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Examining One Mathematics Teacher's Decisions Regarding Mathematics and Language

Ji Yeong I
Iowa State University, jiyeongi@iastate.edu

Zandra de Araujo
University of Missouri–Columbia

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Abstract
Teachers have to make many in-the-moment decisions when teaching. We investigated one teacher's decisions in response to the difference between the intended meaning of a mathematical problem and her student's understanding. The student—an English language learner—had a different interpretation of the mathematical scenario related to one particular clause in the problem that was, ironically, intended to be explanatory but ended up obscuring intended meaning and therefore impacted the student's solution. In order to reflect on the teacher's decisions, we include a vignette that illustrates the teacher's tensions when making her instructional decisions. The vignette is followed by the teacher's rationale for her decisions and an analysis of the episode. We invite readers to participate in her decision-making process and explore impacts of each decision.

Disciplines
Science and Mathematics Education

Comments
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Abstract

Teachers have to make many in-the-moment decisions when teaching. We investigated one teacher's decisions in response to the difference between the intended meaning of a mathematical problem and her student's understanding. The student—an English language learner—had a different interpretation of the mathematical scenario related to one particular clause in the problem that was, ironically, intended to be explanatory but ended up obscuring intended meaning and therefore impacted the student's solution. In order to reflect on the teacher's decisions, we include a vignette that illustrates the teacher's tensions when making her instructional decisions. The vignette is followed by the teacher's rationale for her decisions and an analysis of the episode. We invite readers to participate in her decision-making process and explore impacts of each decision.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Have you ever had a moment when you notice your student interprets a direction or a problem statement differently from the intended meaning when working on mathematical problems? If so, how did you recognize it and what did you do?

2. What types of decisions do teachers of English language learners (ELLs) face when enacting mathematics word problems using complicated language with students?

3. When working with an ELL on mathematics, how do you decide whether your students' misunderstandings, if any, stem from language, mathematics, culture differences, or some combination of these areas?

Ji-Yeong I (jiyeongi@iastate.edu) is an assistant professor of mathematics education at Iowa State University. She taught mathematics and science for diverse students in urban areas before she started her research career. Her research interests include English language learners, teacher education, and culturally responsive teaching.

Zandra de Araujo (dearaujoz@missouri.edu) is an assistant professor of mathematics education at the University of Missouri. Her research examines teachers' use of curriculum, particularly with English language learners, and the preparation of elementary mathematics teachers.

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Teachers make many pedagogical decisions daily. Schoenfeld (2011) described teachers' decision-making processes as "the selection of goals consistent with the teachers' resources and orientations" (p. 460). In other words, a teacher's decisions should be made in accordance to his or her goals for students, while also taking their individual beliefs about learning, access to curriculum materials or technology, and expectations for performance based on cultural and linguistic standards into account. Moreover, the decisions, made consciously or unconsciously, have consequences that may or may not be evident in the short term.

Imagine you are a mathematics teacher and you just discovered your student interpreted a problem statement differently from the intended meaning. Should you address this misinterpretation immediately, or should you wait until he or she realizes it on his or her own? And what if the student is an English language learner (ELL)? How can you figure out if the different interpretation is due to the student's English or to the student's mathematical fluency? Linguists have discussed the importance of distinguishing mistakes from errors when working with language learners. Brown (2007) characterized errors as fixed habits that cannot be self-corrected. Errors stem from a lack of knowledge of language conventions. For example, a student may incorrectly say "my four dog" repeatedly because he or she may not have learned the rule regarding plural nouns. In contrast, mistakes are the result of a temporary stumble. Mistakes, also referred to as slips or lapses, can be self-corrected because they do not result from a lack of understanding (Brown, 2007). For example, if someone writes "Angie is to nice" rather than "Angie is too nice" because they were typing quickly, this constitutes a mistake because the person knows the correct form. Therefore, it is essential to determine if a student's different language use or interpretation is a mistake or an error if a teacher is to enact the proper response.

