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Signal Processing

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Abstract

A signal is a time or space varying quantity that conveys information. It may take the form of a sound wave, a voltage, a current, a light wave, a magnetic field, a displacement, a velocity, an acceleration or a host of other physical quantities. These are examples of continuous signals (the signal is present at all instances of time or space). As a matter of convenience we often convert continuous signals into discrete signals by sampling the phenomena at intervals (Figure 3.1).

Disciplines

Biomechanics | Kinesiology | Motor Control | Psychology of Movement

Comments

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Chapter 3
Signal Processing
T.R. Derrick

What is a signal and why should we process it?

A signal is a time or space varying quantity that conveys information. It may take the form of a sound wave, a voltage, a current, a light wave, a magnetic field, a displacement, a velocity, an acceleration or a host of other physical quantities. These are examples of continuous signals (the signal is present at all instances of time or space). As a matter of convenience we often convert continuous signals into discrete signals by sampling the phenomena at intervals (Figure 3.1).

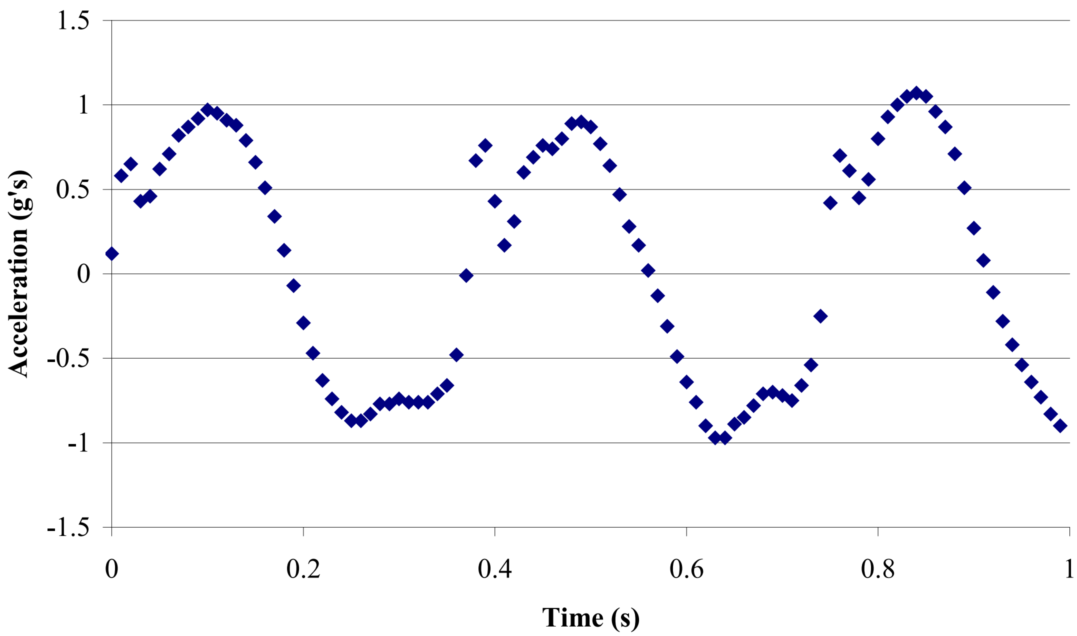


Figure 3. 1. The digitized discrete representation of the acceleration of the head while running. The signal was sampled at 100 Hz (100 samples per second).

Characteristics of a signal

Any time varying signal has four characteristics: frequency (f), amplitude (A), offset (A_0) and phase (θ). These characteristics are depicted in the schematics in Figure 3.2. The frequency represents the number of cycles per second that the signal completes. This is easy to see in a sine wave (Figure 3.2a) but more difficult to visualize in non-cyclic signals with multiple frequencies. The amplitude of a signal is quantified by the magnitude of oscillations (Figure 3.2b). The offset (or DC offset) is the quantity of displacement that the average value of the signal is from zero (Figure 3.2c). The phase angle (or shift) in the signal is the quantity of time displacement (Figure 3.2d).

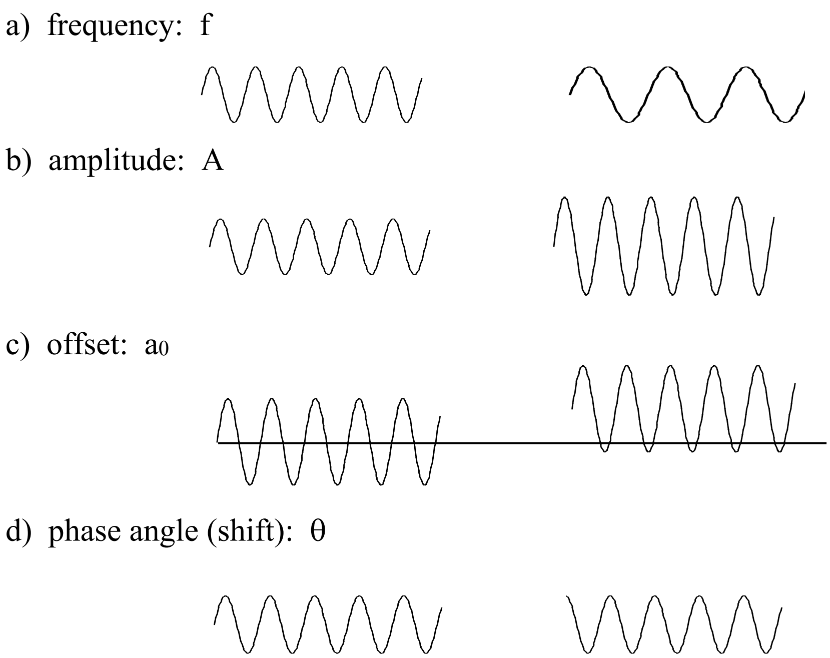


Figure 3. 2. The four essential components of a time varying signal.

