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Ultrasonic beam models for the generation of surface waves and plate waves with angle beam transducers

Lester W. Schmerr Jr.
Iowa State University, lschmerr@iastate.edu

Alexander Sedov
Lakehead University

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Abstract
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Keywords
transducers, ultrasonic applications, dispersion (wave), integral equations, nondestructive evaluation, QNDE, Aerospace Engineering

Disciplines
Aerospace Engineering | Materials Science and Engineering | Mechanical Engineering | Structures and Materials

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ULTRASONIC BEAM MODELS FOR THE GENERATION OF SURFACE WAVES AND PLATE WAVES WITH ANGLE BEAM TRANSDUCERS

Lester W. Schmerr Jr.[1,2] and Alexander Sedov[3]

[1]Center for NDE, Iowa State University, Ames, IA, 50011, USA
[2]Dept. of Aerospace Eng., Iowa State University, Ames, IA, 50011, USA
[3]Dept of Mech. Eng., Lakehead University, Thunder Bay, ON, Canada P7B 5E1

ABSTRACT. A new transducer beam model is developed for the generation of Rayleigh surface wave and Lamb plate waves by an angle beam transducer, which is one of the configurations commonly used for the generation of these wave types. The beam model is a fully 3-D model that takes into account the fields generated by the transducer acting on the surface of a wedge. The model also accounts for the leaky wave nature of the surface or plate waves under the wedge surface through an explicitly defined leaky wave attenuation coefficient.

Keywords: Ultrasonic Beam Model, Surface Waves, Plate Waves
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INTRODUCTION

In ultrasonic inspections with surface waves and plate waves, angle beam wedge transducers are commonly used to generate the interrogating waves. However, to date there are no effective models of such transducers similar to those that have been developed for bulk wave angle beam transducers [1], [2]. Instead, most model studies often rely on the approach of Viktorov [3] where the wedge geometry is replaced by a specified stress distribution on a stress-free surface. As Viktorov points out this procedure is inadequate because: 1) it does not take into account the waves as they are actually generated in the transducer wedge, 2) the simple stress distributions used are not representative of the actual stresses present, and 3) the model ignores the attenuation of the waves as they are generated under the wedge. In addition to using these simplifying assumptions many studies also only consider 2-D models [4]. Here, we will present a fully 3-D model of an angle beam transducer that generates surface or plate waves, a model that addresses all of the inadequacies mentioned. The model uses a combination of high frequency asymptotics and Greens functions for surface and plate waves to describe the waves present. The attenuation of the waves under the wedge is included through an attenuation term that is derived by an elementary power balance relationship.

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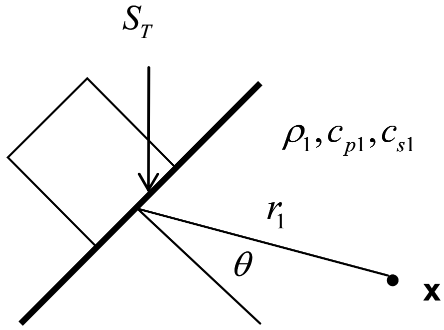

FIGURE 1. The geometry configuration for a compressional wave transducer on the surface of a wedge.

ELEMENTS OF THE BEAM MODELS

Consider first a contact transducer that is placed on the surface of a wedge, as shown in Fig. 1. It is assumed that this transducer acts as a constant pressure source over an area, S_T. Although this transducer generates waves of multiple types, including P-waves, S-waves, head waves, and Rayleigh surface waves, the most significant waves present for generating disturbances in the underlying solid are the P-waves [2]. The vector displacement, $\mathbf{u}(\mathbf{x},\omega)$ at a point, $\mathbf{x}$, in the wedge for these radiated P-waves can be described by a Rayleigh/Sommerfeld type of integral [2]:

$$\mathbf{u}(\mathbf{x},\omega) = \frac{p_0}{2\pi\rho_1 c_{p1}^2} \int_{S_T} K_p(\theta)\, \mathbf{d}_1^p \frac{\exp(ik_{p1}r_1)}{r_1} dS \tag{1}$$

where p_0 is the pressure on the transducer face, (ρ_1, c_{p1}) are the density and compressional wave speed of the wedge, respectively, and $k_{p1} = \omega / c_{p1}$ is the corresponding wave number, where ω is the circular frequency. The unit vector, $\mathbf{d}_1^p$, describes the polarization of the P-wave and $K_p(\theta)$ is a known directivity function [2].

