The Use of Acoustoelastic Measurements to Characterize the Stress States in Cracked Solids

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The Use of Acoustoelastic Measurements to Characterize the Stress States in Cracked Solids

Abstract
The theory of acoustoelasticity predicts that a plane longitudinal acoustic wave passing through a solid which is already in a deformed state will propagate with a velocity (v) which is different from the (v₀) of the same wave propagating through the undeformed medium. It may be shown that Δv/v₀ = (v-v₀)/v₀ = B(σ₁+σ₂) where σ₁ and σ₂ are the principal stress in the plane normal to the wave propagation direction and B is the acoustoelastic constant. Wave transit time measurements allow the relative velocity change Δv/v₀ to be determined, so that contours of constant principal stress sum (σ₁+σ₂) may be mapped by acoustically scanning a stressed solid. We have used the technique described above to characterize the states of stress in cracked and notched aluminum panels. A method for extracting crack stress intensity factors from the acoustic data is proposed and illustrated for center-cracked panel specimens. The results indicate that the technique may offer a promising method for nondestructive testing and evaluation.

Keywords
Nondestructive Evaluation

Disciplines
Materials Science and Engineering

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The theory of acoustoelasticity predicts that a plane longitudinal acoustic wave passing through a solid which is already in a deformed state will propagate with a velocity \( v \) which is different from the \( v_0 \) of the same wave propagating through the undeformed medium. It may be shown that 

\[
\frac{\Delta v}{v_0} = \frac{v - v_0}{v_0} = B(\sigma_1 + \sigma_2)
\]

where \( \sigma_1 \) and \( \sigma_2 \) are the principal stress in the plane normal to the wave propagation direction and \( B \) is the acoustoelastic constant. Wave transit time measurements allow the relative velocity change \( \Delta v/v_0 \) to be determined, so that contours of constant principal stress sum \( (\sigma_1 + \sigma_2) \) may be mapped by acoustically scanning a stressed solid.

We have used the technique described above to characterize the states of stress in cracked and notched aluminum panels. A method for extracting crack stress intensity factors from the acoustic data is proposed and illustrated for center-cracked panel specimens. The results indicate that the technique may offer a promising method for nondestructive testing and evaluation.

Scanning experiments involving both shear and longitudinal acoustic wave probes may, in principle, be used to nondestructively determine the complete state of plane deformation in a stressed solid. We shall point out how one may use such acoustic information to determine the \( J \) integral and the \( M \) integral associated with cracked specimens. The integrands of these elastostatic conservation integrals contain terms involving elastic rotations which are not directly obtainable from the acoustic data, but it is possible to use forward integration of the compatibility equations to obtain the requisite information. An illustration example in which \( J \) and \( M \) are determined using this technique will be presented. This technique may find practical applications in the continuous nondestructive monitoring of critical structural elements.

**INTRODUCTION**

In recent years research has been conducted on development of acoustoelastic measurement techniques, in conjunction with the theory of acoustoelasticity, into a nondestructive experimental stress analysis tool.\(^1\)\(^-\)\(^4\) This paper discusses the application of ultrasonic stress measurements to fracture mechanics. With knowledge of fracture mechanics quantities such as the stress intensity factor and the \( J \) integral, and making use of appropriate fracture criteria, an assessment can be made of how dangerous flaws might be which exist in structural elements. Attempts which have been made to use ultrasonics to nondestructively evaluate these quantities are summarized here. The first part of the paper describes measurement of the stress intensity factor while the \( J \) and \( M \) integrals are discussed in the second.

**MEASUREMENT OF \( K_I \) FROM FAR FIELD ULTRASONIC DATA**

Investigation has been made into the use of longitudinal wave ultrasonic stress measurements to nondestructively determine the stress intensity factor in specimens containing cracks. This work is part of the recently completed doctoral thesis of John Hunter.\(^5\) Discussion will be limited to mode I, or opening mode deformation, although the technique to be described is applicable to mixed mode cases.

