12-12-2018

Working Paper Number 19002

The Fight-or-Flight Response to the Joneses and Income Inequality

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This paper studies the fight-or-flight ambivalence people show towards the success of the proverbial Joneses. If an agent cares about leisure and his consumption relative to a benchmark set by the Joneses, his preferences display the keeping-up-with-the-Joneses (KUJ) property if an increase in the benchmark urges him to substitute away from leisure into work, allowing him to finance more consumption; the opposite is labeled running-away-from-the-Joneses (RAJ). The long literature, thus far, finds a) if any agent's behavior displays KUJ (or RAJ), everyone's will, or b) if an agent displays KUJ (or RAJ) in one portion of the consumption space, so will he everywhere. In an otherwise-standard environment with endowment heterogeneity, we provide conditions under which different agents sharing the same underlying preferences may endogenously respond very differently to the Joneses: while some may choose to keep up, others, possibly their close neighbors, may choose to run away. These choices themselves shape the income distribution, which in turn, determine the identity and fate of the Joneses. The analysis is novel because a) such fight-or-flight conflict does not arise in existing models of consumption externalities, and b) it identifies an endogenous mechanism that may dampen or amplify market income inequality arising from innate heterogeneity.

Keywords

leisure distribution, rat race, amplification, wealth-dependent

Disciplines

Behavioral Economics | Labor Economics

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Abstract

This paper studies the fight-or-flight ambivalence people show towards the success of the proverbial Joneses. If an agent cares about leisure and his consumption relative to a benchmark set by the Joneses, his preferences display the keeping-up-with-the-Joneses (KUJ) property if an increase in the benchmark urges him to substitute away from leisure into work, allowing him to finance more consumption; the opposite is labeled running-away-from-the-Joneses (RAJ). The long literature, thus far, finds a) if any agent's behavior displays KUJ (or RAJ), everyone's will, or b) if an agent displays KUJ (or RAJ) in one portion of the consumption space, so will he everywhere. In an otherwise-standard environment with endowment heterogeneity, we provide conditions under which different agents sharing the same underlying preferences may endogenously respond very differently to the Joneses: while some may choose to keep up, others, possibly their close neighbors, may choose to run away. These choices themselves shape the income distribution, which in turn, determine the identity and fate of the Joneses. The analysis is novel because a) such fight-or-flight conflict does not arise in existing models of consumption externalities, and b) it identifies an endogenous mechanism that may dampen or amplify market income inequality arising from innate heterogeneity.

Keywords: leisure distribution, rat race, amplification, wealth-dependent risk aversion, keeping up with the Joneses, income inequality

JEL classifications: J 22, E2, I 31

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† We thank, without implicating, the editor, Jim Bullard and two anonymous referees for their invaluable input.

1 Introduction

Human beings, by nature, are gregarious. In our daily lives, we share a large portion of the economic space with many others. These generic "others" – the proverbial Joneses – influence our lives. They shape our decision-making, and often, our very definition of happiness. They help set a marker, a benchmark, for what it means for any one of us to "make it" in life. And yet, each of us struggle with mixed feelings about giving the Joneses *this much* power – "positional concern" (Aronsson and Johansson-Stenman, 2010) – over our lives. After all, their influence shapes every facet of our lives, how much we work, how much time we spend with our families, how big a house we live in. For some of us, this influence is motivating in nature, goading us to work harder in an effort to keep up with the Joneses; others simply recoil from the latter's success. Imperceptibly, however, these consumption and leisure choices themselves shape the overall income distribution, which in turn, determine the identity and fate of the Joneses themselves. This paper is an attempt to formalize this fundamental ambivalence, our fight-or-flight response if you will, to the success of the proverbial Joneses and to study how such dialectics affect the income distribution in a society.

Economists have long accepted the notion that our sense of well-being derives in part on the consumption choices of the Joneses – see Luttmer (2005), Dynan and Ravina (2007), Maurer and Meier (2008), Bertrand and Morse (2016, and Alvarez-Cuadrado et. al. (2016) for empirical backing.[1] They have investigated, at length, the notions of consumption externalities and consumption benchmarking, the idea that the Joneses set the benchmark we attempt to emulate or beat. Dupor and Liu (2003) provides an useful taxonomic classification and nomenclature. In their language, the decisions of others can elicit feelings of *jealousy* or *admiration* in us; we respond by *keeping up* with the Joneses (KUJ) or *running away* from the Joneses (RAJ). If a rise in the consumption benchmark goads us to work more so we can consume more, we are said to be keeping up with the Joneses; if it encourages us to work less and consume less, we are running away. As Chugh (2008) puts it "...KUJ is a desire to be similar to others, while RAJ is a desire to be different from others."

This entire line of work uses a conventional, keeping-up preference formulation wherein

[1] Luttmer (2005) investigates whether people care about their relative position and if falling behind the Joneses' diminish their happiness. He find thats, after controlling for individual income, higher earnings of neighbors are associated with lower levels of self-reported happiness, suggesting the negative effect of increases in neighbors' earnings on own well-being is "most likely caused by interpersonal preferences people having utility functions that depend on relative consumption in addition to absolute consumption." Bertrand and Morse (2016) using state-year variation in incomes and consumption for U.S. data to find that poorer ("nonrich") households consume a larger share of their current income on conspicious items when exposed to higher income (and consumption) at the top. They find compelling evidence "consistent with one possible causal pathway: status-seeking".

everyone relishes the idea of keeping up: if benchmark consumption rises, everyone responds by taking whatever action helps increase their own consumption.[2] This is unappealing: there is no reason why responses to social changes be uniform across a large population with differing fundamentals. There is, however, a reason why the literature generates this unattractive uniformity. Much of it – except a few exceptions in finance, such as Chan and Kogan (2002) and Xiouros and Zapatero (2010) – assumes a homogenous population, and as such, the Joneses' decisions are identical, in equilibrium, to that of the individual. More strikingly, in all such models, even those that admit a heterogenous population, every agent ends up responding in one way or the other: either *they all fight or they all flee* with no scope for ambivalence. This is our entry point into this vast literature. To foreshadow, to generate ambivalence, wealth-dependent (or person-specific) risk aversion is necessary (but not sufficient), something the literature on consumption externalities has not touched on.[3]

We begin by asking, under what conditions will some individuals raise, and others reduce, their consumption in response to the consumption of the Joneses? We examine these issues within the broad confines of the Dupor and Liu (2003) framework. In our static model, agents are heterogeneously-endowed with "effective time e" (or innate productive ability, if you will) and they devote part of that time to work (at a fixed wage) and the rest to leisure. All agents share the same underlying preferences – they care about leisure and effective consumption, an amalgamation of own consumption and an economy-wide consumption benchmark. What is different across agents is their risk aversion: it is person-specific (depends on e). For the most part, we stay agnostic (as do Dupor and Liu, 2003) as to the exact origins of the benchmark. Despite having similar preferences, the impact of a change in the benchmark on the slope of an indifference curve – the marginal rate of substitution (MRS) between leisure and consumption – is person-specific because risk aversion is person-specific. Changes in the benchmark have the *potential* – it is by no means routine – to alter the MRS, and via this channel, affect agents' decision-making about their own consumption and leisure. If an increase in the benchmark causes the MRS of agent i to rise (fall), he is said to keep up (run away) in the Dupor and Liu (2003) sense.

We study situations in which diametrically-opposing behavioral responses to others' consumption – we call it fight-or-flight or more formally, *dual response* – can emerge, generating endogenous differences in leisure, income, and consumption based on those influences. When preferences generate dual response behavior, an agent with effective-time endowment, e, may

[2] As Barnett et. al. (2010) phrase it, "the issue of "what if they don't?" has not received much attention".

[3] There is a fair bit of evidence for risk aversion that varies with wealth, time, and other househod characteristics. Recent examples include Guiso and Paiella (2008), Paravisini et. al (2016), and Ampudia et al (2017).

run away from the Joneses while another agent, with $e' \neq e$ but *with the same underlying preferences*, may keep up. In fact, the same agent may flee from the Joneses at one level of the benchmark and keep up with them at another level. We show, if preferences are additively-separable in effective consumption and leisure, then wealth-dependent risk aversion is necessary (but not sufficient) to generate dual response; indeed, the cut-off level of risk aversion, below which the agent behaves one way and above which another way, turns out to be *one*.[4]

What is the *raison d'être* for dual response? Heuristically, an increase in the benchmark evokes jealousy: the same consumption brings less joy knowing the benchmark is higher. (It has no effect on the marginal utility of leisure.) This, in effect, raises the price of effective consumption relative to leisure for all; everyone responds by cutting effective consumption. This textbook-style price effect unleashes income and substitution effects and the size of these effects, of course, depends on the person and his/her risk aversion. For some, the correct response is to work harder and increase own consumption so as to stem the fall in effective consumption; for others, it is the opposite. A novelty of our paper is connecting such keeping-up or running-away behavior with risk aversion under very general preferences.

What is the value-added of dual response? In the existing literature, a change in the benchmark always elicits the same *qualitative* response from, say, the rich and the poor: both respond either by keeping up or running away from the Joneses. Not so, with dual response. Here, the possibility arises that the rich react to an increase in the benchmark by raising their consumption while the poor do the exact opposite, *or vice versa*. Dual response, then, has the potential to act as an *endogenous amplification* (or dampening) mechanism, taking innate differences in people and either amplifying or dampening those differences in terms of what may be *observed* (such as in differences in income or consumption), via the differential, qualitative impact the benchmark has on individual choices.

For the general class of HARA preferences, if risk aversion falls with e, we show the existence of a cut-off e below which agents fight the Joneses and above which they flee. Loosely, your 4000 square-feet house is eye-catching when your neighbors have 2000 square-feet homes. That same house fails to impress in the same way it once did when your neighbors build 3000 square-feet homes. Faced with this in-your-face intrusion of the Joneses, you can choose to work harder, earn more and build a 5000 square-feet house in an effort to successfully stay ahead. Alternatively, you can live with the reduced effective consumption and enjoy a more leisurely existence

[4] If preferences are non-separable, then wealth-dependent risk aversion is neither necessary nor sufficient. Our model also generates a distribution of risk-aversion, one that depends both on the person-specific e and the consumption benchmark. A similar feature is present in Gollier (2004). In Chan and Kogan (2002), the only source of heterogeneity is differences in the (constant) relative risk aversion among agents.

partly because it has gotten so much harder to impress given a 3000 square-feet benchmark. Under one parametrization of HARA preferences, we show the less-affluent in the neighborhood choose to build a "bigger house" so as to compete; others, their "rich" neighbors, may choose not to, and appear to "drop out". Moreover, if the benchmark house becomes bigger, the above mentioned cut-off e rises: all else same, more people join the rat race, work more and consume more. The shape of the overall income distribution changes; in the case just described, income inequality in society goes down. In the process of this societal transformation, the identity and fate of the Joneses is not spared.[5]

To see this more clearly, consider two economies, A and B, identical in almost all innate aspects – they share the same mean and distribution of e, and the utility functions are identical across the two economies – except, for some unspecified reason, the benchmark consumption level in A is higher than in B. In such a setting, suppose the preferences in each country do *not* display dual response but do display *either* KUJ or RAJ. In such a setting we show that the Gini of labor income across the two countries is identical. However, if preferences in each country display dual response, then, ceteris paribus, the Gini of earnings in A could be *lower/higher* than in B. Here, innate inequality in A and B is the same but measured income inequality is lower/higher in the country with the higher consumption benchmark – and this is entirely due to the fight-or-flight dialectics. This last statement may comfort policymakers who initiate action to reduce innate inequality, hoping to reduce income inequality, but are unsuccessful.

The rest of the paper is shaped as follows. Section 2 offers a short review of the literature while Section 3 lays out the basic environment, the general set up of preferences, and the connection with risk aversion. Section 4 lays out the definitions of KUJ and RAJ, local and global, and Sections 5-6 discuss the agent's optimization problem and the surrounding mechanics of flight-or-fight behavior explained in textbook income/substitution effect terms. Section 7 takes up the HARA class of preferences to demonstrate the possibility of a fight-or-flight response. The relation between fundamental and measured income inequality is also discussed. Section 8 illustrates facets of flight-or-flight behavior in the case of endogenous benchmarking. Section 9 contains a discussion of our modeling assumption and concludes with areas for future research. Proofs of results and additional helpful material is in the appendices.

[5]Frank (2009) building on insights from Veblen (1892) isolates the deeper inefficiency underlying this situation. If each of us slog to get the additional satisfaction from having more house than the Joneses next door – Veblen's "ignoble form of emulation" – unaware that, all along, the Joneses are doing the same, then isn't it possible that, for some, the relative satisfaction gain never materializes (Hopkins and Korneinko, 2004).

2 Literature

The literature on consumption benchmarks and other-regarding preferences has done much to expand our understanding of societal influences on consumption choices of individuals. Much of the progress in this line of inquiry has been in the domain of finance and macroeconomics, mainly asset pricing, using dynamic models. Work on the consumption-leisure dimensions of societal influences in static models is relatively sparse. There is a long line of work that studies consumption externalities in macroeconomics; prominent examples are Duesenberry (1949), Abel (1990, 2005), Frank (1999), de la Croix and Michel (1999), Ljunqvist and Uhlig (2000), Dupor and Liu (2003), Gollier (2004), Liu and Turnovsky (2005), García-Peñalosa and Turnovsky (2008), Alvarez-Cuadrado and Van Long (2011), Bishnu (2013), Chen et. al (2015) among many others. While some focus on concomitant asset-pricing issues, others are more interested in the public economics of consumption taxation in the presence of consumption externalities.[6]

Our work follows the strand in the literature that posit utility functions which depend not only on the absolute value of consumption, but also on the benchmark level of consumption. An alternative strand – see Hopkins and Kornienko (2004) and Hopkins (2008), among others – assume people care about their ordinal rank in the consumption distribution alongside their own consumption level. In the latter, each agent must choose how to allocate his income between a visible (positional) good and another (nonpositional) good, the consumption of which is not seen by other agents. A classic result here is that the proportion of income spent on conspicuous consumption increases at each level of income, a global KUJ response of sorts.

A version of our fight-or-flight result appears in Clark and Oswald (1998; specifically, equation (10) in their paper). There, too, agents may conform (be "followers" and raise their action a when the comparison a^* rises) or they may be "deviants" and do the opposite. As in our model, who does what depends on the concavity or convexity of marginal utility from comparison, which we translate into risk aversion. Our focus is more on the possibility that a) follower or deviant behavior changes with income, and b) it may depend on a^* itself. In previous, related work, Barnett, Bhattacharya, and Bunzel (2010) consider a similar framework wherein agents *choose* whether or not to allow the consumption decision of others to influence their own consumption and work decisions. That paper introduces a two-piece, level-dependent utility function in which agents receive a utility-kick if their consumption beats an endogenously-determined consumption benchmark, based on the economy's level of mean consumption. In that setting, an agent can "drop out" thereby insulating himself from the influence of the Joneses. In equilib-

[6]The endogenous amplification mechanism we unearth is potentially of value to researchers studying risk and portfolio choice and to macroeconomists studying distributional consequences of shocks.

rium, endogenous inequality emerges – in levels of consumption, income, and leisure – and is critically dependent on the degree of influence people draw from others as well as on the fundamental inequality in time endowments. The approach mimics, in some sense, the notion of agents *keeping up* or *running away* from the influence of others (some agents will meet the challenge and join the consumption rat race while others will drop out of the race – "self-sufficient types in Strulik, 2015). However, it does not follow the conventional taxonomy in Dupor and Liu (2003).[7] Specifically, individuals dropping out of the rat race draw no influence from the consumption decisions of others, which is conceptually different from Dupor and Liu's notion of running away from Joneses. In keeping with Dupor and Liu, in the present paper an individual cannot shield himself from the influence of his neighbors, but unlike Dupor and Lui, he may respond to them in more ways than one.

In an insightful recent paper, Allen and Chakraborty (2018) allow agents to pursue the consumption benchmark of those who are richer than them. Hoping to catch up, these agents work more and save more energized by how far they fall below the aspirational benchmark. There is no notion of RAJ in their setup, however. Gershman (2014) focuses on an aspect of the afore-discussed ambivalence: some in his model, incentivized by KUJ effects, indulge in conspicuous consumption and overwork, while others "hide their wealth and underinvest, constrained by the fear of malicious envy".

3 The model

3.1 Primitives

Consider a single-good, static economy populated with a continuum of agents – denoted by i – distinguished by their endowment of effective time, e^i, over and above the one unit of time available to all. Assume e is distributed according to a distribution $\mathcal{G}$ with non-negative support $[\underline{\xi}, \overline{\xi}]$, and mean $\bar{e}$, where $\overline{\xi} \leq \infty$. The distribution $\mathcal{G}$ captures fundamental inequality, innate, unchangeable differences between people.

