8-24-2018

Working Paper Number 18021

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Disciplines
Economic Theory | Environmental Policy | Environmental Studies | Public Economics

A WAY TO RESOLVE INTERGENERATIONAL CONFLICT OVER THE ENVIRONMENT UNDER THE PARETO CRITERION USING GREEN BONDS

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This Version: August 24, 2018§

Abstract

Any policy designed to combat climate change will likely create intergenerational winners and losers because the associated costs appear up-front while the benefits come downstream. In such cases, cost benefit analysis seeks *potential* Pareto-improvements in the Kaldor–Hicks sense, the hypothetical potential for the winners to compensate the losers. Of course, once the policy is indeed implemented and has unleashed distortions and general-equilibrium effects, such potential improvements may not lead to *actual* improvements. We ask, which policies, once implemented, would pass the Pareto test that no generation subsequent to policy action be made worse off than before. We study a "business as usual" (BAU) overlapping-generations economy in which pollution is a by-product of productive activity by the current generation but "damages" production for future generations. Over time, the BAU economy gets increasingly polluted, consumption falls, and generational welfare levels decline. A government introduces costly pollution abatement and finances it via distorting taxes and the sale of green bonds. Pollution levels start to decline, generating downstream welfare gains. Some of these gains are taxed – without hurting anyone, in a Pareto sense – to help pay off the debt. Along the transition, every generation faces less pollution, consumes more and is happier than if life had continued in the BAU world.

JEL Classification Numbers: O44, Q56, H5

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§We have benefitted from comments from an anonymous referee, Lint Barrage, Satyajit Chatterjee, Francis Denning, Murat Iyigun, Larry Karp, Pietro Peretto, Elena Irwin, Scott Taylor, Christian Traeger, and participants at the No-Free-Lunch brownbag at Iowa State, the WEAI conference in Portland, 2017 Asian Meeting of the Econometric Society in Hong Kong, and seminar participants at Peking University. We also acknowledge financial support from the Danish Council for Independent Research (Social Sciences) under the Danish Ministry of Science, Technology and Innovation.

1 Introduction

Climate change is an externality – the Stern Review (2007) calls it the greatest market failure and largest externality in history. What is more, both the externality and the measures needed to address it are, necessarily, intergenerational in nature. After all, greenhouse gases are long-lived and their effects linger, long after they appear. Similarly, costly measures adopted today may generate benefits far into the future, well beyond the lifespan of the generations funding them. As such, any response towards combating climate change will require strong political action across generations. By the same token, however, any policy response will likely create intergenerational winners and losers and, in turn, raise thorny questions about intergenerational equity and its trade-off with efficiency. Pearson (2011) phrases it bluntly: "Should we sacrifice our use of cheap fossil fuel energy today so that generations yet unborn, who presumably will be richer than we are, can avoid adjusting to a warmer world?" Even if we could agree the answer is yes, how should the near-term costs of clean-up be allocated across generations in a fair and efficient manner? For if it is not perceived to be fair, why would different generations participate in this cross-generational initiative? There are other concerns of a more practical nature. Specifically, "there is no political institution or mechanism through which the present generation can securely compensate [...future] generations for the consequences of global warming [...and symmetrically,] there is no obvious way for future generations [...] to compensate us for our sacrifices if we take expensive greenhouse abatement measures today." (Pearson, 2011; pp. 23) Is it possible to navigate our way around these concerns?

In this paper, we are conceptually motivated by the same sort of questions as in the Integrated Assessment Models (IAM) of climate economics but our goal is far more modest: it is to produce a simple model capturing intergenerational conflict, an issue IAMs are typically silent on.[1] Specifically, we study an overlapping-generations economy with the following key features: the production technology has standard decreasing marginal productivity; the stock of greenhouse gases reduces this marginal productivity, but has no direct effect on the producing generation's utility; the government contemplates policy action aimed at correcting the underlying intergenerational externality; it can issue debt (green bonds) to finance its abatement activities and impose a distortionary, labor tax to service the debt or the policy itself[2]; the abatement policy can reduce the stock of greenhouse gases at some cost that is increasing and convex in the intensity of the abatement; there is no intergenerational altruism. In this economy, laissez faire – the business-as-usual world (henceforth, "BAU") – is associated with declining consumption, declining welfare and worsening of the environment over generations. Since heat-trapping gases such as carbon dioxide linger for generations, benefits from the government's clean-up effort would take time to

[1] The by-now vast literature on IAMs has united the science and economics of climate change via the damage function, a way of identifying the impacts of climate change and attributing monetary damages to them. Their goal, in many instances, is to maximize a global welfare criterion within the confines of an infinitely-lived-agent, Ramsey-style model and the control variable is an abatement policy. The point is to solve for optimal paths for consumption, investment, and other variables while devoting enough resources to keeping the environment clean. Intergenerational conflict of the type discussed above is absent or a mere sideshow in the one-agent, one-world scenario that IAMs study. See Schneider, Traeger, and Winkler (2012) for an insightful analysis of why insights from infinitely-lived agent IAMS – that are silent on intergenerational conflict – are so different from what overlapping-generations models deliver.

[2] One can think of such an externality as reducing the return of future production in terms of the level of utility it is capable to deliver. This can be a direct erosion of future productivity of inputs of production (such as, land) or an indirect disturbance to the ability of humans to enjoy consumption (for instance, because of extreme weather events).

ramp up thereby creating winners and losers across cohorts.

The standard response in the literature to these inter-generational equity concerns is to postulate a social welfare function, a weighted function of the welfare of current and future generations, the likely losers and winners respectively. Depending on the weights chosen – a matter of considerable importance discussed in the Stern Review (2007) – policy action is justified if it maximizes the generationally-aggregated, social welfare function even if it requires some generations to sacrifice for others.[3] Our approach is different. We do not seek to compute socially optimal policies. Instead, we directly ask, what sorts of policy action can be rationalized by imposing the Pareto criterion, the restriction that no generation subsequent to policy action be made worse off than before (the BAU world)?[4] The imposition of a generation-by-generation Pareto criterion is deliberate and instructive. It allows us to employ insights from overlapping-generation models of deficit financing – see Bovenberg and Heijdra (1998) – to address Pearson's (2011) concern about there being "no obvious way" by which generations can share clean-up costs: we allow for inter-generational compensation for investments in environment-friendly policies via debt financing (green bonds).[5] The general logic is, use debt to make future generations pay for the abatement policy that previous generations implemented via taxation on those future generation's labor. As for the Pareto criterion, in the inaugural period of implementation of the abatement policy, its cost is entirely financed with debt, so no tax needs to be levied on the initial generation and its consumption can remain unchanged. Because productivity improves in the future, thanks to the decreasing stock of pollutants, it is possible to produce more output in every future period and take a share of it through the labor tax so as to simultaneously give each future generation a bit more consumption *and* raise tax revenues to repay the debt. By insisting that such policies meet the generational Pareto criterion, we are in effect arguing that Pearson's other concern – lack of a political institution – is not that critical: after all, it seems natural to think that policies that satisfy the Pareto criterion are less likely to be blocked as they make their way through modern democratic processes.

Why might such a line of questioning be challenging? Standard welfare economics, recently emphasized by Foley (2007) and Heal (2009), suggests that in the presence of a huge uninternalized externality such as climate change, the BAU scenario cannot be Pareto efficient and hence action to correct the externality must, *in principle*, offer a Pareto improvement: "the gains must outweigh the costs so that the gainers could compensate the losers and still gain. We can all come out ahead — whether we actually do is a matter of institutional design." (Heal, 2009) This remark from Heal (2009) captures the essence of the standard approach to check whether a proposed policy intervention is potentially desirable: use the Kaldor-Hicks compensation criterion and verify if it generates an efficiency gain – a *hypothetical* potential for the winners to compensate the losers (and leave the latter as happy as before the policy introduction). If everyone post policy is at their pre-policy utility and there are still some new resources left standing, then the present value of these new, net resources could, in principle, be distributed in some fashion to future generations – this is how the *potential* for Pareto improvement is demonstrated. However, as nu-

[3]See Endress et al. (2014) for an analysis allowing for different intra- and intergenerational discounting. Sachs (2014) argues for "avoiding the overemphasis on a social discount factor to calibrate the interests of different generations".

[4]A similar approach is used in Bovenberg and Heijdra (2002), Hoel et al. (2015), and von Below et al. (2015).

[5]Green bonds are exhaustively discussed in Kaminker and Stewart (2012) and Orlov et al. (2018).

merous welfare economists have noted, and Nishiyama and Smetters (2007) observe, "...constructing a policy that is *actually* Pareto improving from a policy that improves Hicksian efficiency is a tougher task." [emphasis ours]

Why tougher? Because an actual policy would release its own dynamics, produce general-equilibrium gains and losses spread across generations, and those would all have to be properly accounted for if the policy upon implemenation is to be deemed genuinely Pareto improving, generation by generation. We take on this "tougher task" using analytical means. What makes the analysis especially challenging is that the very act of compensating current generations releases its own dynamics. For sure, investments in environment-friendly policies via debt financing allow future generations to reap gains, but they also have to participate, via tax payments and additional debt purchase, in the servicing of the outstanding debt. Debt will be growing at the gross rate of interest (assumed to exceed unity); hence, it is not trivial whether the downstream gains from a better environment can cover the aforediscussed compensation (including interest) *and* prevent the debt from *exploding*. There is an added realism/complication: since the tax instrument we study is distortionary, it affects incentives to produce with feedback effects on both the budget and the environment and other variables influencing welfare. In sum, *one* major contribution of this paper is demonstrating existence of a set of abatement policies which, once implemented, induce a path of Pareto-improvements over the BAU and keeps the associated path of debt well-behaved. It bears emphasis here, that unlike many papers in this line of work, we are not restricting the analysis to abatement policies that are "locally near zero" or arbitrarily small.

There is another important dimension in which we advance the literature. As Karp and Rezai (2014a) argue, a convergent conclusion from the literature emanating from the Stern Review (2007) and the IAMs – summarized in Heal (2009) – is that current generations must sacrifice consumption in order to combat climate change.[6] This conclusion is often blamed as the reason why climate negotiations have proven to be a non-starter.[7] We take on the challenge of studying policies that not only satisfy the generational Pareto criterion in utility terms, but also ensure that no generation has to *sacrifice consumption* along the way. Our results connect up with the broader literature on sustainability – Neumayer (2007), Heal (2009) – that recognizes substitutability between a loss to environmental capital (due to global warming) and gains to incomes/ consumption and argues for the need to maintain at least a minimum critical level of the former. In a way, requiring that consumption not decline ties our hands substantially; it precludes the possibility of exploiting the substitutability of the environment and consumption to leave generations as happy as in the BAU.

The rest of the paper is organized as follows. Section 2 reviews the related literature while Section 3 lays bare our contribution relative to the Kaldor-Hicks criterion. Section 4 describes the model economy

[6]Nordhaus (2007) argues that assumptions in the Stern Review (2007) concerning a "low discount factor" amplifies the harmful impacts of climate change in the distant future and "rationalizes deep cuts in emissions, and indeed in all consumption, today." Rezai (2010) argues this need to cut consumption is an artifact of the constraining assumption made in IAMs that, in spite of knowing about the dangers of climate change, agents in the BAU, invest nothing in mitigation, a constrained-optimal equilibrium. This automatically implies current generations would "attain lower consumption and utility levels if they started investing in mitigation".

[7]"The American way of life is not up for negotiation" – the classic U.S. position outlined by the senior President George Bush at 1992 Earth Summit in Rio de Janeiro.

with an exogenously-specified interest rate, endogenous labor-leisure choice and the BAU environment, and exposes the inefficiency arising from the environment externality. Section 5 studies a constant environmental policy under an intergenerational Pareto criterion and the associated implementation hurdles for the tractable case of quasi-linear preferences. It also analyzes debt dynamics under the Pareto criterion and studies several extensions. Section 6 considers an extension to a case with physical capital and endogenous factor prices, while Section 7 concludes. Proofs of results, some extra derivations, and some details on parameter choices are contained in the appendices.

2 Review of the literature

The literature on the economics of inter-generational equity and efficiency concerns in environmental models is vast.[8] Below, we summarize some of the papers that are closest in spirit to the current endeavor. An early contribution that led the way in terms of the search for Pareto-improving policies is Gerlagh and Keyzer (2001) who study a productive, non-renewable natural resource with amenity value and show that handing over property rights over that resource to an intergenerational trust fund that entitles every generation to the same income claim as in the zero extraction policy can yield a Pareto improvement.[9] Of course, for the initial generation to want to create the fund requires them to care about future generations. An important contribution involving intergenerational borrowing is Bovenberg and Heijdra (1998). In their setup, distant and near generations differ in their reliance on capital income (which translates to non-environmental welfare) versus environmental utility. Taxes on pollution are akin to a tax on capital and benefit distant generations – they enjoy a better environment – but hurt them because they inherit a smaller stock of physical capital. Debt, as in our paper, can be used to allow all generations to share in the efficiency gains of environmental policy, in some cases in a Pareto-improving way. The assumption that pollution hurts utility directly but does not affect production makes our results non-comparable; additionally, only marginal policy changes relative to the BAU are considered which means they can sidestep issues relating to long-run behavior of debt paths.

Karp and Rezai (2014a, b) focus on the "conflict between different types of agents alive when the [mitigation] policy is first implemented". They depart from the usual one-sector OG model and allow for two sectors with an endogenously-evolving relative price between the sectors. They rely on an idea, reminiscent of Poutvaara (2004), that if investments in pollution mitigation by the current young generate increases in future asset values, then the current old – the owners of said assets – can compensate the young from those capitalized benefits leaving everyone better off, just as Heal (2009) argued would be possible. Conceptually, the novelty of their paper rests on the fact that tax-induced increases in asset prices allow market-intermediated, Pareto-improving policies even when the government *cannot* use bonds to redistribute across generations.[10] Dao et al. (2015) study an intergenerational social compact between

[8]Allowing for country differences (North vs South) raises issue of intragenerational distribution which are not considered here, see e.g. Bretschger et al. (2014) and Kverndoll et al. (2014).

[9]Rasmussen (2003) is an early example of a paper using a calibrated OG model to study environmental taxation in a model where environmental quality is held fixed. Leach (2009) is similar in spirit but stays away studying tax or debt policies.

[10]A salient feature of our paper is that, unlike much of the literature, our analysis handles ambitious abatement policies not

generations in which the young invest some of their labor income in mitigation activities in return for a subsidy to their old-age capital income paid for by the next young generation. The compact terms are such that participation in it generates a Pareto improvement compared to non-participation. They also consider compact terms that are self-sustaining, meaning any incentive to default on the contractual terms are eliminated. In an important recent contribution, von Below et al. (2015) revisit the Poutvaara (2004) and Karp and Rezai (2014a) strategy of resolving the conflict between the young and the old at the point the mitigation policy is implemented. In their setup, the old and young suffer losses in rental and wage income when energy use (which is polluting) is curtailed but the old can be offered a compensatory pension by the young in lieu of the future benefit the latter get from a better environment. The added novelty is that the benefit from a cleaner environment accrues not just to the current young in the future, but to all future generations; if the future beneficiaries can be co-opted into the deal between the current young and old, then far more ambitious environmental policies can be attempted without hurting any generation. Their focus, however, is not on the dynamics of the path of pensions/debt, whether they explode, nor are they seeking policies that always improve upon consumption in the BAU.

Our work is part of an important literature that studies the consequences of environmental policies on environmental quality, growth and welfare (Howarth and Norgaard 1992; John and Pecchenino 1994; Jouvet et al. 2000; Gutierrez 2008; Goenka et al. 2012; Dao and Davila 2014; Wang et al. 2015). In many of these papers, environmental quality enters preferences directly. Their primary purpose is to study the role of government for eliminating the dynamic inefficiency in OG economies with environmental externalities. These papers focus on tax-financed mitigation policies and do not allow for debt financing. Intergenerational equity concerns or the search for Pareto-improving policies is not their focus. Fodha and Seegmuller (2014) do allow for debt financing but stay away from studying welfare issues along the transition.

