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Bounded Price Variation, Rational Expectations, and Endogenous Switching in the U.S. Corn Market

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Introduction

In recent years there has been considerable interest in modeling markets that are hypothesized to be in disequilibrium. Methods for estimating disequilibrium models have been investigated by Fair and Jaffee, Fair and Kelejian, Amemiya, and Maddala and Nelson (all 1974). Applications of disequilibrium analysis in specific market contexts have been reported by Laffont and Garcia (1977), Rosen and Quandt (1978), Ziemer and White (1982), Hay and Anderson (1988), and others.

These earlier studies of disequilibrium modeling have recently been extended in several important ways. To begin, Maddala has examined estimation methods for markets that are not in disequilibrium all of the time. For instance, government commodity programs offer guaranteed price supports to participating producers. If the market price falls below the support price, the government purchases stocks and the market is in disequilibrium. However, if the market price is above the support price, the government takes no action and the market is characterized by equilibrium. The result is that markets with government price supports will be in equilibrium part of the time (government stocks are zero) and will be in disequilibrium part of the time (government stocks are positive).[1] These models are referred to as bounded price variation models and are similar to the endogenous switching models examined by Maddala and Nelson (1975).

Another important extension of the basic disequilibrium model has been to producers forming rational expectations. Rational expectations models of markets with bounded price variation have been considered by Chanda and Maddala (1983), Shonkwiler and Maddala (1985), and Baxter (1987). The solution for the rational price predictor is complicated in these instances by the fact that producers must consider, among other things, the probability of market equilibrium. The result is that the rational predictor cannot be solved for in closed form. Although headway has been made in estimating such models by making simplifying assumptions about the expectations process, there has to date been no attempt to incorporate fully all information implied by the rational expectations hypothesis.

Although the task presented by incorporating the cross-equation restrictions implied by rationality in a bounded price variation model is formidable, it is not impossible. Progress has been made in estimating nonlinear rational expectations models. In particular, Fair and Taylor (1983) outline an iterative method for obtaining maximum likelihood (ML) estimates of nonlinear rational expectations models, a method that incorporates all information implied by the rationality hypothesis. The basic idea is that the analytical reduced form for the rational price predictor, as obtained in linear models, can be substituted for by numerical solutions of the rational predictor in a nonlinear model. The result is that all information is used in the estimation and formal tests of the resulting cross-equation restrictions can be conducted.

In this study, we formulate and estimate a model that includes both bounded price variation (e.g., occasional disequilibrium) and rational expectations for the U.S. corn economy, a market that has been influenced substantially by government intervention. Doing this builds upon the earlier foundations and empirical research of Maddala (1983), Shonkwiler and Maddala (1985), and others. A unique feature of this study is that all information implied by the rational expectations hypothesis is used in the estimation, something not yet achieved by previous research in this area. First, the theory of rational expectations for markets with bounded price variation is developed. A discussion of the estimation of these models, including two-stage estimators, full information estimators, and the Fair-Taylor approach, follows. The third section presents the results obtained after applying the estimation framework to the U.S. corn market. The final section reviews important conclusions of the study and their implications for future research.

Rational Expectations and Lower Price Bounds

Consider a market represented by a stochastic supply and demand system and an exogenously set lower price limit $\bar{P}_t$:

$$D_t = \alpha_1' X_{1t} + \alpha^* P_t + e_{1t}, \tag{1}$$

$$S_t = \beta_1' X_{2t} + \beta^* P_t^e + e_{2t}, \tag{2}$$

$$Q_t = D_t = S_t \quad \text{if } P_t \geq \bar{P}_t, \tag{3}$$

$$Q_t = D_t < S_t \quad \text{if } P_t < \bar{P}_t, \tag{4}$$

where D_t is quantity demanded, S_t is quantity supplied, Q_t is quantity transacted, P_t is the market clearing price, P_t^e is the rational expectation of price formed at the time production decisions are made, X_{1t} and X_{2t} are vectors of demand and supply shifters, respectively, and e_{1t} and e_{2t} are joint normally distributed random variables with mean zero and variance-covariance matrix Σ.

