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Pareto-improving transition to fully funded pensions under myopia

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Pareto-improving transition to fully funded pensions under myopia

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1 Introduction

Old-age pension (social security) programs are ubiquitous. Most have a significant unfunded, pay-as-you-go (PAYG) component: the working generations pay taxes to pay for a transfer (pension) to the retired, elderly. Many of these programs have survived a century and often absorb 5-15% of G.D.P. Yet, their *raison d'être* is a matter of some debate. (Blake, 2006).

The debate starts with Aaron (1966) and Samuelson (1975) who show there is no long-run welfare justification for introducing a permanent PAYG pension program if the economy is initially dynamically *efficient*. By their logic, PAYG pensions crowd out private saving, and therefore, can have a welfare rationale only in dynamically *in*efficient economies, those with a capital-overaccumulation problem. Since most real-world economies are thought of as dynamically efficient, the Aaron-Samuelson result leaves open the question, why are PAYG pensions so popular, or more bluntly, why not get rid of them and adopt mandated, fully funded (FF) schemes which offer higher returns under dynamic efficiency?

The literature quickly moved on to a variant of the above questions: how can the economy engineer a transition from an existing PAYG system to a fully funded one? Could such a move ever constitute a Pareto improvement since there would be a cohort that paid into the PAYG system, right before it is demolished, but will not see a benefit in return? Not to mention, the introduction of a PAYG scheme bestows the initial generation with a windfall, a "gift": the first round of retirees receive a pension not having contributed to the system.[1] The problem, as Brunner (1996) notes is that "in the transition phase the active would have to bear a *two-fold burden*: paying for the pensions of the retired and accumulating a sufficient stock of capital from which their own pensions could be financed." The upshot is that the introduction of a PAYG scheme came to be viewed as the "original sin": for *whatever* historical reason the sin has been committed, the PAYG scheme is installed, so the question now is, can we get rid of it, and if so, how?

In a highly influential paper, Boldrin and Montes (2005), and later Andersen and Bhattacharya (2017), argue that the 'original sin' line of thinking is limiting because it fails to acknowledge that PAYG schemes may have been introduced for a good reason, and as such, may play other significant roles, besides their role as a pension program.[2] In their view, PAYG pensions are best viewed, *non*-paternalistically, as *one* arm in a two-armed, intergenerational welfare state, the other arm being public education. The central insight of Boldrin and Montes (2005) is to bring the two arms together: tax the working, middle-aged to finance public education for the young

[1]The literature noted this – see Breyer (1989), Breyer and Straub (1993), Sinn (2000), Lindbeck and Persson (2003), and Blake (2006) and references therein.

[2]More broadly, social security serves multiple functions: it is a pension (or old-age support) program, it provides insurance (e.g., dependent survivor benefits), and it also brings about income redistribution – see Barr and Diamond (2006) and Krueger and Kubler (2006). As Barnett, Bhattacharya and Puhakka (2018) argue, "[...] while a social security system may touch on all three roles, its principal identity is (and has always been) intergenerational, its chief function, pension provision to the elderly. To reiterate, in its identity and function as the chief intermediator of intergenerational transfers, social security is unique." To be fair, the 'original sin' line of thinking focuses solely on the pension role of social security and rightly notes there are other better pension programs that could have been adopted. Our angle is that the forced-saving element in social security offers commitment value to time-inconsistent agents and, hence, the introduction of PAYG pensions may not have been a 'sin'.

and offer *those* middle-aged a compensating pension when old paid for by the publicly-educated next cohort of middle-aged.[3] Viewed this way, Andersen and Bhattacharya (2017) argue that a PAYG pension scheme, far from being the 'original sin', is to be viewed as the *just* compensation to the retired for prior financing of public education, and in the presence of an intergenerational education externality, "once it has served its purpose, it can be phased out and that too in a Pareto-improving manner."

This paper takes to heart the following ideas from Boldrin and Montes (2005) and Andersen and Bhattacharya (2017): a) it is important not to summarily dismiss a PAYG pension scheme as an "original sin" – any discussion of a transition to fully-funded systems must recognize the rationale for introducing the PAYG scheme in the first place,[4] and b) the transition must be Pareto-improving. Where this paper parts ways with Boldrin and Montes (2005) and Andersen and Bhattacharya (2017) is that it argues that the construct of a two-armed welfare state is *not necessary* to satisfy (a)-(b) above.

Specifically, this paper studies *another* popular, albeit paternalistic, rationale for public pensions – present bias – one that is preference-based (and widely observed) and therefore does not rely on the specific institutional structure of a welfare state described in Boldrin and Montes (2005).[5] Following Andersen and Bhattacharya (2011), it shows that in a dynamically efficient economy in which the PAYG scheme is dominated in rate of return by a FF scheme (i) sufficient present bias – specifically, myopia[6] – may rationalize a PAYG scheme in the first place, and as such, its introduction need not be an 'original sin', (ii) it is possible to phase out the PAYG scheme, even if it helps with self-control problems of myopic agents, and replace it eventually with mandated, return-dominant fully-funded pensions, and most importantly, iii) the transition can be Pareto-improving.

In order to develop some intuition on why these results hold and why present bias matters, consider for now a standard overlapping-generations model with exogenous factor prices, dynamic efficiency, no population growth, no present bias, no altruism, with a PAYG scheme al-

[3]In a dynamically efficient economy, PAYG pensions are long-run undesirable in the original-sin sense but are easier to introduce because of the aforementioned gift to the initial retired generation. On the other hand, public education is long-run desirable (especially if it contributes to human capital externalities) but troublesome to usher in because the current generation, educated under the previous regime, will be asked to finance it for the *next* generation but will see no benefit especially if they are not altruistic.

[4]"It is inadequate and potentially misleading to study the effects of Social Security in models in which there is no particular reason for Social Security to exist in the first place." (Diamond, 2004)

[5]Falk et. al. (2017) study global variation in economic preferences using the Global Preference Survey (GPS), an experimentally validated survey dataset of time preference, etc. from 80,000 individuals in 76 countries. In one query, participants were asked, "Would you rather receive amount x today or y in 12 months?" Their analysis of the data reveal substantial heterogeneity in preferences and evidence of present-bias across countries.

[6]Why myopia? "There seems to be an unstated belief that, left to their own devices, a sizeable fraction of households would inadequately save and insure." (Kotlikoff, 1987). Researchers – Feldstein (1985) – have argued that insufficient foresight (myopia) may be blamed, and PAYG pensions, via the forced-saving element inherent in them, may help such agents save adequately for retirement. In fact, such "paternalistically motivated forced savings constitutes an important, and to some the most important, rationale for social security retirement systems." (Kaplow, 2008). While it is true that myopia can, to some extent, explain the undersaving, Andersen and Bhattacharya (2011) argue that it is a leap from there to argue that forced saving via PAYG pensions is the cure: only if agents are *sufficiently* myopic will they welcome PAYG pensions.

ready in place, an original sin if you will. Suppose the economy is in a steady state. Now, consider a possible transition to a FF scheme: a gradual phasing out of the PAYG scheme and its replacement by a FF scheme. The initial young generation are contributing to the status-quo PAYG scheme with return 1 (since the net population growth rate is zero) the proceeds of which are used to pay a pension to the existing generation of retirees. In addition, they are optimally saving on their own at the market return, $R > 1$ (dynamic efficiency). Under the proposed scheme, they are, in addition, mandated to contribute into a FF scheme at return, R. How do the young react? They simply cut their own saving, one for one, and switch to saving via the FF scheme. This switch has no welfare consequences for them as long as their own saving is in the interior. If there was a way to cut their PAYG contribution and channel it toward the FF scheme, then the young would benefit from the return difference, R vs 1, but then the initial generation of retirees would be hurt. As such, a Pareto-neutral transition is not possible under the circumstances.

Now, assume all is same as before except agents suffer from present-bias (myopia) and hence, are saving "too little" on their own. Such agents benefit from the forced-saving character of the existing PAYG scheme.[7] The young, however, would welcome a change to a mandated, FF scheme because, in addition to the forced-saving benefit, it offers a higher return, R vs 1. This generates a welfare gain for the inaugural young which means, under the Pareto criterion, *their* PAYG benefit maybe reduced (parenthetically, the *next* young's PAYG (FF) contribution can be reduced (increased)). The initial generation of retirees is unaffected, future generations contribute less and less to (and receive less and less from) the PAYG and lean more toward the higher return, FF scheme. Eventually, the former is phased out, the latter holds sway, and no one is hurt in the transition.

Matters are considerably more complex when factor prices (wages and interest rates) are endogenously determined. Now, when the initial young are mandated to contribute to a FF scheme, more capital is accumulated but its return next period is lowered. Under a simple condition on the production function that ensures capital income is non-decreasing in capital, we show that the initial young will see more retirement consumption from the FF scheme than under the status-quo PAYG scheme. Also, as before, they welcome this postponement of consumption because it reduces the severity of their present bias problem. Overall, there is a welfare gain which may be "taxed" under the Pareto criterion to reduce the PAYG contribution of the initial young and raise their mandated contribution to the FF scheme. Subsequent generations, in addition to the benefits discussed above, also enjoy higher wages due to the increase in the capital stock. A Pareto-improving transition is set in motion.

The key intuition is that there is a forced-saving element in both schemes. Both are desirable to myopic agents: both help with their undersaving problem by making it easier to postpone consumption. It's just that the FF scheme, in addition, offers *higher* retirement consumption, and hence, is more efficient at the same task; the resultant welfare gain helps pay for the Pareto-

[7]We show below that there does not exist a PAYG scheme that *entirely* cures the undersaving problem; a suitable FF program, however, exists. As such, tweaking with the PAYG program but leaving it in place is not a great option.

restrained transition.[8]

Our analysis informs the discussion on pension policy design in two major ways. First, PAYG programs are being challenged in many countries. The OECD Pensions Outlook (2012) argues pension reforms since the mid 1980s have led "to a reduction in public pension promises in many (OECD) countries, typically between a fifth and a quarter", that these cuts "call for longer working periods and an expanded role for funded, private pensions", and that most OECD countries "have already moved or are moving towards a more diversified system, where pay-as-you-go (PAYG) pensions need to be complemented with fully funded pension arrangements...". And yet, policy makers face the conundrum that establishing/expanding funded schemes takes time, and does not solve the at-hand problem of supporting current pensioners.[9] These hurdles have historically rationalized PAYG schemes, and these rationales remain potent to this day. This paper shows a way to reconcile the immediate needs of current pensioners born under an inefficient, unfunded scheme with the long-run aim of establishing an efficient, funded scheme.

Second, on the subject of pension design, the paper sheds some light on the issue, how should a country usher in old-age security policies if none exist to begin with. Should it, for example, simply start things off with a FF scheme? There again, the answer, depends on the raison d'être for an old-age security policy. If, as was the case in the U.S., "the immediate need to provide immediate assistance to destitute aged individuals" is paramount, then starting with a FF scheme may not be that useful because it does not address the "immediate need".[10,11] Our suggestion would be to usher in both a PAYG and a FF scheme together. The initial young are asked contribute to the PAYG scheme whose proceeds are transferred to the current, retired, they also contribute to the FF scheme. The conventional "double burden" concern argues against our suggestion. We point out why the conventional line of thinking may be limited in its scope because it does not account for the welfare gains due to postponed consumption that accrues to present-biased agents discussed heretofore. It is these gains that allow the FF scheme to enter

[8]Our insistence that both the original PAYG and the subsequent FF scheme be optimal is crucial. Suppose the incumbent PAYG scheme is not optimal, agents have standard preferences (no myopia) and they are at a zero-saving corner. Such agents do not like the PAYG scheme because it force saves for them but they would benefit from a transition to a higher-return, albeit suboptimal FF scheme. In our case, from the vantage of true utility, agents like the policy reforms.

[9]In some countries, there has even been a retreat in that resources from funded schemes are being transferred to current pensioners. Example include Estonia, Latvia, Lithuania, Poland and Romania where a larger share of the contributions are allocated to the PAYG scheme, and Argentina and Hungary having dissolved funded schemes, OECD (2015, 2017).

[10]In the case of the United States: "The Social Security Act established two types of provisions for old-age security: (1) Federal aid to the States to enable them to provide cash pensions to their needy aged, and (2) a system of Federal old-age benefits for retired workers. The first measure was designed to provide immediate assistance to destitute aged individuals." https://www.ssa.gov/history/50ed.html

[11]Interestingly the famous Beveridge report proposed a mandatory funded pension scheme, where contributions paid over the work-life were to be set on an actuarial basis to ensure the pension would be above some absolute poverty threshold. This scheme was not introduced since it would offer no pensions to those already old, something which could be achieved by the universal PAYG pension, see e.g. Bozio et al (2010). Bismarck's initial idea was also to establish a funded scheme, but for the same reason as in the UK, the scheme was set-up as a PAYG scheme, see Scheubel (2013).

and push out the PAYG scheme without hurting anyone.

The rest of the paper is organized as follows. Section 2 reviews the literature while Section 3 lays out the model in its general form, allowing for both exogenous and endogenous factor prices. It derives the agents' decision rules while Section 4 studies the long-run optimal choices of schemes as well as the transition from a PAYG to a FF system assuming exogenous factor prices, the expositionally easier case to study. Section 5 does the same for endogenous factor prices. Some concluding remarks are listed in Section 6. Proofs of all major results are to be found in the appendices.

2 Literature review

A quick review of the surrounding literature is in order. To start with, in the literature on time-inconsistent agents with multi-selves in dynamic conflict, a 'sophisticated self' may seek commitment devices, such as mandatory pensions, to help his future selves stick to his better judgment about retirement saving – see Summers (1989), Laibson et. al. (1998), Imrohoroglu et al (2003), Fehr et. al. (2008), and Kaplow (2008). The agent uses the commitment device, ends up with more retirement wealth, and is made better off. The quantitative side to these issues is studied in Kumru et. al (2011) and Caliendo and Gahramanov (2013). At the same time, it is well understood that, under perfect capital markets, individuals can offset the mandated saving (inherent in PAYG systems) by reducing their own saving – if need be, even borrow against their future pension wealth – leaving total retirement savings unchanged, and inadequate, just as before. Andersen and Bhattacharya (2011) show that the mandated part crowds out voluntary saving and only if they are sufficiently myopic does a welfare case arise.

There is a large literature on the possibility of transiting from PAYG pensions to FF pensions. That literature assumes that the PAYG scheme is in place and discusses whether a transition to FF is possible under the Pareto-criterion even though there is no welfare rationale for the PAYG in the first place. Moreover, it assumed that voluntary retirement savings is adequate which means this literature is somehow detached from the other branch of the pension literature focusing on "under saving".

Sinn (2000) – others, such as Feldstein and Leibman (2002) make similar points – argues that the cost of PAYG pension has to be recovered by future generations either as an implicit debt in the PAYG pension scheme (the return difference is the implicit tax to pay the initial debt) or an explicit debt. It cannot be escaped by transition; once the PAYG scheme has been implemented, it has inevitable consequences. A reduction of tax distortions has been suggested as an side-benefit which may make transition possible under the Pareto-criterion, see Breyer and Straub (1993). The idea is that contributions to PAYG pensions distort labor supply, while contribution to a FF scheme do not. The former does not have an individualized link between contributions and entitlements, while the latter has – see Homburg (1990), Breyer and Straub (1993), Fenge (1995). Damjanovic (2006) provides an overview. Hence, a transition may lower tax distortions thereby producing gains which can be used to make the transition feasible under the Pareto-criterion.