Any time varying signal, h(t) is made up of these 4 characteristics. The following equation incorporates each of the 4 variables:

$$h(t) = a_0 + A\sin(2\pi ft + \theta)$$

but, $2\pi f = \omega$ so another way to write this is:

$$h(t) = a_0 + A\sin(\omega t + \theta)$$

The time (t) is the discrete time value and depends on the sampling frequency or sampling rate. If the sampling frequency is 100 Hz (100 samples per second) then the sampling period is the inverse (100^{-1} or 0.01 seconds). This means that there is a sample or data point every 0.01 seconds. So, t is one of the discrete values in the set (0, 0.01, 0.02, 0.03, 0.04, … T). The variable T is the sampling time and it represents the total time that the signal has been digitized.

Adding the equation for a 2 Hz sine wave and a 20 Hz sine wave will generate the following curve:

$$h(t) = \sin(2\pi 2t) + \sin(2\pi 20t)$$

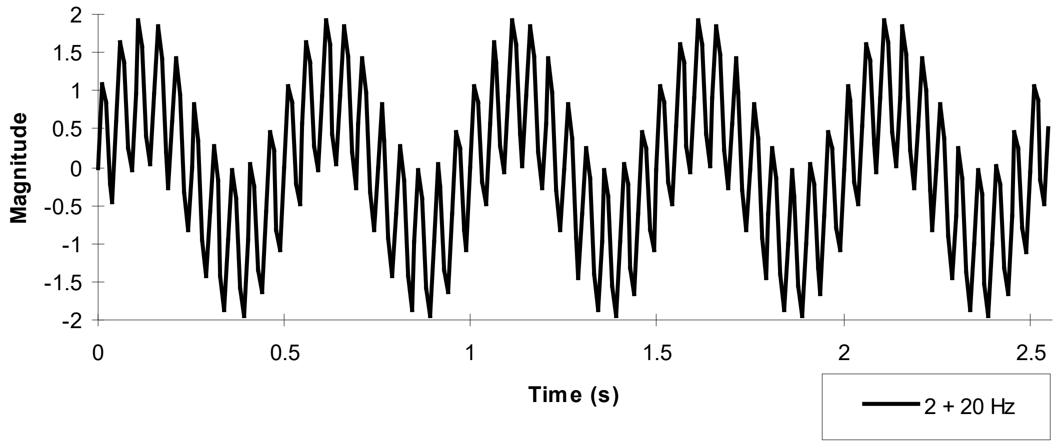


Figure 3. 3. A 2 Hz and a 20 Hz summed over a 2.5 second period. a_0 and θ are zero for both waves and amplitude is 1.

The Fourier Transform

ANY time varying signal can be represented by successively adding all of the individual frequencies present in the signal (Winter, 1990). The A_n and θ_n values may be different for each frequency and may be zero for any given frequency.

$$h(t) = a_0 + \sum a_n \sin(2\pi f_n t + \theta_n)$$

By using the cosine and sine functions this series can be rewritten without the phase variable:

$$h(t) = a_0 + \sum a_n \sin(2\pi f_n t) + b_n \cos(2\pi f_n t)$$

This series is referred to as the Fourier series. The a_n and b_n coefficients are called the Fourier coefficients. They can be calculated using the following formulae:

$$a_0 = \frac{1}{T}\int_0^T h(t)dt$$

$$a_n = \frac{2}{T}\int_0^T h(t)\cos(2\pi f_n t)dt$$

$$b_n = \frac{2}{T}\int_0^T h(t)\sin(2\pi f_n t)dt$$

Here is another way of looking at it. If you want to know how much of a certain frequency (f_n) is present in a signal h(t) you can multiply your signal by the sine wave [$\sin(2\pi f_n t)$], take the mean value and multiply it by 2. If you repeat this process for a cosine wave and then add the square of the sine and cosine values together, you will get an indication of how much of the signal is made up with the frequency f_n. This is the power at frequency f_n.

$$a_0 = \text{mean}[h(t)]$$

$$a_n = 2 * \text{mean}[h(t) * \cos(2\pi f_n t)]$$

$$b_n = 2 * \text{mean}[h(t) * \sin(2\pi f_n t)]$$

$$\text{power}(f_n) = a_n^2 + b_n^2$$

The Fourier coefficients can be calculated from the equally-spaced time varying points with the use of a Discrete Fourier Transformation (DFT) algorithm (appendix A). Given the Fourier coefficients, the original signal can be reconstructed using an inverse DFT algorithm. The DFT is a calculation intensive algorithm. A more elegant and more used class of algorithms are the Fast Fourier Transformations (FFT). An FFT requires that the number of original data points be a multiple of a power of 2 (…16, 32, 64, 128, 256, 512, 1024 …). The usual method of obtaining a "power of 2" number of data points is to pad the data with zeros (add zeros to the data until the number of points is a power of two). Unfortunately this creates two problems that must be overcome:

1. Padding reduces the power of the signal. Parseval's theorem implies that the power in the time domain must equal the power in the frequency domain (Proakis and Manolakis, 1988). If you pad with zeros you reduce the power (a straight line at zero has no power). You can restore the original power by multiplying the power at each frequency by $(N+L)N^{-1}$, where N is the number of non-zero values and L is the number of padded zeros.

