2014

Finite element based hybrid evolutionary optimization approach to solving rigid pavement inversion problem

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Abstract

This paper focuses on the development of a new backcalculation method for concrete road structures based on a hybrid evolutionary global optimization algorithm, namely shuffled complex evolution (SCE). Evolutionary optimization algorithms are ideally suited for intrinsically multi-modal, non-convex, and discontinuous real-world problems such as pavement backcalculation because of their ability to explore very large and complex search spaces and locate the globally optimal solution using a parallel search mechanism as opposed to a point-by-point search mechanism employed by traditional optimization algorithms. SCE, a type of evolutionary optimization algorithms based on the tradeoff of exploration and exploitation, has proved to be an efficient method for many global optimization problems and in some cases it does not suffer the difficulties encountered by other evolutionary computation techniques. The SCE optimization approach is hybridized with a neural networks surrogate finite-element based forward pavement response model to enable rapid computation of global or near-global pavement layer moduli solutions. The proposed rigid pavement backcalculation model is evaluated using field non-destructive test data acquired from a full-scale airport pavement test facility.

Keywords

concrete pavements, evolutionary algorithms, finite elements, neural networks, pavement moduli backcalculation

Disciplines

Civil and Environmental Engineering | Construction Engineering and Management

Comments

This is a manuscript of an article from *Engineering with Computers* 30 (2014): 1, doi: 10.1007/s00366-012-0281-y. The final publication is available at Springer via http://dx.doi.org/10.1007/s00366-012-0281-y.

Reference to this paper should be made as follows: Ceylan, H. and Gopalakrishnan, K. (2014). "*Finite Element based Hybrid Evolutionary Optimization Approach to Solving Rigid Pavement Inversion Problem.*" Engineering with Computers Journal, Vol. 30, No. 1, pp 1-13.

Finite Element based Hybrid Evolutionary Optimization Approach to Solving Rigid Pavement Inversion Problem

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ABSTRACT

This paper focuses on the development of a new backcalculation method for concrete road structures based on a hybrid evolutionary global optimization algorithm, namely Shuffled Complex Evolution (SCE). Evolutionary optimization algorithms are ideally suited for intrinsically multi-modal, non-convex, and discontinuous real-world problems such as pavement backcalculation because of their ability to explore very large and complex search spaces and locate the globally optimal solution using a parallel search mechanism as opposed to a point-by-point search mechanism employed by traditional optimization algorithms. Shuffled Complex Evolution (SCE), a type of evolutionary optimization algorithms based on the tradeoff of exploration and exploitation, has proved to be an efficient method for many global optimization problems and in some cases it does not suffer the difficulties encountered by other evolutionary computation techniques. The SCE optimization approach is hybridized with a Neural Networks (NN) surrogate finite-element based forward pavement response model to enable rapid computation of global or near-global pavement layer moduli solutions. The proposed rigid pavement backcalculation model is evaluated using field non-destructive test data acquired from a full-scale airport pavement test facility.

Key words: Neural networks, pavement moduli backcalculation, finite elements, evolutionary algorithms, concrete pavements.

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Introduction

Transportation agencies across the world, in charge of maintaining the highway systems, frequently evaluate the structural condition of road pavements as part of their routine maintenance schedule. The Falling Weight Deflectometer (FWD) is the most commonly used non-destructive type pavement deflection testing equipment used for such purposes. The measured deflections from FWD can be correlated to in-situ material stiffness of each layer in the pavement structure through a procedure known as backcalculation or inverse analysis. Determination of in-situ material stiffness is essential in assessing the structural condition of exiting pavement for estimation of pavement remaining life and in determining the thickness of new overlay.

The backcalculation methodology is an inverse process to determine in-situ materials stiffness of pavement layer by matching the measured and the theoretical deflection with iteration or optimization schemes. The most common approach in current commercial backcalculation software requires inputting initial seed modulus which is an assumed layer modulus for an iterative process. Thus, the reliability of the final optimized solution is dependent upon the initial seed modulus. It is not uncommon that minor deviations between measured and computed deflections usually result in significantly different moduli and the various combinations of modulus values essentially produce the same deflection basin (Mehta and Roque 2003).

Over the years, numerous pavement backcalculation approaches have been developed. Each approach has its own pros and cons and researchers continue to explore advanced hybrid approaches to pavement moduli backcalculation with the main aim of facilitating speed of convergence, robustness, and computational efficiency (Gopalakrishnan et al. 2010).

Evolutionary optimization algorithms are ideally suited for intrinsically multi-modal, non-convex, and discontinuous real-world problems because of their ability to explore very large and complex search spaces and locate the globally optimal solution using a parallel search mechanism as opposed to a point-by-point search mechanism employed by traditional optimization algorithms (Muttil and Liong 2004). Shuffled Complex Evolution (SCE), a type of evolutionary optimization algorithms based on the tradeoff of exploration and exploitation, has been proved to be an efficient method for many global optimization problems and in some cases it does not suffer the difficulties encountered by other evolutionary computation techniques (Muttil and Liong 2004).

This paper proposes a hybrid SCE-based heuristic optimization algorithm for analysis of rigid pavement non-destructive test data and backcalculation of concrete pavement layer moduli.

Shuffled Complex Evolution (SCE) Algorithm

The SCE algorithm developed at the University of Arizona is reported to be an efficient global optimization method that can be used to handle non-linear problems with high-parameter dimensionality (Duan et al. 1992, Duan et al. 1993, Duan et al. 1994, Muttil and Liong 2004). It consists of all the four principles for global optimization: the controlled random search, the implicit clustering, the complex shuffling, and the competitive evolution. The search for the optimal solution begins with a randomly selected complex of points spanning the entire feasible space. The implicit clustering helps to concentrate the search in the most promising of the regions. The use of complex shuffling provides a freer and more extensive exploration of the search space in different directions, thereby reducing the chances of the search getting trapped in local optima. Three of these principles are coupled with the competitive complex evolution (CCE) algorithm, which is a statistical reproduction process employing the complex geometric shape to direct the search in the correct direction. The synthesis of these concepts makes the SCE algorithm not only effective and robust, but also flexible and efficient (Nunoo and Mrawira 2004).

The SCE algorithm developed at the University of Arizona (also referred to as SCE-UA) was originally to deal with the rainfall – runoff models (Duan et al. 1992). The SCE has been used extensively for the calibration of various rainfall-runoff models and other water-related fields (Muttil and Jayawardena, 2008) since then. In the field of civil engineering, Nunoo and Mrawira (2004) applied the SCE in decision-making process of infrastructure management. Since the decision-making process of infrastructure management involves a large number of sections and a large number of possible treatment alternatives, optimization of infrastructure preservation activities for available resources is a difficult task. The findings of this study showed that the SCE algorithm is very efficient and consistent in simultaneous consideration of the trade-off among various infrastructure preservation strategies. Sanson and Shibayama (2007) also demonstrated that SCE algorithm could be used to optimize the planning of maintenance and rehabilitation of a road network in Japan.

Barakat and Altoubat (2009) evaluated three optimization techniques to solve nonlinear constrained structural optimization problems. These methods are SCE, simulated annealing (SA) and genetic algorithm (GA). They concluded that the robust search capability of SCE algorithm technique is well suited for solving the structural problem in hand. Gopalakrishnan (2010) successfully demonstrated the use of global optimization techniques like Particle Swarm Optimization (PSO) and SCE in the backcalculation of conventional flexible pavement layer moduli. The current study focuses on developing SCE-based rigid pavement backcalculation models.

Basic Algorithm

The SCE control parameters should be determined in advance to achieve the required exploration process. These parameters include the number of points in a complex (m), the number of points in a sub complex (q), the number of complexes (p), the number of consecutive offspring generated by each sub complex (α), and the number of steps in-evolution taken by each complex (β). Duan et al (1994) provides guidelines for proper selection of these parameters.

The basic algorithm for SCE described by Duan et al (1993) is represented in Figure 1 and can be outlined as follows:

1. An initial population of points is sampled randomly from the feasible solution space (Ω) in the real space (R^n).
2. The selected population is partitioned into one or more complexes, each containing a fixed number of points.
3. Each complex evolves according to a competitive complex evolution (CCE) algorithm.
4. The entire population is periodically shuffled and points are reassigned to complexes to share the information from the individual complexes.
5. Evolution and shuffling are repeated so that the entire population is close to convergence criteria, and are stopped if the convergence criteria are satisfied.

The CCE algorithm is a sub-route in SCE algorithm. CCE algorithm employs the downhill simplex method (Nelder and Mead 1965) in generating offsprings. The simplex method facilitates evolution of each complex independently in an improvement direction. The CCE procedure is described by Duan et al (1993).