Given observations on P_t and $\bar{P}_t$, we can classify the data points belonging to equilibrium and those belonging to excess supply. Let τ_1 denote the set of observations where $P_t \geq \bar{P}_t$ and τ_2 the set of observation for which $P_t < \bar{P}_t$. If $P_t \geq \bar{P}_t$, then $Q_t = D_t = S_t$ and we have a simultaneous system given by Equations 1-3 with Q_t and P_t determined endogenously. If $P_t < \bar{P}_t$, then $Q_t = D_t < S_t$ and it is a disequilibrium model. In this case the observed market price is $\bar{P}_t$ but we still observe <u>both</u> D_t and S_t since we know the amount produced and the amount the government buys under the price support program.

This system describes a market for a commodity where government price supports truncate the price distribution. However, we have assumed that agents form rational expectations about the product price and use these expectations when making production decisions. The model must be closed, then, by incorporating the rational expectations assumption. In its general form, the rational price predictor can be written as

$$P_t^e = E_{t-1}(P_t|\Omega_{t-1})$$

where E_{t-1} is the expectation of price taken in period t-1, conditioned on information Ω_{t-1} available to decision makers at the time expectations are formed.

In the typical rational expectations model, the restricted reduced form of the structural system in Equations 1-3 is solved for in terms of expected price and then substituted for P_t^e in the supply equation (Wallis 1980, Goodwin and Sheffrin 1982, Shonkwiler and Emerson 1982). Using Equations 1-3, the restricted reduced form for price is

$$P_t = (\alpha^*)^{-1}(\beta_1'X_{2t} + \beta^* P_t^e - \alpha_1'X_{1t} + e_{2t} - e_{1t}). \qquad (5)$$

Taking conditional expectations of both sides of Equation 5, and collecting terms, gives the rational predictor for price:

$$P_t^* = (\alpha^* - \beta^*)^{-1} \; (\beta_1'X_{2t}^* - \alpha_1'X_{1t}^*), \qquad (6)$$

where X_{1t}^* and X_{2t}^* represent the expectations of demand and supply shifters formed in period t-1. The rational predictor in Equation 6 is not appropriate in the present case, however, since producers must also consider the possibility that the market will be in disequilibrium (that is, the support price is effective).

The first step in deriving the rational predictor for the model in Equations 1-4 is to define the truncated expectation of price. Using standard results for truncated normal distributions (Johnson and Kotz, 1970, pp. 81-87), the truncated rational price expectation can be shown to be

$$P_t^e = \bar{P}_t\Phi(K_t) + \sigma(2\pi)^{-1/2}\exp(-K_t^2/2) + P_{1t}^e[1 - \Phi(K_t)], \qquad (7)$$

where

$$P_{1t}^{e} = (\alpha^{*})^{-1}(\beta_1{}'X_{2t}^{*} + \beta^{*}P_t^{e} - \alpha_1{}'X_{1t}^{*}),$$

$$K_t = [\bar{P}_t - (\alpha^{*})^{-1}(\beta_1'X_{2t}^{*} + \beta^{*}P_t^{e} - \alpha_1'X_{1t}^{*})]/\sigma,$$

$$\sigma^2 = (\alpha^{*})^{-2}\text{var}(e_{2t} - e_{1t}),$$

and $\Phi(\cdot)$ and $\phi(\cdot)$ denote, respectively, the distribution and density functions of the standard normal. Here $1 - \Phi(K_t)$ is the probability π_t that the price support is not effective and $\Phi(K_t)$ is the probability $(1 - \pi_t)$ that the market will be in disequilibrium; that is,

$$\pi_t = 1 - \Phi(K_t) = 1 - \int_{K_t}^{\infty} \phi(z_t)dz_t. \tag{8}$$

By combining terms in Expressions 7 and 8, the rational price predictor P_t^e can be written as

$$P_t^e = (1-\pi_t)\bar{P}_t + \pi_t P_{1t}^{*}, \tag{9}$$

where

$$P_{1t}^{*} = E_{t-1}(P_t | P_t^e \geq \bar{P}_t) = P_{1t}^{e} + \sigma \frac{\phi(K_t)}{1 - \Phi(K_t)}.$$

This result is intuitively appealing since it specifies that, in the bounded price variation model, the rational predictor is a weighted average of the support price $\bar{P}_t$ and the expectation of the market clearing price P_{1t}^{*}, obtained under the assumption that the support price is <u>not</u> effective. Likewise, the weights are simply derived from the probability π_t that the support price will not hold. The rational predictor P_t^e in the bounded price variation model is obtained by the simultaneous solution of Equations 7 and 8. The resulting system is highly nonlinear, though, and an analytical solution cannot be obtained. In particular, note that P_t^e appears in both the right- and left-hand sides of Equation 7.