Agents work and consume. Let l^i denote agent i's labor supply and x^i his leisure. Agent i's time constraint is given by $x^i + l^i = 1 + e^i$.[8] Those with higher e^i – sometimes referred to as "richer" agents – have more effective time to devote to work and leisure. If the wage rate is w

[7] Also, the two-piece utility formulation is somewhat unwieldy, generates non-convexities, and hence, a role for consumption lotteries.

[8] Our framework is nearly identical to that in Dupor and Liu (2003) except they measure the disutility of working while we introduce leisure as a argument directly in the utility function.

units of the consumption good, the budget constraint is written as

$$c^i = wl^i = w\left(1 + e^i - x^i\right). \tag{1}$$

We normalize $w = 1$ implying

$$c^i + x^i = \left(1 + e^i\right). \tag{2}$$

This formulation, arguably, is a little unusual. To see why it is useful, consider an alternative formulation of the budget constraint: $c^i = w^i l^i$ and $l^i + x^i = 1$ with w^i drawn from a wage distribution. In this case, were we to plot c^i on the vertical axis and x^i on the horizontal, the budget line would have a slope $-w^i$ (the budget line would connect the points $\left(0, w^i\right)$ and $(1, 0)$). In contrast, the budget constraint considered in the paper has $c^i + x^i = \left(1 + e^i\right)$ implying a budget line with slope -1 (independent of i). If, in fact, we permitted differences across agents in wages and time endowments, the budget line would connect the points $\left(0, w^i\left(1 + e^i\right)\right)$ and $\left(1 + e^i, 0\right)$. In the case where wages differ, agents with higher wages but the same time endowment have steeper budget lines while in the case where only the time endowment differs, the budget times for agents with higher e have the same slope but are farther out northeast in a parallel fashion. In short, any analysis where the difference across agents is rooted in differences in the value of their time necessitates contending with income and substitution effects, whereas if these differences are restricted to their effective time endowment, only an income effect is present. This makes the analysis much simpler, which is why we adopt it here. Persson and Tabellini (2002) use the same formulation as in the current paper for similar reasons.

The utility of agent i – denoted W – is given by

$$W \equiv W\left(c^{*i}, x^i\right) \tag{3}$$

where W is strictly increasing and strictly concave in each argument, i.e., $W_1 > 0, W_2 > 0$, $W_{11} < 0$, and $W_{22} < 0$; separability, an assumption we will make at places below, requires $W_{21} = 0$. c^{*i} is effective consumption best thought of as an amalgamation of own consumption, c^i, and own consumption relative to a consumption benchmark, c^i/c_b, where c_b is the benchmark. The benchmark may be identified as a reference point which would make $W\left(c^{*i}, x^i\right)$ reference-dependent (Kramer, 2016).

There is a single benchmark, same for all agents, which, loosely speaking, will be identified as the consumption of the proverbial Joneses.[9] Indeed, almost the entire analysis will proceed assuming a single *exogenously*-specified benchmark. In Section 8, we will allow for an endogenously-derived benchmark.

3.2 The consumption aggregator and its properties

Let $c^{*i} \equiv f\left(c^i, c_b\right)$ where f is a consumption aggregator and let $\Delta \subseteq \Re^2_+$ denote the feasible set of $\left(c^i, x^i; c_b\right)$. In this paper, we focus exclusively on a specific multiplicative form[10] for f used by Ljungqvist and Uhlig (2000), Dupor and Liu (2003), and Abel (2005):

Assumption 1

$$f\left(c^i, c_b\right) = c^i\,\mathcal{A}\left(c_b\right) \tag{4}$$

where $\mathcal{A} \equiv \mathcal{A}\left(c_b\right)$ *is a continuously-differentiable function of* c_b *and* $\mathcal{A}'\left(c_b\right) < 0 \in \Delta$.

As we show in Appendix B, this form of amalgamation may be obtained as a *generalized f-mean* of c^i and relative consumption, c^i/c_b, but can permit a broader interpretation of how the benchmark influences consumption.[11] Abel (2005), for instance, uses

$$\mathcal{A}\left(c_b\right) = \left(c_b\right)^{-\eta},\ \eta \in [0, 1). \tag{5}$$

Derivatives of the aggregator function determine behavioral responses to changes in the benchmark. We have:

Jealousy-Admiration (Dupor and Liu, 2003) If $\partial W\left(c^{*i}, x^i\right)/\partial c_b = W_1 f_2 < 0\ (> 0)\ \forall \left(c^i, x^i\right)$ (i.e., $f_2 \lesseqgtr 0$ respectively, everywhere), the utility function displays *jealousy* (*admiration*).

Since

$$f_1 = \mathcal{A};\ f_2 = c^i\mathcal{A}'\left(c_b\right) < 0;\ f_{12} = \mathcal{A}'\left(c_b\right) < 0, \tag{6}$$

it follows that our singular focus on (4) permits only jealousy, not admiration. A few points to note. First, this definition of jealousy measures the effect of the benchmark on one's utility, not on marginal utility. Second, increases in the benchmark induce jealousy in every agent – i.e.,

[9] Multiple benchmarks, specific to different wealth groups, can, in principle be accommodated. There, an agent could keep up with one set of neighbors and run away from another. Our main insights, suitably adapted to that world, would doubtless carry over.

[10] For completeness, Appendix A includes a discussion of the subtractive form of f.

[11] Special cases of the generalized f−mean include the arithmetic, geometric, and harmonic means.

takes some of the joy out of own consumption – in effect reducing their effective consumption, and this is true however high their consumption may be.

3.3 When do the Joneses affect agent i's decisions

Changes in the benchmark have the potential to alter the *slope* of agents' indifference curves, and via this channel, affect their decision-making. To see this, differentiate W to get $dW = \frac{\partial W}{\partial c^{*i}} \frac{\partial c^{*i}}{\partial c^{i}} dc^{i} + \frac{\partial W}{\partial x^{i}} dx^{i}$. Letting z^i denote the marginal rate of substitution (MRS) between leisure and consumption for agent i, we have

$$z^{i} \equiv -\frac{\frac{\partial W}{\partial x^{i}}}{\frac{\partial W}{\partial c^{i}}} = -\frac{W_{2}\left(f\left(c^{i}, c_{b}\right), x^{i}\right)}{W_{1}\left(f\left(c^{i}, c_{b}\right), x^{i}\right) \cdot f_{1}\left(c^{i}, c_{b}\right)}. \tag{7}$$

Dropping the arguments of W and f,

$$\frac{\partial z^{i}}{\partial c_{b}} = \frac{f_{2}}{W_{1} f_{1}} \left(-W_{21} + W_{2} \frac{W_{11}}{W_{1}} + W_{2} \frac{f_{12}}{f_{2} f_{1}}\right). \tag{8}$$

It turns out there is a nice relationship connecting a measure of risk aversion with $\frac{\partial z^{i}}{\partial c_{b}}$. Let

$$\sigma\left(c^{*i}\right) \equiv -\frac{c^{*i} W_{11}}{W_{1}},$$

be the coefficient of relative risk aversion (inverse of the elasticity of substitution) defined on effective consumption, c^{*i}, for person i.[12] Using (6), rewrite (8) in terms of $\sigma\left(c^{*i}\right)$ as

$$\frac{\partial z^{i}}{\partial c_{b}} = \frac{c^{i} \mathcal{A}'\left(c_{b}\right) W_{2}}{W_{1} \mathcal{A} c^{*i}} \left(1 - \sigma\left(c^{*i}\right) - \frac{c^{*i} W_{21}}{W_{2}}\right). \tag{9}$$

Clearly, the Joneses have the ability to affect agent i's decisions iff $\frac{\partial z^{i}}{\partial c_{b}} \neq 0$ for some i. From (9), this can happen when $\left(1 - \sigma\left(c^{*i}\right) - \frac{c^{*i} W_{21}}{W_{2}}\right) \neq 0$. This condition is *necessary and sufficient.* If W is assumed to be separable, i.e., $W_{21} = 0$, then the Joneses can influence agent i's decisions iff $\sigma\left(c^{*i}\right) \neq 1$. Here on, we assume W is separable.[13] Examples where $\partial z^{i} / \partial c_{b} \neq 0$ abound – see Appendix C.2.

Let us take a closer look at the effect of c_b on z by distinguishing its differential effect on the marginal utilities, $\partial W / \partial x$ and $\partial W / \partial c$. Changes in the benchmark have no impact on the

[12] Consider the coefficient of relative risk aversion defined on effective consumption $\sigma\left(c^{*i}\right) \equiv -\frac{c^{*i} W_{11}}{W_{1}}$; the same coefficient, defined on c^{i}, takes the form $\sigma\left(c^{i}\right) \equiv -c^{i} \frac{W_{11} f_{1} + W_{1} f_{11}}{W_{1} f_{1}}$. For (4), $f_{11} = 0$ and it can be checked that $\sigma\left(c^{*i}\right) = \sigma\left(c^{i}\right)$.

[13] Appendix C.1 studies several special functional forms that are commonly used but non-separable.

marginal utility of leisure, i.e., $\frac{\partial(\partial W/\partial x)}{\partial c_b} = 0$. However,

$$\frac{\partial\left(\partial W/\partial c\right)}{\partial c_b} = \underbrace{W_1 \mathcal{A}'\left(c_b\right)}_{-} + \underbrace{W_{11} c \mathcal{A}\left(c_b\right) \mathcal{A}'\left(c_b\right)}_{+} = W_1 \mathcal{A}'\left(c_b\right)\left[1-\sigma\right]. \tag{10}$$

From (10), we see an increase in the benchmark has two distinct effects on marginal utility of consumption, $\partial W/\partial c$. On the one hand, a higher benchmark decreases effective consumption, thereby raising the marginal utility of effective consumption (making any incremental increase in c *more* valuable to the agent) – this effect, expressed in the second middle term on the r.h.s of (10), has a positive impact on $\partial W/\partial c$. On the other hand, a higher benchmark means increases in consumption have a smaller impact on effective consumption – jealousy – making any incremental increase in c *less* valuable to the agent.[14] These conflicting effects are embodied in σ, cf. the last term on the r.h.s of (10). Since the benchmark has no impact on the marginal utility of leisure, those who see a net increase (decrease) in their valuation of c will now be less (more) reluctant to swap c for x. To foreshadow, it is this differential effect on the willingness to substitute c for x that lies at the core of dual response – fight and flight. Below, in Section 5, we offer a detailed examination of when we might see these effects dominate one other in the same economy.

4 KUJ, RAJ

Starting from *any* initial allocation, consider an increase in c_b. Following Dupor and Liu (2003), define (Global) Keeping up with the Joneses (**G-KUJ**) and (Global) Running away from the Joneses (**G-RAJ**) as:[15]

G-KUJ $\frac{\partial z^i}{\partial c_b} > 0 \, \forall \left(c^i, x^i\right) \in \Delta$

G-RAJ $\frac{\partial z^i}{\partial c_b} < 0 \, \forall \left(c^i, x^i\right) \in \Delta$

Heuristically, if the agent's preferences display KUJ, his eagerness to substitute away from leisure into work – the marginal rate of substitution, z – increases when benchmark consumption rises; see how in Figure 1 the indifference curve pivots from the initial black to the orange (flatter curve), with an increase in the benchmark. The opposite is true if his preferences display RAJ, as illustrated by the steeper, red curve.

[14] Continuing with the housing fable from the introduction, at the margin, you get more satisfaction from living in a slightly bigger house when the neighbors have done so. But, at the same time, the slightly bigger house does not do as much for you as before because the goalpost has moved out further.

[15] Gollier (2004) defines the degree of conformism as the increase in an agent's consumption that leaves his marginal utility unchanged for a unit increase in the consumption benchmark.

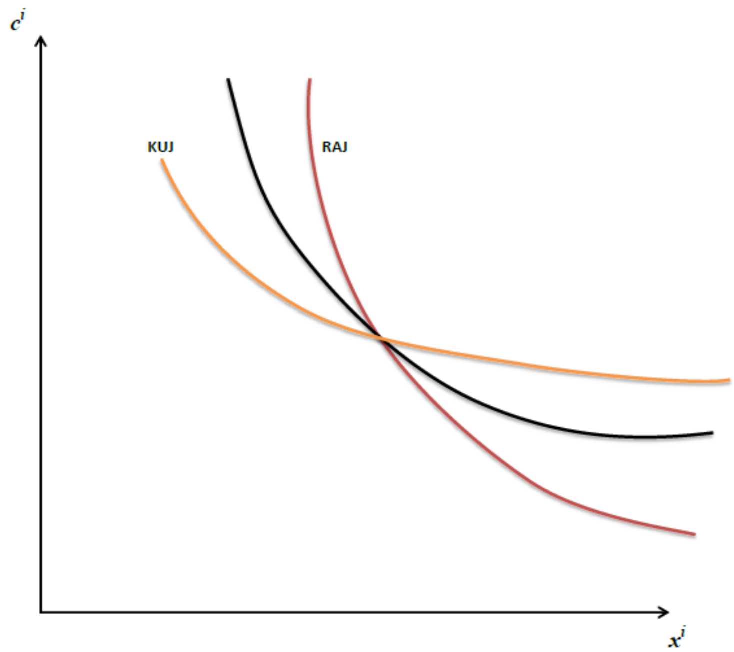

Figure 1: KUJ (orange) & RAJ (red)

It is worth re-emphasizing the definitions of keeping up or running away apply *globally*, i.e., they apply *everywhere in the feasible, positive orthant*, irrespective of the agent's initial allocation; hence the label, global. An implication of framing the definition thusly is if any agent displays KUJ (or RAJ) everyone will. Any fundamental differences between people – here, captured by heterogeneity in e – will not render differences in the *direction of their response* to a change in c_b.

It bears emphasis here that the entire literature on consumption externalities, to date, has studied preferences which satisfy either G-KUJ or G-RAJ.[16] We depart from the existing literature and demonstrate below, that for a large class of preferences, $\partial z^i/\partial c_b$ may be positive (or negative) *locally*, i.e., only for a subset of the feasible, positive orthant, *not everywhere*. For such preferences (see Figure 2) people in different portions of the consumption space, those with different e, may respond very differently – substitute away *or* toward work – to a given change in the benchmark.[17] Define $\Delta \equiv \Delta_K \cup \Delta_R$ and define (Local) Keeping up with the Joneses (**L-KUJ**) over a portion of the commodity space, Δ_K, and (Local) Running away from the Joneses (**L-RAJ**) at some other portion of the commodity space, Δ_R as:[18]

[16] Kawamoto (2009) studies a two-class, two-period overlapping generations model in which all agents have status concerns and some agents keep up with the Joneses and others run away. However, this bifurcation emerges as a result of differences in preferences that are exogenously imposed.

[17] In Figure 2, as drawn, agents at the initial allocation A display KUJ while those at B display RAJ.

[18] A brief comment about the equivalence between two seemingly-different definitions of KUJ and RAJ. One such definition – the one proposed by Dupor and Liu (2003) – is discussed above and relies on how the MRS changes with a change in the benchmark. For example, if the agent's preferences display KUJ, his desire to substitute away from leisure into work increases, which raises his income, but does that mean he chooses to consume more? An

L-KUJ $\frac{\partial z^i}{\partial c_b}|_{(c^i,x^i)\in\Delta_K} > 0$

L-RAJ $\frac{\partial z^i}{\partial c_b}|_{(c^i,x^i)\in\Delta_R} < 0$

Similar global and local notions for jealousy and admiration are possible but not explored further in this paper. As stated above in Assumption 1, we focus exclusively on f forms satisfying jealousy globally.

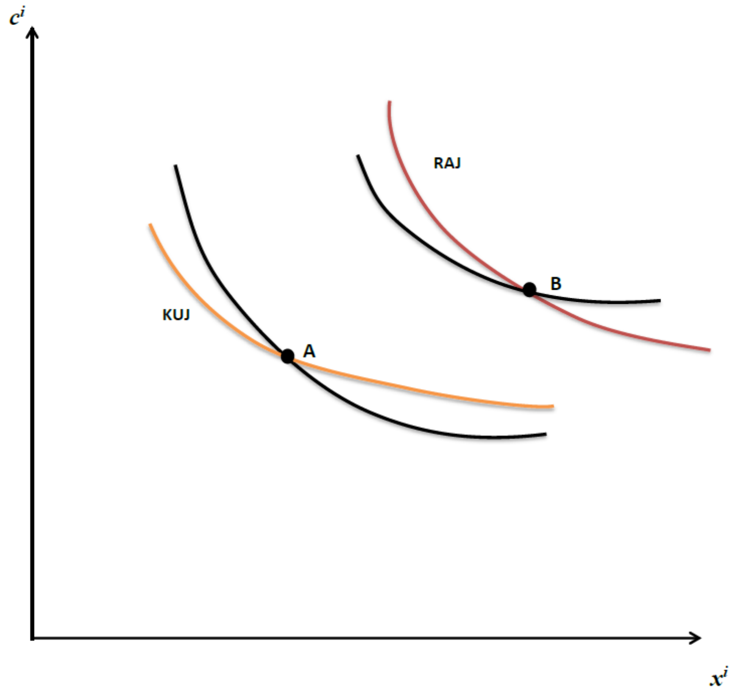

Figure 2: L-KUJ (orange) and L- RAJ (red)

We now turn to formal definitions of dual response and develop understanding of the economics underlying the phenomenon.