The use of ultrasonics to measure stress is based on the theory of acoustoelasticity, which predicts that the velocity of an acoustic wave propagating through a solid depends on the state of deformation and hence the state of stress in the solid. For a plane longitudinal wave at normal incidence to a body in a state of plane stress, with wave speed \( v_0 \) and \( v \) in an undeformed and deformed medium, respectively, acoustoelasticity predicts

\[
\frac{\Delta v}{v_0} = \frac{v - v_0}{v_0} = B(\sigma_1 + \sigma_2) \quad (1)
\]

where \( \sigma_1 \) and \( \sigma_2 \) are the in-plane principal stress components, and \( B \) is a material constant which depends on the elastic constants \( (E, v) \) and the third order Murnaghan constants of the material. For a given material \( B \) can be determined directly using a uniaxial tension calibration test. It is important to calibrate the material under study because \( B \) varies widely among different materials. The relative velocity change, \( (\Delta v/v_0) \) is measured with longitudinal ultrasonic waves using a technique described in detail in Ref. 1. It should be emphasized that longitudinal wave measurements permit the determination of the principal stress sum only. The separate principal stress components and the principal directions cannot be evaluated with longitudinal waves only. The laboratory apparatus used to make ultrasonic measurements is shown in Fig. 1. With the aid of a laboratory minicomputer, this testing machine is capable of making measurements throughout a twodimensional grid. Using our capability of determining the sum of the principal stresses point by point in plane specimens, an attempt has been made to evaluate stress intensity factors. In Fig. 2, an infinite plate containing a crack and subjected to remote uniaxial tension is shown. The elasticity solution for this case has been recently discussed by Efetis et al.\(^5\) and is given in terms of a complex potential function:
\[ \sigma_{xx} = 2 \Re e^4 - 2y \Im m^4 - \sigma_0/z \]
\[ \sigma_{yy} = 2 \Re e^4 + 2y \Im m^4 \]
\[ \sigma_{xy} = -2y \Re e^4 \]  
(2)

where \( \phi(z) = (a \Im m^2 - a^2 - (a \Im m)/z \). (Primes denote differentiation of a function with respect to its argument.) Expanding this solution for small values of \( r \), the radius measured from the crack tip, and introducing the mode I stress intensity factor for an infinite plate

\[ K_I = \sigma_0 \sqrt{a} \]  
leads to

\[ \sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos(\theta/2)(1 - \sin \theta/2 \sin 30/2) \]

\[ - \sigma_0 + O(1/r^2) \]

\[ \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos(\theta/2)(1 + \sin \theta/2 \sin 30/2) + O(1/r^2) \]

\[ \sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \theta/2 \cos \theta/2 \cos 30/2 \]  
(4)

In fracture mechanics the assumption is made that regardless of the geometry and loading on a body containing a crack, the stresses in the near tip region have the same geometric dependence as that shown in Eqs. (4) for the infinite plate, and the boundary conditions only affect the value of the single parameter \( K_I \). Summing the first of Eqs. (3), one obtains

\[ \sigma_{xx} + \sigma_{yy} = \frac{2K_I}{\sqrt{2\pi r}} \cos \theta/2 - \sigma_0 + O(1/r^2) \]  
(5)

and for plane specimens

\[ \sigma_1 + \sigma_2 = \sigma_{xx} + \sigma_{yy} \]  
(6)

Making use of the assumption discussed above, if ultrasonic measurements were available in the near field, the stress intensity factor could be evaluated easily. A method which makes use of several data points is shown in Fig. 3. Referring to the coefficient of \( K_I \) in (5) as the "geometric factor" measured values of the principal stress sum could be plotted versus the value of the geometric factor at those points. A straight line fit through the data would give an estimate of \( K_I \) as its slope. Unfortunately near tip data are difficult to obtain and there are nonlinear effects occurring in this region which are not accounted for by the theory underlying the stress measurement. (This is true of other experimental techniques as well as ultrasonics.) Consequently it is desirable to be able to measure the stress intensity factor from data further away from the crack tip, a region in which the neglected terms in Eq. (5) become important. To do so, it is necessary to extend the concepts discussed thus far. Returning to the expressions for stresses in the infinite plate solution, and summing the first two leads to

\[ \sigma_{xx} + \sigma_{yy} = \sigma_0 \Re e(2z/(z^2 - a^2)^{1/2}) \]  
(7)