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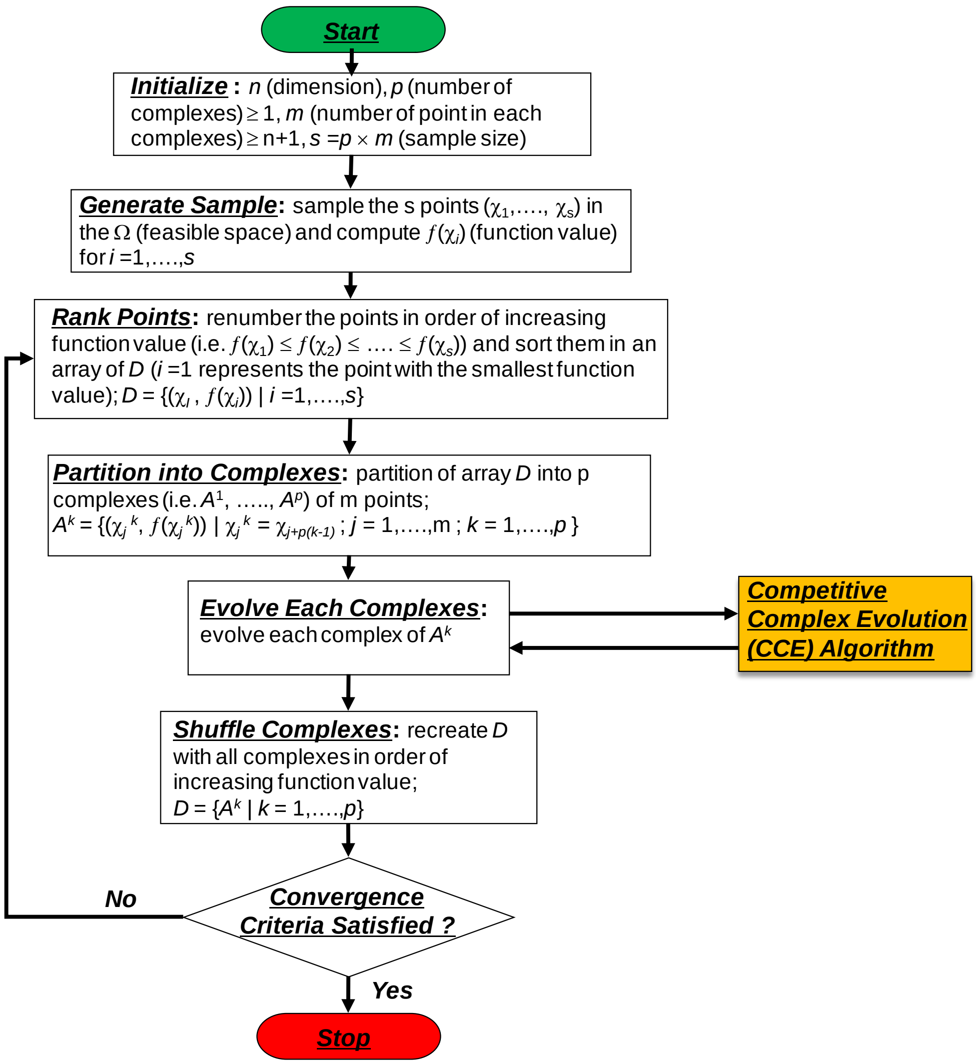

Figure 1. Shuffled complex evolution (SCE) algorithm (after Duan et al. 1993)

SCE-based Rigid Pavement Inverse Analysis

The elastic modulus of the slab, E, and modulus of subgrade reaction, k, are the two most important backcalculated concrete pavement properties. Over the years, researchers have developed many different methodologies for backcalculation of concrete pavement properties from FWD measurements, including the AREA method for rigid pavements (Ioannides et al., 1989; Ioannides, 1990; Barenberg and Petros, 1991), ILLI-BACK (Ioannides, 1994), graphical solution using IILI-SLAB (Foxworthy and Darter, 1989), use of regression analysis to solve AREA method for rigid pavements (Hall, 1992; Hall et al., 1996), use of best fit algorithm to find radius of relative stiffness (l) (Hall et al., 1996; Smith et al., 1996), among others.

Based on a backcalculation study of concrete pavement properties using 277 deflection basins obtained from the Denver International Airport (DIA), Rufino et al. (2002) studied the effect of slab modeling (number of layers, interface condition, and model type) as well as effect of different methodologies and sensor configurations on backcalculated pavement properties. It was found that backcalculated slab modulus of elasticity (E) is lower on average when the pavement layers on top of subgrade are bonded versus unbonded interface. Higher backcalculated k-values are obtained when the slab is modeled as plate compared to modeling the slab as elastic layer. Modeling of the slab and base as elastic layers seem to yield more reasonable backcalculated results since the interface bonding condition can be reflected both in the backcalculated slab elastic modulus and subgrade k-value.

Over the past decade, the use of computational intelligence techniques in pavement systems modeling, analysis, and design, has become increasingly common. Ceylan (2002) employed Artificial Neural Networks (ANNs) in the analysis of concrete pavement systems and developed ANN-based design tools that incorporated the ISLAB 2000 (Tabatabaie and Barenberg, 1978; Khazanovich, 1994; Khazanovich et al, 2000) finite element solutions into routine practical design at several orders of magnitude faster than ISLAB 2000.

Khazanovich and Roesler (1997) developed a program called DIPLOBACK for backcalculation of moduli values of composite pavements based on ANNs. ANNs have also been applied along with dimensional analysis to backcalculate joint properties from FWD testing (Ioannides et al., 1996). The advantage of using ANN and dimensional analysis together is that they both reduce the database size necessary to accurately estimate pavement properties (Rufino et al., 2002). In the development of the new Mechanistic-Empirical Pavement Design Guide (MEPDG) for the American Association of State Highway and Transportation Officials (AASHTO), ANNs were recognized as nontraditional, yet very powerful computing techniques and ANN models were used in preparing the concrete pavement analysis package (Khazanovich et al, 2001). Ceylan et al. (2009) developed a suite of ANN-based flexible, rigid, and composite pavement backcalculation models from comprehensive synthetic databases.

This paper discusses the implementation of the SCE optimization approach for a slab-on-grade rigid pavement structure although it can be used for a variety of pavement geometry and types owing to its flexible and integrated modular systems approach. The objective (fitness) function or the cost function for the proposed SCE optimization approach is the difference between measured FWD deflections and computed pavement surface deflections.

SCE Implementation

The proposed SCE global optimization backcalculation approach is presented in Figure 2. This approach treats backcalculation as a global optimization problem where the cost function to be minimized is defined as the differences in measured and computed deflections. The optimal solution (elastic modulus of the slab, E, and modulus of subgrade reaction, k) is searched for in the multi-modal solution space by the SCE algorithm as described previously. Thus, for every update of the population of moduli solutions in the SCE search scheme, the forward pavement response model has to be invoked to compute the resulting surface deflections. In this paper, the SCE optimization technique is hybridized with a Neural Networks (NN) surrogate forward pavement response model for rapid prediction of surface deflections using elastic moduli and thicknesses of pavement layers as inputs. This reduces the computational time of SCE significantly considering the number of times the surface deflections need to be computed using different sets of pavement layer moduli during the optimization process. Thus, the resulting hybrid backcalculation model, NN-SCE combines the robustness of SCE with the computational efficiency of NNs.

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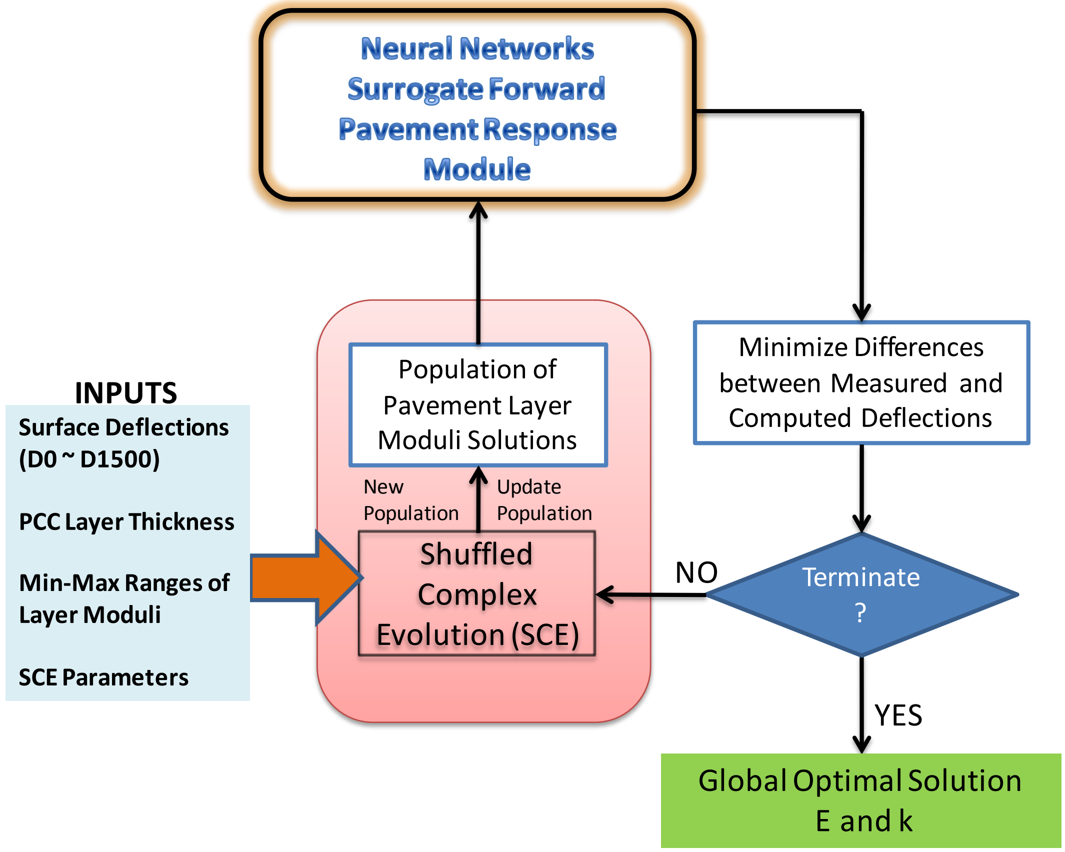

