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# Ultrasonic measurement models for surface wave and plate wave inspections

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**Abstract**
A complete ultrasonic measurement model for surface and plate wave inspections is obtained, where all the electrical, electromechanical, and acoustic/elastic elements are explicitly described. Reciprocity principles are used to describe the acoustic/elastic elements specifically in terms of an integral of the incident and scattered wave fields over the surface of the flaw. As with the case of bulk waves, if one assumes the incident surface waves or plate waves are locally planar at the flaw surface, the overall measurement model reduces to a very modular form where the far-field scattering amplitude of the flaw appears explicitly.

**Keywords**
ultrasonics, surface waves (fluid), nondestructive testing, acoustic field, nondestructive evaluation, QNDE, Aerospace Engineering

**Disciplines**
Aerospace Engineering | Materials Science and Engineering | Mechanical Engineering | Structures and Materials

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# ULTRASONIC MEASUREMENT MODELS FOR SURFACE WAVE AND PLATE WAVE INSPECTIONS

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**ABSTRACT.** A complete ultrasonic measurement model for surface and plate wave inspections is obtained, where all the electrical, electromechanical, and acoustic/elastic elements are explicitly described. Reciprocity principles are used to describe the acoustic/elastic elements specifically in terms of an integral of the incident and scattered wave fields over the surface of the flaw. As with the case of bulk waves, if one assumes the incident surface waves or plate waves are locally planar at the flaw surface, the overall measurement model reduces to a very modular form where the far-field scattering amplitude of the flaw appears explicitly.

**Keywords:** Ultrasonic Measurement Model, Surface Waves, Plate Waves, Reciprocity
**PACS:** 43.35

## INTRODUCTION

Although ultrasonic bulk waves are used in the majority of ultrasonic NDE tests, surface and plate waves also are important because of their ability to travel long distances and interrogate remote flaws. Complete ultrasonic measurement models for ultrasonic bulk wave inspections have been available for some time and have been used effectively for simulating ultrasonic bulk wave tests [1] but the same modeling framework has not been developed for corresponding surface and plate wave inspections. Here, we will, show that a complete measurement model can be similarly defined for surface and plate waves, using general reciprocity principles, that is applicable under very general conditions to many types of surface and plate wave inspections. It is also demonstrated that this new measurement model reduces to a very modular form similar to the type originally obtained for bulk waves by Thompson and Gray where the scattering amplitude of the flaw appears explicitly in the measurement model [1], [2]. This form has been very useful for conducting quantitative flaw characterization analyses with bulk waves and should prove equally valuable for surface and plate wave inspections.

## AN ELECTROACOUSTIC MEASUREMENT MODEL FOR CONTACT TESTS

Previously, we have developed a complete "electroacoustic" measurement model of ultrasonic bulk wave immersion systems that describes explicitly all the components including the pulser/receiver, cabling, transducers, and the acoustic-elastic processes

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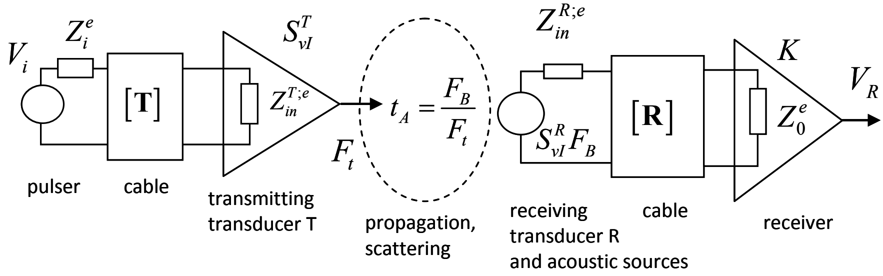


