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Keywords
eddy current testing, finite element analysis, boundary integral equations, nondestructive evaluation, QNDE, Electrical and Computer Engineering

Disciplines
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Comments

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A NOVEL BOUNDARY INTEGRAL EQUATION FOR SURFACE CRACK MODEL

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ABSTRACT. A novel boundary integral equation (BIE) is developed for eddy-current nondestructive evaluation problems with surface crack under a uniform applied magnetic field. Once the field and its normal derivative are obtained for the structure in the absence of cracks, normal derivative of scattered field on the conductor surface can be calculated by solving this equation with the aid of method of moments (MoM). This equation is more efficient than conventional BIEs because of a smaller computational domain needed.

Keywords: Boundary-Integral Method, Cracks, Eddy Current Testing
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INTRODUCTION

Boundary element method (BEM) is one of widely used numerical approaches for solving eddy-current nondestructive evaluation (EC NDE) problems due to its high efficiency and the ability to handle complex geometries [1]. With different boundary integral formulations, this approach has been performed in many kinds of EC NDE problems to investigate the interaction between eddy current and cracks [2-4]. Other numerical methods, such as finite element method (FEM) and volume integral method, are very difficult and expensive to analyze EC NDE problems due to the need of dealing with high conductivity at low frequency. Besides this, BEM is more applicable to the problems with both bounded and open region.

The key part of BEM is to find appropriate boundary integral equations (BIEs). Take the example of the ideal crack. In EC problems, it can be modeled as a current dipole layer on the crack surface and the scattered field can be viewed as being generated by this dipole layer because of its zero thickness. An electric field integral equation (EFIE) has been developed to find the dipole density for the ideal crack [5, 6]. In addition, BIEs for the magnetic vector potential have been developed using an integral kernel with a weaker singularity than that of EFIE approach [7]. However, the formulations have more complicated form. The effect of crack closure has also been studied by combining the variational method with BEM [8].

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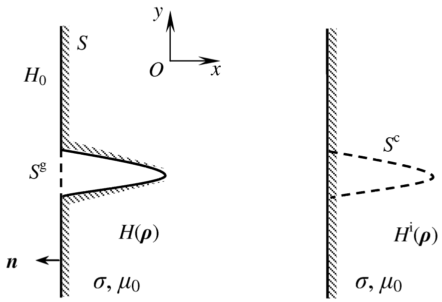


FIGURE 1. Schematic configuration of the surface crack problems with uniform applied field H_0. H^i is the incident magnetic field in the absence of the crack. S^c and S^g represent crack surface and "gap" surface, respectively.

In this paper, thin crack model will be used to examine our equation. Thin crack model involves eddy-current interaction with the infinite deep crack and the square corner, which are two most typical models for EC NDE problems. Kahn *et al.* has solved the semi-infinite crack problem, which is a special case of the thin crack model, with modification of Sommerfeld's diffraction theory and solved the square corner problem with image theory [9]. An approximate closed form solution for a long, surface-breaking crack was obtained using the Wiener-Hopf technique by Harfield and Bowler [10]. Numerically, the conventional BIE has been applied to the ideal crack with different inclination [11]. However, only the average of surface impedance on both sides of the crack was solved.

In order to efficiently solve more general EC NDE problems for both bounded and open region with less number of unknowns, a BIE for the magnetic field will be proposed in this paper. The formulation is based on the solution for the structure in the absence of cracks. In combination with the method of moments (MoM), this integral equation can be solved numerically. The proposed BIE provides an easier way to truncate unbounded computational domain without adding any extra hypothetical boundaries.

FORMULATION

General Formulation

Two-dimensional EC NDE problems with surface cracks are considered here. Assume the conductor is nonmagnetic material, which has the permeability of free space μ_0 and the conductivity σ. A uniform magnetic field, H_0, which has only a *z* component, is applied to the crack as shown in Fig. 1. Here, the time factor, $e^{-i\omega t}$, is suppressed. In the conductor, the total magnetic field, $H(\boldsymbol{\rho})$, satisfies the scalar Helmholtz equation:

$$\left(\nabla^2 + k^2\right) H(\boldsymbol{\rho}) = 0, \qquad (1)$$

where k is the wave propagation constant and $k^2 = i\omega\mu_0\sigma$. The corresponding boundary integral equation can be written as