Estimation Methods

There are two possible alternatives for proceeding with the estimation. The first is to approximate the solution of Equations 7 and 8 with a general function of the form

$$P_t^e = f(X_{1t}^*, X_{2t}^*, \bar{P}_t) \qquad \text{if } P_t \geq \bar{P}_t \tag{10}$$
$$= \bar{P}_t \qquad \text{otherwise}$$

where in practice $f(\cdot)$ can be specified as a low-order polynomial of the expectations of the exogenous variables and the support price. Equation 10 could then be estimated by the tobit method and the predicted values used as instruments for estimating Equation 2.[2] Estimates of the rational price predictor are obtained in a manner similar to that described by Maddala (1983). In particular, the estimates $\hat{P}_t^e$ from the tobit model are substituted in the supply equation for the subset τ_1. For this purpose, note that:

$$\hat{P}_t^e = E(P_t | t \epsilon \tau_1) = \Pi Z_t^* + \sigma_v \frac{\phi(C_t)}{1 - \Phi(C_t)} , \tag{11}$$

where Π is the parameter vector associated with the tobit model in Equation 10; $Z_t^* = (X_{1t}^*, X_{2t}^*)'$ is a vector of expected values of exogenous variables; and $C_t = (\bar{P}_t - \Pi Z_t^*)/\sigma_v$. The instruments for the rational expectation of price can be obtained using the tobit estimates of $\hat{\Pi}$ and $\hat{\sigma}_v$ in combination with Equation 11.

Although this method can be used to obtain consistent estimates of the supply equation parameters, it does not use all information implied by the rationality assumption. The full impact of the prior information implied by rational expectations can only be obtained if the restrictions implied by the simultaneous solution of 7 and 8 are introduced into the estimation. The other alternative, then, is to solve 7 and 8 iteratively and to use these iterative solutions in the estimation.[3]

This is precisely the algorithm suggested by Fair and Taylor (1983) for solving and estimating nonlinear rational expectations models. Their approach is to obtain initial consistent estimates of the parameter vector $\Theta = (\alpha_1', \beta_1', \alpha^*, \beta^*)$. Using these initial estimates, and corresponding instruments for the expectations of the exogenous variables, numerical values of the rational predictor P_t^e can be obtained by solving the system in 7 and 8 using an iterative solution method such as Gauss-Seidel. The resulting expectation is then consistent with the underlying model structure much in the same way that it would be if the calculated restricted reduced form could be solved for explicitly. In this way, cross-equation restrictions resulting from the rational expectations hypothesis are incorporated fully in nonlinear rational expectations models. Maximum likelihood estimation procedures employing numerical derivatives can be used to obtain new estimates of the parameter vector Θ. The entire solution-estimation process is repeated iteratively until convergence is obtained.

It is important to obtain "good" starting values for the iterative estimation. A method for obtaining instruments for the rational price predictor using a tobit approximation to the reduced form has been described. Similar methods can also be used to obtain consistent estimates of demand equation parameters. The procedure is slightly different, though, since the disequilibrium effects of the support price must be incorporated. In this case, a correction for the nonzero means of the residuals in the two regimes (equilibrium versus excess supply) is required.[4] Following the procedures outlined in Maddala (1983), the demand equation can be written as

$$Q_t = \alpha_1' X_{1t} + \alpha^* P_t + \frac{\sigma_{2v}}{\sigma_v} \frac{\phi(C_t)}{1 - \Phi(C_t)} + \eta_t \text{ for } t\epsilon\tau_t, \qquad (12)$$

and

$$D_t = \alpha_1' X_{1t} + \alpha^* \bar{P}_t - \frac{\sigma_{2v}}{\sigma_v} \frac{\phi(C_t)}{\Phi(C_t)} + \eta_t \text{ for } t\epsilon\tau_2, \tag{13}$$

where $C_t = (P_t - \pi Z_t)/\sigma_v$ and the residual η_t now has a mean of zero. The term σ_{2v} represents the covariance between the error term of the demand equation and the error term of an unrestricted reduced form tobit regression used to obtain instruments for P_t. The two-stage consistent estimates of the demand equation can then be obtained by substituting the instruments for P_t and C_t obtained from the tobit regression into Equations 12 and 13 and then applying OLS.

