Continuous Time Control for Bilateral Telemetry Application

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Continuous Time Control for Bilateral Telemetry Application

by

Madhura Kulkarni

A creative component report submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Electrical Engineering

Program of Study Committee:
Dr. Greg R. Luecke, Major Professor

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Ames, Iowa
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DEDICATION

I would like to dedicate this research work to my family for their constant support and encouragement. I would also like to thank my friends and professors for guiding me through the process of this work.
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\[ K_p = 95 \text{ and } K_d = 410 \]
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I would like to express my gratitude to Dr. Greg R. Luecke for his constant support and encouragement. His insights and enthusiasm guided me through the research and the writing of this report. I would also like to thank Dr. Ratnesh Kumar for helping me build an understanding of the research topic through his discussions.
Time delay or dead time is defined as the response time required for a process/ a device when an input is applied. Dead time is a phenomena commonly occurring in industrial processes, biological systems and engineering applications. Transport lags, communication lags, computational delays are various types of time delays which inherently cause improper functioning of the system unless compensated for. Bilateral telemetry is one such application which faces instability in operation when subjected to time delay. The master robot in the virtual environment is ‘motion and force coupled’ with the slave robot in the virtual environment. This bilateral feedback provides better tracking results as compared to the unilateral scenario at the cost of introducing a transport lag in the communication channel. The present work focuses on addressing this time delay using classical control methods to provide for stability in event of large time delays.
CHAPTER 1. OVERVIEW

Teleoperation is defined as the remote control of machines electronically. The arrangement consists of a master robot placed in the virtual environment and a slave robot in the actual environment. The force applied by the operator controls the position of the master robot. This force acts as a position command for the slave robot. The term 'bilateral' arises due to the position data of the slave robot feedback to the master robot to enhance the performance of the teleoperating system.[1] This setup is shown in Figure 1.1.

![Teleoperation Setup](image)

Figure 1.1: Teleoperation Setup

Teleoperation has been in use in the nuclear industry to safeguard and reduce human intervention in the field operating areas. The early applications of bilateral feedback used servo controllers on both master and slave side to reflect the position of the slave to the master as an input force is applied.[5] For applications in undersea or outer space areas the large distances between master and slave robots cause a significant delay in the data transmission. This delay is due to the transmission losses in the communication channels and impacts the stability of the system. Passivity and scattering theory have been used in the past to address this issue.[1] As an advancement, Internet has been used as a communication medium for robot control.
in both unilateral[4] and bilateral operations.[7] The variable signal transmission times of the Internet have been compensated for using event based approach[2] using the non-time based references on the slave and the sensor.[11] In this work, we propose the use of classical control methods in discrete domain to address the stability of the system with time delay. We have used Internet as a communication medium between the master and slave. The communication delay is assumed to be constant. The architecture used for such an experiment consists of two 'Phantom Omni' haptic devices as the master and the slave.
CHAPTER 2. MATHEMATICAL MODELING

2.1 Mathematical Model for the Robotic Arm

Consider a simple master-slave system. We propose the use of two 'Phantom Omni' robots manufactured by Sensable Devices. Since both the robots are identical there is zero kinematic mismatch. Hence, we can represent both the haptic arms as electromechanical motor systems with voltage as the input and position as the output.

![Figure 2.1: Schematic of a DC motor [3]](image)

As shown in the figure 2, a DC motor consists of field wound on an iron core named as stator. The stator is the stationery part of the motor. The iron cylinder placed in between the poles of the magnet is the rotating part of the motor which constitutes as the rotor. The rotating coils wound on the rotor known as armature windings. A commutator consisting of low resistance carbon brushes are connected to the stator which come into contact with the rotor on application of a current source. When a current is passed through the rotor, the rotor rotates which causes each of the armature conductors to exert a torque on the rotor.
For the purpose of modeling we interpret the DC motor as follows:

![Electric and Mechanical representation of a motor](image)

Figure 2.2: Mechanical and Electrical representation of a motor

The armature can be represented as a circuit consisting of armature resistance $R_A$, inductance $L_A$, supply voltage $E_A$ and induced voltage $e_m$. The magnetic field induced can be represented as a circuit consisting of field resistance $R_F$, inductance $L_F$ and a supply voltage $E_F$. The back emf observed in the motor is represented as $K_e$.[3]

The rotor has a moment of inertia $J_m$, damping due to back emf of the motor $B_m$ and a driving torque $\tau_e$ which constitute the mechanical equation for motor as follows:

$$J_m s^2 \theta_m + B_m s \theta_m + \tau_A = \tau_e$$

The electromechanical torque exerted on the rotor is proportional to the current flowing through it:

$$\tau_e = K_m I$$
For the gear input/output the equation can be written as:

$$\frac{\tau_A}{R_1} = N\tau_A$$

∴ $$\frac{R_2}{R_1} = N$$

The output inertia is given by:

$$J_o s^2 \theta_o + B_o s \theta_o = N\tau_A + D$$

Combining the above equations:

$$J_m s^2 \theta_m + B_m s \theta_m + \frac{J_o s^2 \theta_o + B_o s \theta_o - D}{N} = \tau_e$$

