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A novel tuned liquid wall damper for multi-hazard mitigation

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Abstract

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Keywords

tuned liquid wall damper, hazard mitigation, structural control, head loss, computational fluid dynamics

Disciplines

Civil Engineering | Structural Engineering | VLSI and Circuits, Embedded and Hardware Systems

Comments

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A novel tuned liquid wall damper for multi-hazard mitigation

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ABSTRACT

A structural system consists of gravity and lateral load resisting components. Structural walls in the gravity system are typically designed to resist vertical loads only, and are assumed to be inactive to mitigate lateral loads. In this paper, we propose a novel multifunctional wall system, which is embedded with multiple-capillaries containing free-flowing fluids and can act as both a load carrying member and a Tuned Liquid Wall Damper (TLWD). Functioning similarly to a Tuned Liquid Column Damper (TLCD), the damping force of the proposed wall system is provided by the head loss of the fluid between each capillary. An analytical model is derived first to describe the dynamic behavior of the TLWD. The accuracy of the analytical model is verified using Computational Fluid Dynamics (CFD) simulations. The model is further used to compute the reduced response of an assumed primary structure attached with a TLWD to demonstrate the damping capability. Results show that TLWDs can effectively dissipate energy while occupying much less space in buildings compared to TLCDs.

Keywords: tuned liquid wall damper, hazard mitigation, structural control, head loss, computational fluid dynamics

1. INTRODUCTION

Mitigation of structural responses against wind or earthquake hazards is a way to ensure structural serviceability and safety. Supplemental damping systems, include active, semi-active, and passive systems, have been proposed to reduce structural vibrations^{5,6,7}. While active damping systems are able to provide a wide range of control force, passive control systems can produce large damping forces without any external source of energy⁸. Tuned liquid column dampers (TLCDs) are a type of passive damping systems that dissipate energy from earthquake or wind through inertia forces that are out of phase. TLCDs usually consist of liquid-filled U-shaped tubes with orifices in the middle and leverage the liquid's gravity force as a restoring force. Damping forces are provided by the head loss from the liquid motion change near orifices and corners, and the friction between the liquid and columns' inner surfaces. Extensive research has been done regarding both theoretical analyzes¹⁻⁴ and practical applications of TLCDs⁹⁻¹².

The proposed Tuned Liquid Wall Damper (TLWD) is a multi-column liquid damping system that can be built inside structural walls (Figure 1). For example, if the wall is made of reinforced concrete, interior tubes can be placed before casting of concrete. All the tubes (columns) are connected at the bottom to allow liquid flow through; and at the top to relieve the air pressure caused by liquid motion. Similar to TLCDs, TLWDs also rely on liquid movement through the orifices to dissipate energy. Unlike conventional TLCDs, which are usually installed at the top of a building and generate a concentrated force to the structure, TLWDs can be installed within structural walls over multiple floors. Another benefit of TLWDs is that they can help preserve the structure's ambient temperature due to the liquid's large specific heat capacity.

With the TLWD concept, multi-columns are required to provide damping functionality while respecting space limitation. Some researchers investigated the performance of multi-column TLCD. For instance, Shope *et al.*¹³ used a multi-cell liquid damping device to eliminating floor vibrations. Min *et al.*¹⁴ proposed a novel retunable multi-cell TLCD by dividing the tubes into multiple cells. The advantage of this system is that its natural frequency can be adjusted over a wide range, making it easier to tune to the natural frequency of the primary structure.

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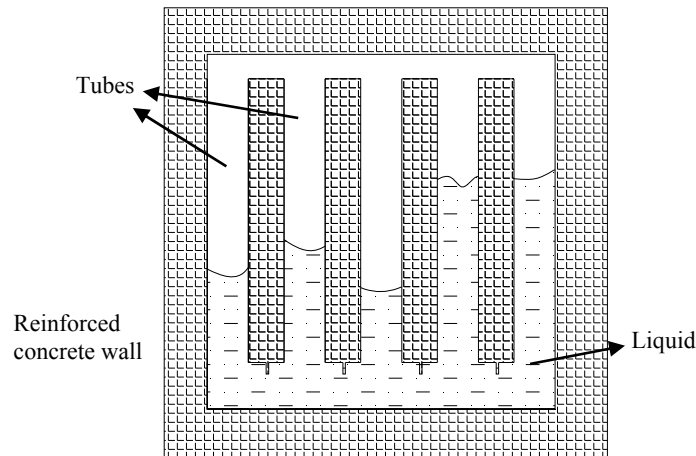


Figure 1. Tuned Liquid Wall Damper

The objective of this paper is to evaluate the effectiveness of the proposed TLWD. To this end, an analytical model is derived to describe the dynamic behavior of the TLWD in Section 2. In Section 3, we use Computational Fluid Dynamics (CFD) method to verify the analytical model. In Section 4, the analytical modal is used to analyze cases with different column sizes and numbers to evaluate the effectiveness of TLWDs to reduce the primary structure's response under excitations. Section 5 concludes the paper.