Privatization of PAYG schemes is analyzed in a number of quantitative analyses – see e.g. Fehr et. al (2008), Nishiyama and Smetters (2007, 2014), Werding et. al. (2018), Frassi et. al. (2018), Kumru and Thanopoulos (2011) and it is generally found that this is generally not possible under the Pareto criterion. These studies also includes various reasons for having a PAYG-pension scheme, including present-biased preferences as well as insurance of both income and longevity.

A number of quantitative studies have considered the transition path following pension reform, including a privatization of PAYG schemes, see Auerbach and Kotlikoff (1987), Nishiyama and Smetters (2005, 2007, 2014), Fehr and Haberman (2008) and Fehr et. al. (2008). The procedure here is, first find the equilibrium trajectory and associated life-time utility for current and future cohorts given the reform. Then, in a separate simulation, impose lump-sum transfers or taxes to equalize post-reform life-time utilities to pre-reform utilities. If the present value of these lump-sum taxes/transfers is positive, the Hicks-Kaldor criterion ensuring the possibility of a welfare improvement is satisfied, i.e., the gainers from the reform can, potentially, compensate the losers. These compensations are hypothetical in the sense that were they to be *actually implemented*, as part of the policy package, the post-reform equilibrium trajectory and associated utilities would be different than the ones used in the Hicks-Kaldor criterion calculations. Our approach differs because we implement the actual policy and explicitly impose that utilities should be no less than in the pre-reform case along the actual, *not hypothetical*, transition path. This is a non-trivial task when market returns are endogenous.

Finally, there is large body of work – Kaganovich and Zilcha (2012), Ono and Uchida (2016), Lancia and Russo (2016), and Bishnu and Wang (2017) – that studies the political economy of co-existence of the twin institutions of public education and public pensions but is not concerned with the transition from PAYG to FF pensions.

3 The model economy

We begin by laying out the model in its general form with endogenous factor prices and use it to present results both for exogenous and endogenous factor prices. The model is also set up to allow for pensions to be PAYG and/or fully funded (FF).

3.1 Primitives

Consider a closed, market economy, in the tradition of Diamond (1965), wherein, at each date $t = 1, 2, .., \infty$, a continuum of identical two period-lived agents is born. There is no population growth and the size of a cohort of newborns at any date is held fixed at 1. Agents consume both as young and old but work only as young. When old, they are retired: they consume whatever they have and die. When young, agents work in competitive labor markets at a wage w, consume, and save (s) for old age in perfect capital markets at the gross rate

Assumption 1 *(Dynamic efficiency)*

$$R_{t+1} > 1 \; \forall t$$

between t and $t+1$. In Section 5 below, we allow for market-determined, endogenous factor prices. There, the single final good is produced using a standard neoclassical production function $F(K_t, L_t)$ where K_t denotes the capital input and L_t denotes the labor input at t. The final good can either be consumed in the period it is produced, or it can be saved to yield capital at the beginning of the following period. Capital is assumed to depreciate 100% between periods. Let $k_t \equiv K_t/L_t$ denote the capital-labor ratio (capital per young agent). Then, output per young agent at time t may be expressed as $f(k_t)$ where $f(k_t) \equiv F(K_t/L_t, 1)$ is the intensive production function. We assume $f(0) = 0$, $f' > 0$ and $f'' < 0$, and that the usual Inada conditions hold. *Until further notice* though, we focus on exogenously-specified and constant w and R.

Following Chetty (2015), we draw a distinction between the "true" and "choice" utility of agents. Agents' behavior is dictated by their choice utility, but their actual well-being, our measure of welfare, is governed by their true lifetime utility. Let c^y denote consumption as young, and c^o consumption as old. The "true" preferences of the cohort who are young in period t, denoted with a "*", is the standard, separable

$$\Omega_t^* \equiv u(c_t^y) + \beta^* u(c_{t+1}^o) \tag{1}$$

where $\beta^* \in (0,1]$ is the true discount factor. The felicity function $u(\cdot)$ is assumed to fulfill standard assumptions, including $u'(\cdot) > 0$ and $u''(\cdot) < 0$ and Inada conditions. At points below, we will use the CES form: $u(c) = \frac{c^{1-\phi}}{1-\phi}, \phi \in (0,1)$.

Our yardstick for welfare is Ω^*. The choice preferences when young are given as $\Omega_t \equiv u(c_t^y) + \beta u(c_{t+1}^o)$ and myopia arises when

Assumption 2 *(Myopia)*

$$\beta < \beta^*$$

which is assumed, henceforth.

The MRS (marginal rate of substitution) measures the rate at which an agent wishes to substitute second period consumption for first period consumption. In our case, the choice MRS of an agent is given by $\frac{-u'(c^y)}{\beta u'(c^o)}$ and the true MRS is $\frac{-u'(c^y)}{\beta^* u'(c^o)}$. A myopic agent places less weight on the future ($\beta < \beta^*$), and therefore, cares relatively more about current consumption. Hence, the compensation (in second period consumption) he seeks for giving up a unit of first period consumption is higher the more myopic he is. His true indifference curve is flatter than his choice indifference curve.

The government is immune to the myopia of agents and is paternalistic – it decides on policy action using Ω^*. All young agents have access to a government-intermediated pension scheme wherein they contribute a lump-sum amount τ_t at date t and receive a pension of b_{t+1} at $t+1$. A pay-as-you-go (PAYG) pension satisfies $b_t = \tau_t$ (since the net population growth rate is assumed zero) while a fully-funded (FF) pension has $b_t = R\tau_{t-1}$.[12] Note that a myopic agent perceives the effective return on private saving as $R\beta$, and that on the PAYG scheme as β. To the government, these returns are higher, $R\beta^*$ and β^*, respectively.

3.2 Decision rules

The budget constraints of an agent are

$$\begin{aligned} c_t^y &= w_t - \tau_t - s_t, && (2) \\ c_{t+1}^o &= R_{t+1}s_t + b_{t+1}. && (3) \end{aligned}$$

The private saving decision is determined by

$$u'(w_t - \tau_t - s_t) = \beta R_{t+1} u'(R_{t+1}s_t + b_{t+1}) \text{ for } s_t > 0, \tag{4}$$

and at the zero private-saving corner by

$$u'(w_t - \tau_t) > \beta R_{t+1} u'(b_{t+1}) \text{ for } s_t = 0. \tag{5}$$

In line with the pensions literature, $s_t \geq 0$ is imposed. Agents do not have any wage income as old; all they have is either interest income from prior savings or pension payouts. Allowing negative saving is tantamount to allowing borrowing against future pensions which we disallow; in any case, such borrowing is not possible/allowed in many countries.[13]

For later use, note

$$\frac{\partial s_t}{\partial R_{t+1}} = -\frac{\beta u'(c_{t+1}^o) + \beta R_{t+1} s_t u''(c_{t+1}^o)}{u''(c_t^y) + \beta R_{t+1}^2 u''(c_{t+1}^o)} = -\frac{\beta u'(c_{t+1}^o)\left[1 + \frac{c_{t+1}^o u''(c_{t+1}^o)}{u'(c_{t+1}^o)}\right]}{u''(c_t^y) + \beta R_{t+1}^2 u''(c_{t+1}^o)} \text{ for } s_t > 0. \tag{6}$$

Hence, $\frac{\partial s_t}{\partial R_{t+1}} > 0$ holds when relative risk aversion is less than one,

Assumption 3 *(Risk-aversion)*

$$\phi(c_{t+1}^o) \equiv -\frac{c_{t+1}^o u''(c_{t+1}^o)}{u'(c_{t+1}^o)} < 1, \tag{7}$$

[12] Mandatory savings funds have access to the same capital market products as do private savers, and hence the returns are assumed to be identical. It may be argued that mandated schemes can deliver higher risk-adjusted net returns due to lower marketing and transactions (economies of scale) costs. On the other hand, the governance structure may distort the objectives of the investment policies in mandated pension funds.

[13] Andersen and Bhattacharya (2018) consider the possibility of borrowing, but at a rate higher than the savings rate. The return difference generates a "corner" solution for the savings decision. The present paper implictly assumes an infinite borrowing rate. In Andersen and Bhattacharya (2018) it is shown that there is no welfare case for compelling the individual to borrow in response to a high mandated savings requirement. As such, there is no loss in generality from the approach taken here.

a standard condition, well-known in the literature. Henceforth, we assume this is true.

In the absence of a pension scheme ($\tau = b = 0$), the private saving decision satisfies $u'(w_t - s_t) = R_{t+1}\beta u'(R_{t+1}s_t)$. In this case, the non-negativity constraint on saving is never binding because of Inada conditions. If $\beta = 0$, then, of course, $s_t = 0$ is possible; not otherwise. From the perspective of true utility, the optimal savings level s_t^* satisfies

$$u'(w_t - s_t^*) = R_{t+1}\beta^* u'(R_{t+1}s_t^*). \tag{8}$$

Myopia implies people place less weight on the future ($\beta < \beta^*$), and therefore, care relatively more about current consumption – such agents save too little, i.e., $s_t < s_t^*$. Indeed, $\frac{\partial s_t}{\partial \beta} > 0$ holds implying as β falls (the agent is more myopic), the less he saves, and the gap between his choice and true saving (s_t vs s_t^*) increases.[14] It is important to note that, in spite of $\frac{\partial s_t}{\partial \beta} > 0$, sufficiently high myopia (low-enough but still $\beta > 0$) *will not* drive agents to the own zero-saving corner; since $b = 0$, and the agent earns nothing when old, Inada conditions will prevent that. For future use, note that when $b > 0$, sufficiently high myopia *will* drive agents to the own zero-saving corner.

4 A role for pensions?

4.1 PAYG

The government is aware that a change in pension benefits affects private saving via changes in the agent's after-tax endowment and his future income. Focus attention on a steady state. The government takes the agent's optimal saving response to its pension into account, $s(b)$ and mandates a pension b by maximizing $\Omega^*(b) = u(w - s(b) - b) + \beta^* u(Rs(b) + b)$. The optimal level of b, if positive, is defined as the solution to

$$\frac{d\Omega^*(b)}{db} = -u'(c^y)\left[1 + \frac{\partial s(b)}{\partial b}\right] + \beta^* u'(c^o)\left[R\frac{\partial s(b)}{\partial b} + 1\right] = 0, \tag{9}$$

Using $u'(c^y) = \beta R u'(c^o)$, $\frac{d\Omega^*(b)}{db}$ reduces to

$$\frac{d\Omega^*(b)}{db} = u'(c^o)\left\{\beta^* - \beta R + R\frac{\partial s(b)}{\partial b}(\beta^* - \beta)\right\} \tag{10}$$

How does private saving respond to policy action? For the PAYG scheme, we find, in general,

$$\frac{\partial s_t}{\partial b} = -\frac{u''(c_t^y) + \beta R u''(c_{t+1}^o)}{u''(c_t^y) + \beta R^2 u''(c_{t+1}^o)} \in [-1, 0) \text{ for } s_t > 0 \tag{11}$$

[14] This neatly captures the argument that present-biased agents "under-save" – leaving "too little" for old-age consumption relative to what their true self wants.

leading to

$$\frac{\partial c^o_{t+1}}{\partial b} = [1-R]\,\frac{u''(c^y_t)}{u''(c^y_t)+\beta R^2 u''(c^o_{t+1})} < 0 \text{ for } s_t > 0 \text{ given } R > 1. \tag{12}$$

The PAYG pension is designed to supplement an agent's own saving for retirement. Recognizing that, the agent cuts his own saving as forced saving via the pension increases. If the forced pension and his voluntary saving earned the same return, he would cut his own saving one-for-one in response to an increase in the pension. However, under dynamic efficiency, an extra unit devoted to the pension brings less future income than what private saving would have. As such, he does not reduce his own saving one for one; the crowding out – cf. eq. (11) – is less than proportionate with the pension increase. Additionally, the present value of lifetime income under the pension, $w + \frac{1-R}{R}b$ falls as b rises (since $R > 1$), the agent's retired consumption falls, cf. eq. (12).

Focus attention on eq. (10). In the absence of myopia $(\beta = \beta^*)$, the second term on the r.h.s. of (10) drops out and hence the sign of $\frac{d\Omega^*(b)}{db}$ is the same as the sign of $1-R$.

Proposition 1 *[Aaron (1966), Samuelson (1975)] Under Assumption 1 (dynamic efficiency),* $\frac{d\Omega^*(b)}{db} < 0$ *if there is no myopia* $(\beta = \beta^*)$ *implying there is no welfare justification for introducing or expanding a PAYG scheme.*

Proof. See Andersen and Bhattacharya (2011). ■

In the absence of myopia, then, the optimal PAYG pension is $b = 0$. The agent dislikes the fact that his *total retirement income*, given by $Rs(b) + b$, *falls* with a rise in b.[15] This is clearly not what the government intended. Thankfully, the fall in $Rs + b$ stops once s hits zero. This is so because of the crowding out of private savings ($\frac{\partial s_t}{\partial b} \leq 0$ for $s_t > 0$); at a sufficiently high level of b, call it $\underline{b}$, the corresponding level of private saving is zero, $s_t = 0$. Thereafter, any further raising of b $(b \geq \underline{b})$ has no effect on s as the non-negativity constraint on s binds – total retirement income is simply b which clearly rises with b!

The question is, how does the presence of myopia help reinstate a role for PAYG pensions? Notice when myopia is absent, the second term on the r.h.s. of (10) drops out implying $\frac{\partial s(b)}{\partial b}$ ceases to have any effect on the choice of b. Intuitively, the envelope theorem washes out the effect of b on s. Not so, when myopia is present. In that case, the choice self views the effect of b on s *differently* from how the true self does – the true self discounts the effect on future saving at rate β^* *greater* than the rate at which the choice self discounts the same.

What about the first term on the r.h.s. of (10)? Since (11) tells us that $\frac{\partial s_t}{\partial b} < 0$ for $s_t > 0$, it follows, that in the presence of myopia, the second term on the r.h.s. of (10) is negative. This means, for $\frac{d\Omega^*(b)}{db} > 0$ or PAYG pensions to have a shot at improving true welfare, the first term on the r.h.s. of (10), $\beta^* - R\beta$, necessarily has to be positive. Equivalently, a necessary condition

[15] It is easy to check that

$$\frac{\partial}{\partial b}\left(Rs_t(b)+b\right)) = R\frac{\partial s_t}{\partial b} + 1 = \frac{u''(c^y_t)\,(1-R)}{u''(c^y_t)+\beta R^2 u''(c^o_{t+1})} < 0$$

for a welfare rationale for PAYG pensions is sufficiently-strong myopia, $\beta^* > R\beta \Leftrightarrow \beta < \beta^*/R$ – ordinary myopia, $\beta < \beta^*$, is not enough![16] Why? This is for the true self to benefit from the pension, the myopic agent's perceived effective return on private saving, $R\beta$, must be at least *less* than his true self's perceived return on the competing PAYG scheme, β^*. (In the absence of myopia, this is not possible under dynamic efficiency.) Otherwise, even the true self would prefer no pensions.

Even when myopia is sufficiently strong, how big does b need to be? Recall $s_t < s_t^*$ – from the standpoint of true utility, a myopic agent is anyway saving too little. From (11), we know the agent cuts s_t in response to the pension when $s_t > 0$. A PAYG pension crowds out own saving which the true self dislikes, but as b rises beyond $\underline{b}$, consumption during retirement rises and that makes such a b attractive from the perspective of true utility. Knowing the true self likes $b > \underline{b}$, what level of b should the government choose? In the present setting with exogenous factor prices, there is no inherent dynamics in the economy. In which case, the pension level may be set, right away, at its long-run optimal value, the one that solves $\max_b u(w-b) + \beta^* u(b)$ where

$$u'(w - b^*) \equiv \beta^* u'(b^*). \tag{13}$$

Note that b^* does not replicate s^*(defined in eq. (8)). We have

Lemma 1 *(Andersen and Bhattacharya, 2011) A necessary condition for the PAYG pension b^* – determined by (13) – to improve true welfare is $\beta^* > \beta R$. For CES utility, $u(c) = \frac{c^{1-\phi}}{1-\phi}, \phi \in (0,1)$, a sufficient condition for true welfare to increase,*

$$\Omega^*(b^*) > \Omega^*(0) \Longleftrightarrow \left(\frac{R + R\beta^{*\frac{1}{\phi}}}{R + (R\beta)^{\frac{1}{\phi}}} \right)^{\phi} > \frac{R + \frac{\beta^*}{\beta}(R\beta)^{\frac{1}{\phi}}}{R + (R\beta)^{\frac{1}{\phi}}} \tag{14}$$

holds.