where

$$\theta_m = N\theta_o$$

$$J_m s^2 \theta_m + B_m s \theta_m + \frac{J_o s^2 \theta_o}{N} + \frac{B_o s \theta_o}{N} \frac{D}{N} = \tau_e$$

$$(J_m N^2 + J_o) s \omega_o + (B_m N^2 + B_o) \omega_o = N\tau_e + D$$

In terms of output shaft:

$$J_{eq} s \omega_o + B_{eq} \omega_o = N\tau_e + D$$

where:

$$J_{eq} = J_m N^2 + J_o$$

$$B_{eq} = B_m N^2 + B_o$$
For the electrical circuit representation we can write the Kirchoff’s voltage equation as follows:

\[ L_A i(t) + R_A i(t) + K_e \omega = V_{in} \]

Laplace transform of the above equation gives:

\[ L_A sI + R_A I + K_e \omega = V_{in} \]

which can be written for the current I as follows:

\[ L_A sI = V_{in} - R_A I - K_e \omega \]

Reducing the loop:

\[ J_{eq} s \omega_0 = N \tau_e + D - B_{eq} \omega_0 \]

For \( \tau_e = K_m \times I \) the block diagram becomes:
Reducing the loop:

Connecting the two blocks we obtain:

If we assume no dependence on motor speed and no disturbance 'D', the block diagram becomes:

Since $R_A$, $NK_m$ and $B_{eq}$ are constants, clubbing them together as a constant 'C':

$$C = \frac{1}{R_A} \times NK_m \times \frac{1}{B_{eq}}$$

The block diagram thus becomes:
The effect of motor inductance $\tau_E$ is negligible and hence can be ignored for this application.

Thus for modeling purposes, the motor equations can be solely represented using the rotor moment of inertia $J_{eq}$ and the damping due to back emf $B_{eq}$.

$$\frac{1}{\tau_E s + 1} \quad \frac{1}{J_{eq} s + B_{eq}}$$

Hence, the complete motor block diagram becomes:

For our application, the disturbance $D$ is in fact operator hand force $F_{hand}$. Hence re-writing the block diagram as follows:

Since, force and voltage are analogous quantities, solving the above block diagram, the transfer function for the robotic arm with force $F_{hand}$ as the input and position $\theta_o$ as the output gives:

$$\frac{\theta_o}{F_{hand}} = \frac{1}{J_{eq} s^2 + B_{eq} s + \frac{N^2 K_m K_e}{R_A}} = \frac{1}{B_{eq}^2} \frac{N^2 K_m K_e s}{s(\tau s + 1)}$$
where:

\[ B_{eq2} = B_{eq8} + \frac{N^2 K_m K_e s}{R_A} \]

\[ \tau = \frac{J_{eq}}{B_{eq2}} \]
2.2 Calculating the Environmental Stiffness

For an elastic body, application of an external force causes deformation which is resisted by the internal forces. For a spring, such an application of force(load) causes deformation in form of displacement.

This type of force acting on the spring or any elastic body is called as the 'spring force' which is given by:

\[ P = Ku \]

where:

- \( P \): spring force
- \( K \): spring constant
- \( u \): displacement due to force \( P \)

Figure 2.3: Spring force experienced by an elastic body when subjected to a load \( 'P' \) [6]

Figure 2.4: Relation between force \( 'P' \) and displacement \( 'u' \) for an elastic body [6]
As the force varies, the displacement produced by the spring varies. This relation remains linear for a small value of the force and as the value of the force increases, the nonlinearity is apparent (Figure 2.4).

The cantilever beam is one such type of elastic body which has been utilized in our experiment to calculate the environmental stiffness. The environmental stiffness can be considered as an analogous quantity to the spring constant. For our application, environmental stiffness is defined as the opposition created by the surrounding which the slave robot has to overcome to follow the desired trajectory as governed by the master robot. The slave system can be analogously depicted as a mass-spring system. The spring force generated when the slave robot is placed in the virtual environment is given as follows:

\[ F_E = K_E \times x_{slave} \]

Using the equation for spring force of the beam the environmental stiffness \( K_E \) is given by:

\[ K_E = \frac{3EI}{L^3} \]

where:

- \( L \) : length of the beam
- \( E \) : Young’s modulus of elasticity
- \( I \) : moment of inertia of the beam

For our experiment, we have represented the cantilever beam with a standard 30 cm \( \equiv 0.3 \)m long steel ruler. The ruler is clamped at one end using a wooden support of 0.04m thickness. The other end of the ruler consists of a fixed Phantom Omni inkwell of measured weight 0.025kg. The inkwell is used for fixing the stylus position. The weight of the stylus is known to be 0.020kg [8]. Thus the total fixed load on the cantilever beam(steel ruler) is 0.045kg. When not in operation, this is the maximum load that the cantilever beam will be subjected to. During the course of the operation, the total load on the beam depends on the mass of the stylus and the variable mass of the operator’s arm - whether the arm is outstretched or bent, whether the haptic device is held tightly or loosely. This setup is shown in Figure 2.5.
Because of the fixed position of the inkwell on the ruler as shown in Figure 2.5, the length of the beam $L$ is calculated as follows:

$$L = \text{clamping position of the ruler} - \text{inkwell position}$$

To observe a sufficient amount of spring force, the clamping position of the ruler was chosen at 0.247m (Figure 2.6). The inkwell was placed $\approx$ at 0.013m location (Figure 2.7). This represents the application of a force on the free end of the cantilever beam.
Thus, the length of the beam ‘\( L \)’ is:

\[
L = 0.234m
\]