$$\int_S \mathrm{d}S(\boldsymbol{\rho}')\, G(\boldsymbol{\rho},\boldsymbol{\rho}') \frac{\partial H(\boldsymbol{\rho}')}{\partial n'} = \frac{1}{2} H_0 + H_0 \int_S \mathrm{d}S(\boldsymbol{\rho}') \frac{\partial G(\boldsymbol{\rho},\boldsymbol{\rho}')}{\partial n'}, \qquad \boldsymbol{\rho} \in S, \qquad (2)$$

where $\boldsymbol{n}$ is the normal direction of the surface S and $G(\boldsymbol{\rho},\boldsymbol{\rho}')$ is the Green's function for two-dimensional Helmholtz wave equation and given by

$$G(\boldsymbol{\rho},\boldsymbol{\rho}') = \frac{i}{4} H_0^{(1)}(k\,|\boldsymbol{\rho}-\boldsymbol{\rho}'|), \tag{3}$$

where $H_0^{(1)}$ is the Hankel function of the first kind of zero order. Equation (2) will be referred to as the conventional BIE. All integrals in Eq. (2) are based on principle values over the whole boundary of the conductor, including the crack surface, S^{c}, which means the whole surface S should be considered when solving Eq. (2). For those problems with infinite domain, such as surface crack in half-space conductor, several extra long enough hypothetical boundaries are needed to truncate the computational domain when numerical approaches are applied to the conventional BIE. This will dramatically increase the unknowns and the CPU time spent on filling and solving matrix.

Let us consider the total magnetic field as the incident magnetic field, $H^{\mathrm{i}}(\boldsymbol{\rho})$, plus the magnetic field scattered by the crack, $H^{\mathrm{s}}(\boldsymbol{\rho})$. The incident field also satisfies Eq. (2) except only for the surface without cracks. Subtracting BIE for the incident magnetic field from Eq. (2), a new BIE can be written as

$$\begin{aligned}&\int_S \mathrm{d}S(\boldsymbol{\rho}')\, G(\boldsymbol{\rho},\boldsymbol{\rho}') \frac{\partial H^{\mathrm{s}}(\boldsymbol{\rho}')}{\partial n'} \\ &= H^{\mathrm{inc}}(\boldsymbol{\rho}) + \int_{S^{\mathrm{c}}-S^{\mathrm{g}}} \mathrm{d}S(\boldsymbol{\rho}') \left[H_0 \frac{\partial G(\boldsymbol{\rho},\boldsymbol{\rho}')}{\partial n'} - G(\boldsymbol{\rho},\boldsymbol{\rho}') \frac{\partial H^{\mathrm{i}}(\boldsymbol{\rho}')}{\partial n'} \right],\end{aligned} \tag{4}$$

where

$$H^{\mathrm{inc}}(\boldsymbol{\rho}) = \begin{cases} \frac{1}{2} H_0 - H^{\mathrm{i}}(\boldsymbol{\rho}) & \boldsymbol{\rho} \in S^{\mathrm{c}}, \\ 0 & \boldsymbol{\rho} \in \mathrm{S} \text{ except } S^{\mathrm{c}}, \end{cases} \tag{5}$$

and the subscript of the integral on the right-hand-side, $S^{\mathrm{c}} - S^{\mathrm{g}}$, represents subtracting the integral over S^{g} from that over S^{c}. The region S^{g} is the surface in the absence of cracks but not included in S. The right-hand-side of Eq. (4) just involves a smaller region compared with Eq. (2). We only need to truncate the domain without adding any extra boundaries. Apparently, it is more efficient and faster than using Eq. (2) to solve EC NDE problems.

Ideal Crack

The ideal crack, which acts as a perfect barrier to the flow of eddy current, has infinitesimal thickness. Thus, only the sum of magnetic fields on opposite sides of the crack, $H_+^{\mathrm{s}} - H_-^{\mathrm{s}}$, could be solved together. Here, the subscripts $\pm$ refer to limiting values approaching each side of the crack surface. The integral over surface S^{g} is zero due to the infinitesimal thickness. Finally, equation (4) can be reduced to