Very recently, we have become aware of three papers that are indeed very close in spirit to our current endeavor. Like us, Sachs (2014) eschews the social discount factor and argues for "intergenerational fiscal transfers to allocate the burdens and benefits of climate change mitigation across generations without the need to trade off one generations's well-being for another's." Many of the ideas in our paper are also discussed in Sachs's which uses an even more barebones model than ours; for example, in his model, production and emissions are exogenous and each generation faces a separate welfare function. Most importantly though, Sachs is only able to write down a discounted PV condition on net taxes which, if satisfied, would mean Pareto improvements are *possible*, hypothetically speaking. Flaherty et al. (2016) extends the Sachs (2014) argument by considering a three-stage, continuous-time finite-horizon model where in the first stage is the BAU with production-induced emissions and in the second stage, mitigation policies are carried out by private agents who are "reimbursed for their effort by the issuance of green bonds". In the final stage, the future generation pays back the bonds through an income tax – they do not experience pollution which makes them happier. Eventually, taxes can be brought down and debt retired. While the paper does traverse some of the same landscape as ours, it is primarily numerical and takes

just those locally around zero. Several of the results in Karp and Rezai (2014a,b) hold locally near the zero tax rate while the general flavor of our results holds more widely.

many ad hoc modeling short-cuts toward that goal. Orlov et al. (2018) extend the Flaherty et al. (2016) line of thinking by examining the quantitative importance of green bonds in financing mitigation in a OG version of a DICE model and whether their use can help the economy reach the socially optimal steady state in a Pareto-improving way. In a sense, Sachs (2014) and the current paper provides the theoretical foundation for Orlov et al. (2018).

3 Kaldor-Hicks: hypothetical and actual

The benchmark of a Pareto-improving policy — one which leaves some generations better off, but hurts no one – is, in many cases too demanding since actual Pareto-improvements are hard to find in practice. Economists attempt, instead, to seek '*potential* Pareto-improvements' in the Kaldor–Hicks sense. Of course, potential improvements that rationalize the policy to begin with may not lead to *actual* improvements, once the policy is implemented and released distortions and general equilibrium effects. A clear discussion of this issue is in order.

The standard cost-benefit (CB) analysis due to Kaldor-Hicks checks whether the PDV of benefits from the policy being proposed exceed the PDV of costs. If it does, but there are winners and losers, then, in principle, compensations can be directed from the winners to the losers so that the losers are no worse off than before. In this sense, any policy that meets the CB analysis criterion with hypothetical compensations, would also generate *potential* Pareto-improvements' in the Kaldor–Hicks sense. The problem is, once the policy is implemented and actual compensations are made, endogenous decision variables may get distorted and factor prices may change, which the standard CB analysis does not keep track of. This means, it is not clear whether a policy that passed the CB-analysis/Pareto test with *hypothetical* compensations at some factor prices *would also pass the same test* under distorted decisions and *changed* factor prices in the *actual*-compensation world.

To see this clearly, consider an economy at the BAU steady state, wherein every generation receives lifetime indirect utility U^{BAU}. A policy, μ, is being considered – at this stage, we are agnostic about what μ is. Assume, the policy to be implemented will impose costs on some early generations but will generate benefits for future ones. Policy makers can, in principle, compute the compensating variations (across generations) to identify winners and losers relative to U^{BAU}. Let factor prices in this hypothetical (h) yet-to-be-implemented μ-policy world be given by $\left(\omega_{t+j}^{h}, R_{t+j}^{h}\right)$ and let taxes be denoted τ_{t+j}^{h}. Denote the compensating variation by $CV_{t+j}\left(\omega_{t+j}^{h}, R_{t+j}^{h}, \tau_{t+j}^{h}, U^{BAU}\right)$, either positive (if a loser) or negative (if a winner). In short, $CV_{t+j}\left(\omega_{t+j}^{h}, R_{t+j}^{h}, \tau_{t+j}^{h}, U^{BAU}\right)$ represent hypothetical compensations and the Kaldor-Hicks criterion checks if $\sum_{j=t}^{\infty}\left(R_{t+j}^{h}\right)^{j} CV_{t+j}\left(\omega_{t+j}^{h}, R_{t+j}^{h}, \tau_{t+j}^{h}, U^{BAU}\right) \gtrless 0$. In these calculations, it is implictly assumed that the capital market may be used to bring about these compensations from the winners to the losers at the return, R_{t+j}^{h}.

Now, suppose these Kaldor-Hicks-inspired compensations are *actually* implemented and the policy gets off the ground. This may require the use of intergenerational-transfer instruments, such as debt. Then, two things happen: a) these compensations introduce tax distortions, and or b) even in the absence of tax distortions, the use of intergenerational transfer instruments, such as, debt, affects the (endogenous) interest rates.[11] It is not important which of these occurs, but what is important is after the policy is actually implemented – denote with an a – the path for R, ω and τ will change to, say, $\left(\omega^a_{t+j}, R^a_{t+j}, \tau^a_{t+j}, U^{BAU}\right)$. The issue is,

$$\sum_{j=t}^{\infty} \left(R^h_{t+j}\right)^j CV_{t+j}\left(\omega^h_{t+j}, R^h_{t+j}, \tau^h_{t+j}, U^{BAU}\right) \neq \sum_{j=t}^{\infty} \left(R^a_{t+j}\right)^j CV_{t+j}\left(\omega^a_{t+j}, R^a_{t+j}, \tau^a_{t+j}, U^{BAU}\right).$$

Our goal below is not to simply demonstrate the above inequality but to compute an actual path of $\left(\omega^a_{t+j}, R^a_{t+j}, \tau^a_{t+j}\right)$ etc. that deliver actual Pareto improvements over U^{BAU}. Of course, it is daunting to compute $\sum_{j=t}^{\infty} \left(R^a_{t+j}\right)^j CV_{t+j}\left(\omega^a_{t+j}, R^a_{t+j}, \tau^a_{t+j}, U^{BAU}\right)$. We find a simple, intuitive way of checking whether the CB/Pareto condition is satisfied without having to compute $\sum_{j=t}^{\infty} \left(R^a_{t+j}\right)^j CV_{t+j}\left(\omega^a_{t+j}, R^a_{t+j}, \tau^a_{t+j}, U^{BAU}\right)$ It turns out the compensations needed under an actual policy, though complicated, map neatly onto the path of debt that operationalizes those compensations. In short, studying the debt dynamics and conditions under which debt is ultimately retired is crucial to establish that the Kaldor-Hicks criterion holds under *actual* compensations.

4 The model economy: BAU

We start by presenting a stylized, barebones model to explain the idea and scope for implementation of environmental policies under the Pareto criterion. Subsequently, this framework is discussed and generalized. The essential ingredients of the model are: environmental damage caused by economic activity, finitely-lived households who do not internalize the consequence of their activities on the environment, and a government which may initiate an abatement policy using taxes and green bonds as financing instruments. In this setting, the no policy scenario – BAU – produces a deterioration of the environment due to "overproduction"; in the long run, the environment is harmed and welfare reduced. A costly abatement policy can be implemented but it would only gradually improve the environment – it takes time to undo the consequences of accumulated environmental damages. Implementation of this policy faces an intergenerational conflict – the costs of the policy come up-front while the benefits come downstream to future generations. The essential question is, can the government implement its abatement policy in such a way as to avoid this fundamental intergenerational conflict?

To help fix ideas and generate analytical insights with manageable dynamics in both the stock of pollution and debt, we start off by studying a model in which labor is the *only* input and postpone introducing capital (and hence, endogenous interest rates along with capital dynamics) until Section 6. We consider a one-good, economy inhabited by two period-lived generations of agents and an infinitely-lived policymaker.

[11] In practice, these distortions or factor-price changes may be insignificant for small, localized policies; for a CB analysis under such settings, these may be safely ignored. Not so for major, enduring policies such as in the areas of health, infrastructure, education, environment, and so on.

4.1 Households

Consider a standard overlapping-generations economy where agents live for two periods – they work as young and consume as old, and there is no population growth.[12] The life-time utility for the generation being young $t+j$ is given by[13]

$$(1) \quad U_{t+j} \equiv u\left(c_{t+j+1}\right) - v\left(L_{t+j}\right)$$

where $u\left(c_{t+j+1}\right)$ is the utility from consuming c_{t+j+1} when old and $v\left(L_{t+j}\right)$ disutility from working L_{t+j} when young. (The time index $t = 0, 1, 2...$ indicates life in the BAU while the time index j indicates life under the μ-policy regime discussed in Section 5 below.) Some results are presented for the CES-utility function

$$(2) \quad u\left(c_{t+j+1}\right) = \frac{\left(c_{t+j+1}\right)^{1-\sigma}}{1-\sigma}; \quad \sigma \geq 0,$$

where the metric of relative risk aversion $\frac{-cU_{cc}(\cdot)}{U_c(\cdot)} = \sigma$ is constant. For future use, the special case of $\sigma = 0$ will be referred to as "quasi-linear utility".

The budget constraint of the household is

$$(3) \quad c_{t+j+1} = R\left[\left[w_{t+j} - \tau_{t+j}\right] L_{t+j} + \pi_{t+j}\right]$$

where $R \geq 1$ is the exogenous, gross return on savings, w the wage rate, π the profit from ownership of firms[14], and τ is a tax on labor supply.[15]

The household chooses labor supply to maximize (1) given (3), taking all factor prices, profit and taxes as given. The optimal labor supply decision satisfies[16]

$$(4) \quad u_c\left(\cdot\right) R\left[w_{t+j} - \tau_{t+j}\right] = v_L\left(L_{t+j}\right).$$

4.2 Production

The representative firm produces using labor (L) as the sole input, and output (y) in period $t+j$ is determined by the production function, $F(.)$:

$$(5) \quad y_{t+j} = H\left(S_{t+j}\right) F\left(L_{t+j}\right)$$

[12] Allowing for consumption in both periods of life would introduce a savings decision, but the present-value of life-time income would simply equal income when young. As such, with exogenous factor prices, removing the saving decision is harmless. Things change, of course, when the rate of return is endogenized (see Section 6 below.)

[13] The functions, $u(\cdot)$ and $v(\cdot)$, satisfy standard properties, including $u_c > 0$, $u_{cc} \leq 0$, $v_L > 0$ and $v_{LL} \geq 0$.

[14] Ownership is inherited from one generation to the next. Below, when factor prices are endogenous, ownership is determined by savings as young.

[15] We keep the analysis tractable by abstracting from the effects of enviromental taxes on the tax bases of other forms of taxation and the use of environmental taxes to fund non-environmental items on the government budget. For a more general treatment of this issue, see Barrage (2015).

[16] Assuming that there is an interior soluton. If we assume $lim_{c\to 0} u_c\left(c\right) = +\infty$, then we must have $L_{t+j} > 0$.

which has two components, and $F(.)$ satisfies standard properties of a production function.[17] Pollution reduces the production of output as captured by a "damage function", $H(S)$, where S is the stock of pollution (environmental quality), see eq. (8) below, and $H(0) = 1$, and $H_S(S) < 0$ for $S > 0$. A particular simple form of the damage function – used for some results below – is the linear approximation

$$(6) \quad H(S) \cong 1 - \rho S; \ \rho > 0 \text{ and } \rho \approx 0$$

which satisfies $H(0) = 1$ and $H_S(S) = -\rho < 0$. Hence, (net) output in period $t + j$ is given by $y(S_{t+j}, L_{t+j}) = H(S_{t+j})\ F(L_{t+j})$, where $y_S < 0$, $y_L > 0$ and $y_{SL} < 0$. Pollution reduces output that is available for consumption.[18] Also, while the marginal product of labor is positive, it declines with pollution: $H(S)$ acts as an adverse productivity "shock".

Profit to the firm is given by

$$\pi_{t+j} = y_{t+j} - w_{t+j} L_{t+j}.$$

The firm chooses labor input (level of production) to maximize profits taken factor prices and the environmental stock as given. Optimal labor demand satisfies

$$(7) \quad w_{t+j} = H(S_{t+j})\, F_L(L_{t+j}).$$

4.3 Pollution

Pollution is generated as a by-product of production. Let the stock of pollutants at the start of $t + j$ be denoted by S_{t+j}. Then, the stock of pollution evolves according to

$$(8) \quad S_{t+j+1} = (1 - \epsilon)\, S_{t+j} + G(L_{t+j}), \ S_0 > 0 \text{ given}$$

where $\epsilon \in (0, 1)$ is a constant that determines the speed with which pollution levels return to zero in the absence of any fresh emissions. Notice how eq. (8) captures the idea that changes to the environment can be very long-lived, spread across many cohorts. Since labor is the only input, we think of it as the polluting input as well: we posit that use of input L_{t+j} generates emissions of amount $G(L_{t+j})$ where $G(0) = 0$ and $G_L(\cdot) > 0$. Emissions can increase more (less) rapidly than input use if G is assumed to be convex (concave) – see Heutel (2012). Pollution is an unintended by-product of productive activity by firms and no firm-level disposal of this by-product is possible. (Murty et al., 2012).

There is a pollution-abatement technology which can improve environmental quality.[19] Specifically, a

[17] This includes $F_L(\cdot) > 0, F_{LL}(\cdot) < 0$ with the implication $\frac{F(\cdot)}{L} > F_L(\cdot)$.

[18] The Stern Review (2007) uses 5% of GDP as the lower bound for the cost of climate change under the BAU scenario. Burke et al. (2015) show that overall economic productivity is nonlinear in temperature for all countries, rich or poor, with productivity falling sharply at temperatures higher than 13C, and that the relationship is true for both agricultural and non-agricultural activity. Dell et al. (2012) finds strong growth effects but only for poor countries.

[19] Our goal is *not* to produce a reasonably accurate model of climate-change economics nor is it to study specific policies aimed at combating climate change.

fraction μ_{t+j} of the total emissions in period $t+j$ $(j \geq 0)$ may be abated at a cost

(9) $\mathcal{A}(\mu_{t+j}) = \Lambda\left(\mu_{t+j}\right) G\left(L_{t+j}\right).$

where $\Lambda(0) = \Lambda_{\mu}(0) = 0, \Lambda_{\mu} > 0, \Lambda_{\mu\mu} > 0$ for $\mu > 0$ – abatement costs increase at an increasing rate, the more ambitious the abatement policy.[20] These assumptions imply average costs ($\frac{\Lambda(\mu)}{\mu}$) are increasing in μ. Abatement costs depend on the level of emissions via $G(\cdot)$. Under such a policy, total emissions in period $t+j$ is given by $\left(1-\mu_{t+j}\right) G\left(L_{t+j}\right)$ and the transition equation for the stock of pollution becomes

(10) $S_{t+j+1} = (1-\epsilon) S_{t+j} + \left(1-\mu_{t+j}\right) G\left(L_{t+j}\right).$

At this stage, it is important to note that agents work when young, save everything in international capital markets that are unaffected by local pollution, and go on to consume only when old. This means, their own second-period consumption is entirely unaffected by the pollution they create. The implication is, private agents have no incentive to care about abatement of pollution in any way. This sets the stage for government action, the entire focus of our paper.

4.4 Government and abatement policy

The government may decide to pursue an abatement policy, characterized by the fraction μ_{t+j} abated (implying the stock of pollution follows (10)). If so, it carries abatement costs of amount, $\Lambda\left(\mu_{t+j}\right) G\left(L_{t+j}\right)$. The total tax revenue raised is $\tau_{t+j} L_{t+j}$. Let B_{t+j} denote the primary (i.e., non-interest) budget balance in $t+j$, the difference between tax revenue and primary expenditure (abatement cost).[21] We have

(11) $B_{t+j} = \tau_{t+j} L_{t+j} - \Lambda\left(\mu_{t+j}\right) G\left(L_{t+j}\right).$

Let D_{t+j} denote public debt at the end of period $t+j$. Then, debt evolves according to

(12) $D_{t+j+1} = R D_{t+j} - B_{t+j}.$

Equation (12) is crucial for the following analysis. If the government cannot fully finance abatement expenses by taxing the current young, there is a budget deficit ($B < 0$) and thus debt is created. Debt increases at the gross rate of return $R > 1$ if debt-servicing is financed by further borrowing. Pending

[20]One may be concerned that assuming the cost of the initial bit of abatement is zero plays a key role in the paper especially since environmental policies are known to involve a significant initial fixed cost. In a more general formulation, we can allow environmental policies to have an initial fixed cost, $\overline{\mathcal{A}} > 0$, so the total cost in period t (the first period when the policy was implemented) is: $\mathcal{A}(\mu_t) = \overline{\mathcal{A}} + \Lambda\left(\mu_t\right) G\left(L_t\right)$, and the total cost in subsequent periods $t+j$ $(j \geq 1)$ equals: $\mathcal{A}(\mu_{t+j}) = \Lambda\left(\mu_{t+j}\right) G\left(L_{t+j}\right)$. We would continue to assume $\Lambda(0) = \Lambda_{\mu}(0) = 0, \Lambda_{\mu} > 0, \Lambda_{\mu\mu} > 0$ for $\mu > 0$: in this case, the variable abatement cost is 0 and minimum in the BAU, and it is increasing and convex for positive levels of abatement. Our analysis below will *not* be restricted to $\mu \simeq 0$.