Since the ruler is made of steel, the Young’s modulus of the ruler is observed as:

\[
E = 69 - 72 \times 10^9 GPa
\]

The steel ruler can be approximated as a rectangular object. Hence, the moment of inertia ‘\( I \)’ of the cantilever beam can be determined using the standard inertia equation for a rectangle given by:

\[
I = \frac{bh^3}{12}
\]

where:

\( b \): base or width of the rectangle

\( h \): height of the rectangle
As shown in Figures 2.8 and 2.9, a vernier calliper was used to measure the beam height and the beam width. The beam height and the beam width were observed to be 0.0015 meters and 0.03 meters respectively.

This leads to the moment of inertia being calculated as:

\[
I = \frac{bh^3}{12} = \frac{0.03 \times (0.0015)^3}{12} = 8.449 \times 10^{-12} m^4
\]

While calculating the height of the steel ruler, the following phenomenon was observed: The etch markings on the steel ruler contributed a significant amount to the measurement of the height \( h = 0.0015 m \). We consider a case where the environmental stiffness \( K_E \) is assumed to be 100N/m. Using the values of:

\( L = 0.234 m, \ E = 69 \times 10^9 GPa, \ b = 0.03 m \)

we reverse calculate the height of the beam as follows:

\[
h = \sqrt[3]{\frac{12KL^3}{3bE}}
\]

which gives \( h \) as:

\[ h = 0.0014 m \]
This error of one ten-thousandth of an inch is due to the etching material coated on the steel ruler.

Assuming $h = 0.0015 \text{m}$ we calculate the environmental stiffness as follows:

$$K_E = \frac{3EI}{L^3}$$

$$K_E = \frac{3 \times 69 \times 10^9 \times 8.449 \times 10^{-12}}{(0.234)^3}$$

which gives the value of environmental stiffness as:

$$K_E \approx 136.50 \text{N/m}$$

To verify this calculated value, we perform the following experiment:

2.2.1 Case 1: Stylus + inkwell attached to the cantilever beam

We consider the case where the cantilever beam is subjected to its maximum non operating load i.e. the inkwell and the stylus are connected. The position at the end of the beam is measured which is termed as the 'original position' = 0.015m from the base(Figure 2.10). Known weights: aluminum disk of 0.077kg and steel ball of 0.0952kg are additively placed
Figure 2.10: With stylus and inkwell connected: original position of the cantilever beam at 0.0212m underneath the inkwell position on the cantilever beam. With each load, the displacement is measured.

Using the standard spring force equation:

\[ F = Kx \]

where: \( F = mg \)

'm' : mass of the known weights in kilograms

'g' : acceleration due to gravity = 9.81m/s\(^2\)

Plotting a graph of force'F' vs displacement’x’ we calculate the slope which is in turn our value of environmental stiffness \( K_E \) (spring rate).
As shown in Figure 2.11, an aluminum disk of known weight 0.077g causes a displacement of 0.0238m. The actual displacement is calculated as:

\[ \text{Actual displacement} = \text{original position} - \text{position due to weights} \]

Thus, the actual displacement observed for a mass of 0.077kg is 0.0026m.

As shown in Figure 2.12, with the steel ball placed alongwith the aluminum disk the total weight becomes 0.1722kg. The actual displacement is calculated as 0.0128m.
Using these data points i.e. displacement with no load, displacement with 0.077kg load and displacement with 0.1722kg load we plot the graph of force \( F \) vs displacement \( x \) as shown in Figure 2.13:

The slope of the line is calculated using the data tips shown in the plot(Figure 2.13).

\[
K_E = \frac{(1.688 - 0.7547)}{(0.0128 - 0.0026)}
\]

\[
\therefore K_E = 91.5 N/m
\]
2.2.2 Case 2: Inkwell attached to the cantilever beam without the Omni stylus

Figure 2.14: With no stylus attached to the cantilever beam, the displacement observed is 0.015m

We now consider a case where the stylus is disconnected from the cantilever beam. In such a case the minimum mass subjected to the cantilever beam is that of the inkwell (0.025g). The original position for such an arrangement is 0.015m as shown in Figure 2.14.

Now, adding a weight of 0.077kg to the cantilever beam causes a displacement of 0.0288. The actual displacement is thus 0.0288 - 0.015 = 0.0138m (Figure 2.15).