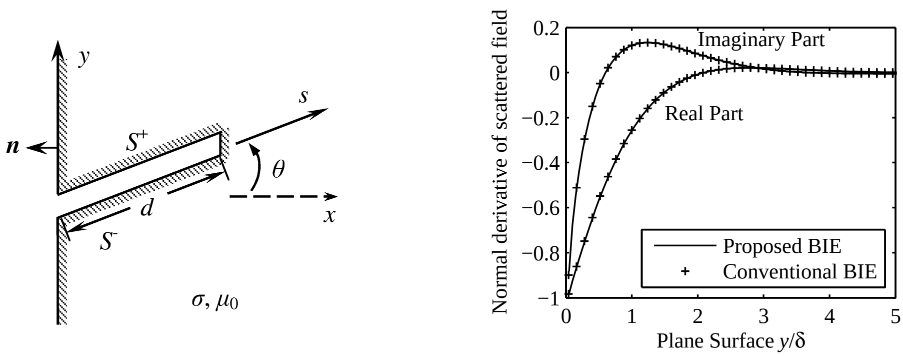
$$\int_{S^c} G \frac{\partial (H_+^s - H_-^s)}{\partial n'} \mathrm{d}S + \int_{S \text{ except } S^c} G \frac{\partial H^s}{\partial n'} \mathrm{d}S = \begin{cases} H_0 - H^i(\boldsymbol{\rho}) & \boldsymbol{\rho} \in S^c, \\ 0 & \boldsymbol{\rho} \in \mathrm{S} \text{ except } S^c, \end{cases} \quad (6)$$

which is same as the formulation found by Kahn [11]. Notice that half of H_0 is added to H^{inc} in Eq. (5) for the point over crack surface S^c, because the corresponding point on the opposite side of the crack which introduces another principle value should be considered when crack's thickness goes to zero.

NUMERICAL RESULTS

The proposed BIE is discretized into matrix equations using the method of moments (MoM) [12]. Pulse basis and the point-matching (collection) method are used in our code. Matrix elements are evaluated with standard six-point Gaussian quadrature rule [13] and the one incorporating logarithmic singularities [14]. Two kinds of two-dimensional NDE problems are tested here: a thin crack in the half-space conductor or in the quarter-space conductor. Normal derivative of scattered field, $\partial H^s / \partial n$, is calculated first using Eq. (4). Then normal derivative of total field, $\partial H / \partial n$, which is proportional to the surface impedance due to the uniform applied magnetic field, is obtained by adding the contribution from incident field together.

In order to verify our formulations and programs, a thin crack in the half-space conductor is examined first. As shown in Fig. 2(a), the depth of the crack is d and the angle between the normal of plane surface and the crack surface is θ. All the lengths are normalized by the skin depth δ. The incident field for half-space conductor without cracks is $H^i(\boldsymbol{\rho}) = H_0 e^{ikx}$. In our implementation, the thin crack model, which thickness is quite small compared with other dimensions and skin depth, is used. The tip of the crack is represented as one single element. As stated above, only the average of surface impedance on opposite sides of the ideal crack could be solved using Eq. (6) by Kahn [11]. However, the surface impedance on each side of the thin crack surface can be determined separately



(a) (b)

FIGURE 2. (a) Configuration of thin crack in a half-space conductor. Surface S^+ and S^- are the upper and lower surfaces of the crack, respectively. (b) Normal derivative of scattered field on the plane surface for a perpendicular crack with depth $d = 4\delta$. Values are normalized by the surface impedance for the half-space conductor. Numerical results calculated from proposed (solid line) and conventional (plus sign) BIE are shown.

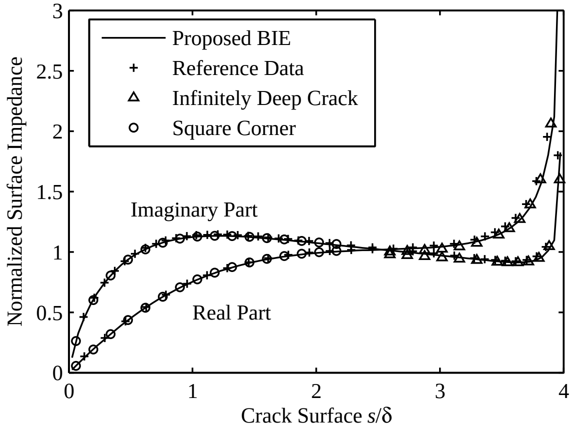


FIGURE 3. Normalized surface impedance on the crack surface for a perpendicular crack with depth $d = 4 \times\delta$. The results (solid line) are calculated using the proposed BIE. Kahn's results [11] for the ideal crack (plus sign) are plotted. The analytical solution for the infinitely deep crack (triangle) and square corner (circle) are also shown for comparison.

in our code. Our results are compared with the Kahn's and a good agreement can be observed not only for the numerical results using Eq. (6), but also for the analytical solution.