[21]The total budget balance (including interest rate payments) is given by

$$\tau_{t+j} L_{t+j} - \left[\Lambda\left(\mu_{t+j}\right) G\left(L_{t+j}\right) + (R-1) D_{t+j-1}\right] = B_{t+j} - (R-1) D_{t+j-1}.$$

further action, this way of financing causes debt levels to explode. The big point is, debt financing may allow the government to postpone the financing, but eventually budget surpluses would be needed to avoid an unsustainable debt trajectory.

Below, we study the *entire range* of abatement policies characterized by μ that can be implemented. This is of paramount significance given that most of the literature studies the effect of abatement intensities locally near zero. In that case, if the marginal cost of the abatement policy is zero when its intensity is zero, it is easy to see that a marginal increase in intensity causes a second order increase in debt but a first order benefit in terms of future productivity gains so that, overall, future taxes can gradually repay the debt. We not only study the "locally zero" case, but also show that a wide range of μ is implementable.

4.5 Equilibrium

Combining labor supply (4) and labor demand (7), equilibrium employment in period $t+j$ comes to depend on the environmental quality (S_{t+j}) and taxes (τ_{t+j}), i.e.

$$L_{t+j} = L\left(S_{t+j}, \tau_{t+j}\right)$$

Given equilibrium employment, output can be determined from (5), and the stock of pollution evolves by (22), and the public budget by (12).

For future use, it is instructive to note how equilibrium employment depends on pollution and taxes. In general, the signs of the comparative static labor supply responses, $\frac{\partial L_{t+j}}{\partial S_{t+j}}$ and $\frac{\partial L_{t+j}}{\partial \tau_{t+j}}$, are ambiguous due to oppositely signed income and substitution effects. For (2), we have

$$(13)\quad \frac{\partial L_{t+j}}{\partial S_{t+j}} < 0 \text{ and } \frac{\partial L_{t+j}}{\partial \tau_{t+j}} < 0 \text{ for } \sigma \leq 1$$

giving standard signs, i.e., more pollution and hence a lower return to labor (lower marginal product) and higher taxes reduces labor supply, when $\sigma \leq 1$. (For proof, see Appendix A.)

Since consumption can be written as $c_{t+j+1} = R\left[\left[w_{t+j} - \tau_{t+j}\right] L_{t+j} + \pi_{t+j}\right] = R\left[H\left(S_{t+j}\right) F\left(L_{t+j}\left(S_{t+j}, \tau_{t+j}\right)\right) - \tau\right.$ $c(S_{t+j}, \tau_{t+j})$, and it can be shown that

$$(14)\quad \frac{\partial c_{t+j+1}}{\partial S_{t+j}} < 0 \text{ and } \frac{\partial c_{t+j+1}}{\partial \tau_{t+j}} < 0$$

i.e., consumption is unambiguously decreasing both in the pollution level and in taxes. (For proof, see Appendix B.)

Finally, life-time utility for cohort $t+j$ can be written as

$$(15)\quad U_{t+j}\left(S_{t+j}, \tau_{t+j}\right) \equiv u\left(c(S_{t+j}, \tau_{t+j})\right) - v\left(L_{t+j}\left(S_{t+j}, \tau_{t+j}\right)\right),$$

where it can be shown

$$(16) \quad \frac{\partial U_{t+j}\left(S_{t+j}, \tau_{t+j}\right)}{\partial S_{t+j}} < 0 \text{ and } \frac{\partial U_{t+j}\left(S_{t+j}, \tau_{t+j}\right)}{\partial \tau_{t+j}} < 0$$

i.e., life-time utility is decreasing in both the level of pollution and in taxes. (For proof, see Appendix C.)

Eq. (16) highlights the intergenerational dilemma. Financing abatement by imposing taxes on the current cohort would harm them – they would pay for the policy, but not benefit from less pollution. The gains will come downstream to future generations as the environment improves and those welfare gains can be taxed without hurting them. But, in the meantime, debt financing would be required. Will the downstream tax revenue be sufficient to service the debt and prevent the debt level from exploding? Does the answer to this question constrain how ambitious the abatement policy can be? An additional challenge lies ahead. Can welfare improvements be delivered by the policy without necessitating *cuts* in consumption along the way? These issues are taken up in Section 5 below.

4.6 Discussion of modeling assumptions

The model setup is necessarily barebones, designed to generate clean qualitative insights taking advantage of a lot of analytical tractability. Environmental quality does not have a direct utility effect but it reduces net output and thus the level of consumption. In John and Pecennino (1994) pollution is a by-product of consumption, and pollution hurts utility directly; therefore, welfare can be improved via major cuts in consumption – we have set our goals to deliver welfare improvements without requiring consumption sacrifices. Not including environmental quality in the utility function also has another advantage. It allows us to sidestep issues relating to the substitutability between environmental and consumption goods, for as Neumeyer (2007) and others have argued, if the substitutability is low, then it may be that "no consumption growth, however high, can compensate" for the damage to the environment – after all, as Heal (2009) points out, "certain ecosystem services or products, such as water and food, are essential to survival and cannot be replaced by produced goods". Similarly, we disallow altruism on the part of agents not because we think people don't care about the welfare of their progeny but because we wish to make a case for environmental action even if they didn't – it stacks the deck against the ability to find a successful policy thereby making the paper's results starker. Allowing altruism would also introduce private transfers from parents to children some of which may be crowded out by the government's debt policy.

The assumption of a single input, labor, is obviously limiting but also keeps the analysis manageable. Studying a neoclassical economy with capital as an additional input, possibly the dirty input, would add another state variable, bring in interest rate effects, and clutter the dynamics considerably. (Such complications are studied using numerical methods in Section 6 below.) As will become clear, our focus is largely on the issue of implementation of environmental policies under an intergenerational Pareto criterion. Adding capital does not fundamentally alter our understanding of *that* issue. But first, we set the stage by laying out the BAU economy with no policy action.

4.7 Business-as-usual (BAU) – no environmental policy

Consider first the business-as-usual situation with no government policy ($\mu = 0, \tau = B = D = 0$). Each cohort decides on the level of economic activity, which in turn, has a negative effect on the environment, affecting future production possibilities and thus welfare of future generations.

It is apparent a social planner, by choosing L to internalize the effects of pollution damage downstream, can improve social welfare. Consider the problem of a social planner who, in steady state, solves

$$\begin{array}{c} \max_L \; U^{SP} \equiv u(c) - v(L) \\ \text{s.t. } c = RH(S)F(L), \; \epsilon S = G(L) \end{array} \qquad \text{(SP)}$$

incorporating the effect of labor supply on the environment. Denote the planner's solution by L^{SP}.

Proposition 1 *In the BAU, the dynamics of pollution is given by*

$$\text{(17)} \quad S^{BAU}_{t+j+1} = (1-\epsilon) S^{BAU}_{t+j} + G\left(L\left(S^{BAU}_{t+j}\right)\right).$$

There exists an unique steady state, S^*_{BAU} where

$$\text{(18)} \quad S^{*BAU} = \frac{G\left(L^{*BAU}\right)}{\epsilon} \textit{ where } L^{*BAU} \equiv L\left(S^{*BAU}\right).$$

Let the steady-state levels of employment and pollution in the planning economy be denoted by L^{SP} and S^{SP}. Then,

$$L^{*BAU} > L^{SP} \textit{ and } S^{*BAU} > S^{SP}.$$

The proof of Proposition 1 is in Appendix D. If, in addition, we assume the history of the economy is such $S_0 < S^*_{BAU}$ and "near" S^*_{BAU}, then it is apparent that the stock of pollution in the BAU is increasing and converging to S^*_{BAU} in the long run. Without intervention, the economy has a higher level of employment and a worse environmental situation than what is socially optimal. Intuitively, agents do not take into account how their own production decisions impact pollution which, in turn, affects the production decisions of their offspring. As a consequence utility is lower, $U^{*BAU}(\cdot) < U^{SP}(\cdot)$.

Starting from a low initial level of pollution, S_0, under the conditions spelt out in (29) – see Appendix D – the stock of pollution continues to rise and approaches a higher level, S^*_{BAU}, in the long run. This implies a steady, unrelenting decline in the quality of the environment. Along such a path of environmental degradation, consumption and thus welfare declines as well. We illustrated this in Figure 1 for the case where labor supply rises along this transition ($\sigma > 1$).

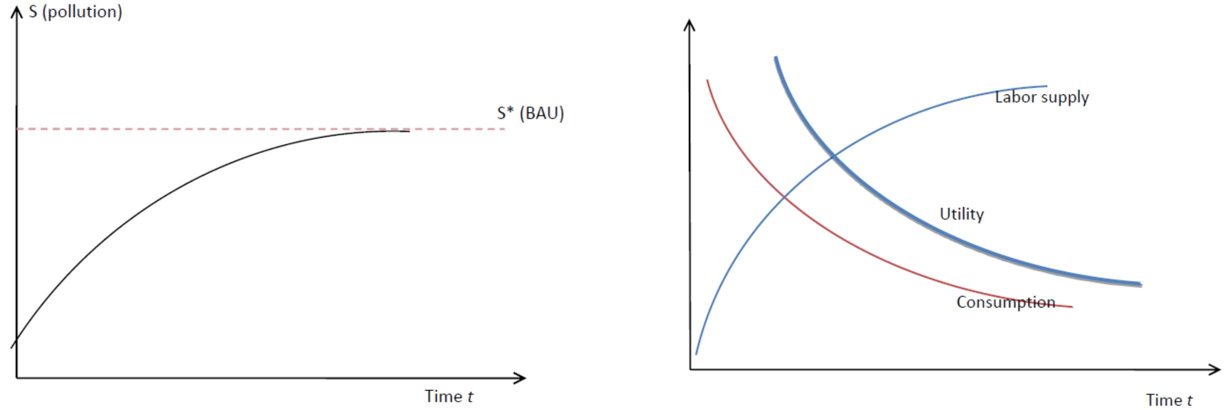


Figure 1: A portrait of the BAU, $\sigma > 1$

Every generation pollutes and leaves a worse environment for its progeny than what it received from its parents. The worsening environmental quality hurts the children but not the parents – the people whose actions generated the pollution – do not care (since they are not altruistic). The big question for us is, can the government initiate a pollution-reduction policy that leaves every generation, post policy, no worse off, possibly better off, compared to their life in the BAU?

5 Pareto-improving policies

In our setup, unregulated market leads to "overproduction" and progressive deterioration of the environment. Assessed in terms of steady-state welfare, there is a case for the government to introduce an abatement policy (μ), but at what level? [22] Even if that is decided upon, there is the thorny issue of implementation: the costs of such a policy come up-front, but the benefits, in terms of less pollution, arrive more gradually. As hinted earlier, bond financing may relieve this problem but it is not, in general, clear whether future generations, are, in net terms, better off: they benefit from less pollution but have to service the public debt.

Below, we outline *a* constant-μ policy that leaves every generation, post policy, no worse off, possibly better off, compared to their life in the BAU; additionally, an unsustainable path of debt path can be avoided. It is, by no means obvious, that such a policy even exists. That is, even though a policy is understood to increase long-run welfare, it may not be implementable under a Pareto condition. By the same token, there may indeed exist many such policies. Our aim is show the existence of *one* such

[22] Steady state welfare under a generic abatement policy μ is

$$U = u\left(H\left(S\right)F\left(L\right) - \Lambda\left(\mu\right)G\left(L\right)\right) - v\left(L\right)$$

where $S = \frac{(1-\mu)G(L)}{\epsilon}$. Using $u_c\left(y(L) - \Lambda\left(\mu\right)G\left(L\right)\right)R\left[H\left(S\right)F_L\left(L\right) - \Lambda\left(\mu\right)G\left(L\right)\right] = v_L\left(L\right)$ it follows

$$\left.\frac{\partial U}{\partial \mu}\right|_{\mu=0} = u_c\left(\cdot\right)H_S\left(S\right)F\left(\cdot\right)\frac{\partial S}{\partial \mu} > 0,$$

and hence, steady state welfare can be improved by introducing an abatement policy.

policy but not to characterize the full set of such policies. By the same token, computing socially optimal μ-policies, ones that maximize some measure of weighted social welfare, is not our focus.

5.1 Abatement and associated debt

Assume the government announces at t and *commits* to a permanent abatement policy $\mu_{t+j} = \mu$ for all $j \geq 0$, given the initial state of pollution $S_t = S_t^{BAU}$ inherited from the BAU-world. Henceforth, use the superscript μ to denote variables under the μ-policy regime.[23] Under this policy, the law of motion for pollution is given by

$$S_{t+j+1}^{\mu} = (1-\epsilon) S_{t+j}^{\mu} + (1-\mu) G\left(L\left(S_{t+j}^{\mu}, \tau_{t+j}\right)\right) \quad , \quad j \geq 0$$

At any date, there are two modes of financing associated abatement costs, either by taxation (τ_{t+j}) or by running budget deficits ($B_{t+j} < 0$) – issuing debt (green bonds) on the international capital market at the gross rate $R > 1$ with associated debt dynamics, cf. (12). Specifically, the primary budget balance B, cf. (11) under the abatement policy is given as

$$B_{t+j} = \begin{cases} -\Lambda(\mu) G\left(L_t^{BAU}\right) < 0 \text{ for } j = 0 \\ \tau_{t+j} L_{t+j}^{\mu} - \Lambda(\mu) G\left(L_{t+j}^{\mu}\right) \lesseqgtr 0 \text{ for } j > 0 \end{cases}$$

The crucial question is, can the μ-policy be implemented under the Pareto condition

(19) $U_{t+j}\left(S_{t+j}^{\mu}, \tau_{t+j}^{\mu}\right) \geq U_{t+j}^{BAU}\left(S_{t+j}^{BAU}, 0\right),$

where the l.h.s of (19) is the life-time utility to cohort $t+j$ under the government policy, and the r.h.s. is the same in the absence of any intervention (continuing BAU).

The Pareto condition (19) effectively determines the largest tax payment τ_{t+j}^{PC} which can be levied on any cohort $t+j$ and still leave them with the same utility as in the status quo by the condition

$$U_{t+j}\left(S_{t+j}, \tau_{t+j}^{PC}\right) = U_{t+j}^{BAU}\left(S_{t+j}^{BAU}, 0\right)$$

i.e., any tax $\tau_{t+j} > \tau_{t+j}^{PC}$ would leave cohorts worse off, and $\tau_{t+j} < \tau_{t+j}^{PC}$ would make them better off compared to the BAU. Two observations: First, at the inaugural date of the policy, the tax capacity is zero $(\tau_t^{PC} = 0)$ since cohort t inherits a stock of pollution, S_t^{BAU}, and the first cohort to experience improvement in the environment due to the policy is cohort $t+1$.[24] It follows that, under the Pareto condition, cohort t cannot be taxed, and so some debt, $D_t = \Lambda(\mu) G\left(L_t^{BAU}\right) > 0$ must be incurred. Second, under the Pareto condition, the μ-policy gradually reduces pollution raising the tax capacity over

[23] As a referee pointed out, commitment to the μ-policy is important. Otherwise, the government could run the μ-policy for a few periods, take it offline thereafter, presumably leaving those future generations no worse off than had the BAU continued all along.