To obtain maximum likelihood estimates, we must consider the effects of the endogenous switching regime on the likelihood function. The appropriate likelihood function for the price support model is given by

$$L = \prod_{\tau\epsilon\tau_1} f(Q_t, P_t) \cdot \prod_{\tau\epsilon\tau_2} g(D_t, S_t), \tag{14}$$

where $f(Q_t, P_t)$ is the joint density of Q_t and P_t derived from the joint density of (e_{1t}, e_{2t}) as in any simultaneous equations model, and $g(D_t, S_t)$ is the joint density of D_t and S_t derived from (e_{1t}, e_{2t}), treating $P_t = \bar{P}_t$ as exogenous. Note also that the Jacobian of the transformation for $f(Q_t, P_t)$ is $|\alpha^*|$, which is expected to be nonzero since α^* is in general nonzero. Likewise, the Jacobian of the transformation for $g(D_t, S_t)$ is unity.

In the present case, an "unconcentrated" log likelihood function must also be used, since changes in Σ affect the solution of Equations 7 and 8, and thereby the computed residuals. Apart from a constant, the unconcentrated log likelihood function can be written (before partitioning) as

$$L^* = \sum_{t=1}^{T} \log|J_t| - \frac{T}{2} \log|\Sigma| - \frac{1}{2} \sum_{t=1}^{T} e_t' \Sigma^{-1} e_t \tag{15}$$

where $e_t = (e_{1t}, e_{2t})'$ and J_t is the Jacobian of the transformation from (e_{1t}, e_{2t}) to (Q_t, P_t) or (D_t, S_t). The ML estimates are then obtained by maximizing L^* with respect to the parameters (Θ, Σ). With the Fair-Taylor method, each evaluation of L^* requires computing the expected value of P_t^e from Equations 7 and 8 for t = 1, ..., T. Nonlinear maximization routines such as the Davidon-Fletcher-Powell algorithm can then be used to maximize L^*.

Model Specification

The preceding procedures are applied to a structural model of the U.S. corn economy. A simplified two-equation, supply-demand framework is used and is similar to the one reported in Shonkwiler and Maddala (1985). Supply S_t is specified as total production: the product of yield and acres harvested. The demand equation represents the total demand for corn including exports, feed and food use, and stocks.

The specific form of the supply equation is then

$$S_t = b_0 + b_1 P_t^e + b_2 SORG_t + b_3 DRY_t + b_4 S_{t-1}.$$

Here P_t^e is the rational price expectation obtained by solving Equations 7 and 8 iteratively. The variable $SORG_t$ is sorghum yields and serves as a proxy variable for corn yields. There have been dramatic improvements in corn yields over the past 30 years due to the widespread adoption of fertilizers, pesticides, and hybrid plant varieties. Sorghum yields are then used primarily as a proxy variable for technological change. Sorghum yields may also serve as a measure of growing conditions during the production period. Initial estimates of the supply equation resulted in residual outliers for 1970, 1974, 1980, and 1983. These extremes were discounted in the final estimation by including a dummy variable, DRY_t. Finally, producers may not be able to adjust production fully during any given year due to fixed rotations, prior fertilizer and chemical applications, and other lags in adjustment. This hypothesized partial adjustment process is accounted for by including the lagged dependent variable S_{t-1} in the supply equation.

This specification for supply is a simplification of the economic decisions facing corn producers and several potentially important variables have been omitted. For instance, acreage diversions and set-asides have been an important feature of government programs throughout the period of analysis (Cochrane and Ryan 1976). In addition, deficiency payments and other direct government subsidies have become important in recent years. While these policy variables have implications for acres planted (Houck et al. 1976), their overall impact on production may be indeterminate because farmers have compensated for reduced acres by using more fertilizer and other inputs on their remaining land (Paarlberg 1980). The result is that total production may not be affected substantially by these land retirement programs.

Additionally, deficiency payments are determined on the basis of historical production patterns. Consequently, the payment of these subsidies will encourage producer participation in the government program, but the immediate impact on production may be negligible. Finally, prices for competing products such as soybeans are not included in the specification. This is because price supports have also been important in the soybean market and the inclusion of soybean price in the supply equation would unnecessarily complicate the model.