In this paper, we analyze a single episode of a teacher experiencing tensions between mathematics and language when deciding how to respond to an ELL who misinterpreted a task statement. This student's response does not fall neatly into the category of mistakes or errors (Brown, 2007) because it was not due to Henry's English structure or grammar but to his interpretation of the problem statement overall. Therefore, we use the term *misinterpretation,* rather than mistake or error, to describe this situation. In addition to our analysis of the episode, we also provide the teacher's insight into how she perceived the student's misinterpretation. By providing both a researcher's and a teacher's perspective, we hope to shed light on differing accounts and the various aspects to which observers might attend (Boaler & Humphreys, 2005). The reader might similarly examine the vignette and consider the instructional decisions he or she might make in the moment regarding the same types of situations.

Context

Henry, a third-grade Chinese student, had been living in the United States for two years when he participated in our study. He attended a public elementary school located in a small city and was identified as an ELL by his school district. Though we did not have access to Henry's English proficiency level, we noted, and his teacher confirmed, that his informal English was fluent when engaging in everyday conversation. Henry was confident in mathematics and often expressed his fondness for the subject. During our interview, Henry was eager to solve the mathematics tasks and demonstrated strong computational and problem-solving skills. However, throughout the interview, Henry typically wrote only his solution; he did not show or describe how he arrived at the solution unless we asked him to explain his thought process.

The purpose of the study from which this paper is drawn was to investigate preservice teachers' use of cognitively demanding tasks with ELLs. When searching for tasks we purposefully selected those that were not solvable by applying a simple algorithm or computation. For the present study, we modified a released item (http://ccsstoolbox.agilemind.com/parcc/elementary_3775_1.html) from Partnership for the Assessment of Readiness for College and Careers (PARCC) as follows.

> Three classes at an elementary school are going on a fieldtrip to the zoo. Mrs. Ruiz's class has 23 people, Mr. Yang's class has 25, and Mrs. Evans' class has 24 people (all numbers include the teacher). They can choose to use buses, vans, and/or cars. Buses have 20 seats, vans have 16 seats, and cars have 5 seats. You are in charge of deciding how to transport all of the classes to the zoo. Explain how you would choose how many of each type of vehicle to take and why. Write a response and explain your thinking.
>
> Extension
> 1. If there cannot be any empty seats in a vehicle, how would you choose the vehicles? Explain your strategy.
> 2. If you can only take less than five vehicles, how many different ways can you choose? Explain your strategy.

The original task only required students to find three combinations of vehicles that could be used to transport the classes to the zoo. It also included images of the vehicles and a table of the relevant data. We modified the task to increase the cognitive demand by making the task more open-ended and adding prompts such as, "Write a response and explain your thinking," and, "Explain your strategy." We also removed the images and table in the original PARCC item to maximize the capacity of modification. Before we investigated the PSTs' implementation of the task with ELLs, we piloted the modified task by enacting it with several ELLs to check if it had an appropriate level of cognitive demand for this age group.

When collecting data for that pilot study, the first author, a former mathematics teacher whose native language is not English, interviewed Henry and encountered an interesting moment. We, the authors, then transcribed the interview, thoroughly reading the transcript several times. Focusing on Henry's misinterpretation and the teacher's corresponding decisions, each author wrote analytic memos and reflections of the interview. After discussing our initial analysis, we summarized the reflections from the teacher's and the researchers' view. Although the initial purpose of Henry's interview was to pilot the task, the first author remained in the role of teacher throughout the interview though she was not Henry's classroom teacher. Hence, we refer to her perspective as the teacher's perspective and juxtapose that with the second author's researcher perspective.

In the following sections, we focus on the teacher's decision-making process. We begin with an excerpt from the teacher's meeting with Henry, and then provide an interpretation of this excerpt from the teacher's viewpoint, followed by the researcher's reflection.