[24] The government's policy affects the start-of-period stock of pollution starting at $t+1$. This implies the inaugural generation t is unaffected by the policy – their production, and hence, emissions, are predetermined from the BAU.

time ($\tau^{PC}_{t+j+1} > \tau^{PC}_{t+j}$) (since $\frac{\partial S}{\partial \mu} < 0$, $\frac{\partial U^{\mu}_{t+j}}{\partial S_{t+j}} < 0$, and $\frac{\partial U^{\mu}_{t+j}}{\partial \tau_{t+j}} < 0$). The abatement policy, thus, releases two dynamic forces. First, debt financing triggers the dynamics implied by (12): $D_{t+j} = RD_{t+j-1} - B_{t+j}$. Since $R > 1$, the debt level will be ever increasing unless sufficiently large budget surpluses can be generated. Secondly, the stock of pollution gradually decreases, benefitting later cohorts, allowing higher taxes which partially defray both debt-service costs and possible reduction in debt. But which dynamic force dominates? Do improvements in the environment come sufficiently quickly and strongly to dominate the debt dynamics? If not, the policy is not implementable under the Pareto criterion. If yes, the intergenerational distribution conflict has been resolved.

Before embarking on characterizing a μ-policy, a few key points on the public finance dynamics are worth noting. First, for debt to decline, a sufficiently large primary budget balance is required, i.e., at some date $j = k$,

$$B_{t+k} > (R-1)\, D_{t+k-1} \Longrightarrow D_{t+k} - D_{t+k-1} < 0$$

must obtain. This means, for debt levels to fall, the primary budget surplus has to be large enough to cover, at least, the interest expense on the outstanding debt. It is clear that such a "**debt turning point – k**" emerges only when taxes are sufficient to outpace the underlying growth in debt and associated interest expenses. Second, if follows from (12) that

(20) $$D_{t+k+1} - D_{t+k} = R\,(D_{t+k} - D_{t+k-1}) - (B_{t+k+1} - B_{t+k})\,.$$

Hence, if a debt turning point has been reached ($D_{t+k} - D_{t+j-k} < 0$), the level of debt would keep decreasing ($D_{t+k+1} - D_{t+k}$) thereafter since the primary budget is non-deteriorating ($B_{t+k+1} - B_{t+k} \geq 0$). If the debt level is declining, it may eventually reach zero or approach a steady state.[25]

5.2 Dynamics of pollution, taxes and debt: quasi-linear utility

It is apparent that the existence of a debt turning point is essential. If such a point can be identified, implementation under the Pareto condition is achieved. No cohorts are worse-off, some may be better-off, and while some debt is necessary to usher in the policy, it can eventually be eliminated. For later reference, denote by k^B the smallest[26] k for which $t + k^B$ is the first time (if it happens) the primary budget displays a surplus ($B_{t+k^B} > 0$), by k^T the smallest k for which there at $t + k^T$ is a turning point in the debt level (if it happens), i.e. $D_{t+k^B} - D_{t+k^B-1} < 0$, and by k^{DF} the smallest k for which debt at $t + k^D$ has been eliminated (if it happens), i.e. $D_{t+k^D} \leq 0$. The next step is to show that for some μ-policy, there is a path of taxes τ_{t+j} satisfying (19) with associated non-exploding debt.

In practice, it is impossible to simply solve the non-linear inequality (19) for τ^{PC}_{t+j}, the path of Pareto-

[25] If it does reach zero, the country is debt free; if it so wishes, it can continue to raise taxes to pay for pollution abatement and any primary surplus may be lent to world markets or it can reduce taxes and allow for welfare improvements to future cohorts. It is not important for our analysis that the debt vanishes. We stay away from taking a stand on any of these issues and do not study the dynamics in the economy once debt has been paid off.

[26] Strictly, at $t + \lceil k^B \rceil$ where $\lceil k^B \rceil$ is the smallest integer not less than k^B, and similarly for the other critical dates. To avoid unnecessary technical complexity, this is implicitly understood in the text.

improving taxes. It is instructive to work out a special case – quasi-linear utility cf. (2) with $\sigma = 0$ – so as to clarify the deeper mechanisms. We also assume the economy is at the BAU steady state when the policy is first implemented, i.e., $S_t^{BAU} = S_{BAU}^*$.[27]

Even though it is impossible to use (19) to compute a path of Pareto-improving taxes, one can create one possible but feasible tax path, call it $\widehat{\tau}_{t+j}$, where, *by construction*, labor supply is the same under the μ-policy as in the BAU. Specifically, the tax $\widehat{\tau}_{t+j}$ is determined by (see Appendix E)

$$(21)\quad \widehat{\tau}_{t+j} = w_{t+j}^{\mu} - w_{t+j}^{BAU} = \left[H\left(S_{t+j}^{\mu}\right) - H\left(S_{BAU}^*\right)\right] F_L\left(L_{BAU}^*\right).$$

The tax takes away the increases in the marginal product of labor caused by environmental improvements under the μ-policy. By construction, then, labor supply under the μ-policy remains unchanged, but it can be shown that consumption (see Appendix E) increases compared to the BAU, $c_{t+j+1}^{\mu} > c_{t+j+1}^{BAU}$. This ensures that, under $\widehat{\tau}_{t+j}$, all future cohorts (from $t+1$ on) are strictly better off than BAU. The question is, does this μ-policy generate sufficient tax revenue ($\widehat{\tau}_{t+j}$) to avoid an unsustainable debt level, and make it possible to eliminate the initially incurred debt at some point in the future.

Since, by construction, the labor supplies are the same pre and post policy, $L_{t+j} = L_{BAU}^*$, the evolution equation for pollution can be written as

$$(22)\quad S_{t+j+1}^{\mu} = (1-\epsilon)\, S_{t+j}^{\mu} + (1-\mu)\, G\left(L_{BAU}^*\right).$$

The dynamics of pollution (and everything else) becomes a lot more tractable since emissions in the μ-policy economy are given by $(1-\mu)\, G\left(L_{BAU}^*\right)$, predetermined. After a bit of routine manipulation, it can be shown

$$S_{t+j+1} = (1-\epsilon)^{j+1} S_t + \frac{1-(1-\epsilon)^{j+1}}{\epsilon}(1-\mu)\, G\left(L_{BAU}^*\right) = \left[(1-\epsilon)^{j+1}\mu + (1-\mu)\right] S_{BAU}^* \quad ; \quad j \geq 0.$$

Clearly, S_{t+j} is declining relative to S_{BAU}^* over time. Therefore $\widehat{\tau}_{t+j} = \left[H\left(S_{t+j}^{\mu}\right) - H\left(S_{BAU}^*\right)\right] F_L\left(L_{BAU}^*\right)$ is rising (H increases for S decreasing) over time.

Next, does satisfying the Pareto criterion render the path explosive? Using $L_{t+j}^{\mu} = L_{BAU}^*$, it follows that[28] the primary budget evolves as

$$(23)\quad B_{t+j+1} - B_{t+j} = \left[H\left(S_{t+j+1}^{\mu}\right) - H\left(S_{t+j}^{\mu}\right)\right] F_L\left(L_{BAU}^*\right) L_{BAU}^* > 0$$

since S^{μ} has been established to be declining over time. Knowing $B_{t+j+1} - B_{t+j} > 0$ for all j, we know from above that if a turning point for debt can be identified, the debt level will keep decreasing and reach 0 in finite time. As discussed above, there are three critical dates of interest.

Proposition 2 *Under quasi-linear utility and an affine damage function, the μ-policy and associated tax*

[27] Later we show our results generalize beyond this simple illustrative case. We show in Appendix E that our analytical results holds for $\sigma < 1$, and numerically we show in Section 5.3 that implementation may also be possible for $\sigma > 1$. The possibility of implementing the policy outside of steady state is explored in Section 5.3 below.

[28] Using that $B_{t+j} = \widehat{\tau}_{t+j}\, L_{t+j}^{\mu} - \Lambda(\mu)\, G\left(L_{t+j}^{\mu}\right) = \left[H\left(S_{t+j}^{\mu}\right) - H\left(S_{BAU}^*\right)\right] F_L\left(L_{BAU}^*\right) L_{BAU}^* - \Lambda(\mu)\, G\left(L_{BAU}^*\right)$

policy $\widehat{\tau}_{t+j}$, the primary balance becomes positive at $t + k^B$ where k^B is given as

$$k^B = \frac{\ln\left(1 - \frac{\epsilon}{\rho F_L(L^{*BAU})L^{*BAU}} \frac{\Lambda(\mu)}{\mu}\right)}{\ln(1-\epsilon)} \text{ for } \mu \in \left(0, \mu^B\right)$$

where μ^B is determined by $1 - \frac{\epsilon}{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}} \frac{\Lambda(\mu^B)}{\mu^B} \equiv 0$. The debt has a turning point at $t + k^T$ where k^T is determined by

$$k^T = \frac{\ln\left(1 - \frac{R-1+\epsilon}{\rho F_L(L^{*BAU})L^{*BAU}} \frac{\Lambda(\mu)}{\mu}\right)}{\ln\left(\frac{1-\epsilon}{R}\right)} \text{for } \mu \in \left(0, \mu^T < \mu^B\right)$$

where μ^T is determined by $1 - \frac{R-1+\epsilon}{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}} \frac{\Lambda(\mu^T)}{\mu^T} \equiv 0$. Finally, the debt is reduced to zero at $t+k^D$ where k^D is determined as the solution to

$$\frac{\rho F_L\left(L^{*BAU}\right)L^{*BAU}}{\epsilon}\left(1 - \frac{R-1}{R-1+\epsilon}\frac{1-\left(\frac{1-\epsilon}{R}\right)^{k^D+1}}{1-\left(\frac{1}{R}\right)^{k^D+1}}\right) = \frac{\Lambda(\mu)}{\mu} \text{ for } \mu \in \left(0, \mu^T\right).$$

The proof of this special case is in Appendix F.

The three dates of interest, k^B, k^T and k^{DF} and the associated μ-ranges, $\mu \in \left(0, \mu^B\right)$, $\mu \in \left(\mu^T, \mu^B\right)$ and $\mu \in \left(0, \mu^T\right)$ are collected in Figure 2.

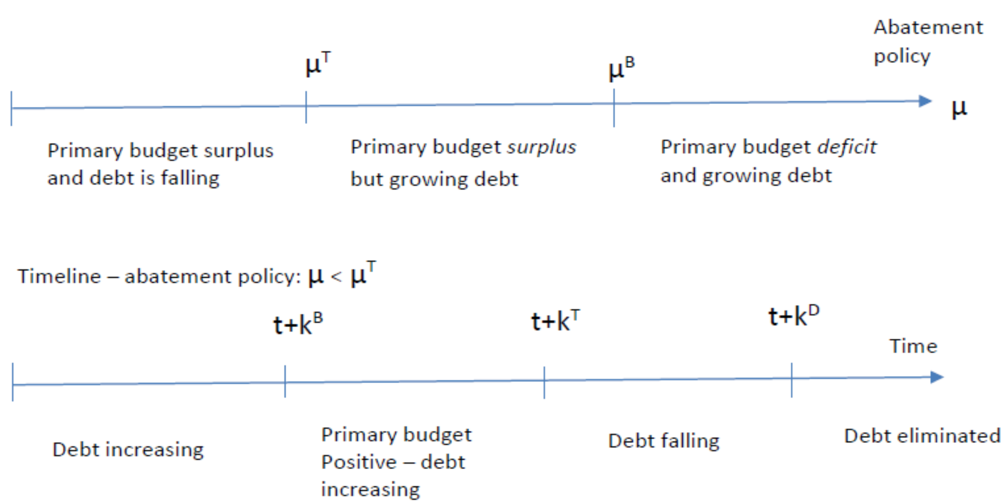


Figure 2: Dates and abatement policies (μ) of interest

The important finding is that there is a set of policies $0 < \mu \leq \mu^T$ consistent with a turning point in debt and, thus, its eventual elimination. Moreover, the associated tax policy satisfies the Pareto condition and private consumption and utility are also higher under this policy than under the BAU.

Note, the policy cannot be too aggressive, if $\mu^T < \mu \leq \mu^B$ it would eventually generate surpluses on the primary balance, but never sufficiently large to induce a turning point in the debt level. The debt level would be on an unsustainable path, and the policy would not be feasible. If $\mu > \mu^B$ the primary balance is never in surplus, and clearly this policy is not sustainable. To further interpret the condition delimiting the implementable policies (μ^T) it is useful to write it as[29]

$$AC(\mu^T) \equiv \frac{\Lambda\left(\mu^T\right)}{\mu^T} = \varphi_2 \equiv \frac{1}{\frac{R-1+\epsilon}{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}}$$

where the l.h.s. is the average abatement cost of abating at the rate μ and the r.h.s. is a constant (depending on parameters and properties of the BAU-equilibrium). The average abatement cost is increasing in μ (recall, $\frac{\Lambda(\mu)}{\mu}$ increasing in μ). Hence, intuitively, the condition is saying that policies which do not have too-high average costs can be implemented. Put differently, the average abatement cost limits how ambitious the abatement policy can be: the burden to be carried depends on the average abatement cost, which in turn, depends on how ambitious the policy is.[30]

5.3 An illustration of debt dynamics and some robustness checks

In this section, we study the robustness of some of our aforediscussed results to some alternative formulations. First and foremost, we wish to demonstrate that the general tenor of our results go through when $\sigma > 1$. Second, we check if the policy can be inaugurated at *any* point in the BAU transition, not necessarily at the steady state. Third, we had, for tractability's sake assumed an affine damage function in the computation of the debt dynamics; here, we relax that restriction as well. The bigger robustness question, the one about extending the analysis to an economy with endogenous factor returns, is postponed to Section 6 below.

The goal here is not a full-blown calibration exercise but rather to paint a picture of the Pareto-improving transition with broad brushstrokes to see if environmental policy can improve matters and the make sure the associated debt paths behave. The following functional forms are used:[31]

[29]Similarily, we have that $\overline{\mu}_1$ is determined by the condition

$$AC(\overline{\mu}_1) = \frac{\Lambda\left(\overline{\mu}_1\right)}{\overline{\mu}_1} = \varphi_1 \equiv \frac{1}{\frac{\epsilon}{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}} = \frac{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}{\epsilon}$$

and since $\varphi_1 > \varphi_2$ it follows that $\overline{\mu}_1 > \overline{\mu}_2$.

[30]The above begs a question on the scope for implementation if there are both fixed and variable costs (e.g. $\mathcal{A}(\mu_t) = \overline{\mathcal{A}} + \Lambda\left(\mu_t\right)G\left(L^\mu_t\right)$, $\overline{\mathcal{A}} > 0$) – hence, some fixed costs have to be incurred before any abatement is possible. In the Appendix G, we show that implementation is still possible under certain conditions for policies satisfying $\overline{\mu}_{21} \leq \mu \leq \overline{\mu}_{22}$ (where $\overline{\mu}_{22} > \overline{\mu}_{21} > 0$). That is, the abatement has to be suffcently large to cover the fixed costs ($\mu \geq \overline{\mu}_{21} > 0$) but not so large that the average costs becomes too high ($\mu \leq \overline{\mu}_{22}$).