2. Padding can introduce discontinuities between the data and the padded zero values if the signal does not end at zero. This discontinuity will show up in the resulting spectrum as increased power in the higher frequencies. To insure that your data start and end at zero you can apply a windowing function or subtract the trend line before

performing the transform. Windowing functions begin at zero, rise to one and then return to zero again. By multiplying your signal by a windowing function you are reducing the endpoints to zero in a gradual manner. Windowing should not be performed on data unless there are multiple cycles. Subtracting a trend line that connects the first point to the last point can be used as an alternative.

Most software packages give the result of an FFT in terms of a real portion and an imaginary portion. For a real discrete signal (most biomechanics applications) the real portion corresponds to the cosine coefficient and the imaginary portion corresponds to the sine coefficient of the Fourier series equation. An FFT results in as many coefficients as there are data points (N), but half of these coefficients are a reflection of the other half. Therefore, the N/2 points will represent frequencies from 0 to one half of the sampling frequency ($F_s/2$). Each frequency bin will have a width of F_s/N Hz. By increasing the number of data points (by padding with zeros or collecting for a longer period of time) you can decrease the bin width. This does not increase the resolution of the FFT but rather it is analogous to interpolating more points from a curve.

Researchers often adjust the bin width so that each bin is 1 Hz wide. This is referred to as normalizing the spectrum. Adjusting the bin width changes the magnitude because the sum of the power frequency bins must equal the power in the time domain. Normalizing the spectrum allows data of differing durations or sampling rates to be compared. The magnitude of a normalized spectrum will be in units of (original units)$^2 \cdot$Hz^{-1}.

A plot of the power at each frequency is referred to as the power spectral density (PSD) plot or simply the power spectrum. A PSD curve contains the same information as its time domain counterpart but it is rearranged to emphasize the frequencies that contain the greatest power rather than when in the cycle the most power occurs. Figure 3.4 shows a leg acceleration curve along with the PSD.

From the Scientific Literature
Spectral analysis of impact shock during running. Shorten, M.R. and Winslow, D.S. (1992). *International Journal of Sport Biomechanics*, 8:288-304.

The purpose of this study was to determine the effects of increasing impact shock levels on the spectral characteristics of impact shock and impact shock wave attenuation in the body during treadmill running. Three frequency ranges were identified in leg acceleration curves that were collected during the stance phase of running. The lowest frequencies (4-8 Hz) were identified as the active region due to muscular activity. The mid range frequencies (12-20 Hz) were due to the impact between the foot and the ground. There was also a high frequency component (60-90 Hz) due to the resonance of the accelerometer attachment. Since these frequencies all occurred at the same time it was impossible to separately analyze them in the time domain. Head accelerations were also calculated so that impact attenuation could be calculated from the transfer functions (TF). Transfer functions were calculated from the power spectral densities at the head (PSD_{head}) and the leg (PSD_{leg}) using the following formula:

$$TF = 10 \log_{10} (PSD_{head}/PSD_{leg}).$$

This formula resulted in positive values when there is a gain in the signal from the leg to the head and negative values when there is an attenuation of the signal from the leg to the head. The results indicated that there was an increase in the leg impact frequencies as running speed increased. There was also an increase in the impact attenuation so that head impact frequencies remained relatively constant.

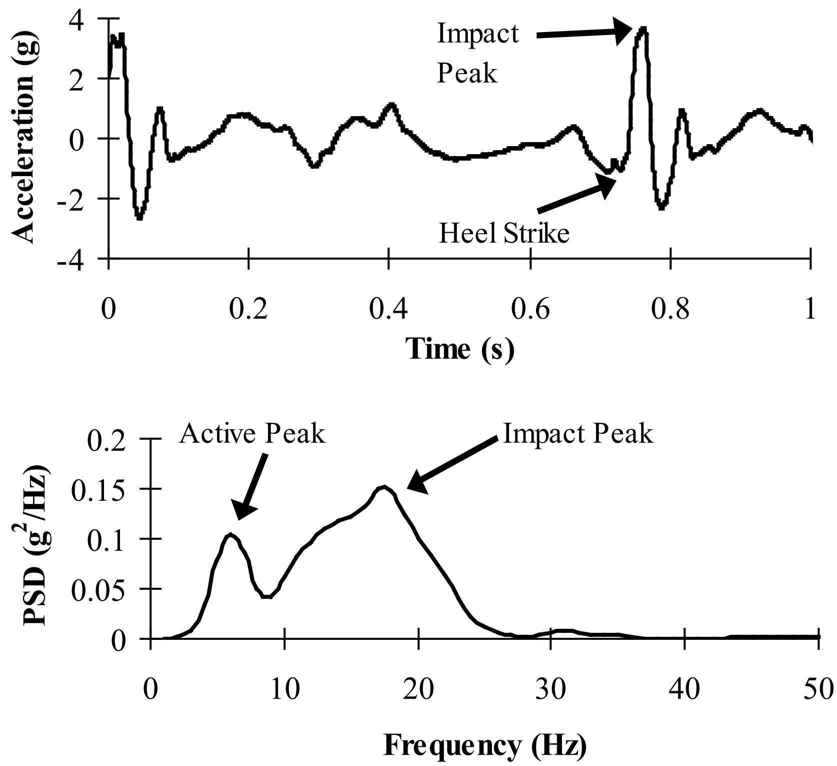


Figure 3. 4. Leg accelerations during running in the time and frequency domains. Time domain graph shows two ground impacts while the frequency domain graph is for a single stance phase.

The Wavelet Transform

The Discrete Fourier Transformation has the advantage that frequencies can be separated no matter when they occur in the signal. Even frequencies that occur at the same time can be separated and quantified. A major disadvantage is that we do not know **WHEN** those frequencies are present. We could overcome this difficulty by separating our signal into sections before applying the DFT to each section. We would then have a better idea of when a particular frequency occurred in the signal. A Wavelet Transform does essentially that.