Figure 2. Neural networks (NN) - shuffled complex evolution (SCE) hybrid global Optimization backcalculation approach

The SCE, in essence, finds the optimal values of the NN inputs (pavement layer moduli) iteratively such that the corresponding values of the network outputs (deflections) match the measured pavement surface deflections to minimize the differences between the measured and computer deflections. Although the error-minimization deflection-based objective function can be defined in a number of ways, a simple objective function representing sum of the squared differences between measured and computed deflections as shown in Equation 1 was selected for this study (where n = 6):

$$f = \sum_{i=1}^{n} (D_i - d_i)^2 \quad (1)$$

The NN-SCE hybrid optimization toolbox for rigid pavement backcalculation was implemented in MATLAB. The input variables to the toolbox include six FWD measured surface deflections at 300-mm radial offsets starting from the center of the FWD loading plate (D0, D300, D600, D900, D1200, and D1500), PCC layer thickness, and the corresponding min-max ranges of pavement layer moduli. For ease of implementation, all values were normalized in the range of 0.1 to 0.9.

A trained NN serves as a surrogate forward pavement response model that has learned the mapping between pavement layer moduli and resulting pavement surface deflections for a variety of case scenarios generated using the ISLAB 2000 pavement finite element program, as described in the next section.

The choice of the SCE algorithm's parameters is crucial in achieving convergence of solution for the problem under consideration. In this study, the following guidelines proposed by Duan et al. (1992) were used for determining the SCE parameters (m = number of points in each complexes, p = number of complexes, q = number of parent solutions, α and β are user specified parameters):

- $m = 2n$, where n = dimension or number of parameters being estimated
- $q = n+1$ and $p = 5$
- $\beta = 1$ and $\alpha = 1$

NN Surrogate Forward Pavement Response Model

A BackPropagation (BP) type NN model was trained in this study with the results from the ISLAB 2000 finite element program to develop the surrogate forward pavement response model. BP NNs are very powerful and versatile networks that can be taught a mapping from one data space to another using example of the mapping to be learned. The term "back-propagation network" actually refers to a multi-layered; feed-forward neural network trained using an error back-propagation algorithm. The learning process performed by this algorithm is called "back-propagation learning" which is mainly an "error minimization technique" (Haykin, 1999). Backpropagation networks excel at data modelling with their superior function approximation capabilities (Haykin, 1999).

A total of 41,106 data vectors generated by modeling slab-on-grade concrete pavement systems using ISLAB 2000 were used for NN training and testing. Concrete pavements analyzed in this study were represented by a six-slab assembly, each slab having dimensions of 6.1 m by 6.1 m (20 ft by 20 ft). The dense liquid model, proposed by Winkler (1867), was used to characterize the subgrade behavior. To maintain the same level of accuracy in the results from all analyses, a standard ISLAB 2000 finite element mesh was constructed for the slab. This mesh consisted of 10,004 elements with 10,209 nodes. The 40-kN (9,000-lb) FWD loading condition was simulated in ISLAB 2000.

The ISLAB 2000 solutions database was generated by varying E, k and thickness of PCC (h_{pcc}) over a range of values representative of realistic variations in the field. The E ranged from 6.9 to 103.4 GPa (1,000 to 15,000 ksi); k ranged from 13.6 to 217 MPa/m (50 to 800 psi/in); and h_{pcc} ranged from 152 to 635 mm (6 to 25 in) considering that most design thicknesses would be in this range. Note that the US design approach for concrete pavements limits k to 0.16-0.20 N/mm^3. A Poisson's ratio (μ) of 0.15 was assumed for PCC. Thus a total of 41,106 ISLAB 2000 analyses (51 different values of E × 31 different values of k × 26 different values of h_{pcc}) were conducted to represent a complete factorial of all the input values.

Histograms and quantile box plots of inputs (E, k, and h_{pcc}) for generating the synthetic database are presented in Figure 3. Frequency histogram is shown on the left side and a box plot is shown on the right side. The rectangular box plot extends from the first to the third quartile with the median marked in the center.

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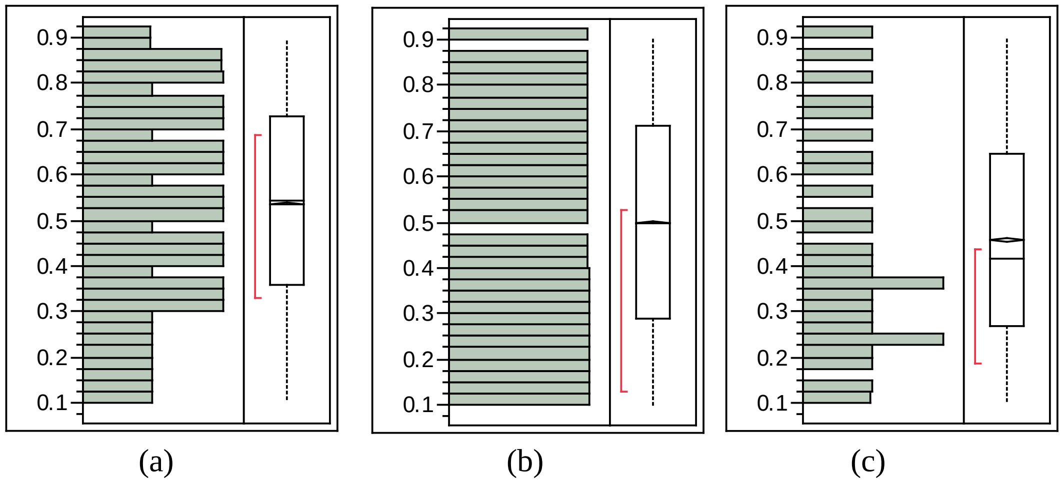

Figure 3. Histograms and quantile box plots of inputs: (a) E_{PCC}; (b) k; and (c) h_{PCC}

A scatterplot for each pair of variables in the synthetic database is displayed in a matrix arrangement in Figure 4. A 95% bivariate normal density ellipse is imposed on each scatterplot with the correlation values shown. If the variables are bivariate normally distributed, this ellipse encloses approximately 95% of the points. The correlation of the variables is seen by the collapsing of the ellipse along the diagonal axis. If the ellipse is fairly round and is not diagonally oriented, the variables are uncorrelated. It is observed that the modulus of subgrade reaction, k, is strongly correlated to deflections at the outer sensors (especially D1200 and D1500). Relatively, E is poorly correlated to all the deflections.

A network with two hidden layers was exclusively chosen for the NN models trained in this study. Satisfactory results were obtained in the previous studies with these types of networks due to their ability to better facilitate the nonlinear functional mapping (Ceylan, 2002; Ceylan et al, 2005).

In the BP NNs used in this study, the connection weights are initially selected at random. Inputs from the mapping examples are propagated forward through each layer of the network to emerge as outputs. The errors between those outputs and the correct answers are then propagated backwards through the network and the connection weights are individually adjusted to reduce the error. After many examples (training patterns) have been propagated through the network many times, the mapping function is learned with some specified error tolerance. This is called supervised learning because the network has to be shown the correct answers for it to learn (Haykin, 1999; Ceylan et al, 2005).

