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Microsecond State Monitoring of Nonlinear Time-Varying Dynamic Systems

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Microsecond State Monitoring of Nonlinear Time-Varying Dynamic Systems

Abstract
Reliable operation of next generation high-speed complex structures (e.g. hypersonic air vehicles, space structures, and weapons) relies on the development of microsecond structural health monitoring (μSHM) systems. High amplitude impacts may damage or alter the structure, and therefore change the underlying system configuration and the dynamic response of these systems. While state-of-the-art structural health monitoring (SHM) systems can measure structures which change on the order of seconds to minutes, there are no real-time methods for detection and characterization of damage in the microsecond timescales.

This paper presents preliminary analysis addressing the need for microsecond detection of state and parameter changes. A background of current SHM methods is presented, and the need for high rate, adaptive state estimators is illustrated. Example observers are tested on simulations of a two-degree of freedom system with a nonlinear, time-varying stiffness coupling the two masses. These results illustrate some of the challenges facing high speed damage detection.

Keywords
Dynamic systems, Structural health monitoring, Damage, Weapons, Simulation, Space frame structures, Engineering simulation, Vehicles, Dynamic response, Stiffness

Disciplines
Civil Engineering | Dynamics and Dynamical Systems | Structural Engineering | VLSI and Circuits, Embedded and Hardware Systems

Comments

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ABSTRACT

Reliable operation of next generation high-speed complex structures (e.g., hypersonic air vehicles, space structures, and weapons) relies on the development of microsecond structural health monitoring ($\mu$SHM) systems. High amplitude impacts may damage or alter the structure, and therefore change the underlying system configuration and the dynamic response of these systems. While state-of-the-art structural health monitoring (SHM) systems can measure structures which change on the order of seconds to minutes, there are no real-time methods for detection and characterization of damage in microseconds.

This paper presents preliminary analysis addressing the need for microsecond detection of state and parameter changes. A background of current SHM methods is presented, and the need for high rate, adaptive state estimators is illustrated. Example observers are tested on simulations of a two-degree of freedom system with a nonlinear, time-varying stiffness coupling the two masses. These results illustrate some of the challenges facing high speed damage detection.

INTRODUCTION

Hypervelocity air vehicles, space structures, and weapon systems are subject to extremely high speed impacts (>4 km/s), causing damage to propagate through the structures in microseconds [1, 2]. Here, damage refers to a change in the structure’s configuration, material failure, or change in the system boundary conditions. High amplitude impacts may cause damage that alters the underlying dynamic response of the structure, resulting in loss of system functionality or other severe consequences. Structural health monitoring (SHM) and prediction systems with integrated sensor networks can identify changes in the operation of a system, predict remaining useful life, and allow for mitigating the risk of destructive circumstances [3]. Current SHM systems can measure and process slowly varying structures on the order of seconds to minutes. While system state estimation and control techniques are capable of operating on a microsecond scale [4], there are no existing, real-time methods that can detect and characterize damage in complex, time-varying structures in microseconds timescales. The development of microsecond SHM ($\mu$SHM) methods on the timescales of 10 $\mu$s to 10 ms will allow for the structural integrity of a system to be monitored on timescales where speed of damage detection is critical.

Several challenges arise when detecting and predicting damage at these timescales. As summarized in a recent newsletter by the authors [5], the feasibility of microsecond prognostics must account for high rate physics of failure, complexity of the rate-dependent structural dynamics, high rate real-time processing requirements, identifying optimal quantity and placement of sensors [6], and uncertainties throughout the system (material, structural, history/lifecycle, etc.).

This paper will outline some of the high rate, state-of-the-art state estimation and SHM techniques currently in use. A comparative study of observers for induction motors is given to demonstrate the feasibility of high rate physics of failure and highlight some of the capabilities and limitations seen in high rate observers. A Luenberger observer and a Kalman filter are developed that use output feedback to adapt the observer parameters. These observers are simulated on a two degree of freedom, time-varying system, and performance limitations are demonstrated.
CURRENT STATE-OF-THE-ART

This section will discuss some of the current methods used to detect system damage and provide examples of high speed estimators for induction motors.

Microsecond Structural Health Monitoring

There is a need for adaptive, model-based approaches which incorporate relevant, nonlinear, rate-dependent phenomena and methods of handling uncertainties of the system dynamics into a fast-running microsecond state estimator.