Again introducing the same stress intensity factor, Eq. (3), used earlier,

\[ \sigma_{xx} + \sigma_{yy} = \frac{2K_I}{\sqrt{2\pi r}} \Re e(2z/(z^2 - a^2)^{1/2}) - \sigma_0 \]  
(8)

In Eq. (8) the stress intensity factor is multiplied by a new geometric factor which will be valid anywhere in the infinite plate. Now, in attempting to measure \( K_I \) for a specimen, an assumption is made similar to that made previously: in the region in which data is being taken, it is assumed that the principal stress sum for the specimen under test has the same geometric dependence exhibited by the infinite plate solution in Eq. (8), the boundary conditions only affecting the value of \( K_I \). If this assumption is valid, and a straight line is fit through a plot at measured \( (\sigma_{xx} + \sigma_{yy}) \) at various points versus the value of the geometric factor at those points, the stress intensity factor would be given by the slope of that line (Fig. 4).

An attempt was made to apply the procedure described above on a center-cracked panel (Fig. 5). It was proposed to take ultrasonic data in a 20 mm square grid in front of the crack. Before the experiment was conducted, the validity of the assumption discussed above was investigated using a numerical (boundary collocation) solution for stresses in a center-cracked panel. The numerical solution was scaled so that the principal stress sum matched the infinite plate solution at one point, and the contours for the stress sum for the numerical solution were plotted on top of those for the infinite plate. These contours matched quite well in the 20 mm square region of interest, indicating the assumption was valid in this case.

Ultrasonic measurements were then made in this region using the experimental apparatus described previously at 400 data points (1 mm point spacing). Since the acoustoelastic \( B \) (Eq. (1)) was known from a calibration test on the material (aluminum 6061-T6), the principal stress sum could be determined from the ultrasonic data. Plots of measured values of principal stress sum versus geometric factor are shown in Fig. 6. For two values of applied load, the straight lines that fit through the data points give an estimate for \( K_I \) within 20% of the handbook value for this geometry.

From these preliminary results we are encouraged that this technique can be developed into a useful tool for determining the stress intensity factor in practical situations.

**DETERMINATION OF THE J AND M INTEGRALS USING ULTRASONIC DATA**

Another area of application of ultrasonic stress measurement capabilities to fracture mechanics is experimental evaluation of the J and M integrals. The definition and physical interpretation of these quantities is discussed in Ref. 10 and will be summarized here. The J integral is a contour integral around the tip of a flaw in a solid and is defined as

\[ J = \int_c (Wn_x - T_{ik}K_{ikx}) \, dz \]  
(9)
where \( W \) is the strain energy density, \( T_k \) is the traction, and \( U_k \) is the displacement along the contour. \( J \) represents the mechanical energy release rate with respect to translation of the tip of the flaw in the \( x \) direction. \( M \) is defined as

\[
M = \frac{\phi}{c} \left( W_{x}^{-1} - T_{ik} U_{k, i} x_{i} \right) \text{ds} \tag{10}
\]

where \( c \) is a contour completely enclosing a flaw. \( M \) represents the energy release rate with respect to self-similar expansion of the flaw. The practical significance of these quantities is similar to that of the stress intensity factor. For elastic deformation \( J \) is the same as the crack extension force \( G \), thus

\[
J = G_1^2 \tag{11}
\]

and knowledge of \( J \) enables determination of \( K_1 \) is more generally useful than \( K_1 \), however, because while \( K \) versus \( K_1 \) as a fracture criterion is restricted to cases of small scale yielding, \( J \) versus \( J_c \) is valid for general yielding so long as no unloading occurs. The \( M \) integral is useful because, using path independence arguments, \( J \) can be determined from \( M \) and \( G \) gives useful results for closed contours, which are sometimes more convenient, while \( J \) gives zero). For instance, it is easily shown for an interior crack that

\[
M = 2aJ \tag{12}
\]

where \( a \) is the half crack length. Applications of the \( M \) integral in determining stress intensity factors have been discussed by Freund.11