Teacher Decision Moment

The following vignette begins after the teacher (denoted T) has provided Henry (H) with the modified task previously stated. Henry was rushing to solve the task as soon as he read it.

T: So is there any word you don't know?
H: No.
T: Okay. Then is there anything you don't understand in this problem?
H: There's about, the teacher included, teacher… Okay. 23 plus, [whispering to himself] Okay. 72 people and add the teachers, 75.
T: What is 75?
H: Because I just can't, all the students first, and then just add the teachers.

T: So, you understand what you have to do for this problem, right?
H: 75, then.
T: Would you explain what you have to do in this problem to me?
H: I don't…
T: Because I do not really understand what we have to do, so you can just add…? Would you explain and help me understand?
H: Um, I found 23, and then add 24,
T: Why did you add those three numbers?
H: So they're students.
T: They're students? Not teachers?
H: Yes. And then, three teachers and then, Mrs. Ruiz, Mr. Yang, Mrs. Evans.

Reflection – You have noticed Henry double counted the teachers. How would you address this misinterpretation of the task statement, if at all (decision)? What might happen next as a result of this decision (outcomes)?

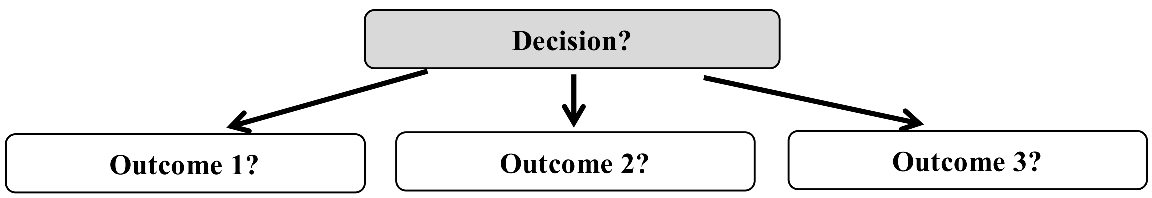

Figure 1. Possible decision and related outcomes stemming from Henry's misinterpretation.

Reflections

Teacher Perspective

At the start of my meeting with Henry, my goal was to find out if he understood all the words and the mathematical situation presented in the task. However, Henry had already started working on the problem and did not attend to my questions. I had planned to discuss what the task was about and to find an entry point together before he began to solve it because I wanted to make sure he fully understood the problem's context. Henry did not approach this task as I had planned, so I changed my plan and asked questions to address his work on the task such as, "Would you explain and help me understand?" (line 13), and, "Why do you have to add those three numbers?" (line 15). While he was responsive to these questions, I noticed he interpreted the task differently (lines 4-5). He added the number of teachers separately, resulting in his arriving at 75 people instead of 72. Hence, following the exchange above, I decided to provide some guidance in the hope that Henry would notice his double counting.

T: Could you read this sentence?
H: All numbers include the teacher is 75, so I got it, so a bus has 20 seats, so I could use… um… about… um.
T: Actually I'm not sure what this means, you know, English is not my first language, either, so I think you can help me understand "all numbers include the teacher" means you have to add three more or you don't have to?
H: Include teacher means the teachers are included.
T: Included where? Included in this number? ((points to the class totals))

H: Yeah.
T: So you have to add one more or you don't have to?
H: You have to add.

Initially, I had assumed Henry knew the meaning of "include," but after he insisted that 75 people were on the trip (line 20), it seemed as though he had misinterpreted it. It is possible that he had not read the sentence carefully at the start of the task because he was busy calculating numbers. He answered my question about what the sentence meant with, "Include teacher means the teachers are included" (line 26), merely repeating the sentence. I was then further convinced of a misunderstanding once I asked Henry whether he had to add the three to the sum in an effort to get him to rethink his answer (line 24), and he responded "You have to add" (line 30). Throughout this exchange Henry was confident in his understanding of and approach to the task.