The demand equation is specified as

$$D_t = a_0 + a_1 P_t + a_2\ EXP_t + a_3\ INC_t,$$

where EXP_t represents corn exports and INC_t represents total disposable income. Income is used to reflect shifts in the derived demand for corn due to increased demand for livestock products. Exports are included as a separate explanatory variable to account for their largely exogenous growth during the period of analysis.[5] For those observations belonging to τ_1, Equation 3 applies and $D_t = S_t + STK_{t-1}$ where STK_{t-1} denotes beginning period stocks in all positions. For observations in τ_2 the above identity no longer holds, but total demand can be derived from total supply by considering total government purchases, denoted as CCC_t. Specifically,

$$D_t = S_t + STK_{t-1} - CCC_t.$$

Data for the 36 crop years 1950 through 1985 were used in the empirical estimation. The market price variable P_t is the average price of corn received by farmers before price support payments. Other data on production, stocks, income, support rates, exports, and sorghum yields were obtained from various USDA sources. Following Rausser and Riboud (1983), we set the market price equal to the support price during periods when the observed market price was below the loan rate. The result was that 19 years in the sample data were identified as belonging to the excess supply regime (τ_2) while 17 years were identified as market clearing (τ_1).

Before estimation, the expected values of the supply and demand shifters must be generated. Simple first-order autoregressive models were estimated and the fitted values were used as instruments for the expectations of EXP_t, INC_t, and $SORG_t$. The estimation results for these autoregressive models are reported in Table 1. The explanatory power of these models is acceptable with R^2 values in all cases exceeding 0.90.

Estimation Results

With this data, the two-stage and ML estimates of the endogenous switching model for the U.S. corn market were obtained. The results of the ML estimates, with rational expectations, are reported in Table 2. To facilitate comparison, an equilibrium model was also estimated with lagged corn price used in place of the rational predictor in the supply equation. The equilibrium model was estimated using two-stage least squares (TSLS). The dependent variable in the demand equation is simply production plus beginning stocks; no adjustment is made for CCC purchases during periods of disequilibrium.

All estimated coefficients have theoretically correct signs in both estimated models. In addition, all parameter estimates for both models are significantly different from zero

Table 1. Autoregressive models for exogenous variables

1. Corn exports

$$EXP_t = \underset{(204.789)}{73.740} + \underset{(0.045)}{0.952}\ EXP_{t-1}$$

$R^2 = 0.929$ $h = 0.776$

2. Total disposable income

$$INC_t = \underset{(28.084)}{-0.348} + \underset{(0.007)}{1.086}\ INC_{t-1}$$

$R^2 = 0.999$ $h = 1.218$

3. Sorghum yields

$$SORG_t = \underset{(4.507)}{3.579} + \underset{(0.051)}{0.942}\ SORG_{t-1}$$

$R^2 = 0.910$ $h = -0.566$

Note: Values in parentheses are standard errors and h is Durbin's h-statistic.

Table 2. Results of a supply and demand model for the U.S. corn market, 1950-85

Variable	Estimated Coefficients	
	TSLS	ML
Demand equation		
Constant	53.733	48.591
	(5.741)[a]	(2.596)
Price of corn, P_t	-16.189	-11.525
	(3.209)	(2.056)
Corn exports, EXP_t	1.680	0.979
	(0.291)	(0.273)
Personal income, INC_t	0.022	0.024
	(0.003)	(0.003)
Supply equation		
Constant	-8.083	-10.300
	(6.040)	(3.841)
Expected corn price, P_t^e	10.346	10.590
	(1.971)	(1.983)
Sorghum yields, $SORG_t$	0.491	0.476
	(0.114)	(0.129)
Lagged production, S_{t-1}	0.407	0.432
	(0.124)	(0.144)
Drought variable, DRY_t	-18.576	-23.415
	(3.584)	(4.784)

Note: P_t^e for the TSLS estimates is lagged corn price P_{t-1} and for the ML estimates is the computed rational expectation. The value of the log likelihood for the ML model is -146.352.

[a]Asymptotic standard errors appear in parentheses.

at usual significance levels. The goodness-of-fit measure for the ML model is the "generalized" R^2 originally proposed by Baxter and Cragg (1970). The coefficient obtained is 0.959, which indicates that the estimated bounded price variation model with rational expectations does a good job of explaining the data. By comparison, the conventional R^2 values for the TSLS model are 0.927 for the demand equation and 0.903 for the supply equation.