2. ANALYTICAL MODEL OF TLWD

A typical TLWD inside a structural wall can be described as a multi-column TLCD, as illustrated in Figure 2. In order to understand TLWD's working mechanism, consider a typical dual-column TLCD (Figure 3). Its equation of motion can be expressed as³:

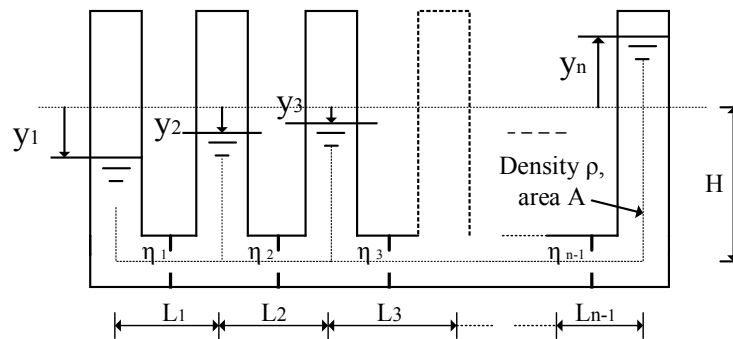


Figure 2. Analytical model of Tuned Liquid Wall Damper

$$M\ddot{x} + m_L\ddot{x} + C\dot{x} + Kx + m_{L_n}\ddot{y} = -F(t) \quad (1)$$

$$m_d\ddot{y} + m_{L_n}\ddot{x} + \frac{1}{2}\rho\eta A\dot{y}|\dot{y}| + 2\rho Agy = 0 \quad (2)$$

where K , M , and C are the stiffness, mass, and damping matrices of the structure, respectively; x is the displacement of the structure; y is the displacement of liquid surface from the balanced position; $F(t)$ is the excitation force;

$m_L = \rho AL$, which is the horizontal mass of tuned liquid column damper; $m_d = \rho A(L + 2H)$, which is the total mass of tuned liquid column damper; and η is the head loss coefficient controlled by orifice, friction force.

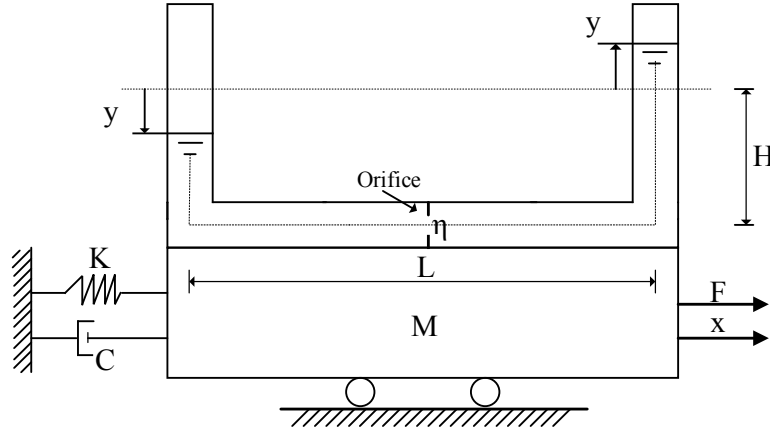


Figure 3. Dual-column TLCD attached to a SODF structure

The governing equation for the fluid flow in a TLWD is similar to that of a conventional dual-column TLCD, consisting of inertia force, damping force, and restoring force due to gravity. However, the nonlinear inertia force for TLWD takes different form, which can be described as:

$$\sum_{i=1}^n \rho A(H - y_i) \ddot{y}_i + \sum_{i=1}^{n-1} \left(\sum_{j=i}^{n-1} \rho AL_i \ddot{y}_i \right) + \sum_{i=1}^{n-1} \rho AL_i \ddot{x} + \frac{1}{2} \rho A \sum_{i=1}^{n-1} (\eta_i \left(\sum_{i=1}^i \dot{y}_i \right) \left| \sum_{i=1}^i \dot{y}_i \right|) + 2\rho Ag \sum_{i=1}^n |y_i| = 0 \quad (3)$$