Proof. *See Appendix A* ■

For it to be optimal, the PAYG pension has to be large enough to drive voluntary private saving to the corner. Increasing the PAYG beyond that point makes it possible to increase old-age consumption, and thus, counteract the effect of the myopia. However, since the PAYG scheme is return-dominated, myopia ($\frac{\beta^*}{\beta} > 1$) alone is not enough to deliver a welfare rationale for a PAYG pension. *Sufficiently strong* myopia relative to the rate of return ($\frac{\beta^*}{\beta} > R$) is required for a welfare improvement to be possible.

Note from (14) that risk-aversion (or intertemporal substitution) plays a role: if $\phi = 1$ or $\phi = 0$, then $\Omega^*(b^*) = \Omega^*(0)$ and in either case, b^* does not improve welfare. In Appendix A, we present a numerical example of a configuration of parameters satisfying $\beta^* > \beta R$ that also satisfies (14) for all $\phi \in (0,1)$.

[16] To see this clearly, suppose $\beta^* = (1+\epsilon)\beta$ where $\epsilon > 0$ but arbitrarily small. Then, $\beta^* > \beta$. But $\beta^* > \beta R$ requires $(1+\epsilon)\beta > \beta R \Leftrightarrow \epsilon > R - 1$ implying the gap between β and β^* cannot be arbitrarily small.

4.1.1 Fully funded pensions

Consider, next, a mandated fully funded pension scheme with contribution rate d ($\tau_t = d$ and $b_{t+1} = Rd$). The FF-pension also crowds out voluntary saving, and since the returns are the same, the crowding out is one-to-one, i.e., analogous to eq. (11), we have

$$\frac{\partial s}{\partial d} = -1 \text{ for } s > 0.$$

Hence, a mandated FF pension contribution only affects total saving if it is sufficiently large, $d \geq \underline{d}$. The critical contribution level $\underline{d}$ is defined by $u'(w - \underline{d}) \equiv R\beta u'(R\underline{d})$. The contribution rate maximizing long-run true welfare is determined by (assuming voluntary saving is driven to zero, $d \geq \underline{d}$)

$$\max_d u(w-d) + \beta^* u(Rd)$$

and the optimal level d^* is determined by the first order condition

$$u'(w - d^*) \equiv R\beta^* u'(Rd^*) \tag{15}$$

Notice $d^* = s^*$. This means a FF program with contribution equal to s^* can exactly replicate the desired retirement saving of the true self. Of course, private saving is zero but retirement saving under the FF scheme is *exactly* what true utility demands. It follows directly that

Lemma 2 *A fully funded pension with contribution rate* d^* *– determined by (15) – generates higher true steady-state utility when compared either to what is possible under laissez faire* ($\tau = b = 0$) *or an optimal PAYG pension* (b^*).

Note, the relation between the optimal PAYG pension, b^*, and the FF pension, d^*, is, in general, ambiguous ($b^* \gtreqless d^*$). [17]

4.2 Transition from a PAYG to a FF pension system

A PAYG pension has the advantage that it provides, up-front, the current old with a pension, and therefore offers an immediate remedy to their low old-age consumption problem. To that end, suppose the government introduces the long-run optimal PAYG pension at level b^*, cf. (13).

[17] The comparison is not trivial: the benefit of a higher return on the FF pension compared to the PAYG pension has to be weighed against the fact that an income effect may reduce the optimal saving/contribution level. We have

$$\frac{\partial d^*}{\partial R} = -\frac{\beta^* u'(Rd^*) + R\beta^* u''(Rd^*)d^*}{u''(w-d^*) + R^2\beta^* u''(Rd^*)} = -\frac{\beta^* u'(Rd^*)\left[1 - \phi(c^o)\right]}{u''(w-d^*) + R^2\beta^* u''(Rd^*)}$$

where, recall, $\phi(c^o) \equiv -\frac{c^o u''(c^o)}{u'(c^o)}$ is the measure of relative risk aversion (recall, $c^o = Rd^*$). The optimal mandated savings is increasing in the rate of return ($\frac{\partial d^*}{\partial R} > 0$) if the coefficient of relative risk aversion is less than one, c.f. Assumption 3. This, in turn, implies $d^* > b^*$, that is, the optimal FF pension is larger than the optimal PAYG pension since their size is the same if $R = 1$.

From Lemma 2, we know that continuing this program is not in the interest of long-run welfare: an optimal FF scheme would do better. The question is: is it possible to make the transition to a FF scheme under the Pareto criterion, the constraint that utility for each cohort remain at least as high had the PAYG pension scheme b^* persisted?

To operationalize this question, assume that the long-run optimal PAYG scheme (b^*) installed at t and kept in place up to date $t+m$ ($m>0$). Recall, this is consistent with every cohort up to $t+m$ being at the zero private-saving corner. Also recall for b^* to be welfare-improving, agents must display sufficiently strong myopia, i.e., $\frac{\beta^*}{\beta}>R$. At $t+m$, the government ushers in a FF scheme by mandating the *then* young to contribute $d_{t+m}>0$ to the scheme. (It is possible that private saving re-emerges, we denote it $s(b^*,d_{t+m})$); recall, though, any increase in d is crowded out one-for-one by a decline in s). Denote the PAYG pension to be received by these retirees by b_{t+m+1}, i.e., we allow for the possibility that the level of the PAYG scheme is changed after the FF scheme is introduced. True life-time utility for the cohort born at $t+m$ under this policy package is:

$$\Omega^*_{t+m}=u(w-b^*-d_{t+m}-s(b^*,d_{t+m}))+\beta^*u(Rs(b^*,d_{t+m})+Rd_{t+m}+b_{t+m+1}),$$

which may be rewritten as

$$\Omega^*_{t+m}=u(w-b^*-d_{t+m}-s(b^*,d_{t+m}))+\beta^*u(Rs(b^*,d_{t+m})+(R-1)\,d_{t+m}+d_{t+m}+b_{t+m+1}).$$

Note that the term $(R-1)\,d_{t+m}$ captures the return gain to switching from the PAYG to the FF scheme. To foreshadow, this extra income/welfare will be crucial for a successful transition under the Pareto criterion.

The first issue is whether the cohort born at $t+m$ sees a welfare gain from the mandate of a FF pension contribution, d_{t+m}, on top of their PAYG pension contribution b^*?

Lemma 3 *True life-time utility of cohort $t+m$, Ω^*_{t+m}, can be improved by mandating them to contribute at the margin to a FF scheme in addition to their PAYG contribution of b^*:*

$$\left.\frac{\partial\Omega^*_{t+m}}{\partial d_{t+m}}\right|_{d_{t+m}=0,b_{t+m+1}=b^*}=[R-1]\,\beta^*u'(b^*)>0 \textit{ given } R>1$$

Proof. *See Appendix B* ■

Note that with the PAYG pension at b^*, private saving is already at zero; hence adding on an incremental FF pension does not distort saving: $\frac{\partial s(b^*,d_{t+m})}{\partial d_{t+m}}=0$. The marginal unit earns R via the FF scheme which is better than what it would have earned under the PAYG scheme. Hence, on the margin, continuing the initial PAYG scheme and adding a (small) mandated FF pension makes the inaugural set of agents better off.

The welfare gain may be used to phase out the PAYG scheme under the Pareto criterion.

Define

$$\Omega_{t+m}^{*\text{PAYG}} \equiv u(w - b^*) + \beta^* u(b^*)$$

as the lifetime utility to cohort $t+m$ had the PAYG pension b^* continued unchanged. It follows, that for cohort $t+m$, the Pareto-condition is

$$\Omega_{t+m}^{*} \geq \Omega_{t+m}^{*\text{PAYG}}$$

where

$$\Omega_{t+m}^{*} \equiv u(w - b^* - d_{t+m}) + \beta^* u(Rd_{t+m} + b_{t+m+1})$$

where Ω_{t+m}^{*} captures the changes ushered in by adding a mandated FF contribution, d_{t+m}, over and above the PAYG contribution of b^* *as well as* the possibility that this cohort will see a PAYG benefit of b_{t+m+1} instead of b^*. Lemma 3 implies $\exists d_{t+m} > 0$ such that $\Omega_{t+m}^{*} \geq \Omega_{t+m}^{*\text{PAYG}}$ holds. Then, there exists a $b_{t+m+1} < b^*$ such that

$$u(w - b^* - d_{t+m}) + \beta^* u(Rd_{t+m} + b_{t+m+1}) = u(y - b^*) + \beta^* u(b^*) \equiv \Omega_{t+m}^{*\text{PAYG}}$$

Hence, the gains from phasing in the FF pension ($d_{t+m} > 0$) may be used, under the Pareto criterion, to bring *down* the level of the return-dominated PAYG pension ($b_{t+m+1} < b^*$).

The upshot is that both PAYG and mandated FF schemes are appropriate government interventions for myopic agents. The FF scheme is just better: an unit of funds taken from the PAYG contribution and shifted to the FF scheme produces a return gain. Yes, in a literal sense, it is true that a transition from a PAYG to a FF scheme requires some cohorts to "pay twice" but, unlike in the classical results (see Proposition 1) derived with time-consistent agents, such cohorts are not worse off.

What happens to generations further down the transition, indexed $t+m+j$ with $j > 0$? What does the trajectory of d and b look like under the Pareto criterion? Consider, for the sake of argument, *a* very simple, stylized scheme that sets $d_{t+m} = \kappa b^*$, $\kappa \in (0,1)$ so that

$$d_{t+m} + b^* = (1+\kappa)b^*$$
$$d_{t+m+j} + b_{t+m+j} = (1+\kappa)b^*; \ j > 0$$

i.e., right from the start of the transition, the overall contribution rate (summed across both pensions) is raised relative to the PAYG world. (Many other such schemes can be constructed – see below.) This implies, for example,

$$d_{t+m+1} = (1+\kappa)b^* - b_{t+m+1}$$

that is, if we generate a declining sequence for b_{t+m+1}, we automatically generate an increasing sequence for d_{t+m+1} – if the PAYG is phased out, the FF is phased in. The equal utility condition for period $t+m$, $\Omega^*_{t+m} = \Omega^{*\text{PAYG}}_{t+m}$ now reads

$$u(w-(1+\kappa)b^*) + \beta^* u(R\kappa b^* + b_{t+m+1}) = u(w-b^*) + \beta^* u(b^*);$$

apropos Lemma 3, there exists a $\kappa > 0$ ensuring $b_{t+m+1} < b^*$, the start if the declining sequence for b. Below, we show it is possible to engineer a transition which leaves *every* cohort along the transition at least as well off as in the counterfactual persistent PAYG scheme. This is our flagship result.

Proposition 2 *For an economy with an existing PAYG pension b^*, there exists a policy package $\{b_{t+m+j}, d_{t+m+j}\}_{j=0}^{\infty}$ implemented at $t+m$ which satisfies the Pareto condition*

$$\Omega^{*\textit{TRANSITION}}_{t+m+j} \equiv u(w-b_{t+m+j}-d_{t+m+j})+\beta^* u(Rd_{t+m+j}+b_{t+m+j+1}) \geq \Omega^{*\textit{PAYG}}_{t+m+j} \equiv u(w-b^*)+\beta^* u(b^*) \textit{ for all } j \geq 0 \tag{16}$$

with b_{t+m+j} following a decreasing sequence and reaching 0 in finite time, and d_{t+m+j} following an increasing sequence converging to d^*, allowing d^* to be implemented.

Proof. See Appendix C. ■

Along the transition path, the PAYG-pension is gradually phased out, and the FF pension expanded, ensuring cohorts have the same life-time true utility had the PAYG-world persisted. Eventually, the PAYG pension is fully phased out $(b_{t+m+j} = 0)$, and replaced by a FF pension $(d_{t+m+j} > 0$ for all $j \geq 0)$. Once that happens, all cohorts are necessarily better off than under the PAYG scheme. Notably, it is possible to implement d^* which delivers the optimal level of saving, s^*, from the point of view of the true self.

Of course, there may be multiple Pareto-improving transition paths. Proposition 2 outlines *a* particular path where utility for cohorts is kept at the PAYG level until the pension is fully phased out, after which, subsequent future cohorts get to enjoy higher utility. Other paths, where some of the future gains are distributed up-front such that all cohorts are strictly better off, are possible.

5 Endogenous factor prices

It is instructive to take stock of what we know up to this point and lay out the path for where we are headed. Under dynamic efficiency, the "empirically relevant" case, we know from Aaron (1966) and Samuelson (1975) that there is no welfare justification for introducing a PAYG scheme if $\beta = \beta^*$ (no myopia). Indeed, the optimal long-run PAYG pension is zero. It is also known that myopic agents save too little compared to what their true utility demands. Introducing a PAYG pension worsens matters: it causes them to further reduce their saving, as long as own saving is positive, thereby reducing old-age consumption and hurting true utility. Sufficiently strong myopia is necessary to justify PAYG pensions under dynamic efficiency. Once the long-run optimal PAYG pension is installed, it may be gotten rid of and gradually replaced by a FF scheme that offers a better return. The welfare of generations caught in the transition is not hurt relative to their life under the optimal PAYG scheme.

All this was accomplished for the case of exogenous factor prices. The case with endogenous factor prices is obviously more challenging since changes in the pension system trigger general equilibrium responses to wages and interest rates, which in turn, impact saving decisions. Recall, our approach differs from usual Kaldor-Hicks one because we implement the actual policy and explicitly impose that utilities be no less than in the pre-reform case along the actual, *not hypothetical*, transition path. This is a non-trivial task when factor prices are endogenous.

Most of the steady-state welfare results derived earlier retain their flavor. The focus though, as before, is on the role of PAYG and FF pensions in the lives of myopic agents, and the possibility of engineering a transition from a PAYG to a FF scheme under the Pareto criterion. However, before we get there, we settle up some issues regarding dynamic competitive equilibria for this economy.

5.1 Equilibrium

Henceforth, we assume factor markets are perfectly competitive, and thus, factors of production are paid their marginal product in each period, i.e.,

$$R_{t+1} = f'\left(k_{t+1}\right) \equiv R\left(k_{t+1}\right) \tag{17}$$

and

$$w_t = f\left(k_t\right) - k_t f'\left(k_t\right) \equiv \omega\left(k_t\right), \tag{18}$$

where $f'\left(k_t\right) > 0$ and $f''\left(k_t\right) < 0$ and, hence $R'\left(k_t\right) = f''\left(k_t\right) < 0$ and $\omega'\left(k_t\right) = -k_t f''\left(k_t\right) = -k_t R'\left(k_t\right) > 0$.

In several places below, we will assume

Assumption 4

$$\eta(k) \equiv \frac{R'\left(k\right)k}{R(k)} > -1$$

which holds for a standard production function such as the Cobb-Douglas, $f\left(k\right) = k^{\alpha}$, $\alpha \in (0,1)$ where $\eta(k) = \alpha - 1$. It also holds for a CES production function $f\left(k\right) = \left[\alpha k^{\frac{1-\sigma}{\sigma}} + (1-\alpha)\right]^{\frac{\sigma}{\sigma-1}}$ with $\alpha \in (0,1)$ and $\sigma \geq 1$.