Interestingly, the own-price elasticity of demand for the TSLS model is -0.456, while the own-price elasticity for the endogenous switching model is -0.346. This result closely parallels that of Ziemer and White (1982); own-price demand elasticities are smaller for the disequilibrium model than those implied by the equilibrium specification. The other striking result is that the estimated price coefficients in the supply equations are remarkably close in magnitude. The own-price elasticity of supply for the equilibrium/cobweb model is 0.362, while the same elasticity for the endogenous switching/rational expectations model is 0.346. This would suggest that, at a minimum, the rational expectations hypothesis is operationally equivalent to the more commonly used naive expectations framework for modeling supply response in the corn market.

Testing Expectations

Additional insight can be obtained by formally testing the restrictions implied by rational expectations. As Fair and Taylor (1983, p. 1170) indicate, the usual likelihood ratio test of the cross-equation restrictions implied by rational expectations can be performed with nonlinear models when the expectations are computed iteratively. The calculated test statistic was 10.76, which is below the appropriate chi-square statistic with three degrees of freedom at the .01 level (11.34) but above the same statistic at the .05 significance level (7.82). While these results are somewhat mixed, they do provide additional evidence that the rational expectations hypothesis with bounded price variation is appropriate in the present context.

To further examine the implication of the rational expectations hypothesis, a single-equation least squares estimation of the supply equation using the computed rational expectation P_t^e from the ML model was performed. The validity of the single-equation version of the supply model with computed rational expectations was then checked against the TSLS supply equation with naive expectations and the supply model, which uses the tobit approximation to the rational price predictor.[6]

The relative performance of each model in relation to the alternatives was determined using non-nested hypotheses tests. The results of pairwise and joint Davidson and MacKinnon (1981) J tests are reported in Table 3. For the pairwise tests, the statistics reported in the column labeled Test 1 are for the null hypothesis that the first of the two compared models is true. The statistics in the column labeled Test 2 derive from the null hypothesis that the second model is true. An $\alpha = .05$ significance level was used for each of the tests.

Considering first the specification with computed rational expectations, the results in Table 3 indicate this model dominated all others in the pairwise comparisons. That is, the null hypothesis of rational expectations could not be rejected in favor of the alternative expectations models. Likewise, the specifications using the tobit approximation to the rational predictor and naive expectations are always rejected when the supply model with computed rational expectations is the alternative.

The lower half of Table 3 presents the results for joint J tests. With the joint tests, the null hypothesis that a specification is true is tested against all other alternatives simultaneously.[7]

The likelihood ratio test statistics resulting from these joint tests are reported in the column labeled Test 1. Again, the results indicate that the model specification using the computed rational price expectations could not be rejected when tested jointly against the alternatives. At the same time, the null hypotheses for the supply models, which used the tobit approximation to the expectations and lagged corn prices, were rejected in both instances.

Table 3. Pairwise and joint non-nested tests of alternative expectations hypotheses

Test Type	Models Compared	Test 1[a]	Test 2
Pairwise	computed rat. exp.-tobit approx.	-1.84[b]	6.59*
Pairwise	computed rat. exp.-naive exp.	-0.42	3.49*
Pairwise	tobit approx.-naive exp.	4.56*	1.52
Joint	computed rational expectations	0.22	--
Joint	tobit approximation	32.50**	--
Joint	naive expectations	12.29**	--

[a]The test statistics for Test 1 are for the null hypothesis that the first model listed in the comparison is true. Alternatively, the test statistics for the Test 2 column are derived under the null hypothesis that the second listed model is true.

[b]Under the null hypothesis for the pairwise tests, the test statistic is distributed as standard normal with a critical value of 1.96 at the .05 level. A single asterisk indicates the null hypothesis in the pairwise tests could be rejected. Under the null hypothesis for the joint tests, the test statistic is distributed as chi-square with two degrees of freedom, which is 5.99 at the .05 level.

The test results are conclusive. The supply equation, which uses computed rational expectations, dominates both the model that uses instruments derived from a tobit approximation to the expectation and the model that uses naive expectations. In addition, the results of a pairwise test indicate that the supply equation that uses a tobit approximation to the rational predictor dominates a model that uses naive expectations. Although these results were not obtained using the ML version of the bounded price variation model, they provide important evidence for the relevancy of this model in the U.S. corn market. There is also a long tradition of using lagged prices and other types of extrapolative predictors to generate expectations variables in models of agricultural supply (Askari and Cummings 1977). The results here suggest that these methods are inferior for estimating supply response in the corn market compared with the more informationally efficient rational expectations assumption. This is true even when the rational predictor is approximated using an unrestricted reduced form.