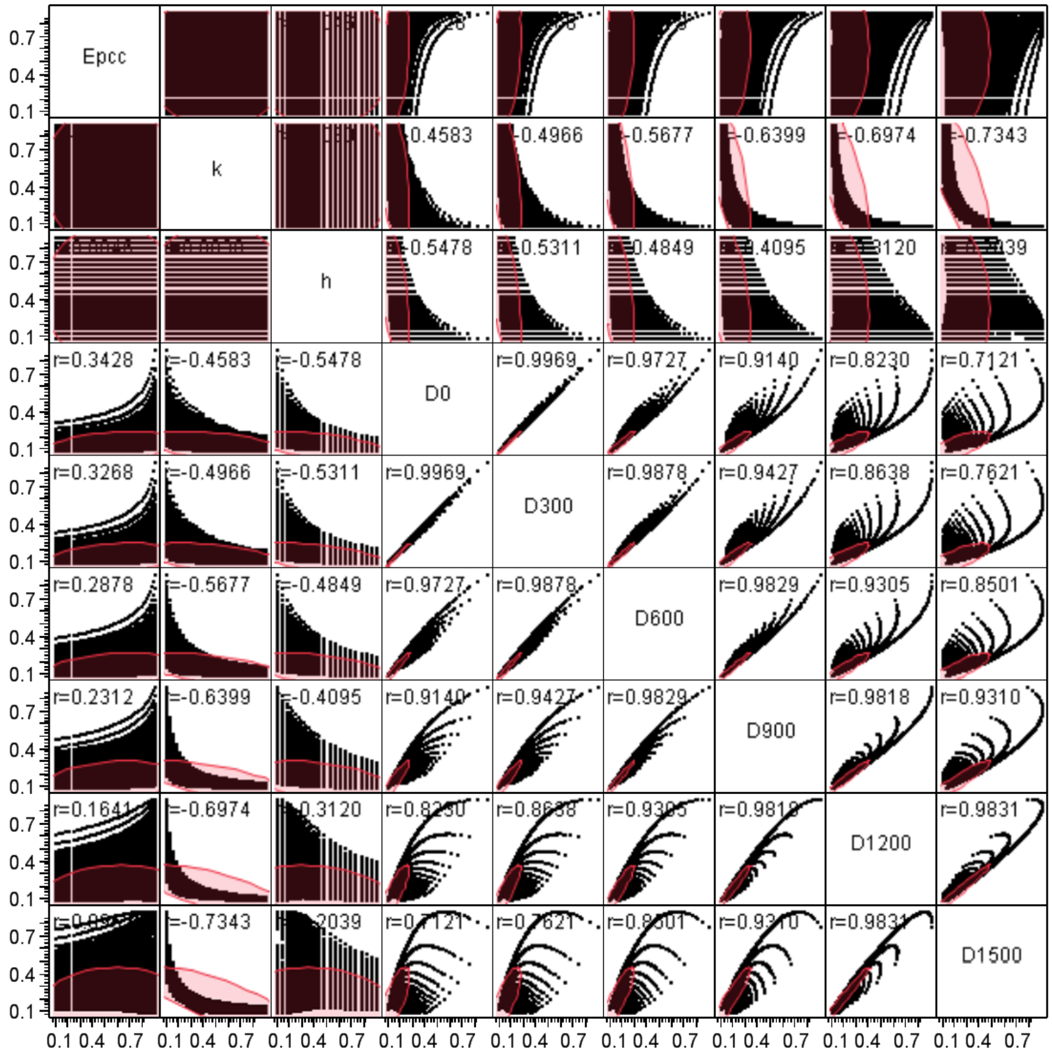

Figure 4. Scatterplot matrix of input and output variables in the synthetic database

The 3-60-60-6 architecture (see Figure 5: 3 inputs [E, k, and PCC layer thickness], 60 nodes in the first and second hidden layers, and 6 outputs [D0 ~ D1500], respectively) was chosen as the best architecture based on its lowest training and testing Mean Squared Errors (MSEs). The NN training and testing were conducted with the MATLAB NN toolbox using the Levenberg-Marquardt training algorithm. In the BP learning algorithm, the error energy used for monitoring the progress toward convergence is the generalized value of all errors that is calculated by the least-squares formulation and represented by a Mean Squared Error (MSE) as follows (Haykin 1999):

$$MSE = \frac{1}{MP}\sum_{1}^{P}\sum_{k=1}^{M}\left(d_k - y_k\right)^2 \tag{2}$$

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where y_k and d_k are actual and desired outputs, respectively, M is the number of neurons in the output layer and P represents the total number of training patterns. Other performance measures such as the Root Mean Squared Error (RMSE), Average Absolute Error (AAE), etc. are also used.

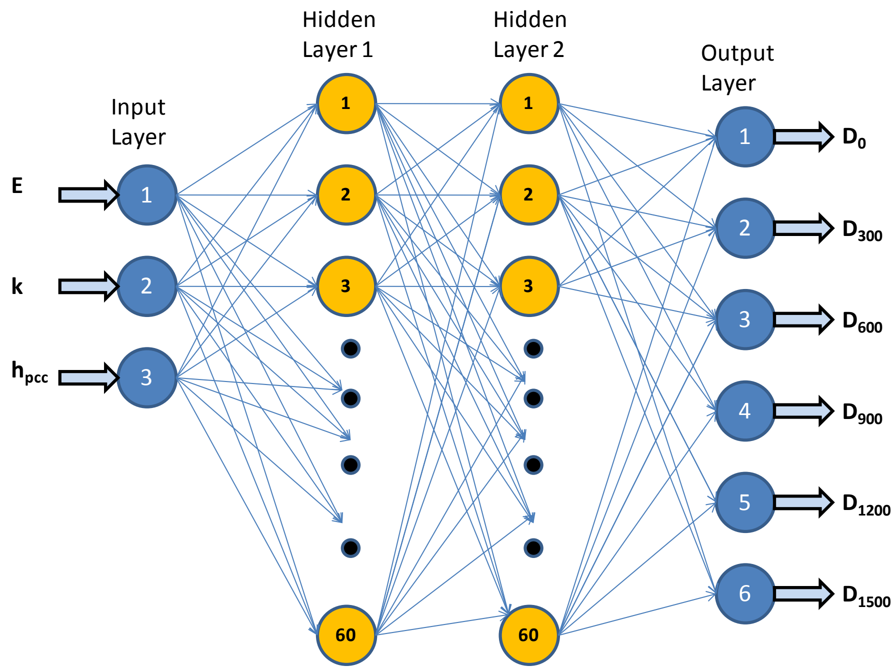

Figure 5. NN surrogate forward pavement response model architecture

Figure 6 shows the training and testing MSE progress curves for the 3-60-60-6 network. Both the training and testing curves for the output are in the same order of magnitude thus depicting proper training. The almost constant MSEs obtained for the last few iterations also provide a good indication of adequate training of this network. Exceptional prediction performance of the NN surrogate response models were achieved validating their suitability for use in SCE hybrid optimization approach.

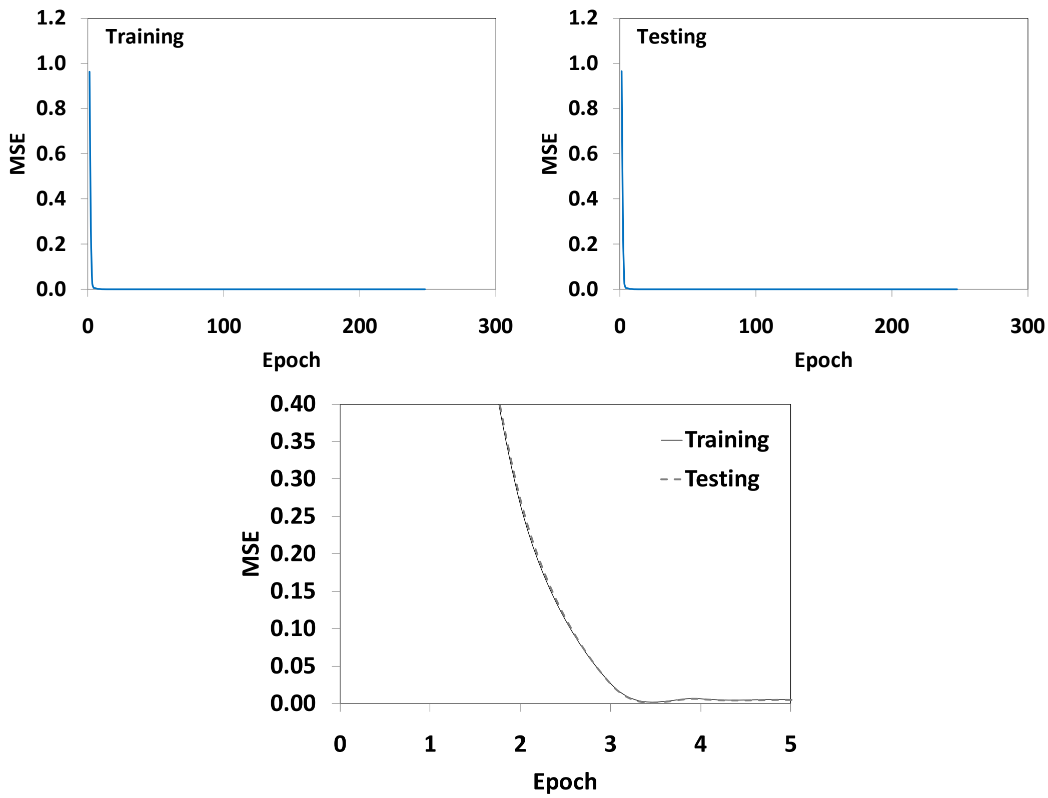

Figure 6. NN surrogate forward pavement response model training progress curve

Prediction Performance of NN-SCE Backcalculation Approach

Hypothetical data covering wide ranges of layer thicknesses and FWD deflections commonly encountered in the field were first used to evaluate the prediction accuracy of the developed NN-SCE hybrid concrete pavement backcalculation tool. A total of about 150 datasets were independently selected from the comprehensive synthetic FE solutions database to assess the prediction performance. The performance of NN-SCE optimization approach in backcalculating concrete pavement layer moduli is reported in Figure 7. As shown in the plots, all 150 NESCE backcalculation predictions fell on the line of equality for the two pavement layer moduli (concrete modulus, E and modulus of subgrade reaction, k) thus indicating a proper training and excellent performance of the proposed hybrid backcalculation model.

