF} (P_i^F, P_i', M')$$

$$+ P_{HII} (1 - P_{HII1} - P_{HII2}) P_{HII3} n_H$$

$$[X_{IF} (P_i^F, P_i', M') - Q_{IF} (P_i^F, P_i', M') ]$$

$$+ (1 - P_{HI} - P_{HII}) (1 - P_{HII1}) n_H X_{IF} (P_i^F, P_i', M')$$
The aggregate grain demand of all agricultural households in the free market is a function of the relative frequencies $P_{HI}$, $P_{HII}$, $P_{HIII}$, $P_{HII1}$, $P_{HII2}$, $P_{HII31}$, $P_{HII31}$, the total number of agricultural households, the free market prices $P_{IF}$, the prices of other goods, and disposable income.

**Aggregate grain supply of agricultural households in the free market**

Similarly, we first aggregate $S_{HIF}$, $S_{HIIF}$, and $S_{HIIIF}$, grain supply of the first, the second, and the third type of agricultural households in the free market, then combine these together to obtain $S_{HF}$, aggregate grain supply of all agricultural households in the free market:

$$S_{HF} = S_{HIF} + S_{HIIF} + S_{HIIIF} \quad (4.210)$$

Using formulas (4.192) through (4.201) and referring to formulas (4.164), (4.169), and (4.171), we obtain

$$S_{HIF} = S_{HIF1} + S_{HIF2} \quad (4.211)$$

$$= 0 + (1 - P_{HI1}) P_{HI13} n_H Q_{JF12}$$

$$- P_{HI} (1 - P_{HI1}) P_{HI13} n_H$$

$$[Q_{JF} (P''_1 F', P'_1, M') - K_{JF} (P''_1 F', P'_1, M') ]$$

$$- S_{HIF} (P_{HI}, P_{HII}, P_{HIII}, P_{HII1}, P_{HII2}, P_{HII31}, P_{HII31}, n_H, P''_1 F', P'_1, M') \quad (4.212)$$

For the first type of agricultural household, the aggregate grain supply in the free market is a function of the relative frequencies $P_{HI}$, $P_{HII}$, $P_{HIII}$, and $P_{HII1}$, $P_{HII2}$, $P_{HII31}$, $P_{HII31}$ [which is $(1 - P_{HI1} - P_{HII2})$], the total number
of agricultural households, the free market price \( P_{1F}' \), the prices of other goods, and disposable income.

\[
S_{HIIIF} = S_{HIIIF1} + S_{HIIIF2} + S_{HIIIF3} \tag{4.213}
\]

\[
= 0 + 0 + P_{HII1} (1 - P_{HII1} - P_{HII2}) n_H JQ_{FII3}
\]

\[
- P_{HII} (1 - P_{HII1} - P_{HII2}) (1 - P_{HII33}) n_H
\]

\[
+ [Q_{jF} (P_{1F}', P_i', M') - X_{jF} (P_{1F}', P_i', M')]
\]

\[
= S_{HIIIF} (P_{HII1}, P_{HII2}, P_{HII33} n_H' P_{1F}', P_i', M') \tag{4.214}
\]

The aggregate grain supply in the free market for the second type of agricultural households is a function of the relative frequencies \( P_{HII1}' \), \( P_{HII2}' \) and \( P_{HII33}' \), the total number of agricultural households, the free market prices, the prices of other goods, and the disposable income.

\[
S_{HIIIF} = S_{HIIIF1} + S_{HIIIF2} \tag{4.215}
\]

\[
= 0 \tag{4.216}
\]

The aggregate grain supply in the free market for the third type of agricultural households is zero. Substituting (4.212), (4.214) and (4.216) into (4.210) we obtain

\[
S_{HF} = P_{HI} (1 - P_{HII}) P_{HII3 n_H}
\]

\[
+ [Q_{jF} (P_{1F}', P_i', M') - X_{jF} (P_{1F}', P_i', M')]
\]

\[
+ P_{HII} (1 - P_{HII1} - P_{HII2}) P_{HII33} n_H
\]

\[
+ [Q_{jF} (P_{1F}', P_i', M') - X_{jF} (P_{1F}', P_i', M')]
\]

\[
= S_{HF} (P_{HII1}, P_{HII2}, P_{HII33} n_H' P_{1F}', P_i', M') \tag{4.217}
\]
For agricultural households, the aggregate grain supply in the free market is a function of the relative frequencies $P_{HI}$, $P_{HI1}$, $P_{HI11}$, $P_{HI2}$, $P_{HI21}$, $P_{HI3}$, $P_{HI31}$, and $P_{HI33}$, the total number of agricultural households, the free market price $P^*_F$, the prices of other goods, and disposable income.

**Comparative statics for aggregate grain demands and supplies**

Comparative statics are made for three types of agricultural households separately.

1. Nonbuyers

Using formulas (4.203) and (4.211) we have

\[
\begin{align*}
\frac{\partial S_{HIF}}{\partial P_{HI}} &= \frac{\partial S_{HIF1}}{\partial P_{HI}} + \frac{\partial S_{HIF2}}{\partial P_{HI}} > 0 \\
\frac{\partial S_{HIF}}{\partial P_{HI1}} &= \frac{\partial S_{HIF1}}{\partial P_{HI1}} + \frac{\partial S_{HIF2}}{\partial P_{HI1}} < 0 \\
\frac{\partial S_{HIF}}{\partial P_{H11}} &= \frac{\partial S_{HIF2}}{\partial P_{H11}} > 0 \\
\frac{\partial S_{HIF}}{\partial n_H} &= \frac{\partial S_{HIF1}}{\partial n_H} + \frac{\partial S_{HIF2}}{\partial n_H} > 0 \\
\frac{\partial S_{HIF}}{\partial P_{js}} &= \frac{\partial S_{HIF1}}{\partial P_{js}} + \frac{\partial S_{HIF2}}{\partial P_{js}} < 0 \\
\frac{\partial S_{HIF}}{\partial P_{jF}} &= \frac{\partial S_{HIF1}}{\partial P_{jF}} + \frac{\partial S_{HIF2}}{\partial P_{jF}} < 0 \\
\frac{\partial S_{HIF}}{\partial Q_{js}} &= \frac{\partial S_{HIF1}}{\partial Q_{js}} + \frac{\partial S_{HIF2}}{\partial Q_{js}} < 0
\end{align*}
\]

The results indicate that, for the first type of agricultural
households in the free market, (1) both the aggregate grain demand, \( D_{HIF} \), and the aggregate grain supply, \( S_{HIF} \), increase if the relative frequency \( P_{HI} \) or the total number of agricultural households increases; (2) \( D_{HIF} \) is uncertain, but \( S_{HIF} \) decreases, if the relative frequency \( P_{HI} \) increases; (3) \( D_{HIF} \) increases, but \( S_{HIF} \) decreases, if the relative frequency \( P_{HII} \) increases; and (4) \( D_{HIF} \) decreases, but \( S_{HIF} \) increases, if the state market price \( P_{JS} \), or the free market price \( P_{JF} \), or the sales quote \( Q_{JS} \) increases.

2. Partial buyers

Using formulas (4.205) and (4.213) we have

\[
\frac{\partial D_{HII}}{\partial P_{HII}} = \frac{\partial D_{HIF2}}{\partial P_{HII}} + \frac{\partial D_{HIF3}}{\partial P_{HII}} > 0 \quad \frac{\partial S_{HIF}}{\partial P_{HII}} = \frac{\partial S_{HIF3}}{\partial P_{HII}} > 0
\]

\[
\frac{\partial D_{HII}}{\partial P_{HII1}} = \frac{\partial D_{HIF3}}{\partial P_{HII1}} < 0 \quad \frac{\partial S_{HIF}}{\partial P_{HII1}} = \frac{\partial S_{HIF3}}{\partial P_{HII1}} < 0
\]

\[
\frac{\partial D_{HII}}{\partial P_{HII2}} = \frac{\partial D_{HIF2}}{\partial P_{HII2}} + \frac{\partial D_{HIF3}}{\partial P_{HII2}} > 0 \quad \frac{\partial S_{HIF}}{\partial P_{HII2}} = \frac{\partial S_{HIF3}}{\partial P_{HII2}} < 0
\]

\[
\frac{\partial D_{HII}}{\partial P_{HII31}} = \frac{\partial D_{HIF3}}{\partial P_{HII31}} < 0 \quad \frac{\partial S_{HIF}}{\partial P_{HII31}} = \frac{\partial S_{HIF3}}{\partial P_{HII31}} < 0
\]

\[
\frac{\partial D_{HII}}{\partial n_H} = \frac{\partial D_{HIF2}}{\partial n_H} + \frac{\partial D_{HIF3}}{\partial n_H} > 0 \quad \frac{\partial S_{HIF}}{\partial n_H} = \frac{\partial S_{HIF3}}{\partial n_H} > 0
\]

\[
\frac{\partial D_{HII}}{\partial P_{JS}} = \frac{\partial D_{HIF2}}{\partial P_{JS}} + \frac{\partial D_{HIF3}}{\partial P_{JS}} < 0 \quad \frac{\partial S_{HIF}}{\partial P_{JS}} = \frac{\partial S_{HIF3}}{\partial P_{JS}} > 0
\]

\[
\frac{\partial D_{HII}}{\partial P_{JF}} = \frac{\partial D_{HIF2}}{\partial P_{JF}} + \frac{\partial D_{HIF3}}{\partial P_{JF}} < 0 \quad \frac{\partial S_{HIF}}{\partial P_{JF}} = \frac{\partial S_{HIF3}}{\partial P_{JF}} > 0
\]
The above results show that, for the second type of agricultural households in the free market, (1) both the aggregate grain demand, $D_{HIIF}$, and the aggregate grain supply, $S_{HIIF}$, increase when the relative frequency $P_{HI}$, or the total number of agricultural households increases; (2) both $D_{HIIF}$ and $S_{HIIF}$ decrease when the relative frequency $P_{HI}$, or $P_{HI31}$, increases; (3) $D_{HIIF}$ is uncertain, but $S_{HIIF}$ decreases, when the relative frequency $P_{HI}$ decreases; and (4) $D_{HIIF}$ decreases, but $S_{HIIF}$ increases, when the state market price $P_{JS}$, or the free market price $P_{jP}$, or the ration, or the sales quota increases.

3. Complete buyers

From formulas (4.207) and (4.213) we have

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]

\[
\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF2}}{\partial P_{HI}} < 0
\]
These results illustrate that, for the third type of agricultural households in the free market, the aggregate grain demand, $D_{HIIF}$, declines if the relative frequency $P_{HI}^\prime$ or $P_{HII}^\prime$ or $P_{HIII}^\prime$, or the total number of agricultural households, or the state market price, or the free market price, or the state ration rises.