In order to solve the equation above, we need to determine the liquid height, i.e., y_i , in each column. One approach is to apply energy conservation law at each T joint, relating the kinematic energy (liquid velocity) to gravity potential energy (liquid height in each column). However, this approach may be complex because both the head loss at the T joint and the work done by the friction resistance force are highly nonlinear. In addition, the law governing the splitting of the flow velocity at the joints varies for different fluid turbulence intensity. Another approach is to assume a linear relationship of the liquid heights based on the distances between each column. The liquid height in one column can be taken as proportional to the distance between the column and geometry center of the TLWD, which means that the outermost columns have the largest liquid height. However, it should be noted that this assumption is only valid when the orifice blocking ratios between each columns are similar and relatively small. A heavily damped case may behave differently. For a four-column TLWD, if such linear relationship is assumed, Eq. (3) reduces to:

$$(L_1 + H - y_1) \ddot{y}_1 + L_2 (\ddot{y}_1 + \ddot{y}_2) + (H - y_2) \ddot{y}_2 + (H + y_3) \ddot{y}_3 + (L_3 + H + y_4) \ddot{y}_4 + (L_1 + L_2 + L_3) \ddot{x} + \frac{1}{2} \eta (\dot{y}_1 |\dot{y}_1| + \dot{y}_3 |\dot{y}_3| + \dot{y}_4 |\dot{y}_4|) + 2g(y_1 + y_2 + y_3 + y_4) = 0 \quad (4)$$

The relationship between liquid heights in different columns can be expressed as:

$$y_1 + y_2 = y_3 + y_4 \quad (5)$$

$$\frac{y_1 - y_2}{L_1} = \frac{y_3 + y_1}{L_1 + L_2} = \frac{y_4 + y_1}{L_1 + L_2 + L_3} \quad (6)$$

Then the liquid heights in each column, except for the first one, can be interpreted as a function of y_1 :

$$y_2 = \frac{2L_2 + L_3 - 2L_1}{2L_1 + 2L_2 + L_3} y_1, \quad y_3 = \frac{2L_1 + 2L_2 - L_3}{2L_1 + 2L_2 + L_3} y_1, \quad y_4 = \frac{2L_1 + 2L_2 + 3L_3}{2L_1 + 2L_2 + L_3} y_1 \quad (7)$$

Substituting Eq. (7) into Eq. (4) results:

$$(10/3L + 4H)\ddot{y}_1 + 3L\ddot{x} + 2.125\eta\dot{y}_1|\dot{y}_1| + 8/3gy_1 = 0 \quad (8)$$

Eq. (8) is similar to the dual-column TLCDC equation, which can be solved using numerical methods or analytical solutions provided by past research. The performance of the damper can be evaluated using the transfer function:

$$x(t) = \frac{\hat{p}}{K} H e^{i\Omega t} \quad (9)$$

If the excitation is harmonic, the transfer function has an explicit solution⁸:

$$H(\rho) = \sqrt{\frac{(\gamma^2 - \rho^2)^2 + 4\xi_d^2 \rho^2 \gamma^2}{(\gamma^2 - \rho^2)(1 - (1 + \bar{m})\rho^2) - 4\xi\xi_d \rho^2 \gamma - p^2 \rho^4 \bar{m}^2 + 4(\xi_d \rho \gamma (1 - (1 + \bar{m})\rho^2) + \xi \rho (\gamma^2 - \rho^2))^2}} \quad (10)$$

where $x(t)$ is the time history of the primary structure's response; \hat{p} is the amplitude of the excitation force if the force is harmonic; $\bar{m} = \rho A(10/3L + 4H)/M$ is the mass ratio of damper versus structure; ξ is the damping ratio of structure; ξ_d is the equivalent damping ratio of TLWD, which can be adjusted based on orifice blocking ratios (see Section 3.1); $p = 3L/(10/3L + 4H)$, the length ratio of horizontal length versus equivalent total length of the damper; $\rho = \Omega/\omega$, which is the frequency ratio of excitation frequency versus structure's frequency; and $\gamma = f_{\text{damper}}/f_{\text{structure}}$, which is the tuning ratio of damper frequency versus structure frequencies.