In passing, note that $\frac{d(kR(k))}{dk} = R\left(k\right) + kR'\left(k\right) > 0 \Leftrightarrow \frac{kR'(k)}{R(k)} > -1 \Leftrightarrow \eta(k) > -1$. This means if k rises, capital income $\left(R\left(k\right) * k\right)$ also rises if Assumption 4 holds. This fact will be useful in

Proposition 5-6 below.

Throughout, a dynamically-efficient economy is assumed

Assumption 5 $R(k_t) > 1\,\forall t$

The economy without any government intervention – laissez faire – is identical to that studied in Diamond (1965). We have from (4), and using (17)-(18),

$$u'(\omega(k_t) - k_{t+1}) = R(k_{t+1})\,\beta u'(R(k_{t+1})\,k_{t+1})$$

which implicitly defines the equilibrium law of motion for $k: k_{t+1} = \psi(k_t, \beta, 0)$. In the case of a time-invariant PAYG scheme ($\tau_t = b_{t+1} = b > 0$), the equilibrium condition in the capital market is $k_{t+1} = s_t$, and hence, we have

$$u'(\omega(k_t) - k_{t+1} - b) = R(k_{t+1})\,\beta u'(R(k_{t+1})\,k_{t+1} + b) \tag{19}$$

which implicitly defines the equilibrium law of motion for k :

$$k_{t+1} = \psi(k_t, \beta, b). \tag{20}$$

All competitive equilibria with PAYG pensions are characterized by the sequence $\{k_{t+1}\}_{t=1}^{\infty}$ defined by (20) and the government budget constraint. For a FF scheme with contribution rate d ($\tau_t = d$ and $b_{t+1} = Rd$), the equilibrium condition in the capital market is $k_{t+1} = s_t + d_t$, and hence, the corresponding equilibrium law of motion for k is

$$u'(\omega(k_t) - k_{t+1}) = R(k_{t+1})\,\beta u'(R(k_{t+1})\,k_{t+1})$$

Define

1. k : the steady state capital-labor ratio in the absence of any government intervention, defined as laissez faire, the solution to $k = \psi(k, \beta, 0)$.

2. k^b : the steady state capital-labor ratio for a given PAYG pension b which solves $k^b = \psi\left(k^b, \beta, b\right)$.

3. k^* : the steady state capital-labor ratio in the absence of both myopia and pension which solves $k^* = \psi(k^*, \beta^*, 0)$.

We make all standard assumptions ensuring the existence of a unique and stable – see Appendix D – steady-state equilibrium, see e.g., de la Croix and Michel (2004):

Assumption 6 *(Stability)*

$$R'(k)\,\beta u'(c^o) + R(k)\,\beta u''(c^o)\left[R'(k)\,k + R(k)\right] + u''(c^y)\left[1 + R'(k)\,k\right] < 0 \tag{21}$$

In particular, assume $0 < \psi_k\left(k^b, \beta, b\right) < 1$ (this assumption is necessary for k^b – see below – to be locally stable).

5.2 PAYG pensions

As before, we start by establishing the impact on private saving (or capital) of myopia and pensions. It is easy to verify (see Appendix D) that

$$\frac{\partial k_{t+1}}{\partial \beta} = -\frac{R\left(k_{t+1}\right) u'(c_{t+1}^o)}{R'\left(k_{t+1}\right) \beta u'(c_{t+1}^o) + R\left(k_{t+1}\right) \beta u''(c_{t+1}^o)\left[R'\left(k_{t+1}\right) k_{t+1} + R\left(k_{t+1}\right)\right] + u''(c_t^y)} > 0$$

implying the bigger the weight (β) agents assign to the future, the larger the saving due to consumption smoothing, and therefore larger the capital-labor ratio at any point in time. Similarly, in Appendix D, we show

$$\frac{\partial k_{t+1}}{\partial b} = -\frac{u''(c_t^y) + R\left(k_{t+1}\right) \beta u''(c_{t+1}^o)}{R'\left(k_{t+1}\right) \beta u'(c_{t+1}^o) + R\left(k_{t+1}\right) \beta u''(c_{t+1}^o)\left[R'\left(k_{t+1}\right) k_{t+1} + R\left(k_{t+1}\right)\right] + u''(c_t^y)} < 0 \quad (22)$$

meaning that the PAYG pension crowds out saving and leads to a lower capital-labor ratio. Assume a PAYG scheme is introduced (unanticipated) in period t such that each young pays $b > 0$ to each old in that and all future periods. From eq. (22), it follows that upon introduction of the PAYG pension, the capital stock is declining along the equilibrium trajectory and eventually reaches k^b, defined in Section 5.1. Since these results hold in steady state, the next result is immediate.

Lemma 4 $k^b < k < k^*$

Proof. Since $\frac{\partial k}{\partial \beta} > 0$ (see Appendix D) and $\beta < \beta^*$, k is smaller than the capital stock in a corresponding economy with non-myopic households. Myopia implies agents save too little, and hence, the capital stock is lower. Since $\frac{\partial k}{\partial b} < 0$ (see Appendix D), k^b is smaller than the capital stock in the absence of a PAYG pensions system, $k^b < k$. ■

From Lemma 4, recall $k < k^*$ holds, meaning the underlying "undersaving" issue faced by myopic agents persists in the case with endogenous factor prices: myopic agents hold too little capital. Under dynamic efficiency ($R(k) > 1$), if policy action can incentivize these agents to hold more capital, then steady-state welfare would rise. The problem, as before, is that a higher pension *reduces* the capital stock. There is, however, *one big difference* as compared to the case with exogenous factor prices. With endogenous factor prices, as the pension crowds out physical capital, the return to capital would rise (an effect absent earlier), raising the incentive to hold more of it. The equilibrium, therefore, has both voluntary savings and the PAYG pension.

Proposition 3 *Suppose Assumptions 1 - 6 hold. a) For introduction of a PAYG pension scheme to increase true steady-state welfare over laissez faire, it is necessary that*

$$\beta^* > R(k)\beta$$

holds. b) It is possible that, upon introduction, true welfare improves both for the inaugural generation and for each and every subsequent cohort, and
c) k^* *(corresponding to* β^**) cannot be replicated by a PAYG pension.*

Proof. See Appendix E. ■

Below, we present a simple numerical example to show that the optimal PAYG pension may be positive. Recall such an example is interesting because a) voluntary saving (in the form of capital) and public pensions co-exist, and b) the optimal pension is positive even under dynamic efficiency.

Example 1 *Suppose* $f(k) = 2k^{0.2}$, $\phi = 0.9$ *and* $\beta^* = 1$. *Using (19), we have*

$$k(b) = -\frac{b - (R\beta)^{\frac{1}{\sigma}}(w(k) - b)}{R(k) + (R(k)\beta)^{\frac{1}{\sigma}}}$$

and true steady state welfare is $\Omega^*(b) = u(w(k(b)) - k(b) - b) + \beta^* u(Rk(b)k(b) + b)$. *Here, optimal* $b \simeq 0.3$ *when* $\beta = 0.8$.

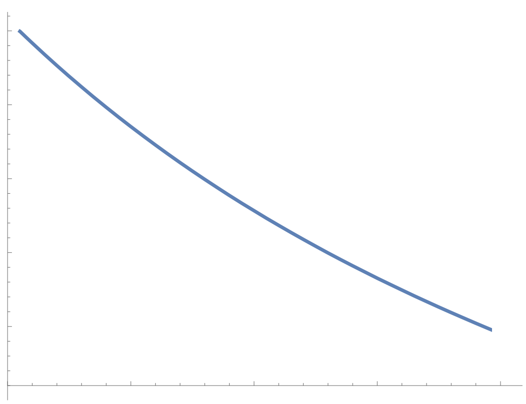

Figure 1a: $k(b)$

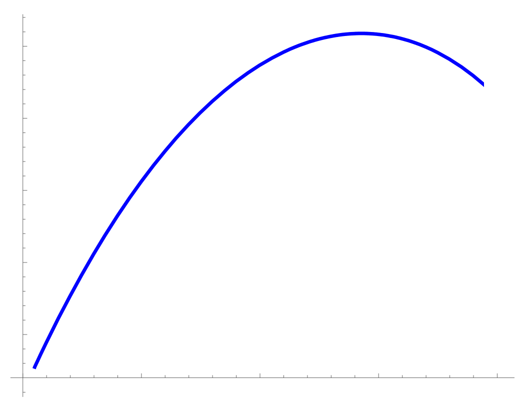

Figure 1b: $\Omega^*(b)$

5.3 FF pensions

Under a FF scheme, the decision problem for an individual born in period t is

$$\max_{s_t} \Omega_t = u(w_t - s_t - d) + \beta u(Rs_t + Rd)$$

Since s and d have the same return, they are perfect substitutes and only total saving $k = s + d$ matters. It follows that voluntary savings s decreases one-for-one with an increase in d for $s > 0$. For d so high that s is driven to the zero corner, we have $u'(\omega(k_t) - d) > \beta R(k_{t+1})u'(R(k_{t+1})d)$.

Proposition 4 *Setting mandatory pension savings at the level* $d = k^* > k$ *implements what the long-run true self wants. Steady-state welfare under such a program is, therefore, higher than in laissez faire and for any PAYG pension.*

The undersaving problem can thus be addressed by a proper choice of the mandated pension contribution. This, however, is a steady-state result, and therefore not of much use in solving the immediate problem for households with inadequate savings.

5.4 Transition to a fully-funded system

The FF pensions system is superior to a PAYG pensions system in terms of true steady state welfare, and the former can even be designed so as to completely eliminate the consequences of myopia. But is it possible with endogenous factor prices to make a transition from a PAYG to a FF system under the Pareto-criterion so that no cohorts are made worse off along the transition path?

5.4.1 Gain from introducing a FF pension

Let the PAYG pension scheme b be introduced in period t. Is there at some point in time – during adjustment to steady state or in the new steady state – a welfare gain from introducing a FF pension? To answer this question, consider introduction of a mandatory contribution larger than the initial capital stock, $d_{t+m} > k_{t+m}$ in period $t+m$, implying full crowding out of private savings ($s_{t+m} = 0$ and $k_{t+m+1} = d_{t+m}$), where the capital is predetermined at k_{t+m}. We have

Proposition 5 *At any point in time $t+m$, $m > 0$ after the introduction of the PAYG pension scheme, true life-time utility Ω^*_{t+m} can be improved by introducing a FF pension contribution $d_{t+m} > k_{t+m}$, under assumption 4 and*

$$\beta^* > \frac{\beta}{1+\eta(k)} \tag{23}$$

Proof. See Appendix F. ■

For Cobb-Douglas technology, the condition in (23) reduces to a simple restriction on parameters, $\beta^* > \frac{\beta}{\alpha}$.

The assumption on $\eta(k_{t+m})$ – see Assumption 4 – ensures that the return to capital is not "too sensitive" to the capital stock. Sufficiently-strong myopia ($\beta^* > \frac{\beta}{1+\eta}$) is necessary and sufficient for phasing in of a FF pensions system ($d_{t+m} > 0$) to have positive welfare effects on all subsequent cohorts.[18] Assuming this holds, it is possible to reduce the PAYG pension while satisfying the Pareto condition, i.e., there exists a $b_{t+m+1} < b$ and $d_{t+m} > k_{t+m}$ such that

$$\Omega^*_{t+m} \equiv u(\omega(k_{t+m}) - b - d_{t+m}) + \beta^* u(R(k_{t+m+1})\, d_{t+m} + b_{t+m+1}) = \Omega^{*\text{PAYG}}_{t+m}.$$

This is the first step in a transition out of the PAYG scheme.

[18]Note when $\eta = 0$ ($R'(k) = 0$) the condition reduces to $\beta^* > \beta$, cf. the finding for the case with exogenous factor prices (Lemma 3).

5.4.2 Complete phasing out of PAYG pensions

To work out an explicit case with full phasing out of a PAYG pension under the Pareto criterion, we first analyze an economy that has reached a steady-state with a PAYG pension $b > 0$ and the associated capital stock, k^b, and associated life-time utility, $\Omega^{*\text{PAYG}}$. [19] At $t + m > t$, an unanticipated announcement is made that a phasing out of the PAYG scheme and a transition to a FF system is underway with a goal of achieving the optimal long-run level of capital $d = k^*$.

We are looking for a policy sequence $\{b_{t+m+j}, d_{t+m+j}\}_{j=0}^{\infty}$ which satisfies the Pareto condition

$$\Omega_{t+m+j}^{*\text{TRANSITION}} \equiv u(\omega\left(k_{t+m+j}\right) - b_{t+m+j} - d_{t+m+j}) + \beta^* u(R\left(k_{t+m+j+1}\right) d_{t+m+j} + b_{t+m+j+1}) \geq \Omega_{t+m+j}^{*\text{PAYG}} (j \geq 0)$$

where $k_{t+m} = k^b$ and $k_{t+m+j+1} = d_{t+m+j}$, implies

$$b_{t+m} = b > b_{t+m+1} > b_{t+m+2} > ... > .... = 0$$

and the introduction of increasing contribution to or phasing in of the FF pensions system:

$$k^b < d_{t+m} < d_{t+m+1} < d_{t+m+2} < ... < .... < d = k^*$$

i.e. the pension to the current old (in period $t + m$) is the pension from the PAYG regime and the current young finance this, where the contribution is at the PAYG steady state level. The current young are also required to contribute to the FF scheme with a contribution d_{t+m}.

Proposition 5 gives conditions ensuring that cohort $t + m$ are better off when a FF pension is introduced on top of the PAYG pension. Under the Pareto criterion, this welfare gain may be used to bring down the PAYG pension this cohort receives (without hurting them). The next cohort sees an increase in their wage income due to a higher capital stock. That, as well as the reduced contribution to the PAYG pension, enables further increases in FF pension contribution resulting in additional increases in the capital stock and enabling a greater reduction in the PAYG pension they receive. Hence, the first step has more savings and a reduction in the PAYG pension. Downstream the change in savings and thus the capital stock also affects wage. Cohort $t+m+1$ will have a higher wage rate because the capital stock has increased (compared to status quo), and this make them better-off. Under the Pareto-condition this creates room to decrease the PAYG pension further. Working out this dynamics in detail generates the following result.

Proposition 6 *Assume the transition starts from a steady-state equilibrium with a PAYG pension b and associated capital, k^b. Under the assumption – see Assumption 4 – that $\eta(k^b) > -1$ and $\beta^* > \frac{\beta}{1+\eta(k^b)}$ there exists a trajectory satisfying the Pareto criterion, where the PAYG pension is entirely phased out, and the FF pension expanded so that k^* is reached in the long run.*

Proof. See Appendix G ■

[19] The capital stock is determined by the first order condition for savings $u'(\omega\left(k^b\right) - k^b - b) = R\left(k^b\right)\beta u'(R\left(k^b\right) k^b + b)$ and the associated welfare level $\Omega^{*\text{PAYG}} \equiv u\left(\omega\left(k^b\right) - k^b - b\right) + \beta^* u(R\left(k^b\right) k^b + b)$.

The above shows the existence of a transition path assuming that the economy is initially in steady state equilibrium with a PAYG pension b. The result can be considerably generalized.