Lastly, using formulas (4.209) and (4.217) we have

$$\frac{\partial D_{HIIF}}{\partial P_{HI}} = \frac{\partial D_{HIIF}}{\partial P_{HI}} + \frac{\partial D_{HIIF}}{\partial P_{HI}} + \frac{\partial D_{HIIF}}{\partial P_{HI}} > 0$$

$$\frac{\partial D_{HIIF}}{\partial P_{HII}} = \frac{\partial D_{HIIF}}{\partial P_{HII}} + \frac{\partial D_{HIIF}}{\partial P_{HII}} + \frac{\partial D_{HIIF}}{\partial P_{HII}} < 0$$

$$\frac{\partial D_{HIIF}}{\partial P_{HIIL}} = \frac{\partial D_{HIIF}}{\partial P_{HIIL}} < 0$$

$$\frac{\partial D_{HIIF}}{\partial P_{HIIL}} = \frac{\partial D_{HIIF}}{\partial P_{HIIL}} > 0$$

$$\frac{\partial D_{HIIF}}{\partial P_{HIIL}} = \frac{\partial D_{HIIF}}{\partial P_{HIIL}} < 0$$

$$\frac{\partial D_{HIIF}}{\partial P_{HIIL}} = \frac{\partial D_{HIIF}}{\partial P_{HIIL}} < 0$$
\[
\frac{\partial D_{HF}}{\partial p_{HII1}} = \frac{\partial D_{HIIF}}{\partial p_{HII1}} < 0
\]
\[
\frac{\partial D_{HF}}{\partial q_{HI}} = \frac{\partial D_{HIIF}}{\partial q_{HI}} + \frac{\partial D_{HIIIF}}{\partial q_{HI}} > 0
\]
\[
\frac{\partial D_{HF}}{\partial p_{JF}} = \frac{\partial D_{HIIF}}{\partial p_{JF}} + \frac{\partial D_{HIIIF}}{\partial p_{JF}} < 0
\]
\[
\frac{\partial D_{HF}}{\partial q_{JS}} = \frac{\partial D_{HIIF}}{\partial q_{JS}} < 0
\]
\[
\frac{\partial D_{HF}}{\partial p_{HI1}} = \frac{\partial D_{HIIF}}{\partial p_{HI1}} > 0
\]
\[
\frac{\partial D_{HF}}{\partial p_{HII1}} = \frac{\partial D_{HIIF}}{\partial p_{HII1}} < 0
\]
\[
\frac{\partial D_{HF}}{\partial p_{HI13}} = \frac{\partial D_{HIIF}}{\partial p_{HI13}} > 0
\]
\[
\frac{\partial D_{HF}}{\partial q_{HI13}} = \frac{\partial D_{HIIF}}{\partial q_{HI13}} < 0
\]
These results tell us that, for agricultural households in the free market, (1) the aggregate grain demand is uncertain, but the aggregate grain supply decreases if the relative frequency, \( P_{H1} \) or \( P_{HII} \), increases; (2) \( D^{HF} \) decreases, but \( S^{HF} \) increases, if the relative frequency \( P_{H1} \) or \( P_{HII3} \), the state market price \( P_{JS} \), or the free market price \( P_{JF} \), or the state ration or the sales quota increases; (3) \( D^{HF} \) increases, but \( S^{HF} \) decreases, if the relative frequency \( P_{H1} \) or \( P_{H} \), increases; (4) both \( D^{HF} \) and \( S^{HF} \) decrease if the relative frequency, \( P_{HIII} \) or \( P_{HII2} \), increases; or (5) both \( D^{HF} \) and \( S^{HF} \) increase if the total number of
agricultural households increases.

Graphic analysis To convert a graph in the united market, \((Q^u, P^u)\) subspace into a graph in the free market, \((Q^F, P^F)\) subspace, we only need to draw an excess demand or an excess supply from the \((Q^u, P^u)\) subspace as a demand or a supply in the \((Q^F, P^F)\) subspace. By using this procedure, it is easy to give a graphic analysis for all three types of agricultural households in this part.

1. Nonbuyers

There are two subtypes of nonbuyers in the free market. For the first subtype, the aggregate grain demand \(D^{HIF1}\) is a downward-sloping line, and aggregate grain supply \(S^{HIF1}\) does not exist in the \((Q^F, P^F)\) subspace (Figure 4.17a).

- \(D^{HIF1}\)

\[ Q^F \]
\[ P^F \]
\[ P^F \]
\[ P^F \]

On the other hand, Figure 4.17b shows that, for the second subtype, aggregate grain demand is downward-sloping and aggregate grain supply is upward-sloping in the \((Q^F, P^F)\) subspace. Adding both Figure 4.17a and
Figure 4.17b, we obtain Figure 4.17c, which shows a downward-sloping aggregate grain demand $D^HIF$ and an upward-sloping aggregate grain supply $S^HIF$.

2. Partial buyers

There are three subtypes of partial buyers in the free market. For the first subtype, the aggregate grain demand is zero; but the demands are downward-sloping in the $(Q^F, P^F)$ subspace for both the second and third subtypes. On the other hand, for both the first and the second subtype, the aggregate grain supply does not exist; but it is upward-sloping in the $(Q^F, P^F)$ subspace for the third subtype (Figure 4.18a, b). Figure 4.18c shows the situation for the second type of agricultural household in the free market.

![Diagram](image)

(a) (b) (c)

Figure 4.18. Aggregation of grain demands and supplies of the second type of agricultural households in the free market

3. Complete buyers

There are two subtypes of complete buyers in the free market. For the first subtype, both grain demand and supply are zero. For the second
subtype, grain demand is downward-sloping in the \((Q^F, P^F)\) subspace, and grain supply is zero (Figure 4.19). Thus, aggregate grain demand and supply of complete buyers are similar to that shown in Figure 4.19.

![Diagram](image)

**Figure 4.19.** Aggregate grain demand and supply of the third type of agricultural households in the free market

Lastly, adding Figure 4.17c, Figure 4.18c and Figure 4.19 together, we obtain Figure 4.20, which shows the normal demand \(D^{HF}\) and supply \(S^{HF}\) in the \((Q^F, P^F)\) subspace.

![Diagram](image)

**Figure 4.20.** Aggregate grain demand and supply of all agricultural households in the free market
Theory of International Grain Trade
in the Mixed Economy

In the Chinese mixed economy, the state government has applied an organizational reform to decentralize the state trading system. Without changing the feature of the state monopoly in foreign trade management, the government still has significant power to use trade to achieve its goals.

The discussion of international grain trade in the Chinese mixed economy consists of four major parts: policy environment, the Ministry of Foreign Trade (MOFT) model, aggregate grain import demand or export supply, and interactions of plans and markets. Under the discussion of policy environment, the changed decision environment faced by the MOFT in the mixed economy is provided so that theoretical work can be further developed. Later, individual grain import demands and/or export supplies are derived from the MOFT model in which the MOFT is assumed to maximize its utility subject to its constraints due to planning control. The aggregate grain import demand or export supply is summed from all individual grain import demands or export supplies. Lastly, the plan's impacts on both the domestic grain market(s) (including the state market and the free market) and the international grain market are inspected and evaluated.

Policy environment

State monopoly and decentralization of foreign trade Since the economic reform, the Chinese government has decentralized foreign trade
management in two directions: horizontally to several ministries other than the MOFT and vertically to local governments. This is an attempt to provide flexibility and improve efficiency for the MOFT's planning.

At the ministerial level, several new export and import corporations (FTCs) have been established under the guidance of some production ministries to conduct foreign trade directly. Among these corporations, China National Seeds Import and Export Corporation (CNSIEC) is the only one relevant to the foreign grain trade. At the local level, the governments of several provinces and cities have been granted authority to organize local corporations to direct the operations of local FTC branches. These branches still need to report their plan and budget matters to national FTCs and, hence, the MOFT.

Although there is an organizational reform in the state trading system, individuals still are not allowed to do an import-export business in the Chinese mixed economy. The bulk of China's foreign trade, especially foreign grain trade, still is mainly carried out by FTCs and, hence, the MOFT.

Objective of foreign trade With a more flexible attitude, the Chinese government still thinks that foreign trade can play a very important role in China's industrialization. Thus, the main task of foreign trade in the national economy has not changed fundamentally.

The government tries, to the maximum feasible extent, to export available goods to earn valuable foreign currency to ensure its import capacity for industrial plants and equipment. On the other hand, it tries to encourage China's own production of agricultural and light-
industrial commodities to satisfy basic needs in the domestic markets and needs in the international markets if possible.

In the international market, the MOFT may still behave as a specific consumer with a utility function within imported commodity space subject to constraints due to planning controls. The assumed objective for the MOFT still is to maximize its utility from purchasing commodities in the international markets.

**Plan control** The MOFT still makes its trade plan which illustrates its revenues and expenditures, imposes quotas for imports and exports, and target quantities of goods to be exported and imported.

The MOFT still is concerned with its external balance, which is its budget in the foreign exchange. In addition, although its loss can still be covered by the State Treasurer, it now pays considerable attention to its internal balance, which is its budget in the domestic currency. The reason the MOFT does this is that the State Council sets the profit target in the domestic currency to the MOFT and will reward the income bonus as a money incentive to the staff members of the MOFT if the MOFT can achieve at least or above the target.

The remaining concern for the MOFT, such as the planned balance (rather than real balance) of domestic demand and supply in the state market (rather than the combined market), has not changed from that under the planned economy.

**Market operation** Because this section concentrates on the foreign grain trade in the mixed economy only, we simply assume that the MOFT is a price taker in all international markets except the
international grain markets. The behavior of the MOFT in the foreign grain markets in the mixed economy is the same as that in the planned economy. In detail, the MOFT acts as a price taker when it purchases grain from the foreign markets and acts as a price follower when it sells grain to the foreign markets.

Other bilateral trade agreements In the mixed economy, the MOFT has several bilateral agreements with other countries. In general, the contents of the agreements may be very similar to those carried out in the planned economy.

Model of the Ministry of Foreign Trade

In the Chinese mixed economy, the MOFT still is assumed to maximize its utility in the import commodity space, subject to given constraints. Suppose the MOFT continuously has a utility function \( U(X_{11}, ..., X_{j1}, X_{k1}, ..., X_{n1}) \) where \( X_{i1} \) \((i = 1, ..., n)\) is the \( i^{th} \) imported good. Furthermore, to guarantee utility maximization, we still suppose \( U(X_{i1}) \) has all the proper properties such as completeness, reflexivity, transivity, continuity, nonsatiation, and strict convexity. The new Lagrangian function is formulated as follows:

\[
L = U(X_{11}, ..., X_{j1}, X_{k1}, ..., X_{n1}) + \lambda_1 \left( \sum_{i=1}^{n} p_{1i} X_{iE} - \kappa FR + \eta FL - \sum_{i=1}^{n} p_{1i} X_{iI} \right) + \lambda_2 \left[ \sum_{i=1}^{n} (e p_{1i} - p_{1i}^S) X_{iE} - \sum_{i=1}^{n} (e p_{1i} - p_{1i}^S) X_{iI} - \pi \right] + \lambda_3 \left[ \sum_{i=1}^{j} (X_{iE} - X_{iI}) - (S_{PS}^G - D_{PS}^G) \right]
\]
\[ + \sum_{i=1}^{n} \lambda_{4i}[(x_{iE} - x_{iI}) - (s_{i}^{PS} - d_{i}^{PS})] \]
\[ + \sum_{i=1}^{n} \lambda_{5i}(x_{iI} - x_{iI}) \]

(4.218)

where

- \( x_{iI} \) = the \( i \)th imported commodity
- \( x_{iI}^{-} \) = the import quota for the \( i \)th commodity, in case there is no desire to import, \( x_{iI}^{-} = 0 \)
- \( x_{iE}^{-} \) = the export quota for the \( i \)th commodity, in case there is no potential to export, \( x_{iE}^{-} = 0 \)
- \( p_{i}^{I} \) = international price for the \( i \)th commodity
- \( p_{i}^{S} \) = domestic (the state market) price for the \( i \)th commodity
- \( e \) = the exchange rate in Yuan/$
- \( \bar{\pi} \) = the internal profit target for the MOFT
- \( FR \) = the foreign currency reserves
- \( FL \) = the foreign loans
- \( S_{G}^{PS} \) = planned domestic grain supply in the state market
- \( D_{G}^{PS} \) = planned domestic grain demand in the state market
- \( S_{i}^{PS} \) = planned domestic supply of \( i \)th commodity in the state market
- \( D_{i}^{PS} \) = planned domestic demand of \( i \)th commodity in the state market

Yuan (1 Yuan = 100 Fen) is the basic unit of the Chinese currency -- RenMingBe (RMB), according to the current exchange rate, 1 Yuan = $0.268 or $1 = 3.73 Yuan.
\( \kappa, \eta \) - fixed proportional coefficients  \( 0 \leq \kappa \leq 1, \ 0 \leq \eta \leq 1 \)

The constraints in the Lagrangian function can be classified into four categories. Three of these four categories are exactly the same as those listed in the MOFT model in the planned economy, and are not explained in detail in this section. The remaining one is the newly added internal balance in the current MOFT model. Now we introduce these four categories as follows:

1) External balance. \( \sum_{i=1}^{n} P_{i}^{I} X_{iE} + \kappa FR + \eta FL - \sum_{i=1}^{n} P_{i}^{I} X_{iI} \) is an external balance constraint. It requires that total expenditures for importing goods must be covered by total earnings from exporting goods plus other possible sources of foreign currency.