3. VERIFICATION OF THE ANALYTICAL MODEL USING CFD METHODS

3.1 Verification of CFD model

In this study, CFD models are created in academic ANSYS FLUENT 17.2 software to investigate the behavior of TLWDs. To ensure the credibility of the models, the CFD model is validated using experimental results from Wu *et al.*⁴. In their experiment, TLCDCs were attached to a shaking table. The cross section of the TLCDC was 15x 15 cm² square; horizontal and vertical lengths were 85 cm and 63.5 cm, respectively; and the orifice blocking ratio varied from 20% to 80%.

A standard k-epsilon model is adopted in the CFD model. Structured mesh is generated with a maximum element length of 0.01 m. The TLCDC was subjected to harmonic excitations with its own natural frequency and a displacement amplitude of 4 cm. Considering the unevenness of liquid surface in the vertical columns, the liquid surface displacement y is defined as the distance between the horizontal centerline of the liquid surface (Figure 4) to its original balance position. Results from the CFD are compared against results reported in Ref.⁴, listed in Table 1. Results show good agreement overall between the experimental, analytical, and simulation results. However, the discrepancy between results increases with decreasing blocking ratio. In the analytical model, the head loss due to the liquid velocity change at the transition elbow, friction force between liquid and tube surface, and flow turbulence intensity are not considered. All these factors may lead to differences between the simulation and analytical results.

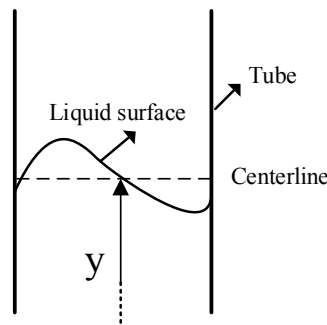


Figure 4. Liquid surface displacement

Table 1 Comparison of liquid surface displacement amplitudes (cm) between CFD and experimental results

Orifice blocking ratio(%)	20	40	60	80
Experimental results ⁴	15.8	13.1	9.2	4.3
Analytical results (Eq. (9),(10))	15.1	12.1	8.6	4.6
CFD simulation results	13.4	11.3	8.3	4.3

3.2 CFD verification of TLWD

Next, the analytical and CFD model are used to study a 4-column TLWD, with $L_1=L_2=L_3=H=48.3$ cm, a cross section area of 225 cm^2 , and an orifice blocking ratio of 20%. As shown in Figure 5, the linear relationship assumption is confirmed by the CFD model, where the liquid heights in the four columns are proportional to each other. Solving Eq. (8) using the State-Space method, the amplitude of the liquid height in the first column y_1 is 9.1 cm. y_2 and y_3 can be computed based on the linear relationship as $y_2 = y_3 = y_1 / 3 = 3.03$ cm, which is close the liquid height amplitudes from CFD model, where $y_1 = y_4 = 8.3$ cm, $y_2 = y_3 = 3.1$ cm. The difference between the analytical and CFD results for the outer columns is slightly larger than that for the inner columns.

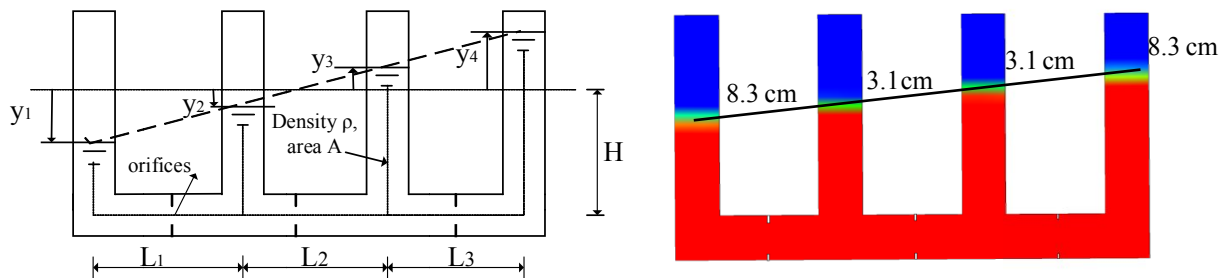


Figure 5. Four-column TLWD analytical and CFD model comparison

3.3 Factors that may induce errors in CFD simulation

Other than the orifice blocking ratio as discussed in Section 3.1, it is known that any drastic variations in the liquid velocity may result in additional head loss in the damping system. For example, the transition zone between the vertical columns and horizontal tube is another source of head loss. A round corner elbow usually provides a smoother transition for fluid flow than for a sharp corner. In some recent work, it has been experimentally noted that differences between experimental and analytical results arose with large transition zones. CFD models of both sharp and round corner TLCDs are created and compared to investigate this problem. We slightly increase the width and height of round corner TLCD to ensure that both TLCDs have the same liquid volume inside the tube, as well as the same natural frequency. A comparison of results in the CFD models show that the TLCD with round corners has a 3.05% larger liquid displacement amplitude. In models where the cross section area is large compared to the length of TLCD, the difference is more significant.