Figure 2.15: Adding a known weight of 0.077kg to the cantilever beam. The beam is now positioned at 0.0288m

Further addition of 75g to the cantilever beam causes the beam location to shift to 0.0308m as shown in Figure 2.16. The actual displacement recorded is 0.0158m.
Figure 2.16: Adding another known weight of 0.075kg to the cantilever beam. The total load on the beam is now 0.152kg.

Figure 2.17: Placing a steel ball and aluminum disk of 0.0952kg and 0.077kg respectively the cantilever beam. The total load on the beam is now 0.1722kg.

For a steel ball and an aluminum disk placed on the cantilever beam, the displacement observed is 0.0358m. The actual displacement is therefore calculated as 0.0208m (Figure 2.17).
Using these data points, the plot of force $F$ vs displacement $x$ is given by:

![Figure 2.18: Plot of Force $F$ vs displacement $x$ for only inkwell placement on the cantilever beam.](image)

The data is curve fitted (shown by a red line) to estimate the environmental stiffness as follows:

$$K_E = \frac{(1.586 - 1.004)}{(0.01995 - 0.01285)}$$

$$K_E \approx 82 N/m$$
2.2.3 Case 3: Inkwell attached to the cantilever beam with the weights placed on top of the inkwell

We now consider the case where only the inkwell is connected to the cantilever beam and the weights are placed on top of the inkwell.

Figure 2.19: With a minimum load from the inkwell, the cantilever beam is placed at 0.017m

With no weight attached to the cantilever beam, the only load is that of the inkwell with measured weight 0.025kg. As shown in Figure 2.19 the original position recorded is 0.017m.

Now by placing two aluminum disks of 0.077kg and 0.075kg on top of the inkwell, the total load to which the cantilever beam is subjected is 0.152kg. The actual displacement observed is 0.0175m. This is shown in Figure 2.20.

Figure 2.20: With load of 0.077kg + 0.075kg = 0.152kg placed on top of the inkwell, the cantilever beam is placed at 0.0345m
As shown in Figure 2.21 or a total external load of 0.0952kg (steel ball) + 0.077kg (aluminum disk 1) + 0.075kg (aluminum disk 2) = 0.2472kg, the actual displacement recorded is 0.0251m.

Using this data, the graph of force $F$ vs displacement $x$ is plotted as follows:

![Graph of force $F$ vs displacement $x$](image)

Figure 2.22: With the load placed on top of the inkwell the plot for force $F$ vs displacement $x$ is observed as above

The environmental stiffness is thus calculated as:

$$K_E = \frac{(2.423 - 1.49)}{(0.0251 - 0.0175)}$$

$$\therefore K_E = 122.76 \text{N/m}$$
From the above three experiments we observe the environmental stiffness value to be 91.5N/m, 82N/m and 122.76N/m respectively. Accounting for frictional forces and round off errors we can approximate the environmental stiffness value as an average of the three values obtained experimentally.

Averaging these values thus yields $K_E = 98.7N/m$ which for all practical purposes is approximated as $K_E = 100N/m$.

Further, to observe the relation between 'force' and 'displacement', we consider a one dimensional application of the Phantom Omni. We send out a force in negative 'y' direction to observe the displacement along the y-axis. With no damping i.e. $\zeta = 0$ the force inputted is a sinusoidal waveform given by:

$$F_y = a(1 - \cos(\omega t))$$

For $\omega = 0.25$ and $\omega = 0.5$, we vary the value of the amplitude factor 'a' from 0.5 to 5.0 in steps of 0.5. This results in the force value $F_y$ in the range of -1N to -10N.

<table>
<thead>
<tr>
<th>a</th>
<th>$F_y$(N)</th>
<th>$Displacement_y$(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-0.99993</td>
<td>-9.73909</td>
</tr>
<tr>
<td>1.0</td>
<td>-2</td>
<td>-14.0298</td>
</tr>
<tr>
<td>1.5</td>
<td>-2.99963</td>
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<td>-20.3996</td>
</tr>
<tr>
<td>2.5</td>
<td>-4.9997</td>
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</tr>
<tr>
<td>5.0</td>
<td>-9.99985</td>
<td>-19.6352</td>
</tr>
</tbody>
</table>

Table 2.1: Maximum force and displacement values corresponding to various values of amplitude factor 'a' for $\omega = 0.5$.
Figure 2.23: For various values of amplitude factor ‘a’ the saturation is observed at -4N

For $F_y = -1$ to -4N, the relation between displacement in y-direction and $F_y$ is linear. For a force beyond -4N, there is saturation in the spring force and hence the nonlinearity is apparent (Figure 2.23).

Figure 2.24: For an input force the spring force exerted by the cantilever beam is observed

As can be seen from the Figure 2.24 there is a decrease in the amplitude of the sine wave observed at the output of the Phantom Omni. This decrease in the amplitude is due to the frictional forces present inside the Phantom Omni.
2.3 Calculating the natural frequency and damping ratio

Figure 2.25: Step response of the Phantom Omni for forces in the range from 1N to 4N

For the one dimensional application of the Phantom Omni, a force in the negative y-direction ranging from 1N to 4N was inputted to observe the displacement in y-direction. This force was inputted as a step force decreasing in magnitude with every 10 seconds (Figure 2.25). This resulted in a scenario where the Phantom Omni was subjected to disturbances of varied amplitudes. For every transition, the response of the Omni was observed. This was useful in determining the transient characteristics of the system.