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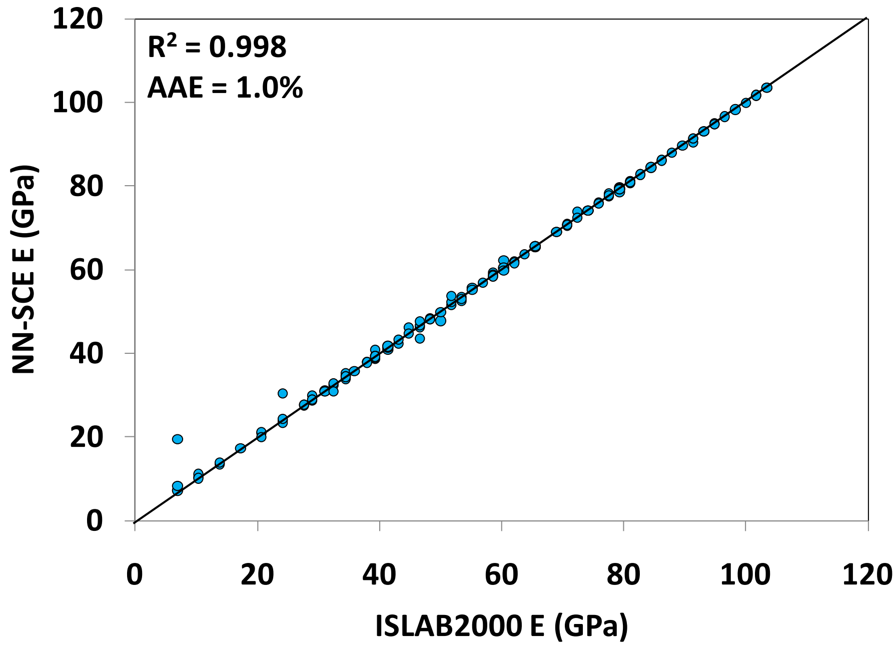
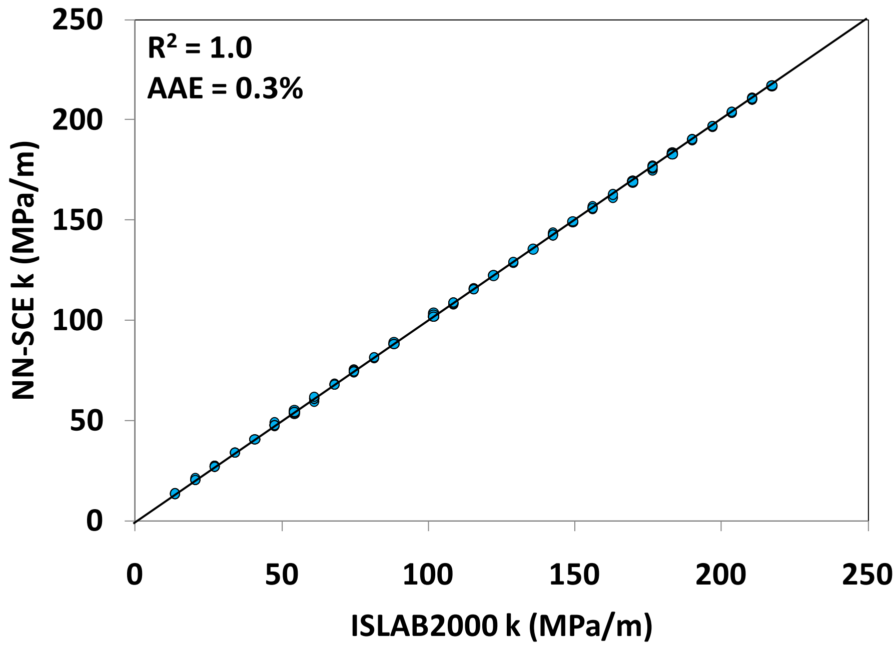


(a)



(b)

Figure 7. Prediction performance of NN-SCE backcalculation tool with hypothetical data: (a) E; (b) k

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The *goodness-of-fit* statistics for the NN-SCE model predictions were performed using statistical parameters such as the correlation coefficient (R^2) and the average absolute error (AAE). Average Absolute Errors (AAEs) were calculated as sum of the individual absolute relative errors divided by the number of independent testing patterns:

$$Average\ Absolute\ Error\ (AAE), \% =$$

$$\sum_{i=1}^{n} \left| \frac{y_{actual} - y_{predicted}}{y_{actual}} \right|_i * 100 \tag{3}$$

where i is the ith testing pattern among n testing patterns.

The R^2 is a measure of correlation between the predicted and the measured values and therefore, determines accuracy of the fitting model (higher R^2 equates to higher accuracy). The AAE indicates the relative improvement in accuracy and thus a smaller value is indicative of better accuracy.

The modulus of subgrade reaction (k) is the stress that will cause one unit of deflection in the underlying soil. Soils such as clay will have a lower k-value compared to cement treated or asphalt treated bases. Research has shown that the value of k depends on certain soil characteristics such as density, moisture, soil texture and other factors that influence the strength of the soils. The k-value of a particular soil will also vary with size of the loaded area and the amount of deflection. The modulus of subgrade reaction is directly proportional to the loaded area and inversely proportional to the deflection. Modulus of subgrade reaction is obtained by conventional plate bearing tests as described in AASHTO T222, correlation with soil properties or other soil tests and also by backcalculation from FWD testing on concrete pavements.

For slab-on-grade systems, Ioannides (1990) proposed a closed-form procedure for backcalculating foundation properties based on principles of dimensional analysis by recognizing the existence of a unique relationship between AREA (Hoffman and Thompson, 1981) and radius of relative stiffness (l_k). Once the radius of relative stiffness is known, Westergaard's (1926) maximum deflection solution for interior loading can be used to backcalculate the subgrade k-value. Once the subgrade properties and radius of relative stiffness are known, the slab modulus of elasticity (E) can also be determined. This approach has been coded in a computer program called ILLI-BACK by Ioannides (1990). The following equations are used:

$$AREA = 6 * \left[1 + 2\left(\frac{D_{12}}{D_0}\right) + 2\left(\frac{D_{24}}{D_0}\right) + \left(\frac{D_{36}}{D_0}\right) \right] \tag{4}$$

$$l_k = \left[\frac{\ln\left(\frac{36 - AREA}{1812.279133}\right)}{-2.559340} \right]^{4.387009} \tag{5}$$

$$k = \left(\frac{P}{8D_0 l_k^2}\right) \left\{ 1 + \left(\frac{1}{2\pi}\right) \left[\ln\left(\frac{a}{2l_k}\right) - 0.673 \right] \left(\frac{a}{l_k}\right)^2 \right\} \tag{6}$$

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$$E = \left(\frac{12 l_k^{\ 4} k (1 - \mu^2)}{h^3} \right) \tag{7}$$

where the deflections are in inches; P is the FWD/HWD load in lbs; a is the radius of load plate (usually 6 inches); and h is the effective slab thickness in inches. Note that the k-value obtained from this backcalculation procedure is the dynamic-k value which is typically twice the k value obtained from plate bearing tests. Using this closed-form backcalculation procedure, the k-values were backcalculated using the same hypothetical data used for testing the prediction performance of NN-SCE approach.

For the sake of illustration, the comparison between the closed-form procedure k values and NN-SCE predicted k values are presented in Figure 8 for the hypothetical test data. There is a very good agreement between the two, especially at lower k values. The NN-SCE NN predictions were also in agreement with the NN based backcalculation results reported by Ceylan et al. (2008).