The P-waves generated by the transducer interact with the lower surface of the wedge, which is assumed to be in smooth contact with the underlying solid. In practice a thin fluid couplant layer exists at this interface, but the thickness of the couplant is neglected in this model. The incident P-wave generates reflected P- and S-waves in the wedge. It is assumed that these reflected waves can be obtained from the incident spherically spreading P-waves contained in Eq. (1) by assuming the interactions of these waves with the lower surface of the wedge can be treated locally as plane wave interactions. This is a high frequency, Kirchhoff-like assumption and allows one to explicitly obtain the pressure, $p(\mathbf{x}_s,\omega)$ generated at any point, $\mathbf{x}_s$, on the lower surface of the wedge as

$$p(\mathbf{x}_s,\omega) = \frac{-i\omega p_0}{2\pi\rho_1 c_{p1}^2} \rho_2 c_{s2} T \int_{S_T} K_p(\theta) \frac{\exp(ik_{p1}r_1)}{r_1} dS \tag{2}$$

where (ρ_2, c_{p2}, c_{s2}) are the density, compressional wave speed, and shear wave speed, respectively, of the solid beneath the wedge, and T is given by

$$T = \frac{c_{s2}}{c_{p2}} T^{P;P} \left(\frac{c_{p2}^2}{c_{s2}^2} - 2\frac{c_{p2}^2}{c_n^2} \right) - 2iT^{S;P} \frac{c_{s2}}{c_n} \sqrt{\frac{c_{s2}^2}{c_n^2} - 1} \tag{3}$$

where $(T^{P;P}, T^{S;P})$ are ordinary plane wave transmission coefficients for P-waves and S-waves, respectively for two planar elastic solids in smooth contact [2].

If the pressure at the interface between the wedge and the underlying solid is known, this pressure can be used in conjunction with the Green's function for surface waves or plate waves, $G_{ij}(\mathbf{x}_s, \mathbf{x})$, in an integral representation integral [2] to obtain the velocity components, $v_i(\mathbf{x})$, at any point, $\mathbf{x}$, in the underlying solid as

$$v_i(\mathbf{x}) = \int_S -i\omega p(\mathbf{x}_s, \omega) G_{3i}(\mathbf{x}, \mathbf{x}_s) dS(\mathbf{x}_s) \tag{4}$$

This is an approach similar to that of Viktorov [3], but with two major differences. First, the pressure distribution being used here comes directly from a transducer beam model, instead of a simple, prescribed distribution. Second, the Green's function being used is an explicit 3-D function, valid for a high frequency Rayleigh wave or Lamb wave mode traveling in an isotropic elastic solid given by [5]

$$G_{ij}(\mathbf{x}, \mathbf{x}_s) = \frac{1}{4P_n c_n} \frac{\exp(ik_n r_2 + i\pi/4)}{\sqrt{2\pi k_n r_2}} p_{ni}^*(x_{s3}) p_{nj}(x_3) \tag{5}$$

where the free surface (for a Rayleigh wave) or the free surfaces of a plate (for Lamb waves) are described in (x_1, x_2) coordinates. For Rayleigh waves the x_3 axis is taken from the free surface into the underlying solid while for Lamb waves the x_3 axis is taken from the center of the plate, which is assumed to be of thickness, $2h$. The radius $r_2 = \sqrt{(x_1 - x_{s1})^2 + (x_2 - x_{s2})^2}$ and c_n is the phase velocity for the nth traveling mode. Although for plate waves an angle beam transducer may generate more than one mode, we will consider only a single generated mode in our beam model. Results for multiple generated modes can then be obtained by simply introducing a summation. The "polarization" terms in Eq. (5) are given as

$$\mathbf{p}_n(x_3)=\begin{Bmatrix}\hat{v}_{n1}(x_3)\dfrac{(x_1-x_{s1})}{r_2}\\ \hat{v}_{n1}(x_3)\dfrac{(x_2-x_{s2})}{r_2}\\ i\hat{v}_{n2}(x_3)\end{Bmatrix},\ \mathbf{p}_n(x_{s3})=\begin{Bmatrix}\hat{v}_{n1}(x_{s3})\dfrac{(x_1-x_{s1})}{r_2}\\ \hat{v}_{n1}(x_{s3})\dfrac{(x_2-x_{s2})}{r_2}\\ i\hat{v}_{n2}(x_{s3})\end{Bmatrix} \tag{6}$$