Predictive Capability

Although formal tests of alternative expectations hypotheses provide important insights, it follows that the best measure of model adequacy is its predictive capability. Simulation performance is especially important in the present case since several alternative hypotheses are embedded in each model; that is, bounded price variation and rational expectations versus equilibrium and naive expectations. Historical simulations were used to evaluate the ability of both the ML and TSLS models to explain movements in the endogenous variables.

Two common measures of forecast performance were used to assess the simulation performances of the estimated models. The first is root-mean-square error (RMSE), which is a measure of the deviation of the simulated value from its actual time path. The second evaluation measure involves an auxiliary regression of observed values for the endogenous variables on their respective simulated values (Cohen and Cyert 1961) of the form

$$Y_t = \delta_0 + \delta_1 \hat{Y}_t, \tag{16}$$

where Y_t is the actual value of an endogenous variable; $\hat{Y}_t$ is the corresponding estimated value from a nonstochastic model simulation; and δ_0 and δ_1 are parameters. Perfect simulation performance would be indicated by $\delta_0 = 0$, $\delta_1 = 1$, and $R^2 = 1$.

Performance measures were computed for production and price using both the ML and TSLS models; the results are presented in Table 4. Test results of the auxiliary regression equation for production indicate that both models generate unbiased supply predictions. That is, the null hypotheses that $\delta_0 = 0$ and $\delta_1 = 1$ could not be rejected at the $\alpha = .10$ level for either the ML or TSLS models. However, the R^2 for the ML model is 0.942 which is somewhat higher than the R^2 for the TSLS model (0.903). Alternatively, the computed RMSE for production is lower for the ML model. These results suggest that the ML model does a better job of predicting production than does the TSLS model, a conclusion that is consistent with the reported results.

The auxiliary regression equations and RMSEs for price were estimated only for equilibrium points. This is because price is not endogenous in the ML model during periods of disequilibrium and, by construction, the simulated price is identical to the observed price (the support rate). An immediate observation is that the ability to simulate price, as indicated by the R^2 values from the auxiliary regression estimates, is lower in both instances than is the ability to simulate production.[8] The R^2 for the ML auxiliary price equation is 0.697, while the same measure for the TSLS model is 0.615. Likewise, the RMSE for price from the ML model is 0.481, while the same value for the TSLS model is 0.489. Tests of the hypothesis that the auxiliary regression equation lies on a 45-degree line could not be rejected for the ML model, while the hypothesis that $\delta_1 = 1$ was rejected for the TSLS model. Thus, the ML model apparently produces unbiased price simulations during periods of equilibrium while the TSLS model does not. In addition, the ML model has a slight advantage in

Table 4. Predictive performance evaluations of the estimated supply and demand models.

Supply

ML Model

$$S_t = -2.012 + 1.073\ \hat{S}_{t1}$$
$$\quad (4.446)^{*} \quad (0.046)^{**}$$

$R^2 = 0.942$ RMSE = 4.696

TSLS Model

$$S_t = 0.423\text{x}10^{-5} + 1.000\ \hat{S}_{t2}$$
$$\quad (2.893)^{*} \quad (0.056)^{**}$$

$R^2 = 0.903$ RMSE = 5.605

Price

ML Model (Equilibrium)

$$P_t = 0.243 + 0.795\ \hat{P}_{t1}$$
$$\quad (0.415)^{*} (0.136)^{**}$$

$R^2 = 0.697$ RMSE = 0.481

TSLS Model (Equilibrium)

$$P_t = 0.566 + 0.722\ \hat{P}_{t2}$$
$$\quad (0.468)^{*} (0.147)^{**}$$

$R^2 = 0.615$ RMSE = 0.489

Note: $\hat{S}_{t1}$ and $\hat{P}_{t1}$ refer, respectively, to the production and price predictions from the ML model. S_{t2} and P_{t2} are the same predictions from the TSLS model. A single asterisk indicates not significantly different from zero at the $\alpha = .10$ level. A double asterisk indicates not significantly different from one, $\alpha = .10$. Values in parentheses are standard errors.

simulating price levels relative to the TSLS model. These results suggest that the bounded price variation model with rational expectations does a better job of simulating actual values than does the equilibrium model with naive expectations.

Conclusions

In recent years, there have been several important extensions of the basic disequilibrium model. In particular, disequilibrium analysis has been extended to markets with bounded price variation. These are markets subjected to exogenously determined price limits or supports. In addition, the rational expectations hypothesis also has been investigated in a bounded price variation framework. Although some progress has been made in applying these models to empirical data, we know of no attempt to incorporate fully all information implied by the rationality assumption.