Proposition 7 *Assume that the transition starts from an arbitrary date t with a PAYG pension b_t. Assume*

$$\beta^* > \frac{\beta}{1+\eta(k^b_{t+j})} \textit{ for } j \geq 0$$

where $-1 < \eta(k^b_{t+j}) \equiv \frac{R'(k_{t+j})k_{t+j}}{R(k_{t+j})} \mid_{k_{t+j}=k^b_{t+j}} < 0$. *Then there exists a trajectory satisfying the Pareto criterion, where the PAYG pension is phased out, and the FF pension is expanded.*

Proof. See Appendix H ■

The bottom line is this. Starting from an initial setting with a PAYG pension in place, it is possible to replace it with another mandated scheme, the FF scheme, which not only preserves (even increases) the benefits of the PAYG in terms of its forced-saving character but also generates a higher return. And along the transition, no one is hurt. Note that once the PAYG scheme is fully phased out, cohorts further downstream are made strictly better off.

6 Conclusion

In the pensions literature it is well-established that a PAYG pension has the advantage of delivering pensions up front to current pensioners. The downside is that this scheme is return-dominated by a funded scheme, which thus delivers higher long-run welfare. However, the phasing in of such a scheme runs over several decades. This disadvantage of the PAYG scheme has prompted the question whether it can be phased out without hurting any cohorts along the transition. The literature has largely answered this question in the negative.

This paper argues that the discussion on pension system transition has overlooked the reasons why pension schemes were introduced in the first place. A key argument is that agents do not save enough due to their present bias. Starting from this observation, we show that it may be optimal to introduce a PAYG scheme in the first place, not only because it is beneficial to the inaugural old, but also because it addresses an undersaving problem. However, this scheme is return dominated by a funded scheme. A switch to the latter scheme is a good idea but it would endanger the incomes of the current generation of retirees. We show that a transition is possible and yet no cohorts are worse off. In a way, our results speak to a “division of labor” between PAYG and FF pensions: the former takes care of the needs of the current retirees and the latter, because of present bias, proves beneficial to both current and future generations. This last statement has important implications for pension policy design.

As outlined in the introduction, our analysis informs the debate on pension policy design. The classic conundrum facing policymakers has been the following. There is a generation of current retirees that need a pension. At the same time, the current working generation needs to be transferred to a FF scheme. How to get the young to contribute to paying a pension to the initial generation of retirees *and* get them to contribute to their own FF scheme? Conventional thinking stops here because the burden on the transition generation from having to pay twice is believed to be too much for any generation to have to endure. This has been the major sticking point in the discussion about pension reform. Our analysis argues the transition may not be as burdensome as believed.

In the present paper we have focussed on the basic differences between PAYG and funded pension schemes to address the fundamental transition issue. It should be stressed that distributional and insurance aspects have been overlooked, and they may give arguments for PAYG schemes, also in the long run. Our focus, of course, is on inadequacies in saving-for-retirement alone and the use of mandated schemes to that end.

As we have shown, once the mandate is high enough, the voluntary retirement saving disappears and further increases in the contribution mandate raises agents' welfare. Problems would emerge if the government mandate was so aggressive as to warrant borrowing by the young, but as Andersen and Bhattacharya (2018) have shown, there is no welfare case to choose such a high mandate. An implication of this idea is the following. Suppose there were some agents who did not suffer from time inconsistency. The welfare of such agents under laissez faire and under the government mandate would be identical.

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Appendix

A Proof of Lemma 1

The contribution rate under a PAYG pension scheme is $\tau_t = b_t$, and true life-time utility therefore reads

$$\Omega_t^*(b_t) = u(w - b_t - s(b_t)) + \beta^* u(Rs(b_{t+1}) + b_{t+1})$$

Notice that with exogenous factor prices we immediately reach steady state if implementing a time-invariant pension ($b_{t+j} = b$ for all $j \geq 0$). Denote this level of PAYG benefits for b, which we consider in the following (and, hence, suppress time subscripts). Agents are better-off under the PAYG scheme compared to laissez-faire if

$$\Omega^*(b) > \Omega^*(0) \tag{24}$$

Private savings given the level of pension, $s(b)$, is

$$u'(w - b - s(b)) = R\beta u'(Rs(b) + b) \text{ if } s > 0,$$

$$u'(w - b) > R\beta u'(b) \text{ if } s = 0.$$

For $s > 0$ we have

$$\frac{\partial s(b)}{\partial b} = -\frac{u''(c^y) + R\beta u''(c^o)}{u''(c^y) + R^2\beta u''(c^o)}$$

We have, using (1)-(3), the results above and assuming time-invariant PAYG pensions system, that (remember, $R > 1$ is assumed)

$$\begin{aligned} \frac{\partial \Omega^*(b)}{\partial b} &= -u'(w - b - s(b)) \left[1 + \frac{\partial s(b)}{\partial b}\right] + \beta^* u'(Rs(b) + b) \left[R\frac{\partial s(b)}{\partial b} + 1\right] \\ &= [1 - R]\, u'(c^o) \left[\frac{\beta^* u''(c^y) + \beta^2 R^2 u''(c^o)}{u''(c^y) + R^2\beta u''(c^o)}\right] < 0 \text{ for } s > 0 \end{aligned}$$

Hence, if the PAYG pension b should be welfare improving it is necessary that it be high enough that private voluntary pensions savings is fully crowded out. Define $\underline{b} : s(\underline{b}) = 0$. If private savings is zero (implying $u'(w - b) > R\beta u'(b)$), the optimal b is a solution to

$$\max_b u(w - b) + \beta^* u(b)$$

and the associated first order condition is $u'(w - b) = \beta^* u'(b)$. Hence, for $b^* > \underline{b}$ being socially optimal when $s = 0$ it is necessary that

$$\beta^* u'(b) > R\beta u'(b)$$

which requires $\beta^* > R\beta$, i.e. with sufficiently strong myopia there is a welfare case for a PAYG pension. Note, this is a necessary condition; for true utility to increase, it is required that

$$\Omega^*(b^*) > \Omega^*(0)$$

see Andersen and Bhattacharya (2011) for details.

Assuming a CES utility function $u\left(c\right) = \frac{c^{1-\phi}}{1-\phi}$, $\phi \in (0, 1)$,we have that savings in equilibrium s

and optimal PAYG pension b^* are given by a solution to $(w-s)^{-\phi} = R\beta\,(Rs)^{-\phi}$ and $(w-b^*)^{-\phi} = \beta^* b^{*-\phi}$, respectively. These give

$$
\begin{aligned}
s &= \frac{(R\beta)^{\frac{1}{\phi}}}{R+(R\beta)^{\frac{1}{\phi}}}w \Longrightarrow c^y = \frac{R}{R+(R\beta)^{\frac{1}{\phi}}}w,\, c^o = \frac{R\,(R\beta)^{\frac{1}{\phi}}}{R+(R\beta)^{\frac{1}{\phi}}}w \\
b^* &= \frac{\beta^{*\frac{1}{\phi}}}{1+\beta^{*\frac{1}{\phi}}}w \Longrightarrow c^y = \frac{1}{1+\beta^{*\frac{1}{\phi}}}w,\, c^o = \frac{\beta^{*\frac{1}{\phi}}}{1+\beta^{*\frac{1}{\phi}}}w
\end{aligned}
$$

Substituting these into the true utility function $\Omega^* = \frac{(c^y)^{1-\phi}}{1-\phi} + \beta^* \frac{(c^o)^{1-\phi}}{1-\phi}$ and doing some calculations gives

$$
\begin{aligned}
\Omega^*(s) &= \frac{w^{1-\phi}}{1-\phi}\left(\frac{R}{R+(R\beta)^{\frac{1}{\phi}}}\right)^{1-\phi}\left(1+\beta^*(R\beta)^{\frac{1-\phi}{\phi}}\right) \\
\Omega^*(b^*) &= \frac{w^{1-\phi}}{1-\phi}\left(1+\beta^{*\frac{1}{\phi}}\right)^{\phi}
\end{aligned}
$$

Hence, for $\Omega^*(b^*) > \Omega^*(s)$ it is necessary and sufficient that (remember, $\phi < 1$ is assumed)

$$
\frac{w^{1-\phi}}{1-\phi}\left(1+\beta^{*\frac{1}{\phi}}\right)^{\phi} > \frac{w^{1-\phi}}{1-\phi}\left(\frac{R}{R+(R\beta)^{\frac{1}{\phi}}}\right)^{1-\phi}\left(1+\beta^*(R\beta)^{\frac{1-\phi}{\phi}}\right)
$$

or

$$
\left(\frac{R+R\beta^{*\frac{1}{\phi}}}{R+(R\beta)^{\frac{1}{\phi}}}\right)^{\phi} > \frac{R+\frac{\beta^*}{\beta}(R\beta)^{\frac{1}{\phi}}}{R+(R\beta)^{\frac{1}{\phi}}}
$$

One period in the model is approximately 30 calendar years. Assuming an annual subjective rate of time preference of about 10% gives the one period discount factor $\beta \approx 0.06$. Further, assuming that $\beta^* = 1$ and $R = 2.42$, which implies annual real interest rates of 3%, satisfying the necessary condition $\beta^* > R\beta$, gives the following parameter values: $\beta = 0.06$; $R = 2.43$; $\beta^* = 1$. These give the following results:

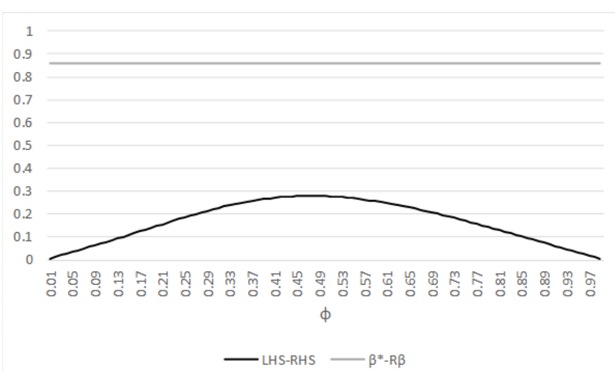

Figure 2a

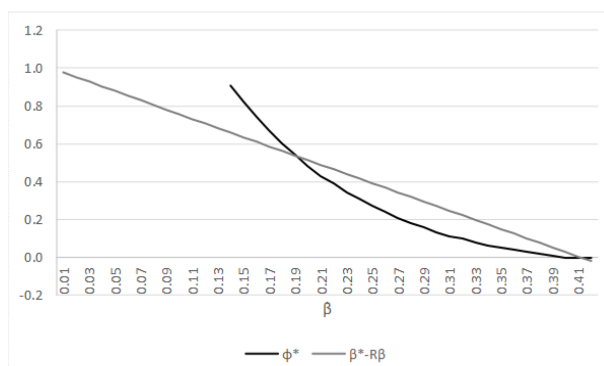

Figure 2b

As can be seen from Figure 2.a, the $LHS > RHS$ holds for all $\phi \in (0,1)$ implying that a PAYG pensions system is welfare improving. This result holds for $\beta = 0.06$ and it is interesting to see how the result depends on the value of β.

The black line in Figure 2b shows the maximum value of ϕ, i.e. ϕ^*, for which $LHS > RHS$ for different β. The line starts $\beta \approx 0.14$ implying that $LHS > RHS$ for all $\phi \in (0,1)$ and $0 < \beta \leq 0.14$. For $\beta > 0.14$, ϕ^* is decreasing in β implying that the parameter space in ϕ for which $LHS > RHS$ is decreasing in β. The necessary condition $\beta^* > R\beta$ is fulfilled for all βs where $LHS > RHS$ (which is logical since it is a necessary condition).

B Proof of Lemma 3

True life-time utility is affected by mandated contributions to a FF pension scheme as follows

$$\frac{\partial \Omega^*_{t+m}}{\partial d_{t+m}} = -u'(c^y_{t+m})\left[1 + \frac{\partial s(\cdot)}{\partial d_{t+m}}\right] + \beta^* u'(c^o_{t+m+1})\left[R + R\frac{\partial s(\cdot)}{\partial d_{t+m}}\right]$$

With PAYG contribution $b_{t+m+1} = b^*$, we have $s = 0$ implying $\frac{\partial s(\cdot)}{\partial d_{t+m}} = 0$. Hence, evaluating the welfare effect for $d_{t+m} = 0, b_{t+m+1} = b^*$ and using that $u'(w - b^*) = \beta^* u'(b^*)$ gives

$$\left.\frac{\partial \Omega^*_{t+m}}{\partial d_{t+m}}\right|_{d_{t+m}=0, b_{t+m+1}=b^*} = -u'(w - b^*) + R\beta^* u'(b^*) = [R-1]\,\beta^* u'(b^*) > 0 \text{ given } R > 1$$

Hence, on the margin, continuing the initial PAYG scheme and adding a (small) mandated FF pension makes agents better off.

C Proof of Proposition 2

Let

$$d_{t+m} + b = (1+\kappa)b^*$$
$$d_{t+m+j} + b_{t+m+j} = (1+\kappa)b^*;\ j > 0$$

i.e. from the start of transition the contribution rate is raised relative to the PAYG world, $\kappa > 0$. This implies

$$d_{t+m+1} = (1+\kappa)b^* - b_{t+m+1}$$

i.e. if we have a declining sequence for b_{t+m+1} we get an increasing sequence for d_{t+m+1}; if PAYG is phased out, the FF scheme is phased in. The equal utility condition for period $t+m$ reads

$$u(w - b^* - d_{t+m}) + \beta^* u(Rd_{t+m} + b_{t+m+1}) = u(w - b^*) + \beta^* u(b^*)$$

or

$$u(w - (1+\kappa)b^*) + \beta^* u(R\kappa b^* + b_{t+m+1}) = u(w - b^*) + \beta^* u(b^*)$$

and given Lemma 3 there exists a $\kappa > 0$ ensuring $b_{t+m+1} < b^*$. For period $t+m+1$ the equal utility condition can now be written

$$u(w - b_{t+m+1} - d_{t+m+1}) + \beta^* u(Rd_{t+m+1} + b_{t+m+2}) = u(w - b^*) + \beta^* u(b^*)$$

or

$$u(w-(1+\kappa)b^*)+\beta^* u(R\left((1+\kappa)b^*-b_{t+m+1}\right)+b_{t+m+2})=u(w-b^*)+\beta^* u(b^*)$$

or

$$u(w-(1+\kappa)b^*)+\beta^* u(R\kappa b^*+R\left(b^*-b_{t+m+1}\right)+b_{t+m+2})=u(w-b^*)+\beta^* u(b^*)$$

Since

$$u(w-(1+\kappa)b^*)+\beta^* u(R\kappa b^*+b_{t+m+1})=u(w-b^*)+\beta^* u(b^*)$$

it is implied

$$u(R\kappa b^*+R\left(b^*-b_{t+m+1}\right)+b_{t+m+2})=u(R\kappa b^*+b_{t+m+1})$$

and since $b^*-b_{t+m+1}>0$ this requires $b_{t+m+2}<b_{t+m+1}$. Similar reasoning for subsequent periods. If d^* has not been reached along this transition path, then clearly it is possible to increase d to reach this level, since no compensation is needed any longer when the PAYG pension has been eliminated.

D Proof of Lemma 4

It is easy to verify that the partial derivatives of (20) are

$$\begin{aligned}
\frac{\partial k_{t+1}}{\partial k_t} &= \frac{u''(c_t^y)\omega'(k_t)}{R'(k_{t+1})\beta u'(c_{t+1}^o)+R(k_{t+1})\beta u''(c_{t+1}^o)\left[R'(k_{t+1})k_{t+1}+R(k_{t+1})\right]+u''(c_t^y)}>0\\
\frac{\partial k_{t+1}}{\partial \beta} &= -\frac{R(k_{t+1})u'(c_{t+1}^o)}{R'(k_{t+1})\beta u'(c_{t+1}^o)+R(k_{t+1})\beta u''(c_{t+1}^o)\left[R'(k_{t+1})k_{t+1}+R(k_{t+1})\right]+u''(c_t^y)}>0\\
\frac{\partial k_{t+1}}{\partial b} &= -\frac{u''(c_t^y)+R(k_{t+1})\beta u''(c_{t+1}^o)}{R'(k_{t+1})\beta u'(c_{t+1}^o)+R(k_{t+1})\beta u''(c_{t+1}^o)\left[R'(k_{t+1})k_{t+1}+R(k_{t+1})\right]+u''(c_t^y)}<0
\end{aligned}$$

It is assumed that the denominator in these expressions is strictly negative, which under Assumption 6.