where $(\ )^*$ in Eq. (5) denotes the complex conjugate and the "power flow" term for the *n*th mode, P_n, is

$$P_n=\frac{1}{2}\rho_2 c_{gn}\int\left(\left|\hat{v}_{n1}\right|^2+\left|\hat{v}_{n2}\right|^2\right)dx_3 \tag{7}$$

with the integration limits taken from zero to infinity for Rayleigh waves and from $-h$ to $+h$ for Lamb plate waves and where c_{gn} is the group velocity for the *n*th mode. Physically, P_n represents a normalized power/unit width in a traveling wave mode [6] (normalized because the functions in the integrand are non-dimensional). Both the polarization and power terms are given in terms of the $\left(\hat{v}_{n1}(x_3),\hat{v}_{n2}(x_3)\right)$ functions. These functions are just proportional to the ordinary modal functions used to describe the propagation of 2-D surface waves or plate waves in many textbooks [2]. Specifically, here we have for Rayleigh waves

$$\begin{aligned}\hat{v}_{n1}(x_3)&=\exp(-\alpha_{n1}x_3)-\frac{(2c_2^2-c_n^2)}{2c_2^2}\exp(-\alpha_{n2}x_3)\\ \hat{v}_{n2}(x_3)&=\frac{c_n\alpha_{n1}}{\omega}\exp(-\alpha_{n1}x_3)-\frac{\omega}{c_n\alpha_{n2}}\frac{(2c_2^2-c_n^2)}{2c_2^2}\exp(-\alpha_{n2}x_3)\end{aligned} \tag{8}$$

and for symmetric Lamb waves

$$\begin{aligned}\hat{v}_{n1}(x_3)&=\cosh(\alpha_{n1}x_3)-\frac{(2c_2^2-c_n^2)}{2c_2^2}\frac{\cosh(\alpha_{n1}h)}{\cosh(\alpha_{n2}h)}\cosh(\alpha_{n2}x_3)\\ \hat{v}_{n2}(x_3)&=\frac{-c_n\alpha_{n1}}{\omega}\sinh(\alpha_{n1}x_3)+\frac{\omega}{c_n\alpha_{n2}}\frac{(2c_2^2-c_n^2)}{2c_2^2}\frac{\cosh(\alpha_{n1}h)}{\cosh(\alpha_{n2}h)}\sinh(\alpha_{n2}x_3)\end{aligned} \tag{9}$$

For anti-symmetric Lamb waves, one can again use Eq. (9) if one replaces all the cosh functions with sinh functions and sinh functions with cosh functions. In both Eq. (8) and Eq. (9) we have

$$\alpha_{n1}=\omega\sqrt{\frac{1}{c_n^2}-\frac{1}{c_1^2}},\quad \alpha_{n2}=\omega\sqrt{\frac{1}{c_n^2}-\frac{1}{c_2^2}} \tag{10}$$

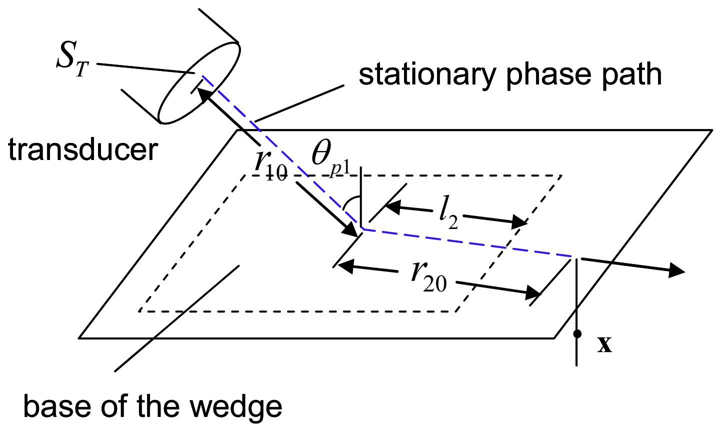

FIGURE 2. Geometry for an angle beam transducer generating a surface or plate wave, showing the stationary phase path from a point on the transducer to a point on the surface of the underlying solid.