5 Dual response: Fight or Flight

Henceforth, refer to the set of (c^i, x^i) in Δ in which L-KUJ holds as Δ_K, and L-RAJ holds as Δ_R, respectively. Note, these sets depend on c_b.

Definition (dual response) Preferences are said to display *dual response* if both Δ_K and Δ_R are non-empty.

alternative definition, briefly commented on by Dupor and Liu (2003), says yes; an agent exhibits KUJ if an increase in the consumption of the Joneses incentivizes the agent to choose to increase his own consumption. In Appendix E , we show these definitions are equivalent, that is the sign of $\partial z^i/\partial c_b$ at $(c^{*i}, x^i) = (\hat{c}^{*i}, \hat{x}^i)$ is the same (opposite) sign as $\partial \hat{c}^i/\partial c_b$ $(\partial \hat{x}^i/\partial c_b)$.

Dual response (or the fight-or-flight response) implies $\frac{\partial z^i}{\partial c_b}$ has one sign for some (c, x) and the opposite sign at some other (c, x) in the feasible space. In particular, an agent with endowment e can be seen running away from the Joneses – the *flight* response – while another agent with $e' \neq e$ but *with the same underlying preferences*, keeps up – the *fight* response. Indeed, as will be apparent below, it is even possible for the *same* person to respond differently to a change in the benchmark depending on the initial level of the benchmark.

5.1 When can $\partial z^i/\partial c_b$ change sign?

We would like to know more about whether $\partial z^i/\partial c_b$ can *change sign* over the expanse of the consumption possibilities set, Δ. Specifically, it follows from (9) that when W is separable, for $\frac{\partial z^i}{\partial c_b}$ to change sign, $1 - \sigma\left(c^{*i}\right)$ must as well. This, in turn, requires $\sigma\left(c^{*i}\right)$ to vary sufficiently with c^{*i} and for c^{*i} itself to vary with e^i. The latter can happen in one of two ways.

1. The sign of $\frac{\partial z^i}{\partial c_b}$ varies across individuals: If $\sigma\left(c^{*i}\right)$ *varies* with e^i and $\sigma\left(c^{*i}\right) > 1$ for some i and $\sigma\left(c^{*j}\right) < 1$ for some j, then $\frac{\partial z^i}{\partial c_b}$ is of one sign for i and a different sign for j. Clearly, wealth-dependent risk aversion, in fact sufficient heterogeneity in it – enough to straddle *unity* – is necessary to getting $\partial z^i/\partial c_b$ to change sign over the consumption landscape.

2. The sign of $\frac{\partial z^i}{\partial c_b}$ varies for a given individual as c_b changes. If $\sigma\left(c^{*i}\right)$ *varies* with c^{*i} which, in turn, varies with c_b, $\frac{\partial z^i}{\partial c_b}$ can be of one sign for i at some level of c_b and have a different sign for the same i at another level of c_b. That is, for a given i, $1 - \sigma(\cdot)$ may change sign with a change in the benchmark.

Before proceeding further, we supply an example where $\frac{\partial z^i}{\partial c_b}$ does change sign:

Example 1 *(Dual response) Consider a simple form of additively-separable, Stone-Geary preferences:*

$$W\left(c^{*i}, x^i\right) = \left(c^{*i} - \varphi_c\right)^{\beta} + \left(x^i\right)^{1-\beta}, \; \beta \in (0,1), \; \varphi_c > 0$$

where $\left(c^{*i}, x\right)$ must satisfy $c^{*i} - \varphi_c > 0$ and $x^i > 0$. In Appendix F, we show $\sigma\left(c^{*i}\right) = (1-\beta)\frac{c^{*i}}{c^{*i}-\varphi_c}$ and $\frac{\partial z^i}{\partial c_b} = \frac{c^{*i}\beta - \varphi_c}{c^{*i}-\varphi_c}$. Clearly, $\frac{\partial z^i}{\partial c_b}$ can change sign. Since $c^{*i} - \varphi_c > 0$ must hold, it follows $\frac{\partial z^i}{\partial c_b}$ changes sign depending on the level of effective consumption:

$$\frac{\partial z^i}{\partial c_b} < 0 \Longleftrightarrow \varphi_c < c^{*i} < \frac{\varphi_c}{\beta}$$

$$\frac{\partial z^i}{\partial c_b} > 0 \Longleftrightarrow c^{*i} > \frac{\varphi_c}{\beta} > \varphi_c$$

Apropos Point 1 above, there can be many reasons why $\sigma\left(c^{*i}\right)$ varies with e^i. The simplest such reason would be non-homotheticity of W as exemplified by the Stone-Geary form in Example 1. Here c^{*i} depends on e^i (as would be expected, for most utility functions) but due to the non-homotheticity of W, this dependence carries over to c^{*i}, and hence to $\sigma\left(c^{*i}\right) = -\frac{c^{*i}W_{11}}{W_1}$. We formalize this idea in the following proposition.

Proposition 1 *(No dual response) a. (C.E.S class) Suppose W is of the C.E.S form such that*

$$W\left(c^{i*}, x^i\right) = \Lambda\left(\lambda\left(c^{i*}\right)^\rho + (1-\lambda)\left(x^i\right)^\rho\right)^{k/\rho}$$

where $\Lambda > 0$ is a constant, $0 < \lambda < 1$, $\rho \leq 1$ and $0 < k \leq 1$. Then, preferences exhibit G-KUJ if $\rho < 0$ and G-RAJ if $\rho > 0$.[19]
b. (No dual response) Suppose W is homothetic and additively separable in $\left(c^i, x^i\right)$ for any given c_b. Then, dual response does not obtain.

The proof can be found in Appendix C.2, Example 6. [20]

Where do this leave us in our quest to find the possibility of dual response? Two important utility classes – one homothetic, the other additively-separable and C.E.S – *fail* to generate dual response. Additionally, multiplicatively-separable preferences, $W\left(c^{i*}, x^i\right) = h\left(c^{i*}\right)v\left(x^i\right)$, cannot generate dual response – see Appendix D. Our search for dual response must take us *outside* the realm of these preference classes.

In passing, it is instructive to ask the question, how is any of this potentially useful? In the existing literature, a change in the benchmark always elicits an identical qualitative response from the rich and the poor; both respond either by keeping up or running away. With dual response, the possibility arises that the rich react to the Joneses by raising their consumption and the poor do the opposite, or vice versa. Dual response, then, can act as an endogenous amplification or dampening mechanism, taking fundamental differences and either amplifying or dampening them via individual choices and market outcomes. This is the promise that dual response holds out.

[19] Of course, the elasticity of substitution in this case is $\epsilon = 1/(1-\rho)$. Proposition (1) can then be restated in terms of ϵ, i.e., preferences are G-KUJ or G-RAJ depending on whether ϵ is less than or greater than 1, respectively. When $k = \rho$, the coefficient of relative risk aversion is constant; $\sigma\left(c^{*i}\right) = 1/\epsilon$

[20] Note also, homotheticity of W in $\left(c^{*i}, x^i\right)$, in general, does not necessarily imply homotheticity of W in $\left(c^i, x^i\right)$. The two are the same if and only if, as in (4), f is homogenous of degree 1 in c^i for a given benchmark c_b.

Part b of Proposition 1 is really a corollary to Part a; it follows from Bergson's Theorem which states that if a utility function is quasi-concave, increasing, and separable, it is homothetic if and only if it is of the C.E.S. form.

6 The agent's problem redux

The first order conditions for the agent's problem of maximizing (3) subject to (2) and the aggregator, $c^{*i} \equiv f\left(c^i, c_b\right)$ can be summarized by

$$W_1\left(c^{*i}, x^i\right) f_1 - W_2\left(c^{*i}, x^i\right) = 0 \Leftrightarrow \frac{W_2\left(\hat{c}^{*i}, \hat{x}^i\right)}{W_1\left(\hat{c}^{*i}, \hat{x}^i\right)} \equiv f_1, \tag{11}$$

where a hat ("^") denotes optimal choices; also, in writing (11), we have assumed interior solutions for choice variables c^i and x^i, and $\hat{c}^{*i} \equiv f\left(\hat{c}^i, c_b\right)$. The second order condition is $W_{11}\left(\hat{c}^{*i}, \hat{x}^i\right) f_1^2 + W_1\left(\hat{c}^{*i}, \hat{x}^i\right) f_{11} + W_{22}\left(\hat{c}^{*i}, \hat{x}^i\right) < 0$.

Before getting into the mathematical nitty-gritties, a simple textbook approach to our problem is in order. First, reinterpret the agent's problem as one of choosing c^* and x to maximize $W\left(c^*, x\right)$ subject to the budget constraint $\frac{c^{*i}}{\mathcal{A}(c_b)} + x^i = \left(1 + e^i\right)$ implying the relative price of effective consumption in terms of leisure is $p \equiv 1/\mathcal{A}\left(c_b\right)$. Then (11) gives $W_1\left(c^{*i}, x^i\right) = pW_2\left(c^{*i}, x^i\right)$.

Figure 3a summarizes this basic problem for a given agent i drawn in (x, c^*) space. The slope of the original budget line, FE, is $-\mathcal{A}\left(c_b\right)$. In this formulation, the effect of an exogenous increase in c_b, by assumption, reduces $\mathcal{A}\left(c_b\right)$ – raises p – meaning the relative price of c^* has gone up or equivalently, the relative price of leisure has *fallen*; consequently, the budget line pivots inward, as indicated by the new budget line $F'E$. Consider an agent who chose the point H before the benchmark increase. After the change, the same agent chooses H'. His new bundle has less c^* but more x. Alternatively, if the agent was originally at I, that agent, after the benchmark change, would be at I' : his new bundle has less c^* and *less* x.

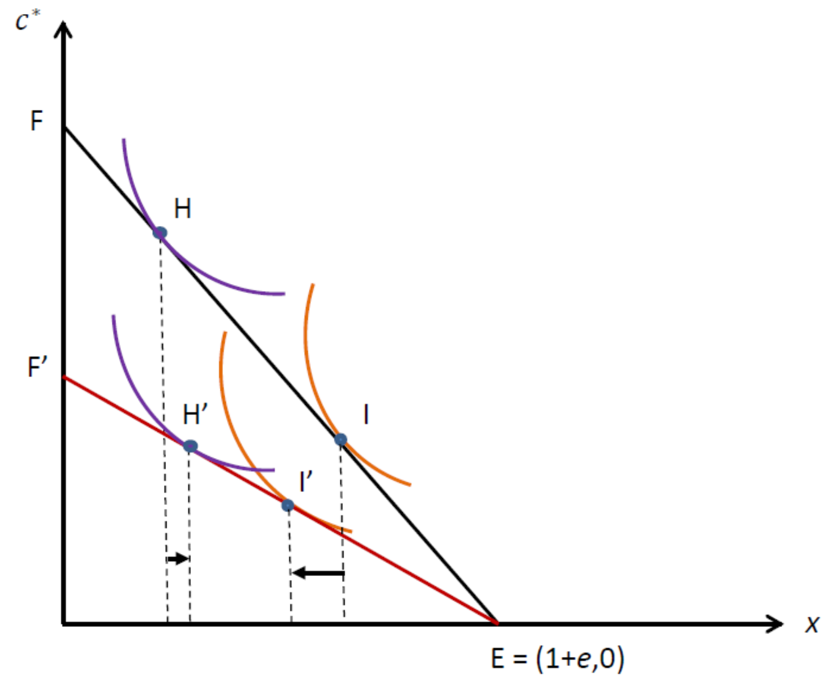

Figure 3a: KUJ/RAJ for an agent i

Why the difference between the two responses? Two effects are at play here. First, an increase in c_b, as noted, reduces the relative price of leisure making it more attractive. On the other hand, the increase in p causes the budget set to shrink (reduces the purchasing power of the endowment in terms of c^*) rendering the original bundle of (x, c^*) unaffordable. This is the income effect. We don't know which effect will dominate, in general. When the agent moves from point H to H', it is the substitution effect that dominates; the opposite is true if the agent moves from I to I'. In the former case, he works less so his c is lower; he is *running away from the Joneses*. In the latter, he is *keeping up*; his c rises because he works more. In either case, his c^* is lower, though c may rise or fall, depending on the leisure response. Evidently, the curvature of the indifference curves – steepness at point H vs. I – plays an important part in determining whether he keeps up or runs away.

Figure 3b makes the point that two agents with very different e^i may react differently to a change in the benchmark. In Figure 3b, the agent with the higher e is initially at H. After the change in c_b, he is at H' : the substitution effect of an increase in c_b causes him to move to H'' and the income effect from H'' to H'. The poorer agent moves from L to L' : for this agent, the substitution effect of an increase in c_b causes him to move to L'' and the income effect from L'' to L'. Notice though, the magnitude of these effects for the same change in c_b is vastly different across the two agents. This is why an agent with endowment e may exhibit RAJ while another with endowment $e' \neq e$ may exhibit KUJ even though they share the same underlying preferences.

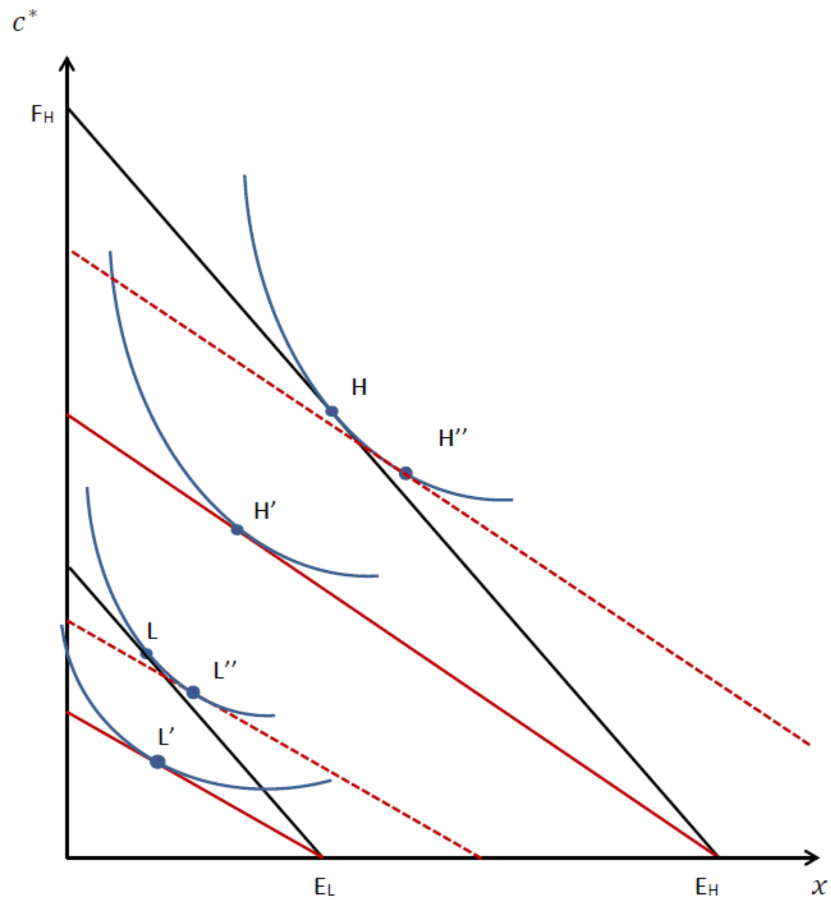

Figure 3b: KUJ/RAJ for two differently-endowed agents

The discussion above, framed as a choice over x and c^*, is useful for grasping the general intuition of dual response because the analysis of changes in the benchmark can be cast in terms of textbook price effects. However, since the agent's actual choice is over x and c, it may be instructive to understand dual response within the latter context, one where the agent chooses x and c by maximizing $W(c\mathcal{A}(c_b), x)$ subject to $c + x = 1 + e$. Note, here a change in c_b has no effect on the budget line; instead, it changes the shape of the indifference curves. Figure 4 portrays the choice problem of two different agents, one poor, the other rich. The dashed lines represent budget lines of these agents, the black curves, labeled $I(c_b)$, $J(c_b)$, represent the highest-attainable indifference curves for each agent, for a given benchmark, c_b.