The authors [7-10] and collaborators [11, 12] have previously conducted preliminary work on health monitoring methods for detecting time-varying damage at the microsecond timescale (10 µs to 10 ms) and have illustrated areas which need further technical development. One empirical-based damage detection method used milliseconds of time data, but the damage was stationary in the system and existed before the dynamics of the structure illustrated damage [8]. Other damage detection methods examine structures that are damaged during an impact event (therefore creating time-varying systems). In one work, a model was used with strain energy methods to detect local stiffness changes in a plate, but the algorithm required large computational resources [7]. Another technique uses electromechanical impedance (EMI) on the microsecond time scale to detect damage [11, 12]. However this EMI method can only detect nearby defects in the structure because the high-frequency vibration used for excitation is heavily damped in typical structures.

There is a clear need for a model-based data fusion method to incorporate local and global measurements in order to implement microsecond monitoring and prognosis on a full structure.

Nonlinear State Observers

Observers are used in the parameter estimation and feedback control communities for estimating the internal states and parameters of a dynamic system using measurements of the system and any applied forces. A block diagram of an observer used for damage detection is shown in Figure 1. The observer takes measurements of the system’s inputs and responses, computes estimates of internal system states and parameters, and feeds this information to an algorithm to determine the presence and severity of structural damage.

There are three main categories of observers for nonlinear systems: data-based, statistical, and model-based methods [13, 14]. Data-based methods, such as fuzzy logic and neural network estimators, are methods that process information without knowledge of a system’s dynamics. These methods are particularly useful for handling highly complex systems. The performance of data-based methods is dependent on the quality of data mining and interpretation algorithms, and these methods are limited by the lengthy computational time required to achieve an appropriate estimate [15]. In applications of state estimation for fault detection and prognosis, data-based methods generally only provide a sequential measure of fault based on pattern recognition and classification. Alternatively, they require precise examples and extensive training over available data sets to associate data features to types and measures of fault.

Statistical methods, such as the least squares estimator (LSE), the maximum likelihood estimator (MLE), and particle filters, provide probabilistic predictions based on known parameters. They bypass the need for linearized dynamic equations allowing global convergence of estimations. A general limitation of statistical methods is their reliance on available data sets for training and/or extraction of probability distribution functions. Statistical methods can identify faults through a probabilistic measure, and they may be used to conduct prognosis by evaluating the probability of faults, but require knowledge of probability distribution functions.

Model-based observers use a numerical representation of the system to be identified. Model-based methods include the Kalman filter (KF), sliding mode observer (SMO), Luenburger Observers (LO), and variations of these methods for nonlinear systems (e.g., Extended Kalman Filter). Additionally adaptive observers (AO) are variations of traditional model-based or data-based observers and continuously change the gains and system parameters with a pre-determined feedback rule. These methods have attracted much attention because they produce accurate state estimations when it is not possible or practical to have sensors to characterize every state [16]. Furthermore, the nominal models required for control and estimation purposes are readily available [17]. Model-based methods have the advantage of providing precise measures of damage due to the availability of models, therefore enabling condition assessment and system prognosis. However, they require knowledge of the physical model, which may add significant burden on computational time. Nevertheless, these methods are able to provide measures of parameter changes following a fault, which is desirable in damage estimation.

Figure 1. Block diagram of an observer used for damage detection.
Adaptive Observers

Adaptive observers (AO) are a set of methods that are an extension of the traditional observers by adding feedback rules to continuously update the parameters of the observer based on measurements of the system’s inputs and outputs, see Figure 2 [18, 19]. This quality makes them ideal for handling uncertainty in state estimation [20]. In particular, adaptive observers have been proposed to estimate the un-measurable states for different classes of nonlinear systems [21]. The nature of AOs gives them a unique advantage of asymptotic system performance without unnecessary dependence on models and in the presence of uncertainties through their ability to adapt to unexpected changes in system conditions.

Comparative Study of Observers for Induction Motors

This section compares observers for applications restricted to induction motors. The induction motor’s highly nonlinear and uncertain dynamics make it a useful baseline for comparing observers applicable to other time-varying dynamic system [22].

In [23], a comparative study of an adaptive sliding observer (ASO) and an extended Kalman filter (EKF) was conducted for a sensor-less motor drive. The dynamic performance of the estimators was tested by accelerating the motor from 478 rpm to 1260 rpm. The results showed the ASO had a computation time of 19 µs, was easy to implement, and produced an accurate estimation. The results of the EKF showed a computation time of 86 µs, was complicated to implement, and produced a more accurate estimation than the adaptive sliding observer. This study exhibits the tradeoff between accuracy and computation time.