In this study we have applied these recent advances in disequilibrium modeling and rational expectations theory to estimate a simple model of the U.S. corn market. An important feature was that <u>all</u> information implied by rational expectations was incorporated in the estimation. This was accomplished by using the Fair-Taylor iterative estimation method for obtaining maximum likelihood estimates of nonlinear rational expectations models.

The results are encouraging and the empirical evidence indicates that the bounded price variation model with rational expectations performs better than a traditional equilibrium model with naive expectations. A formal test of the restrictions implied by the rational expectations hypothesis was conducted, with the result confirming the restrictions implied by rationality at the $\alpha = .01$ level. In addition, a single equation version of the supply model that used the calculated rational price predictor was tested against the TSLS supply equation and a supply equation that used a tobit approximation of the expectation using non-nested hypotheses tests. The results provide more evidence of the validity of the rational expectations hypothesis in the corn market.

Finally, the simulation performance of the bounded price variation model was contrasted with the equilibrium model. Again, the results indicate that the bounded price variation model simulates past price and production levels more accurately than does the equilibrium model.

Perhaps the most encouraging aspect of this study is that complicated, nonlinear rational expectations models can be successfully estimated and applied. In the past, researchers have been restricted to using linear structures or making simplifying assumptions to implement and test the rational expectations hypothesis. The results here indicate that the rationality assumption can be incorporated in a broader range of model specifications.

Endnotes

1. Throughout this study we abstract from problems created by the voluntary nature of price support programs. In practice, price supports are only offered to participating producers and, consequently, the price support may or may not be effective during any given year (in fact, average market prices were frequently below support prices during the 1950s and early 1960s). The implicit assumption used throughout this analysis, however, is that all producers are eligible to receive the support price.

2. This instrumental variables approach provides consistent estimates and is frequently used in practice to estimate complicated rational expectations models (Sargent 1975, Sargent and Wallace 1978).

3. A third alternative, pursued by Shonkwiler and Maddala (1985), is to assume that producers have perfect foresight with regards to whether or not the support price will be effective. Consequently, the probability π_t will either be one or zero, and the resulting model is linear. Equation 9 then applies and estimation proceeds as usual. The iterative solution-estimation method proposed here is superior to this approach, though, since π_t is determined endogenously.

4. Recall that supply is not a function of current price; hence no correction is needed.

5. The possibility exists that exports are also endogenous, so the demand equation is misspecified. To test for this possibility, we used a specification error test suggested by Spencer and Berk (1981) in which two versions of the demand equation are estimated. Under the null hypothesis, exports are assumed to be exogenous and thus orthogonal to

the error vector $\underline{e}_1$. For the alternative hypothesis, exports are removed from the instrument set and treated as an additional endogenous variable in the two-stage estimation. The specification error test yielded a chi-square test statistic of 4.64. Since the critical value of the chi-square distribution with five degrees of freedom at the 10 percent level is 9.24, we fail to reject the null hypothesis. This provides strong evidence that the demand equation used in this study is correctly specified and that exports can be taken as exogenous.

6. The OLS estimates of the supply equation with computed rational expectations are

$$
\begin{aligned}
S_t = & \underset{(3.261)}{-13.196} + \underset{(1.736)}{12.386}\, P_t^e + \underset{(0.096)}{0.492}\, SORG_t - \underset{(3.123)}{24.976}\, DRY_t \\
& + \underset{(0.100)}{0.415}\, S_{t-1}
\end{aligned}
$$

$$R^2 = 0.930,$$

where values in parentheses are standard errors. Likewise, the OLS estimates using the tobit approximation to the expectation are

$$
\begin{aligned}
S_t = & \underset{(4.484)}{-5.591} + \underset{(3.060)}{7.628}\, P_t^* + \underset{(0.143)}{0.455}\, SORG_t - \underset{(4.552)}{21.366}\, DRY_t \\
& + \underset{(0.170)}{0.492}\, S_{t-1}
\end{aligned}
$$

$$R^2 = 0.847.$$

The estimation results for the naive expectations model are listed in Table 2.

7. As pointed out by Davidson and MacKinnon (1981), the joint tests should, in general, be more useful than the pairwise tests, since inconclusive results will not be encountered.

8. Most of the simulation error for price occurred in one year, which apparently represents an outlier in the sample data.

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