Since we are already able to separate frequencies, we will use that to build an intuitive feel for how this transform works. If we take a signal that contains frequencies from 0-100 Hz our first step is to separate the frequencies into two portions, one below 50 Hz and one above 50 Hz. Next we take these two sections and separate them into two portions each. We now have sections of 0-50, 50-100, 0-25, 25-50, 50-75 and 75-100 Hz. This process continues to a predefined level. The procedure is called decomposition. At this point we have several time series representations of the original signal, each containing different frequencies. We can plot these representations on a 3-D graph with time on one axis, frequency on a second axis and magnitude on the third (Figure 3.5). There are problems with this process that involve Heisenberg's uncertainty principle (both the time and frequency cannot be known at a point on the time-frequency plane). This is essentially a resolution problem that Wavelet Transformation techniques overcome with the use of variable resolution.

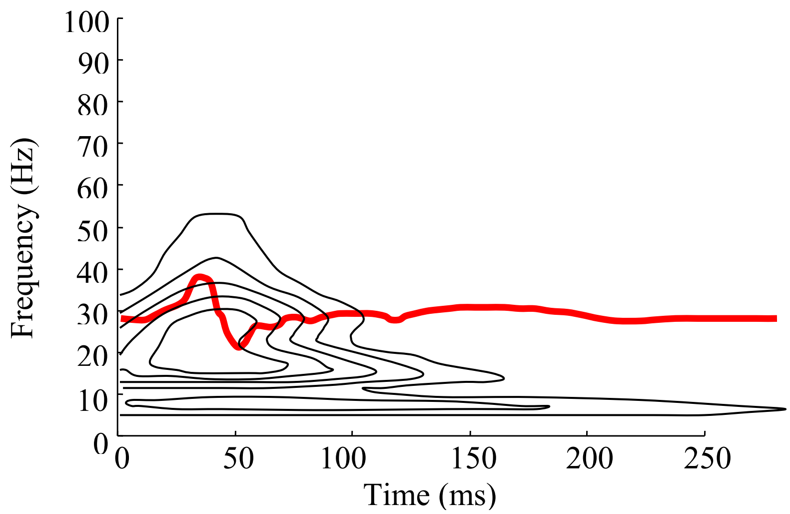


Figure 3. 5. A 3-dimensional contour map of the frequency by time values of a leg acceleration curve during running. The time domain curve is superimposed on the contour. There are two peaks in this curve. The high frequency peak (~20 Hz) occurs between 20 and 60 ms. The lower frequency peak (~8 Hz) occurs between 0 and 180 ms.

From the Scientific Literature

Wakeling, J.M, Von Tscharner V., Nigg, B.M. (2001). Muscle activity in the leg is tuned in response to ground reaction forces. *Journal of Applied Physiology* 91: 1307-1317.

The purpose of this study was to investigate the response of muscle activity in the leg to different impact forces. A human pendulum apparatus was used to control leg geometry and initial conditions as the pendulum impacted a force platform. The loading

rate was varied by changing the viscoelastic properties of the shoe midsole. Myoelectrical signals were recorded from the tibialis anterior, medial gastrocnemius, vastus medialis and biceps femoris muscles. These signals were resolved by wavelet analysis into there magnitudes in time and frequency space. Traditional Fourier-transformations would be inadequate to describe a non-stationary signal such as would be anticipated during the impact. Differences occurred in the magnitude, time and frequency content of the myoelectric signals during the period 50 ms before impact and 50 ms after impact. These differences justified the use of the wavelet technique to accomplish the decomposition. It was speculated that the change in myoelectric patterns that occurred with different loading rates was due to differences in muscle fiber type recruitment. It was concluded that the levels of muscle activity were adjusted in response to the loading rate of the impact forces.

Sampling Theorem

The process signal must be sampled at a frequency greater than twice as high as the highest frequency present in the signal itself.

The minimum sampling rate is called the Nyquist sampling frequency (F_N). In human locomotion, the highest voluntary frequency is less than 10 Hz*, thus a 20 Hz sampling rate should be satisfactory however, in reality biomechanists usually sample at 5 to 10 times the highest frequency in the signal. This insures that the signal is accurately portrayed in the time domain without missing peak values.

The sampling theorem indicates that if the signal is sampled greater than 2 times the highest frequency, then the signal is completely determined by these data points. In fact the original signal is given explicitly by the following formula (also see Appendix B):

$$h(t) = \Delta \sum h_n \left[\frac{\sin[2\pi f_c (t - n\Delta)]}{\pi (t - n\Delta)} \right]$$

where,

Δ = sampling period (1/sampling frequency)
$f_c = 1/2\Delta$
h(t) = original signal
h_n = n^{th} sampled data point
t = time

By using this formula (Shannon's reconstruction formula) it is possible to collect data at slightly greater than twice the highest frequency and then apply the reconstruction formula to "resample" the data at a higher rate (Marks, 1993). Figure 3.6 illustrates the

* Frequencies greater than 10 Hz occur in human movement whenever segments collide with other objects.

use of the resampling formula to reconstruct a running vertical ground reaction force curve. The signal was originally sampled at 1000 Hz. As indicated in the figure the impact peak was measured at 1345 N. Every 20^{th} point was then extracted to simulate data sampled at 50 Hz. The peak value occurred between samples and the nearest data point was 1316 N. This also changed the time of occurrence of the impact peak. After applying the reconstruction formula to the 50 Hz data the peak was restored to 1344 N with the same time of occurrence as the originally sampled data. With modern computers there is little reason to under-sample a signal unless there is a limitation to the hardware. Such is often the case when collecting kinematic data from video. The video cameras sampling rate may be limited to 60 or 120 Hz.