2) Internal balance. \( \sum_{i=1}^{n} (e P_{i}^{I} - P_{i}^{S}) X_{iE} - \sum_{i=1}^{n} (e P_{i}^{I} - P_{i}^{S}) X_{iI} - \pi \) is an internal balance constraint. It says that \( \sum_{i=1}^{n} (e P_{i}^{I} - P_{i}^{S}) X_{iE} \) total earnings converted to domestic currency from importing goods, minus total expenditures converted to domestic currency for importing goods, \( \sum_{i=1}^{n} (e P_{i}^{I} - P_{i}^{S}) X_{iI} \), must reach at least \( \pi \), the profit target set by the state government.

3) Balance of domestic demand and supply. \( (X_{iE}^{I} - X_{iI}^{I}) - (S_{i}^{PS} - D_{i}^{PS}) = 0 \) is a planned balance constraint for the domestic demand and supply of the \( i^{th} \) commodity. It means that, for the \( i^{th} \) commodity, the planned domestic surplus or shortage in the state market must be disposed of or
be covered by using the international market. In particular, $\sum_{i=1}^{n} (X_{iE} - X_{ii}) - (S_{G}^{PS} - D_{G}^{PS})$ is designed for the planned balance of domestic demand and supply for a composite good—grain.

4) Import quota. $X_{ii} - \bar{X}_{ii}$ is the import quota constraint imposed for some commodities. That is, import of the $i^{th}$ commodity must not exceed the import quota $\bar{X}_{ii}$.

The Kuhn-Tucker conditions are then:

$$\frac{\partial L}{\partial X_{iI}} = \frac{\partial U}{\partial X_{iI}} - \lambda_{1} P_{iI} - \lambda_{2} (e_{P}^{I} - P_{s}^{I}) + \lambda_{3} + \lambda_{5i} \leq 0$$

for $X_{ii} \neq 0$, $i = 1, \ldots, j$ (4.219)

$$\frac{\partial L}{\partial X_{iI}} = \frac{\partial U}{\partial X_{iI}} - \lambda_{1} P_{iI} - \lambda_{2} (e_{P}^{I} - P_{s}^{I}) + \lambda_{3} \leq 0$$

for $X_{ii} = 0$, $i = 1, \ldots, j$ (4.220)

$$\frac{\partial L}{\partial X_{iI}} = \frac{\partial U}{\partial X_{iI}} - \lambda_{1} P_{iI} - \lambda_{2} (e_{P}^{I} - P_{s}^{I}) + \lambda_{4i} + \lambda_{5i} \leq 0$$

for $X_{ii} \neq 0$, $i = k, \ldots, n$ (4.221)

$$\frac{\partial L}{\partial X_{iI}} = \frac{\partial U}{\partial X_{iI}} - \lambda_{1} P_{iI} - \lambda_{2} (e_{P}^{I} - P_{s}^{I}) + \lambda_{4i} \leq 0$$

for $X_{ii} = 0$, $i = k, \ldots, n$ (4.222)

$$X_{ii} \frac{\partial L}{\partial X_{iI}} = 0$$

for $i = 1, \ldots, n$ (4.223)

$$\frac{\partial L}{\partial \lambda_{1}} = \sum_{i=1}^{n} P_{i} X_{iE} + \kappa FR + \eta FL - \sum_{i=1}^{n} P_{i} X_{ii} \geq 0$$

(4.224)

$$\lambda_{1} \frac{\partial L}{\partial \lambda_{1}} = 0$$

(4.225)
\[
\frac{\partial L}{\partial \lambda_2} = \sum_{i=1}^{n} (e \bar{P}_i - P_i^S) \bar{X}_{1i} - \sum_{i=1}^{n} (e \bar{P}_i - P_i^S) \bar{X}_{1i} = 0 \quad (4.226)
\]

\[
\lambda_2 \frac{\partial L}{\partial \lambda_2} = 0 \quad (4.227)
\]

\[
\frac{\partial L}{\partial \lambda_3} = \sum_{i=1}^{n} (\bar{X}_{1i} - \bar{X}_{1i}) - (S_{G}^{PS} - D_{G}^{PS}) \geq 0 \quad (4.228)
\]

\[
\lambda_3 \frac{\partial L}{\partial \lambda_3} = 0 \quad (4.229)
\]

\[
\frac{\partial L}{\partial \lambda_{4i}} = (\bar{X}_{1i} - \bar{X}_{1i}) - (S_{i}^{PS} - D_{i}^{PS}) \geq 0 \quad \text{for } i = k, \ldots, n \quad (4.230)
\]

\[
\lambda_{4i} \frac{\partial L}{\partial \lambda_{4i}} = 0 \quad i = k, \ldots, n \quad (4.231)
\]

\[
\frac{\partial L}{\partial \lambda_{5i}} = (\bar{X}_{1i} - \bar{X}_{1i}) \geq 0 \quad \text{for } \bar{X}_{1i} \neq 0, i = 1, \ldots, n \quad (4.232)
\]

\[
\lambda_{5i} \frac{\partial L}{\partial \lambda_{5i}} = 0 \quad \text{for } \bar{X}_{1i} \neq 0, i = 1, \ldots, n \quad (4.233)
\]

\[
\lambda_1, \lambda_2, \lambda_3, \lambda_5 \geq 0 \quad \text{for } i = 1, \ldots, n \quad (4.234)
\]

\[
\lambda_{4i} \geq 0 \quad i = k, \ldots, n \quad (4.235)
\]

Suppose the MOFT maintains both its external balance and internal balance,

\[
\frac{\partial L}{\partial \lambda_1} = \sum_{i=1}^{n} (e \bar{P}_i - P_i^S) \bar{X}_{1i} + \eta_{FR} + \eta_{FL} - \sum_{i=1}^{n} (e \bar{P}_i - P_i^S) \bar{X}_{1i} = 0 \quad (4.236)
\]

\[
\frac{\partial L}{\partial \lambda_2} = \sum_{i=1}^{n} (e \bar{P}_i - P_i^S) \bar{X}_{1i} - \sum_{i=1}^{n} (e \bar{P}_i - P_i^S) \bar{X}_{1i} = 0 \quad (4.237)
\]

then, (4.224) through (4.227) can be dropped out.

Similarly, suppose the MOFT maintains grain as well as all other...
commodities at the planned balance level, we then have

\[ \frac{\partial L}{\partial \lambda_3} = \sum_{i=1}^j (x_{iE} - x_{iI}) - (S^{PS} - D^{PS}) = 0 \] (4.238)

\[ \frac{\partial L}{\partial \lambda_{4i}} = (x_{iE} - x_{iI}) - (S^{PS}_i - D^{PS}_i) = 0 \quad i = k, \ldots, n \] (4.239)

These results lead to omitting (4.228) through (4.231) from our model.

Furthermore, suppose the second-order conditions are satisfied; solving the first-order conditions, we find three possible ways for \( x_{iI} \):

1. \( x_{iI} = 0 \) (4.240)

2. \( x_{iI} = x_{iI}(P^I, P^S, E, FR, FL, \pi, x_E, x_I, S^{PS}_G - D^{PS}_G, S^{PS} - D^{PS}) \) (4.241)

3. \( x_{iI} = \bar{x}_{iI} \) (4.242)

Apparently, the first formula (4.240) implies a corner solution, that is, the MOFT does not import any amount of good \( i \). The second formula (4.241) illustrates that import demand for good \( i \) is a function of international commodity prices, domestic commodity prices (in the state market), the exchange rate, the foreign reserves in the country, the foreign loans borrowed by the government, the profit target for the MOFT, the export quotas, the import quotas, the planned balance of domestic grain supply and demand in the state market, and the planned balances of domestic supply and demand of other commodities in the state market. The last formula (4.242) indicates that imported good \( i \) is just equal to its quota.

Once \( x_{iI} \) is determined, the corresponding quantities of either net export supply \( x_{iNE} \) or net import demand \( x_{iNI} \) are immediately determined.
Comparing our new model (4.218) in the mixed economy with the previous model (3.163) in the planned economy, we find out that the only difference is the constraint of internal balance. Furthermore, comparing the results of (4.240) through (4.242) of our new model with the corresponding results (3.182) through (3.184), we find out that the only difference is that formula (4.241) for \( X_{1i} \) contains three more factors: the domestic state market prices \( P^S \), the exchange rate \( e \), and the profit target \( \pi \) for the MOFT. Because we use a similar method to solve a similar new model in this section, we can easily use the results provided in the previous model with (4.241) to replace corresponding (3.183). Thus, except for these different formulas, we only list all results of our new model without detailed explanations.

1. \( X_{1i} = 0 \)
   a. \( X_{1E} = 0 \) \hspace{1cm} (4.243)
   This case is dropped out because it is meaningless.
   b. \( X_{1E} > 0 \)

   \( X_{1NE} = \bar{X}_{1E} \) \hspace{1cm} (4.244)
   The MOFT is a command exporter for good \( i \).

2. \( X_{1I} = X_{1I}(P^I, P^S, e, FR, FL, \bar{\pi}, \bar{X}_E, \bar{X}_I, S^G_G - D^G, S^P - D^P) \)
   a. \( \bar{X}_{1E} = 0 \)

   \( X_{1NI} = X_{1I}(P^I, P^S, e, FR, FL, \bar{\pi}, \bar{X}_E, \bar{X}_I, S^G_G - D^G, S^P - D^P) \) (4.245)

   Net import demand for good \( i \) is a function of the international price vector, the domestic state market price vector, the exchange rate, foreign reserves, foreign loans, the profit target set for the MOFT.
export quotas, import quotas, the planned balance of domestic grain supply and demand in the state market, and the planned balance of domestic supply and demand of other commodities in the state market.

Thus, the MOFT is a flexible importer for good 1.

b. $\bar{X}_{1E} > 0$

1) if $\bar{X}_{1E} > \bar{X}_{1I}$

$$X_{1NE} = \bar{X}_{1E} - \bar{X}_{1I}(P^I, P^S, e, FR, FL, \bar{\pi}, \bar{X}_E, \bar{X}_I, S^G_{PS-D^PS}, S^S_{PS-D^PS})$$

$$= X_{1NE}(P^I, P^S, e, FR, FL, \bar{\pi}, \bar{X}_E, \bar{X}_I, S^G_{PS-D^PS}, S^S_{PS-D^PS})(4.246)$$

2) if $\bar{X}_{1E} < \bar{X}_{1I}$

$$X_{1NI} = \bar{X}_{1I} - \bar{X}_{1E}$$

$$= X_{1I}(P^I, P^S, e, FR, FL, \bar{\pi}, \bar{X}_E, \bar{X}_I, S^G_{PS-D^PS}, S^S_{PS-D^PS}) - \bar{X}_E$$

$$= X_{1NI}(P^I, P^S, e, FR, FL, \bar{\pi}, \bar{X}_E, \bar{X}_I, S^G_{PS-D^PS}, S^S_{PS-D^PS})(4.247)$$

These results indicate that either net export supply or net import demand for good 1 is a function of international prices, the domestic state market prices, the exchange rate, foreign reserves, foreign loans, the profit target for the MOFT, export quotas, import quotas, the planned balance of domestic grain supply and demand in the state market, and the planned balance of domestic supply and demand for other commodities in the state market. The MOFT is either a restricted flexible exporter or restricted flexible importer for goods in these cases.