The main operational device inside the Phantom Omni is a DC motor. Using section 2.1 we can say that the equation governing the robotic arm is a standard second order system. This allows us to use the standard transient response characteristics associated with a second order system. The peak overshoot for the system was calculated using the reading in green highlighted box shown in Figure 2.25.
Using the data points in Figure 2.26 we calculate the peak overshoot $M_p$ as follows:

$$M_p = \frac{(-7.507 - (-6.324))}{(-7.507)}$$

$$M_p = 0.1575$$

Using the standard formula:

$$M_p = \exp \frac{-\zeta \pi \sqrt{1 - \zeta^2}}{\pi} \times 100$$

the damping ratio was calculated as:

$$\zeta = 0.899 \approx 0.9$$
Figure 2.27: For a step input with reference to the $\sigma - j\omega$ axis, the x-axis represents the attenuation $\sigma = \zeta \omega_n$.

The displacement along the x-axis which is equivalent to $\zeta \omega_n$ or the attenuation was found out to be 0.3.

$$\zeta \omega_n = 0.3$$

$$\therefore \omega_n = \frac{0.3}{\zeta} = 0.3333 \text{ rad/sec}$$

Using these values of $\omega_n = 0.3333 \text{ rad/sec}$, $\zeta = 0.9$ and $K_E = 100 \text{ N/m}$ we have designed the second order representation for the Phantom Omni.

Since, two Phantom Omnis are utilized to represent the master and slave systems respectively; their architectures are same i.e. they are represented using the same second order system.
CHAPTER 3. CONTROL SYSTEM DESIGN

With the experimentally calculated values of damping ratio \( \zeta \), natural frequency \( \omega_n \) and environmental stiffness \( K_E \) we now construct a block diagram structure to observe the effects of various dynamics introduced by the mathematical modeling.

![Figure 3.1: A basic block diagram representing the master-slave system](image)

Figure 3.1 shows the basic model of a LTI SISO system with force as the input variable and position as the output variable. The operator in the virtual environment controls the motion of the master robot. Hence, the input to the system is the hand force commanded by the operator. The hand force is variable in nature depending on whether the handle is held firmly or lightly, whether the arm is bent or outstretched.

The actions performed by the master robot are conveyed using a communication channel (represented as \( \Delta T \)) to the slave robot. The slave robot in the actual environment experiences certain oppositions. These oppositions as mentioned in section 2.2 are primarily the environmental stiffness and the frictional forces experienced by the Phantom Omni.
The slave robot tries to imitate the actions of the master robot in the actual environment. This slave imitation data is feedback to the master robot via a communication channel. The data contains the current position of the slave robot which is incorporated with the environmental stiffness $'K_E'$. Thus, the spring force experienced by the slave robot in the environment forms the feedback loop.

This feedback spring force is applied to the master robot in terms of voltage. This conversion of force to voltage is done using a control gain of the form:

$$\text{Control Gain} = \frac{R_A}{K_{mN}}$$

The architecture of the Phantom Omni is such that it allows for an internal adjustment of voltage to force conversion. Hence, it is no longer impertinent to represent the control gain and the conversion factor from voltage to force.

Compensating the force to voltage conversion blocks we can form the block diagram as follows:

![Block Diagram](image)

**Figure 3.2: Modified block diagram representing the master-slave system**

The force from the master robot is sent to the slave robot using a communication channel and vice versa. The communication channel proposed is User Datagram Protocol(UDP). It is observed that the communication channel creates a delay in the transmission of data from master robot to the slave robot and vice versa. This communication delay significantly impacts the stability of the system thus making the loop prone to undesirable behaviors.
For the slave system, the standard second order equation has been used as follows:

\[
\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]

If we represent our slave system as a mass spring damper system, the second order equation becomes:

\[
1 = \frac{1}{Ms^2 + Bs + KE}
\]

Thus, the above equations are equivalent and hence we can express mass \( M \), damping \( B \) as follows:

\[
M = \frac{KE}{\omega_n^2}
\]

\[
B = 2 \times \zeta \times \omega_n \times M
\]

In regards to the time constant form used to represent the master system this can be equivalently written as:

\[
\tau = \frac{M}{B}
\]

\[
C = \frac{1}{B}
\]
### 3.0.1 Case 1: No delay and no dynamics

We first consider the ideal case where the system experiences no instability. This consists of the master system and an accurate execution of the operator's commands in the actual environment. The system equation thus becomes:

$$\frac{x}{F_{\text{hand}}} = \frac{C}{\tau s^2 + s + CK_E}$$

At steady state the transfer function becomes:

$$\frac{x}{F_{\text{hand}}(s=0)} = \frac{1}{K_E}$$

which shows that the system behavior solely depends on the environmental stiffness and its effects on the manipulator behavior. Any instability due to the environment will affect the stability of the system. It also shows that any amount of operator's force will include a factor of the environmental stiffness. For e.g. if we command the Phantom Omni to travel for 1m with $K_E = 100\, N/m$ the actual travel will be 0.01m.