[31]The parameters are chosen as follows: $\sigma = 2$, $\beta = 0.4$, $\gamma = -2$, $A = 280$, $\alpha = 0.64$, $\rho = 4 \times 10^{-7}$, $\lambda = 0.039$, $\phi = 1.5$, $\delta = 1000$, $\theta = 0.9$, $R = 2$ and $\epsilon = 0.126$.

utility: $U_{t+j}\left(c_{t+j+1}, L_{t+j}\right) = \frac{c_{t+j+1}^{1-\sigma}}{1-\sigma} - \beta \frac{L_{t+j}^{1-\gamma}}{1-\gamma};\ \sigma \geq 0, \beta > 0, \gamma < 0.$

production: $F\left(L_{t+j}\right) = AL_{t+j}^{\alpha};\ A > 0, 0 < \alpha < 1.$

damage: $H\left(S_{t+j}\right) = \frac{1}{1+\rho S_{t+j}^{2}};\ \rho > 0.$

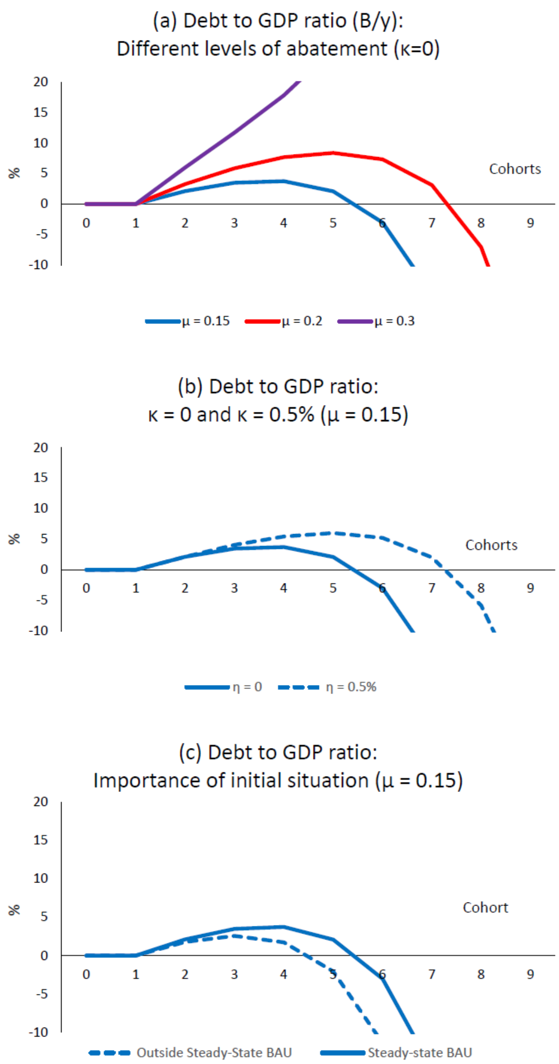
abatement cost: $\Lambda\left(\mu\right) = \lambda \mu^{\phi};\ \lambda > 0, \phi > 1.$

emission : $G\left(L_{t+j}\right) = \delta L_{t+j}^{\theta};\ \delta > 0, \theta > 0.$

Implementation under the Pareto criterion depends critically on debt dynamics, as explained above. To illustrate some key insights, Figure 3 reports the debt paths for various scenarios. First, implementation depends on the level of abatement (μ). As illustrated in Figure 3a, low levels of abatement does not require much debt and the turning point is reached quickly, whereas more ambitious policies cause more debt and it takes longer to reach the turning point. There is a critical level of abatement above which implementation is not possible. When implementation is possible, there is scope to provide gains up-front[32] as illustrated in Figure 3b, where $\eta = 0$ corresponds to unchanged utility relative to the BAU steady-state (the Pareto neutral criterion) and $\eta = 0.5\%$ allow all cohorts to have a utility gain of 0.5% relative to the BAU steady state. As expected, front-loading of the gains imply higher debt and a later turning point, i.e., implementation becomes more difficult. Finally, if the policy is implemented before the BAU has reached steady-state, implementation becomes easier (Figure 3c) showing that delay of abatement is costly.

[32]As explained earlier, the way we've set things up, the generation born at the inaugural date cannot benefit from this environmental policy, unless the government borrows to makes transfers to them (which we disallow). The inaugural generations' utility is held at the BAU level; all others are made strictly better off.

Figure 3: Debt dynamics: Abatement policies, utility criterion and initial situation



Note: In all simulations, the initial situation is the BAU steady state. In (c), the initial situation is away from the BAU steady state: we set $S_{j=0} = 0.85 \times S^*_{BAU}$.

6 An extension: endogenous factor prices

How does our previous discussion change if the assumption of an exogenously-specified interest rate is abandoned in favour of an economy with neoclassical production? With neoclassical production, factor prices, especially interest rates, are endogenous and depend on environmental variables, and the effects of policy choices at the initial date will, via its general equilibrium effects on endogenous variables such as saving (and hence the capital stock, and factor returns), will linger. It is apparent that debt will, initially, crowd out private saving thereby reducing the capital stock and help bring down production-related emissions. This would cause the wage rate to decrease and the interest rate to increase. The dynamics of debt becomes immensely complicated since it gets coupled with the dynamics of both the endogenously-evolving capital stock and the pollution stock. Whether these general equilibrium effects ease the implementation hurdles at future dates is not at all clear. Below, we briefly sketch this more general model and study it numerically to check if the general tenor of our results derived earlier is preserved.

6.1 The model economy

We consider an otherwise standard, Diamond (1965) model and augment it to include pollution.[33] Agents supply their entire unit of time to competitive labor markets when young and are retired when old. There is no population growth, and the population size is normalized to 1. Firms use capital and labor to produce the single, consumption good using a standard CRS technology $F\left(K_{t+j}, L_{t+j}\right)$ where K is capital. Emissions, in this case, are generated from the use of capital. At time t $(j=0)$, the government starts to implement an environmental policy so as to abate a fraction $\mu > 0$ of the fresh emissions generated in each period. To finance the abatement-related expenses, the government may impose a tax τ^K on firms for the use of the dirty input, *now capital*. If needed, the government can also issue green bonds. All trade is conducted in perfectly competitive markets and there are no market imperfections.

For firms, the net output in period $t+j$ equals

$$y_{t+j} = H\left(S_{t+j}\right) F\left(K_{t+j}, L_{t+j}\right) = H\left(S_{t+j}\right) A K_{t+j}^{\alpha} L_{t+j}^{1-\alpha},$$

where $A > 0$ and $0 < \alpha < 1$. At $t+j$, firms take pollution stock S_{t+j}, interest rate R_{t+j}, wage rate w_{t+j} and tax rate τ_{t+j}^{K} as given and solve

$$\max_{K_{t+j}^{d}, L_{t+j}^{d}} H\left(S_{t+j}\right) F\left(K_{t+j}^{d}, L_{t+j}^{d}\right) - \left(R_{t+j} + \tau_{t+j}^{K}\right) K_{t+j}^{d} - w_{t+j} L_{t+j}^{d}.$$

[33] The basic structure is the same as that discussed above. The Diamond model with a pollution externality is a workhorse environment in the macro-environement literature – see Gutierrez (2008), Goenka et al. (2012) and Farmer and Bednar-Friedl (2010).

The first order conditions read

(24) $R_{t+j}+\tau^K_{t+j}=\alpha AH\left(S_{t+j}\right)\left(K^d_{t+j}\right)^{\alpha-1}\left(L^d_{t+j}\right)^{1-\alpha},$

(25) $w_{t+j}=\left(1-\alpha\right)AH\left(S_{t+j}\right)\left(K^d_{t+j}\right)^{\alpha}\left(L^d_{t+j}\right)^{-\alpha}.$

Agents born in period $t+j$ care about young-age consumption (c^y_{t+j}) and old-age consumption (c^o_{t+j+1}).[34] Since the return on public debt is the same as return on capital, agents are indifferent between the two forms of savings. Let s_{t+j} be the total saving by generation $t+j$. Agents in generation $t+j$ take w_{t+j}, R_{t+j+1} as given and solve

$$\max_{c^y_{t+j},c^o_{t+j+1}} U_{t+j}\left(c^y_{t+j},c^o_{t+j+1}\right)=\frac{\left(c^y_{t+j}\right)^{1-\sigma}}{1-\sigma}+\beta\frac{\left(c^o_{t+j+1}\right)^{1-\sigma}}{1-\sigma},\sigma>0$$
$$\text{s.t. } c^y_{t+j}=w_{t+j}-s_{t+j},$$
$$c^o_{t+j+1}=R_{t+j+1}s_{t+j}.$$

from which we derive the optimal saving function, $s_{t+j}=\frac{w_{t+j}}{1+\beta^{-\frac{1}{\sigma}}\left(R_{t+j+1}\right)^{1-\frac{1}{\sigma}}}$ and the indirect utility $\widetilde{U}_{t+j}$.

In any period $t+j$, polluting emissions are generated as a by-product of capital use, $G\left(K_{t+j}\right)$. The pollution stock evolves according to

(26) $S_{t+j+1}=\left(1-\epsilon\right)S_{t+j}+\left(1-\mu\right)G\left(K_{t+j}\right).$

Government incurs abatement costs of amount $\mathcal{A}_{t+j}=\Lambda\left(\mu\right)G\left(K_{t+j}\right)$ in period $t+j$ where, as before, μ is constant. The stock of debt at the end of $t+j$ equals

$$D_{t+j}=R_{t+j}D_{t+j-1}+\mathcal{A}_{t+j}-\tau^K_{t+j}K_{t+j}.$$

Also, the next period's capital stock is determined by $K^s_{t+j+1}=s_{t+j}-D_{t+j}$.[35]

In equilibrium, $L^d_{t+j}=L^s_{t+j}=1$, and $K^d_{t+j}=K^s_{t+j}=K_{t+j}$, so factor prices are $R_{t+j}=\alpha AH\left(S_{t+j}\right)K^{\alpha-1}_{t+j}-\tau^K_{t+j}$,and $w_{t+j}=\left(1-\alpha\right)AH\left(S_{t+j}\right)K^{\alpha}_{t+j}$ using which we derive optimal saving to be

$$s_{t+j}=\frac{\left(1-\alpha\right)AH\left(S_{t+j}\right)K^{\alpha}_{t+j}}{1+\beta^{-\frac{1}{\sigma}}\left(\alpha AH\left(S_{t+j+1}\right)K^{\alpha-1}_{t+j+1}-\tau^K_{t+j+1}\right)^{1-\frac{1}{\sigma}}}.$$

Combining the above equilibrium relationships, we derive the endogenously-evolving laws of motion for the economy. At the start of period $t+j$, the pollution stock S_{t+j}, the capital stock K_{t+j}, the public debt level D_{t+j-1} and the sequence of tax rates $\left\{\tau^K_{t+k}\right\}_{k\geq j}$ are known – the government announces and commits to this path of taxes, which are in turn computed under the Pareto criterion $\left(\widetilde{U}^{\mu}_{t+j}\geq\widetilde{U}^{\mu=0}_{t+j}\right)$.

[34] We stay away from allowing an endogenous labor-leisure choice since that adds *another* dimension to the ensuing 3-D dynamics without the potential for great, additional insight.

[35] Notice, allowing debt to go negative would have the direct effect of raising the capital stock thereby hurting the environment.

Then $\{K_{t+j+1}, S_{t+j+1}, D_{t+j}\}$ are determined by

$$S_{t+j+1} = (1-\epsilon) S_{t+j} + (1-\mu) G(K_{t+j}),$$
$$D_{t+j} = \left(\alpha AH(S_{t+j}) K_{t+j}^{\alpha-1} - \tau_{t+j}^K\right) D_{t+j-1} + \Lambda(\mu) G(K_{t+j}) - \tau_{t+j}^K K_{t+j},$$
$$K_{t+j+1} = \frac{(1-\alpha) AH(S_{t+j}) K_{t+j}^{\alpha}}{1+\beta^{-\frac{1}{\sigma}}\left(\alpha AH(S_{t+j+1}) K_{t+j+1}^{\alpha-1} - \tau_{t+j+1}^K\right)^{1-\frac{1}{\sigma}}} - D_{t+j}.$$

What is important for our purposes is that the path of debt constrained by the Pareto criterion eventually turns around and is headed downward thereafter. For concreteness sake, we discontinue the government policy once the debt has been paid off; it is not important whether the economy has reached the new steady state by then. For completeness, note that in the BAU of this economy, the dynamics are given by

$$S_{t+j+1} = (1-\epsilon) S_{t+j} + G(K_{t+j}),$$
$$K_{t+j+1} = \frac{(1-\alpha) AH(S_{t+j}) K_{t+j}^{\alpha}}{1+\beta^{-\frac{1}{\sigma}}\left(\alpha AH(S_{t+j+1}) K_{t+j+1}^{\alpha-1}\right)^{1-\frac{1}{\sigma}}}.$$

The unique BAU steady state is characterized by

(27) $S = \dfrac{G(K)}{\epsilon},$

(28) $K = \dfrac{(1-\alpha) AH(S) K^{\alpha}}{1+\beta^{-\frac{1}{\sigma}}\left(\alpha AH(S) K^{\alpha-1}\right)^{1-\frac{1}{\sigma}}}.$

6.2 Numerics

Resorting to numerical analysis allows us to check the robustness of our theory to alternative assumptions, many of which had been made, to begin with, for analytical tractability. Our analysis below is designed to offer qualitative insight into the Pareto-improving transition and see if environmental policy can improve matters and associated debt paths do not explode. We start by assigning functional forms and parameter values that are in line with established practice in the literature. The following functional forms are used:

utility: $U_{t+j}\left(c_{t+j}^y, c_{t+j+1}^o\right) = \dfrac{\left(c_{t+j}^y\right)^{1-\sigma}}{1-\sigma} + \beta\dfrac{\left(c_{t+j+1}^o\right)^{1-\sigma}}{1-\sigma};\ \sigma > 0, 0 < \beta < 1.$

production: $F(L_{t+j}) = AK_{t+j}^{\alpha} L_{t+j}^{1-\alpha};\ A > 0,\ 0 < \alpha < 1.$

damage: $H(S_{t+j}) = \dfrac{1}{1+\rho S_{t+j}^2};\ \rho > 0.$

abatement cost: $\Lambda(\mu) = \lambda\mu^{\phi};\ \lambda > 0,\ \phi > 1.$

emissions : $G(K_{t+j}) = \delta K_{t+j}^{\theta};\ \delta > 0, \theta > 0.$

The parameters of the model are chosen as follows:

UTILITY	PRODUCTION	ABATEMENT COST
$\sigma = 2$	$A = 327$	$\lambda = 0.866$
$\beta = 0.7$	$\alpha = 0.36$	$\phi = 2.5$
EMISSION	DECAY RATE	DAMAGE
$\delta = 1.165$	$\epsilon = 0.126$	$\rho = 4 \times 10^{-7}$
$\theta = 0.9$		