Another significant factor to be considered is the friction force at the tube interface. For laminar flows, the friction resistance is only related to the flow's Reynolds number and is insensitive to surface roughness. However, when the flow is turbulent, the friction force becomes dependent on the roughness of the interior tube surface. The roughness has a negative influence on the liquid motion velocity. ANSYS FLUENT allows the roughness height of walls to be considered. The roughness heights for typical materials such as stainless steel, PVC and plastic pipes, and ordinary concrete are 0.015, 0.0015 – 0.007, and 0.3-0.5 mm, respectively. The liquid motion speed can be reduced by a maximum of 13% if a large roughness height is set. In the model discussed in Section 3.1, we set the roughness height to be 0.02 mm, which is an unknown parameter in the shaking table test reported in Ref.⁴ and may be a source of error. If the liquid in TLWDs is designed to be contained by plastic tubes or other smooth surface tubes, the friction force will be very small.

4. APPLICATION OF THE ANALYTICAL MODEL

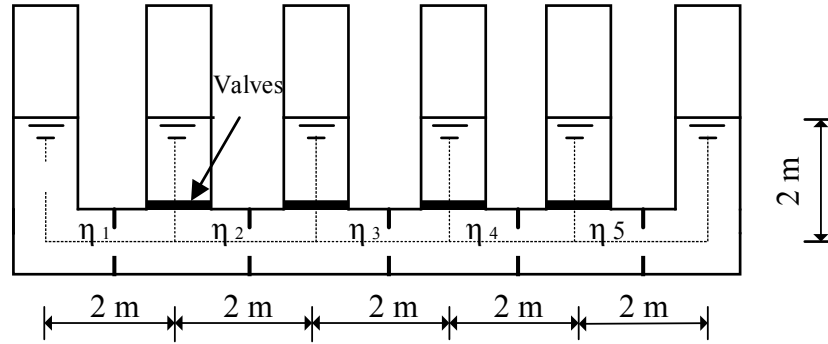


Figure 6. A 6-column TLWD with valves in the inner columns

In this section, the analytical model derived in Section 2 is used to evaluate the effectiveness of the TLWD. The first example is to determine whether the number of columns affects their performance. A 6-column TLWD is taken, as shown in Figure 6. The geometry parameters are: $L_1=L_2=L_3=L_4=L_5=2$ m, $H=2$ m, cross-section area $A=2$ m². If the inner columns are blocked by valves, the liquid inside the columns are ineffective and is considered as part of the structure's mass. The 6-column TLWD is then reduced to a 4-column or 2-column TLWD. For all the cases, we assume that the TLWD is attached to a SDOF structure with mass, stiffness and damping ratio of 2×10^6 kg, 348.2kN/m, and 0.02, respectively. The SDOF structure is subjected to a harmonic excitation with a frequency of 0.208 Hz.

Using the analytical model, the liquid motion equation of a TLWD can be simplified into the form similar to a 2-column TLCD (Eqs. (12), (13), and (14)). Although the velocity of liquid inside a TLWD is not uniform, it can be treated as an dual-column TLCD with an equivalent mass of ρAL_e and an equivalent stiffness of ρAk_e (Eq. (11)):

$$\rho A(L_e \ddot{y} + L_h \ddot{x} + C\eta |\dot{y}| \dot{y} + k_e y) = 0 \quad (11)$$

When the two end columns at the end are filled with liquid, Eq. (3) becomes:

$$14 \ddot{y} + 10 \ddot{x} + 0.5 \mu |\dot{y}| \dot{y} + 2gy = 0 \quad (12)$$

When the outside four columns are filled with liquid, Eq. (3) becomes:

$$20 \ddot{y} + 10 \ddot{x} + 4.84 \mu |\dot{y}| \dot{y} + 3.2gy = 0 \quad (13)$$