**FIGURE 1.** Elements of an electroacoustic measurement model for an ultrasonic immersion setup.

present [1],[3-4]. This model is shown graphically in Fig. 1. The pulser is modeled as a Thevenin equivalent voltage source, $V_i$, and electrical impedance, $Z_i^e$, and the receiver is modeled as an electrical impedance, $Z_0^e$, and gain factor, *K*. The cabling is modeled as 2x2 transfer matrices, [T] and [R], during sound generation and reception, respectively. The sending transducer, T, in an immersion setup is modeled as an input electrical impedance, $Z_{in}^{T;e}$, and a sensitivity, $S_{vI}^T$, that is defined as the ratio of the average output velocity, *v*, on the face of the transducer to the driving current, *I*. On reception, the receiving transducer, R, and the acoustic waves that drive it are modeled as a voltage source of magnitude $S_{vI}^R F_B$ and an electrical impedance, $Z_{in}^{R;e}$, where $S_{vI}^R$ is the sensitivity of the receiving transducer (of the same type used to describe the transmitting transducer) and $Z_{in}^{R;e}$ is the input impedance of the receiving transducer. The term $F_B$ is the blocked force acting at the face of the receiving transducer, which is defined to be the force at this transducer when its face is held rigidly fixed [1]. Between the sending and receiving transducers, all the wave propagation and scattering processes present are described in terms of an acoustic-elastic transfer function, $t_A = F_B / F_t$, which is defined as the ratio of the received blocked force to the force at the face of the transmitting transducer. It should be noted that all of these elements are defined in the frequency domain and hence, in general, depend of the frequency, *f*, but this dependency has not been shown explicitly. It has been shown previously that all of these elements except for the acoustic-elastic transfer function can be measured explicitly with a series of purely electrical measurements. Alternatively, it has also been shown that all these electrical and electromechanical terms can be lumped into a single "system function", $s(f)$, that can be obtained by a single measurement in a calibration setup where $t_A$ is known [1]. Thus, the received voltage, $V_R$, appearing in Fig. 1 is then given simply as [1]

$$V_R(f) = s(f)\, t_A(f) \tag{1}$$

For a surface wave or plate wave inspection, the pulser/receiver and cabling elements present can be described by exactly the same terms found in Fig. 1 for an immersion setup, so we need to only examine what differences may be present at the

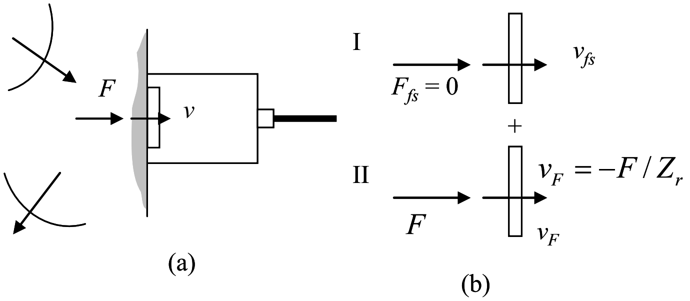


**FIGURE 2.** (a) A contact receiving transducer, and (b) the decomposition of the reception process into problems I and II. In problem I the surface is completely free and the incident waves are present while in problem II the incident waves are absent and the force, *F*, acting on the transducer face is present.

sending and receiving transducers and in the definition of the acoustic-elastic transfer function. At the sending transducer, in an immersion setup the transducer is often modeled as a piston (constant velocity) source. For surface and plate wave inspection setups, however, the sending transducer is often either in direct contact to the surface of the part being inspected or placed in contact with the surface of a wedge. In these contact cases, it is more appropriate to model the transducer output as a pressure distribution on the surface. However, in both the immersion and contact cases the sending transducer can be modeled as an electrical impedance and sensitivity as shown in Fig. 1. Also, the output of the transducer in both cases can be taken either as the total force, $F_t$, at the face of the transducer, as shown in Fig. 1, or the output could be described in terms of the average velocity at the face, $v_t$. Since the force and velocity are related through the acoustic radiation impedance, $Z_r^{T;a}$, of the transmitter, where $F_t = Z_r^{T;a} v_t$, the choice of force or velocity is immaterial. In fact, note that in the immersion case of Fig. 1 the transducer output (and acoustic-elastic transfer function) was defined in terms of the output force although the transducer was explicitly modeled as a velocity source. Thus, in a contact or an immersion setup the elements on the sound generation side of Fig. 1 are identical.