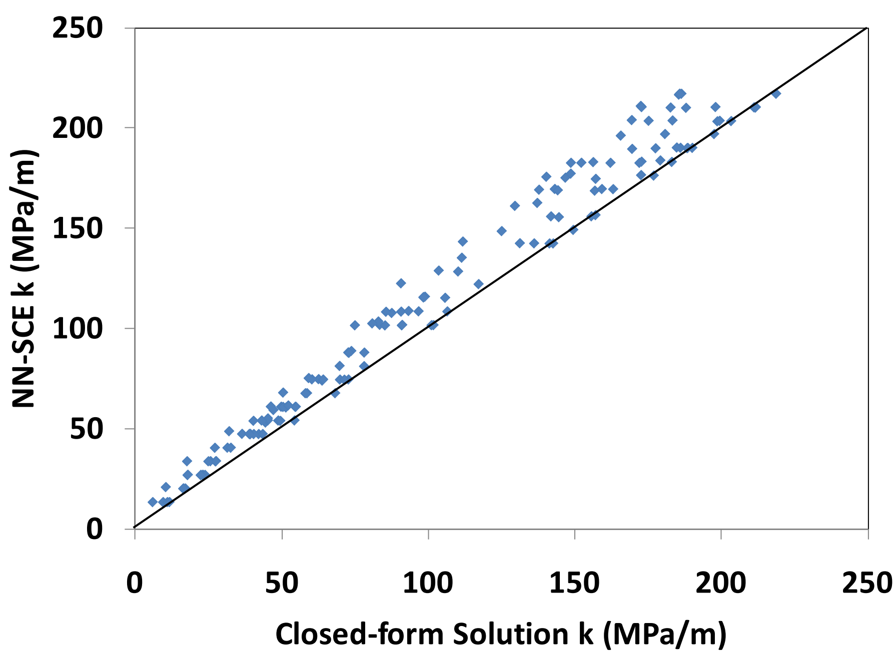

Figure 8. NN-SCE k predictions compared with closed-form solutions

NN-SCE Backcalculation Using Field Data

The National Airport Pavement Test Facility (NAPTF) is located at the Federal Aviation Administration's (FAA) William J. Hughes Technical Center near Atlantic City International Airport, New Jersey, USA. It was constructed to support the development of advanced mechanistic-based airport pavement design procedures based on sound theoretical principles and with models verified from appropriate full-scale test data. The NAPTF is a fully enclosed state-of-the-art full-scale test facility. The first series of test pavements, referred to as Construction Cycle 1 (CC-1), consisted of nine instrumented test pavements (six flexible and three rigid) that were 18.3 m (60 ft) wide and total 274.3 m (900 ft) in length. The nine test pavements were built on three different subgrade materials: low-strength (target California Bearing Ratio [CBR] of 4), medium-strength (target CBR of 8), and high-strength (target CBR of 20). All NAPTF data referenced in this paper are available for public download at the NAPTF web site: http://www.airporttech.tc.faa.gov/naptf/.

This study focuses on the NAPTF CC1 rigid pavement sections. The three rigid pavement sections are designated as follows: (a) LRS – rigid pavement with stabilized base over low-strength subgrade, (b) MRS – rigid pavement with stabilized base over medium-strength subgrade, and (c) HRS – rigid pavement with stabilized base over high-strength subgrade. Cross-sections of the as-constructed PCC test pavements are presented in Figure 9.

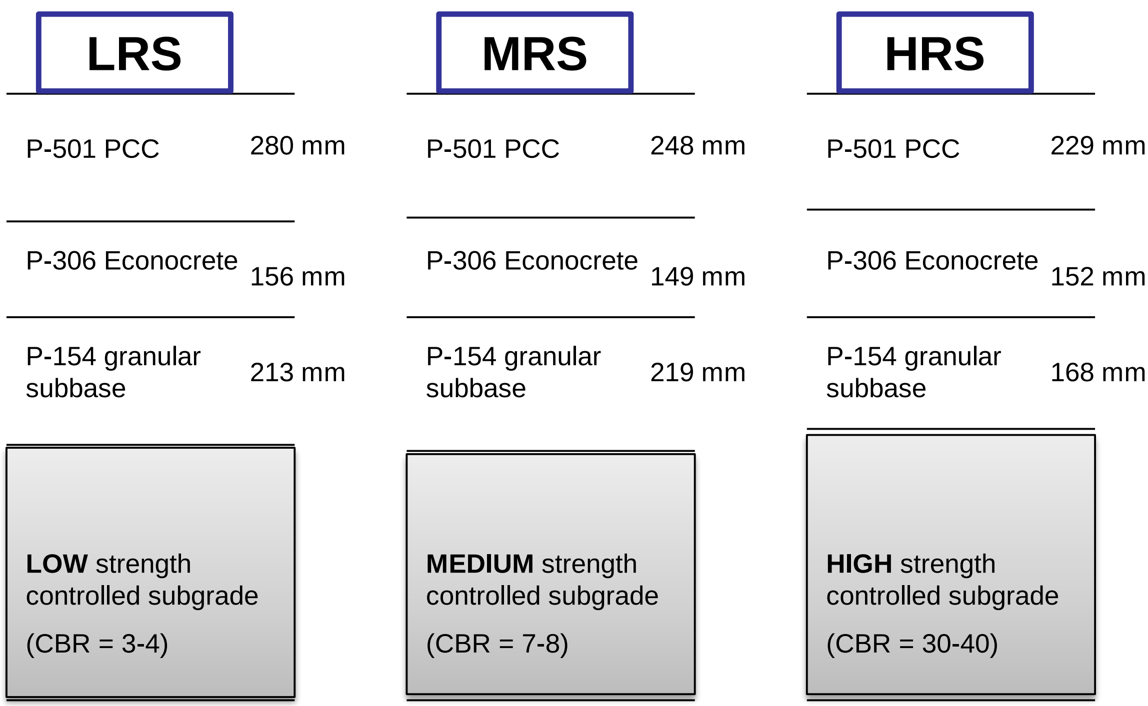

Figure 9. Schematic of NAPTF CC1 rigid pavement items cross-sections

During CC-1 traffic testing, a simulated 6-wheel dual-tridem (Boeing 777) landing gear, with 1,372-mm (54-in) dual spacing and 1,448-mm (57-in) tandem spacing was loaded on the north wheel track (LANE 2) while the south side (LANE 5) was loaded with a 4-wheel dual-tandem (Boeing 747) landing gear having 1,118-mm (44-in) dual spacing and 1,473-mm (58-in) tandem spacing. The wheel loads were set to 20.4 metric tons (45,000 lbs) each and the tire pressure was 1,295 kPa (188 psi). The traffic speed was 8 km/h (5 mph) throughout the traffic test program. To realistically simulate transverse aircraft movements, a wander pattern consisting of a fixed sequence of 66 vehicle passes (33 traveling in the east direction and 33 traveling in the west direction), arranged in nine equally spaced wander positions (or tracks) at intervals of 260 mm (10.25 in), was used during NAPTF traffic testing. For rigid pavement test sections, failure was defined in terms of structural cracking initiating at the joints at the bottom of the PCC layer (McQueen et al. 2002).

The FAA HWD equipment has seismometers (for measuring deflections) which utilize a spring for reference and Linear Variable Differential Transformer (LVDT) for the sensor. The deflections were measured at offsets of 0-mm (D_0), 305-mm (D_{300}), 610-mm (D_{600}), 914-mm (D_{900}), 1219-mm (D_{1200}), and 1524-mm (D_{1500}) intervals from the center of the load. The HWD tests were conducted at nominal force amplitudes of 53.4-kN (12,000-lb), 106.7-kN (24,000-lb), and 160-kN (36,000-lb). The HWD test locations which are of interest in this study (slab center locations) are shown in Figure 10. Note that LANE 1 was trafficked by six-wheel B777 gear loading while LANE 3 was trafficked by four-wheel B747 gear loading. LANE 2 was untrafficked. The same set of FWD/HWD testing points were used in all three rigid pavement sections.

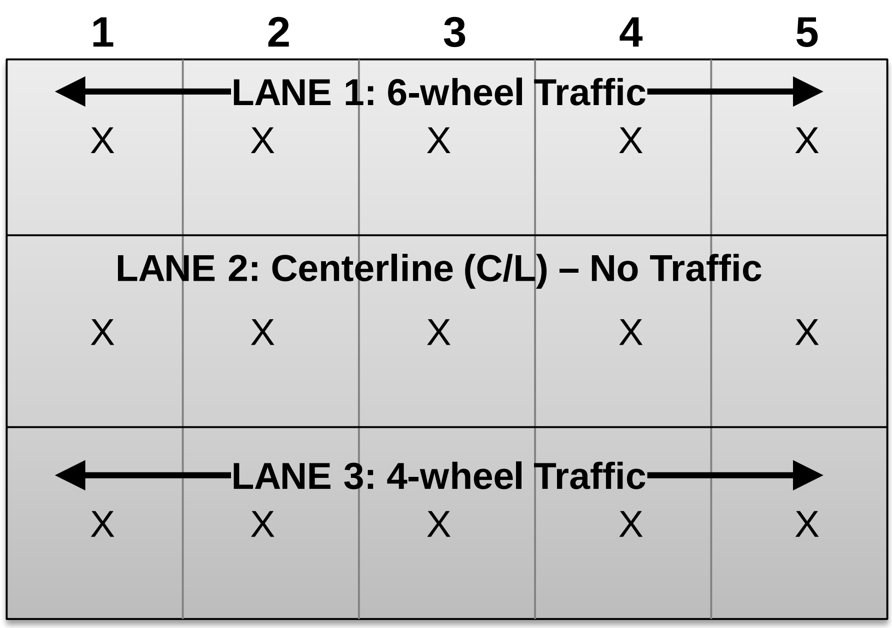

Figure 10. Schematic of NAPTF CC1 rigid pavement NDT slab center test locations

The developed NN-SCE backcalculation methodology was applied on the actual HWD deflection basins acquired at NAPTF during CC1 traffic testing. Since the NN-SCE backcalculation models have been developed for slab-on-grade pavement systems, P-306 Econocrete layer underlying the PCC slab in NAPTF rigid test items were converted into an equivalent PCC slab thickness using a procedure discussed by Ceylan (2002). This equivalent thickness was used as the h_{pcc} input for the NN-SCE backcalculation models.