Insuring Circular Continuity

In order for the resampling formula to work correctly, the data must have circular continuity. To understand circular continuity draw a curve on a piece of paper and then form a tube with the curve on the outside. Circular continuity exists if there is no "discontinuity" where the start of the curve meets the end of the curve. This means the first point on the curve must be equal to the last point. But the principle of circular continuity goes further, the slope of the curve at the start must equal the slope of the curve at the end. The slope of the slopes (the second derivative) must also be continuous. If you do not have circular continuity and you apply Shannon's reconstruction algorithm you may be violating the assumption that only frequencies less than half the sampling frequency are present in the data. Discontinuities are by definition high frequency changes in the data. If this occurs you will see oscillations in the reconstructed data that have high amplitudes at the endpoints of the curve. These oscillations will become smaller (damped) the further you get from the endpoints and they will become much more evident if derivatives are calculated.

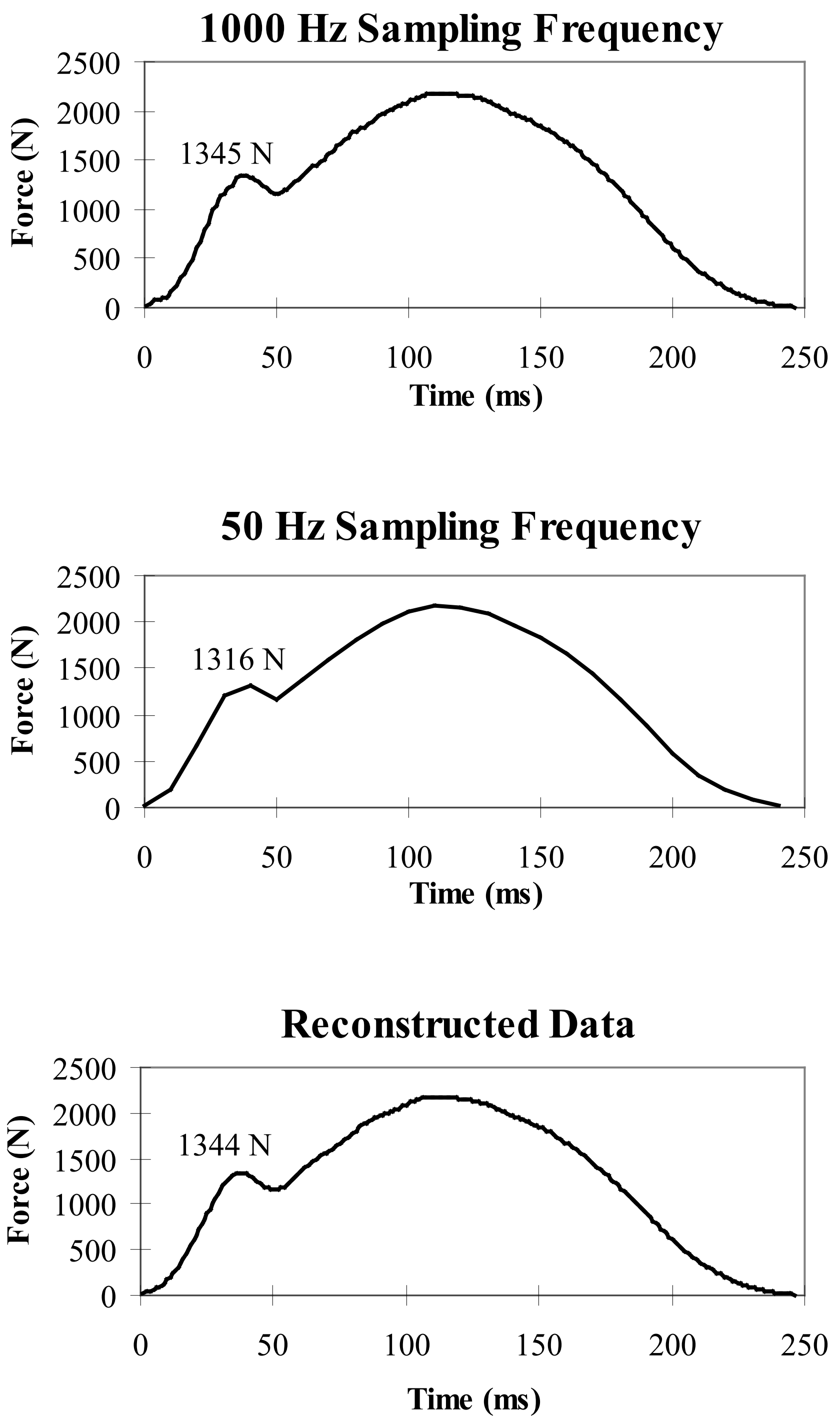


Figure 3. 6. From Hamill, Caldwell and Derrick, 1997. A running vertical ground reaction force curve sampled at 1000 Hz, sampled at 50 Hz, and sampled at 50 Hz then reconstructed at 1000 Hz. The magnitude of the impact peak is identified in each graph. Reconstructing the signal results in near a peak value very close to the original.

The following steps (Derrick, 1998) can be taken to approximate circular continuity (see Figure 3.7):

1. Split the data into two halves.
2. Copy the first half of the data and reverse and invert it. Attach this segment to the front of the original data.
3. Copy the second half of the data and reverse and invert it. Attach this segment to the back of the original data.
4. Subtract the trend line from the first data point to the last data point.