At this point, I wondered if I should point out this misinterpretation to Henry. I was hesitant to tell him he had misinterpreted the task because he was confident in his understanding of English and was actively solving the task. Although stopping to clarify his misinterpretation would allow him to proceed with the intended quantities, I was afraid that it would decrease his self-efficacy (Ramdass & Zimmerman, 2008). Moreover, because his mathematical thinking was on the right track, I did not want to interrupt his problem solving process. Moreover, he may not have been ready to listen because he was very focused on solving the problem. In light of these factors, I decided to wait until he would be more responsive to listening.

Finally, I found a chance to address the meaning of the clause, "*all numbers include the teacher,"* as he worked on the second extension. In order to take fewer than five vehicles, Henry found that the 75 people could either take four buses or take three buses and a van to the zoo. I knew that using the intended amount of 72 people would yield more possibilities. I took this opportunity to take him back to the original task and reconsider the clause:

T: Let's go back to the beginning. Okay. This little sentence, just one sentence, "all numbers include the teachers," which means the teachers are already in there. One of the 24 people is the teacher. One of the 25 people is the teacher. Then, 23 people, one of those are teachers.
H: Oh. I know. So, there's only 32 people.
T: 32?
H: Hmmm. Yeah.
T: What is 32?
H: Because I just, to show that I add them out.
T: So, how many people are total?
H: Hmm
T: Before, you said it's 75. Now you figured out how many people in three classes?
H: Now. Then there are 72.

He quickly understood how this realization impacted his initial solution and changed the number of people going on the trip to 72. He then completed the final extension using 72 people and was able to, with some assistance, use a table to find all the ways to transport the people. Although I was torn over whether to intervene sooner, I did not want to stop him when he was engaged in mathematical activity because my primary goal was to support Henry's mathematical learning.

Researcher Perspective

While analyzing this situation, I first noted three important observations. First, an ELL (or any student) with sufficient mathematical capabilities could arrive at a different solution than intended because he or she misinterpreted a single clause or phrase. Second, determining whether a student's different solution to a task stems from a misinterpretation of mathematics, language, culture, or some combination of the factors is difficult. Finally, determining the most effective way to address a student's misinterpretation is challenging, particularly at the moment it occurs.

As Henry came to his initial conclusion that 75 people were going on the field trip, the teacher could have proceeded in a number of ways. For example, she could have immediately addressed Henry's misinterpretation of the clause. This intervention may have helped Henry circumvent future challenges when solving the extensions, but he may have experienced frustration because his mathematical work was overshadowed by his language misinterpretation. Alternately, she could have decided to ignore the misinterpretation completely because it was unrelated to his ability to meet the mathematical learning goal of the problem. Or, the teacher could have waited until Henry completed the task using his interpretation and then go back through the problem, asking questions such as, "What if the numbers meant the students and the teacher, would that change your answer?" Such questions may have allowed Henry to continue with the problem's intent while addressing language issues afterward.

What we see that the teacher chose a fourth approach: to wait until there was a seemingly appropriate teaching moment to address Henry's misinterpretation. The decision to focus on the context and language seemed appropriate to her in this instance because Henry was mathematically correct within his interpretation of the problem. His method was to find the total number of people and split that number into groups of 20, 16, and 5. Henry's proper approach caused her to delay addressing Henry's misinterpretation because her focus was on his mathematical thinking rather than his English vocabulary (Moschkovich, 1999, 2010). However, when working on the final extension, the teacher did intervene by telling Henry that the quantities listed contained the teachers.

In retrospect, it seems as though it would have been relatively easy for the teacher to address the misinterpretation immediately. However, it is not clear whether an earlier intervention would have resulted in Henry solving the task as intended, Henry being discouraged and losing interest as the teacher had feared, or some other outcome. Although the teacher was able to find a time to intervene, the teacher might not have addressed the misinterpretation at all if a seemingly appropriate moment had not arisen.