Nondestructive measurement of the \( J \) and \( M \) integrals involves evaluation of the integrand at several points along a contour and then numerically integrating to determine \( J \) or \( M \). By expanding the \( J \) integral in Eq. (9) it becomes clear what this entails for in plane stress:

\[
J = \frac{1}{2} \int_{C} \left[ \frac{1}{2} \left( \sigma_{yy} - \sigma_{xx} \right) dy + \sigma_{xy} \left( \frac{1}{2} \sigma_{xx} + \sigma_{yy} \right) dx \right.
\]

\[
+ \int_{C} \omega_{xy} \left( \sigma_{xx} dy - \sigma_{yy} dx \right) \tag{13}
\]

Thus determination of \( J \) requires knowledge of the entire stress tensor along the contour as well as the rotation component \( \omega_{xy} \). The same holds true for \( M \). While in some geometries the integrand simplifies considerably and knowledge of the principal stress sum is sufficient to evaluate the integrand,2 in general this is not true. However, shear wave measurements are capable of determining the full state of stress point by point in a plane specimen. Unfortunately it is difficult to take shear wave data at many points due to the problem of coupling the wave to the specimen.2,3 A shear wave scanning apparatus similar in principle to that shown in Fig. 1 but using direct mechanical coupling of the transducer to the specimen has been constructed at Stanford University. Preliminary testing is underway, and it is hoped this device will enable the use of shear waves to determine the stress state in plane stress specimens in the near future. Of the terms in the integrand in Eq. (13), it would then remain to determine \( \omega_{xy} \). It is not clear how to directly measure this quantity at many points nondestructively by any experimental technique. The importance of the rotation term was investigated for both the \( J \) and \( M \) for two analytical cases, and it was found to contribute from 50% - 75% of the results; thus it is certainly not negligible.

A method for determining \( \omega_{xy} \) from stress data by making use of compatibility relations is proposed. Using the definitions of stress and rotation,

\[
e_{11} = \frac{1}{2} \left( u_{1,1} + u_{2,1} \right)
\]

\[
e_{22} = \frac{1}{2} \left( u_{1,2} + u_{2,2} \right)
\]

\[
e_{12} = \frac{1}{2} \left( u_{1,2} - u_{2,1} \right)
\]

the gradients of rotation can be expressed in terms of strain gradients by

\[
\omega_{xy,x} = \frac{1}{2} \left( \sigma_{xx} - \sigma_{yy} \right) - \frac{1}{2} \left( \sigma_{xx} + \sigma_{yy} \right)
\]

\[
\omega_{xy,y} = \frac{1}{2} \left( \sigma_{xx} - \sigma_{yy} \right) - \frac{1}{2} \left( \sigma_{xx} + \sigma_{yy} \right)
\]

Note the \( \omega_{xy,x} \) and \( \omega_{xy,y} \) terms are required because, \( J \) versus \( J_c \) is valid for general yielding so long as no unloading occurs. This requires that \( \omega_{xy} \) be determined numerically by using Eq. (18), and the \( J \) and \( M \) integrals were determined by integrating forward from a point at which rotation is known:

\[
\omega_{xy}(x,y) = \omega_{xy}(x_0,y_0) + \int_{x_0}^{x} \int_{y_0}^{y} \omega_{xy} \text{ds} \tag{18}
\]