Reversal of the first or second half of data is a procedure by which the first data point becomes the last data point of the segment, the second data point become the second to last, etc. Inversion is a procedure that flips the magnitudes about a pivot point. The pivot point is the point closest in proximity to the original data. The figure below shows a schematic diagram of the data after the front and back segments are added and before the trend line is subtracted.

Step 2 insures continuity at the start of the original data set. Step 3 insures continuity at the end of the original data set. Step 2 and 3 together insure that the slopes at the start and end of the new data set are continuous but it is still possible to have a gap between the magnitude of the first point and the magnitude of the last point of the new data set. Step 4 removes this gap by calculating the difference between the trend line and each data point. Thus the first and last points will be equal to zero.

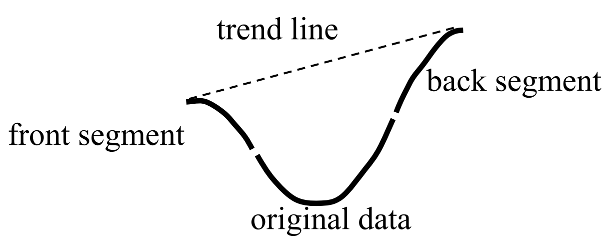


Figure 3. 7. Schematic diagram of the procedure used to insure a signal has the property of circular continuity.

If you fail to heed the sampling theorem you not only lose the higher frequencies, but the frequencies above the $\frac{1}{2}F_N$ actually fold back into the spectrum. In the time domain this is referred to as alaising. An anti-alaising filter is sometimes applied to a signal before it is digitized to insure that there are no frequencies above $\frac{1}{2}F_N$.

Smoothing Data

Why do we need to smooth the data?

There is error associated with the measurement of a signal. This error may be the result of skin movement, incorrect digitization, electrical interference, movement artifact in moving wires, etc. This error or 'noise' often has characteristics that are different from the signal. It is typically non-deterministic, lower in amplitude and often in a frequency range that is different than the signal. For instance, error associated with errors in the digitizing process is generally higher in frequency than human movement. Noise that has frequencies different than those in the signal can be removed. If you were to plot the signal and the signal + noise, it would look like figure 3.8. The goal of smoothing is to eliminate the noise while leaving the signal unaffected.

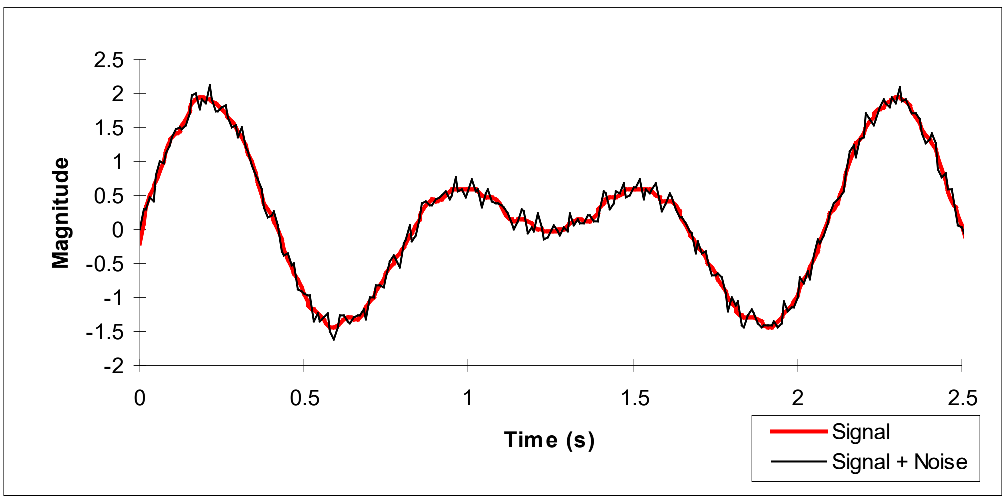


Figure 3. 8. A biological signal with and without noise.

<u>Smoothing Techniques</u>

Polynomial smoothing

Any *n* data points can be fit with a polynomial of degree *n-1* of the following form:

$$x(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + \ldots\ldots\ldots\ldots + a_{n-1}t^{n-1}$$

This polynomial will go through each of the *n* data points, thus no smoothing has been accomplished. Smoothing occurs by eliminating the higher order terms. This restricts the polynomial to lower frequency changes and thus it will not be able to go through all of the data points. Most human movement can be described with a 9th order polynomial or less. This method produces a single set of coefficients that represent the entire data set, so there is a large savings in storage space. The polynomial also has the advantages of allowing you to interpolate points at different time intervals and it makes the calculation of derivatives relatively easy.

Splines

A spline function consists of a number of low order polynomials that are pieced together in such a way that they form a smooth line. Cubic (3^{rd} order) and quintic (4^{th} order) splines are most popular for biomechanics applications.

Fourier smoothing

Fourier smoothing consists of transforming the data into the frequency domain, eliminating the unwanted frequency coefficients and then performing an inverse transform to reconstruct the original data without the noise.

Moving average

A 3-point moving average can be accomplished by replacing each data point (n) by the average of $n-1$, n and $n+1$. A 5-point moving average will utilize the data points $n-2$, $n-1$, n, $n+1$, $n+2$ and will result in more smoothing than a 3-point moving average. Note that there will be undefined values at the start and end of the series.

Digital filtering

A digital filter is a type of weighted moving average. The points that are averaged are weighted by coefficients in a manner that the cut-off frequency can be determined. In the case of a low-pass filter, frequencies below the cut-off are attenuated while frequencies above the cut-off are unaffected.

Digital Filters

The type of digital filter is determined by the frequencies that are passed through. The following digital filters are all implemented in the same manner but the coefficients are adjusted for a particular cut-off frequency. Signals that are band-passed or notched are run through the filter with both a low-pass and a high-pass cut-off frequency.