In [24], another comparative study was conducted on a Luenberger observer (LO), sliding mode observer (SMO), and the EKF. All observers delivered high accuracy estimations at high motor speeds. The LO and SMO results were more robust to parameter variations than the EKF. The EKF’s performance proved to be immune to external noise while the LO and SMO’s performance was excellent only in the presence of small external noise. The calculation complexity of LO and SMO were essentially identical and were much simpler than the complex calculations of the EKF. Using a 150 MHz processor, one update took the LO and SMO about 5 µs to achieve while the EKF took about 100 µs.

Table 1 summarizes the main results from these two studies. Although these numbers are based on the programmer’s proficiency, it serves as a good example for the feasibility of high rate estimation and health monitoring of complex systems using observers.

Table 1. Summary of comparative study of observers for induction motors.

<table>
<thead>
<tr>
<th>Observer Type</th>
<th>Computational Time</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luenberger Observer (LO)</td>
<td>5 µs for one update using 150 MHz processor [24]</td>
<td></td>
</tr>
<tr>
<td>Sliding Mode Observer (SMO)</td>
<td>5 µs for one update using 150 MHz processor [24]</td>
<td></td>
</tr>
<tr>
<td>Adaptive Sliding Observer (ASO)</td>
<td>19 µs computational time. [23]</td>
<td></td>
</tr>
<tr>
<td>Extended Kalman Filter (EKF)</td>
<td>86 µs computational time and 100 µs for one update using 150 MHz processor. [23, 24]</td>
<td></td>
</tr>
</tbody>
</table>

MODEL-BASED OBSERVERS

The use of adaptive, model-based observers for parameter estimation will be demonstrated on an example system. Two adaptive observers will be examined: one based on a Luenberger observer and the other based on a Kalman filter. Both estimators will incorporate time-varying system dynamics.

Description of a Mechanical System

Consider a second-order, n degree of freedom, time-varying dynamic system representing a mechanical structure with displacement vector \( q(t) \), input vector \( u(t) \), and measurements vector \( y(t) \) of the form

\[
M(t)\ddot{q}(t) + S(t)\dot{q}(t) + K(t)q(t) = H(t)u(t),
\]

\[
y(t) = P(t)q(t) + N(t)\dot{q}(t) + w(t),
\]

where \( \dot{q}(t) \) is the velocity vector, \( \ddot{q}(t) \) is the acceleration vector, \( M(t), S(t), \) and \( K(t) \) are the mass, damping and stiffness matrices respectively, \( H(t) \) is the input matrix, \( P(t) \) and \( N(t) \) are the output matrices associated with the measurement of the displacement and velocity respectively, and the overdot denotes time derivatives. The term \( w(t) \) represents measurement noise. Many of the state estimation methods in the literature are designed for systems of first-order equations. Noting this, define the state vector \( x(t) \) as

\[
x(t) = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \]

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The second-order system above can also be written as the first-order (state-space) system
\[
\dot{x}(t) = A(t)x(t) + B(t)u(t) + G(t)v(t),
\]
where \( \dot{x}(t) \) is time derivative of the state vector \( x(t) \), \( A(t) \) is the state matrix, \( B(t) \) is the input matrix, and \( C(t) \) is the measurement matrix. These state-space matrices are defined in terms of the matrices describing the physical system as
\[
A(t) = \begin{bmatrix} 0 & I \\ -M(t)^{-1}K(t) & -M(t)^{-1}S(t) \end{bmatrix},
\]
\[
B(t) = \begin{bmatrix} 0 \\ M(t)^{-1}H(t) \end{bmatrix},
\]
\[
C(t) = \begin{bmatrix} P(t) & N(t) \end{bmatrix},
\]
where \( 0 \) is a \( n \times n \) matrix of zeros and \( I \) is a \( n \times n \) identity matrix. The term \( G(t)v(t) \) is added to equation (4) to represent un-modeled dynamics or unmeasured input sources. The term \( v(t) \) is this process noise, and \( G(t) \) is a noise matrix. This system and the state matrices will be used in the following sections for defining the Luenberger observer and Kalman filter.

**Luenberger Observer**

The Luenberger observer (LO) is a common estimator used for state estimation of linear, deterministic systems. The Luenberger observer finds an estimate of the state vector, \( \hat{x}(t) \), through the equations
\[
\begin{bmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \end{bmatrix} = \begin{bmatrix} A(t) & 0 \\ L\hat{C}(t) & A(t) - L\hat{C}(t) \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} + \begin{bmatrix} B(t) \\ B(t) \end{bmatrix} u(t),
\]
where \( L \) is a constant observer gain and the hat denotes the vector or matrix is the estimate used in the model. These estimates may be adapted over time using an additional adaptation law. The first row of equation (9) is the original system dynamics, while the second row describes the dynamics of the state estimate. Here the process noise \( G(t)v(t) \) and measurement noise \( w(t) \) are ignored. The Luenberger observer can have fast convergence rates when given an accurate system model and appropriate selection of \( L \), but the observer is not robust in the presence of noise. The fixed observer gain \( L \) can be calculated from the desired eigenvalues of the matrix \( [A(t) - L\hat{C}(t)] \) at some point in time, such as when the system is in its initial, undamaged configuration.