When all the six columns are filled with liquid, Eq. (3) becomes:

$$21.2 \ddot{y} + 10 \ddot{x} + 5.18 \mu |\dot{y}| \dot{y} + 3.6gy = 0 \quad (14)$$

The response of the structure equipped with a TLWD can be determined by the transfer function (Eq. (10)): the lower is the transfer function's peak value, the better is the damper's performance. By adjusting ξ_d value (Figure 7(a)), which is controlled by the orifice blocking ratio, the optimized transfer function curves can be obtained. The optimized parameters of different cases are shown in Table 2. The natural frequencies of TLWD decrease with the increase of the number of columns. By comparing the optimized transfer functions curves showed in Figure 7(b), it can be seen that the 6-column TLWD has the minimum optimized transfer function peak value, which indicates that if the orifices of the dampers are properly tuned, the 6-column TLWD generates a larger damping force than other TLWDs with fewer columns constructed from the same geometry (i.e., non-fully optimized TLWDs).

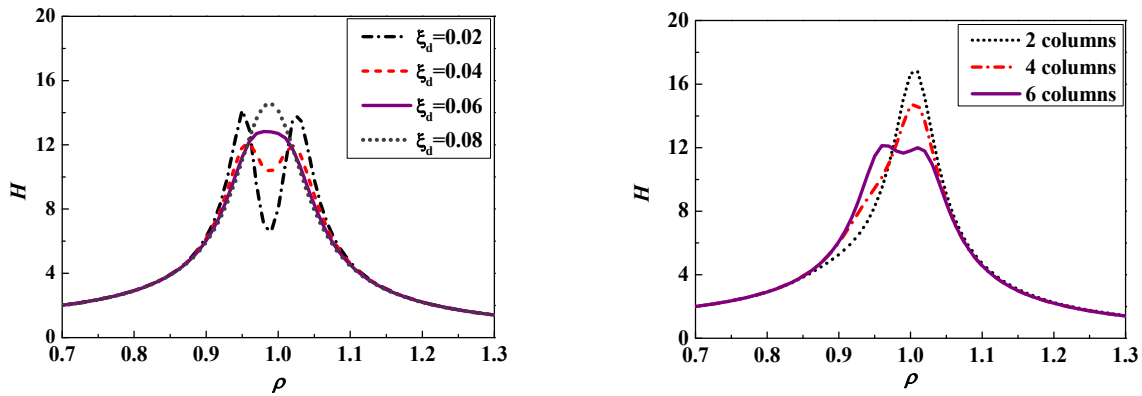


Figure 7. (a) Optimizing the 6-columns TLWD transfer function by altering ξ_d ; (b) Optimized transfer functions curves with different numbers of effective columns

Table 2 Structural parameters of TLWDs with different numbers of effective columns

Cases	Effective mass of liquid (kg)	Equivalent mass (kg)	Equivalent mass ratio	Tuning ratio	Length ratio
2 columns	28000	28000	0.0140	0.904	0.714
4 columns	36000	40000	0.0200	0.957	0.500
6 columns	44000	42400	0.0212	0.986	0.472

Next, we will evaluate the TLWD's damping efficiency. It is assumed that all TLWDs with different numbers of columns have the same amount of liquid and the distances between each columns remain unchanged. Then the transfer function of each case can be optimized by altering the vertical liquid height in the columns and the cross section area of the tubes at the same time. The optimized effective mass, tuning and length ratios are shown in Table 3. Based on the optimized transfer function curves (Figure 8), the 2-column TLWD design has the best performance and achieves a structural response that is 30% and 21.4% less than the 6-column and 4-column TLWDs, respectively. The reason is that, in multi-column TLWDs, the liquid motion in inner columns is slower than that of outer columns, making the liquid in the inner columns less effective. The increased number of inner columns will reduce of the efficiency of TLWDs. A possible improvement is adjusting the orifice blocking ratios between the columns to make the liquid heights more evenly. However, it is noted that the 2-column design requires a very large tube cross-sectional area, which may be impractical for making wall dampers. The tube areas can be further reduced if more columns in the TLWD are added.