The stress gradients in Eq. (17) are determined numerically using a central difference formula. It now appears that shear wave measurements will permit determination of all quantities in the integrand of \( J \) and \( M \) for plane specimens. This technique was tested on the infinite plate solution. The stress values from this solution were stored in a two-dimensional grid in the first quadrant (Fig. 7), taking advantage of symmetry. The rotation was determined using Eq. (18), and the \( J \) and \( M \) integrals were determined by integrating numerically. The results for the rotations are shown in Table I. There is an apparent smoothing effect in numerically integrating the data resulting in more accurate determination of the \( J \) and \( M \) integrals than for the rotations themselves (Table III). In an effort to simulate experimental data, random noise was introduced into the experimental data. The technique appeared quite capable of handling this as shown in Table III. Consequently it is hoped that when shear wave measurements are available it will be possible to use this method to evaluate the \( J \) and \( M \) integral for practical plane specimens.

**CONCLUSIONS**

Application of ultrasonic stress measurements to the nondestructive evaluation of fracture mechanics quantities has been presented. It is expected
that these techniques can be developed into a useful tool for assessing the status of flaws in practical situations.

ACKNOWLEDGMENTS

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REFERENCES


Fig. 1. Diagram of testing machine for acoustoelastic measurements.

Fig. 2. Stress field in an infinite cracked panel under remote uniaxial tension.

Fig. 3. Calculation of Kt for a mode I crack using near field data.
Fig. 4. Calculation of $K_1$ for a mode I crack using extended field solution.

Fig. 5. Center cracked panel showing scanned area.

Fig. 6. Calculation of $K_1$ for a center cracked panel for two values of applied load.

Fig. 7. Contours used for evaluating J and M using the infinite plate solution.

**Table I.** Comparison of rotations determined numerically versus theoretical values.

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<th>X</th>
<th>Y</th>
<th>EXACT</th>
<th>APPROX</th>
<th>% ERROR</th>
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**AVE, ERROR:** 5.63%
Table II. Results for the J and M integrals evaluated using theoretical data.

<table>
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<tr>
<th>NOISE LEVEL</th>
<th>J INTEGRAL (EXACT - 2.014)</th>
<th>% ERROR</th>
<th>M INTEGRAL (EXACT - 20.14)</th>
<th>% ERROR</th>
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<td>2.035</td>
<td>.50</td>
<td>20.50</td>
<td>2.20</td>
</tr>
</tbody>
</table>

Table III. Results for the J and M integrals using simulated experimental data.
Unidentified Speaker: What was the sensitivity of these measurements?

R. King (Stanford University): We usually do measurements on aluminum because we feel our system will do a better job on that. It depends on the value of the \( P \) constant I showed, the proportionality constant. For aluminum that constant is high, which means there is a high relative velocity change with stress. I'm not sure percentage-wise how accurately we can measure stress. I would say probably plus or minus ten-percent. With steel, the effect is smaller, and although we can make stress measurements in steel, it is somewhat more difficult. The trick we have used is to first attempt to make stress measurements on a certain specimen configuration on aluminum, and then go back and try it on steel.

Unidentified Speaker: I wanted to congratulate you. That was really beautiful work. But I wanted to ask you - you did \( K \) measurements, as a function of crack depth. Now, was it a real, true fatigue crack, or did you guys give a saw cut and change the depth of the saw cut?

R. King: Actually, this one was a fatigue crack. I didn't mention it was grown by a colleague, Mike Resch, who helped us make a laser burn on the specimen to create a small cavity. We fatigued the specimen and grew a crack outwards from this cavity. We have also made measurements on "phony" cracks made by saw cuts.

P. Holler (Inst. fur Zerstorungsfreie Prufverfahren): You can do it on C.O.D. specimens. Do you have any idea how to get the proper radiation in the normal case? You need an angle of incidence parallel to the crack face, and also to the crack growth, or is it mainly a thought for C.O.D. experiments and things like that?

R. King: We need an angle of incidence normal to the plane of the specimen. Everything I have done is restricted to that. We are looking into methods of three-dimensional stress determination. It's more complicated. We're going to lick the two-D case first.

P. Holler: Thank you very much again.

# #