The steady state capital stock k^b for a given PAYG pension b is given by the k^b solving

$$k^b=\psi\left(k^b,\beta,b\right)$$

Hence,

$$\begin{aligned}
\frac{\partial k^b}{\partial \beta} &= \frac{\psi_\beta\left(k^b,\beta,b\right)}{1-\psi_k\left(k^b,\beta,b\right)}>0 \text{ for } b\geq 0\\
\frac{\partial k^b}{\partial b} &= \frac{\psi_b\left(k^b,\beta,b\right)}{1-\psi_k\left(k^b,\beta,b\right)}<0 \text{ for } b\geq 0
\end{aligned}$$

where it is assumed that $0<\psi_k\left(k^b,\beta,b\right)<1$, which holds under Assumption 6. This implies that $k^b<k^*$, where k^* is the steady state capital stock in the absence of myopia ($\beta=\beta^*$).

E Proof of Proposition 3

E.1 Part A

Steady state

True steady state welfare is

$$\Omega^* = u(\omega\left(k\left(b\right)\right) - k\left(b\right) - b) + \beta^* u(R\left(k\left(b\right)\right) k\left(b\right) + b)$$

The effect of an introduction of a PAYG system on welfare is given by:

$$\begin{aligned} \left.\frac{\partial \Omega^*}{\partial b}\right|_{b=0} &= \left.\left\{ u'(c^y)\left[\left[\omega'\left(k\right) - 1\right]\frac{\partial k}{\partial b} - 1\right] + \beta^* u'(c^o)\left[\left[R'\left(k\right)k + R\left(k\right)\right]\frac{\partial k}{\partial b} + 1\right]\right\}\right|_{b=0} \\ &= \left. u'(c^o)\left\{ \left[\beta^* - R\left(k\right)\beta\right]\left[1 + R'\left(k\right)k\frac{\partial k}{\partial b}\right] + \left[\beta^* - \beta\right]R\left(k\right)\frac{\partial k}{\partial b}\right\}\right|_{b=0} \end{aligned}$$

where the second line uses the steady state version of (19), (2) and (3) for a PAYG pension $\tau = b$ and that $\omega'\left(k\right) = -kf''\left(k\right) = -R'\left(k\right)k$ (as can be seen from (17) and (18)). Since

$$\begin{aligned} \left.\left[\beta^* - \beta\right]R\left(k\right)\frac{\partial k}{\partial b}\right|_{b=0} &< 0 \\ \left.R'\left(k\right)k\frac{\partial k}{\partial b}\right|_{b=0} &> 0 \end{aligned}$$

and $\frac{\partial k}{\partial b}\mid_{b=0} < 0$ and $R'\left(k\right) < 0$, a necessary condition for introducing a PAYG pensions system to increase steady state welfare is

$$\beta^* > R\left(k(0)\right)\beta \tag{25}$$

Assuming $R\left(k\right) > 1$, sufficient myopia is necessary for a PAYG pension to increase steady state welfare – see Andersen and Bhattacharya (2011).

Further, a sufficient condition for introduction of a PAYG scheme to increase steady state welfare is:

$$\left\{ \left[\beta^* - R\left(k\right)\beta\right]\left[1 + R'\left(k\right)k\frac{\partial k}{\partial b}\right] + \left[\beta^* - \beta\right]R\left(k\right)\frac{\partial k}{\partial b}\right\}\mid_{b=0} > 0$$

or

$$\beta^* > R\left(k\left(0\right)\right)\beta\frac{1 + \left[R'\left(k\left(0\right)\right)k\left(0\right) + 1\right]\left.\frac{\partial k}{\partial b}\right|_{b=0}}{1 + \left[R'\left(k\left(0\right)\right)k\left(0\right) + R\left(k\left(0\right)\right)\right]\left.\frac{\partial k}{\partial b}\right|_{b=0}}$$

or

$$\beta^*\left.\left[1 + \left[R'\left(k\right)k + R\left(k\right)\right]\frac{\partial k}{\partial b}\right]\right|_{b=0} > \beta\left.\left\{R\left(k\right)\left[1 + \left[R'\left(k\right)k + 1\right]\frac{\partial k}{\partial b}\right]\right\}\right|_{b=0}$$

Using the results from Appendix D, and that $\omega'\left(k\right) = -kf''\left(k\right) = -R'\left(k\right)k$ gives

$$\frac{\partial k}{\partial b} = -\frac{u''(c^y) + R\left(k\right)\beta u''(c^o)}{R'\left(k\right)\beta u'(c^o) + R\left(k\right)\beta u''(c^o)\left[R'\left(k\right)k + R\left(k\right)\right] + u''(c^y)\left[1 + R'\left(k\right)k\right]}$$

and, hence

$$
\begin{aligned}
& 1+\left[R'(k)\,k+R(k)\right]\frac{\partial k}{\partial b} \\
= \quad & \frac{R'(k)\,\beta u'(c^o)-[R(k)-1]\,u''(c^y)}{R'(k)\,\beta u'(c^o)+R(k)\,\beta u''(c^o)\,[R'(k)\,k+R(k)]+u''(c^y)\,[1+R'(k)\,k]}
\end{aligned}
$$

and

$$
\begin{aligned}
& 1+\left[R'(k)\,k+1\right]\frac{\partial k}{\partial b} \\
= \quad & \beta\frac{R'(k)\,u'(c^o)+[R(k)-1]\,R(k)\,u''(c^o)}{R'(k)\,\beta u'(c^o)+R(k)\,\beta u''(c^o)\,[R'(k)\,k+R(k)]+u''(c^y)\,[1+R'(k)\,k]}
\end{aligned}
$$

From Assumption 6, we have that

$$
R'(k)\,\beta u'(c^o)+R(k)\,\beta u''(c^o)\left[R'(k)\,k+R(k)\right]+u''(c^y)\left[1+R'(k)\,k\right]<0
$$

Hence,$1+[R'(k)\,k+R(k)]\,\frac{\partial k}{\partial b}>0$ iff $R'(k)\,\beta u'(c^o)-[R(k)-1]\,u''(c^y)<0$. This holds iff (note that $R'(k)<0$ and $R(k)>1$)

$$
\frac{R'(k)\,\beta}{R(k)-1}<\frac{u''(c^y)}{u'(c^o)} \tag{26}
$$

We therefore have that a sufficient condition for a PAYG scheme to increase steady state welfare is

$$
\beta^*>R(k(0))\,\beta\frac{1+[R'(k(0))\,k(0)+1]\,\frac{\partial k}{\partial b}\big|_{b=0}}{1+[R'(k(0))\,k(0)+R(k(0))]\,\frac{\partial k}{\partial b}\big|_{b=0}} \tag{27}
$$

assuming a stable steady state and that (26) holds when evaluated at $b=0$. Since the right-hand side is independent of β^*, it can be concluded that there exists β^* such that the introduction of a PAYG scheme increases steady state welfare. Further, since $R(k)>1$ we have that

$$
\frac{1+[R'(k(0))\,k(0)+1]\,\frac{\partial k}{\partial b}\big|_{b=0}}{1+[R'(k(0))\,k(0)+R(k(0))]\,\frac{\partial k}{\partial b}\big|_{b=0}}>1
$$

then (27) says we require β^* to be sufficiently higher than β,or sufficiently high myopia.

E.2 Part B

Assuming a PAYG scheme is introduced in time t, the welfare of an individual born in time t ($k_t=k$) is

$$
\Omega_t^*=u(\omega(k)-k_{t+1}-b)+\beta^* u(R(k_{t+1})k_{t+1}+b)
$$

and the welfare effects for the individual from introducing a PAYG pensions system are:

$$\begin{aligned}\left.\frac{\partial \Omega_t^*}{\partial b}\right|_{b=0} &= -u'\left(c^y\right)\frac{\partial k_{t+1}}{\partial b}\,|_{b=0} - u'\left(c^y\right) \\ &\quad + \beta^* u'(c^o)\left[R'\left(k\right)k + R\left(k\right)\right]\left.\frac{\partial k_{t+1}}{\partial b}\right|_{b=0} + \beta^* u'(c^o) \\ &= u'(c^o)\left[\left[\beta^* - R\left(k\right)\beta\right] + \beta^* R'\left(k\right)k\left.\frac{\partial k_{t+1}}{\partial b}\right|_{b=0} + \left[\beta^* - \beta\right]R\left(k\right)\left.\frac{\partial k_{t+1}}{\partial b}\right|_{b=0}\right]\end{aligned}$$

where the second equality uses that $u'\left(c^y\right) = R\beta u'\left(c^o\right)$. The first term inside the bracket is strictly positive under dynamic efficiency.The second term is strictly positive while the third term is strictly negative.

Further, a sufficient condition for introducing a PAYG system in period t to have positive effects on welfare of individuals born in period t is:

$$\left[\beta^* - R\left(k\right)\beta\right] + \beta^* R'\left(k\right)k\left.\frac{\partial k_{t+1}}{\partial b}\right|_{b=0} + \left[\beta^* - \beta\right]R\left(k\right)\left.\frac{\partial k_{t+1}}{\partial b}\right|_{b=0} > 0$$

or

$$\beta^* > R\left(k\right)\beta\frac{1 + \left.\frac{\partial k_{t+1}}{\partial b}\right|_{b=0}}{1 + \left[R'\left(k\right)k + R\left(k\right)\right]\left.\frac{\partial k_{t+1}}{\partial b}\right|_{b=0}} \tag{28}$$

Individuals born in period $t+j$ and after

For subsequent periods $t+j$ $(j \geq 1)$ we have

$$u'(\omega\left(k_{t+j}\right) - k_{t+j+1} - b) = R\left(k_{t+j+1}\right)\beta u'(R\left(k_{t+j+1}\right)k_{t+j+1} + b) \tag{29}$$

and the welfare of an individual born in period $t+j$ $(j \geq 1)$ is

$$\Omega_{t+j}^* = u\left(\omega\left(k_{t+j}\right) - k_{t+j+1} - b\right) + \beta^* u(R\left(k_{t+j+1}\right)k_{t+j+1} + b)$$

and the welfare effects for the individual from introducing a PAYG pensions system are:

$$\begin{aligned}\frac{\partial \Omega_{t+j}^*}{\partial b}\quad|\quad_{b=0} &= \left[u'\left(c^y\right)\left[\omega'\left(k_{t+j}\right)\frac{\partial k_{t+j}}{\partial b} - \left.\frac{\partial k_{t+j+1}}{\partial b}\right|_{b=0}\right] + \beta^* u'(c^o)\left[R'\left(k\right)k + R\left(k\right)\right]\left.\frac{\partial k_{t+j+1}}{\partial b}\right|_{b=0}\right] \\ &\quad - u'\left(c^y\right) + \beta^* u'\left(c^o\right) \\ &= u'(c^o)\left[\beta R\left[\omega'\left(k_{t+j}\right)\frac{\partial k_{t+j}}{\partial b} - \left.\frac{\partial k_{t+j+1}}{\partial b}\right|_{b=0}\right] + \beta^*\left[R'\left(k\right)k + R\left(k\right)\right]\left.\frac{\partial k_{t+j+1}}{\partial b}\right|_{b=0} - R\beta + \beta^*\right] \\ &= u'(c^o)\left[\left[\beta^* - R\beta\right] + R\left[\beta^* - \beta\right]\left.\frac{\partial k_{t+j+1}}{\partial b}\right|_{b=0} + \beta^* R'\left(k\right)k\left.\frac{\partial k_{t+j+1}}{\partial b}\right|_{b=0} + \beta R\omega'\left(k_{t+j}\right)\frac{\partial k_{t+j}}{\partial b}\right]\end{aligned}$$

Note that the first three terms are similar to the terms in the expression for $\left.\frac{\partial \Omega_t^*}{\partial b}\right|_{b=0}$. In addition there is now the term $\beta R\omega'\left(k_{t+j}\right)\frac{\partial k_{t+j}}{\partial b} < 0$ capturing the fact that the pension, by lowering capital, also reduces the wage rate.

E.3 Part C

Does there exists a $b > 0$ ensuring that $k^b = k^*$, i.e. is it possible that a PAYG pensions scheme gives the optimal steady state capital stock k^*? Using steady state versions of (8) (using that

$s = k$), (19), (17) and (18) we have that k^* and k^b are given by solutions to

$$u'(\omega (k^*) - k^*) = R (k^*) \beta^* u'(R (k^*) k^*)$$

and

$$u'(\omega \left(k^b\right) - k^b - b) = R \left(k^b\right) \beta u'(R \left(k^b\right) k^b + b)$$

respectively. For $k^b = k^*$ to hold, this requires

$$\frac{u'(\omega (k^*) - k^* - b)}{\beta u'(R (k^*) k^* + b)} = \frac{u'(\omega (k^*) - k^*)}{\beta^* u'(R (k^*) k^*)}$$

or

$$\frac{\beta^*}{\beta} = \frac{u'(\omega (k^*) - k^*) u'(R (k^*) k^* + b)}{u'(\omega (k^*) - k^* - b) u'(R (k^*) k^*)}$$

Since $\beta^* > \beta$, it is required that

$$\underbrace{\frac{u'(\omega (k^*) - k^*) u'(R (k^*) k^* + b)}{u'(\omega (k^*) - k^* - b) u'(R (k^*) k^*)}}_{LHS} > 1 \tag{30}$$

Using $b > 0$ and

$$\lim_{b \to 0} LHS = 1; \quad \frac{\partial LHS}{\partial b} < 0$$

it follows that

$LHS < 1$ for all $b > 0$

and hence condition (30) never holds. The optimal steady state capital stock (k^*) is not attainable under myopia ($\beta < \beta^*$) by an appropriate choice of the PAYG pension (b).

F Proof of Proposition 5

In the presence of a PAYG pension b the period $t + j + 1$ capital stock is given as the period $t + j$ savings, i.e.

$$k_{t+j+1} = s (k_{t+j}, b)$$

The true life-time utility for the cohort being young in period $t + j$ ($j \geq 1$) is:

$$\Omega^{*\text{PAYG}}_{t+j} = u (\omega (k_{t+j}) - s (k_{t+j}, b) - b) + \beta^* u(R (s (k_{t+j}, b)) s (k_{t+j}, b) + b)$$

where $s (k_{t+j}, b)$ gives equilibrium savings for the young in $t + j$.