**Kalman Filter**

Assume the process noise \( v(t) \) and measurement noise \( w(t) \) are white, Gaussian, zero mean, and independent of each other. From these assumptions, the Kalman filter equations can be written as
\[
\begin{bmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \end{bmatrix} = \begin{bmatrix} A(t) & 0 \\ F(t)\hat{C}(t) & A(t) - F(t)\hat{C}(t) \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} + \begin{bmatrix} B(t) \\ B(t) \end{bmatrix} u(t) + \begin{bmatrix} G(t) v(t) \\ F(t) w(t) \end{bmatrix},
\]
where \( F(t) \) is the time-varying observer gain. The optimal observer gain, \( F^{opt}(t) \), is given by
\[
F^{opt}(t) = \Pi(t)\hat{C}^T(t)\hat{C} W(t)^{-1},
\]
where \( W(t) \) is the covariance matrix of the measurement noise \( w(t) \) and \( \Pi(t) \) is the covariance of the state error \( x(t) - \hat{x}(t) \). The state error covariance is the solution to the time-dependent Ricatti equation
\[
\dot{\Pi}(t) = A(t)\Pi(t) + \Pi(t)A^T(t) - \Pi(t)\hat{C}^T(t)W(t)^{-1}\hat{C}(t)\Pi(t),
\]
where \( \dot{\Pi}(t) \) is the time derivative of \( \Pi(t) \). Given enough convergence time, this estimator can compensate for disturbances or inaccuracies in the initial state estimate.

**Parameter Adaptation Rule**

The matrices in the Luenberger observer and Kalman filter, \( A(t), B(t), \) and \( \hat{C}(t) \), can be updated using measurements from the system to calculate estimates of the system parameters. Consider the equation (1) written as
\[
\dot{q}(t) = [M(t)^{-1}[H(t)u(t) - S(t)\dot{q}(t) - K(t)q(t)]],
\]
which can be expressed as
\[
\dot{\theta}(t) = \Psi(t)[\Theta(t)]^T,
\]
where \( \Theta(t) \) is a parameter vector composed of the stiffness and damping parameters and \( \Psi(t) \) is a matrix formed from the state vector and the input forces. Next an adaptive back-propagation rule for updating \( \Theta(t) \) is calculated using measurements of the velocity vector \( \dot{q}(t) \) and the estimated velocity vector \( \hat{q}(t) \), namely
\[
\dot{\theta}(t) = -\Gamma [\Psi(t)]^T[\dot{q}(t) - \hat{q}(t)],
\]
where \( \Gamma \) is a pre-defined learning rate matrix [25, 26]. The performance of this rule will depend on the value of the learning rate matrix.
SIMULATION STUDY

A preliminary study is used to evaluate the estimators’ performance and illustrate opportunities and limitations of high rate estimation. The previously mentioned Luenberger observer and Kalman filter are evaluated on simulations from a time-varying system. The simulation consists of a two degrees-of-freedom (DOF) system shown in Figure 3. The simulations use 1 kg for both masses \( m_1 \) and \( m_2 \), damping values of 0.1 N s m\(^{-1} \) for both dampers \( s_1 \) and \( s_2 \), and a simulation rate of 50 kHz. Both stiffness values \( k_1 \) and \( k_2 \) are initially set to 400 N m\(^{-1} \). The stiffness of the spring between the masses \( k_2 \) suddenly decreases at the moment an external force is applied to the second mass. This stiffness change simulates damage to the linkage between the two masses.