3) if $\bar{X}_{1E} = \bar{X}_{1I}$

$$X_{1NI} = X_{1NE} = 0 \quad (4.248)$$

We simply drop this case out.
The MOFT is a command importer in this case.

The MOFT is a command exporter for good i.

The MOFT is a dual command importer.

The MOFT is a dual command exporter.

We simply neglect this case.

All of these results reflect that the MOFT tries to decide all net imports and net exports for all goods together, although it may pay prior attention to export quotas. The MOFT now is concerned with the international prices as well as the domestic state market prices.

Comparative statics From the above discussion, we can classify MOFT's importing or exporting behaviors. We show comparative statics of only the newly added factors $P^S$, $e$, and $\pi$ for only flexible importer and restricted flexible exporter or importer. For the remaining factors within the previous three functions and the remaining functions, the
readers can refer to the comparative statics for the corresponding part in Chapter III because of similarity.

1. Importer
   a. dual command importer
      \[ X_{INI1} = \bar{X}_{II} - \bar{X}_{IE} \]
   b. command importer
      \[ X_{INI2} = \bar{X}_{II} \]
   c. flexible importer
      \[ X_{INI3} = \bar{X}_{II} (P^I, P^S, e, FR, FL, \bar{\pi}, \bar{X}_E, \bar{X}_I, s_{PS}^G, d_{PS}^G, s_{PS-DPS}^G) \]

\[ \frac{\partial X_{INI3}}{\partial P^I} = \frac{\partial X_{II}}{\partial P^I} < 0 \]
\[ \frac{\partial X_{INI3}}{\partial P^S} = \frac{\partial X_{II}}{\partial P^S} < 0 \]
\[ \frac{\partial X_{INI3}}{\partial e} = \frac{\partial X_{II}}{\partial e} < 0 \]
\[ \frac{\partial X_{INI3}}{\partial \bar{\pi}} = \frac{\partial X_{II}}{\partial \bar{\pi}} > 0 \]

Net import demand for the ith good decreases if the exchange rate in yuan/dollar, the international price, or the state market price increases, and it is uncertain if the profit target is raised.

Moreover, we have the following results which are similar to those in Chapter III.

\[ \frac{\partial X_{INI3}}{\partial (s_{PS}^G - d_{PS}^G)} > 0 \]
but

$$\frac{\partial X_{GNI}}{\partial (S_G - D_S)} < 0$$

d. restricted flexible importer

$$X_{INI4} = X_{II}(P^I, P^S, e, FR, FL, \bar{\pi}, X_E, \bar{X}_I, S_G^{PS}, D_G^{PS}, S_S^{PS}) - X_{IE}$$

The partial derivatives for $X_{INI4}$ are exactly the same as those for $X_{INI3}$ and are omitted here.

2. Exporter

a. dual command exporter

$$X_{INE1} = \bar{X}_I - X_{II}$$

b. command exporter

$$X_{INE2} = \bar{X}_{IE}$$

c. restricted flexible exporter

$$X_{INE3} = \bar{X}_{IE} - X_{II} (P^I, P^S, e, FR, FL, \bar{\pi}, X_E, \bar{X}_I, S_G^{PS}, D_G^{PS}, S_S^{PS})$$

The partial derivatives for $X_{INE3}$ are the same as those for $X_{INI3}$ but with opposite signs.

Graphic analysis The graphic analysis also is the same as that in the similar part in Chapter III and, hence, is omitted in this section.

Aggregate import grain demand and export grain supply

Within the first $j$ grain commodities, suppose that the first $e$ are net importing grain commodities and the remaining $j-e$ are net exporting grain commodities. In the mixed economy, the aggregate import grain
demand $D^{MM}$ is then

$$D^{MM} = \sum_{i=1}^{4} X^{INi1} + \sum_{i=1}^{4} X^{INi2} + \sum_{i=1}^{4} X^{INi3} + \sum_{i=1}^{4} X^{INi4}$$

\[ (4.254) \]

\[ = \sum_{i=1}^{4} (\bar{X}^{INi1} - \bar{X}^{INi1}) + \sum_{i=1}^{4} \bar{X}^{INi2} + \sum_{i=1}^{4} X^{INi3} (P^{I}, P^{S}, e, FR, FL, \bar{\pi}, \bar{X}_{E}, \bar{X}_{I}, S_{G}^{P_{PS} - D_{PS}}, S_{PS}^{P_{PS} - D_{PS}}) + \sum_{i=1}^{4} [X^{INi4}(P^{I}, P^{S}, e, FR, FL, \bar{\pi}, \bar{X}_{E}, \bar{X}_{I}, S_{G}^{P_{PS} - D_{PS}}, S_{PS}^{P_{PS} - D_{PS}} - \bar{X}^{INi4})] \]

\[ = D^{MM} (P^{I}, P^{S}, e, FR, FL, \bar{\pi}, \bar{X}_{E}, \bar{X}_{I}, S_{G}^{P_{PS} - D_{PS}}, S_{PS}^{P_{PS} - D_{PS}}) \]  \[ (4.255) \]

That is, as long as the MOFT behaves as a flexible or a restricted flexible importer for at least one grain commodity, the aggregate import grain demand is a function of international prices of commodities, the domestic state market prices of commodities, the exchange rate, the foreign reserves, foreign loans, the profit target for the MOFT, export quotas, import quotas, the planned balance of domestic grain supply and demand in the state market, and the planned balances of domestic supplies and demands of other commodities in the state market.

Similarly, we can obtain $S^{XM}$, the aggregate export grain supply in the mixed economy as the following:

$$S^{XM} = \sum_{j=1}^{3} X^{INE1} + \sum_{j=1}^{3} X^{INE2} + \sum_{j=1}^{3} X^{INE3}$$

\[ (4.256) \]

\[ = \sum_{j=1}^{3} (\bar{X}^{INE1} - \bar{X}^{INE1}) + \sum_{j=1}^{3} \bar{X}^{INE2} + \sum_{j=1}^{3} X^{INE3} (P^{I}, P^{S}, e, FR, FL, \bar{\pi}, \bar{X}_{E}, \bar{X}_{I}, S_{G}^{P_{PS} - D_{PS}}, S_{PS}^{P_{PS} - D_{PS}}) \]

\[ + \sum_{j=1}^{3} [X^{INE3}(P^{I}, P^{S}, e, FR, FL, \bar{\pi}, \bar{X}_{E}, \bar{X}_{I}, S_{G}^{P_{PS} - D_{PS}}, S_{PS}^{P_{PS} - D_{PS}})] \]
As long as the MOFT behaves as a restricted flexible exporter for at least one grain commodity, the aggregate export grain supply is similar to the aggregate grain import demand as a function of international prices of commodities, the domestic state market prices of commodities, the exchange rate, the foreign currency reserves, the foreign loans, the profit target for the MOFT, the export quotas, the import quotas, the planned balance of domestic grain supply and demand in the state market, and the planned balances of domestic supplies and demands of other goods in the state market.

Three important things need to be noted here. First, both the aggregate import grain demand and the aggregate export grain supply of the MOFT do contain both the international price vector, including international grain prices, and the domestic state market price vectors, including the state grain market prices. This reflects the fact that the MOFT's behavior is affected not only by international prices but also by the domestic state market prices. Thus, there is a certain connection between prices in the state grain market and prices in the international grain market in the mixed economy.

Second, both $D^m$ and $S^m$ contain the exchange rate. The exchange rate plays an important role in the mixed economy to adjust the import grain demand and export grain supply. Thus, the government can use the exchange rate as a new policy instrument to affect the grain economy.

Third, whether China becomes a net importing grain country or a net
exporting country still depends on the difference between $D^{MM}$ and $S^{XM}$.

In case $D^{MM} > S^{XM}$, the MOFT is a net grain importer; otherwise, it is a net grain exporter. In today's world, both domestic and foreign agricultural economists who specialize in Chinese agricultural development recognize that a shortage in the domestic grain market will persist in the long run in the Chinese economy. If this is true, we obtain the net grain import demand $D^{NMM}$ as follows:

$$D^{NMM} = D^{MM} - S^{XM}$$

(4.258)

It is certain that $D^{NMM}$ is a function that contains all the factors $D^{MM}$ and/or $S^{XM}$ have/has.

Comparative statics The comparative statics for both $D^{MM}$ and $S^{XM}$ are the same as those for $D^{MP}$ and $S^{XP}$ in the corresponding part of Chapter III except for those relationships that include the domestic state market prices, the exchange rate, and the profit target for the MOFT. Thus, we mainly consider only those factors in the comparative statics in this subpart of Chapter IV.

1. Aggregate import grain demand

$$\frac{\partial D^{IMM}}{\partial P_I} = \Sigma \frac{\partial x^{INI3}_1}{\partial P_I} + \Sigma \frac{\partial x^{INI4}_1}{\partial P_I} > 0$$

$$\frac{\partial D^{IMM}}{\partial P_S} = \Sigma \frac{\partial x^{INI3}_1}{\partial P_S} + \Sigma \frac{\partial x^{INI4}_1}{\partial P_S} < 0$$

$$\frac{\partial D^{IMM}}{\partial P_G} = \Sigma \frac{\partial x^{INI3}_1}{\partial P_G} + \Sigma \frac{\partial x^{INI4}_1}{\partial P_G} < 0$$
\[
\frac{\partial D^M}{\partial P^S} = \frac{\partial x^{INI3}}{1-e} + \frac{\partial x^{INI4}}{1-e} < 0
\]

\[
\frac{\partial D^M}{\partial e} = \frac{\partial x^{INI3}}{1-e} + \frac{\partial x^{INI4}}{1-e} > 0
\]

\[
\frac{\partial D^M}{\partial \pi} = \frac{\partial x^{INI3}}{1-e} + \frac{\partial x^{INI4}}{1-e} > 0
\]

\[
\frac{\partial D^M}{\partial (S^P_G - D^P_G)} = \frac{\partial x^{INI3}}{1-e} \frac{\partial (S^P_G - D^P_G)}{1-e} < 0
\]

The above results indicate that the aggregate import grain demand
(1) is uncertain if the state market price, the international market
price of a grain commodity, the exchange rate in yuan/dollar, or the
profit target for the MOFT increases; and (2) decreases if the average
state grain market price or the average international grain price in­
creases.

2. Aggregate export grain supply

\[
\frac{\partial S^M}{\partial P^I} = \frac{\partial x^{INE3}}{f-j} \frac{\partial P^I}{\partial P^I} > 0
\]

\[
\frac{\partial S^M}{\partial P^S} = \frac{\partial x^{INE3}}{f-j} \frac{\partial P^S}{\partial P^S} < 0
\]

\[
\frac{\partial S^M}{\partial P^G} = \frac{\partial x^{INE3}}{f-j} \frac{\partial P^G}{\partial P^G} < 0
\]

\[
\frac{\partial S^M}{\partial S^P_G} = \frac{\partial x^{INE3}}{f-j} \frac{\partial S^P_G}{\partial P^G} < 0
\]
The above results illustrate that (1) the aggregate export grain supply is uncertain if the state market price, the international market price of a grain commodity, the exchange rate, or the profit target for the MOFT increases; (2) the aggregate export grain supply increases if the average state grain market price increases; and (3) aggregate export grain supply decreases if the average state grain market price increases.