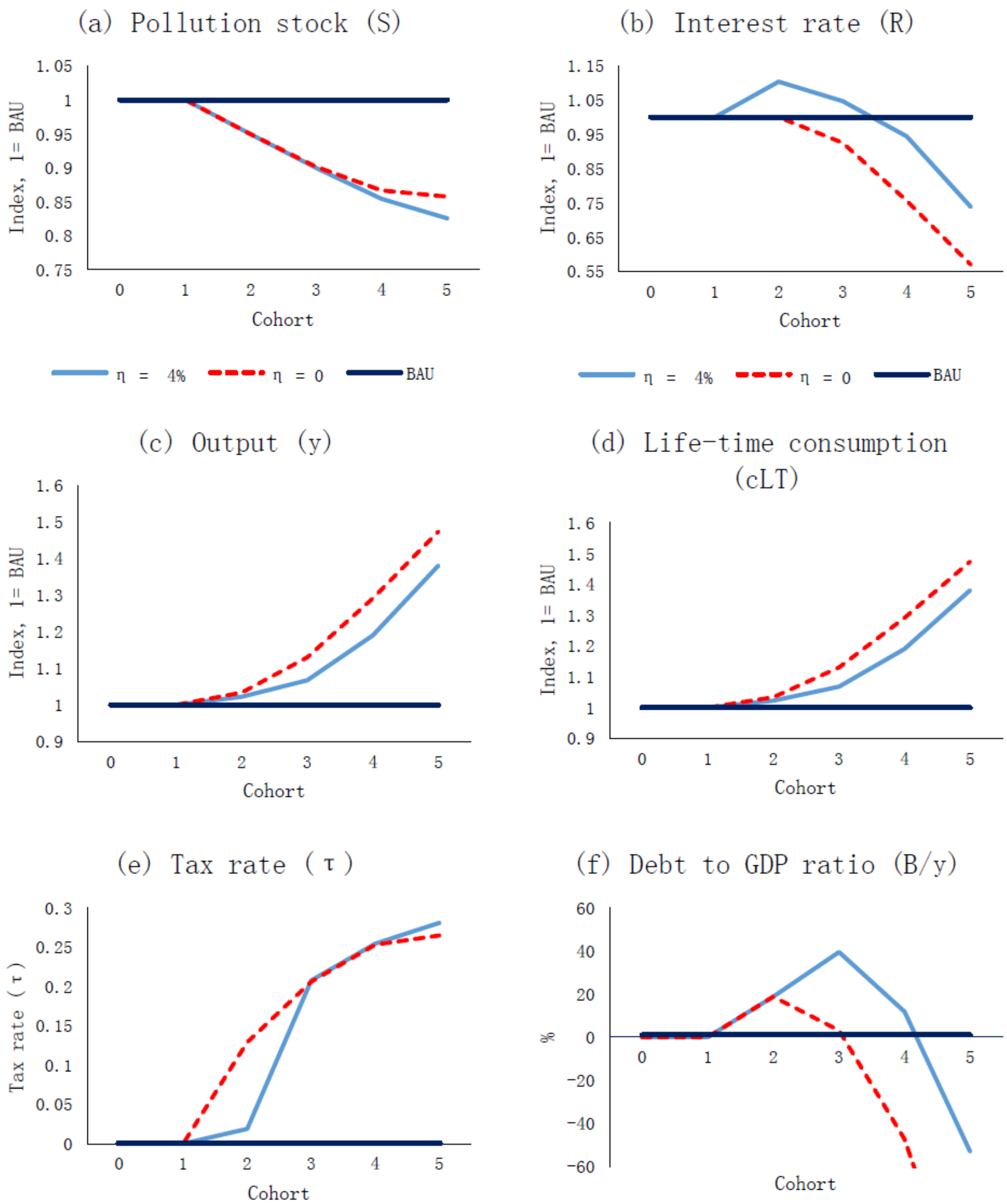
Table 1: Parameter Values

Appendix H contains a detailed discussion explaining the rationale for these parameter choices, many of which are taken from Karp and Rezai (2014b). We consider the following environmental policy: the government starts to abate 40% of the emissions generated in each period (hence, $\mu = 0.4$) in a situation where the economy is in the BAU steady state. Figure 4 displays the adjustment paths for some key variables for two scenarios; one under the Pareto neutral criterion (labelled $\eta = 0$) and one where gains are released up-front by allowing all cohorts a utility improvement of 4% relative to the BAU steady state ($\eta = 4\%$). The specifics of the algorithm used to compute the paths is discussed in Appendix I. We show a case where the policy can be implemented, see Figure 4f, and in accordance with above, the up-front utility gains imply higher debt and a later turning point. Along the adjustment path, taxes can be increased (Figure 4e) since agents are better off, and the tax revenue is sufficient to ensure the implementation of the policy, even if some up-front utility gains are allowed. The abatement policy ensures a lower pollution stock (Figure 4a), and importantly higher output (Figure 4c) and life-time consumption (Figure 4d).[36] The path for the interest rate (Figure 4b) reflects that the issuance of public debt crowds out private capital resulting in a higher interest rate. Along the adjustment path this effect is turned around alongside reductions in public debt.

The upshot our numerical exercise is that all the major takeaways of the paper derived for the tractable case of exogenous interest rates continue to hold in the endogenous interest rate setting.

[36]Defined as the present value of life-time consumption, i.e., $c^{LT}_{t+j} \equiv c^{y}_{t+j} + \frac{c^{o}_{t+j+1}}{R_{t+j+1}}$.

Figure 4: Adjustment to abatement policies with endogenous factor prices



7 Concluding remarks

This paper studies a tractable economy populated by overlapping generations of agents facing a standard stock externality from pollution caused by productive activities. In the laissez faire equilibrium, environmental quality gets worse over time, and consumption and utility falls. The business-as-usual situation is a grim one and presents an opportunity for government intervention in the form of pollution abatement. The catch is that such policies are costly and it takes a while for the benefits to start appearing in a substantial way. The government can borrow to start the abatement and can tax some of the downstream welfare gains to help pay down the debt. The big question is, can the government usher in such an environmental policy that makes sure that no generation is hurt (indeed "all" are better off) and the debt is paid off in finite time? We show, the answer is in the affirmative. The new equilibrium has lower pollution levels than in the business as usual world. Along the transition, every generation is better off (at least no worse off) in utility terms and consumption is also rising.

Our analysis has also stayed away from studying alternative policies that put a direct cap on labor supply (through mandatory length of work week laws) or imposes capital controls, see e.g. Knight et al. (2013). Also, instead of using debt, the generations could work out a corresponding path of intergenerational transfers as in von Below et al. (2015). It is our conjecture that any attempt to introduce such policies under the Pareto criterion would presumably face similar implementation hurdles as raised here.

In future work, we are exploring the quantitative margins of the paper by incorporating it into a North-South, two-country calibrated integrated assessment model of climate change. Interactions and coordination on abatement policies between the North and the South as well as a study of carbon-tax policies are interesting issues we take up in that setting.

Appendix

A Proof of Eq. (13)-(14)

From the first order condition (4), we can calculate

$$U_{LL} = u_{cc}(\cdot) R^2 \left[H(S_{t+j}) F_L(L_{t+j}) - \tau_{t+j}\right]^2 + u_c(\cdot) RH(S_{t+j}) F_{LL}(L_{t+j}) - v_{LL}(L_{t+j}) < 0,$$

$$\begin{aligned} U_{LS} &= u_{cc}(\cdot) R\left[H(S_{t+j}) F_L(L_{t+j}) - \tau_{t+j}\right] RH_S(S_{t+j}) F(L_{t+j}) + u_c(\cdot) RH_S(S_{t+j}) F_L(L_{t+j}) \\ &= u_c(\cdot) RH_S(S_{t+j}) F_L(L_{t+j}) \left[1 - \underbrace{\frac{-c_{t+j+1}u_{cc}(\cdot)}{u_c(\cdot)}}_{=\sigma \text{ for } (2)} \underbrace{\frac{H(S_{t+j}) F(L_{t+j}) - \tau_{t+j}\frac{F(L_{t+j})}{F_L(L_{t+j})}}{H(S_{t+j}) F(L_{t+j}) - \tau_{t+j} L_{t+j}}}_{<1 \text{ since } \frac{F(\cdot)}{F_L(\cdot)}>L}\right], \end{aligned}$$

and

$$\begin{aligned} U_{L\tau} &= -u_{cc}(\cdot) RL_{t+j} R\left[H(S_{t+j}) F_L(L_{t+j}) - \tau_{t+j}\right] - u_c(\cdot) R \\ &= -u_c(\cdot) R\left[1 - \underbrace{\frac{-c_{t+j+1}u_{cc}(\cdot)}{u_c(\cdot)}}_{=\sigma \text{ for } (2)} \underbrace{\frac{H(S_{t+j}) F_L(L_{t+j}) L_{t+j} - \tau_{t+j} L_{t+j}}{H(S_{t+j}) F(L_{t+j}) - \tau_{t+j} L_{t+j}}}_{<1 \text{ since } F_L(\cdot)L<F(\cdot)}\right]. \end{aligned}$$

It follows that $\frac{\partial L_{t+j}}{\partial S_{t+j}} = -\frac{U_{LS}}{U_{LL}}$ and $\frac{\partial L_{t+j}}{\partial \tau_{t+j}} = -\frac{U_{L\tau}}{U_{LL}}$. In general, we cannot sign $\frac{\partial L_{t+j}}{\partial S_{t+j}}$ or $\frac{\partial L_{t+j}}{\partial \tau_{t+j}}$. For (2), if $\sigma \le 1$, we have $\frac{\partial L_{t+j}}{\partial S_{t+j}} < 0$ and $\frac{\partial L_{t+j}}{\partial \tau_{t+j}} < 0$.

In the BAU, $\tau = 0$, and then $\frac{dL_{t+j}}{dS_{t+j}} \left(\frac{\partial L_{t+j}}{\partial S_{t+j}}\right)$ can be simplified to

$$\left.\frac{dL_{t+j}}{dS_{t+j}}\right|_{BAU} = -\frac{u_c(\cdot) RH_S(S_{t+j}) F_L(L_{t+j}) \left[1 - \frac{-c_{t+j+1}u_{cc}(\cdot)}{u_c(\cdot)}\right]}{U_{LL}}.$$

The rest follows.

B Proof of Eq (14)

Using $c_{t+j+1} = R\left[H(S_{t+j}) F(L_{t+j}(S_{t+j}, \tau_{t+j})) - \tau_{t+j} L_{t+j}(S_{t+j}, \tau_{t+j})\right]$, we have

$$\begin{aligned} \frac{\partial c_{t+j+1}}{\partial S_{t+j}} &= R\left[H_S(S_{t+j}) F(L_{t+j}) + \left(H(S_{t+j}) F_L(L_{t+j}) - \tau_{t+j}\right) \frac{\partial L_{t+j}}{\partial S_{t+j}}\right] \\ &= R\left[H_S(S_{t+j}) F(L_{t+j}) - \left(H(S_{t+j}) F_L(L_{t+j}) - \tau_{t+j}\right) \frac{U_{LS}}{U_{LL}}\right] \\ &= R\frac{H_S(S_{t+j}) F(L_{t+j}) U_{LL} - \left(H(S_{t+j}) F_L(L_{t+j}) - \tau_{t+j}\right) U_{LS}}{U_{LL}}. \end{aligned}$$

Plug in the expressions for U_{LL} and U_{LS} (see Appendix A) and we get

$$\begin{aligned}\frac{\partial c_{t+j+1}}{\partial S_{t+j}} &= RH_S(\cdot)F(\cdot)\frac{u_c(\cdot)RH(S_{t+j})F_{LL}(L_{t+j}) - u_c(\cdot)R\frac{F_L(L_{t+j})}{F(L_{t+j})}(H(S_{t+j})F_L(L_{t+j}) - \tau_{t+j}) - v_{LL}(L_{t+j})}{U_{LL}} \\ &< 0.\end{aligned}$$

Similarly,

$$\begin{aligned}&\frac{\partial c_{t+j+1}}{\partial \tau_{t+j}} = R\left[(H(S_{t+j})F_L(L_{t+j}) - \tau_{t+j})\frac{\partial L_{t+j}}{\partial \tau_{t+j}} - L_{t+j}\right] \\ &= R\left[-(H(S_{t+j})F_L(L_{t+j}) - \tau_{t+j})\frac{U_{L\tau}}{U_{LL}} - L_{t+j}\right] \\ &= -R\frac{(H(S_{t+j})F_L(L_{t+j}) - \tau_{t+j})U_{L\tau} + L_{t+j}U_{LL}}{U_{LL}} \\ &= -R\frac{-u_c(\cdot)R(H(S_{t+j})F_L(L_{t+j}) - \tau_{t+j}) + u_c(\cdot)RH(S_{t+j})F_{LL}(L_{t+j})L_{t+j} - v_{LL}(L_{t+j})L_{t+j}}{U_{LL}} \\ &< 0.\end{aligned}$$

C Proof of Eq. (16)

By the envelope theorem, we have

$$\frac{\partial \tilde{U}_{t+j}}{\partial S_{t+j}} = RU_c(\cdot)H_s(\cdot)F(\cdot) < 0,$$

and

$$\frac{\partial \tilde{U}_{t+j}}{\partial \tau_{t+j}} = -RU_c(\cdot)L_{t+j} < 0.$$

D Proof of Proposition 1

Existence and uniqueness issues surrounding S^*_{BAU} are dealt with using standard techniques.[37] Similarly, imposing parametric restrictions such that

$$(29)\quad 0 < \left.\frac{dS^{BAU}_{t+j+1}}{dS^{BAU}_{t+j}}\right|_{S^*_{BAU}} < 1 \iff 0 < 1 - \epsilon + G_L\left(L\left(S^{*BAU}\right)\right)\left.\frac{dL\left(S^{BAU}_{t+j}\right)}{dS^{BAU}_{t+j}}\right|_{S^*_{BAU}} < 1$$

holds ensures local stability of S^{*BAU}. (Recall, sign $\left.\frac{dL_{t+j}}{dS_{t+j}}\right|_{BAU} = \text{sign}(\sigma - 1)$.

In the BAU equilibrium, labor supply and hence equilibrium employment is determined by the first

[37] Define $J(S) \equiv G(L(S)) - \epsilon S$. Then, it follows $J(0) = G(L(0)) > 0$ and $\lim_{S\to+\infty} J(S) = -\infty$ if G has a finite upper bound. If $\sigma \leq 1$, then $L_S \leq 0$, so $J_S(S) = G_L(\cdot)L_S(S) - \epsilon < 0$ in which case S^*_{BAU} is unique.

order condition of agents (which take S as given):

$$u_c\left(RH(S^{*BAU})F(L^{*BAU})\right)RH\left(S^{*BAU}\right)F_L\left(L^{*BAU}\right)-v_L\left(L^{*BAU}\right)=0$$

which, combined with $\epsilon S^{*BAU}=G\left(L^{*BAU}\right)$ – see (eq. (18)) yields

$$u_c\left(RH\left(\frac{G\left(L^{*BAU}\right)}{\epsilon}\right)F\left(L^{*BAU}\right)\right)RH\left(\frac{G\left(L^{*BAU}\right)}{\epsilon}\right)F_L\left(L^{*BAU}\right)-v_L\left(L^{*BAU}\right)\equiv 0$$

where L^{*BAU} is the steady state equilibrium.[38] Now consider the problem of a social planner who, in steady state, solves

$$\begin{array}{c}\max_L\ U^{SP}\equiv u\left(c\right)-v\left(L\right)\\ \text{s.t. } c=RH\left(S\right)F\left(L\right),\ \epsilon S=G\left(L\right)\end{array}\qquad\text{(SP)}$$

incorporating the effect of labor supply on the environment. Denote the planner's solution by L^{SP}. Then,

$$\frac{dU^{SP}}{dL}=u_c\left(RH\left(\cdot\right)F\left(L^{SP}\right)\right)R\left[\underbrace{H_S\left(.\right)\frac{G_L\left(L^{SP}\right)}{\epsilon}F\left(L^{SP}\right)}_{<0}+H\left(\frac{G\left(L^{SP}\right)}{\epsilon}\right)F_L\left(L^{SP}\right)\right]-v_L\left(L^{SP}\right)\equiv 0.$$

For L^{SP} to be well-defined, assume $\frac{d^2U^{SP}}{dL^2}<0$ holds. (Even though $\frac{d^2U^{P}}{dL^2}<0$ obtains, it does not follow that $\frac{d^2U^{SP}}{dL^2}<0$ holds, precisely because of the externality.) Notice, the underscored term arises because the planner takes into account the effect of his choice of L on S which, in turn, affects $H\left(S\right)$, i.e. more employment and thus economic activity harms the environment. It follows that

$$\begin{aligned}L^{SP}&<L^{*BAU}\\ S^{SP}&<S^{*BAU}\end{aligned}$$

E Path of taxes under quasi-linear utility and policy-invariant labor supply

As discussed above, the inaugural generation t ($j=0$) is unaffected by the policy. Because of the government's abatement activity during period t, the start-of-period stock of pollution next period ($j=1$) satisfies $S^{\mu}_{t+1}<S^{BAU}_{t+1}$. If the government imposes no taxes in $t+1$, then it follows from Lemma **??** that generation $t+1$ will be strictly better off, i.e., $\widetilde{U}_{t+1}\left(S^{\mu}_{t+1},\tau_{t+1}=0\right)>\widetilde{U}^{BAU}_{t+1}\left(S^{BAU}_{t+1}\right)$. Some or all of this welfare gain may be taxed away by the government to help defray (part of) the abatement and debt service costs in that period. Once the tax is imposed, welfare will fall (relative to $\widetilde{U}_{t+1}\left(S^{\mu}_{t+1},0\right)$). Hence there exists a range for the tax rate, say $\tau_{t+1}\in[0,\overline{\tau}_{t+1}]$ such that $\widetilde{U}_{t+1}\left(S^{\mu}_{t+1},\tau_{t+1}\right)\geq\widetilde{U}^{BAU}_{t+1}\left(S^{BAU}_{t+1}\right)$. When $\tau_{t+1}=\overline{\tau}_{t+1}$, $\widetilde{U}_{t+1}=\widetilde{U}^{BAU}_{t+1}$ (a Pareto-neutral choice of tax) and if $\tau_{t+1}=0$, the government leaves all the welfare gains to generation $t+1$.