On the receiving transducer side, however, there are some differences between an immersion inspection and a contact inspection. Figure 2(a) shows a contact receiving transducer lying on an otherwise free surface and the incident and scattered waves at the receiving transducer. The force and average velocity from all the waves acting on the face of this transducer are given as *F* and *v*, respectively. This setup can be decomposed into the solution of two separate problems as shown in Fig. 2(b), where in problem I the pressure (and force) on the face of the face of the transducer is zero and the average velocity on the transducer face is just the free surface velocity, $v_{fs}$, due to the incident and reflected waves. In problem II incident waves are absent and the total force, *F*, present in the original problem of Fig. 2(a) is placed on the face of the transducer and the resulting average velocity on the transducer face is $v_F$. Since problem II is identical to that of a transmitting transducer problem, F and $v_F$ are related through the acoustic radiation

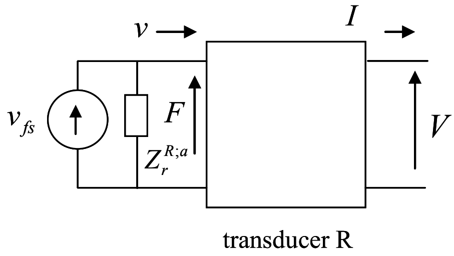


**FIGURE 3.** A lumped parameter model of a contact receiving transducer as a "mechanical current" source in parallel with the acoustic radiation impedance of the transducer.

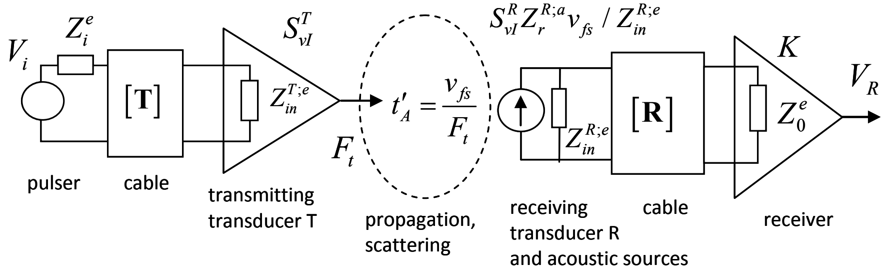


**FIGURE 4.** Elements of an electroacoustic measurement model for an ultrasonic contact testing setup.

impedance of the receiving transducer, $Z_r^{R;a}$, i.e. we have $F = -Z_r^{R;a} v_F$, where the minus sign is present because we have taken the velocity $v_F$ to be in the direction towards the face of the transducer. Since $v = v_{fs} + v_F$ we find

$$v = v_{fs} - \frac{F}{Z_r^{R;a}} \tag{2}$$

In the immersion case, a similar decomposition yields

$$F = F_B - Z_r^{R;a} v \tag{3}$$

where the blocked force, $F_B$, appears instead [1].

If we model the transducer as a 2x2 transfer matrix that transforms the force, *F*, and velocity, *v*, at its input port to voltage, *V*, and current, *I*, at its output port, then Eq. (2) implies the lumped parameter model of Fig. 3, where a "mechanical current" source, $v_{fs}$, is placed in parallel with the acoustical radiation impedance of the receiving transducer (Fig. 3). As in the immersion case, the transfer matrix and these source and impedance terms can be combined into a Thevenin equivalent current source whose magnitude is $S_{vI}^R Z_r^{R;a} v_{fs} / Z_{in}^{R;e}$ in parallel with the electrical impedance of the receiving transducer, $Z_{in}^{R;e}$.