Assume that transition starts in period $t + m$, and the mandated mandated FF contribution to be exactly equal to the voluntary savings in the PAYG regime, i.e.

$$d_{t+m} = s\left(k_{t+m}, b\right)$$

Since voluntary and mandatory FF savings are perfect substitutes for $s > 0$ (mandated savings crowds out voluntary savings one-to-one), total savings and thus the capital stock are unaffected. For $d_{t+m} \geq \underline{d}_{t+m} \equiv s\left(k_{t+m}, b\right)$ individuals are at the corner for voluntary savings ($s = 0$ and $k_{t+m+1} = d_{t+m}$), and the true life-time utility of cohort $t+m$ can be written

$$\Omega_{t+m}^{*\text{TRANSITION}} = u\left(\omega\left(k_{t+m}\right) - d_{t+m} - b_{t+m}\right) + \beta^* u(R\left(d_{t+m}\right) d_{t+m} + b_{t+m+1}) \tag{31}$$

which implies

$$\frac{\partial \Omega_{t+m}^{*\text{TRANSITION}}}{\partial d_{t+m}} = -u'\left(\omega\left(k_{t+m}\right) - d_{t+m} - b_{t+m}\right) + \beta^* u'(R\left(d_{t+m}\right) d_{t+m} + b_{t+m+1})\left[R'\left(d_{t+m}\right) d_{t+m} + R\right]$$

Assessing this for $d_{t+m} = \underline{d}_{t+m}$ we can exploit that $u\prime\left(\omega\left(k_{t+m}\right) - \underline{d}_{t+m} - b_{t+m}\right) = R\beta u(R\left(\underline{d}_{t+m}\right)\underline{d}_{t+m} + b_{t+m+1})$ and hence

$$\begin{aligned}\left.\frac{\partial \Omega_{t+m}^{*\text{TRANSITION}}}{\partial d_{t+m}}\right|_{d_{t+m}=\underline{d}_{t+m}} &= \left.\left\{-\beta u'(c^o)R + \beta^* u'(c^o)\left[R'\left(d_{t+m}\right) d_{t+m} + R\right]\right\}\right|_{d_{t+m}=\underline{d}_{t+m}} \\ &= \left.\left\{u'(c^o)\left[\beta^*\left[R'\left(k_{t+m+1}\right) k_{t+m+1} + R\right] - \beta R\right]\right\}\right|_{d_{t+m}=\underline{d}_{t+m}}\end{aligned}$$

It follows that $\left.\frac{\partial \Omega_{t+m}^{*\text{TRANSITION}}}{\partial d_{t+m}}\right|_{d_{t+m}=\underline{d}_{t+m}} > 0$ is ensured if

$$\beta^* \left.\left[\frac{R'\left(k_{t+m+1}\right) k_{t+m+1}}{R(k_{t+m+1})} + 1\right]\right|_{d_{t+m}=\underline{d}_{t+m}} > \beta$$

Defining $\eta(k_{t+m+1}) \equiv \left.\left[\frac{R'(k_{t+m+1})k_{t+m+1}}{R(k_{t+m+1})}\right]\right|_{d_{t+m}=\underline{d}_{t+m}} < 0$. It is assumed that $\eta(k_{t+m+1}) > -1$ (see Assumption 4).

$$\beta^* > \frac{\beta}{1 + \eta\left(k_{t+m+1}\right)}$$

Sufficiently strong myopia ($\beta^* > \beta$) is necessary and sufficient for phasing in of a FF pensions system ($d_{t+m} > 0$) to have positive welfare effects on an individual born in period $t+m$. Assuming that this holds, it is possible to reduce the PAYG pension while satisfying the Pareto condition, i.e. there exists a $b_{t+m+1} < b$ and $d_{t+m} > 0$ such that $\Omega_{t+m}^{*\text{TRANSITION}} = \Omega_{t+m}^{*\text{PAYG}}$. This can be seen as the first step in a transition out of the PAYG scheme. Importantly, the above result holds for any initial situation, i.e., the economy needs not to be in steady state.

G Proof of Proposition 6

To work out a case where there is a complete transition from a PAYG to a FF pension under the Pareto condition, assume that the economy is initially in steady state equilibrium with a PAYG pension b. Denote the steady-state level of capital by k^b.

Generation $t+m$

From Appendix F we have that $\left.\frac{\partial\Omega^{*\text{TRANSITION}}_{t+m}}{\partial d_{t+m}}\right|_{d_{t+m}=\underline{d}_{t+m}} > 0$. Setting $\underline{d}_{t+m} = k^b$, his implies that introducing a transition to a FF scheme where the initial contribution to the FF scheme is $d_{t+m} > k^b$ increases the welfare of generation $t+m$

$$\begin{aligned}\Omega^{*\text{TRANSITION}}_{t+m} &= u\left(\omega\left(k^b\right) - d_{t+m} - b\right) + \beta^* u(R\left(d_{t+m}\right)d_{t+m} + b) \\ &> u\left(\omega\left(k^b\right) - k^b - b\right) + \beta^* u(R\left(k^b\right)k^b + b) \\ &= \Omega^{*\text{PAYG}}\end{aligned}$$

implying that there exists a $b_{t+m+1} < b$ such that

$$\begin{aligned}\Omega^{*\text{TRANSITION}}_{t+m} &= u\left(\omega\left(k^b\right) - d_{t+m} - b\right) + \beta^* u(R\left(d_{t+m}\right)d_{t+m} + b_{t+m+1}) \\ &= \Omega^{*\text{PAYG}}\end{aligned} \tag{32}$$

We therefore have that there exits

$$\begin{aligned} d_{t+m} &> k^b \\ b_{t+m+1} &< b\end{aligned}$$

such that generation $t+m$ is no worse off ((32) holds) and the phasing in of FF and out of PAYG has started.

Generation $t+m+1$

For generation $t+m+1$ to be no worse off the following must hold

$$\begin{aligned}\Omega^{*\text{TRANSITION}}_{t+m+1} &= u\left(\omega\left(d_{t+m}\right) - d_{t+m+1} - b_{t+m+1}\right) + \beta^* u(R\left(d_{t+m+1}\right)d_{t+m+1} + b_{t+m+2}) \\ &= u\left(\omega\left(k^b\right) - k^b - b\right) + \beta^* u(R\left(k^b\right)k^b + b) \\ &= \Omega^{*\text{PAYG}}\end{aligned} \tag{33}$$

From (32), this requires

$$\begin{aligned}\Omega^{*\text{TRANSITION}}_{t+m+1} &= u\left(\omega\left(d_{t+m}\right) - d_{t+m+1} - b_{t+m+1}\right) + \beta^* u(R\left(d_{t+m+1}\right)d_{t+m+1} + b_{t+m+2}) \\ &= u\left(\omega\left(k^b\right) - d_{t+m} - b\right) + \beta^* u(R\left(d_{t+m}\right)d_{t+m} + b_{t+m+1}) \\ &= \Omega^{*\text{T}}_{t+m}\end{aligned}$$

Since $d_{t+m} > k^b$ we have that $\omega\left(d_{t+m}\right) > \omega\left(k^b\right)$ (remember that $\omega'\left(,\right) > 0$). Using this, $b_{t+m+1} < b$ and $u'\left(,\right) > 0$ we have

$$u\left(\omega\left(d_{t+m}\right) - d_{t+m} - b_{t+m+1}\right) > u\left(\omega\left(k^b\right) - d_{t+m} - b\right)$$

Therefore, there exists $d_{t+m+1} > d_{t+m}$ such that

$$u\left(\omega\left(d_{t+m}\right) - d_{t+m+1} - b_{t+m+1}\right) = u\left(\omega\left(k^b\right) - d_{t+m} - b\right)$$

Using Assumption 4 (which implies that $\frac{\partial[R(k_t)k_t]}{\partial k_t} > 0$)

$$u(R\left(d_{t+m+1}\right)d_{t+m+1} + b_{t+m+1}) > u(R\left(d_{t+m}\right)d_{t+m} + b_{t+m+1})$$

implying that there exists a $b_{t+m+2} < b_{t+m+1}$ such that:

$$u(R\left(d_{t+m+1}\right)d_{t+m+1} + b_{t+m+2}) = u(R\left(d_{t+m}\right)d_{t+m} + b_{t+m+1})$$

We therefore have that there exits:

$$\begin{aligned} d_{t+m+1} &> d_{t+m} > k^b \\ b_{t+m+2} &< b_{t+m+1} < b \end{aligned}$$

such that generations $t+m$ and $t+m+1$ are no worse off ((32) and (33) hold) and phasing in of FF and out of PAYG continues.

Generation $t+m+2$

For generation $t+m+2$ to be no worse off the following must hold:

$$\begin{aligned} \Omega^{*\text{TRANSITION}}_{t+m+2} &= u\left(\omega\left(d_{t+m+1}\right) - d_{t+m+2} - b_{t+m+2}\right) + \beta^* u(R\left(d_{t+m+2}\right) d_{t+m+2} + b_{t+m+3}) \\ &= u\left(\omega\left(k^b\right) - k^b - b\right) + \beta^* u(R\left(k^b\right) k^b + b) \\ &= \Omega^{*\text{PAYG}} \end{aligned} \tag{34}$$

From (33), this requires:

$$\begin{aligned} \Omega^{*\text{TRANSITION}}_{t+m+2} &= u\left(\omega\left(d_{t+m+1}\right) - d_{t+m+2} - b_{t+m+2}\right) + \beta^* u(R\left(d_{t+m+2}\right) d_{t+m+2} + b_{t+m+3}) \\ &= u\left(\omega\left(d_{t+m}\right) - d_{t+m+1} - b_{t+m+1}\right) + \beta^* u(R\left(d_{t+m+1}\right) d_{t+m+1} + b_{t+m+2}) \\ &= \Omega^{*\text{T}}_{t+m+1} \end{aligned}$$

Since $d_{t+m+1} > d_{t+m}$ we have that $\omega\left(d_{t+m+1}\right) > \omega\left(d_{t+m}\right)$. Using this, $b_{t+m+2} < b_{t+m+1}$ and $u'\left(,\right) > 0$ we have:

$$u\left(\omega\left(d_{t+m+1}\right) - d_{t+m+1} - b_{t+m+2}\right) > u\left(\omega\left(d_{t+m}\right) - d_{t+m+1} - b_{t+m+1}\right)$$

Therefore, there exists $d_{t+m+2} > d_{t+m+1}$ such that:

$$u\left(\omega\left(d_{t+m+1}\right) - d_{t+m+2} - b_{t+m+2}\right) = u\left(\omega\left(d_{t+m}\right) - d_{t+m+1} - b_{t+m+1}\right)$$

Using Assumption 4 (which implies that $\frac{\partial[R(k_t)k_t]}{\partial k_t} > 0$):

$$u(R\left(d_{t+m+2}\right) d_{t+m+2} + b_{t+m+2}) > u(R\left(d_{t+m+1}\right) d_{t+m+1} + b_{t+m+2})$$

implying that there exists a $b_{t+m+3} < b_{t+m+2}$ such that:

$$u(R\left(d_{t+m+2}\right) d_{t+m+2} + b_{t+m+3}) = u(R\left(d_{t+m+1}\right) d_{t+m+1} + b_{t+m+2})$$

We therefore have there exists:

$$\begin{aligned} d_{t+m+2} &> d_{t+m+1} > d_{t+m} > k^b \\ b_{t+m+3} &< b_{t+m+2} < b_{t+m+1} < b \end{aligned}$$

such that generation $t+m$, $t+m+1$ and $t+m+2$ are no worse off ((32), (33) hold (34) hold) and phasing in of FF and out of PAYG continues.

Continuing this for the following generations givens the same result and we have decreasing sequence of b and increasing sequence of d implying phasing out of the PAYG scheme and phasing in of the FF scheme satisfying the Pareto condition.

H The existence of Pareto improving transition to a FF scheme starting from an arbitrary date

Recall, the equilibrium law of motion for the capital stock is given by $k_{t+1} = \psi\left(k_t, \beta, b\right)$ where $\frac{\partial k_{t+1}}{\partial k_t} > 0$, $\frac{\partial k_{t+1}}{\partial \beta} > 0$ and $\frac{\partial k_{t+1}}{\partial b} < 0$. Steady state capital stock is defined as the k solving $k = \psi\left(k, \beta, b\right)$. The steady state capital stock with and without a PAYG scheme under myopia are k^b and k, respectively, and the steady state capital stock without a PAYG scheme and in the absence of myopia is k^*. From Lemma 4 in the paper, we have that $k^b < k < k^*$.

Assume that the myopic economy is in a steady state without a PAYG scheme and the capital stock is k. Then at time t a PAYG scheme is introduced with constant contribution and benefits b. This results in the economy developing towards a new steady state with capital stock $k^b < k$. Since $\frac{\partial k_{t+1}}{\partial k_t} > 0$ and $\frac{\partial k_{t+1}}{\partial b} < 0$, this results in the capital stock decreasing towards the new steady state

$$k = k_t^b > k_{t+1}^b > ... > k^b$$

where true life-time utility is

$$\Omega_{t+j}^{*\text{PAYG}} = u\left(\omega\left(k_{t+j}^b\right) - k_{t+j+1}^b - b\right) + \beta^* u(R\left(k_{t+j+1}^b\right) k_{t+j+1}^b + b) \text{ for } j \geq 0 \tag{35}$$

and the development of the capital stock is given by

$$u'\left(\omega\left(k_{t+j}^b\right) - k_{t+j+1}^b - b\right) = R\left(k_{t+j+1}^b\right) \beta u'(R\left(k_{t+j+1}^b\right) k_{t+j+1}^b + b) \text{ for } j \geq 0 \tag{36}$$

or

$$u'\left(c_{t+j}^{yb}\right) = R\left(k_{t+j+1}^b\right) \beta u'(c_{t+j+1}^{ob}) \text{ for } j \geq 0$$

FF scheme is introduced at time $t+m$ with increasing contribution to the FF scheme $d_{t+m} < d_{t+m+1} < ... < d$ and decreasing contribution to the PAYG scheme $b_{t+m} = b > b_{t+m+1} > ... > 0$. At time of introduction $t+m$, the economy can either be on transition path towards the PAYG steady state with capital stock k_{t+m} or at the PAYG steady state with capital stock k^b, where $k^b < k_{t+m} < k$.

Generation $t+m$

We have that (due to private and FF pension savings being perfect substitutes) $d_{t+m} > k_{t+m+1}^b$ is necessary for generation $t+m$ to be better off by introduction the FF scheme pushing private savings to the zero corner. Hence, $k_{t+m+1}^d = d_{t+m} > k_{t+m+1}^b$ and true life-time utility of generation $t+m$ is therefore

$$\Omega_{t+m}^{*\text{TRANSITION}} = u\left(\omega\left(k_{t+m}^b\right) - d_{t+m} - b\right) + \beta^* u(R\left(d_{t+m}\right) d_{t+m} + b_{t+m+1}) \tag{37}$$

First order Taylor approximation of (37) to the PAYG allocation in (36) gives

$$\begin{aligned}
\hat{\Omega}_{t+m}^{*\text{TRANSITION}} &= -u'\left(c_{t+m}^{yb}\right)\hat{d}_{t+m} + \beta^* u'(c_{t+m+1}^{ob})\left[R'\left(k_{t+m+1}^b\right)k_{t+m+1}^b + R\left(k_{t+m+1}^b\right)\right]\hat{d}_{t+m} \\
&\quad +\beta^* u'(c_{t+m+1}^{ob})\hat{b}_{t+m+1} \\
&= u'(c_{t+m+1}^{ob})R\left(k_{t+m+1}^b\right)\left\{\beta^*\left[1+\eta(k_{t+m+1}^b)\right] - \beta\right\}\hat{d}_{t+m} \\
&\quad +\beta^* u'(c_{t+m+1}^{ob})\hat{b}_{t+m+1}
\end{aligned}$$

where $\hat{\Omega}_{t+m}^{*\text{TRANSITION}} \equiv \Omega_{t+m}^{*\text{TRANSITION}} - \Omega_{t+m}^{*\text{PAYG}}$, $\hat{d}_{t+m} \equiv d_{t+m} - k_{t+m+1}^b$ and $\hat{b}_{t+m+1} \equiv b_{t+m+1} - b$. Welfare of an individual born in period $t+m$ is therefore unchanged iff

$$\hat{\Omega}_{t+m}^{*\text{TRANSITION}} = 0$$

which requires

$$R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right]-\beta\right\}\hat{d}_{t+m}+\beta^*\hat{b}_{t+m+1}=0$$

or

$$\hat{b}_{t+m+1}=-\frac{R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right]-\beta\right\}}{\beta^*}\hat{d}_{t+m} \tag{38}$$

Hence, given Assumption 4 there exists a $\hat{b}_{t+m+1}<0$ or $b_{t+m+1}<b=b_{t+m}$ for every $\hat{d}_{t+m}>0$ or $d_{t+m}>k^b_{t+m+1}$ satisfying (38). Therefore, the FF pension can be introduced and the PAYG pension lowered such that life-time utility for individuals born in period $t+m$ (cohort $t+m$) is unaffected satisfying the Pareto criterion. Note that

1. the initial contribution to the FF scheme is greater than the capital stock under the PAYG scheme $d_{t+m}>k^b_{t+m+1}$ and, hence, non-trivial, and
2. this result holds independent of whether the economy is already at the PAYG steady state ($k^b_{t+m}=k^b_{t+m+1}=k^b$) or on the transition path towards it ($k^b_{t+m}>k^b_{t+m+1}>k^b$) when the FF scheme is introduced.