For now, we set aside Pareto criterion (19) and our search for Pareto-improving taxes and instead

[38]As discussed earlier, $\frac{d^2U^P}{dL^2}<0$ holds and so L^{*BAU} is well defined.

focus on a subset of taxes, $\{\widehat{\tau}_{t+j}\}_{j=0}^{\infty}$ such that $L_{t+j}^{\mu} = L\left(S_{t+j}^{\mu}, \widehat{\tau}_{t+j}\right) = L\left(S_{t+j}^{BAU}\right) = L_{BAU}^{*}\ \forall j > 0$. In other words, $\widehat{\tau}_{t+j}$ is chosen to keep labor supply under the government's policy the same as its level in the BAU. This helps fix the second argument of $\widetilde{U}(.,.)$ in (19). We wish to investigate what implication this may have for the first argument, consumption, and via this channel, dig deeper into (19). The associated debt dynamics are a separate matter which we will turn to further below.

Start with the optimality conditions for labor supply, pre and post policy, and use them to back out the necessary path of taxes using the fact $L_{t+j}^{\mu} = L\left(S_{t+j}^{BAU}\right)$. This means

$$\text{BAU: } u_c\left(\underbrace{RH\left(S_{t+j}^{BAU}\right)F\left(L_{t+j}^{BAU}\right)}_{c_{t+j+1}^{BAU}}\right)RH\left(S_{t+j}^{BAU}\right)F_L\left(L_{t+j}^{BAU}\right) - v_L\left(L_{t+j}^{BAU}\right) = 0$$

$$\text{policy: } u_c\left(\underbrace{R\left[H\left(S_{t+j}^{\mu}\right)F\left(L_{t+j}^{\mu}\right) - \widehat{\tau}_{t+j}L_{t+j}^{\mu}\right]}_{c_{t+j+1}^{\mu}}\right)R\left[H\left(S_{t+j}^{\mu}\right)F_L\left(L_{t+j}^{\mu}\right) - \widehat{\tau}_{t+j}\right] - v_L\left(L_{t+j}^{\mu}\right) = 0$$

$$\text{with } L_{t+j}^{\mu} = L_{t+j}^{BAU}$$

For (2), these equations reduce to

$$(30)\quad \left(\frac{H\left(S_{t+j}^{\mu}\right)}{H\left(S_{t+j}^{BAU}\right)} - \frac{\widehat{\tau}_{t+j}}{H\left(S_{t+j}^{BAU}\right)\frac{F\left(L_{t+j}^{BAU}\right)}{L_{t+j}^{BAU}}}\right)^{\sigma} = \frac{H\left(S_{t+j}^{\mu}\right)}{H\left(S_{t+j}^{BAU}\right)} - \frac{\widehat{\tau}_{t+j}}{H\left(S_{t+j}^{BAU}\right)F_L\left(L_{t+j}^{BAU}\right)}.$$

For a moment, *consider an example* where $\frac{F(L_t)}{L_t} = F_L(L_t)$, i.e., $F(L_t)$ is a linear function of L_t. Then, for all $\sigma \neq 1$, (30) becomes

$$\left(\frac{H\left(S_{t+j}^{\mu}\right)}{H\left(S_{t+j}^{BAU}\right)} - \frac{\widehat{\tau}_{t+j}}{H\left(S_{t+j}^{BAU}\right)F_L\left(L_{t+j}^{BAU}\right)}\right)^{\sigma-1} = 1 \Leftrightarrow \widehat{\tau}_{t+j} = \left[H\left(S_{t+j}^{\mu}\right) - H\left(S_{t+j}^{BAU}\right)\right]F_L\left(L_{t+j}^{BAU}\right)$$

and

$$c_{t+j+1}^{\mu} = R\left[H\left(S_{t+j}^{\mu}\right)F\left(L_{t+j}^{BAU}\right) - \widehat{\tau}_{t+j}L_{t+j}^{BAU}\right] = RH\left(S_{t+j}^{BAU}\right)F\left(L_{t+j}^{BAU}\right) = c_{t+j+1}^{BAU}.$$

In this special case, under the tax policy, $\widehat{\tau}_{t+j}$, labor supply (by construction) and consumption are exactly the same as in the BAU. In that case, the equal utility condition (19) is satisfied and then $\widehat{\tau}_{t+j}$, *for this special case*, is also the path of Pareto-neutral taxes. Note, however, this does not mean pollution levels are the same pre and post policy.

More generally, $\frac{F(L_t)}{L_t} > F_L(L_t)$ holds. In this more general setting, first note, when $\sigma = 1$, there does not exist $\widehat{\tau}_{t+j} > 0$ satisfying (30) and therefore, labor supply pre and post policy cannot be the same. This is because with a logarithmic utility, a better environment has no direct effect on the labor supply. If the government nevertheless collects taxes anyway, labor supply would change.

If $\sigma \neq 1$, then with $\widehat{\tau}_{t+j} > 0$, eq. (30) implies

$$(31)\quad \left(\frac{H\left(S^{\mu}_{t+j}\right)}{H\left(S^{BAU}_{t+j}\right)} - \frac{\widehat{\tau}_{t+j}}{H\left(S^{BAU}_{t+j}\right)\frac{F\left(L^{BAU}_{t+j}\right)}{L^{BAU}_{t+j}}} \right)^{\sigma-1} < 1.$$

If $\sigma < 1$, then it follows from (31) that $\widehat{\tau}_{t+j}$ must satisfy $\widehat{\tau}_{t+j} < \left[H\left(S^{\mu}_{t+j}\right) - H\left(S^{BAU}_{t+j}\right)\right]\frac{F\left(L^{BAU}_{t+j}\right)}{L^{BAU}_{t+j}}$, in which case

$$(32)\quad c^{\mu}_{t+j+1} = R\left[H\left(S^{\mu}_{t+j}\right)F\left(L^{BAU}_{t+j}\right) - \widehat{\tau}_{t+j}L^{BAU}_{t+j}\right] > RH\left(S^{BAU}_{t+j}\right)F\left(L^{BAU}_{t+j}\right) = c^{BAU}_{t+j+1}$$

must hold. This means tax rates that keep labor supply unchanged pre and post policy ($\widehat{\tau}_{t+j}$) will benefit agents in consumption terms and offer higher utility relative to what they would get in the BAU. Since $\frac{\partial \tilde{U}_{t+j}}{\partial \tau_{t+j}} < 0$, the government can raise tax rates relative to $\widehat{\tau}_{t+j}$ so as to bring utility down to the BAU level – this would be the "Pareto-neutral tax rate". Such an action would decrease labor supply relative to BAU: recall, at $\widehat{\tau}_{t+j}$, the labor supply pre and post policy are the same; also, we have established that $\frac{\partial L_{t+j}}{\partial \tau_{t+j}} < 0$ if $\sigma < 1$. Hence if $\sigma < 1$, imposing *Pareto-neutral tax rate* reduces both labor supply and consumption relative to the BAU.

In general, we cannot solve $\widehat{\tau}_{t+j}$ explicitly from (30). In the special case of quasi-linear utility ($\sigma = 0$), using (30) we get $\widehat{\tau}_{t+j} = \left[H\left(S^{\mu}_{t+j}\right) - H\left(S^{BAU}_{t+j}\right)\right]F_L\left(L^{BAU}_{t+j}\right)$. And if the starting point is the steady state, $\widehat{\tau}_{t+j} = \left[H\left(S^{\mu}_{t+j}\right) - H\left(S^{*}_{BAU}\right)\right]F_L\left(L^{*}_{BAU}\right)$.

If $\sigma > 1$, it follows from (31) that $\widehat{\tau}_{t+j}$ must satisfy $\widehat{\tau}_{t+j} > \left[H\left(S^{\mu}_{t+j}\right) - H\left(S^{BAU}_{t+j}\right)\right]\frac{F\left(L^{BAU}_{t+j}\right)}{L^{BAU}_{t+j}}$, in which case

$$c^{\mu}_{t+j+1} = R\left[H\left(S^{\mu}_{t+j}\right)F\left(L^{BAU}_{t+j}\right) - \widehat{\tau}_{t+j}L^{BAU}_{t+j}\right] < RH\left(S^{BAU}_{t+j}\right)F\left(L^{BAU}_{t+j}\right) = c^{BAU}_{t+j+1}.$$

So if $\sigma > 1$, imposing tax rates that keep labor supply pre and post policy unchanged makes every post-inaugural generation in the policy regime worse off than they would be if the BAU had continued. Since $\frac{\partial \tilde{U}_{t+j}}{\partial \tau_{t+j}} < 0$, the government can reduce tax rates relative to $\widehat{\tau}_{t+j}$ so as to bring utility up to the BAU level or higher.

F Proof of Proposition 2

$$B_{t+j} = \epsilon\mu S^{*}_{BAU}\left(\frac{\rho F_L\left(L^{*}_{BAU}\right)L^{*}_{BAU}}{\epsilon}\left(1-(1-\epsilon)^{j}\right) - \frac{\Lambda(\mu)}{\mu}\right).$$

Because $B_t < 0$ and B_{t+j} is increasing over time, to calculate the first date when it turns positive, we solve

$$B_{t+k^B} = 0$$

and get (for $\mu > 0$)

$$(1-\epsilon)^{k^B} = 1 - \frac{\epsilon}{\rho F_L\left(L^*_{BAU}\right) L^*_{BAU}} \frac{\Lambda\left(\mu\right)}{\mu}.$$

Only when $1 - \frac{\epsilon}{\rho F_L\left(L^*_{BAU}\right) L^*_{BAU}} \frac{\Lambda(\mu)}{\mu} > 0$ can we have a solution for k^B. Because $\frac{\Lambda(\mu)}{\mu}$ is increasing in μ, this sets an upper bound for μ, denoted by μ^B, which satisfies

$$1 - \frac{\epsilon}{\rho F_L\left(L^*_{BAU}\right) L^*_{BAU}} \frac{\Lambda\left(\mu^B\right)}{\mu^B} = 0.$$

Within the range $\left(0, \mu^B\right)$, we can solve

$$k^B = \frac{\ln\left(1 - \frac{\epsilon}{\rho F_L\left(L^*_D\right) L^*_D} \frac{\Lambda(\mu)}{\mu}\right)}{\ln\left(1-\epsilon\right)}.$$

We get the first date when B turns positive ($k^{B>0}$) by rounding up (taking the ceiling of k^B): $k^{B>0} = t + \left\lceil k^B \right\rceil$. It can be easily shown that $\frac{dk^B}{d\mu} > 0$. Next,

$$\Delta D_{t+j+1} = -R^{j+1} \epsilon \mu S^*_{BAU} \left(\frac{\rho F_L\left(L^*_{BAU}\right) L^*_{BAU}}{R-1+\epsilon} \left(1 - \left(\frac{1-\epsilon}{R}\right)^{j+1}\right) - \frac{\Lambda\left(\mu\right)}{\mu} \right).$$

Recall $D_t = -B_t > 0$. Because once debt begins to fall, it falls forever, to calculate the first date when it declines, we solve

$$\Delta D_{t+k^T} = 0$$

and get (for $\mu > 0$)

$$\left(\frac{1-\epsilon}{R}\right)^{k^T} = 1 - \frac{R-1+\epsilon}{\rho F_L\left(L^*_{BAU}\right) L^*_{BAU}} \frac{\Lambda\left(\mu\right)}{\mu}.$$

Only when $1 - \frac{R-1+\epsilon}{\rho F_L\left(L^*_{BAU}\right) L^*_{BAU}} \frac{\Lambda(\mu)}{\mu} > 0$ can we have a solution for k^T. Because $\frac{\Lambda(\mu)}{\mu}$ is increasing in μ, this sets an upper bound for μ, denoted by μ^T, which satisfies

$$1 - \frac{R-1+\epsilon}{\rho F_L\left(L^*_{BAU}\right) L^*_{BAU}} \frac{\Lambda\left(\mu^T\right)}{\mu^T} = 0.$$

Since $R > 1$, $\mu^T < \mu^B$. This is consistent with our understanding that a necessary condition for debt decline is to have a positive B at an earlier date. Within the range $\left(0, \mu^T\right)$, we can solve

$$k^T = \frac{\ln\left(1 - \frac{R-1+\epsilon}{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}\frac{\Lambda(\mu)}{\mu}\right)}{\ln\left(\frac{1-\epsilon}{R}\right)}.$$

We get the first date when debt declines ($k^{\triangle D<0}$) by rounding up (taking the ceiling of k^T): $k^{\triangle D<0} = t + \lceil k^T \rceil$. It can be easily shown that $\frac{dk^T}{d\mu} > 0$. Finally,

$$D_{t+j} = -\epsilon\mu S^*_D \frac{R^{j+1}-1}{R-1}\left(\frac{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}{\epsilon}\left(1 - \frac{R-1}{R-1+\epsilon}\frac{1-\left(\frac{1-\epsilon}{R}\right)^{j+1}}{1-\left(\frac{1}{R}\right)^{j+1}}\right) - \frac{\Lambda\left(\mu\right)}{\mu}\right)$$

Recall $D_t = -B_t > 0$. To calculate the first date when debt hits zero, we solve $D_{t+k^D} = 0$ and get (for $\mu > 0$)

$$\frac{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}{\epsilon}\left(1 - \frac{R-1}{R-1+\epsilon}\frac{1-\left(\frac{1-\epsilon}{R}\right)^{k^D+1}}{1-\left(\frac{1}{R}\right)^{k^D+1}}\right) = \frac{\Lambda\left(\mu\right)}{\mu}$$

If we know the range of $\frac{1-\left(\frac{1-\epsilon}{R}\right)^{k^D+1}}{1-\left(\frac{1}{R}\right)^{k^D+1}}$, then using the above equation we can find the range of μ that ensures the existence of k^D. To proceed, define $z\left(k^D\right) \equiv \frac{1-\left(\frac{1-\epsilon}{R}\right)^{k^D+1}}{1-\left(\frac{1}{R}\right)^{k^D+1}}, k^D > 0$.

$$\begin{aligned}
\frac{dz\left(k^D\right)}{dk^D} &= \frac{-\left(\frac{1-\epsilon}{R}\right)^{k^D+1}\left(1-\left(\frac{1}{R}\right)^{k^D+1}\right)\ln\frac{1-\epsilon}{R} + \left(1-\left(\frac{1-\epsilon}{R}\right)^{k^D+1}\right)\left(\frac{1}{R}\right)^{k^D+1}\ln\frac{1}{R}}{\left(1-\left(\frac{1}{R}\right)^{k^D+1}\right)^2} \\
&= \frac{\left(1-\left(\frac{1-\epsilon}{R}\right)^{k^D+1}\right)\left(1-\left(\frac{1}{R}\right)^{k^D+1}\right)}{\left(1-\left(\frac{1}{R}\right)^{k^D+1}\right)^2}\left(\frac{\left(\frac{1}{R}\right)^{k^D+1}\ln\frac{1}{R}}{1-\left(\frac{1}{R}\right)^{k^D+1}} - \frac{\left(\frac{1-\epsilon}{R}\right)^{k^D+1}\ln\frac{1-\epsilon}{R}}{1-\left(\frac{1-\epsilon}{R}\right)^{k^D+1}}\right)
\end{aligned}$$

Define $Q\left(x\right) = \frac{x\ln x}{1-x}$. It can be easily shown that $\frac{dQ(x)}{dx} = \frac{\ln x - x + 1}{(x-1)^2} < 0$ for $0 < x < 1$. For $k^D > 0$, $0 < \left(\frac{1-\epsilon}{R}\right)^{k^D+1} < \left(\frac{1}{R}\right)^{k^D+1} < 1$, so

$$\frac{\left(\frac{1}{R}\right)^{k^D+1}\ln\frac{1}{R}}{1-\left(\frac{1}{R}\right)^{k^D+1}} - \frac{\left(\frac{1-\epsilon}{R}\right)^{k^D+1}\ln\frac{1-\epsilon}{R}}{1-\left(\frac{1-\epsilon}{R}\right)^{k^D+1}} = \frac{Q\left(\left(\frac{1}{R}\right)^{k^D+1}\right) - Q\left(\left(\frac{1-\epsilon}{R}\right)^{k^D+1}\right)}{k^D+1} < 0,$$

which implies $\frac{dz\left(k^D\right)}{dk^D} < 0$ for $0 < k^D < +\infty$. So $z\left(k^D\right) = \frac{1-\left(\frac{1-\epsilon}{R}\right)^{k^D+1}}{1-\left(\frac{1}{R}\right)^{k^D+1}} \in \left(1, \frac{R-1+\epsilon}{R-1}\right)$. To ensure the existence of k^D, we require

$$\frac{\Lambda\left(\mu\right)}{\mu} < \frac{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}{R-1+\epsilon},$$

i.e., $\mu \in \left(0, \mu^T\right)$, the same range of μ for the existence of a turning point. We get the first date when debt hits zero ($k^{D<0}$) by rounding up (taking the ceiling of k^D): $k^{D<0} = t + \lceil k^D \rceil$. Also, since $\frac{dz\left(k^D\right)}{dk^D} < 0$, it can be easily shown that $\frac{dk^D}{d\mu} > 0$.