The details are very similar to the immersion case so we will omit them here. Since the free surface velocity in the contact case plays a similar role to that of the blocked force for immersion tests, it is reasonable for the contact case to define the acoustic-elastic transfer function instead as $t'_A = v_{fs} / F_t$. One then has the complete electroacoustic measurement model of Fig. 4 for contact test corresponding to the immersion model of Fig. 1. As with the immersion case one can lump all the electrical and electromechanical elements into a single system function and write

$$V_R(f) = s'(f) t'_A(f) \tag{4}$$

If one compares the system function, $s'(f)$, for the contact case with the system function, $s(f)$, for the immersion case, one finds simply $s'(f) = Z_r^{R;a} s(f)$ i.e. they differ only by the acoustic radiation impedance of the receiving transducer.

## THE ACOUSTIC-ELASTIC TRANSFER FUNCTION

To complete the electroacoustic measurement model of Fig. 4 one must define in more explicit terms the acoustic-elastic transfer function since this term depends on the incident and scattered wave fields and must be obtained through ultrasonic beam and flaw scattering models. For either the immersion or contact cases one can use general mechanical reciprocity principles to relate forces and velocity at the receiving transducer to fields at the surface of the flaw [1]. One finds

$$F_R^f v_R^{(2)} - F_R^{(2)} v_R^f = \int_{S_f} \left( \mathbf{t}^{(1)} \cdot \mathbf{v}^{(2)} - \mathbf{t}^{(2)} \cdot \mathbf{v}^{(1)} \right) dS \tag{5}$$

where $F_R^f, v_R^f$ are the force and velocity generated on the receiving transducer from interactions of the incident waves with the flaw in state (1) where the transmitting transducer is firing and the flaw is present and, $F_R^{(2)}, v_R^{(2)}$ are the force and velocity on the receiving transducer in state (2) where the receiving transducer is assumed to act as a transmitter and the flaw is absent. Similarly, $\mathbf{t}^{(1)}, \mathbf{v}^{(1)}$ are the stress vector and velocity vector on the surface of the flaw, $S_f$, for state (1) and $\mathbf{t}^{(2)}, \mathbf{v}^{(2)}$ are the stress vector and velocity vector for state (2). Since in state (2) at the receiver the force and velocity are related through the acoustic radiation impedance, i.e. $F_R^{(2)} = Z_r^{R;a} v_R^{(2)}$, one finds

$$\left( \frac{F_R^f}{Z_r^{R;a}} - v_R^f \right) F_R^{(2)} = \int_{S_f} \left( \mathbf{t}^{(1)} \cdot \mathbf{v}^{(2)} - \mathbf{t}^{(2)} \cdot \mathbf{v}^{(1)} \right) dS \tag{6}$$

But setting $v_R^f = -v, F_R^{(2)} = F$ (the minus sign is present since the positive direction for $v_R^f$ is opposite to that for $v$) the terms in brackets in Eq. (6) can be recognized from Eq. (2) to be just the free surface velocity at the receiving transducer produced by the flaw. Thus, Eq. (6) gives us an explicit relationship for the transfer function:

$$t'_A = \frac{v_{fs}}{F_T^{(1)}} = \frac{1}{F_T^{(1)} F_R^{(2)}} \int_{S_f} \left( \mathbf{t}^{(1)} \cdot \mathbf{v}^{(2)} - \mathbf{t}^{(2)} \cdot \mathbf{v}^{(1)} \right) dS \tag{7}$$

where $F_T^{(1)} = F_t$ is the force on the transmitting transducer shown previously in our models. From Eq. (4) we then have a complete ultrasonic measurement model for a contact test given by

$$V_R(f) = s'(f) \left[ \frac{1}{F_T^{(1)} F_R^{(2)}} \int_{S_f} \left( \mathbf{t}^{(1)} \cdot \mathbf{v}^{(2)} - \mathbf{t}^{(2)} \cdot \mathbf{v}^{(1)} \right) dS \right] \tag{8}$$