Discussion

Our purpose in analyzing the teacher's decision making is to encourage teachers to reflect on and consider situations when students interpret tasks differently than intended. Making purposeful decisions with regard to these instances while remaining mindful of the mathematical goals is imperative to supporting ELLs' learning. It is harder in a typical classroom setting to notice these types of instances than in an interaction with only one student. Nevertheless, teachers should keep in mind that students' misinterpretation of a single word, phrase, or clause can change their solution, so they need to pay close attention to students' reasoning process and deliberately implement strategies to uncover student's misconceptions as well as provide multiple supports to avoid the misconceptions (Sorto, Mejia Colindres, & Wilson, 2014).

Furthermore, the twofold structure of this study, attending two different perspectives of the teacher and the researcher, helped us analyze Henry's misinterpretation in depth. The teacher's perspective evidenced concern for Henry's confidence and her desire to allow him to correct his own misinterpretation, though she did ultimately intervene. The researcher voiced similar concerns, but her perspective was driven by an analysis of the pros and cons of the different approaches.

We acknowledge that every decision a teacher makes will have pros and cons; however, Ramdass and Zimmerman (2008) assert that taking a self-correction approach helps students increase their mathematical accuracy along with their self-esteem. From this perspective, waiting to intervene until Henry encountered difficulty may be appropriate. However, the best decision is probably to prevent this possible misinterpretation in advance. Henry's misinterpretation occurred during piloting tasks in which we intentionally removed all visual representations. In a classroom setting, teachers could design the task with clearly labeled visuals that show both students and teachers in classrooms as stated in the task, so students can see that the number of teachers was included in the given numbers. Another approach is using a table to show the given number information clearly. More importantly, teachers should notice the clause, "*All numbers include the teacher,"* contains semantic confusion because numbers cannot include people. Teachers could rewrite this clause to make its meaning clear, such as "There are 23 people in Mrs. Yang's class, including the teacher."

Many scholars (e.g., Coggins, Kravin, Coates, & Carroll, 2007; Moschkovich, 2002) have supported the notion of allowing students to use informal language while acquiring academic language. For example, if a student describes an angle as big rather than using the term *obtuse*, teachers can allow them to use the everyday language while reasoning and then bring in the mathematical language later. In Henry's case the word *include* was neither content nor everyday vocabulary, but a *function word* with meaning central to a task context (Echevarria, Vogt, & Short, 2010). Thus, teachers should attend to these words because ELLs need to learn these words as they become fluent in mathematical discourse (Cobb, Stephan, McClain, & Gravemeijer, 2001; Khisty & Chval, 2002; Vomvoridi-Ivanovic & Razfar, 2013).

Moschkovich (1999) suggested that focusing on correcting vocabulary or grammatical errors obscures the mathematical content in what ELLs communicate mathematically. Henry's misinterpretation of the clause impacted his answer to the task, but not his reasoning. Thus, the teacher in this study did not address the unexpected misinterpretation of the clause immediately. This implies the teacher focused initially on Henry's mathematical discourse rather than on his language misinterpretation. If ELLs experience difficulty solving a task because they are not able to make sense of the problem statement, teachers should intervene and help them understand the situation embedded in the problem (I, 2015). However, when the misinterpretation does not affect their core mathematical process, teachers can be flexible, especially during assessments. When teachers stop listening to ELLs' mathematical thinking, both parties may lose sight of the mathematical goals. We encourage teachers to consider each instance individually in attending to the unique needs of ELLs.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Consider a moment when you noticed your student made a misinterpretation either mathematically or linguistically. How did you react/interact in that situation? Are there any different decisions you could have made? How might each option have impacted the outcome?

2. What supports or opportunities would be helpful for teachers when enacting complicated mathematics word problems with ELLs?

3. What are some words that might impede students' mathematical reasoning or problem solving if they do not know the definition of the words? How would you build meaning for those words in teaching mathematics?

4. How can you create mathematical tasks that minimize possibilities of students' misinterpretations?

5. In what ways, if any, do you think the teacher's approach may have differed if she were teaching an entire class rather than one student?