Only the results from HWD tests conducted on the slab centers were used in the analysis. Since the NN-SCE backcalculation models been developed for 40-kN (9,000-lb) loading, the 160-kN (36,000-lb) HWD deflection basins were normalized to a load level of 40-kN (9,000-lb). This is justified since the results from the sensitivity studies conducted at the NAPTF and Denver International Airport (DIA) showed linear FWD/HWD load-deflection behavior for both flexible and rigid pavements and it has been suggested that the amplitude of the FWD/HWD load is not critical provided the generated deflections are within the limits of all deflection sensors (Lee et al. 1997, McQueen et al. 2001, and Hoffman and Thompson 1981).

Guo et al. (2002) analyzed the NAPTF CC1 rigid pavement traffic test data. After completion of the initial twenty-eight passes of NAPTF trafficking, corner cracks were observed in test items MRS and HRS. No corner cracks were found in LRS after the first 28 passes. However, longitudinal cracks were observed in all slabs in LANE 2 (C/L) of LRS. Ultimately, several slabs were cracked into five and even six pieces (Guo et al. 2002). Trafficking was stopped in HRS test item at 849 passes; in MRS test item at 891 passes; and in LRS at 1,195 passes.

Crack measurements showed that the HRS slabs exhibited the largest, and the LRS slabs exhibited the smallest, corner cracks. Furthermore, it was found that all pavement slabs were curled up at the corners, with the HRS slabs exhibiting the greatest amount of curling and the LRS slabs exhibiting the least amount of curling. The larger cracked corner areas observed on the HRS slabs are consistent with greater separation of the PCC slab corners from the Econocrete base (Guo et al. 2002). The HWD data collected prior to trafficking indicated significant increase of curling up of the slabs from the summer to the winter of 1999. The HRS test slabs (resting on the high-strength subgrade) were curled more than the LRS test slabs (resting on the low strength of subgrade) (Guo and Marsey 2001). These findings indicate that the curling and warping of PCC slabs can have significant impact on their traffic load-bearing capacity and cracking performance.

The normalized HWD surface deflections and the equivalent PCC slab thicknesses of the LRS, MRS, and HRS rigid test pavements were used in the NN-SCE backcalculation to predict the k values. The NN-SCE predictions of k values for the three test items are shown in Figures 11 to 13, respectively. In these figures, N denotes the number of load/traffic repetitions at the time of HWD testing. HWD testing was conducted at fixed intervals along the length of the slab. The results are plotted for 6-wheel and 4-wheel traffic lanes as well as untrafficked centerline (C/L) at all test points. Also, the temperature at the time of HWD testing is displayed together with the HWD test dates and load repetitions. The variability in predicted k values increased significantly in all three test items as the number of load repetitions (N) increased due to the formation of cracks.

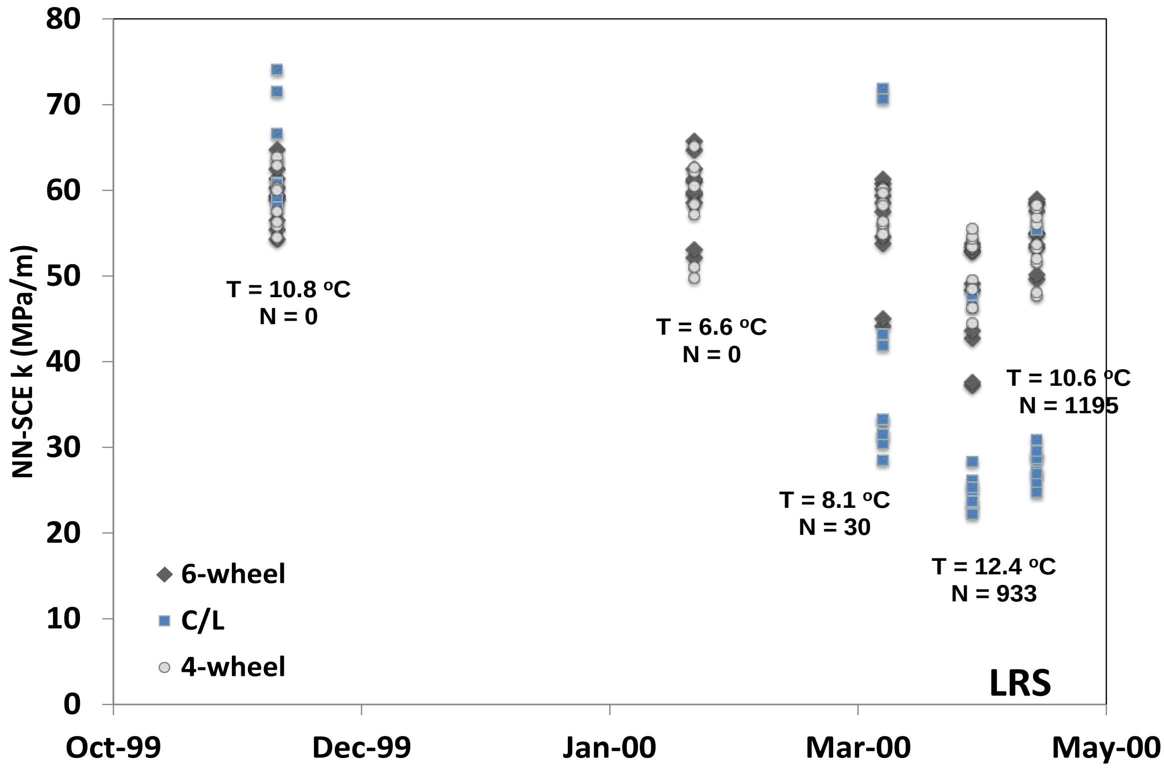

Figure 11. NN-SCE predicted k-values for LRS test section during traffic testing

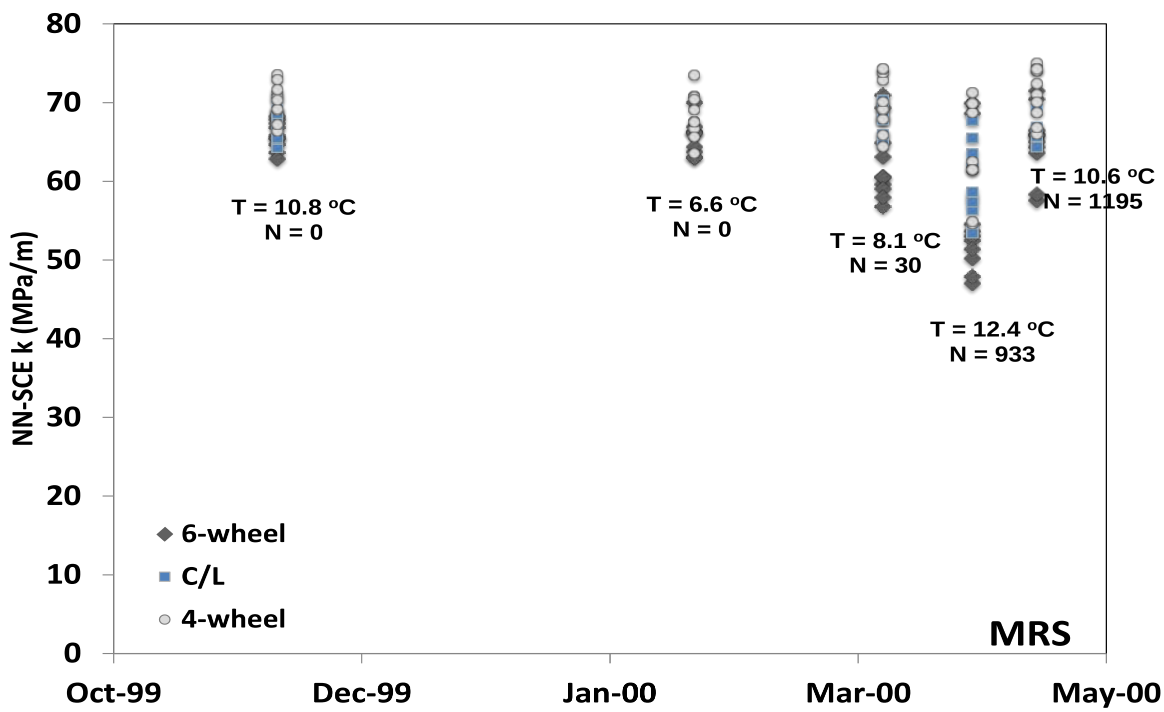

Figure 12. NN-SCE predicted k-values for MRS during traffic testing

Reference to this paper should be made as follows: Ceylan, H. and Gopalakrishnan, K. (2014). "*Finite Element based Hybrid Evolutionary Optimization Approach to Solving Rigid Pavement Inversion Problem.*" Engineering with Computers Journal, Vol. 30, No. 1, pp 1-13.