It is useful to compare Eq. (8) with the similar result for immersion tests given by

$$V_R(f) = s(f) \left[ \frac{1}{Z_r^{T;a} v_T^{(1)} v_R^{(2)}} \int_{S_f} \left( \mathbf{t}^{(1)} \cdot \mathbf{v}^{(2)} - \mathbf{t}^{(2)} \cdot \mathbf{v}^{(1)} \right) dS \right] \tag{9}$$

From $F_T^{(1)} = Z_r^{T;a} v_T^{(1)}$, $F_R^{(2)} = Z_r^{R;a} v_R^{(2)}$ and the relationship between these system functions we see that Eq. (8) and Eq. (9) are identical. Thus, it is possible to use either of these equations as a complete measurement model, although Eq. (8) is in a more "natural" form for contact tests since the driving forces on the face of the transducers appear explicitly while Eq. (9) is in more natural form for immersion tests where the driving velocity fields appear instead.

## REDUCED MEASUREMENT MODEL

Equations (8) and (9) are very general results applicable to any ultrasonic immersion or contact setup. For bulk waves, if one assumes that the incident waves in solution (2) on the surface of the flaw behave locally like plane waves, then these equations reduce to the Thompson-Gray measurement model where the far-field scattering amplitude of the flaw appears explicitly [1]. Here, we will show that the same reduction is possible for surface and plate wave inspections. The details are rather lengthy so that we will just briefly outline the main results. First, we rewrite Eq. (8) in more explicit terms as

$$V_R(f) = s'(f) \left[ \frac{1}{F_T^{(1)} F_R^{(2)}} \int_{S_f} \left( C_{klij} \frac{\partial u_i^{(1)}}{\partial x_{sj}} n_k v_l^{(2)} - C_{klij} \frac{\partial u_i^{(2)}}{\partial x_{sj}} n_k v_l^{(1)} \right) dS(\mathbf{x}_s) \right] \tag{10}$$

where $C_{ijkl}$ are the tensor elastic constants and $u_i^{(m)} = v_i^{(m)} / (-i\omega)$ $(m = 1,2)$ are the displacements in solutions (1) and (2). If, for both solutions (1) and (2) we assume the incident waves on the flaw can be approximated locally as either a plane traveling Rayleigh surface wave or a plane plate wave mode, the form for the velocity of these incident waves is

$$\begin{aligned} \mathbf{v}^{(1);inc} &= V_0^{\beta} \mathbf{p}_{\beta}^{(1)} \exp\left( ik_{\beta} \mathbf{e}_{\beta}^{(1)} \cdot \mathbf{x}_s \right) \\ \mathbf{v}^{(2)} &= V_0^{\alpha} \mathbf{p}_{\alpha}^{(2)} \exp\left( ik_{\alpha} \mathbf{e}_{\alpha}^{(2)} \cdot \mathbf{x}_s \right) \end{aligned} \tag{11}$$

where $V_0^q\ (q=\alpha,\beta)$ is a velocity amplitude for a mode of type $\alpha$ or $\beta$, $\mathbf{e}_\beta^{(1)},\mathbf{e}_\alpha^{(2)}$ are 2-D unit vectors in the direction of propagation for these solutions, $k_q\ (q=\alpha,\beta)$ is the wave number of the propagating wave, and the polarization vectors, $\mathbf{p}_\beta^{(1)}, \mathbf{p}_\alpha^{(2)}$, are given by

$$\mathbf{p}_\beta^{(1)}=\begin{Bmatrix} v_1^\beta(x_{s3})e_{\beta1}^{(1)} \\ v_1^\beta(x_{s3})e_{\beta2}^{(1)} \\ iv_2^\beta(x_{s3}) \end{Bmatrix},\quad \mathbf{p}_\alpha^{(2)}=\begin{Bmatrix} v_1^\alpha(x_{s3})e_{\alpha1}^{(2)} \\ v_1^\alpha(x_{s3})e_{\alpha2}^{(2)} \\ iv_2^\alpha(x_{s3}) \end{Bmatrix} \tag{12}$$