### 3.0.2 Case 2: No delay with dynamics

We now introduce the dynamics caused by the environment and its association with the slave system. Using the experimental data from section 2.2 we have determined the values for damping ratio $\zeta = 0.9$, natural frequency $\omega_n = 0.3333 \, \text{rad/sec}$ and environmental stiffness $K_E = 100\, N/m$. The damping ratio '$\zeta$' and '$\omega_n$' have been measured within the environment.
Since the environmental stiffness is observed to be varying the damping ratio $\zeta$ and natural
frequency $\omega_n$ add dynamics which can destabilize the system. This is shown in Figure 4.6.

![Figure 3.4: No delay with dynamics: master-slave system](image)

The transfer function for such a system becomes:

$$\frac{x_{\text{slave}}}{F_{\text{hand}}} = \frac{C\omega_n^2}{(\tau s^2 + s)(s^2 + 2\zeta\omega_ns + \omega_n^2) + (C\omega_n^2K_E)}$$

For the steady state condition the transfer function becomes:

$$\frac{x_{\text{slave}}}{F_{\text{hand}}(s=0)} = \frac{1}{K_E}$$

This shows that the stability of the system depends on dynamic nature of the environment
of operation.

### 3.0.3 Case 3: With delay and dynamics

The delay included in the block diagram is shown as $\Delta T$. However, in actual calculations
the delay is represented as $e^{-sT}$. To express this delay of $e^{-sT}$, mathematically we use Pade approximation to represent the Taylor series with an approximate value. In the present work, we have used the first order Pade approximation to represent the time delay as:

$$e^{-sT} = \frac{-s + a}{s + a}$$

Since the time delay model introduces a zero in the right half s-plane, the system is subjected
to instabilities. The system thus becomes a non-minimum phase system.[9] This is shown in
Figure 3.5: With delay and dynamics: master-slave system

The system transfer function for such a system is calculated to be as follows:

\[
\frac{x_{slave}}{F_{hand}} = \frac{C\omega_n^2(-s+a)(s+a)}{(\tau s + 1)(s+a)(s^2 + 2\zeta\omega_n s + \omega_n^2)(s+a) + C K_E (-s+a)^2 \omega_n^2}
\]

At steady state:

\[
\frac{x_{slave}}{F_{hand}(s=0)} = \frac{1}{K_E}
\]

Thus it can be seen that the effects of the non-minimum phase contribute to the transient effect of the system. The steady state solely depends on the effects of the environment in which the slave performs its operation.
3.0.4 Case 4: With delay, dynamics and compensation

To compensate for the varying nature of the environmental stiffness and the time delays, we introduce a PD controller. The controller is designed in association with the slave system and hence the slave system can be modeled as a position input and position output system.

The PD controller is able to stabilize the system although unstable poles are observed.

The transfer function thus becomes:

\[
\frac{x_{slave}}{F_{hand}} = \frac{C\omega_n^2(-s + a)(s + a)}{(\tau s + 1)(s)(s + a)^2(s^2 + 2\zeta\omega_n s + \omega_n^2) + CK_E\omega_n^2(-s + a)^2(K_p(s + a_c))}
\]

At steady state:

\[
\frac{x_{slave}}{F_{hand}(s=0)} = \frac{1}{K_p a_c K_E}
\]

Thus the proportional gain and the derivative gain act as a compensation for any instabilities due to the environmental stiffness.
CHAPTER 4. RESULTS

With reference to the design scenarios discussed in chapter 3, we now use MATLAB to simulate the results for the closed loop system response.

\[ F_{\text{hand}} \xrightarrow{F_{\text{net}}} C = \frac{1}{\tau s + 1} \xrightarrow{\dot{x}_{\text{master}}} \frac{1}{s} x_{\text{master}} \rightarrow \Delta T \rightarrow \frac{1}{K_E} \rightarrow \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \rightarrow x_{\text{slave}} \]

Figure 4.1: Basic block diagram representing the master-slave system

With damping ratio \( \zeta = 0.9 \), natural frequency \( \omega_n = 0.3333 rad/sec \) and environmental stiffness \( K_E = 100 N/m \), the values of 'C', 'τ', 'M' and 'B' are computed as follows:

\[
M = \frac{K_E}{\omega_n^2}
\]

\[
M = 900 kg
\]

\[
B = 2 \times \zeta \times \omega_n \times M
\]

\[
B = 540 Ns/m
\]
Using these values, we compute the transfer functions for each of the cases mentioned in chapter 3. The root locus technique has been utilized to observe the behavior of the closed loop system and the impact of delay on such systems. The step response is used to analyze the accuracy of our design in the event of disturbances due to varying nature of the environment or the delays in communication channel.

All the results tested so far are for a time delay of $T = 0.0001$ seconds

4.0.1 Case 1: No delay, no dynamics

\[
\frac{x}{F_{\text{hand}}} = \frac{0.001852}{1.667s^2 + s + 0.1852}
\]

The root locus plot observed for this block diagram is as follows:
The root locus shows a pair of complex conjugate poles in the left half s-plane indicating that the system is stable.