Placing the pressure expression of Eq. (2) and the Green's function expression of Eq. (5) into Eq. (4), we then have an explicit expression for the velocity of the generated surface or plate wave given by

$$v_i(\mathbf{x}) = \frac{-\omega^2 p_0 \rho_2 c_{s2}}{8\pi P_n c_n \rho_1 c_{p1}^2} \frac{\exp(i\pi/4)}{\sqrt{2\pi k_n}} \int_{S_T}\int_{S} T\, K_p(\theta)\, p_3^*(0)\, p_i(x_3) \cdot \frac{\exp(ik_{p1}r_1)}{r_1} \frac{\exp(ik_n r_2)}{\sqrt{r_2}}\, dS\, dS_T \tag{11}$$

However, the integral over the plane surface which the wedge is in contact with can be done approximately at high frequencies by the method of stationary phase [2] to obtain

$$v_i(\mathbf{x}) = \frac{\omega^2 p_0 \rho_2 c_{s2}}{4 P_n c_n \rho_1 c_{p1} \cos\theta_{p1}} \frac{\exp(i\pi/4)}{\sqrt{2\pi k_n}} \int_{S_T} T\, K_p(\theta)\, p_3^*(0)\, p_i(x_3) \frac{\exp(ik_{p1}r_{10} + ik_n r_0)}{\sqrt{r_{20} + c_{p1} r_{10}/c_n}}\, dS_T \tag{12}$$

Here the distances (r_{10}, r_{20}) are taken along a stationary phase path from a point on the transducer surface to a point on the free surface directly above the point **x** at which we are evaluating the surface or plate wave fields, as shown in Fig. 2.

Equation (12) is almost a complete beam model for the generation of surface waves and plate waves with an angle beam wedge transducer. It addresses the first two deficiencies of the Viktorov approach mentioned in the introduction but it still does not account for the fact that the surface or plate waves can "leak" energy back into the wedge as long as they are propagating under the wedge surface. To account for this phenomena from a completely fundamental standpoint is difficult but one can take a more ad-hoc type of approach and include these leaky wave losses in the form of an attenuation coefficient,

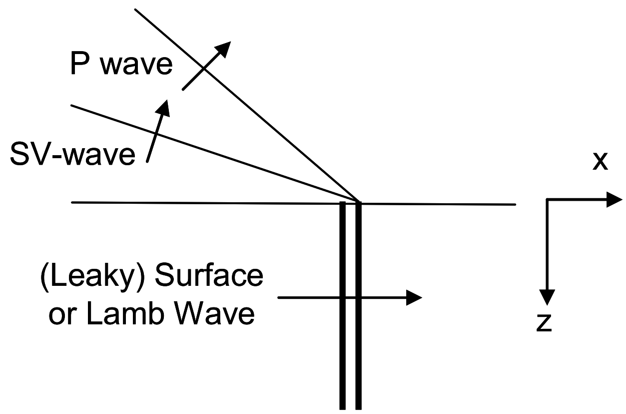

FIGURE 3. A 2-D propagating surface or plate wave and the compressional and shear waves they radiate into the wedge while propagating under the wedge surface.

similar to what is done to account for material attenuation [2]. The key, of course, is to obtain the leaky wave attenuation term explicitly. Fortunately, this can be done by considering a 2-D propagating surface or plate wave mode and the P- and SV-waves they radiate back into the wedge, as shown in Fig. 3. Through a simple power flow balance at the interface one can show that the linear attenuation coefficient, α_{nl} , for a leaky propagating surface or plate wave can be expressed as

$$\alpha_{nl} = \frac{Z_r \left|\hat{v}_{n3}(0)\right|^2}{4P_n} \tag{13}$$

where Z_r is the surface impedance for the waves radiated into the wedge, defined as

$$Z_r = \frac{-\sigma_{zz}\big|_{z=0}}{-\sum\limits_{\alpha=p,sv} v_z^{\alpha}} \tag{14}$$

where σ_{zz} is a normal stress and v_z^{α} is the normal velocity at the surface for a P- or SV-wave $(\alpha = p, sv)$. This surface impedance is, therefore, just the ratio of the pressure acting on the surface to the total normal velocity at the surface. Since both the pressure and velocity terms can be calculated explicitly, one finds

$$Z_r = \frac{\rho_1 c_{p1}}{\cos\theta_p}\left[\left(1-2\sin^2\theta_s\right)^2 + 4\left(\frac{c_{s1}}{c_{p1}}\right)\sin^2\theta_s\cos\theta_s\cos\theta_p\right] \tag{15}$$

with

$$\sin\theta_p = \frac{c_{p1}}{c_n}, \sin\theta_s = \frac{c_{s1}}{c_n} \tag{16}$$