**Graphic analysis** The graphic analyses are made for both aggregate import grain demand and aggregate export grain supply.

1. Aggregate import grain demand

Given the average international grain price $P_i$, we can draw aggregate import grain demand. As Figure 4.21a shows, $D^{\text{MMI}}$, the sum of aggregate import grain demands by both dual command importers and command importers, is a vertical line whose intercept on $Q_G$ is $Q_G = \sum (X_{i1I} + X_{i2I}) + \sum X_{i2I}$ in the $(Q_G, P_G)$ space.

Figure 4.21b gives $D^{\text{MMII}}$, aggregate import grain demand by flexible and restricted flexible importers. Finally, $D^{\text{MM}}$, aggregate import grain demand by the MOFT, is obtained in Figure 4.21c.
Figure 4.21. Aggregation of import grain demands in the mixed economy

2. Aggregate export grain supply

Similarly, Figure 4.22 describes the aggregation of export grain supplies. On the left, \( S_{XMI} \), the sum of aggregate export grain supplies of both dual command exporters and command exporters is given as a vertical line with an intercept \( \bar{Q}_{GE} = \sum \bar{X}_{1E1} - \bar{X}_{1II} + \sum \bar{X}_{1E2} \) on the \( Q_G \) axis. In the middle, \( S_{XMII} \), aggregate grain supply of restricted flexible exporters is given. Lastly, \( S_{XM} \), aggregate export grain supply of the MOFT is drawn as an upward sloping line in the \( (Q_G, P_G) \) space on the right of Figure 4.22.
Interactions of plans and markets

In the mixed grain economy, the state market has not been changed fundamentally. The government sets state market prices by its own evaluation. Moreover, on the consumption side, the government still uses ration coupons in the urban area to restrict urban grain demand and grants permission to the minority of agricultural households to limit the rural grain demand in the state market. On the production side, the government sets contracts (somewhat different new quotas) to guarantee rural grain supply in the state market. Thus, the state government still controls not only the prices in the state market but also the quantities exchanged in the state market. In this sense, the state market can still be seen as an extension of the government policies.

According to the official opinion, the free market, as newly reopened during the economic reform, is only an adjustment and a supplement to the state market. The price in the free market is determined by the interaction of market forces. However, the quantity exchanged in the free grain market is partly determined by price and is partly influenced by the government policies. It is known that grain demand or supply in the free market is only a residual grain demand or supply of the sum of grain demands and supplies in both the household market and the state market. Thus, the government has the power to use a policy instrument to change the exchanged quantities in the state market, and, hence, change the exchanged quantities in the free market indirectly. In this sense, we should not overestimate the roles played by the free market in such a mixed economy. In the international grain market, the Chinese government
still has no power to decide the prices, but has full monopoly power to
decide the varieties and quantity imported and exported. Thus, interna-
tional grain markets still can be partially seen partly as an extension
of the government policies.

Market equilibrium situations In the mixed economy, the disequi-
librium feature of the state market has not changed. It still is
believed that the grain price in the state market is well below its
equilibrium price and, hence, creates a restricted excess demand under
the rationing coupon system in the state grain market.

Because the exchange activity occurring in the free market must obey
the economic principles, noting that the free market price is well above
the state market price, we may conclude that there is an equilibrium
existing in the free grain market, and that it is decided by interaction
of both demand and supply in the market. The average grain price may be
considered as approximately the equilibrium price.

In the international grain market, there is an equilibrium. Because
China is generally a grain importing country and it behaves as a perfect-
ly competitive or a price follower in the international market, the ex-
change prices are determined by excess grain supplies of the rest of the
world, and the exchange quantity is mainly determined by the excess grain
demand of the MOFT.

Domestic policy instruments In the domestic market of the mixed
economy, the main policy instruments include the ration coupons, the
ration permissions, the sales quotas, and the state market prices.

The mechanism of the ration permissions in the state market is the
same as that of the ration coupons, so we are discussing the roles of only the ration coupons, the sales quotas, and the state market price. Apparently, the first two are policy instruments for quantity adjustment, the last one is a policy instrument for price adjustment. Next are comparative studies for these domestic policy instruments.

1. The ration coupon

Figure 4.23 shows how an increase in the issued ration coupon per capita in the state market affects both the free market and the international market.

![Diagrams showing the effects of a change in the ration level in the mixed economy](image)

Figure 4.23. The effects of a change in the ration level in the mixed economy

We now illustrate the initial situations in the domestic markets and international market. To save space, we separate our dual graph and move the free market to the left. Figure 4.23a shows the situation in the free market. $D^F$ and $S^F$ are aggregate grain demand and supply in the free market and cross each other to determine the initial equilibrium price, $P_1^F$, and the equilibrium quantity, $Q_1^F$. Figure 4.23b gives the situation in the state market. $D^M$ and $S^M$ represent the aggregate grain demand and
supply in the state market in the mixed economy. Given the state market price $P^S$, which is less than $P^F$, the grain demand is $Q^D_1$ and grain supply is $Q^S_1$. In general, $Q^D_1$ does not need to equal $Q^S_1$. The planned grain demand is $Q^{PD}_1$ and the planned grain supply is $Q^{PS}_1$. They are not affected by the state market price $P^S$. Obviously, $Q^{PD}_1 > Q^D_1$ and $Q^{PS}_1 = Q^S_1$ because the government has to guarantee that planned balance fully covers the real balance in the state market. Lastly, Figure 4.23c indicates the situation in the international market. $ED$ and $ES(R)$ are excess grain demand by the MOFT and excess grain supply by the rest of the world, respectively. Given an international grain price, $P^I_1$, we have the equilibrium grain quantity, $Q^M_1$, in the international market. All of the previous descriptions about the initial situations of the markets are continuously used in the remaining discussions of all other policy instruments but are not illustrated anymore.

Consider that the government raises the ration coupons per capita, in the state market, grain demand, $D^M$, shifts to the right $D^M'$. At the same time, the planned grain demand, $Q^{PD}_1$, shifts to right $Q^{PD}_2$. Thus, the balance of total grain demand and supply as well as the planned balance of total grain demand and supply in the state market is increasing. As a result of shift in $D^M$, in the free market, grain demand $D^F$ shifts back to $D^F'$. Immediately, the equilibrium price $P^F_1$ goes down to $P^F_2$, and the equilibrium quantity $Q^F_1$ goes down to $Q^F_2$. In the international market, because of changes in the planned balance rather than real balance of grain demand and supply in the domestic state market, excess grain demand $ED$ shifts back to $ED'$. The import grain demand is expanded to $Q^M_1$ at the
given international grain price $P^I_1$.

2. The sales quotas.

The sales quotas are set for agricultural households in the mixed economy. The government uses the sales quota to ensure planned grain supply in the state market. Figure 4.24 describes the impacts on the domestic markets and the international market when the government raises the sales quotas.

![Figure 4.24](image)

**Figure 4.24.** The effects of a change in the sales quota in the mixed economy

The beginning situations are as discussed before. As the government tries to increase the sales quota $Q^s_{1S}$ to $Q^s_{1S}'$, both the grain supply and planned grain supply in the state market increase from $Q^s_1$ to $Q^s_2$ (Figure 4.24b). Because the grain supply in the state market increases, that causes a decrease in grain supply in the free market. In Figure 4.24a, $S^F$, grain supply in the free market, shifts to the left $S^F'$ and grain demand shifts to left $D^F'$. Thus, the equilibrium price $P^F_1$ is decreasing.
to $P_2^F$ but the equilibrium quantity, $Q_1^F$, is not certain and possibly decreases. In the international market, an increase in $P^S_1$ in the state market and a decrease in $P^I_1 - P^S_1$ may cause a left shift in excess demand $ED$. Hence, import grain demand decreased from $Q^M_1$ to $Q^M_2$ (Figure 4.24c).

3. The state market price

The state market prices are set for both domestic consumers and producers. By adjusting the state market price, the government can influence the situation in the markets. Figure 4.25 indicates the effects of a change in the state market price on the domestic markets and international market.

![Diagrams](image)

Figure 4.25. The effects of a change in the state market price in the mixed economy

The initial situations remain the same. Suppose the government decides to increase the state market price, say from $P^S_1$ to $P^S_2$; as a result, grain demand decreases from $Q^D_1$ to $Q^D_2$ in the state market (Figure 4.25b). In the free market, $D^F$ shifts left but $S^F$ shifts right. The
equilibrium quantity is not certain, but the equilibrium price increases (Figure 4.25a). Keeping $P^I_1$ constant as $P^S_1$ rises, the price difference $P^I_1 - P^S_1$ may decrease, and, hence, the MOFT may suffer more loss in importing grain. If this is true, the excess grain demand $ED$ shifts back, and the import grain demand decreases from $Q^M_1$ to $Q^M_2$ (Figure 4.25c).

**International policy instruments** In the international grain markets, the main policy instruments used by the Chinese government are the exchange rate, the import quotas, and the export quotas. Comparative studies of these three policy instruments are discussed next.

1. **Exchange rate**

   The exchange rate is used to shift excess grain demand in the international grain market. Because of the bonus, if the government raises the exchange rate in yuan/dollar, the MOFT may earn more profit from importing grain, hence more bonus from the State Council. Thus, the excess grain demand shifts up. On the other hand, an increase in the exchange rate causes the government to reduce its purchasing interest in the international market, then shifts $ED$ down, the final results is not certain. There is no change in either the domestic state market or free market (Figure 4.26).

2. **The import quota**

   We discuss the impact of all import quotas for the total grain demand only. As Figure 4.27c shows, if the import quota $Q_C$ increases, the excess grain demanded, $ED$, in the international market shifts to the right; the import grain demand is increasing from $Q^M_1$ to $Q^M_2$. There is no
change in the domestic markets.

Figure 4.26. The effects of a change in the exchange rate in the mixed economy

Figure 4.27. The effects of a change in the import quota or export quota in the mixed economy

3. The export quotas

The impact of export quotas is similar to that of the import quotas, but in the opposite direction. As the government increases all export quotas for grain together, it results in a left shift in excess grain demand, ED, and a decrease in import grain demand (Figure 4.27c).
CHAPTER V. MODELING THE CHINESE GRAIN ECONOMY

In keeping with the objective of this study to assess the impacts of government policy on China's grain trade and domestic grain consumption and production, and in attempting to develop empirical analyses of the Chinese grain economy in the future, this chapter will first formulate a complete grain model for China, covering both periods of planned and mixed economy. The differences of individual corresponding equation(s) between the two periods are illustrated; and, therefore, the differences between corresponding submodels for both planned and mixed economy also are illustrated. The most important impact channels in this model are via the domestic grain prices in both the state market and the free market, as well as international grain prices. In addition, the other government policy instruments, such as ration coupons and sales quotas in the domestic grain market(s), and import quotas and export quotas in the international grain market(s), are built into the model so that their impacts also can be further evaluated by estimating this model.