G Abatement with a fixed cost

In this section, we consider the case where "the environmental policies involve a significant initial fixed cost", $\overline{\mathcal{A}} > 0$. (we assume the fixed cost only happens once when the policy was initially implemented, not every period.) So the total cost in period t (the first period when the policy was implemented) is

$$\mathcal{A}(\mu_t) = \overline{\mathcal{A}} + \Lambda\left(\mu_t\right) G\left(L_t^\mu\right)$$

and the total cost in period $t + j$ ($j \geq 1$) equals

$$\mathcal{A}(\mu_{t+j}) = \Lambda\left(\mu_{t+j}\right) G\left(L_{t+j}^\mu\right)$$

We still assume $\Lambda(0) = \Lambda_\mu(0) = 0, \Lambda_\mu > 0, \Lambda_{\mu\mu} > 0$ for $\mu > 0$: the **variable** abatement cost is 0 and minimum in the BAU, and it is increasing and convex for positive levels of abatement. The transition equation for the stock of pollution is given by

$$S_{t+j+1}^\mu = (1 - \epsilon) S_{t+j}^\mu + \left(1 - \mu_{t+j}\right) G\left(L_{t+j}^\mu\right).$$

With a fixed cost, it is equivalent to the case where the government starts with a strictly positive debt $D_{t-1} = \overline{\mathcal{A}}/R > 0$. In another word, adding a fixed cost only changes the debt turning point and the first date when debt levels reach zero (if any). We use the same notations as before. Then,

$$B_t = \tau_t L_t^\mu - \overline{\mathcal{A}} - \Lambda\left(\mu_t\right) G\left(L_t^\mu\right)$$

$$B_{t+j} = \tau_{t+j} L_{t+j}^\mu - \Lambda\left(\mu_{t+j}\right) G\left(L_{t+j}^\mu\right), j \geq 1$$

$$D_{t-1} = 0$$

$$D_{t+j} = R D_{t+j-1} - B_{t+j}, j \geq 0$$

Here we consider the special case of quasi-linear utility and the same policy $\tau_{t+j} = \left[H\left(S_{t+j}^\mu\right) - H\left(S_{BAU}^*\right)\right] F_L\left(L_B^*\right.$ under which the labor supply is the same pre and post policy while consumption is strictly higher.

The first date when the debt starts to decline: the debt turning point

The change in the level of debt between $t + j$ and $t + j + 1$ is:

$$\Delta D_{t+j+1} \equiv D_{t+j+1} - D_{t+j} = R\left(D_{t+j} - D_{t+j-1}\right) - \left(B_{t+j+1} - B_{t+j}\right)$$

which upon repeated iteration yields

$$\begin{aligned}\Delta D_{t+j+1} &= -\sum_{i=0}^{j} R^{j-i}\left(B_{t+i+1}-B_{t+i}\right)+R^{j+1}\left(D_t-D_{t-1}\right)\\ &= -R^{j+1}\left(\sum_{i=0}^{j}\frac{B_{t+i+1}-B_{t+i}}{R^{i+1}}+B_t\right).\end{aligned}$$

It can be easily shown that

$$B_t=-\overline{\mathcal{A}}-\Lambda\left(\mu\right)G\left(L^*_{BAU}\right)=-\overline{\mathcal{A}}-\Lambda\left(\mu\right)\epsilon S^*_{BAU}$$

$$B_{t+1}-B_t=\epsilon\mu\rho S^*_{BAU}F_L\left(L^*_{BAU}\right)L^*_{BAU}+\overline{\mathcal{A}}$$

$$B_{t+i+1}-B_{t+i}=\epsilon\mu\rho S^*_{BAU}F_L\left(L^*_{BAU}\right)L^*_{BAU}\left(1-\epsilon\right)^i, i\geq 1$$

which leads to

$$\begin{aligned}\Delta D_{t+j+1} &= -R^{j+1}\left(\sum_{i=1}^{j}\frac{B_{t+i+1}-B_{t+i}}{R^{i+1}}+\frac{B_{t+1}-B_t}{R}+B_t\right)\\ &= -R^{j+1}\epsilon\mu S^*_{BAU}\left(\frac{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}{R-1+\epsilon}\left(1-\left(\frac{1-\epsilon}{R}\right)^{j+1}\right)-\frac{\Lambda\left(\mu\right)}{\mu}\right)+R^j\left(R-1\right)\overline{\mathcal{A}}\end{aligned}$$

Or, we can think this way. This initial cost generate extra debt in each period, $R^j\overline{\mathcal{A}}$ in period $t+j$. So the change of debt ΔD_{t+j+1} will be the previous change plus the change caused by this initial fixed cost, $R^{j+1}\overline{\mathcal{A}}-R^j\overline{\mathcal{A}}$.

$\Delta D_{t+j+1}<0$ requires

$$-R^{j+1}\epsilon\mu S^*_{BAU}\left(\frac{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}{R-1+\epsilon}\left(1-\left(\frac{1-\epsilon}{R}\right)^{j+1}\right)-\frac{\Lambda\left(\mu\right)}{\mu}\right)+R^j\left(R-1\right)\overline{\mathcal{A}}<0$$

$$\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda\left(\mu\right)}{\mu}<\frac{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}{R-1+\epsilon}\left(1-\left(\frac{1-\epsilon}{R}\right)^{j+1}\right)$$

Suppose the first date when the debt declines is $j=k^T$. Because once debt begins to fall, it falls forever, to calculate k^T, we only need to solve

$$\Delta D_{t+k^T}=0$$

which yields (for $\mu>0$)

$$\left(\frac{1-\epsilon}{R}\right)^{k^T}=1-\frac{R-1+\epsilon}{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda\left(\mu\right)}{\mu}$$

Only when $1-\frac{R-1+\epsilon}{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda(\mu)}{\mu}>0$ can we have a solution for k^T.

Obviously, with an initial cost $\overline{\mathcal{A}}>0$, the implementation of the policy becomes harder under the same non-exploding-debt constraint. Now we discuss whether the implementability set for μ, Ω, is non-empty.

$\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda(\mu)}{\mu}$: when $\mu \to 0$, $\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda(\mu)}{\mu} \to \infty$, which indicates a very mild policy can not be implementable because of the initial fixed cost. The gains from the abatement is too small to even cover the fixed cost.

Also,

$$d\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda(\mu)}{\mu}/d\mu = \frac{\Lambda'(\mu)\mu - \Lambda(\mu) - \frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}}{\mu^2}$$

$\Lambda'(\mu)\mu - \Lambda(\mu)$ is an increasing function of μ, and $\Lambda'(\mu)\mu - \Lambda(\mu) \in (0,\infty)$ for $\mu \in (0,\infty)$, so there exists a unique solution for $d\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda(\mu)}{\mu}/d\mu = 0$. We denote the solution by $\widehat{\mu}$, and it satisfies

$$\Lambda'(\widehat{\mu})\widehat{\mu} - \Lambda(\widehat{\mu}) - \frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}} = 0$$

$\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda(\mu)}{\mu}$ reaches the minimum when $\mu = \widehat{\mu}$.

Two cases: (1) If $1 - \frac{R-1+\epsilon}{\rho F_L(L^*_{BAU})L^*_{BAU}}\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda(\widehat{\mu})}{\widehat{\mu}} \leq 0$, we can not find any μ such that the policy is implementable, i.e., Ω is empty.

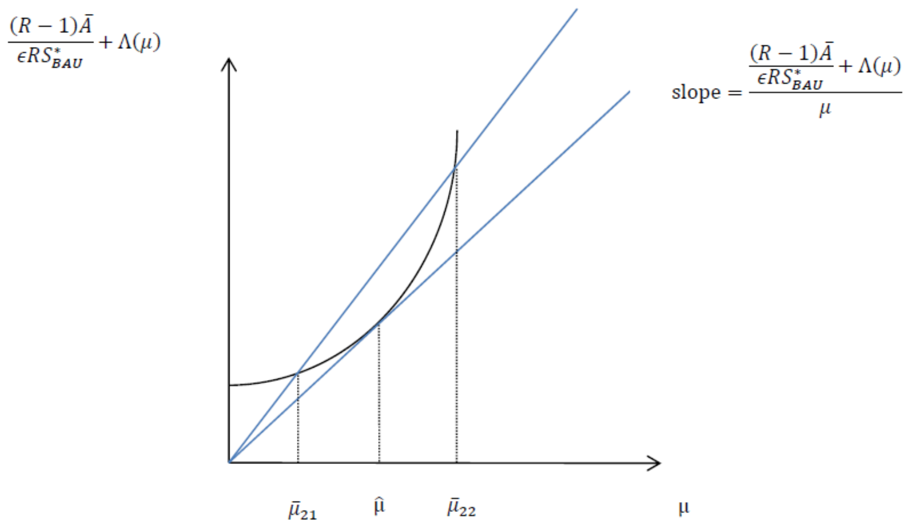
(2) $1 - \frac{R-1+\epsilon}{\rho F_L(L^*_{BAU})L^*_{BAU}}\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda(\widehat{\mu})}{\widehat{\mu}} > 0$, Ω is non-empty. Let $\overline{\mu}_{21}$ and $\overline{\mu}_{22}$ denote the two roots of

$$1 - \frac{R-1+\epsilon}{\rho F_L\left(L^*_{BAU}\right)L^*_{BAU}}\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda(\mu)}{\mu} = 0$$

Then within the range of $(\overline{\mu}_{21}, \overline{\mu}_{22})$, the policy is implementable. We must have $0 < \overline{\mu}_{21} < \widehat{\mu} < \overline{\mu}_{22}$. In this case, we solve

$$k^T = \frac{\ln\left(1 - \frac{R-1+\epsilon}{\rho F_L(L^*_{BAU})L^*_{BAU}}\frac{\frac{(R-1)\overline{\mathcal{A}}}{\epsilon RS^*_{BAU}}+\Lambda(\mu)}{\mu}\right)}{\ln\left(\frac{1-\epsilon}{R}\right)}$$

We can use the graph to illustrate.



The first date when debt levels reach zero

The debt in period $t+j$ (iterating $D_{t+j} = RD_{t+j-1} - B_{t+j}$ and using $D_{t-1} = 0$) is given by

$$D_{t+j} = -\epsilon\mu S_{BAU}^{*} \frac{R^{j+1}-1}{R-1} \left(\frac{\rho F_L \left(L_{BAU}^{*}\right) L_{BAU}^{*}}{\epsilon} \left(1 - \frac{R-1}{R-1+\epsilon} \frac{1-\left(\frac{1-\epsilon}{R}\right)^{j+1}}{1-\left(\frac{1}{R}\right)^{j+1}} \right) - \frac{\Lambda(\mu)}{\mu} \right) + R^{j-1}\overline{\mathcal{A}}.$$

In the case (b) above, the debt will reach zero in finite periods. Suppose the date at which the debt hits zero is $j = k^D$. Then k^D solves $D_{t+k^D} = 0$.

H Discussion of parameter choices

The functional forms for utility $U(\cdot)$, damage $H(\cdot)$ and corresponding parameter values (σ, β and ρ) and decay rate (ϵ) are in line with those used in Karp and Rezai (2014b). α is chosen to make capital's share of output equal to 0.36 as in Heutel (2012). $\theta = 0.9$, indicating emissions a concave function of capital used in the production.

The total factor productivity A and emission intensity δ are calibrated following the same way in Karp and Rezai (2014b). "We scale nominal units by 10^9 2010 USD (\$$T$). Current capital stock, K_0, is roughly 200 \$$T$. Yearly world output is roughly 63 \$$T$, so output in one 35-year period is $y_0 = 35 \times 63 \approx 2200$ \$$T$. [...] Currently, 8.36 $Gt\ C$ are burnt per year. This corresponds to an annual increase in atmospheric CO_2 of 3.92 *ppBv*." (Karp and Rezai 2014b; pp. 20) So total factor productivity is calibrated to $A = \frac{y_0}{K_0^{\alpha}} = 327$, and the emission intensity is $\delta = \frac{3.92 \times 35}{K_0^{\theta}} = 1.165$.

The total abatement cost is $\mathcal{A}_{t+j} = \lambda\mu^{\phi}\delta K_{t+j}^{\theta}$. We set $\phi = 2.5$. λ is calibrated such that it costs 5.4% of GDP to abate all emissions today (Nordhaus, 2008). Recall $\delta = \frac{3.92\times 35}{K_0^{\theta}}$, so $\lambda = \frac{\mathcal{A}_0}{\delta K_0^{\theta}} = \frac{0.054 y_0}{\delta K_0^{\theta}} = \frac{0.054\times 2200}{3.92\times 35} = 0.866$.

I Algorithm for the computations

Given the abatement policy μ initiated in t, we first want to figure out the path of taxes $\left\{\tau_{t+j}^{K}\right\}_{j=0}^{\infty}$. The initial old, generation $t-1$, consume their savings from young. Pareto criterion requires the interest rate cannot be lower, that is, $R_t \geq R_t^{BAU}$. Since $R_t = H(S_t) F_K(K_t) - \tau_{t+j}^{K}$, and $S_t = S_t^{BAU}, K_t = K_t^{BAU}$, we can only set $\tau_t^K = 0$, which implies $R_t = R_t^{BAU}$, and the utility of the initial old is held at the BAU level. We use the following steps to calculate the dynamics for $j > 0$.

Step 1: at the beginning of $t+j\,(j \geq 0)$, $S_{t+j}, K_{t+j},, D_{t+j-1}$ and τ_{t+j}^{K} are given. w_{t+j}, R_{t+j}, S_{t+j+1}, and D_{t+j} are given by

$$w_{t+j} = (1-\alpha) H(S_{t+j}) A K_{t+j}^{\alpha}.$$

$$R_{t+j} = \alpha H(S_{t+j}) A K_{t+j}^{\alpha-1} - \tau_{t+j}^{K}.$$

$$S_{t+j+1} = (1-\epsilon) S_{t+j} + (1-\mu) G(K_{t+j}).$$

$$D_{t+j} = R_{t+j} D_{t+j-1} + \Lambda(\mu) G(K_{t+j}) - \tau_{t+j}^{K} K_{t+j}.$$

Step 2: using Pareto-improving condition

$$\widetilde{U}_{t+j}(w_{t+j}, R_{t+j+1}) = \widetilde{U}_{t+j}^{BAU} + \eta_{t+j}\left|\widetilde{U}_{t+j}^{BAU}\right|, \eta_{t+j} \geq 0,$$

we solve R_{t+j+1}, and then the saving function $s_{t+j}(w_{t+j}, R_{t+j+1})$ is known.

Step 3: calculate K_{t+j+1} and τ_{t+j+1}^{K} using

$$K_{t+j+1} = s_{t+j} - D_{t+j}.$$

$$\tau_{t+j+1}^{K} = \alpha H(S_{t+j+1}) A K_{t+j+1}^{\alpha-1} - R_{t+j+1}.$$

If $\tau_{t+j+1}^{K} < 0$, i.e., the economy cannot achieve the required utility improvement without transfers, then we set $\tau_{t+j+1}^{K} = 0$. In that case, we need to re-calculate K_{t+j+1}. Since S_{t+j+1} and D_{t+j} are known, the capital stock K_{t+j+1} can be solved from ($\tau_{t+j+1}^{K} = 0$):

$$K_{t+j+1} = \frac{(1-\alpha) H(S_{t+j}) A K_{t+j}^{\alpha}}{1 + \beta^{-\frac{1}{\sigma}}\left(\alpha A H(S_{t+j+1}) K_{t+j+1}^{\alpha-1}\right)^{1-\frac{1}{\sigma}}} - D_{t+j}.$$

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