Fig. 2(b) shows the normal derivative of scattered field on the plane surface for the case with $d = 4\delta$ and $\theta = 0^{\circ}$. Numerical results calculated from the conventional BIE with domain truncation are also plotted for comparison. All values are normalized by the surface impedance for the half-space conductor $-ikH_0$. Curves in Fig. 2(b) clearly indicate eddy-current interaction with a square corner can extend to about 2.5δ. Thus, at least a range of 5δ is needed to separate the mutual influence between two neighbor corners. In solving the conventional BIE (2), the solution domain should be truncated to a $15\times15\delta^2$ square to ensure the correctness of the results for this case. As a result, the evaluation of integrals in Eq. (2) should be implemented over a length of about 68δ, but 23δ is enough for the proposed BIE (4). This means a large computational region and a lot of extra unknowns should be required if the conventional BIE with domain truncation, which introduces several corners to truncate the infinite region, is used to solve this kind of problems. Computational region, however, can be just confined to the region one needs without adding any hypothetical boundaries using Eq. (4). The number of unknowns and CPU time also can be significantly decreased.

Fig. 3 shows a comparison of the numerical results calculated from Eq. (4) and the published results [11] for the same case as that in Fig. 2. Because of the symmetrical structure, surface impedance on the upper crack surface S^+ and the lower one S^- are the same as their average value. The results on the crack surface are also compared with the analytical solution for infinitely deep crack ($2.5 < s/d < 4$) and a square corner ($0 < s/d < 2$) [9]. A good agreement can be obtained when the crack is deep enough so that contributions from the corner and the tip can be separated with each other.

Figs. 4 and 5 show the normalized surface impedance for the case with $d = 3\delta$ and $\theta = 30^{\circ}$. Fig. 4(a) shows the surface impedance on both sides of the crack separately. Different current distribution can be observed. The change of eddy current becomes more rapid around the sharp corner. Fig. 4(b) shows the average of them compared with the

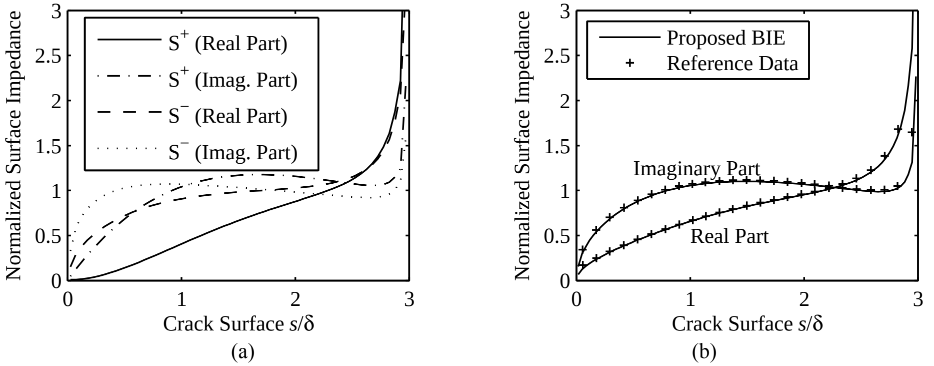


FIGURE 4. Normalized surface impedance over the crack surface for a crack with $d = 3\delta$, and $\theta = 30^o$. The results on both sides of the crack (a) and the average value of them (b) are plotted. Also Kahn's results [11] for the ideal crack (plus sign) are shown in (b).

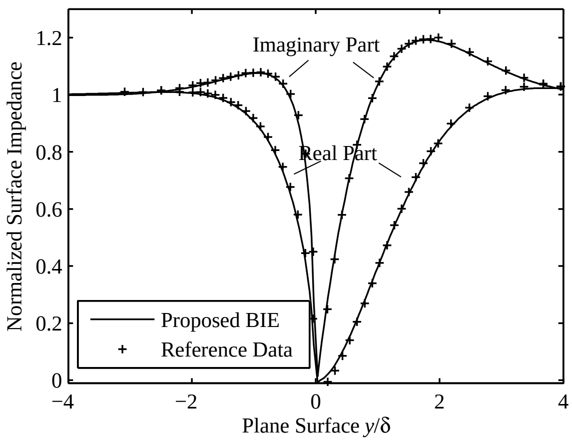


FIGURE 5. Comparison of result from Kahn (plus sign) [11] and that calculated from the proposed BIE (solid line) for the normalized surface impedance on the plane surface: $d = 3\delta$, and $\theta = 30^o$.

published data for the ideal crack with same depth. The average of surface impedance on both sides for each sampling point is shown in Fig. 5. Again, it is compared with Kahn's results [11]. It's clear to see the disturbance of eddy current stretches out further in the positive y-direction, which is the direction of the crack inclination.