For a step input the response is recorded as follows:

The step response indicates that the system stabilizes within a settling time of 14.1 seconds since no delay or dynamics interfere with it’s stability.

This shows that for any given input from the master robot, the slave robot will take 14.1 seconds to respond to the input.
4.0.2 Case 2: No delay, with dynamics

\[
\begin{aligned}
F_{\text{hand}} & \rightarrow C \frac{1}{\tau s + 1} x_{\text{master}} \rightarrow \frac{1}{s} x_{\text{master}} \rightarrow \frac{1}{K_E} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \rightarrow x_{\text{slave}} \\
&\downarrow \quad F_E \\
&\downarrow \quad K_E
\end{aligned}
\]

Figure 4.5: No delay with dynamics: master-slave system

\[
\frac{x_{\text{slave}}}{F_{\text{hand}}} = \frac{2.058 \times 10^{-6}}{1.667s^4 + 2s^3 + 0.7852s^2 + 0.1111s + 0.0002058}
\]

With the dynamics of the slave + environment system added to the closed loop transfer function, the root locus now becomes:

Figure 4.6: Root locus of the system with dynamics and no delay

The two complex conjugate poles are due to the slave system while the two single poles: one at the origin and the other at $\frac{1}{\tau}$ belong to the master system. The pole at the origin and one of the complex conjugate poles converge to the right half of the $s$-plane thus marking that region as unstable.
Figure 4.7: Step response of the system with dynamics and no delay

Due to the pole at the origin and the complex pole converging in the right half s-plane the system takes longer time to stabilize (Settling time = $2.09 \times 10^3$ seconds). This response to a step input is shown in Figure 4.7.
4.0.3 Case 3: With delay and dynamics

With delay introduced into the system as a Pade approximation of 1st order, we now compute the transfer function of the system as follows:

\[
x_{\text{slave}} = \frac{-2.058 \times 10^{-6}s^2 + 823}{1.667s^6 + 6.667 \times 10^4s^5 + 6.667 \times 10^8s^4 + 8 \times 10^8s^3 + 3.141 \times 10^8s^2 + 4.444 \times 10^7s + 8.23 \times 10^4}
\]

This block diagram shows that there is a proportional gain \( K_p = 1 \) present in the system.

The root locus for such a system was observed to be as follows:

Figure 4.9: Root locus of the system with delay and dynamics
Due to the Pade approximation, there is an introduction of zero in the right half s-plane. The pole of the Pade approximation is canceled by another zero created during the formulation of the transfer function.

For a system with time delay, the system settles at $1.17 \times 10^3$ seconds. This settling time is less than the one observed in section 4.0.2. It is because the dominant roots belong to the Pade approximated time delay system and the slave dynamics due to environment have very little effect on the stability of the system.
4.0.4 Case 4: With delay, dynamics and compensation

To tune the PD controller, Ziegler-Nichols method was used. The integral gain $K_I$ and the derivative gain $K_d$ were made zero. The proportional gain $K_p$ was increased to a value such that sustained oscillations were observed.\[9\]

The sustained oscillations were observed for $K_p = 122$. This is the critical gain $'K_u'$. The period of oscillation for this critical gain is $'T_u = 30sec'$. This is shown in Figures 4.13 and 4.14.

Using the predefined formulae for designing a Ziegler Nichols based PD controller \[10\]:

$$\frac{F_{hand}}{\tau s + 1} x_{master} \left( \frac{1}{s} \right) x_{master} \left( \frac{-s + a}{s + a} \right) \frac{1}{K_E} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} x_{slave}$$

Figure 4.12: With delay, dynamics and compensation: master-slave system

Figure 4.13: Ziegler Nichols tuning method: sustained oscillations for a critical gain value $K_u = 122$
Figure 4.14: Ziegler Nichols tuning method: Measuring the period of sustained oscillations as $T_u = 30\text{sec}$

\[
K_p = 0.8K_u = 97.6
\]

\[
T_d = \frac{T_u}{8} = 3.75
\]

the PD controller equation becomes:

\[
C(s) = K_p(1 + T_d s)
\]

\[
C(s) = 97.6(1 + 3.75s)
\]

\[
C(s) = 97.6 + 366s
\]

which becomes:

\[
K_p(s + a_c) = 26.02(s + 0.266)
\]
\[
\frac{x_{\text{slave}}}{F_{\text{hand}}} = \frac{-2.058 \times 10^{-6} s^2 + 823}{1.667 s^6 + 6.667 \times 10^4 s^5 + 6.667 \times 10^8 s^4 + 8 \times 10^8 s^3 + 3.141 \times 10^8 s^2 + 7.457 \times 10^7 s + 8.033 \times 10^6}
\]

Using this controller, the root locus becomes:

Figure 4.15: Root locus of PD controller designed using Ziegler Nichols tuning method: \(K_p = 97.6\) and \(K_d = 366\)

Figure 4.16: Zoomed Root locus of PD controller designed using Ziegler Nichols tuning method: \(K_p = 97.6\) and \(K_d = 366\)
Figure 4.17: Step response of PD controller designed using Ziegler Nichols tuning method: \( K_p = 97.6 \) and \( K_d = 366 \)

The step response is observed in Figure 4.17. This shows that even though there are unstable poles and zeros in the transfer function of the master-slave system, the PD controller is able to compensate for these instabilities and provide for a settling time of 24.9 seconds.