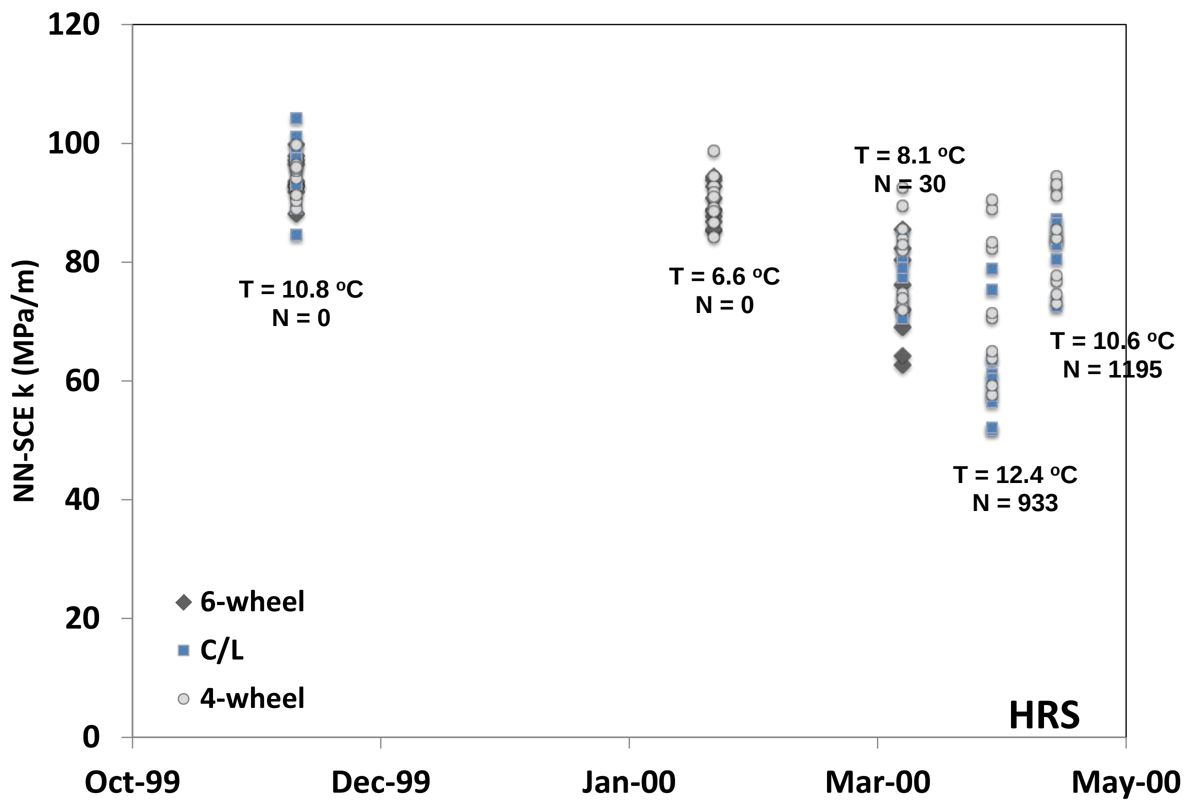

Figure 13. NN-SCE predicted k-values for HRS during traffic testing

Unfortunately, the E values could not be successfully predicted using the NN-SCE backcalculation models. This was as expected since backcalculated E values are very sensitive to pavement layer thicknesses and knowledge of actual degree of bonding in the field is required to make accurate moduli predictions. In addition, the interactions of numerous factors involved, such as slab curling and warping even before trafficking began, crack formations, etc. during trafficking further complicated the backcalculation analysis. Ceylan et al. (2008) encountered similar difficulties when trying to backcalculate E values of NAPTF CC1 rigid pavement test items using NN-based inverse models.

The k-values for the NAPF CC1 rigid pavement test items were also calculated using the closed-form solutions (see Eq. 7) for comparison. The NN-SCE predicted k-values, were in general, more consistent than those predicted using the closed-form solutions. Comparison of Coefficient of Variations (COV = [Mean/St. dev.] x 100) of predicted k-values between the two methodologies are summarized in Table 1. These results demonstrate the robustness of NN-SCE prediction model.

Table 1. Comparison of COV values between closed-form solutions and NN-SCE k predictions

		Coefficient of Variation (COV), %					
		Closed-form Solutions			NN-SCE		
Section	Date	6-wheel	C/L	4-wheel	6-wheel	C/L	4-wheel
LRS	Nov-99	16.7	12.7	10.3	5.4	8.8	5.8
	Feb-00	18.3		18.7	7.4		9.0
	Mar-00	32.3	78.6	31.8	11.3	39.3	3.4
	Apr-00	38.5	34.5	29.5	13.7	33.2	7.6
	Apr-00	32.7	43.6	22.7	6.2	35.6	7.1
MRS	Nov-99	7.4	7.1	9.9	2.7	3.1	3.3
	Feb-00	5.9		8.2	3.2		4.3
	Mar-00	30.7	12.2	12.4	7.7	2.8	5.0
	Apr-00	56.8	27.2	31.3	14.5	8.2	9.6
	Apr-00	23.8	6.2	9.1	6.8	2.2	4.6
HRS	Nov-99	17.6	20.3	12.5	3.6	6.1	3.6
	Feb-00	18.8		12.0	3.7		4.5
	Mar-00	53.8	50.2	43.8	11.4	6.5	9.0
	Apr-00	41.4	65.7	59.2	14.6	14.6	16.6
	Apr-00	33.0	33.7	40.8	10.0	6.2	9.9

Summary and Conclusions

The backcalculation methodology is an inverse process to determine in-situ materials stiffness of pavement layer by matching the measured and the theoretical deflection with iteration or optimization schemes. Over the years, numerous pavement backcalculation approaches have been developed. Each approach has its own pros and cons. This paper focused on the development of a new backcalculation method for concrete pavements based on a hybrid evolutionary global optimization algorithm, namely Shuffled Complex Evolution (SCE).

The SCE algorithm developed at the University of Arizona is reported to be an efficient global optimization method that can be used to handle non-linear problems with high-parameter dimensionality. This approach treats backcalculation as a global optimization problem where the cost function to be minimized is defined as the differences in measured and computed deflections. The optimal solution (elastic modulus of the slab, E, and modulus of subgrade reaction, k) is searched for in the multi-modal solution space by the SCE algorithm. Hypothetical data covering wide ranges of layer thicknesses and FWD deflections commonly encountered in the field were first used to evaluate the prediction accuracy of the developed NN-SCE hybrid concrete pavement backcalculation tool. The results demonstrated the excellent performance of the developed backcalculation tool.

The study also illustrated the complexity of backcalculating properties of rigid pavements subjected to full-scale dynamic simulated aircraft traffic testing with gear wander. Apart from other complicating factors such as the slab curling and warping behavior, the test pavements exhibited corner cracks within few passes of traffic loading which further complicated the interpretation of FWD/HWD test results. The NN-SCE predicted k values were in good agreement with those obtained using the closed-form solutions.

Reference to this paper should be made as follows: Ceylan, H. and Gopalakrishnan, K. (2014). "*Finite Element based Hybrid Evolutionary Optimization Approach to Solving Rigid Pavement Inversion Problem.*" Engineering with Computers Journal, Vol. 30, No. 1, pp 1-13.

Acknowledgements

The authors would like to sincere acknowledge the significant contributions made by Dr. Sunghwan Kim at Iowa State University towards this paper.

he authors gratefully acknowledge the FAA Airport Technology Research and Development Branch for providing the NAPTF field data referenced in this paper. Dr. Satish Agrawal is the Manager of the FAA Airport Technology R & D Branch. The contents of this paper reflect the views of the authors who are responsible for the facts and accuracy of the data presented within. The contents do not necessarily reflect the official views and policies of the Federal Aviation Administration. This paper does not constitute a standard, specification, or regulation.

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