The grain model for China, CHNGMODI, includes four sections: domestic grain demand, domestic grain supply, international grain trade, and government grain price subsidy. The first two sections deal with the domestic markets. The market equilibrium conditions are given for the state market and the free market. In the third section, which treats the international markets, import grain demand/export grain supply are specified. Finally, the fourth section is used to simulate the government grain price subsidy.
To close this chapter, a brief summary about the components of the model is provided. In particular, flexibility of the model CHNGMODI is discussed first. After that, classifications of the equations and variables within CHNGMODI are given in this section.

Domestic Grain Demand

The submodel of domestic grain demand in China is divided into four parts: domestic food grain demand; domestic feed grain demand, domestic seed grain demand, and, finally, domestic inventory grain demand.

Food grain demand

Because of the differences in the government's grain policy between the urban and the rural areas in China, the domestic food grain demand (GFOOD) consists of domestic urban food grain demand (GFOODUB) and domestic rural food grain demand (GFOODRU):

\[ G_{\text{FOOD}} = G_{\text{FOODUB}} + G_{\text{FOODRU}} \]  \hfill (5.1)

Urban food grain demand

The urban food grain demand (GFOODUB) is the sum of the urban food grain demands in the state market and the free market (GFOODUBSM and GFOODUBFM):

\[ G_{\text{FOODUB}} = G_{\text{FOODUBSM}} + G_{\text{FOODUBFM}} \]  \hfill (5.2)

The equations for GFOODUBSM and GFOODUBFM are given by

\[ G_{\text{FOODUBSM}} = f(P_{\text{GSC}}, P_{\text{cross}}, \bar{R}_{\text{GU}}, UBPOP, UBPM) \]  \hfill (5.3)

and

\[ G_{\text{FOODUBFM}} = f(P_{\text{GF}}, P_{\text{cross}}, \bar{R}_{\text{GU}}, UBPOP, UBPM, DUM) \]  \hfill (5.4)

where \( P_{\text{GSC}} \) and \( P_{\text{GF}} \) represent own price of grain to the consumer in the state market and to the consumer and producer in the free market,
respectively, $P_{cross}$ represents the prices of other relevant goods, $\overline{R}_{GU}$ represents the average annual amount of ration coupons per urban person, and $UBPOP$ represents the number of the urban population. $UBPM$ represents income per urban consumer and $DUM$ represents a dummy variable for the structural change.

Obviously, the difference between the explanations of urban grain demand in the planned economy and in the mixed economy is formula (5.4). In particular, there are two new factors, $P_{GF}$ and $DUM$, appearing within (5.4). The former implies the emergence of the free market after the economic reform, and the latter is used to distinguish the periods of the grain economy before and after the reform.

**Rural food grain demand** Similar to the urban food grain demand, the rural food grain demand is the sum of the rural food grain demand in the state market ($GFOODRUSM$) and rural food grain demand in the own (and the free) market ($GFOODRUOM$). By the own market we mean the self-production and self-consumption of grain by many agricultural households, as described in Chapters III and IV. Thus, we have

$$GFOODRU = GFOODRUSM + GFOODRUOM$$

(5.5)

The functional forms of $GFOODRUSM$ and $GFOODRUOM$ are given by

$$GFOODRUSM = f(P_{GSC}, \overline{R}_{GR}, RUPOP, WI)$$

(5.6)

and

$$GFOODRUOM = f(P_{GSC}, P_{GF}, P_{cross}, RUPOP, WI, RUPM, DUM)$$

(5.7)

where $\overline{R}_{GR}$ represents the amount of ration per rural person, $RUPOP$ represents number of rural population, $WI$ represents a weather condition index, and $RUPM$ represents real income per rural consumer. The other
variables, \( P_{GSC} \), \( P_{GF} \), \( P_{cross} \) and DUM, are the same as described earlier in this section.

Clearly, there is no change in formula (5.6) in going from the planned economy to the mixed economy. On the other hand, within formula (5.7), \( P_{GF} \) and DUM are used to illustrate the difference between the rural grain demand in the own market and the free market, and, therefore, the difference between the rural grain demands in the planned economy and the mixed economy.

**Food grain demand in the state market**  
The total food grain demand in the state market (GFOODSM) is provided by

\[
GFOODSM = GFOODUBSM + GFOODRUSM
\]  
(5.8)

where GFOODUBSM and GFOODRUSM are previously specified.

**Food grain demand in the own market (and free market)**  
The total food grain demand in the own (and the free) market (GFOODOM) is provided by

\[
GFOODOM = GFOODUBFM + GFOODRUOM
\]  
(5.9)

where GFOODUBFM and GFOODRUOM are as previously specified.

**Feed grain demand**

**Importance of feed grain**  
Feed grain is very important within the Chinese grain economy because China has one of the world's largest year-end livestock inventories, especially inventories of hogs, which provide over 90 percent of the meat in China and consume a relatively large amount of feed grain to compete with human needs. For example, according to a study by the World Bank (1987), about 87 million tons of grain,
including cereals, tubers, and soybeans, was used to feed livestock in China in 1984. This was equivalent to about 33 percent of the total Chinese human grain consumption in the same year.

**Circulation of feed grain** In general, the government assigns feed grain rations for the production teams or rural households, but does not provide feed grain to them from the state stock. The feed grain will mainly depend upon the collective reserve system or private households. In case of an emergency, with the government’s permission, the household may buy feed grain from the state market. In the mixed economy, however, the rural households may purchase additional feed grain from the free market.

Additionally, the government applies rationing policy to sell feed grain to those state farms, or collective farms, or individual households, that are specialized in livestock production and deliver meat products to the state market.

**Feed grain demand** Because feed is used as input in livestock production, the theoretical specification of feed demand follows the derived demand approach, which has been developed in both the agricultural production team model and the agricultural household model in this study.

In addition, the feed grain demand (GFEED) is separated into two subparts: the feed grain demand in the state market (GFEEDSM) and the feed grain demand in the own (and free) market (GFEEDOM). Thus,

\[ GFEED = GFEEDSM + GFEEDOM \]  

Furthermore, GFEEDSM is specified as a function of price of grain in
the state market ($P_{GSC}$), price of livestock product ($P_{MS}$), meat consumed in the urban area ($UBMC$), per capita income of urban residents ($UBPM$), and number of livestock ($LN$). Thus, the functional form of feed demand in the state market is

$$GFEEDSM = f(P_{GSC}, P_{MS}, UBMC, UBPM, LN)$$ (5.11)

Similarly, $GFEEDOM$ is specified as a function of grain price in the state market ($P_{GSC}$), grain price in the free market ($P_{GF}$), meat price in the state market ($P_{MS}$), meat price in the free market ($P_{MF}$), meat consumed in the rural areas ($RUMC$), per capita income in the rural area ($RUPM$), number of livestock ($LN$), and dummy variable ($DUM$). Thus, the functional form of feed demand in the free market is

$$GFEEDOM = f(P_{GSC}, P_{GF}, P_{MS}, P_{MF}, RUMC, RUPM, LN, DUM)$$ (5.12)

Needless to say, $P_{GF}$ and $DUM$ are factors newly added to formula (5.12) because of the economic reform.

**Seed grain demand**

The demand for seed grain ($GSEED$) is specified as a function of sown area ($GAS$) and a time trend ($T$). The behavioral relationship is given as

$$GSEED = f(GAS, T)$$ (5.13)

**Inventory grain demand**

Total grain inventories in China ($GSTOCK$) are further disaggregated into the state inventory ($GSTOCKS$) and the rural inventory ($GSTOCKRU$). The state inventory managed by the state plan is mainly used for nationwide disasters and other unpredictable events such as war. The rural
inventory owned and managed by production teams or agricultural households is mainly used to compensate their losses in bad harvests. Thus,

\[ \text{GSTOCK} = \text{GSTOCKS} + \text{GSTOCKRU} \quad (5.14) \]

The behavioral relationship for the state stock is specified as

\[ \text{GSTOCKS} = f(\text{UBPOP}, \bar{R}_\text{GU}, \text{GSQ}, \text{WI}) \quad (5.15) \]

where GSQ is grain sales quota, and the other variables, UBPOP, \( R_{GU} \), and WI, are the same as in earlier equations.

The behavioral function for the private stock is specified as

\[ \text{GSTOCKRU} = f(\text{RUPOP}, \text{GPROD}, \text{GSQ}, \text{WI}) \quad (5.16) \]

where GPROD is grain production and the other variables are remaining the same as before.

### Domestic Grain Supply

The submodel of domestic grain supply in China consists of three parts: grain production, grain supply in the state market, and grain supply in the own market (and the free market).

#### Grain production

The grain production part includes three equations explaining area sown to grain, grain yield, and grain production, respectively.

**Area sown to grain** Area sown to grain in China (GAS) is expressed as a function of sown grain area planned by the government (PGAS), the state grain market price for producer, \( P_{\text{GSP}} \), the free grain market price, \( P_{\text{GF}} \), and the structural dummy variable, \( \text{DUM} \), in other words,

\[ \text{GAS} = f(\text{PGAS}, P_{\text{GSP}}, P_{\text{GF}}, \text{DUM}) \quad (5.17) \]

The variables \( P_{\text{GF}} \) and \( \text{DUM} \) are newly added to explain GAS within the
mixed economy.

**Grain yield**  Grain yield in China (GY) is expressed as a function of the state market price $P_{GSP}$, the free market price $P_{GF}$, input uses $X$, the sales quota $GSQ$, a time trend, $T$, to represent technical change, a dummy variable, $DUM$, to represent structural change, and a weather index, $WI$:

$$GY = f(P_{GSP}, P_{GF}, X, GSQ, T, DUM, WI) \quad (5.18)$$

Undoubtedly, $P_{GF}$ and $DUM$, of course, are new variables added in formula (5.18) for $GY$ in the mixed economy.

**Grain production**  China's grain production (GPROD) is expressed as a product of area sown to grain (GAS) and grain yield (GY):

$$GPROD = GAS \times GY \quad (5.19)$$

**Grain supply in the state market**  Grain supply in the state market (GSSM) is expressed as the sum of sales quota (GSQ), and imports (GIM):

$$GSSM = GSQ + GIM \quad (5.20)$$

where the sales quota is a function of urban population (UBPOP), rural population (RUPOP), and grain production (GPROD). Thus,

$$GSQ = f(UBPOP, RUPOP, GPROD) \quad (5.21)$$

**Grain supply in the own market (and the free market)**  Grain supply in the own (and the free) market (GSOM) is the difference between grain production and the sales quota. Thus,

$$GSOM = GPROD - GSQ \quad (5.22)$$
International Grain Trade

The international grain trade submodel for China represents the market linkage between grain markets of China and the rest of the world. This submodel approaches a partial equilibrium because only grain commodities are included. To simplify the model, the international grain prices are assumed to be exogenous because the Ministry of Foreign Trade is assumed to be either a perfect competitor or a price follower in the international grain markets.

The submodel is formed by two parts: net grain import (export) and domestic quantity balance.

Net grain import (export)

For a specific grain commodity $i$, its import demand ($G_{Mi}$) or export supply ($G_{XPi}$) can be specified as a function of its international market price ($P_{Gi}$), its state market price ($P_{Gi}$), the international market price and the state market price of other relevant goods ($P_{cross IN}$ and $P_{cross S}$), the domestic excess demand/supply of grain in the state market (EXGFOODSM) and dummy variable (DUM). Thus we have

$$G_{MI} (G_{XPi}) = f(P_{Gi}, P_{Gi}, P_{cross IN}, P_{cross S}, EXGFOODSM, DUM)$$

Note that the $P_{Gi}$ and $P_{cross S}$ are new variables added to (5.23) due to policy change within the mixed economy. In addition, EXGFOODSM is defined by

$$EXGFOODSM = GFOODSM + GFEEDSM - GSSM$$

where all variables, GFOODSM, GFEEDSM, and GSSM, are specified the same...
as before.