For the two degree of freedom system in Figure 3, the equations of motion can be written as

\[
\begin{bmatrix}
\ddot{q}_1(t) \\
\ddot{q}_2(t)
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & -q_1(t) \\
0 & 0 & 0 & -q_2(t)
\end{bmatrix} \begin{bmatrix}
q_1(t) \\
q_2(t) \\
\dot{q}_1(t) \\
\dot{q}_2(t)
\end{bmatrix} + \begin{bmatrix}
-k_1 \\
-k_2
\end{bmatrix} \begin{bmatrix}
\dot{u}_1(t) \\
\dot{u}_2(t)
\end{bmatrix},
\]

which forms the matrix \( \Psi(t) \) used in the adaption rule given in equation (15). For the Luenberger observer, a rough optimization is performed on the both the constant observer gain \( L \) and learning matrix gain \( \Gamma \) to produce the best estimate. Because the Kalman filter already includes rules for optimizing the observer gain, \( F_{opt}(t) \), the only learning matrix is tuned for optimal parameter estimation.

Figure 4 shows a representative simulation of the system experiencing an impulsive force of amplitude 50 N over a duration of 0.2 ms. During this simulation, the initial \( k_2 \) stiffness value is reduced by 5\% (to 380 N m\(^{-1} \)) at the onset of the force input to simulate damage. Process and measurement noises are included in the Kalman filter implementation as they are needed to calculate the gain in equation (11), and without noise the calculation becomes numerically unstable. The Luenberger is found to become unstable when noise is added to the system and therefore the case of Luenberger with noise is excluded from this paper. The Luenberger estimator is capable to converge to the new, correct stiffness in approximately 16.5 ms, and a 2\% change in stiffness (indicative of damage to the system) is detectable in approximately 12 ms. The Kalman filter is not able to converge to the new, correct stiffness in the simulation time. However, it reaches 80\% of the stiffness change in about 51.5 ms in the presence of process and measurement noise, and a 2\% change in stiffness is detectable in approximately 25 ms. A summary of the high rate estimation simulation results are shown in Table 2.

![Figure 3](image-url)  
**Figure 3.** Two degree of freedom dynamic system with variable stiffness \( k_2 \).

![Figure 4](image-url)  
**Figure 4.** Simulation and estimation results for the two degree of freedom system with changing stiffness. The top plot shows the impulse force \( u_2(t) \) on the second mass, the middle plot shows the velocity of the two masses \( \dot{q}_1(t) \) and \( \dot{q}_2(t) \), and the bottom plot shows the real and estimated values for the stiffness of the connecting spring \( k_2 \).

<table>
<thead>
<tr>
<th></th>
<th>Luenberger Observer</th>
<th>Kalman Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final value</td>
<td>380 N/m</td>
<td>384 N/m</td>
</tr>
<tr>
<td>Final error</td>
<td>0%</td>
<td>20%</td>
</tr>
<tr>
<td>Convergence time</td>
<td>16.5 ms</td>
<td>-</td>
</tr>
<tr>
<td>Time for 2% value change</td>
<td>12 ms</td>
<td>25 ms</td>
</tr>
<tr>
<td>Robust with noise</td>
<td>No – unstable</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The performance of the estimators is sensitive to tuning of observer and learning rate gains, input amplitudes, and the rate of change of system parameters. Figure 5 plots estimates of the stiffness \( k_2 \) from both observers for input amplitudes of 50 N and 100 N. Doubling the input amplitude decreases the estimation time of the Luenberger observer and worsens the...
convergence rate of the Kalman filter. The rate of parameter change can also affect results. Figure 6 plots the estimations during different durations for the stiffness change. The original input force of 50 N is used here. Both observers are better at tracking a slow stiffness change than a fast one.

The implementation of the Kalman filter and Luenberger observer state estimates on this time-varying mechanical system illustrate a few issues with high rate state estimation that will need to be addressed in estimators for more complex systems. Characteristics of the input and the rate of change of parameters produce different qualities of estimation. Adaptation laws that can account for this broader array of inputs and parameter changes are needed for more robust SHM at high speeds. In addition, noise affects the estimation results. The high rate Luenberger observer became unstable in the presence of large noise in the system. New methods are needed which account for system and measurement noise while balancing the convergence time. The Kalman filter was able to give a stiffness estimate in the presence of noise, but with our selected learning rate matrix, the parameter estimates were slow to converge to the final estimate.

CONCLUSIONS

This paper discussed the motivation for developing structural health monitoring (SHM) systems that can detect and characterize damage in timescales on the order of tens or hundreds of microseconds. Such systems are needed in a range of aerospace, automotive, and military applications. High rate SHM methods will have to handle system complexities, system uncertainties, and real-time processing requirements. A case study with observers for induction motors and simulation examples presented here showed the feasibility of damage detection using adaptive observers on a linear system in the millisecond timescales. The estimation and detection of damage in the high rate loading with changing nonlinear dynamics and large noise will require implementing advanced observer design methods. Nonetheless, these simulations have shown that the approach of developing a microsecond state detection method is possible.

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