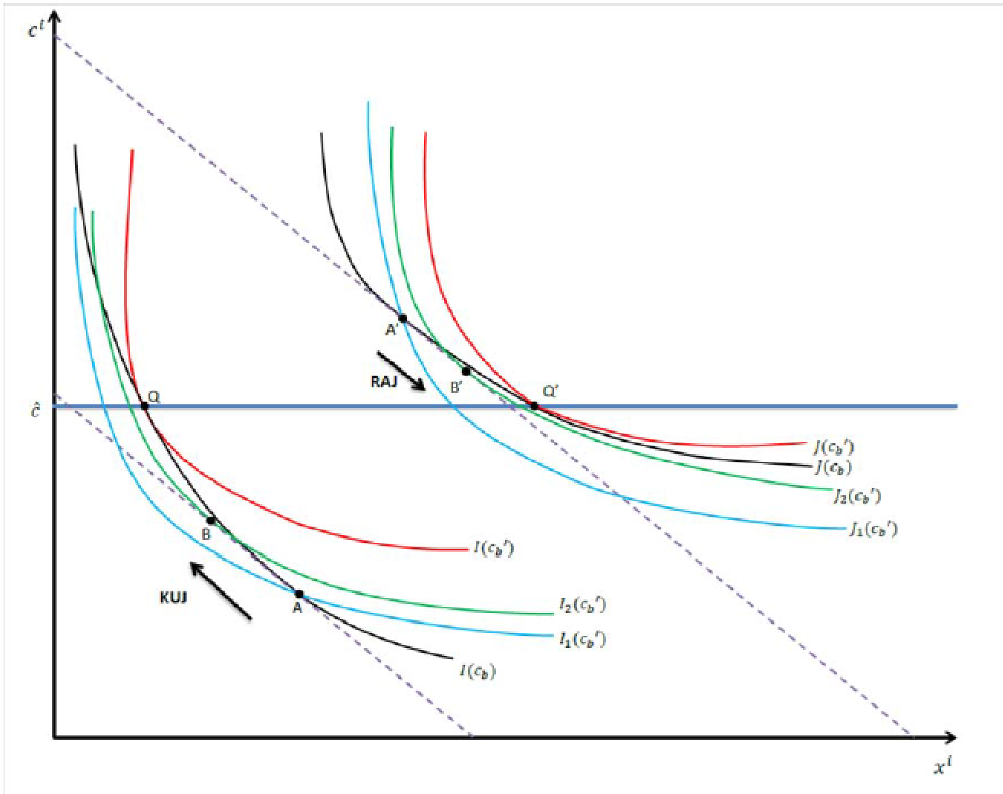

Figure 4: KUJ and RAJ and associated price & income effects

The poorer (wealthier) agent's choice $\left(\hat{c}^i, \hat{x}^i\right)$ is summarized by point $A(A')$. Now consider an increase in the benchmark, from c_b to c_b'. The red curves in the figure, $I\left(c_b'\right)$, $J\left(c_b'\right)$ represent new indifference curves associated with the higher benchmark. We identify a level of consumption $\hat{c}$ at which these new indifference curves are tangent to the old curves $I\left(c_b\right)$, $J\left(c_b\right)$, shown as points Q and $Q\prime$, respectively. Consider now the indifference curve – the blue curve, $I_1\left(c_b'\right)$ – that passes through the initial choice for the poorer agent, point A. The slope at A is flatter than the slope of the budget line, meaning the agent will choose to substitute away from leisure (work and consume more) in response to the increase in c_b, i.e., she keeps up with the Joneses (locally, for this agent, $\partial z^i/\partial c_b > 0$, where for a small change in c_b, the slope of $I_1\left(c_b'\right)$ at A is $\partial z^i/\partial c_b$). The agent's new choice is represented at point B, where the indifference curve $I_2\left(c_b'\right)$ is tangent to the budget line (and has the slope -1). The opposite is true for the wealthier agent. Here the curve $J\left(c_b'\right)$, passing through the initial choice A', is steeper, meaning for this agent, locally, $\partial z^i/\partial c_b < 0$. The agent then chooses to work and consume less; he runs way from the Joneses, as shown by point B', where the indifference curve $J_2\left(c_b'\right)$ is tangent to this agent's budget line.

Next, we formally flesh out the Slutsky decomposition for a cross price increase in p – capturing the income and substitution effects of a change in p (via a change in c_b) on $\hat{x}^i$ where $\hat{x}^i$ is defined in (11):

$$\frac{\partial \hat{x}^i}{\partial p} = \left(\frac{\partial \hat{x}^i}{\partial p}\right)_{U \text{ fixed}} - \hat{c}^{*i} \frac{\partial \hat{x}^i}{\partial \left(1+e^i\right)}$$

Here, the income effect is captured by $-\hat{c}^{*i}\frac{\partial \hat{x}^i}{\partial(1+e^i)}$, the substitution effect by $\left(\frac{\partial \hat{x}^i}{\partial p}\right)_{U \text{ fixed}}$ and the total effect is $\frac{\partial \hat{x}^i}{\partial p}$. It is useful to recall that the effect of an exogenous increase in c_b raises p meaning the relative price of c^* has gone up or that the relative price of leisure has fallen.

Proposition 2 *The total effect of a change in the benchmark on leisure is given by*

$$\frac{\partial \hat{x}^i}{\partial p} = \frac{\frac{1}{p}\hat{c}^{*i} W_{11}\left(\cdot\right) + W_2\left(\cdot\right)}{-pW_{22}\left(\cdot\right) - \frac{1}{p}W_{11}\left(\cdot\right)},$$

the substitution effect by

$$\left(\frac{\partial \hat{x}^i}{\partial p}\right)_{U \text{ fixed}} = \frac{W_2\left(\cdot\right)}{-pW_{22}\left(\cdot\right) - \frac{1}{p}W_{11}\left(\cdot\right)} > 0,$$

and the income effect by

$$-\hat{c}^{*i}\frac{-\frac{1}{p}W_{11}\left(\cdot\right)}{-pW_{22}\left(\cdot\right) - \frac{1}{p}W_{11}\left(\cdot\right)}$$

where the denominator, $-pW_{22}\left(\cdot\right) - \frac{1}{p}W_{11}\left(\cdot\right) > 0$ *by the second order condition. The substitution effect,* $\frac{W_2(\cdot)}{-pW_{22}(\cdot)-\frac{1}{p}W_{11}(\cdot)} > 0$*, and the income effect,* $\frac{\frac{1}{p}\hat{c}^{*i}W_{11}(\cdot)}{-pW_{22}(\cdot)-\frac{1}{p}W_{11}(\cdot)} < 0$.

The substitution effect is always positive; this means as the relative price of effective consumption (p) increases, the agent substitutes out of effective consumption into leisure. The income effect is negative meaning as the relative price of effective consumption (p) increases, the purchasing power of his endowment in terms of effective consumption decreases – the budget set shrinks. In isolation, this income effect will decrease leisure if leisure is a normal good. (This means for dual response to obtain, it is necessary that leisure be a normal good, which is assured under the assumption of separability in our two-good setting (see Kubler et. al., 2014). The size of these effects depends on i. Notice, the total effect of a change in the benchmark is given by

$$\frac{1}{p}\frac{\hat{c}^{*i}W_{11}\left(\cdot\right) + pW_2\left(\cdot\right)}{-pW_{22}\left(\cdot\right) - \frac{1}{p}W_{11}\left(\cdot\right)} = -\frac{1}{p}\frac{\hat{c}^{*i}W_{11}\left(\cdot\right) + W_1\left(\cdot\right)}{pW_{22}\left(\cdot\right) + \frac{1}{p}W_{11}\left(\cdot\right)} = -\frac{1}{p}\frac{W_1\left(\cdot\right)\left[\frac{\hat{c}^{*i}W_{11}(\cdot)}{W_1(\cdot)} + 1\right]}{pW_{22}\left(\cdot\right) + \frac{1}{p}W_{11}\left(\cdot\right)} = -\frac{1}{p}\frac{W_1\left(\cdot\right)\left[1 - \sigma\left(c^{*i}\right)\right]}{pW_{22}\left(\cdot\right) + \frac{1}{p}W_{11}\left(\cdot\right)} \tag{12}$$

using $W_1(\cdot) = pW_2(\cdot)$. Dual response refers to a situation where the direction of this total effect changes with i. As (12) makes clear, this is impossible if $\sigma\left(c^{*i}\right) = \sigma\ \forall i$ or $\sigma\left(c^{*i}\right) \lesseqgtr 1\ \forall i$. This explains why apropos Appendix C.2 (Example 6), dual response cannot obtain under C.E.S preferences or, more generally, under homothetic utility which produces person-invariant σ.

7 HARA utility

The dual-response result discussed above has the potential to split (endogenously) the population into separate camps, those that keep up and those that run away from the Joneses. We explore this below for the concrete example of hyperbolic absolute risk aversion (HARA) preferences.[21] HARA, as is well known, is the most general class of utility functions used in practice.

A utility function exhibits hyperbolic absolute risk aversion (HARA) if and only if the reciprocal of absolute risk aversion is a linear function of wealth. A general, additively-separable form of HARA preferences can be written as

$$W(c^*, x) = \frac{1-\gamma}{\gamma}\left(\varphi_c + \frac{\alpha_c c^*}{1-\gamma}\right)^{\gamma} + \theta\frac{1-\gamma}{\gamma}\left(\varphi_x + \frac{\alpha_x x}{1-\gamma}\right)^{\gamma};\ \alpha_c, \alpha_x > 0,\ \theta > 0 \tag{13}$$

where (c^*, x) have to satisfy $\varphi_c + \frac{\alpha_c c^*}{1-\gamma} > 0$ and $\varphi_x + \frac{\alpha_x x}{1-\gamma} > 0$; φ_c and φ_x can be negative but $\varphi_c(\gamma - 1) > 0$ must hold. Under various restrictions on γ, φ_c,and φ_x, HARA encompasses a wide class of commonly-used utility functions such as the linear, quadratic, exponential, power (including isoelastic), Stone-Geary, and logarithmic. If $\varphi_c = \varphi_x = 0$, then (13) is homothetic in (c^*, x); otherwise not. Also $\varphi_c < 0$ has the flavor of a minimum consumption requirement – cf. Appendix F.

Define $k \equiv \left(\frac{\theta\alpha_x}{\mathcal{A}\alpha_c}\right)^{\frac{1}{\gamma-1}}$. Using (4), it is easy to check that (c^*, x) satisfy $\varphi_c + \frac{\alpha_c c^*}{1-\gamma} > 0$ and $\varphi_x + \frac{\alpha_x x}{1-\gamma} > 0$ if $c^i \geq -\frac{\varphi_c}{\mathcal{A}\alpha_c}(1-\gamma)$ and $x^i \geq -\frac{\varphi_x}{\alpha_x}(1-\gamma)$. Using

$$W_1 = \alpha_c\left(\varphi_c + \frac{\alpha_c\mathcal{A}c^i}{1-\gamma}\right)^{\gamma-1} \text{ and } W_2 = \theta\alpha_x\left(\varphi_x + \frac{x^i\alpha_x}{(1-\gamma)}\right)^{\gamma-1}, \tag{14}$$

(7) yields

$$z^i = -\left(\frac{k\left(\varphi_x + \frac{\alpha_x x^i}{1-\gamma}\right)}{\varphi_c + \frac{\alpha_c c^{i*}}{1-\gamma}}\right)^{\gamma-1}. \tag{15}$$

For subsequent use, the exact expressions for the solution to the agent's problem under (13) are

[21] Appendix F works out the special but popular and analytically tractable case of Stone-Geary utility.

as follows:

$$\hat{c}^i = \frac{(k\varphi_x - \varphi_c)(1-\gamma) + k\alpha_x\left(1+e^i\right)}{(\alpha_c\mathcal{A} + k\alpha_x)} \tag{16}$$

and

$$\hat{x}^i = \frac{\alpha_c\mathcal{A}\left(1+e^i\right) - (k\varphi_x - \varphi_c)(1-\gamma)}{(\alpha_c\mathcal{A} + k\alpha_x)}. \tag{17}$$

For future use, write $\hat{c}^i$ compactly as

$$\begin{aligned}
&\hat{c}^i = a + b\left(1+e^i\right) \text{ where} \qquad (18)\\
&a \equiv (k\varphi_x - \varphi_c)(1-\gamma)/(\alpha_c\mathcal{A} + k\alpha_x)\\
&b \equiv k\alpha_x/(\alpha_c\mathcal{A} + k\alpha_x) > 0\\
&k \equiv \left(\frac{\theta\alpha_x}{\mathcal{A}\alpha_c}\right)^{\frac{1}{\gamma-1}}
\end{aligned}$$

Clearly, an advantage of the HARA form is that while it permits non-homotheticity (if either or both of φ_c and φ_x are non-zero), it nevertheless delivers decision rules that are linear in e. So as to not get overly taxonomic, below we focus our attention on a subset of parameters that satisfy[22]

Assumption 2 $\varphi_c < 0$ *and* $\gamma < 1$.

Note, even under Assumption 2, the sign of a is, in general, indeterminate. If $\varphi_x > 0$, then Assumption 2 guarantees $a > 0$.

7.1 Dual response and HARA

For future use,

$$\sigma^i\left(c^{*i}\right) = \frac{(1-\gamma)\,\alpha_c c^{*i}}{\alpha_c c^{*i} + \varphi_c(1-\gamma)}, \tag{19}$$

for the HARA, which, as well-known, equals a constant $1-\gamma$ when $\varphi_c = 0$. This clarifies the importance of φ_c : when $\varphi_c = 0$, $\sigma^i\left(c^{*i}\right) = (1-\gamma)\ \forall i$ and dual response cannot obtain. Note though if $\varphi_x \neq 0$, even with $\varphi_c = 0$, (13) is non-homothetic revealing that non-homotheticity is *not* sufficient to generate dual response.

[22]For (13) to be well-defined, $\varphi_c(\gamma-1) > 0$ must hold. Our point is to try and reduce clutter by focusing attention on $\varphi_c < 0$ and $\gamma < 1$. We have worked out the case where $\varphi_c > 0$ and $\gamma > 1$ and verified that fight-or-flight behavior is observed in the latter case as well.

Recall from (9) what is ultimately responsible for dual response is the sign of $1-\sigma\left(c^{*i}\right)$. As evident from (19), if $\varphi_c=\varphi_x=0$ (i.e., (13) is homothetic), then, as previously discussed, $\partial z^i/\partial c_b$ does not change sign: dual-response does not obtain. Hence, it is imperative we explore some shifters of $\sigma\left(c^{*i}\right)$ as well as summarize some of its key properties.

Lemma 1 *Assume agents' preferences are represented by (13). The coefficient of relative aversion $\sigma^i\left(c^{*i}\right)$ displays the following properties:*

i. $\partial\sigma^i\left(c^{*i}\right)/\partial\varphi_c<0$, *and* $\lim\limits_{\varphi_c\to 0}\sigma^i\left(c^{*i}\right)=1-\gamma$.

ii. $\partial\sigma^i\left(c^{*i}\right)/\partial c^{*i}<0$.

iii. $\partial\sigma^i\left(c^{*i}\right)/\partial\mathcal{A}>0$.

iv. $\partial^2\sigma^i\left(c^{*i}\right)/\partial c^{*i}\partial\mathcal{A}\gtrless 0$.

One such shifter is φ_c. Note $\sigma^i\left(c^{*i}\right)$ is decreasing in φ_c – risk aversion is lower for all levels of consumption, when $\varphi_c<0$ (i.e., $1-\gamma<\sigma^i\left(c^{*i}\right)$ when $\varphi_c<0$). If one interprets φ_c as subsistence consumption, then it apparent the rich are willing to accept more risk, because any given reduction in c^{*i} (induced by jealousy) will not compromise their ability to consume the subsistence consumption as much as it would for the poor whose consumption is closer to φ_c.

(ii) above states that $\partial\sigma^i\left(c^{*i}\right)/\partial c^{*i}<0$. For a *given* c_b, this implies agents are less risk averse at higher consumption levels. Raising the benchmark decreases agents' aversion to risk. It also implies risk aversion is decreasing in e.