Low-pass

The cut-off is selected so that low frequencies are unchanged while higher frequencies are attenuated. This is the most common filter type. It is often used to remove high frequencies from digitized kinematic data or as a digital anti-alaising filter.

High-pass

The cut-off is selected so that high frequencies are unchanged while lower frequencies are attenuated. Used as a component in band-pass and notch filters or to remove low-frequency movement artifact from low-voltage signals in wires that are attached to the body.

Band-pass

A band of frequencies are unchanged. A low cutoff indicates attenuation of frequencies below that level and a high cutoff indicates attenuation of frequencies above that level. Often used in electromyography when there is movement artifact in the low frequency range and noise in the high frequency range.

Notch

A band of frequencies are attenuated. A low cutoff indicates passing of frequencies below that level and a high cutoff indicates passing of frequencies above that level. Often used to remove 60 Hz powerline noise or other specific frequencies from a signal.

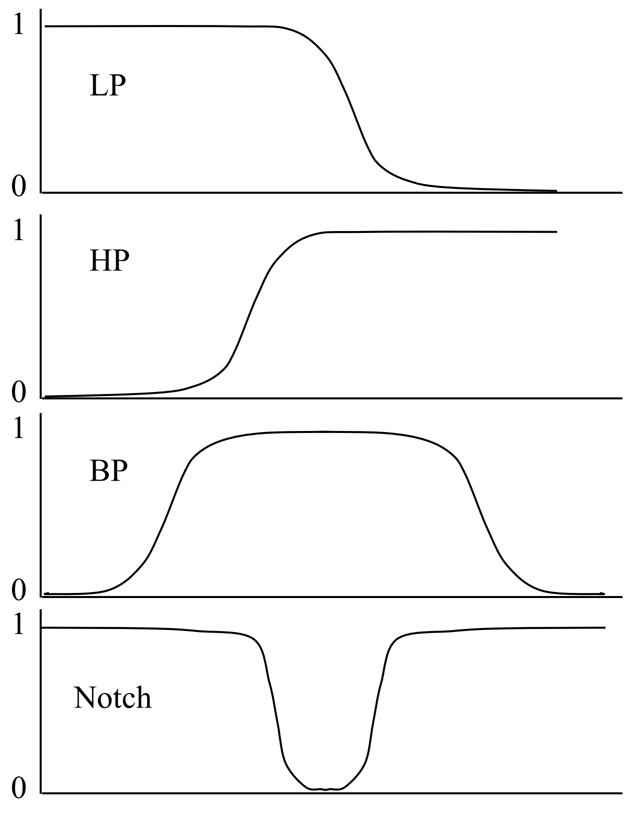


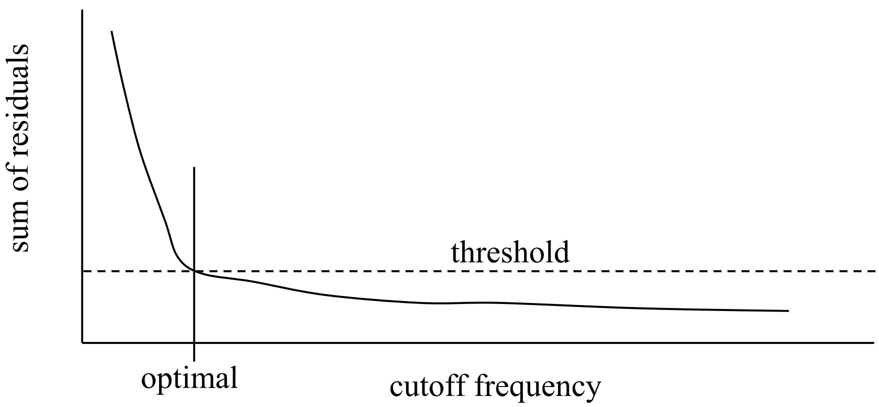
Figure 3.9. Frequency responses of different types of digital filters. The digital filter is implemented in the time domain but it can be visualized in the frequency domain. The frequency response function is multiplied by the signal in the frequency domain and the transformed back into the time domain.

The **roll-off** indicates the steepness of the frequency response. The higher the order (more coefficients) or the greater the number of times the signal is passed through the filter the sharper the roll-off. **Recursive filters** use some raw data and some data that has already been smoothed to calculate each new data point. Recursive filters are sometimes called infinite impulse response (IIR) filters. Non-recursive filters use only raw data points and are also called finite impulse response (FIR) filters. It is theoretically possible that a recursive filter will show some oscillations in some data sets but they have a sharper roll-off. Data that is smoothed using a recursive filter will have a phase lag. This phase lag can be removed by putting the data through the filter twice - once in the forward direction and once in the reverse direction. The filter is considered a 'zero lag' filter if the net phase shift is zero. Digital filters will distort the data at the start and end of a signal. To minimize these distortions extra data should be collected before and after the portion that will be analyzed.

Optimizing the cutoff

The selection of a cutoff frequency is very important when filtering the data. This is a somewhat subjective determination that is based on your knowledge of the signal and the noise. There have been a number of algorithms used to try to find a more objective criterion for determining the cutoff frequency (Jackson, 1979). These optimizing algorithms are typically based on an analysis of the residuals. The residuals are what is left over when you subtract the smooth data from the raw data. Some of these values should be greater than zero and some should be less than zero as long as only noise is being filtered. The sum of all of the residuals should equal zero (or at least be close). When the signal starts to be filtered along with the noise, the sum of the residuals will no longer equal zero. Some optimization routines use this fact to determine at what frequency you can best distinguish signal from noise (Figure 3.10). These algorithms are not completely objective because you must still determine how close to zero the sum of the residual is before selecting the optimal cutoff frequency.