For the second comparison, numerical results for a thin crack in the quarter-space conductor with different depth are shown in Fig. 6. The angle between crack surface and horizontal plane is θ. The incident field and its normal derivative used in Eq. (4) are solved analytically by image theory [11]. Comparisons are made with the results calculated from the conventional BIE. The results for a crack in the half-space conductor with the same inclination angle ($\theta = 45^o$) are also plotted. It's reasonable to see the results agree well with each other when the crack depth is long enough ($d = 8\delta$ is used here) as shown in Figs. 6(a) and 6(b), whereas the effect on account of square corner can be observed when the crack becomes shorter as illustrated in Figs. 6(c) and 6(d). Fig. 6(d) indicates about 3δ should be considered to seperate the interaction between two neighbor corners and roughly $8 \times 8\delta^2$ truncated area should be included for the conventional BIE. Therefore, a length of 8δ for the proposed equation is still much smaller than that for the conventional BIE.

CONCLUSION

A new BIE has been proposed to solve two-dimensional EC NDE problems for the given incident field and its normal derivative. The formulation, implemented with the MoM, has been validated with conventional BEM results for two different kinds of structures. This approach not only can be used to solve the infinite domain problems without adding any hypothetical boundaries, but also is more efficient than the conventional BEM because of the smaller computational domain needed.

Although the results discussed in this paper pertain to two-dimensional EC NDE problems with Dirichlet boundary condition, the concept described can be applied with three-dimensional and nonuniform applied fields to reduce the number of unknowns.

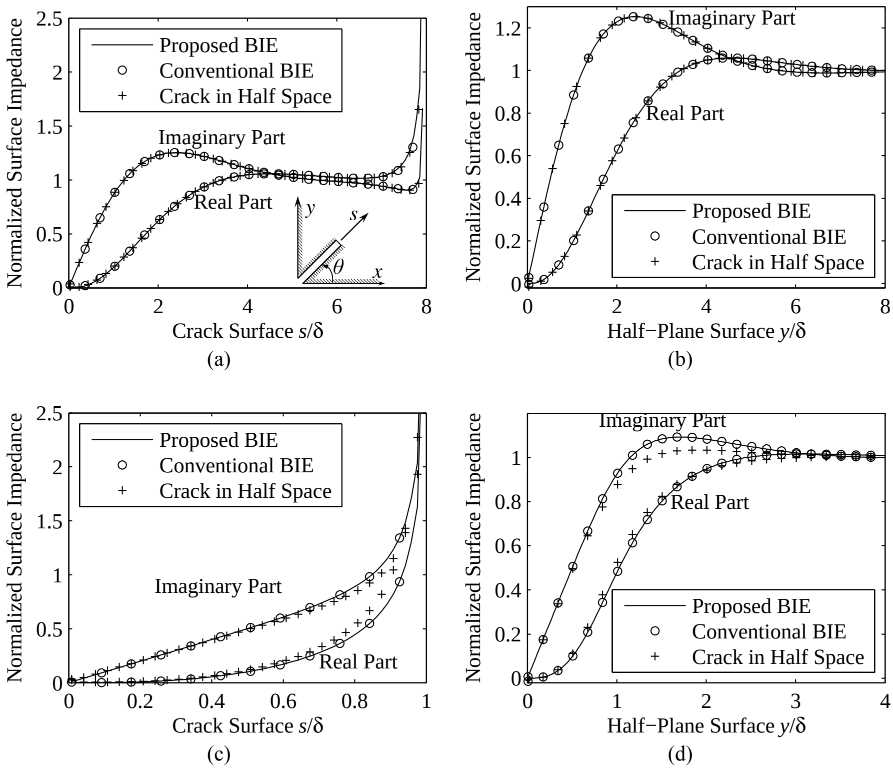


FIGURE 6. Normalized surface impedance for cases with $d = 8\delta$, $\theta = 45^{o}$ (a, b) and $d = \delta$, $\theta = 45^{o}$ (c, d). Numerical results from proposed (solid line) and conventional (circle) BIEs are shown. The results for the surface crack in a half-space conductor with same inclination angle (plus sign) are also plotted.

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