At the cutoff $c^{*i}=\tilde{c}_b^*\equiv\mathcal{A}\left(\tilde{c}_b\right)$, by definition, $\sigma^i\left(\tilde{c}_b^*\right)=1$. Since $\sigma^i\left(c^{*i}\right)$ is monotone in c^{*i}, it follows that $\sigma^i\left(c^{*i}\right)\gtrless 1$ for $c^{*i}\gtrless\tilde{c}_b^*$.

The following proposition establishes the possibility of dual response for HARA preferences.

Proposition 3 *a)The HARA utility function exhibits dual response iff $\varphi_c\neq 0$. Specifically,*

$$\frac{\partial z^i}{\partial c_b}=\begin{cases}>0 \text{ for } c^i<\delta_c: \text{ L-KUJ} \\ <0 \text{ for } c^i>\delta_c: \text{ L-RAJ}\end{cases}\quad \text{for } \gamma<1 \quad ,$$

where, given a c_b,

$$\delta_c\equiv\frac{\varphi_c\left(\gamma-1\right)}{\gamma\alpha_c\mathcal{A}\left(c_b\right)} \tag{20}$$

and δ_c falls in the allowable range of consumption defined earlier.

b) From (20), along with (18), identify an endowment δ_e associated with consumption δ_c,

$$\delta_e \equiv \max\left(\frac{\varphi_c(\gamma-1)}{\gamma\alpha_c\mathcal{A}b} - \frac{a}{b} - 1, \underline{\xi}\right). \tag{21}$$

For $\gamma < 1$, $\partial z^i/\partial c_b > 0$ for all agents with $e^i \leq \delta_e$, and $\partial z^i/\partial c_b < 0$ if $e^i > \delta_e$.

The proof is in Appendix 3. Notice $\varphi_c = 0 \Rightarrow \delta_c = 0$, so we have G-KUJ in the entire consumption range; no dual response here. Of paramount importance is the convenience that we can "turn on" dual response or turn it off by changing φ_c.

Proposition 3 establishes that a subset of the positive orthant – the demarcation happens at δ_e – exhibits one kind of behavior while the remaining portion exhibits the other kind. That is, when $\varphi_c(\gamma-1) > 0$, the preferences in (13) display dual response; they cannot be characterized as either G-KUJ or G-RAJ. This mirrors what is depicted in Figure 4 (the solid blue line in the figure represents δ_c).

Finally, note that cut-off consumption $\delta_c \equiv \frac{\varphi_c(\gamma-1)}{\gamma\alpha_c\mathcal{A}}$ also depends on the benchmark, with $\frac{\partial\delta_c}{\partial c_b} = \frac{\varphi_c(\gamma-1)}{\gamma\alpha_c\mathcal{A}}\mathcal{A}'(c_b) > 0$. (This effect is not shown in Figure 4 so as to keep the discussion simple).

The upshot is the following. For HARA preferences, we derive some clear predictions. Agents with innate ability below a threshold keep up with the Joneses while those above it run away. Loosely speaking, the poor react to an increase in benchmark consumption by working harder to raise their consumption; the rich do the exact opposite. While some in the neighborhood choose to build a "bigger house" to compete, others, their slightly more-affluent neighbors, may choose not to and appear to "drop out". Moreover, if the consumption of the Joneses goes up, the above mentioned threshold rises: all else same, more people respond by keeping up.

7.2 Dual response and effects on inequality

Recall $\mathcal{G}$, the distribution of e, captures fundamental inequality, the innate differences between people. Our framework allows us to examine inequality in measured incomes given this fundamental inequality.

To be precise, agents are endowed with $1+e^i$, not e^i. Recall, optimal consumption (and hence, income, y^i, in this static framework) is of the form: $y^i = a + b\left(1+e^i\right)$ where a and b are constants which depend on the preference primitives as well as on the benchmark, c_b. Suppose the measure of inequality is chosen to be the commonly-used Gini coefficient. One can ask, how does the Gini of y^i,call it ϕ_y, change when the benchmark changes? Does it matter to inequality whether there is dual response?[23] Below, we examine the properties of the Gini and its link to dual response.

It is easy to check (see Appendix H) that if y^i is of the form $a + b\left(1 + e^i\right)$ then we can write the Gini as

$$\phi_y = \frac{b\left(1+\overline{e}\right)}{a + b\left(1+\overline{e}\right)}\phi_{1+e},$$

where $\overline{e}$ is the mean endowment. Evidently, the difference between ϕ_y and ϕ_{1+e} depends on b and, most critically, on the magnitude and sign of a. In particular, if $a < 0$, measured income will display greater inequality than in e. On the other hand, measured income displays lower inequality (when $a > 0$), i.e., $\phi_y < \phi_{1+e}$. For the HARA form, using the explicit form of a and b (defined in eq. (18)) we obtain

$$\phi_y = \frac{k\alpha_x\left(1+\overline{e}\right)}{\left(k\varphi_x - \varphi_c\right)\left(1-\gamma\right) + k\alpha_x\left(1+\overline{e}\right)}\phi_{1+e}.$$

Below, we study settings in which the benchmark changes, changing with it both a and b. Therein will lie the possibility that as c_b increases, the economy will switch from one where income inequality is greater than fundamental inequality to one where it is less. The following proposition summarizes the effect of the benchmark on the Gini:

Proposition 4 *a. The effect on the Gini of y of an increase in the exogenous benchmark, in the presence of dual response, is given by*

$$\frac{\partial\phi_y}{\partial c_b} = \frac{\left(1+\overline{e}\right)}{\left[a + b\left(1+\overline{e}\right)\right]^2}\left\{-\frac{\varphi_c\alpha_x k}{\mathcal{A}\left(\alpha_c\mathcal{A} + k\alpha_x\right)^2}\mathcal{A}'\left(c_b\right)\right\}\phi_{1+e} < 0$$

b. If $\varphi_c = 0$, i.e., in the absence of dual response, the benchmark does not impact the Gini of y.

The proof of Proposition (4) is in Appendix I. To get some intuition, examine the change in individual consumption with respect to the benchmark, i.e. the derivative $\partial c^i/\partial c_b$. Using (18), we have

$$\frac{\partial c^i}{\partial c_b} = \frac{\partial a}{\partial c_b} + \frac{\partial b}{\partial c_b}\left(1+e^i\right).$$

We've shown $\partial b/\partial c_b < 0$ iff $\gamma < 1$. Notice this *negative effect* on c^i is more pronounced the

[23] Our analysis here is cast in a partial equilibrium setting with w held fixed at 1. The inequality in income being discussed here is not of the market-driven kind. If wages were endogenously determined, say, in a competitive market, additional effects on inequality would doubtless emerge which would also influence the pass-through.

larger the time endowment $\left(1+e^i\right)$. On the other hand, the derivative $\partial a/\partial c_b$ can take either sign (see Appendix I) and is independent of e^i. Since $\partial b/\partial c_b < 0$, richer agents will be more likely to decrease their consumption as the benchmark increases, consistent with the result that higher-endowment agents display RAJ. On the other hand, agents with lower endowments will increase their incomes, consistent with keeping up, making it clear how dual response is driving the decrease in inequality.

All the action in the model works off of three features of preferences: homotheticity, dual response, and the consumption benchmark. How do each of these contribute to inequality? Below, we discuss the possibility that as c_b increases, the economy will switch from one where income inequality is less than fundamental inequality to one where it is greater.

1. **Homothetic with or without consumption externalities:** Whenever preferences are homothetic, consumption is linear and proportional to the gross endowment; as a result, income inequality is identical to fundamental inequality. In the particular case of HARA, the preference is homothetic in (c^*, x) when $\varphi_x = \varphi_c = 0$; in this case, $a = 0$, and $\phi_y = \phi_{1+e}$. Whether or not there are consumption externalities, $a = 0$ and b does not affect the inequality, so any change in the distribution of the endowments gets mirrored exactly as a change in observed inequality.

2. **Non-homothetic preferences that do not permit dual response:** As shown above, if we set $\varphi_x \neq 0$ but $\varphi_c = 0$, preferences are not homothetic and there is no dual response. In this case, $a = -k\varphi_x(\gamma-1)/(\alpha_c + k\alpha_x) < 0$ and inequality will exceed fundamental inequality. Allowing for preferences that permit dual response ($\varphi_c \neq 0$) raises a, dampening the magnitude of income inequality. Income inequality may still be greater than fundamental inequality (if $a < 0$) but it is definitely smaller than when $\varphi_c = 0$ (no dual response). This showcases the effect dual response has on measured inequality.

3. The benchmark, in and of itself, will also directly impact a and b, and hence income inequality.

8 Endogenous benchmarking

Now that the barebones structure has been analyzed, we allow for an endogenous benchmark by setting it at a proportion of the cross-sectional average consumption (which, in turn, is endogenously derived). Setting the benchmark to a simple affine function of mean consumption has the advantage that all consumption levels can be expressed as a linear function of the first moment of the distribution of e. [24]

Assume benchmark consumption is the aspirational $\kappa\bar{c}$, where $\bar{c}$ is mean consumption (which in this case, will correspond to the consumption of the agent with mean endowment $1+\bar{e}$, if the consumption decision rule is linear in e^i), and $\kappa > 0$ (typically, $\kappa \geq 1$) is a constant. That is, set $c_b = \kappa\bar{c}$. Let $\Phi\left(1+e^i, c_b\right)$ denote the closed-form solution for the consumption decision rule for the agent with e^i. Mean consumption $\bar{c}$ is then a fixed point of the following equation, $\bar{c} = \Phi\left(\bar{e}, \kappa\bar{c}\right)$. Once $\bar{c}$ is computed, we can set the benchmark to be $c_b = \kappa\bar{c}$. Changing κ allows us to change the endogenous benchmark. These issues are developed below in the context of specific preferences.

Example 2 *Let preferences be of the HARA form – eq. (13) – with* $\gamma = 0.5, \alpha_c = \alpha_x = 1, \theta = 1.5, \varphi_c = -0.5, \varphi_x = -0.5, \underline{\xi} = 1/2, \eta = 1/2$ *and* $\mathcal{A}\left(c_b\right) = \left(c_b\right)^{-\eta}$. *The underlying distribution of* e *is Fréchet with minimum value* $\underline{\xi} = 1/2$, *shape parameter* $\alpha = 2$, *and scale parameter* $\beta = 1$. *For this constellation of parameters, the benchmark* c_b *is set at* κ *times the mean consumption in the economy. In the figures below, we vary* κ *from* $1/5$ *to* 5 *to see the importance of the influences of the Joneses.*

In order to effectively illustrate the impact of changes in the benchmark across a broad stratum of the population, we express, in the left-hand panel, the ratio of consumption against a benchmark consumption value associated with $\eta = 0$ (no consumption externality case). [25] The figure shows agents placed at different points of the e distribution: the 10%, 30%, 50%, 70%, and 90% and their response to changes in the benchmark as we vary κ from $1/5$ to 5. The right-hand panel displays a similar ratio for leisure. The graphs for consumption, for the larger values of κ, are in ascending order (10%, 30%,...) starting from the top down. Initially as the benchmark rises, agents at all points in the distribution cut their consumption – they exhibit LRAJ. As the benchmark rises further, say, crosses 1.8, dual response obtains: some agents continue to exhibit LRAJ while others are starting to exhibit LKUJ; eventually for a high-enough benchmark, everyone in this example, except for those at the 90th percentile of the distribution, exhibits LKUJ. The panel on the right-hand side, depicting leisure, provides a look at the flip-side of consumption dual response. Here, the graphs for the larger values of κ are in descending order (90%, 70%,...) starting from the top. Initially, all agents choose to consume more leisure as κ rises; eventually, however, all but the agent at the 90th percentile enjoy less leisure as κ increases.

[24] On the other hand, adopting the economy's median (or other quantile) of the consumption level as the benchmark brings into play aspects of the distribution, other than the mean (such as, the spread of the distribution).

[25] The Abel form $\mathcal{A}\left(c_b\right) = \left(c_b\right)^{-\eta}$ does not permit us to normalize against consumption values associated with no benchmark, i.e., $c_b = 0$.

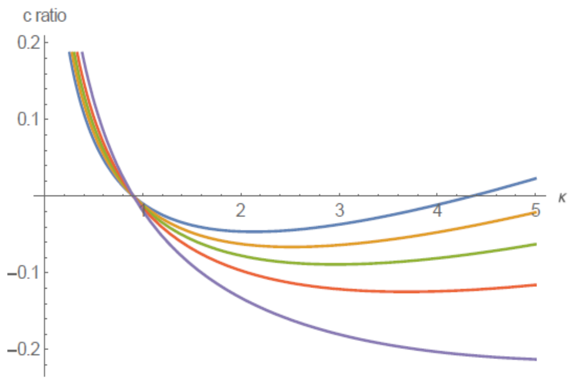

Figure 5a. Consumption relative to baseline

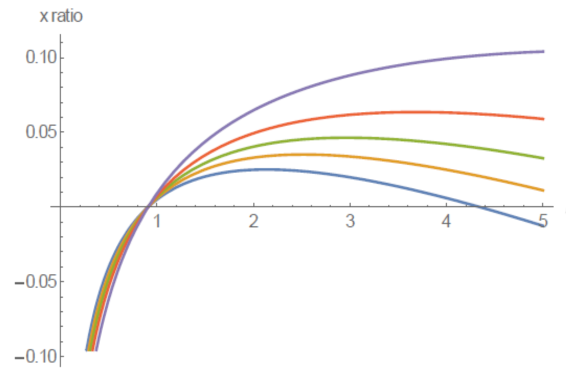

Figure 5b. Leisure relative to baseline.

9 Discussion and concluding remarks

This paper explores the possibility of generating keeping up and running away – from the Joneses behavior among agents who share the same underlying preferences for leisure and consumption relative to a benchmark set by the Joneses. Previous work had utilized with models in which all agents either keep up or they all flee from the Joneses. This is at odds with common observation that some enjoy keeping up on the hedonic treadmill while others choose to stay away. The analysis is novel because a) such fight-or-flight conflict does not arise in existing models of consumption externalities, b) it arises endogenously here, and c) it exposes a deep connection between the fight-or flight-response and wealth-dependent risk aversion of agents and explains the behavior in terms of textbook income/substitution effects. This last point deserves a bit more attention. The existing literature on asset pricing in finance has long studied the importance of agent heterogeneity in generating reasonable dynamics in asset prices. While Chan and Kogan (2002) assume a continuum of investors who differ from each other with respect to the curvature of their utility functions, Barberis and Shleifer (2003) and Hong and Stein (2009) simply assume two different kinds of traders. Our setup *generates* differences in curvature of utility functions by assuming heterogeneity in "innate ability" (not unlike that assumed in Gomes and Michaelides, 2007) and by assuming agents care about their consumption relative to a benchmark.

The notion of a single benchmark for all can be reinterpreted as a threshold, which, upon crossing it leaves one with the feeling of having "made it" in life. Frank (1985) disagrees and argues why per-capita wealth may not be a good wealth reference point for every agent, i.e., why local status may be of more concern to consumers than global status. Presumably, such local status concerns are easily adopted in our setup by allowing for multiple benchmarks. The issue of "choosing one's pond" that occupies Frank's (1985) attention could then be addressed.

Similarly, following Azariadis et. al (2013), in principle one could incorporate benchmarks relating to leisure, and have utility be defined on effective leisure. In such a setting, the feeling of having made it in life would come to those who *get to* enjoy more leisure than their neighbors.[26] Since our notion of dual response is largely driven by relative price effects, it is our conjecture that similar sorts of effects would also arise and generate fight-or-flight in this other setting.

While the paper studies the fight or flight response in the context of consumption-leisure responses, it is easy to imagine that the essentials of the analysis can be applicable in other economic arena, such as the consumption of environmental or health goods. Similarly, as we do in a separate paper, one can study the consequences of such preferences for risk-taking behavior (along the lines explored in Hopkins, 2016) and portfolio choice. Extending the analysis to agents with time-inconsistent preferences and looking for optimal taxation along the lines of Guo and Krause (2015) is a worthy exercise. Finally, a way to extend the analysis in the paper would be to take inspiration from Kubler et.al. (2014) and ask, could we generate preferences that render a good as normal in one part of the income space and inferior in another. These are interesting issues to take up in subsequent research.