The terms $v_m^{(\alpha,\beta)}(x_{s3})\ (m=1,2)$ are just the ordinary modal functions defined for 2-D Rayleigh waves or Lamb waves as a function of depth (in the case of Rayleigh waves) or as a function of the location in the thickness direction (for Lamb waves) [5]. If one uses Eq. (11) in Eq. (10) and assumes the velocity amplitudes are constant over the flaw surface then Eq. (10) can be written in the form

$$V_R^{\alpha\beta}(f)=s'(f)\frac{V_0^\beta}{F_T^{(1)}}\frac{V_0^\alpha}{F_R^{(2)}}A^{\alpha\beta}\left(\mathbf{e}_\beta^{(1)};-\mathbf{e}_\alpha^{(2)}\right)\left[\frac{4P^\alpha\sqrt{2\pi k_\alpha}}{k_\alpha\exp(i\pi/4)}\right] \tag{13}$$

where $V_R^{\alpha\beta}(f)$ is the received voltage (for a received mode of type $\alpha$ due to a mode of type $\beta$ incident on the flaw) and $A^{\alpha\beta}\left(\mathbf{e}_\beta^{(1)},-\mathbf{e}_\alpha^{(2)}\right)$ is the far-field scattering amplitude of the flaw for a scattered wave of mode type $\alpha$ traveling in the $-\mathbf{e}_\alpha^{(2)}$ direction due to an incident plane wave of mode type $\beta$ traveling in the $\mathbf{e}_\beta^{(1)}$ direction [6]. The term $P^\alpha$ is a normalized power/unit width in the $\alpha$ mode given by

$$P^\alpha=\frac{1}{2}\rho_s c_g^\alpha\int\left(\left|v_1^\alpha\right|^2+\left|v_2^\alpha\right|^2\right)dx_3 \tag{14}$$

where $\rho_s$ is the density of the solid and $c_g^\alpha$ is the group velocity. Equation (13) is written for a single incident and scattered mode so if multiple modes are present then the corresponding terms must all be added. Equation (13) is valid for either Rayleigh waves or Lamb waves but a very similar form can be written for Love waves or SH-mode plate waves as well. Equation (13) is very similar to the Thompson-Gray form obtained for bulk waves, which is given by [1]:

$$V_R^{\alpha\beta}(f)=s(f)\frac{V_0^\beta}{v_T^{(1)}}\frac{V_0^\alpha}{v_R^{(2)}}\left[\mathbf{A}^{\alpha\beta}\left(\mathbf{e}_\beta^{(1)};-\mathbf{e}_\alpha^{(2)}\right)\cdot\left(-\mathbf{p}_\alpha^{(2)}\right)\right]\left[\frac{4\pi\rho_s c_\alpha}{-ik_\alpha Z_r^{T;\alpha}}\right] \tag{15}$$

## SUMMARY AND CONCLUSIONS

We have shown that the same general ultrasonic measurement model can be used for immersion and contact tests involving waves of a general (bulk, surface or plate wave) type. We also showed that for both Rayleigh waves and Lamb waves this general measurement model reduces to a form very similar to the Thompson-Gray measurement model for bulk waves. This is an important result because the scattering amplitude of the flaw appears explicitly in this reduced form, showing how the flaw response explicitly contributes to the measured voltage. This reduced form also allows us to separate the calculation of the incident fields with beam models from the flaw scattering calculations and allows us to use deconvolution procedures to obtain measured flaw responses that are independent of the other parts of the ultrasonic measurement system. These new results will be used in the future for developing models of both Rayleigh and Lamb wave inspections.

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