We should note that if one sets the shear wave speed of the wedge equal to zero, Eq. (15) reduces to the simple result $Z_r = \rho_1 c_{p1} / \cos\theta_p$, which is the well-known result for a fluid-solid interface [6]. Since the surface impedance is known and the other terms in Eq. (13) can be calculated for a Rayleigh wave or Lamb wave mode, this attenuation coefficient can be found explicitly. For example, for a leaky Rayleigh wave we obtain for the remaining terms in Eq. (13)

$$\left|\hat{v}_{n3}(0)\right|^2 = \left|\alpha_1' - \frac{1}{2\alpha_2'}\left(2 - \frac{c_{r2}^2}{c_{s2}^2}\right)\right|^2 \tag{17}$$

and

$$P_n = \frac{\rho_2 c_{r2}}{16\left|k_{r2}\right|\alpha_2'}\left[8\left(2 - \frac{c_{r2}^2}{c_{s2}^2}\right) - 8\alpha_1'\alpha_2' - 4\frac{\alpha_1'}{\alpha_2'} - 4\frac{\alpha_2'}{\alpha_1'}\right] \tag{18}$$

where $c_n = c_{r2}$ is the Rayleigh wave speed for the material beneath the wedge and

$$\alpha_1' = \sqrt{1 - \frac{c_{r2}^2}{c_{p2}^2}}, \quad \alpha_2' = \sqrt{1 - \frac{c_{r2}^2}{c_{s2}^2}} \tag{19}$$

Similar explicit but more complex expressions for symmetrical and anti-symmetrical Lamb waves can also be obtained.

By including these leaky wave attenuation coefficients as well as material attenuation coefficients into the beam model of Eq. (12), one obtains a complete beam model for the generation of surface or plate waves with a wedge transducer as

$$\begin{aligned} v_i(\mathbf{x}) = {} & \frac{\omega^2 p_0 \rho_2 c_{s2}}{4P_n c_n \rho_1 c_{p1}\cos\theta_{p1}} \frac{\exp(i\pi/4)}{\sqrt{2\pi k_n}} \int_{S_T} \Big\{ T K_p(\theta) p_{n3}^*(0) p_{ni}(x_3) \\ & \cdot \frac{\exp(ik_{p1}r_{10} + ik_n r_0)}{\sqrt{r_{20} + c_{p1}r_{10}/c_n}} \exp(-\alpha_{p1}r_{10} - \alpha_n r_{20} - \alpha_{nl} l_2) dS_T \Big\} \end{aligned} \tag{20}$$

where α_{p1} is the material attenuation coefficient for P-waves in the wedge, α_n is the material attenuation for a wave mode n in the underlying solid, and l_2 is the portion of the distance traveled beneath the wedge surface along the stationary phase path (see Fig. 2). Equation (20) is similar to bulk wave angle beam transducer models where the transducer wave field is expressed as a superposition of spherical spreading bulk waves from point sources on the face of the transducer [2]. However, here we are the superimposing cylindrically spreading surface or plate waves from those point sources instead.

SUMMARY AND CONCLUSIONS

We have shown how to develop a beam model for the generation of Rayleigh waves or Lamb waves with an angle beam wedge transducer that addresses all the deficiencies of the simple model proposed by Viktorov. Equation (20) is based on having an explicit form of the Green's function for surface or plate waves (see Eq. (5)). That Green's function is valid for planar surfaces, planar plates, and other planar layered media geometries. Thus, our beam model is strictly only valid for those types of planar geometries. However, Eq. (20) could likely be applied to curved geometries if the curvature is not too severe. Also, Eq. (20) can be applied to angle beam inspections with other types of waves other than the Rayleigh waves or Lamb waves discussed here such as the dispersive Rayleigh waves that exist at a solid with a low speed surface layer [7]. Thus, we have developed a new and highly useful beam model for a variety of inspection problems.

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