The net grain import (export) NGIM (NGXP) is the sum of all GIMi and GXPi, i.e.,

\[ NGIM (NGXP) = \sum (NGIM_i - NGXP_i) \]  \hspace{1cm} (5.25)

**Domestic quantity balance**

It is known that the domestic state market is usually in disequilibrium. However, there is a quantity balance for the volume traded in the state market as well as that in the whole domestic market.

**Domestic quantity balance** The domestic quantity balance is expressed by the following equation:

\[ NGIM (NGXP) = GFOOD + GFEED + GSEED + GSTOCK - GPROD \]  \hspace{1cm} (5.26)

It is said that NGIM (NGXP) is the sum of all domestic grain demands, including GFOOD, GFEED, GSEED and GSTOCK, minus domestic grain production (GPROD).

**Domestic quantity balance in the state market** The equation of domestic quantity balance in the state market is expressed as the following:

\[ NGIM (NGXP) = GFOODSM + GSTOCKS - GSSM \]  \hspace{1cm} (5.27)

where GFOODS, GSTOCKS, and GSSM are the same as given before.

**Domestic quantity balance in the own market (the free market)**

The equation of domestic quantity balance in the own market (the free market) is expressed as the following:

\[ GSOM = GFOODOM + GFEEDOM + GSEED + GSTOCKRU \]  \hspace{1cm} (5.28)

where all the variables are defined the same as before.
Government's Grain Price Subsidy

Because the Chinese government wants to keep political stability within the country, it provides heavy subsidies in the form of a difference between its purchase price and sale price for grain in the state market; and, therefore, it is bearing a great pressure to balance its budget. The government’s grain price subsidy can be classified into two categories: 1) grain price subsidy to the Ministry of Food (MOF) due to its grain trade in the domestic state market, and 2) grain price subsidy to the Ministry of Foreign Trade (MOFT) due to its grain trade in the international market.

Obviously, the total government’s grain price subsidy (GPS) is the summation of the above two parts of the grain price subsidies:

\[
GPS = GPS_{MOF} + GPS_{MOFT}
\]  

(5.29)

where GPS_{MOF} is the government’s grain price subsidy to the MOF and GPS_{MOFT} is the government’s grain price subsidy to the MOFT.

Grain price subsidy to the Ministry of Food

In the domestic state grain market, the government has to maintain the lower stable consumer price in order to win the support from the urban consumers; on the other hand, the government has to increase the producer price in order to stimulate the rural producers’ incentive to expand their production. The difference in the consumer price and the producer price in the state grain market has created a financial problem in the government’s budget.

The function explaining the grain price subsidy to the MOF (GPS_{MOF})
is given as

\[ \text{GPSMOF} = (P_{\text{GSP}} - P_{\text{GSC}}) \times \text{GSQ} \tag{5.30} \]

where \( P_{\text{GSP}} \), \( P_{\text{GSC}} \) and GSQ are the same as before.

**Grain price subsidy to the Ministry of Foreign Trade**

In the international market, the government has made grain trades without paying much attention to the domestic state market price, especially in the planned economy. But the price difference between the domestic state market and the international market may create a loss (or an earning) to the MOFT on its internal account. Thus, the MOFT receives a grain price subsidy, if any, (or has given a grain price credit), to compensate (or to turn over) its loss (or earning) from the grain trade. The function explaining grain price subsidy to the MOFT (GPSMOFT) is given by

\[ \text{GPSMOFT} = \sum_{i} (P_{\text{GIN}_{i}} - P_{\text{GSI}_{i}}) \times \text{NGIM}_{i} \tag{5.31} \]

and the function explaining the grain price credit from the MOFT (GPMC-MOFT) is given by

\[ \text{GPMC-MOFT} = \sum_{i} (P_{\text{GIN}_{i}} - P_{\text{GSI}_{i}}) \times \text{NGXP}_{i} \tag{5.32} \]

where \( P_{\text{GIN}_{i}} \), \( P_{\text{GSI}_{i}} \), \( \text{NGIM}_{i} \) and \( \text{NGXP}_{i} \) are the same as before in this model.

**A Brief Summary**

**Flexibility of the grain model for China**

The grain model for China, CHNGMODI, is built to include both periods of the planned economy and the mixed economy by distinguishing
different explanatory variables and time dummy variables. Thus, it can be used to project China's grain demand, grain supply, and foreign grain trade by using time-series data simulating the structural change within the economy.

In addition, by taking out all added new variables and equations mentioned earlier in this chapter, CHNGMODI can be adjusted to become CHNGMODIA, which can be well fitted to the case of the Chinese planned grain economy only. For the case of the Chinese mixed grain economy only, our model CHNGMODI is identical with CHNGMODIB, which can be estimated by using time-series data covering only the period of the mixed economy.

Classification of equations

CHNGMODI consists of 18 (19) behavioral or technical equations (B-type) and 14 (15) identities (I-type) that are arranged in the sections set out next.

<table>
<thead>
<tr>
<th>Section name</th>
<th>No. of relationship</th>
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<td>Food Grain Demand</td>
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<td>Inventory Grain Demand</td>
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<td>Domestic Grain Supply</td>
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<tr>
<td>International Grain Trade</td>
<td>2(3)</td>
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<tr>
<td>Government's Grain Price Subsidy</td>
<td>3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>18(19)</td>
</tr>
</tbody>
</table>
Classification of variables

CHNGMODI contains 32(34) endogenous variables and 25 exogenous variables. The variables are classified and listed below:

**Endogenous variables** The endogenous variables include: GAS, EXGFOODSM, GFEED, GFEEDOM, GFEEDSM, GFOOD, GFOODOM, GFOODRU, GFOODRUOM, GFOODRUSM, GFOODSM, GFOODUB, GFOODUBFM, GFOODUBSM, GIM, GIMi, GSOM, GPS, GSMOF, GPCMOFT, GPSMOFT, NGIM, (NGXP), GPROD, GSSM, GSEED, GSTOCK, GSTOCKS, GSTOCKRU, GXP, GXPI, GY, GSQ, and PGF.

**Exogenous variables** The exogenous variables in CHNGMODI include: GPAS, DUM, LN, Pcross, PcrossIN, PcrossS, PGIN, PGIMi, PGCSC, PGSCi, PGSP, PPGSPi, PMF, PM, PGR, PGU, RUMC, RUPM, RUPOP, T, UBMC, UBPM, UBPOP, WI, and X.
CHAPTER VI. SUMMARY, CONCLUSIONS, AND
SUGGESTIONS FOR FURTHER RESEARCH

Summary

The Chinese grain economy is one of the most important and complex in the world. The main objective of this study is to develop a theoretical framework for analyzing the Chinese grain model in both planned and mixed economy regimes, in order to understand changing relationships among the government, the firms, and the households, and, furthermore, to understand the changing features of the operational mechanism of the socialist economy.

Generally speaking, the Chinese grain economy was/is formed and operated by the government and by basic economic units--urban households (as consumers) and rural firms (as producers and consumers). By inspecting the changing organizations of these basic units and exploring the changing behaviors of the households, the firms, and the government, a theoretical model was constructed and used to evaluate the impacts of changing government policies in both the domestic and international markets. The development of the theoretical model is, in a certain sense, a breakthrough and is quite successful. By analyzing the separate parts of the model, we determine the nature of the whole. To reflect and stress the changes over time due to economic reform, the theoretical discussions of the Chinese grain economy under two scenarios were separated into Chapters III and IV. The former focused on the operational mechanism of the planned grain economy, and the latter emphasized the
The discussion in each Chapter (III or IV) was naturally divided into two sections: the domestic economy and international trade. The domestic economy was separated into urban and rural parts, because of the dual feature of the domestic grain economy. Furthermore, the rural economy was broken into two subparts for state farms (state farm households) and agricultural production teams (agricultural households) due to the different features of ownerships of production means and resources.

Within the domestic economy, first of all, different types of behavior of urban consumers, as well as rural producers and consumers, were analyzed, and different demand/supply functions were derived according to their activities in the market(s). Second, these individual demand/supply functions were synthesized to determine the effects of some government policy instruments. Particularly, in contrast to previous studies, the method of two-stage aggregation was used to obtain the total grain demand and supply functions. That is, we aggregated demands or supplies of individual units into group demands or supplies by the behavioral classifications, then those group demands or supplies were aggregated into a total grain demand or supply.

In our sections on international trade, the different behaviors of the MOFT were classified and aggregated import grain demand/export grain supply functions were derived for both the planned economy and the mixed economy.

By using the method described, in the Chinese grain economy, the roles of key policy instruments, which mainly include the ration coupon,
the state market price, the sales quota, and the import/export quota, were fully and clearly recognized.

To complete this theoretical study, in Chapter V, a grain model for China was formulated by combining the results obtained in Chapters III and IV. This model has several features that were neglected in previous studies by others. First, it covers periods of both planned economy and mixed economy by using a time dummy variable as well as other distinctive variables. Second, it contains the international markets and either two (planned economy) or three (mixed economy) domestic markets with different market characteristics. Third, the differences in methods of determining the domestic market prices (endogenous or exogenous) and international market price (exogenous) associated with the different market characteristics are explicitly presented, and, of course, these three different prices are not necessarily equivalent. Fourth, the model embodies a structural feature that is very important for the purpose of assessing the impacts of rationing policy. Overall, this model provides a theoretical foundation for a full understanding of the Chinese grain economy and (in principle) empirical measurement of the impacts of government policy.

For the associated objectives, this study provides a tool to analyze the interactions of domestic and international markets under both planned and mixed economy assumptions and to assess the impacts of rationing systems, domestic prices (including state market prices and free market prices) and international prices on China's grain trade and domestic market. The grain model for China incorporates all required instruments
and variables; therefore, such impacts can be evaluated theoretically with the model.

Conclusions

The theoretical understanding and findings gained from the study of the features of the Chinese economy can be summarized in the following categories.

Market economy

The basic features of the market economy may be presented by using these key words: private ownership, market mechanism, and government's regulation. Despite some different opinions, in general, economists think that the price mechanism (the market mechanism) in its micro aspect leads private firms to the efficient allocation of resources in the market economy. On the other hand, the government sets policies to deal with problems of macro stability in the economy. In other words, the private firms and households have a right to make their own decisions for their consumption, production, and distribution, whereas the government uses regulation to influence and change the economy. This is the general framework of the market economy in today's world.

Planned economy

The basic features of the planned economy may be described by these key words: state or collective ownership, government's central plan, and the state market. Yun Chen (1984), a Chinese communist leader and authority on the planned economy, vividly depicted the planned economy as
"the economy of the bird cage." That means, the bird can live within the cage, but cannot fly out of the cage. In the planned economy, the government uses the central plan plus the state market, that is, the cage, through state or collective firms to allocate resources to achieve its national economic goals. On the micro level, within the cage, either state or collective firms have a limited freedom to decide what, how, and how much to produce because the main signals in the economy are not market prices but the government's plans, and the means of production are owned by either state or collective firms. The market mechanism plays no role, or a relatively small one, in such an economy.

**Plan and domestic state market** The key to understanding the planned economy is the state market. The state market, controlled and operated by the state government, is not a market in the usual sense, and can be seen as an extension of the government's plan. The prices in the state market are not determined by market forces but by the government's sole opinion.

For the important commodities such as grain, cotton, and steel, the government uses additional policy instruments to guarantee volume traded in the state market. On the demand side, the government uses rationing to restrict consumers' demand. On the supply side, the government uses output targets and sown area plans to lead firms' production and uses sales quotas to guarantee producers' supply.