Our analysis has been conducted entirely in a static framework. How would dual response manifest itself in a two-period setting? To keep matters simple, suppose agents live for two periods (today and tomorrow) and consume and work in each period (the wage rate is still fixed at 1); more crucially, suppose the benchmark is the same in both periods. To continue with our leading example involving housing, suppose the Joneses build a bigger house today. The agent, in the current setting, has more choices than before. He could respond by working harder today, save more, and build a bigger house tomorrow; he could work harder today, save more, and live on in the same house tomorrow; work harder today, save less, and live on in the same house tomorrow; cut down on work today, save more and build a bigger house tomorrow; and so on. The part about saving less or more will depend, as usual, on his income and the interest rate and whether consumption in each period is a normal good. If wage rates change over time, then the agent will incorporate that into his decision-making and work harder in periods where the wage is higher. (See more on this below.) The upshot is that dual response can easily emerge in this dynamic setting. It is quite possible a person exhibits KUJ behavior in his youth and RAJ behav-

[26] Godwin (2018) pontificates on how the rich and famous have made leisure competitive for the rest of us. "Increasingly, our leisure time is not leisure as our parents or grandparents might have enjoyed it: time away from the productive demands of work for pottering, ambling, collecting, socialising. It is leisure with an imperative, self-imposed or otherwise, to maximise relaxation yield, compete over hobby production and co-opt every activity – exercising, meditating, making Halloween costumes for the kids – into a dynamic of human perfectibility."

ior in his later years. By implication, it seems possible a parent may exhibit KUJ in his lifetime but the child may go the other way in his. A more complete study of the dynamic choices would be a worthy topic for future study.

A key component of the preference structure, at least in the HARA framework, used to obtain dual response is the parameter, φ_c. Most commonly, this parameter (if negative) is linked to a minimum or subsistence level of consumption. A pertinent query, then, is whether or not dual response can prevail in economies with economic growth. A simple extension of our model, one that preserves most of the underlying features of the static model we employed, may help illuminate the discussion. Consider a two-period lived overlapping generations model with each cohort consisting of a continuum of agents with time endowments when young, defined over the interval $[\underline{\xi}, \overline{\xi}]$ by a stationary distribution $\mathcal{G}$, much as we have done here. In the first period of life, agents allocate this time endowment to leisure or work. Labor, along with capital (owned by the old at each date) are used to produce output, along traditional lines, and each factor is paid its marginal product. If we assume agents consume the single consumption good only when old, earnings when young, which is saved, becomes next period's capital, assuming 100% depreciation of the capital stock in the production process. This framework, along with the preferences similar to what is used in the static model, make up a cohesive dynamic model from which we can address the issue of growth and dual response within a narrow sense of the Dupor and Liu (2003) taxonomy.

Note, first, if the production technology displays neoclassical constant returns to scale, the economy converges to a steady state with no sustained growth. Along the growth path, and in the steady state, the economy may experience dual response, as consumption for all agent types converges to a steady state; the size of the consumption minimum φ_c relative to these steady state consumption levels playing a critical rule regarding whether or not the economy exhibits dual response. Moreover, in this environment, with growth in the capital stock, the mass of agents running away should rise while those that keep up falls, converging to a steady state distribution (and it is certainly possible that with growth, the offspring of those agents that keep up at any given date may *eventually* run away).

On the other hand, if the economy exhibits sustained endogenous growth, such as in a modified AK growth model, the issue of the relevance of φ_c and its role in delivering dual response becomes more problematic. Of course, if an economy exhibits sustained, positive growth, wage-growth will be sufficient, at some point, to raise the income of all agents above the threshold level that demarks those that keep up and those that run away. More succinctly, a constant substance-level of consumption φ_c eventually becomes mute in an affluent society. However, if one reinterprets this minimum to be a time-varying minimum standard of living that depends on output (i.e., $\varphi_t = \varphi_c Y_t$), the minimum will grow with wages and the a similar sort of population dichotomy present in the static model may also present itself along the steady state growth path. Álvarez-Peláez and Díaz (2005) employ a similar consumption minimum in their study on wealth dynamics.

It is worth noting, however, that introducing variable factor prices in the model, as in a standard growth environment, presents new channels for the consumption benchmark to impact on individual consumption and work/leisure decisions. For example, different aspiration levels (different values of κ) will affect both aggregate labor and capital, and these in turn feedback on the agents' decisions through changes in factor prices. But these effects of the Joneses' are beyond the scope of the Dupor-Liu description of KUJ/RAJ. KUJ/RAJ within the Dupor-Liu taxonomy is limited to outlining how changes in the consumption benchmark impacts *relative* consumption and the impacts this has on the agent's consumption/leisure/work decision. It does not embody the general equilibrium impact of a change in the benchmark, that is, the secondary effects the benchmark may have on market-determined factor prices. This, in turn, suggests a broader, more comprehensive interpretation of KUJ/RAJ is needed to understand the fuller ramifications that consumption aspirations may have in actual economies. Whether these considerations make a dual response (in a broader sense) more or less prevalent is an open question.

Appendices

A Subtractive benchmarking

Suppose $c^{*i} = f\left(c^i, c_b\right) = c^i - \mathcal{B}\left(c_b\right)$ and $W\left(\cdot, x^i\right)$ is defined for $c^i > \mathcal{B}\left(c_b\right)$. In this case

$$\frac{\partial z^i}{\partial c_b} = \frac{\mathcal{B}'(c_b)\left[W_{21}\left(c^i - \mathcal{B}(c_b), x^i\right) W_1\left(c^i - \mathcal{B}(c_b), x^i\right) - W_{11}\left(c^i - \mathcal{B}(c_b), x^i\right) W_2\left(c^i - \mathcal{B}(c_b), x^i\right)\right]}{\left(W_1\left(c^i - \mathcal{B}(c_b), x^i\right)\right)^2}$$

written without the explicit dependencies:

$$\begin{aligned}\frac{\partial z^i}{\partial c_b} &= \frac{\mathcal{B}'(c_b)\left[W_{21}W_1 - W_{11}W_2\right]}{\left(W_1\right)^2} \\ &= \frac{\mathcal{B}'(c_b)}{c^{*i}W_1}\left(\frac{W_{21}}{W_2} - c^{*i}\frac{W_{11}}{W_1}\right)\end{aligned}$$

Clearly whether this term is non-zero depends on the specific utility function. We can, see however, that if $W_{21} \geq 0$, then Sign$\left(\frac{\partial z^i}{\partial c_b}\right)$ = Sign $\mathcal{B}'(c_b)$ which is the same for all individuals and consumption levels. This naturally leads to the following result: Assume f is of the additive form, $f\left(c^i, c_b\right) = c^i - \mathcal{B}\left(c_b\right)$, then a necessary condition for dual response is $W_{21} < 0$.

B Generalized f-mean

The generalized $f-$mean of two real numbers x and y is defined as $g^{-1}\left(\theta g\left(x\right) + \left(1-\theta\right) g\left(y\right)\right),\ \theta \in (0,1)$ where g is continuously differentiable with a well-defined inverse. Note, if we take f to be any linear function, the $f-$mean is the arithmetic mean, if $g\left(x\right) = \log\left(x\right)$ the $f-$mean is the geometric mean and if $g\left(x\right) = 1/x$ the $f-$mean is the harmonic mean. To show that the aggregator in Assumption 1 can be obtained from the generalized $f-$mean assume that g is homogenous of degree n and let $f\left(c^i, c_b\right) = g^{-1}\left(\theta g\left(c^i\right) + \left(1-\theta\right) g\left(\frac{c^i}{c_b}\right)\right)$. Then, write

$$\begin{aligned}f\left(c^i, c_b\right) &= g^{-1}\left(\theta g\left(c^i\right) + \left(1-\theta\right) g\left(\frac{c^i}{c_b}\right)\right) \\ &= g^{-1}\left(\left(c^i\right)^n\left(\theta g\left(1\right) + \left(1-\theta\right) g\left(\frac{1}{c_b}\right)\right)\right) \\ &= c^i g^{-1}\left(\left(\theta g\left(1\right) + \left(1-\theta\right) g\left(\frac{1}{c_b}\right)\right)\right)\end{aligned}$$

Defining

$$\mathcal{A}\left(c_b\right) \equiv g^{-1}\left(\left(\theta g\left(1\right) + \left(1-\theta\right) g\left(\frac{1}{c_b}\right)\right)\right),$$

we have $f\left(c^i, c_b\right) = c^i\,\mathcal{A}\left(c_b\right)$. Finally, note g is monotone because it is invertible:

$$\mathcal{A}'\left(c_b\right) = \frac{-\frac{1}{c_b^2}\left(1-\theta\right) g'\left(\frac{1}{c_b}\right)}{g'\left(\left(\theta g\left(1\right) + \left(1-\theta\right) g\left(\frac{1}{c_b}\right)\right)\right)} < 0$$

C Examples

C.1 Non-separable preferences

Example 3 *(No dual response) Suppose* $W\left(c^{*i},x^{i}\right) = \frac{\left(c^{*i}\right)^{1-\sigma}\left(1-a\left(1+e^{i}-x^{i}\right)\right)^{d}-1}{1-\sigma}$ *as in King et. al (1998). Then,* $\sigma\left(c^{*i}\right)=\sigma$, $W_{21}\neq 0$ *(non-separable) but* $\frac{c^{*i}W_{21}}{W_2}=1-\sigma$; *in this case,* $\frac{\partial z^i}{\partial c_b}=0\,\forall i$.

Example 4 *(No dual response) Suppose* $W\left(c^{*i},x^{i}\right)=\frac{1}{1-\gamma}\left[c^{*i}-\psi\frac{\left(1-x^{i}\right)^{1+\theta}}{1+\theta}\right]^{1-\gamma}$, *the GHH preferences defined in Greenwood et al. 1998. Then* $W_{21}\neq 0$ *(non-separable) and* $\sigma\left(c^{*i}\right)=\frac{c^{*i}\gamma(\theta+1)}{\left(c^{*i}(1+\theta)-\psi(1-x^{i})^{\theta+1}\right)}$ *but* $\frac{\partial z^i}{\partial c_b}=1\,\forall i$.

C.2 Separable preferences

Example 5 *(No dual response) Suppose* $W\left(c^{*i},x^{i}\right)=\frac{\left(c^{*i}\right)^{1-\sigma}-1}{1-\sigma}-\psi\frac{\left(1+e^{i}-x^{i}\right)^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$ *as in MaCurdy (1981). Then,* $W_{21}=0$ *and* $\sigma\left(c^{*i}\right)=\sigma\,\forall i$. *In this case,* $\frac{\partial z^i}{\partial c_b}=1-\sigma\,\forall i$.

Example 6 *(No dual response) Consider the general C.E.S class of additively-separable preferences:*

$$W\left(c^{i*},x^{i}\right)=\Lambda\left(\lambda\left(c^{i*}\right)^{\rho}+(1-\lambda)\left(x^{i}\right)^{\rho}\right)^{k/\rho}$$

where $\Lambda>0$ *is a constant,* $0<\lambda<1$, $\rho\leq 1$ *and* $0<k\leq 1$. *In this case, the term in parenthesis on the r.h.s of (9) is*

$$1-\sigma\left(c^{*i}\right)-\frac{c^{*i}W_{21}}{W_2}=\rho.$$

Clearly $\frac{\partial z^i}{\partial c_b}\neq 0$ *; the Joneses always affect agent* i*'s decisions. Here* $W_1=k\lambda\left(c^{i*}\right)^{\rho-1}W^{k/\rho-1}$, $W_{11}=\lambda\rho\left(k/\rho-1\right)\left(c^{i*}\right)^{\rho-1}W^{-1}W_1+(\rho-1)\left(c^{i*}\right)^{-1}W_1$, $W_2=k\left(1-\lambda\right)\left(x^{i}\right)^{\rho-1}W^{k/\rho-1}$, *and* $W_{21}=\rho k\lambda\left(1-\lambda\right)\left(k/\rho-1\right)$ *Using (4), we get* $f_1f_2/f_{12}=c^{i*}$, *and so the coefficient of relative risk aversion is*

$$\sigma\left(c^{*i}\right)=-c^{i*}W_{11}/W_1=-\rho\lambda\left(k/\rho-1\right)\left(c^{i*}\right)^{\rho}W^{-1}-(\rho-1)\,. \tag{22}$$

Additionally, $c^{*i}W_{21}/W_2=\rho\lambda\left(k/\rho-1\right)\left(c^{i*}\right)^{\rho}W^{-1}$. *Substituting these expressions into (9) yields* $1-\sigma\left(c^{*i}\right)-\frac{c^{*i}W_{21}}{W_2}=\rho$. *These preferences exhibit G-KUJ if* $\rho<0$ *and G-RAJ if* $\rho>0$.

Example 7 *(Dual response) Suppose* $W\left(c^{*i},x^{i}\right)=\frac{\left[\left(c^{*i}\right)^{\phi}\left(x^{i}\right)^{1-\phi}\right]^{1-\gamma}}{1-\gamma}$ *where* $\gamma>(<)\,1$ *implies* c^{*i}

and x^i are substitutes (complements). Then, $\sigma\left(c^{*i}\right)=\gamma\phi-\phi+1\;\forall i$ and

$$\frac{\partial z^i}{\partial c_b}\;\gtrless\;0\Longleftrightarrow\left(\frac{(1-\phi)}{\mathcal{A}\left(c_b\right)\phi+(1-\phi)}\left(1+e^i\right)\right)^{-\phi}\gtrless 1 \text{ for } \gamma>1$$
$$\frac{\partial z^i}{\partial c_b}\;\gtrless\;0\Longleftrightarrow\left[\left(\frac{(1-\phi)}{\mathcal{A}\left(c_b\right)\phi+(1-\phi)}\left(1+e^i\right)\right)^{-\phi}-1\right]\lesseqgtr 1 \text{ for } \gamma<1$$

meaning $\frac{\partial z^i}{\partial c_b}$ may change sign depending on the level of e^i.

Example 8 *(Dual response) Let*

$$W^i\left(c^{*i},x^i\right)=-e^{-\lambda c^{*i}}-\theta e^{-\gamma x^i};\;\theta,\lambda,\gamma>0. \tag{23}$$

In this case, $z^i=-\frac{\theta\gamma}{\lambda A(c_b)}e^{\lambda c^i\mathcal{A}(c_b)-\gamma x^i}$ and

$$\frac{\partial z^i}{\partial c_b}=e^{\lambda c^i\mathcal{A}(c_b)-\gamma x^i}\frac{\theta\gamma\mathcal{A}'\left(c_b\right)}{\lambda\mathcal{A}\left(c_b\right)}\left[1-\lambda\mathcal{A}\left(c_b\right)c^i\right], \tag{24}$$

with

$$\frac{\partial z^i}{\partial c_b}\gtreqless 0\Longleftrightarrow\tilde{c}_b\equiv\frac{1}{\lambda\mathcal{A}\left(c_b\right)}\lesseqgtr c^i.$$

Given (2), and assuming interior solutions for c^i and x^i, the agent's consumption decision satisfies

$$\lambda e^{-\lambda c^i\mathcal{A}(c_b)}\mathcal{A}\left(c_b\right)=\theta\gamma e^{-\gamma\left[1+e^i-c^i\right]} \tag{25}$$

Define $\mathcal{A}_b\equiv\mathcal{A}\left(c_b\right)$ and $m_b\equiv\ln\lambda\mathcal{A}_b/\gamma\theta$. The optimal choices for consumption are linear in e^i and are given by

$$\hat{c}^i=\frac{m_b+\gamma\left(1+e^i\right)}{\lambda\mathcal{A}_b+\gamma} \tag{26}$$

and

$$\hat{x}^i=\frac{-m_b+\lambda\mathcal{A}_b\left(1+e^i\right)}{\lambda\mathcal{A}_b+\gamma}, \tag{27}$$

where the following restriction is assumed to hold for each i: $\left(1+e^i\right)>\max\left\{-m_b/\gamma,m_b/\lambda\mathcal{A}_b\right\}$. From (24) and (26), it follows

$$\frac{\partial z^i}{\partial c_b}=\begin{cases}<0 \text{ for } 1+e^i<1+\tilde{e}\equiv\frac{\lambda\mathcal{A}_b+\gamma}{\lambda\gamma}-\frac{m_b}{\gamma}\;:\;\text{L-RAJ}\\ \geq 0 \text{ for } 1+e^i\geq 1+\tilde{e}\equiv\frac{\lambda\mathcal{A}_b+\gamma}{\lambda\gamma}-\frac{m_b}{\gamma}\;:\;\text{L-KUJ}\end{cases}. \tag{28}$$

Notice, in this case, regardless of values of preference parameters, all agents with low-enough time endowments will run away from the Joneses, while those with large-enough time endowments *keep up.*

A final example of how the sign of $\partial z^i/\partial c_b$ can vary across individuals in the population:

Example 9 *(Dual response) Suppose the population is split in two otherwise identical groups – e is same for all i – except one group has preferences defined by $W\left(c^{*i}, x^i\right) = \frac{\left(c^{1*}\right)^{1-\gamma_1}}{1-\gamma_1} + \frac{\left(x^1\right)^{1-\gamma_1}}{1-\gamma_1}$ and the other $W\left(c^{*i}, x^i\right) = \frac{\left(c^{1*}\right)^{1-\gamma_2}}{1-\gamma_2} + \frac{\left(x^1\right)^{1-\gamma_2}}{1-\gamma_2}$ where $\gamma_1 < 1$ and $\gamma_2 \geq 1$. Then, $\partial z^i/\partial c_b$ has a different sign across the two groups.*

While this example may seem "contrived" it is nonetheless important because this is the channel large parts of the asset-pricing literature use to generate variation in risk preferences.