Further using some manual computation, the \( K_p \) value was adjusted to 95 and the \( K_d \) value was changed to 410. This yielded a system with better settling time.

Figure 4.18: Root locus of PD controller designed using manual parameters: \( K_p = 95 \) and \( K_d = 410 \)
Figure 4.19: Zoomed Root locus of PD controller designed using manual parameters: $K_p = 95$ and $K_d = 410$

The settling time of this system was observed to be 17.3 seconds which is better compared to that obtained using the Ziegler-Nichols tuning method (Figure 4.20).

Figure 4.20: Step response of PD controller designed using manual parameters: $K_p = 95$ and $K_d = 410$
CHAPTER 5. CONCLUSION

In this report we introduce the concept of time delay and its implications to various fields of applications requiring human safety and intervention. We introduced the Phantom Omni haptic devices as the hardware to design the system identification parameters: damping ratio $\zeta$ and natural frequency of oscillation $\omega_n$. We further use the Phantom Omni haptic device to determine the environmental stiffness value. This forms our system identification process. Using these parameters has helped us in simplifying the mathematical model for our master-slave system. Developing the master and slave systems as standard second order systems has allowed us to use the standard transient response characteristics to study the system behavior.

To simplify the analysis, we propose the use of classical control methods to determine the system stability regions. Using the root locus technique helps us in building an intuition of the system behavior in event of large time delays. As can be seen, a proportional plus derivative (PD) controller can provide for a sufficient control during time delayed operation. To enhance the design further, a lead compensator can be designed to accurately design the location of the compensator pole to cancel out the unwanted effects of the time delay system zeros.
APPENDIX A. SPECIFICATION SHEET
Specifications for the
PHANTOM Omni® haptic device

The SensAble Technologies PHANTOM® product line of haptic devices makes it possible for users to touch and manipulate virtual objects. Different PHANTOM devices meet varying needs. The Premium models are high-precision instruments and, within the PHANTOM product line, provide the largest workspaces and highest forces, and some offer 6DOF (6 degrees of freedom) output capabilities. The PHANTOM Omni model is the most cost-effective haptic device available today. Portable design, compact footprint, and IEEE-1394a FireWire® port interface ensure quick installation and ease-of-use.

<table>
<thead>
<tr>
<th>Model</th>
<th>The PHANTOM Omni Device</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force feedback workspace</td>
<td>~6.4 W x 4.8 H x 2.8 D in&lt;br&gt; &gt; 160 W x 120 H x 70 D mm</td>
</tr>
<tr>
<td>Footprint</td>
<td></td>
</tr>
<tr>
<td>Physical area the base of device occupies on the desk</td>
<td>6 5/8 W x 8 D in&lt;br&gt; ~168 W x 203 D mm</td>
</tr>
<tr>
<td>Weight (device only)</td>
<td>3 lb 15 oz</td>
</tr>
<tr>
<td>Range of motion</td>
<td>Hand movement pivoting at wrist</td>
</tr>
<tr>
<td>Nominal position resolution</td>
<td>&gt; 450 dpi&lt;br&gt; ~ 0.055 mm</td>
</tr>
<tr>
<td>Backdrive friction</td>
<td>&lt;1 oz (0.26 N)</td>
</tr>
<tr>
<td>Maximum exertable force at nominal (orthogonal arms) position</td>
<td>0.75 lbf. (3.3 N)</td>
</tr>
<tr>
<td>Continuous exertable force (24 hrs.)</td>
<td>&gt; 0.2 lbf. (0.88 N)</td>
</tr>
<tr>
<td>Stiffness</td>
<td>X axis &gt; 7.3 lb/in (1.26 N/mm)&lt;br&gt; Y axis &gt; 13.4 lb/in (2.31 N/mm)&lt;br&gt; Z axis &gt; 5.9 lb/in (1.02 N/mm)</td>
</tr>
<tr>
<td>Inertia (apparent mass at tip)</td>
<td>~0.101 lbm. (45 g)</td>
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<td>Force feedback</td>
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<tr>
<td>Position sensing</td>
<td>x, y, z (digital encoders)</td>
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<td>[Stylus gimbal]</td>
<td>[Pitch, roll, yaw (± 5% linearity potentiometers)]</td>
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<tr>
<td>Interface</td>
<td>IEEE-1394 FireWire® port: 6-pin to 6-pin*</td>
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<td>Supported platforms</td>
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<tr>
<td>OpenHaptics® SDK compatibility</td>
<td>Yes</td>
</tr>
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</table>

*Please visit the Support and Resources section of our website for more information www.sensable.com/support-overview.htm.
Bibliography