Usually, there is a disequilibrium in the state market because of the exogenous character of the state market price, which cannot drive an equilibrium in the market. The concept of equilibrium used by communist
economists means only equilibrium for quantity but not necessarily for price.

In the state market, for those important goods, the government usually sets a relative low price, which is believed to be below the equilibrium price and which will create a shortage problem. In this sense, some economists call the planned economy a shortage economy.

Additionally, the procurement price may not be equal to the retail price in the state market. In most cases, the procurement price is higher than the retail price, because the government faces the pressure from the producer to raise the former and from the consumer not to raise the latter. As a result, the government has to cover a large subsidy in its budget.

The new findings in this section are: (1) either the ration or the state price has only a partial effect on the consumer demand if the ration is not fulfilled, (2) the state price has only a partial effect on the producers' supply.

Plan and the international market(s) In the international market, the government reserves for itself the exclusive right to purchase or sell important goods such as grain. The government makes a foreign trade plan to set import and export quotas and targets. Given the plan, assuming the MOFT is either a price taker or a price follower in the international market, it may adjust the real volume traded according to the given international prices and other factors such as balance of foreign currency reserves. An equilibrium exists in the international market. The equilibrium quantity is determined by the
MOFT, whereas the equilibrium price is determined by the rest of the world.

The new findings in this part are that, (1) the international price, but not the domestic state market price, can affect the quantity traded in the international market, (2) the domestic balance in the state market but not in the united market, can directly affect the quantity of the foreign trade, and (3) the domestic balance of composite grain commodity, but not the domestic balance of a single grain commodity, is the main concern in the foreign grain market. These findings are consistent with the conjectures in previous studies and are clearly expressed in the mathematical analysis.

**Mixed economy**

The basic features of the mixed economy are dual ownership, government's central plan, and a dual market (a triple market if the own market is included). The most remarkable changes in the mixed economy are emergence of semi-private/private enterprises and the free market. Although the government now still uses a central plan, plus the state market through the state, collective, or even semi-private/private ownership of production means, to allocate resources to achieve its national goals, the former rigid control over the economy has been somewhat loosened because of reopening of the free market. From the micro view, within the cage, the state, collective, and private firms have more freedom to make their own decisions on their consumption, production, and distribution, especially in the free market. The
activities in the free market are mainly led by the market mechanism. Obviously, the mixed economy possesses dual features of both planned and market economy. Hence, it can be seen as transitional between the planned economy and the market economy. The exact location of mixed economy between the two, roughly speaking, will mainly depend upon the proportions of state and private ownership and the relative sizes of the state market and the free market within the economy. At this moment, the Chinese mixed economy still belongs to the socialist category because the central plan, plus the state ownership, still plays a dominant role, and the free market is only an adjustment and supplement to the state market. In this sense, the mixed economy can be considered an enlarged "economy of the bird cage."

**Plan and domestic state and free market** The coexistence of the state market and the free market, especially the coexistence of the state market price and the free market price, has brought some changes in the government plan and the situation in the state market. Based on political considerations, the government has to maintain the price differences between the state market and free market and to create price differences between consumers and producers for the necessary goods such as grain in the state market. Thus, it has to set income subsidy plans to carry this heavier burden. In addition, some government officials may use their power to buy goods in the state market then sell goods in the free market to make their own profits. This may partly damage the government plan in the state market.

The free market within the mixed economy is somewhat different from
the free market in the usual sense. In fact, it is a residual market to
the state market, especially for the most important commodities. The
government can directly influence consumer demand and producer supply in
the state market but not in the free market. However, through the
interaction of the state market and the free market, the demand and
supply in the free market may be affected indirectly. In addition, the
price difference between the state market and the free market may lead to
flows from the state market to the free market, therefore changing the
situation in the free market.

There is no change from disequilibrium situation in the state
market. However, an equilibrium for both price and quantity may exist in
the free market because of effects of market forces. Although the excess
demands for those important goods still exist in the state market within
the mixed economy, overall, the nationwide shortage situation has been
somewhat improved by the price mechanism of the free market.

The new findings in this section are: (1) the ration level, or the
state price, or the free market price plays only a partial role in
affecting the consumers' demands and (2) the state price has no effect on
the producers' supplies, but the free market price does.

Plan and the international market(s) The government still has
monopoly power in the international market. The plan procedure and
policy instruments used are basically unchanged. The market situation
also is assumed to be the same as before. Hence, the exchanged quantity
is decided by the government, given the exchange price in the interna-
tional grain markets.
The important new finding for the mixed economy in this section is that both domestic price and international price can influence the quantity exchanged in the international market. This is different from the first corresponding finding in the planned economy and provides mathematical support for the hypotheses in previous studies. Other new findings for the mixed economy are the same as the second, and the third listed for the planned economy.

Suggestions for Further Research

Although the results of this study were theoretically satisfactory and all the objectives were accomplished, there are still some areas that can be explored for further research. First, an empirical econometric model can be implemented based on the theoretical model developed in Chapter V. At present, an empirical model is more important than ever for the following purposes: (1) to fill in the element missing from the previous studies with the first econometric model covering both the planned economy and the mixed economy on a theoretical basis, (2) to be used statistically to test and examine the validity of the theoretical model, (3) to explore the relationship between availability of data and rationality of the empirical model, (4) to evaluate and estimate the policy instruments impacts on both the domestic and international market(s), (5) to evaluate policy alternatives for structural changes within economies, and (6) to estimate and to project China's grain demand and supply in the domestic market(s) and grain trade in the international market(s).
The empirical work can be done in alternative ways: (1) by constructing and estimating a simplified grain model for China, CHNGRMODI, then developing it into a more detailed grain model for China, CHNGR-MODII, or (2) constructing and estimating several different grain crop models such as a wheat model for China, CHNWHMOD, a rice model for China, CHNRIMOD, etc.; and then combining them to form a grain model for China, CHNGRMODIII.

Second, within the crop sector, this study can be extended to include commercial crops such as cotton. Although we have considered prices of commercial crops in our model, the roles of commercial crops in the grain economy were not thoroughly explored. It is acknowledged that the commercial crops are competitive with grain crops. This is especially true in China because all cultivatable lands are extensively sown each year. In addition, China has been the largest consumer and producer of cotton in the world since 1982. Thus, it is very important to construct a cotton submodel to associate with the grain model to evaluate the interactions of the cotton model and the grain model in both domestic market(s) and international market(s).

Third, within the agricultural sector as a whole, this study also can be extended to include the livestock sector. Even though feed grain was built into the model, the role of the livestock sector in the grain economy was not sufficiently investigated in both theoretical and empirical studies. On the basis of this study, a theoretical livestock model can be developed to fit the policy environment of the livestock sector. Furthermore, the empirical livestock model can be merged with
our grain model to evaluate their relations and interactions in the market(s). By doing this, better estimations and projections of demand, supply and foreign trade for both feed grain and food grain in the Chinese grain economy can be obtained.

Fourth, within the national economy as a whole, this study can be further developed to include the industrial sector, as well as other sectors of the economy. Although the agricultural sector as a whole has been considered, the model is still a partial equilibrium model. To improve the situation, detailed domestic markets for each important good (or composite good) within the country should be constructed, and the corresponding international markets can be treated in the same way used in this study. Thus, a computable general equilibrium (CGE) model can be formulated and estimated so that interactions among the plans, domestic markets and international markets can be further recognized.

Last, this study should be extended to connect the proposed model with the world market(s). Although world grain prices have been considered in the model, the model is still national. To improve the situation, a detailed domestic market for each major country or group of countries should be included. Import demand/export supply can be formulated by using the definition of excess demand/excess supply in each domestic market. In comparison with our national model, such a non-spatial model provides more realistic information for international trade and policy analysis.
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A Glossary of Some Economic Terms

The following economic terms are either commonly used in China or defined by the author in this study. The meanings of the terms are briefly explained in order to familiarize the readers with the text of this study.

**Agricultural household**

Agricultural household is defined by the author as a type of rural household that is a basic unit of production and consumption and belonged to the agricultural production team before the economic reform.

**Bao Can Dau Hu**

A form of the responsibility system in agricultural production with the character of individual operation. By contract, each household uses land owned by the collective and has to pay the state tax and fulfill the state procurement quotas in specified crops.

**Bonus**

A method practiced by social enterprises or administrators to give workers and staffs material incentive or additional income according to their work. In general, bonuses are given as rewards to those who overfulfill their tasks and make special contributions.
**Combined market**

A market that is assumed to combine both the state market and the team market in the planned economy.

**Dual market**

Dual market implies the state market and the free market in the mixed economy.

**Dual market graph**

Dual market graph is a graph created in this study to describe the coexistence of dual markets.

**Free market**

Free market is a form of commodity exchange among individuals and/or firms under the state administration. Prices in the free market are negotiable between buyer and seller.

**Free market analysis**

A market analysis is used to describe the exchange activities in the free market only.

**Grain**

Grain, as defined by the Chinese State Statistics Bureau, includes rice, wheat, corn, soybeans, sorghum, millet, sweet potatoes, and other coarse grains.
Grain price subsidy

Grain price subsidy is the subsidy to either the Ministry of Food or the Ministry of Foreign Trade to compensate their loss in exchanging grain due to the difference between their purchase price and the sale price.

Household market

A market that is assumed to have all agricultural households' own consumption occurring.

Household-own market

A market that is assumed to have an agricultural household's own consumption occurring.

Market mechanism

The process by which individuals and firms carry out economic activities on the basis of information provided by the market. The market mechanism is capable of balancing supply and demand in the market.

Mixed economy

Mixed economy is a transitional economy between planned economy and market economy. The basic features of a mixed economy are coexistence of state/collective ownership and private ownership and coexistence of the state market and the free market.
**Own market**

A market that is assumed (in Chapter V) to circulate own-consumed grain for either production teams or agricultural households without going through the state market and/or the free market.

**Planned economy**

Planned economy is/was an economy set in the socialist country. The basic features of a planned economy are that, the government applies the central plan through the state market to lead the state firms or collective firms to allocate resources and to distribute products and incomes in order to achieve its national goals.

**Production responsibility system**

A system that links labor management and evaluation, and results in proper labor payments from collective according to the job or the final output.

**Production team**

The production team is generally the basic collective accounting and production unit in rural areas in the planned economy. The team owns some collective properties such as land, draft animals, and production means, and, under the guidance of local government, takes care of its own operations and distributes income among its members.

**Ration coupons**

Ration coupons are used by government to limit distribution of a certain good(s) such as grain in the state market. According to the
government's rule, the residents (or urban resident only) may obtain a
certain amount of coupons, and buy the good in the state market only with
both money (by price and quantity) and coupons (by quantity). The
coupons may not be permitted to exchange in the market.

**State farm**

A state farm is an agricultural enterprise under public ownership.
The state owns all means of production on the farm, and controls all the
products. The workers and staffs of the state farm are paid in wages.

**State farm household**

A state farm household is defined by the author as a type of rural
household that is a basic unit of production and consumption and belonged
to the state farm before the economic reform.

**State market**

The state market is a market operated and controlled by the govern-
ment. The prices in the state market are decided by the government's
opinions.

**State market analysis**

A market analysis is used to describe the exchange activities in the
state market.

**Team market**

A team market is the market assumed to have all grain distribution
activities in all of the team-own markets be added together.
**Team-own market**

A team-own market is the market assumed for the distribution activities within the production teams.

**Urban household**

Urban household is the household that lives in an urban area by the government’s permission and has the right to share some benefits such as ration coupons provided by the government.

**United market**

United market is the market assumed to combine both the free market and the household market in the mixed economy.