D Multiplicatively separable preferences

Assume $W\left(c^{i*}, x^i\right) = h\left(c^{i*}\right) v\left(x^i\right)$, where $h\left(tc^{i*}\right) = t^r \cdot h\left(c^{i*}\right)$, and v is increasing and concave. Then, we show $\frac{\partial z^i}{\partial c_b} = 0$. To see this, note in this case,

$$z^i = -\frac{W_2\left(f\left(c^i, c_b\right), x^i\right)}{W_1\left(f\left(c^i, c_b\right), x^i\right) \cdot f_1\left(c^i, c_b\right)} = -\frac{v'\left(x^i\right)}{v\left(x^i\right)} \frac{h\left(c^{i*}\right)}{h'\left(c^{i*}\right) \cdot f_1\left(c^i, c_b\right)}.$$

Calculating the derivative of z^i, we get:

$$\begin{aligned}
\frac{\partial z^i}{\partial c_b} &= -\frac{v'\left(x^i\right)}{v\left(x^i\right)} \frac{h'\left(c^{i*}\right)^2 \cdot f_1\left(c^i, c_b\right) \cdot f_2\left(c^i, c_b\right) - \left[h''\left(c^{i*}\right) \cdot f_1\left(c^i, c_b\right) \cdot f_2\left(c^i, c_b\right) + h'\left(c^{i*}\right) \cdot f_{12}\left(c^i, c_b\right)\right] h\left(c^{i*}\right)}{\left(h'\left(c^{i*}\right) \cdot f_1\left(c^i, c_b\right)\right)^2} \\
&= -\frac{v'\left(x^i\right) \mathcal{A}'}{v\left(x^i\right) \mathcal{A}^2} \left(c^{i*} - \frac{h''\left(c^{i*}\right)}{h'\left(c^{i*}\right)} \frac{h\left(c^{i*}\right)}{h'\left(c^{i*}\right)} \cdot c^{i*} - \frac{h\left(c^{i*}\right)}{h'\left(c^{i*}\right)}\right)
\end{aligned}$$

Using Euler's Theorem, we have the following properties for h:

$$h'\left(c^{i*}\right) = r\frac{h\left(c^{i*}\right)}{c^{i*}}, \qquad h''\left(c^{i*}\right) = (r-1)\frac{h'\left(c^{i*}\right)}{c^{i*}}$$

And finally:

$$\begin{aligned}
\frac{\partial z^i}{\partial c_b} &= -\frac{v'\left(x^i\right) \mathcal{A}'}{v\left(x^i\right) \mathcal{A}^2} \left(c^{i*} - \frac{(r-1)\frac{h'\left(c^{i*}\right)}{c^{i*}}}{h'\left(c^{i*}\right)} \frac{h\left(c^{i*}\right)}{r\frac{h\left(c^{i*}\right)}{c^{i*}}} \cdot c^{i*} - \frac{h\left(c^{i*}\right)}{r\frac{h\left(c^{i*}\right)}{c^{i*}}}\right) \\
&= -\frac{v'\left(x^i\right) \mathcal{A}'}{v\left(x^i\right) \mathcal{A}^2} c^{i*} \left(1 - \frac{r-1}{r} - \frac{1}{r}\right) = 0
\end{aligned}$$

E Reconciling two notions of KUJ/RAJ

At the optimal, $W_1\left(c^{*i}, 1+e^i-c^i\right) f_1 = W_2\left(c^{*i}, 1+e^i-c^i\right)$. Differentiating with respect to c_b, we have:

$$\begin{aligned}&\left(W_{11}f_1 - W_{12}\right) f_1 \partial \hat{c}^i/\partial c_b + W_{11}f_1 f_2 + W_1 f_{11}\partial \hat{c}^i/\partial c_b + W_1 f_{12} \\ &= W_{21}f_1 \partial \hat{c}^i/\partial c_b + W_{21}f_2 - W_{22}\partial \hat{c}^i/\partial c_b\end{aligned}$$

or

$$\partial \hat{c}^i/\partial c_b = -\frac{W_{11}f_1 f_2 + W_1 f_{12} - W_{21}f_2}{\left(W_{11}f_1 - W_{12}\right) f_1 + W_1 f_{11} - W_{21}f_1 + W_{22}} \tag{29}$$

Since $W_{12} = W_{21}$, the denominator of (29) is simply the second-order condition of the agent's optimization problem, which we assume holds and is negative at $\left(c^{*i}, x^i\right) = \left(\hat{c}^{*i}, \hat{x}^i\right)$. Hence, the sign of $\partial \hat{c}^i/\partial c_b$ is the same as the sign of the term $W_{11}f_1 f_2 + W_1 f_{12} - W_{21}f_2$.

From (8), $\partial z^i/\partial c_b = \frac{f_2}{W_1 f_1}\left(-W_{21} + W_2 \frac{W_{11}}{W_1} + W_2 \frac{f_{12}}{f_2 f_1}\right)$. Making use of the fact that $W_1 f_1 = W_2$ at the optimum, we can rewrite this as:

$$\begin{aligned}\partial z^i/\partial c_b &= \frac{f_2}{W_1 f_1}\left(-W_{21} + f_1 W_{11} + \frac{W_1 f_{12}}{f_2}\right) \\ &= \frac{1}{W_1 f_1}\left(-W_{21}f_2 + f_1 f_2 W_{11} + W_1 f_{12}\right).\end{aligned}$$

Since $W_1 f_1 > 0$, the sign of $\partial z^i/\partial c_b$ at the optimum is the same sign as $\partial \hat{c}^i/\partial c_b$. From the budget constraint, $\hat{x}^i = 1 + e^i - \hat{c}^i$; it follows that $\partial z^i/\partial c_b$ and $\partial \hat{x}^i/\partial c_b$ have opposite signs.

F Stone-Geary utility and dual response

A fairly tractable form of preferences that generate dual response is the Stone-Geary kind:

$$W\left(c^{*i}, x^i\right) = \left(c^{*i} - \varphi\right)^{\beta} + \left(x^i\right)^{1-\beta}, \ \beta \in (0,1), \ \varphi > 0 \tag{30}$$

where $\left(c^{*i}, x\right)$ have to satisfy $c^{*i} - \varphi > 0$ and $x^i > 0$. As discussed in Appendix 3.5 of Bertola et. al (2014), it is sometimes useful to think of φ_c as representing some sort of a subsistence (minimum) level of consumption. If $\varphi = 0$, then (30) is homothetic in (c^*, x); otherwise not. For future use, note

$$\begin{aligned}W_1 &= \beta\left(c^{*i} - \varphi\right)^{\beta-1}; \quad W_2 = (1-\beta)\left(x^i\right)^{-\beta} \\ W_{11} &= \beta(\beta-1)\left(c^{*i} - \varphi\right)^{\beta-2}; \quad W_{22} = \beta(\beta-1)\left(x^i\right)^{\beta-2}; \quad W_{21} = 0\end{aligned}$$

and

$$\sigma\left(c^{*i}\right) \equiv -\frac{c^{*i}W_{11}}{W_1} = (1-\beta)\frac{c^{*i}}{c^{*i} - \varphi},$$

$$\frac{\partial z^i}{\partial c_b} = \frac{c^i \mathcal{A}'(c_b) W_2}{W_1 \mathcal{A} c^{*i}} \left(1 - \sigma\left(c^{*i}\right) - \frac{c^{*i} W_{21}}{W_2}\right) = \frac{c^{*i}\beta - \varphi}{c^{*i} - \varphi}.$$

Evidently, $\frac{\partial z^i}{\partial c_b}$ can change sign. Since we need $c^{*i} - \varphi > 0$, it follows dual response requires

$$\begin{aligned} \frac{\partial z^i}{\partial c_b} &< 0 \Longleftrightarrow \varphi < c^{*i} < \frac{\varphi}{\beta} \\ \frac{\partial z^i}{\partial c_b} &> 0 \Longleftrightarrow c^{*i} > \frac{\varphi}{\beta} > \varphi_c \end{aligned}$$

Using the first order condition $W_2 = \mathcal{A}(c_b) W_1$, one can derive

$$c^{*i} = \frac{\varphi + (\mathcal{A}(c_b))^{\frac{1}{1-\beta}}\left(1+e^i\right)}{\left(1 + (\mathcal{A}(c_b))^{\frac{\beta}{1-\beta}}\right)},\ \hat{c}^i = \frac{\varphi + (\mathcal{A}(c_b))^{\frac{1}{1-\beta}}\left(1+e^i\right)}{\mathcal{A}(c_b) + (\mathcal{A}(c_b))^{\frac{1}{1-\beta}}} \text{ and } x^i = \frac{\left(1+e^i\right)\mathcal{A}(c_b) - \varphi}{\mathcal{A}(c_b) + (\mathcal{A}(c_b))^{\frac{1}{1-\beta}}}.$$

For future use, write

$$\begin{aligned} \hat{c}^i &= a + b\left(1+e^i\right) \qquad (31) \\ a &\equiv \frac{\varphi}{\mathcal{A}(c_b) + (\mathcal{A}(c_b))^{\frac{1}{1-\beta}}} > 0 \\ b &\equiv \frac{(\mathcal{A}(c_b))^{\frac{1}{1-\beta}}}{\mathcal{A}(c_b) + (\mathcal{A}(c_b))^{\frac{1}{1-\beta}}} = \frac{(\mathcal{A}(c_b))^{\frac{\beta}{1-\beta}}}{1 + (\mathcal{A}(c_b))^{\frac{\beta}{1-\beta}}} > 0 \end{aligned}$$

Notice these decision rules are linear in e^i.

From (31), one can show

$$\frac{\partial a}{\partial c_b} = -a \frac{1 + \frac{1}{1-\beta}(\mathcal{A}(c_b))^{\frac{\beta}{1-\beta}}}{\mathcal{A}(c_b) + (\mathcal{A}(c_b))^{\frac{1}{1-\beta}}} \frac{\partial \mathcal{A}(c_b)}{\partial c_b} > 0; \quad \frac{\partial b}{\partial c_b} = \frac{\beta}{1-\beta} \frac{b}{\mathcal{A}(c_b)\left(1 + (\mathcal{A}(c_b))^{\frac{\beta}{1-\beta}}\right)} \frac{\partial \mathcal{A}(c_b)}{\partial c_b} < 0 \qquad (32)$$

since $\frac{\partial \mathcal{A}(c_b)}{\partial c_b} < 0$. Using (31), it follows that $V\left(y^i\right) = b^2 V\left(1+e^i\right)$. Since the sign of $\partial b / \partial c_b < 0$, it follows that $V\left(y^i\right)$ falls with an increase in the benchmark. How does the Gini of income ϕ_y respond? One can check, using (32) and for exogenous c_b,

$$\text{sign} \frac{\partial}{\partial c_b}\left(\phi_y\right) = \text{ sign } \frac{\partial \mathcal{A}(c_b)}{\partial c_b} < 0$$

which is negative. In this case, measured income inequality is reduced relative to fundamental inequality as the exogenous benchmark rises.

G Proof of Proposition 3

First note, δ_h provides a valid consumption level when

$$\begin{aligned}\varphi_c + \frac{\alpha_c A \delta_h}{1-\gamma} &> 0 \Leftrightarrow \varphi_c + \frac{\alpha_c A \frac{\varphi_c}{\frac{\gamma}{\gamma-1}\alpha_c A}}{1-\gamma} > 0 \Leftrightarrow \\ \varphi_c - \frac{\varphi_c}{\gamma} &> 0 \Leftrightarrow \varphi_c\left(1-\frac{1}{\gamma}\right) > 0.\end{aligned}$$

This is satisfied by Assumption (2). Now proceed to take the derivative of z^i with respect to c_b. To begin with,

$$\begin{aligned}&\frac{\partial k}{\partial c_b} = -\frac{1}{\gamma-1}\left(\frac{\theta\alpha_x}{\mathcal{A}\alpha_c}\right)^{\frac{1}{\gamma-1}-1}\frac{\theta\alpha_x}{\mathcal{A}^2\alpha_c}\mathcal{A}' \\ &= \frac{1}{1-\gamma}\frac{\mathcal{A}'}{\mathcal{A}}k\end{aligned} \tag{33}$$

then using

$$z^i = -\left(\frac{k\left(\varphi_x + \frac{\alpha_x x^i}{1-\gamma}\right)}{\varphi_c + \frac{\alpha_c c^{i*}}{1-\gamma}}\right)^{\gamma-1}$$

we have

$$\begin{aligned}\frac{\partial z^i}{\partial c_b} &= -(\gamma-1)\left(\frac{k\left(\varphi_x + \frac{\alpha_x x^i}{1-\gamma}\right)}{\varphi_c + \frac{\alpha_c c^{i*}}{1-\gamma}}\right)^{\gamma-2}\frac{\frac{\partial k}{\partial c_b}\left(\varphi_x + \frac{\alpha_x x^i}{1-\gamma}\right)\left(\varphi_c + \frac{\alpha_c c^{i*}}{1-\gamma}\right) - k\left(\varphi_x + \frac{\alpha_x x^i}{1-\gamma}\right)\frac{\alpha_c}{1-\gamma}\frac{\partial c^{i*}}{\partial c_b}}{\left(\varphi_c + \frac{\alpha_c c^{i*}}{1-\gamma}\right)^2} \\ &= -(\gamma-1)\left(\frac{(k)^{\gamma-2}\left(\varphi_x + \frac{\alpha_x x^i}{1-\gamma}\right)^{\gamma-1}}{\left(\varphi_c + \frac{\alpha_c c^{i*}}{1-\gamma}\right)^{\gamma}}\right)\left[\frac{1}{1-\gamma}\frac{\mathcal{A}'}{\mathcal{A}}k\left(\varphi_c + \frac{\alpha_c c^{i*}}{1-\gamma}\right) - k\frac{\alpha_c}{1-\gamma}c^i\mathcal{A}'\right] \\ &= \left(\frac{(k)^{\gamma-1}\left(\varphi_x + \frac{\alpha_x x^i}{1-\gamma}\right)^{\gamma-1}}{\left(\varphi_c + \frac{\alpha_c c^{i*}}{1-\gamma}\right)^{\gamma}}\right)\left[\varphi_c - \frac{\gamma}{\gamma-1}\alpha_c c^{i*}\right]\frac{\mathcal{A}'}{\mathcal{A}}\end{aligned}$$

Since the first term is positive and $\frac{\mathcal{A}'}{\mathcal{A}} < 0$, we have $\frac{\partial z^i}{\partial c_b}$ has the same sign as $-\varphi_c + \frac{\gamma}{\gamma-1}\alpha_c c^{i*}$, and therefore

$$\left\{\begin{array}{ll}\frac{\partial z^i}{\partial c_b} \geq 0 & \text{if } -\varphi_c + \frac{\gamma}{\gamma-1}\alpha_c c^{i*} \geq 0 \\ \frac{\partial z^i}{\partial c_b} \leq 0 & \text{if } -\varphi_c + \frac{\gamma}{\gamma-1}\alpha_c c^{i*} \leq 0\end{array}\right\} \Leftrightarrow \left\{\begin{array}{ll}\frac{\partial z^i}{\partial c_b} \geq 0 & \text{if } \frac{\gamma}{\gamma-1}\alpha_c c^{i*} \geq \varphi_c \\ \frac{\partial z^i}{\partial c_b} \leq 0 & \text{if } \frac{\gamma}{\gamma-1}\alpha_c c^{i*} \leq \varphi_c\end{array}\right\} \Leftrightarrow$$

$$\left\{\begin{array}{ll}\frac{\partial z^i}{\partial c_b} \geq 0 & \begin{array}{l}\text{if } \gamma\alpha_c c^{i*} \geq \varphi_c(\gamma-1) \text{ and } \gamma \geq 1 \\ \text{if } \gamma\alpha_c c^{i*} \leq \varphi_c(\gamma-1) \text{ and } \gamma \leq 1\end{array} \\ \frac{\partial z^i}{\partial c_b} \leq 0 & \begin{array}{l}\text{if } \gamma\alpha_c c^{i*} \leq \varphi_c(\gamma-1) \text{ and } \gamma \geq 1 \\ \text{if } \gamma\alpha_c c^{i*} \geq \varphi_c(\gamma-1) \text{ and } \gamma \leq 1\end{array}\end{array}\right\} \Leftrightarrow \left\{\begin{array}{ll}\frac{\partial z^i}{\partial c_b} \geq 0 & \begin{array}{l}\text{if } c^{i*} \geq \frac{\varphi_c(\gamma-1)}{\gamma\alpha_c} \text{ and } \gamma \geq 1 \\ \text{if } c^{i*} \leq \frac{\varphi_c(\gamma-1)}{\gamma\alpha_c} \text{ and } \gamma \leq 1\end{array} \\ \frac{\partial z^i}{\partial c_b} \leq 0 & \begin{array}{l}\text{if } c^{i*} \leq \frac{\varphi_c(\gamma-1)}{\gamma\alpha_c} \text{ and } \gamma \geq 1 \\ \text{if } c^{i*} \geq \frac{\varphi_c(\gamma-1)}{\gamma\alpha_c} \text{ and } \gamma \leq 1\end{array}\end{array}\right\}.$$

This proves the result.