Generation $t+m+j$**,** $j>0$

Since $k^d_{t+m+1}=d_{t+m}$ and $d_{t+m+j}>d_{t+m}$ we have that $k^d_{t+m+j+1}=d_{t+m+j}$ implying that we have full crowding out of private savings. True life-time welfare of generation $t+m+j$ can therefore be written as

$$\Omega^{*\text{TRANSITION}}_{t+m+j}=u\left(\omega\left(d_{t+m+j-1}\right)-d_{t+m+j}-b_{t+m+j}\right)+\beta^*u(R\left(d_{t+m+j}\right)d_{t+m+j}+b_{t+m+j+1})\text{ for }j>0 \tag{39}$$

The benefit from introducing the FF scheme for generation $t+m+j$ is given by a first order Taylor approximation

$$\hat{\Omega}^{*\text{TRANSITION}}_{t+m+j}=\Omega^{*\text{TRANSITION}}_{t+m+j}-\Omega^{*\text{PAYG}}_{t+m+j}$$

, i.e. the difference between true life-time utility from introducing the FF scheme and the true life-time utility under the PAYG scheme, and is here calculated as a first order Taylor approximation of (39) to the PAYG allocation in (36). Following an introduction of the FF scheme the economy starts to diverge away from its PAYG path with increasing capital stock whereas the capital stock decreases under the PAYG scheme ($k^d_{t+m+j+1}=d_{t+m+j}>d_{t+m}>k^b_{t+m+1}>k^b_{t+m+j+1}$). This questions whether it is appropriate to approximate (39) to the PAYG allocation in (36) when $j>0$. We therefore write the benefits as the difference between two approximations $\hat{\Omega}^{*\text{TRANSITION}}_{t+m+j}\equiv\tilde{\Omega}^{*\text{TRANSITION}}_{t+m+j}-\tilde{\Omega}^{*\text{PAYG}}_{t+m+j}$ where

$$\begin{aligned}\tilde{\Omega}^{*\text{TRANSITION}}_{t+m+j}&\equiv\Omega^{*\text{TRANSITION}}_{t+m+j}-\Omega^{*\text{PAYG}}_{t+m}\\ \tilde{\Omega}^{*\text{PAYG}}_{t+m+j}&\equiv\Omega^{*\text{PAYG}}_{t+m+j}-\Omega^{*\text{PAYG}}_{t+m}\end{aligned}$$

and the first one gives the true life-time utility from introducing the FF scheme and the second gives the true life-time utility of from the PAYG scheme, both approximated to the true life-time utility of generation $t+m$ under the PAYG scheme, under which capital stock accumulation is given by

$$u'\left(\omega\left(k^b_{t+m}\right)-k^b_{t+m+1}-b\right)=R\left(k^b_{t+m+1}\right)\beta u'(R\left(k^b_{t+m+1}\right)k^b_{t+m+1}+b) \tag{40}$$

or

$$u'\left(c^{yb}_{t+m}\right) = R\left(k^b_{t+m+1}\right)\beta u'(c^{ob}_{t+m+1})$$

First order Taylor approximation of (39) and (35) to (40) gives

$$\begin{aligned}
\tilde{\Omega}^{*\text{TRANSITION}}_{t+m+j} &= u'\left(c^{yb}_{t+m}\right)\omega'\left(k^b_{t+m}\right)\hat{d}_{t+m+j-1} - u'\left(c^{yb}_{t+m}\right)\hat{d}_{t+m+j} - u'\left(c^{yb}_{t+m}\right)\hat{b}_{t+m+j} \\
&\quad +\beta^* u'(c^{ob}_{t+m+1})\left[R'\left(k^b_{t+m+1}\right)k^b_{t+m+1} + R\left(k^b_{t+m+1}\right)\right]\hat{d}_{t+m+j} + \beta^* u'(c^{ob}_{t+m+1})\hat{b}_{t+m+j+1} \\
&= u'(c^{ob}_{t+m+1})\left\{\begin{array}{c} R\left(k^b_{t+m+1}\right)\beta\left[\omega'\left(k^b_{t+m}\right)\hat{d}_{t+m+j-1} - \hat{b}_{t+m+j}\right] \\ +R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right] - \beta\right\}\hat{d}_{t+m+j} + \beta^*\hat{b}_{t+m+j+1} \end{array}\right\}
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\Omega}^{*\text{PAYG}}_{t+m+j} &= u'\left(c^{yb}_{t+m}\right)\omega'\left(k^b_{t+m}\right)\hat{k}^b_{t+m+j} - u'\left(c^{yb}_{t+m}\right)\hat{k}^b_{t+m+j+1} \\
&\quad +\beta^* u'(c^{ob}_{t+m+1})\left[R'\left(k^b_{t+m+1}\right)k^b_{t+m+1} + R\left(k^b_{t+m+1}\right)\right]\hat{k}^b_{t+m+j+1} \\
&= u'(c^{ob}_{t+m+1})\left\{\begin{array}{c} R\left(k^b_{t+m+1}\right)\beta\omega'\left(k^b_{t+m}\right)\hat{k}^b_{t+m+j} \\ +R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right] - \beta\right\}\hat{k}^b_{t+m+j+1} \end{array}\right\}
\end{aligned}$$

where $\hat{d}_{t+m+j-1} \equiv d_{t+m+j-1} - k^b_{t+m}$, $\hat{d}_{t+m+j} \equiv d_{t+m+j} - k^b_{t+m+1}$, $\hat{b}_{t+m+j} \equiv b_{t+m+j} - b$, , $\hat{b}_{t+m+j+1} \equiv b_{t+m+j+1} - b$, $\hat{k}^b_{t+m+j} \equiv k^b_{t+m+j} - k^b_{t+m}$ and $\hat{k}^b_{t+m+j+1} \equiv k^b_{t+m+j+1} - k^b_{t+m+1}$.

For $\hat{\Omega}^{*\text{TRANSITION}}_{t+m+j} = 0$ to hold, we have $\tilde{\Omega}^{*\text{TRANSITION}}_{t+m+j} = \tilde{\Omega}^{*\text{PAYG}}_{t+m+j}$ or

$$\begin{aligned}
& u'(c^{ob}_{t+m+1})\left\{\begin{array}{c} R\left(k^b_{t+m+1}\right)\beta\left[\omega'\left(k^b_{t+m}\right)\hat{d}_{t+m+j-1} - \hat{b}_{t+m+j}\right] \\ +R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right] - \beta\right\}\hat{d}_{t+m+j} + \beta^*\hat{b}_{t+m+j+1} \end{array}\right\} \\
= \; & u'(c^{ob}_{t+m+1})\left\{\begin{array}{c} R\left(k^b_{t+m+1}\right)\beta\omega'\left(k^b_{t+m}\right)\hat{k}^b_{t+m+j} \\ +R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right] - \beta\right\}\hat{k}^b_{t+m+j+1} \end{array}\right\}
\end{aligned}$$

and, hence

$$\begin{aligned}
\hat{b}_{t+m+j+1} &= \frac{R\left(k^b_{t+m+1}\right)\beta\omega'\left(k^b_{t+m}\right)}{\beta^*}\hat{k}^b_{t+m+j} + \frac{R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right] - \beta\right\}}{\beta^*}\hat{k}^b_{t+m+j+1} \\
&\quad -\frac{R\left(k^b_{t+m+1}\right)\beta}{\beta^*}\left[\omega'\left(k^b_{t+m}\right)\hat{d}_{t+m+j-1} - \hat{b}_{t+m+j}\right] \\
&\quad -\frac{R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right] - \beta\right\}}{\beta^*}\hat{d}_{t+m+j}
\end{aligned}$$

or

$$\begin{aligned}
\hat{b}_{t+m+j+1} &= \frac{R\left(k^b_{t+m+1}\right)\beta\omega'\left(k^b_{t+m}\right)}{\beta^*}\left(\hat{k}^b_{t+m+j} - \hat{d}_{t+m+j-1}\right) \\
&\quad +\frac{R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right] - \beta\right\}}{\beta^*}\left(\hat{k}^b_{t+m+j+1} - \hat{d}_{t+m+j}\right) \\
&\quad +\frac{R\left(k^b_{t+m+1}\right)\beta}{\beta^*}\hat{b}_{t+m+j}
\end{aligned}$$

or

$$
\begin{aligned}
b_{t+m+j+1}-b &= \frac{R\left(k^b_{t+m+1}\right)\beta\omega'\left(k^b_{t+m}\right)}{\beta^*}\left(k^b_{t+m+j}-d_{t+m+j-1}\right)\\
&+\frac{R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right]-\beta\right\}}{\beta^*}\left(k^b_{t+m+j+1}-d_{t+m+j}\right)\\
&+\frac{R\left(k^b_{t+m+1}\right)\beta}{\beta^*}\left(b_{t+m+j}-b\right)
\end{aligned}
$$

where it is used that $\hat{b}_{t+m+j}\equiv b_{t+m+j}-b$, $\hat{k}^b_{t+m+j}-\hat{d}_{t+m+j-1}=\left(k^b_{t+m+j}-k^b_{t+m}\right)-\left(d_{t+m+j-1}-k^b_{t+m}\right)=k^b_{t+m+j}-d_{t+m+j-1}$ and $\hat{k}^b_{t+m+j+1}-\hat{d}_{t+m+j}=\left(k^b_{t+m+j+1}-k^b_{t+m+1}\right)-\left(d_{t+m+j}-k^b_{t+m+1}\right)=k^b_{t+m+j+1}-d_{t+m+j}$. Further, we have

$$
\begin{aligned}
\Delta b_{t+m+j+1} &= \frac{R\left(k^b_{t+m+1}\right)\beta\omega'\left(k^b_{t+m}\right)}{\beta^*}\left(\Delta k^b_{t+m+j}-\Delta d_{t+m+j-1}\right)\\
&+\frac{R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right]-\beta\right\}}{\beta^*}\left(\Delta k^b_{t+m+j+1}-\Delta d_{t+m+j}\right)\\
&+\frac{R\left(k^b_{t+m+1}\right)\beta}{\beta^*}\Delta b_{t+m+j} \qquad (41)
\end{aligned}
$$

where $\Delta k^b_{t+m+j}\equiv k^b_{t+m+j}-k^b_{t+m+j-1}$ etc.

Individuals born in period $t+m+1$ $(j=1)$

From (41) for $j=1$

$$
\begin{aligned}
\Delta b_{t+m+2} &= \frac{R\left(k^b_{t+m+1}\right)\beta\omega'\left(k^b_{t+m}\right)}{\beta^*}\left(\Delta k^b_{t+m+1}-\Delta d_{t+m}\right)\\
&+\frac{R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right]-\beta\right\}}{\beta^*}\left(\Delta k^b_{t+m+2}-\Delta d_{t+m+1}\right)\\
&+\frac{R\left(k^b_{t+m+1}\right)\beta}{\beta^*}\Delta b_{t+m+1}
\end{aligned}
$$

We have that $\frac{R\left(k^b_{t+m+1}\right)\beta\omega'\left(k^b_{t+m}\right)}{\beta^*}>0$ since $\omega'\left(k^b_{t+m}\right)>0$, $\frac{R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right]-\beta\right\}}{\beta^*}>0$ given Assumption 4 and $\frac{R\left(k^b_{t+m+1}\right)\beta}{\beta^*}>0$. Since $\Delta k^b_{t+m+1}<0$, $\Delta k^b_{t+m+2}<0$ and we have from above that there exists a $\Delta b_{t+m+1}=b_{t+m+1}-b_{t+m}=b_{t+m+1}-b<0$ for $\Delta d_{t+m}=d_{t+m}-d_{t+m-1}=d_{t+m}>k^b_{t+m+1}>0$ such that the true life-time utility of generation $t+m$ is unaffected, it follows that there exists $\Delta b_{t+m+2}<0$ for every $\Delta d_{t+m+1}>0$ such that the true life-time utility of generation $t+m+1$ is unaffected.

We have therefore shown that there exists $b_{t+m}=b>b_{t+m+1}>b_{t+m+2}$ and $d_{t+m}<d_{t+m+1}$ satisfying the Pareto criterion. Note that this result holds independent of whether the economy would have been at the PAYG steady state ($\Delta k^b_{t+m+2}=\Delta k^b_{t+m+1}=0$) or on the transition path towards it ($\Delta k^b_{t+m+2}<0$, $\Delta k^b_{t+m+1}<0$) in the absence of the FF scheme being introduced.

Individuals born in period $t+m+2$ *(*$j=2$*)*

From (41) for $j=2$

$$
\begin{aligned}
\Delta b_{t+m+3} &= \frac{R\left(k^b_{t+m+1}\right)\beta\omega'\left(k^b_{t+m}\right)}{\beta^*}\left(\Delta k^b_{t+m+2}-\Delta d_{t+m+1}\right)\\
&+\frac{R\left(k^b_{t+m+1}\right)\left\{\beta^*\left[1+\eta(k^b_{t+m+1})\right]-\beta\right\}}{\beta^*}\left(\Delta k^b_{t+m+3}-\Delta d_{t+m+2}\right)\\
&+\frac{R\left(k^b_{t+m+1}\right)\beta}{\beta^*}\Delta b_{t+m+2}
\end{aligned}
$$

Since $\Delta k^b_{t+m+2} < 0$, $\Delta k^b_{t+m+3} < 0$ and we have from above that there exists a $\Delta b_{t+m+2} < 0$ for $\Delta d_{t+m+1} > 0$ such that the true life-time utility of generations $t+m$ and $t+m+1$ are unaffected, it follows that there exists $\Delta b_{t+m+3} < 0$ for every $\Delta d_{t+m+2} > 0$ such that the true life-time utility of generation $t+m+2$ is unaffected.

We have therefore shown that there exists $b_{t+m} = b > b_{t+m+1} > b_{t+m+2} > b_{t+m+3}$ and $d_{t+m} < d_{t+m+1} < d_{t+m+2}$ satisfying the Pareto criterion. Note that this result holds independent of whether the economy would have been at the PAYG steady state ($\Delta k^b_{t+m+3} = \Delta k^b_{t+m+2} = 0$) or on the transition path towards it ($\Delta k^b_{t+m+3} <$, $\Delta k^b_{t+m+2} < 0$) in the absence of the FF scheme being introduced.

Individuals born in period $t+m+j$ ($j > 2$)

Repeating the above for generations $t+m+3, t+m+4, ...$ we have that there exist sequences $b_{t+m} = b > b_{t+m+1} > b_{t+m+2} > b_{t+m+3} > .... > 0$ and $d_{t+m} < d_{t+m+1} < d_{t+m+2} < .... < d$ satisfying the Pareto criterion showing that the PAYG scheme can be phased out and a FF scheme phased in without decreasing the true life-time utility of any generation during the transition.