Figure 3.10. Selection of an optimal cutoff using residual analysis.



Steps For Designing a Digital Filter

The following steps can be used to design an underdamped, low-pass, recursive digital filter. Modifications for a critically damped and high-pass filter are also discussed.

Step 1: Convert the cutoff frequency (f_c) from Hertz to radians·seconds^{-1}.

$$\omega_A = 2\pi f_c$$

Step 2: Adjust the cutoff frequency to reduce "warping" due to the bilinear transformation.

$$\Omega_A = \tan\left[\frac{\omega_A}{2 \cdot \text{sample rate}}\right]$$

Step 3: Adjust the cutoff frequency for the number of passes (P). A pass is made each time the weighted averages are applied to the data. For every pass through the data a second pass must be made in the reverse direction to correct for the phase shift. Increasing the number of passes will increase the sharpness of the rolloff.

$$\Omega_N = \frac{\Omega_A}{\left[2^{\left(\frac{1}{P}\right)} - 1\right]^{0.25}}$$

Step 4: Calculate the coefficients.

$$C_1 = \frac{\Omega_N^2}{\left(1 + \sqrt{2}\Omega_N + \Omega_N^2\right)}$$

$$C_2 = \frac{2\Omega_N^2}{\left(1 + \sqrt{2}\Omega_N + \Omega_N^2\right)}$$

$$C_3 = \frac{\Omega_N^2}{\left(1 + \sqrt{2}\Omega_N + \Omega_N^2\right)}$$

$$C_4 = \frac{2\left(1 - \Omega_N\right)^2}{\left(1 + \sqrt{2}\Omega_N + \Omega_N^2\right)}$$

$$C_5 = \frac{\left(\sqrt{2}\Omega_N - \Omega_N^2 - 1\right)}{\left(1 + \sqrt{2}\Omega_N + \Omega_N^2\right)}$$

Step 5: Apply the coefficients to the data to implement the weighted moving average. Y_n values are filtered data and X_n values are unfiltered data. The filter is recursive because previously filtered data (Y_{n-1}) and Y_{n-2}) are used to calculate the current filtered data point (Y_n).

$$Y_n = C_1 X_{n-2} + C_2 X_{n-1} + C_3 X_n + C_4 Y_{n-1} + C_5 Y_{n-2}$$

This filter is underdamped (damping ratio = 0.707). A critically damped filter can be designed (damping ratio = 1) by changing √2 to 2 in each equation in step 4. The warping function must also be altered as follows:

$$\Omega_N = \frac{\Omega_A}{\left[2^{\left(\frac{1}{2P}\right)} - 1\right]^{0.5}}$$

In practice there is little difference between the underdamped and critically damped filter. The distinction can be seen in response to a step input (a function that transitions from 0 to 1 in a single step). The underdamped filter will produce an artificial minimum before the step and an artificial maximum after the step (Robertson, Barden and Dowling, 1992).

It is possible to calculate the coefficients such that the filter becomes a high-pass filter instead of a low-pass filter (Murphy and Robertson, 1992). The first step is to adjust the cutoff frequency by the following:

$$f_c = \frac{sf}{2} - f_{c\text{-old}}$$

where f_c is the new cut-off frequency, $f_{c\text{-old}}$ is the old cutoff frequency and sf is the sampling frequency.

The coefficients (C_1-C_5) are then calculated the same way that they were for the low-pass filter and then the following adjustments are made:

$$c_1 = C_1,\ c_2 = -C_2,\ c_3 = C_3,\ c_4 = -C_4 \text{ and } c_5 = C_5$$

where $c_1 - c_5$ are the coefficients for the high-pass filter.

Appendix A

Discrete Fourier Transform Subroutine

Visual Basic code to calculate the power spectrum. This method is slow but simple.

```
DIM s(numpnts), c(numpnts), h(numpnts), power(numpnts)

w = (2 * pi) / numpnts
m = numpnts / 2 + 1
FOR k = 1 TO m
  k1w = (k - 1) * w
  FOR j = 1 TO numpnts
    alpha = k1w * (j - 1)
    s(k) = s(k) + h(j) * SIN(alpha)
    c(k) = c(k) + h(j) * COS(alpha)
  NEXT j
  s(k) = 2 * s(k)
  c(k) = 2 * c(k)
  power(k) = s(k)^2 + c(k)^2
NEXT k
```

Appendix B

Shannon’s Reconstruction Subroutine

This is Visual Basic code to implement Shannon’s formula for reconstructing data sampled above the Nyquist rate.

‘olddelta = original sampling rate
‘newdelta = new sampling rate
‘samptime = duration of the trial
‘fc = Nyquist frequency
‘newpoints = number of reconstructed data points
‘oldpoints = original number of points
‘d!(n) = nth point of the original signal
‘s!(i) = ith point in the reconstructed signal

```
pi = 3.141159
samptime = (oldpoints - 1) * olddelta
newpoints = samptime / newdelta
fc = 1 / (2 * olddelta)
fc2 = 2 * fc
For i = 1 To newpoints
  t = (i - 1) * newdelta
  For n = 1 To oldpoints
   If (t - (n - 1) * olddelta) <> 0 Then
     m = Sin(fc2 * pi * (t - (n - 1) * olddelta)) / (pi * t - (n
     - 1) * olddelta))
   Else
     m = 1 / olddelta
   End If
   newdata(i) = newdata(i) + olddata(n) * m
  Next n
  newdata(i) = newdata(i) * olddelta
Next i
```

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