H Proof of Eq. (??)

Let y be a continuous random variable with cumulative probability distribution, $F(t) \equiv \Pr\{y \leq t\}$, and support $[\underline{y}, \overline{y}]$ with $0 \leq \underline{y}$ ($\overline{y}$ can be ∞). The Gini is defined as

$$\phi_y = \frac{1}{\mu_y} \int_{\underline{y}}^{\overline{y}} F(t)(1 - F(t))\, dt$$

where μ_y is the mean of y. In our case, $y \equiv a + b(1+e)$. So $\underline{y} = a + b\left(1 + \underline{\xi}\right)$ and $\overline{y} = a + b\left(1 + \overline{\xi}\right)$. With a change of variables, $F(t) \equiv \Pr\{a + b(1+e) \leq t\} = \Pr\left\{(1+e) \leq \frac{t-a}{b}\right\} = \mathcal{G}\left(\frac{t-a}{b}\right)$, or $\mathcal{G}(z)$, where $\mathcal{G}$ is the CDF for the variable $(1+e)$ and $z \equiv \frac{t-a}{b}$. With this change of variables, $bdz = dt$ and when $t = \underline{y}$, $z = 1 + \underline{\xi}$; likewise for $\overline{y}$. Hence,

$$\phi_y = \frac{b}{\mu_y} \int_{1+\underline{\xi}}^{1+\overline{\xi}} \mathcal{G}(z)(1 - \mathcal{G}(z))\, dz$$

By definition,

$$\phi_{1+e} = \frac{1}{1+\bar{e}} \int_{1+\underline{\xi}}^{1+\overline{\xi}} \mathcal{G}(z)(1 - \mathcal{G}(z))\, dz$$

So,

$$\phi_y = \frac{b(1+\bar{e})\,\phi_{1+e}}{\mu_y}$$

Since $\mu_y = a + b(1+\bar{e})$, $\phi_y = \frac{b(1+\bar{e})}{a+b(1+\bar{e})}\phi_{1+e}$.

I Proof of Proposition 4

Recall $\frac{\phi_y}{\phi_{1+e}} = \frac{b(1+\bar{e})}{a+b(1+\bar{e})}$. Then we have

$$\begin{aligned}
&\frac{\partial}{\partial c_b}\left(\frac{\phi_y}{\phi_{1+e}}\right) = \frac{\partial}{\partial c_b}\frac{b(1+\bar{e})}{a+b(1+\bar{e})} \\
&= \frac{(1+\bar{e})}{[a+b(1+\bar{e})]^2}\left\{[a+b(1+\bar{e})]\frac{\partial b}{\partial c_b} - \frac{\partial a}{\partial c_b}b - \frac{\partial b}{\partial c_b}(1+\bar{e})\,b\right\} \\
&= \frac{(1+\bar{e})}{[a+b(1+\bar{e})]^2}\left\{a\frac{\partial b}{\partial c_b} - \frac{\partial a}{\partial c_b}b\right\}
\end{aligned}$$

Now, from (33) we have $\frac{\partial k}{\partial c_b} = \frac{1}{1-\gamma}\frac{\mathcal{A}'}{\mathcal{A}}k$, and we can calculate $\frac{\partial a}{\partial c_b}$ and $\frac{\partial b}{\partial c_b}$

$$\frac{\partial a}{\partial c_b} = \frac{(1-\gamma)\,\varphi_x \frac{\partial k}{\partial c_b}\left(\alpha_c\mathcal{A} + k\alpha_x\right) - \left(\alpha_c\mathcal{A}' + \alpha_x\frac{\partial k}{\partial c_b}\right)(1-\gamma)\left(k\varphi_x - \varphi_c\right)}{\left(\alpha_c\mathcal{A} + k\alpha_x\right)^2}$$
$$= \frac{\mathcal{A}'}{\mathcal{A}\left(\alpha_c\mathcal{A} + k\alpha_x\right)^2}\left[\alpha_c\varphi_x\mathcal{A}k + \alpha_x\varphi_x k - (1-\gamma)\,\varphi_x\alpha_c\mathcal{A}k - \varphi_x\alpha_x k + (1-\gamma)\,\varphi_c\alpha_c\mathcal{A} + \varphi_c\alpha_x k\right]$$
$$= \frac{\mathcal{A}'\left[\gamma\alpha_c\mathcal{A}\left(\varphi_x k - \varphi_c\right) + \varphi_c\left(\alpha_c\mathcal{A} + \alpha_x k\right)\right]}{A\left(\alpha_c\mathcal{A} + k\alpha_x\right)^2}$$

$$\frac{\partial b}{\partial c_b} = \frac{\alpha_x\frac{\partial k}{\partial c_b}\left(\alpha_c\mathcal{A} + k\alpha_x\right) - \left(\alpha_c\mathcal{A}' + \alpha_x\frac{\partial k}{\partial c_b}\right)k\alpha_x}{\left(\alpha_c\mathcal{A} + k\alpha_x\right)^2}$$
$$= \frac{\gamma}{1-\gamma}\frac{\mathcal{A}'\alpha_c\alpha_x k}{\left(\alpha_c\mathcal{A} + k\alpha_x\right)^2}$$

We can now find $\left(a\frac{\partial b}{\partial c_b} - \frac{\partial a}{\partial c_b}b\right)$:

$$a\frac{\partial b}{\partial c_b} - \frac{\partial a}{\partial c_b}b = \frac{\left(k\varphi_x - \varphi_c\right)(1-\gamma)}{\left(\alpha_c\mathcal{A} + k\alpha_x\right)}\frac{\gamma}{1-\gamma}\frac{\mathcal{A}'\alpha_c\alpha_x k}{\left(\alpha_c\mathcal{A} + k\alpha_x\right)^2}$$
$$-\frac{k\alpha_x}{\left(\alpha_c\mathcal{A} + k\alpha_x\right)}\frac{\mathcal{A}'\left[\gamma\alpha_c\left(\varphi_x k - \varphi_c\right)\mathcal{A} + \varphi_c\left(\alpha_c\mathcal{A} + \alpha_x k\right)\right]}{A\left(\alpha_c\mathcal{A} + k\alpha_x\right)^2} = -\frac{\varphi_c\alpha_x k}{\mathcal{A}\left(\alpha_c\mathcal{A} + k\alpha_x\right)^2}\mathcal{A}'$$

Finally, we obtain:

$$\frac{\partial}{\partial c_b}\left(\frac{\phi_y}{\phi_{1+e}}\right) = \frac{(1+\bar{e})}{\left[a + b\,(1+\bar{e})\right]^2}\left\{a\frac{\partial b}{\partial c_b} - \frac{\partial a}{\partial c_b}b\right\} = -\frac{(1+\bar{e})}{\left[a + b\,(1+\bar{e})\right]^2}\left\{\frac{\varphi_c\alpha_x k}{\mathcal{A}\left(\alpha_c\mathcal{A} + k\alpha_x\right)^2}\mathcal{A}'\left(c_b\right)\right\}$$

It is clear from this expression that if $\varphi_c = 0$, then $\frac{\partial}{\partial c_b}\left(\frac{\phi_y}{\phi_{1+e}}\right) = 0$.

References

[1] Abel, A.B. (1990), Asset Prices under Habit Formation and Catching Up with the Joneses, American Economic Review, 80, 38-42.

[2] Abel, A.B. (2005), Optimal Taxation when Consumers Have Endogenous Benchmark Levels of Consumption, Review of Economic Studies 75, 21-42.

[3] Allen, Jeff and Chakraborty, Shankha. (2018) Aspirations, Health and the Cost of Inequality, Journal of Economic Dynamics and Control, 86, 144-64

[4] Alvarez-Cuadrado, F., & Van Long, N. (2011). The relative income hypothesis. Journal of Economic Dynamics and Control, 35(9), 1489-1501.

[5] Álvarez-Peláez, María J., Díaz, Antonia. (2005) Minimum consumption and transitional dynamics in wealth distribution. Journal of Monetary Economics 52(3), 633-667

[6] Alvarez-Cuadrado, F., Casado, J. M., & Labeaga, J. M. (2016). Envy and habits: panel data estimates of interdependent preferences. Oxford Bulletin of Economics and Statistics, 78(4), 443-469.

[7] Ampudia, M., & Ehrmann, M. (2017). Macroeconomic experiences and risk taking of euro area households. European Economic Review, 91, 146-156.

[8] Aronsson, T., O. Johansson-Stenman (2010) Positional concerns in an OLG model: optimal labor and capital income taxation, International Economic Review, 51, 1071–1095

[9] Azariadis, C., Chen, B. L., Lu, C. H., & Wang, Y. C. (2013). A two-sector model of endogenous growth with leisure externalities. Journal of Economic Theory, 148(2), 843-857.

[10] Barberis, N. and A. Shleifer (2003). Style investing. Journal of Financial Economics, 68, 161-199.

[11] Barnett, R. C., Bhattacharya, J., and Bunzel, H. (2010) Choosing to keep up with the Joneses and income inequality, Economic Theory, 45(3), 469-496

[12] Bertola, G., Foellmi, R., & Zweimüller, J. (2014). *Income distribution in macroeconomic models.* Princeton University Press.

[13] Bertrand, M, and A. Morse. (2016) Trickle-down consumption. Review of Economics and Statistics 98(5), 863-879.

[14] Bishnu, M. (2013). 'Linking consumption externalities with optimal accumulation of human and physical capital and intergenerational transfers', Journal of Economic Theory, 148(2), 720-742.

[15] Bucciol, Alessandro, and Raffaele Miniaci. (2011) Household portfolios and implicit risk preference. Review of Economics and Statistics 93(4), 1235-1250.

[16] Bucciol, Alessandro, Raffaele Miniaci, and Sergio Pastorello. (2017) Return expectations and risk aversion heterogeneity in household portfolios. Journal of Empirical Finance 40, 201-219.

[17] Chan, Y. L., and L. Kogan (2002). Catching Up with the Joneses: Heterogeneous Preferences and the Dynamics of Asset Prices. Journal of Political Economy 110:1255–85.

[18] Chen, B. L., Hsu, Y. S., and Mino, K. (2015). Welfare implications and equilibrium indeterminacy in a two-sector growth model with consumption externalities. Macroeconomic Dynamics, 19(03), 535-577.

[19] Chugh, Sanjay K.(2008) Relative consumption benchmarks. Economics Letters 100(2), 204-207.

[20] Clark, Andrew E., and Andrew J. Oswald. (1998). Comparison-concave utility and following behaviour in social and economic settings. Journal of Public Economics 70(1): 133-155.

[21] Croix, D. de la and Michel, P, 1999. Optimal growth when tastes are inherited, Journal of Economic Dynamics and Control, 23(4), 519-537

[22] Duesenberry, J., Income, Saving, and the Theory of Consumer Behavior. Cambridge, Mass.: Harvard University Press, 1949.

[23] Dupor, B. and Liu, W. F. (2003). Jealousy and Equilibrium Overconsumption, American Economic Review, 93, 423-428.

[24] Dynan, K. E., Ravina, E (2007) Increasing Income Inequality, External Habits, and Self-Reported Happiness. American Economic Review, 97, 226–231.

[25] Frank, R. H. Choosing the Right Pond, New York: Oxford University Press, 1985.

[26] ____________Frank, R.H., 1999. Luxury Fever. Why Money Fails to Satisfy in an Era of Excess. The Free Press, New York.

[27] García-Peñalosa, C., Turnovsky, S.J. (2008) Consumption externalities: a representative consumer model when agents are heterogeneous, Economic Theory, 37, 439-467

[28] Gershman, B. (2014). The two sides of envy. Journal of Economic Growth, 19(4), 407-438.

[29] Godwin, Richard (2018) Triathlons, ultramarathons and ambitious baking: why is modern leisure so competitive? The Guardian, 21 Nov

[30] Gollier, C. (2004), Misery Loves Company: Equilibrium Portfolios with Heterogenous Consumption Externalities, International Economic Review, 43, 1169-1192.

[31] Gomes, F., & Michaelides, A. (2007). Asset pricing with limited risk sharing and heterogeneous agents. Review of Financial Studies, 21(1), 415-448.

[32] Greenwood, J., Hercowitz, Z., & Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. American Economic Review, 402-417.

[33] Guo, Jan Ting., Krause, Alan (2015). Dynamic nonlinear income taxation with quasi-hyperbolic discounting and no commitment. Journal of Economic Behavior & Organization, 109, 101-119.

[34] Guiso, L., & Paiella, M. (2008). Risk aversion, wealth, and background risk. Journal of the European Economic Association, 6(6), 1109-1150.

[35] Guvenen, F., (2009). A parsimonious macroeconomic model for asset pricing. Econometrica 77, 1711–1750.

[36] Hopkins, E (2016) Inequality and Risk-Taking Behaviour, mimeo

[37] Hong, H. and J. Stein (1999). A unified theory of underreaction, momentum trading, and overreaction in asset markets. Journal of Finance, 54, 2143-2184

[38] Kawamoto, K. (2009) Status-seeking behavior, the evolution of income inequality, and growth. Economic Theory, 39, 269–289

[39] King, R. G., Plosser, C. I., & Rebelo, S. T. (1988). Production, growth and business cycles: I. The basic neoclassical model. Journal of Monetary Economics, 21(2-3), 195-232.

[40] Kramer, B. (2016). When expectations become aspirations: reference-dependent preferences and liquidity constraints. Economic Theory, 61(4), 685-721.

[41] Kubler, F., Selden, L., & Wei, X. (2014). When Is a Risky Asset "Urgently Needed"?. American Economic Journal: Microeconomics, 6(2), 131-162.

[42] Lahiri, A., M. Puhakka (1998) Habit persistence in overlapping generations economies under pure exchange, Journal of Economic Theory, 78, 176–186

[43] Ljungqvist, L. and Uhlig, H. (2000), Tax Policy and Aggregate Demand Management Under Catching Up with the Joneses, American Economic Review 90, 356-366.

[44] Liu, W.F. and S.J. Turnovsky (2005), Consumption Externalities, Production Externalities, and Long Run Macroeconomic Efficiency, Journal of Public Economics, 89, 1098-1129.

[45] Luttmer, Erzo FP. (2005) Neighbors as negatives: Relative earnings and well-being. Quarterly Journal of Economics 120(3), 963-1002.

[46] Maurer, J. and Meier, A. (2008). 'Smooth it like the Joneses'? estimating peer-group effects in intertemporal consumption choice', Economic Journal, 118, pp. 454–476

[47] MaCurdy, T. E. (1981). An empirical model of labor supply in a life-cycle setting. Journal of Political Economy, 89(6), 1059-1085.

[48] Paravisini, D., Rappoport, V., & Ravina, E. (2016). Risk aversion and wealth: Evidence from person-to-person lending portfolios. Management Science, 63(2), 279-297.

[49] Persson, Torsten, and Tabellini, Guido., (2002). Political economics: explaining economic policy. Cambridge: MIT.

[50] Strulik, H. (2015), How Status Concerns Can Make Us Rich and Happy. Economica, 82: 1217–1240.

[51] Veblen, T. ([1892] 1919). "Some neglected points in the theory of socialism," reprinted in *The place of science in modern civilization*, New York: B.W. Huebsch, 387–408

[52] Wang, J. (1993). A model of intertemporal asset prices under asymmetric information, Review of Economic Studies, 60, 249-282.

[53] Wendner, R (2003) Do habits raise consumption growth? Research in Economics 57(2), 151-163.

[54] Xiouros, C. and F. Zapatero (2010). The Representative Agent of an Economy with External Habit Formation and Heterogeneous Risk Aversion, Review of Financial Studies 23 (8), 3017-3047