Table 3 Optimized parameters of when the liquid amount is kept constant

Number of columns filled with liquid	Effective liquid mass(kg)	Equivalent mass(kg)	Equivalent mass ratio	Vertical height (m)	Tuning ratio	Length ratio	Cross section area(m ²)
2 columns	28000	28000	0.0140	0.90	0.985	0.847	2.38
4 columns	28000	31937	0.0156	1.61	0.988	0.533	1.70
6 columns	28000	27122	0.0136	1.95	0.990	0.476	1.29

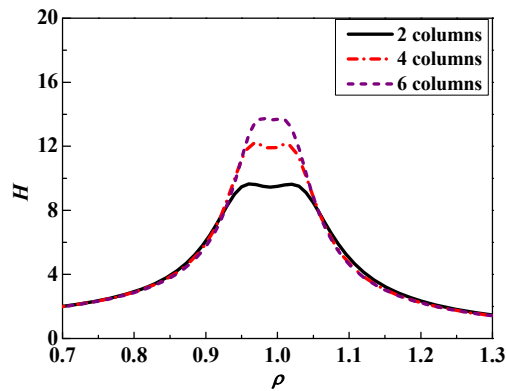


Figure 8. Optimized transfer functions of TLWDs with different numbers of columns and the same amount of liquid

5. CONCLUSIONS

Based on this study, it can be concluded that TLWDs can be used as passive damping devices to dissipate the vibrational energy of structures. The behavior of TLWDs can be described by an analytical model. A CFD simulation method, which is validated against experimental results from a conventional TLCD, is used to verify the analytical model. Various factors that influence the results of CFD simulation are evaluated. The orifice blocking ratio is identified as the main parameter that affects the performance of dampers.

The performance of TLWDs with different numbers of columns in reducing the primary structure's response is studied. It is found that adding inner columns may enhance the damper's performance for fixed geometries. Another comparison shows that if the liquid amount is constant and the dampers are properly tuned, the TLWD with more columns will result in lower efficiency, but yet requires much smaller tube size.

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REFERENCES

- [1] Gao, H., Kwok, K. C. S., and Samali, B., "Optimization of tuned liquid column dampers," *Engineering structures* 19(6), 476-486(1997).
- [2] Chang, C. C., and Hsu, C. T., "Control performance of liquid column vibration absorbers," *Engineering Structures* 20(7), 580-586(1998).
- [3] Yalla, S. K., and Kareem, A., "Optimum absorber parameters for tuned liquid column dampers," *Journal of Structural Engineering* 126(8), 906-915(2000).
- [4] Wu, J-C., Shih, M-H., Lin, Y-Y., and Shen, Y. C., "Design guidelines for tuned liquid column damper for structures responding to wind," *Engineering Structures* 27(13), 1893-1905(2005).
- [5] Laflamme, S., Taylor, D., Maane, M. A., and Connor J. J., "Modified friction device for control of large-scale systems," *Structural control and health monitoring*, 19(4), 548-564(2012).
- [6] Cao, L., Laflamme, S., Taylor, D., and Ricles, J., "Simulations of a variable friction device for multi-hazard mitigation," *Journal of Structural Engineering* 142(12), H4016001(2016).
- [7] Cao, L., Downey, A., Laflamme, S., Taylor, D., and Ricles, J., "Variable friction device for structural control based on duo-servo vehicle brake: Modeling and experimental validation," *Journal of Sound and Vibration*, 348, 41-56(2015).

- [8] Connor, J., and Laflamme, S., [Structural motion engineering], Springer, New York, 367-370(2014).
- [9] Balendra, T., Wang, C., and Rakesh, G., "Vibration control of various types of buildings using TLCD, " *Journal of Wind Engineering and Industrial Aerodynamics*, 83(1), 197-208(1999).
- [10] Min, K.-W., Kim, H.-S., Lee, S.-H., Kim, H., and Kyung, A., S., "Performance evaluation of tuned liquid column dampers for response control of a 76-story benchmark building," *Engineering Structures*. 27(7), 1101-12(2005).
- [11] Lee, H.H., Wong, S.H., and Lee, R.S., "Response mitigation on the offshore floating platform system with tuned liquid column damper," *Ocean Engineering*, 33(8), 1118-42(2006).
- [12] Colwell, S., and Basu, B., "Tuned liquid column dampers in offshore wind turbines for structural control," *Engineering Structures*, 31(2), 358-68(2009).
- [13] Shope, R.L., and Murray, T.M., "Multi-celled liquid dampers to eliminate annoying floor vibrations," *Canadian Acoustics*, 22(3), 167-168(1994).
- [14] Min, K-W., Park, C-S., and Kim, J. "Easy-to-tune reconfigurable liquid column vibration absorbers with multiple cells," *Smart Materials and Structures*, 24(6), 065041(2015).