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# Reformulation of U.S. day-ahead wholesale power markets for improved intertemporal operations

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## **Abstract**

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## **Keywords**

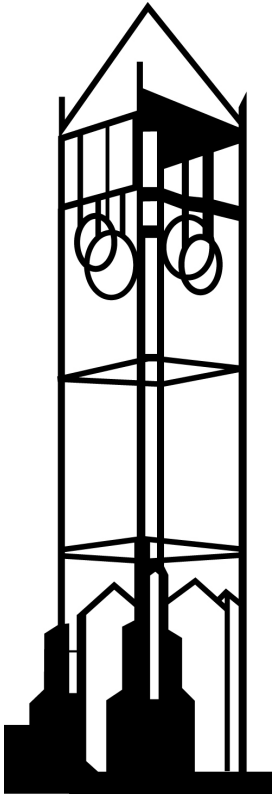
day-ahead market, economic dispatch, electrical energy, financial contracts, forecasting, forward planning, unit commitment, virtual demand bid, virtual supply offer

## **Disciplines**

Economics

# **Reformulation of U.S. Day-Ahead Wholesale Power Markets for Improved Intertemporal Operations**

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Leigh Tesfatsion · Dionysios C. Aliprantis

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**Keywords** Day-ahead market · Economic dispatch · Electrical energy

**JEL Classification** D40 · L1 · Q40

## 1 Introduction

Market operators managing wholesale electric power markets face a daunting multi-objective task: to ensure an appropriate availability of resources at each point in time to maintain both system reliability (adequacy and security) and system efficiency (non-wastage of resources). This task will surely become even more arduous as penetration of renewable energy resources increases, bringing greater dispatch uncertainties.

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A full description of this multi-objective task requires the specification of an incredibly complicated collection of spatially and intertemporally coupled stochastic decision problems (Rajagopal et al. 2013). The degree to which a market operator should optimally clear new financially binding contracts at any current time  $t$  to achieve power balance at a future operating time  $t'$  can only be determined after careful consideration of all available opportunities. These opportunities might entail, for example, the use of energy and the exercise of reserve cleared in processes prior to time  $t$  as well as the use of energy cleared in reliability assessment processes between times  $t$  and  $t'$ .<sup>1</sup> Also, the market operator must take into account at time  $t$  the possibility that uncertainties regarding variable generation and loads could change as the operating time  $t'$  approaches. Procurement costs as well as risks of non-procurement could be higher at later times, providing an incentive to clear energy and/or reserve sooner rather than later. On the other hand, variable generation and load forecasts made at later times could be more accurate, providing an incentive to wait.

In U.S. practice today, however, the design of day-ahead markets (DAMs) encourages Independent System Operators (ISOs)<sup>2</sup> to engage in intertemporally decoupled decision-making. Attention is focused on decisions that minimize immediate net costs, based on market participant (MP) offers/bids and day-ahead forecasts. The possibility that future decisions based on more accurate forecasts could be taken in place of current decisions is not explicitly considered. Moreover, with the exception of ISO-NE's Forward Reserve Market, there is no availability of longer-term forward markets that permit MPs to engage in longer-term energy/reserve procurement to manage private risk but that also include system constraints aimed at ensuring reliable and efficient system operations.

This study proposes initial practical steps toward the ultimate goal of enabling a sequence of linked ISO-managed forward markets to facilitate a more reliable and efficient procurement of energy and reserve over time. A full treatment of this issue would require a careful consideration of energy and reserve product definitions, contract forms, and multiple market formulations with planning horizons ranging from multiple-year to hourly or even intrahourly. Here we address a narrower issue critical for this larger task: namely, if a DAM is temporally positioned within a sequence of market and administrative processes for energy/reserve procurement, how can the ISO managing the DAM practically ensure that both pre-DAM procurement and post-DAM procurement opportunities are given due consideration within the DAM?

For concreteness, Section 2 starts by considering a standard ISO-managed DAM in operation on some day Day D–1, as depicted in Fig. 1. In Section 3, this DAM is assumed to be temporally positioned within a sequence of energy/reserve procurement processes. The standard DAM constraints are then reformulated to take into account the existence of these pre-DAM and post-DAM procurement processes, and the ISO is assumed to function as the clearinghouse for this linked DAM. That is, the ISO solves a DAM optimal power flow (OPF) problem with reformulated DAM constraints to determine unit commitments, cleared levels for dispatchable generation and price-sensitive loads, and locational marginal prices (LMPs) for the next day D.

<sup>1</sup> Following standard usage (Ellison et al. 2012), “energy” is used in this study to refer to the actual production of electrical energy measured in Megawatt hours (MWh). In contrast, “reserve” is used as shorthand for capacity availability, meaning an up/down incremental amount of electrical energy that a market participant could provide if signaled by a market operator to do so for system reliability purposes.

<sup>2</sup> Of the seven centrally-managed U.S. wholesale power markets, currently three (CAISO, ERCOT, NY-ISO) are organized as ISOs and four (ISO-NE, MISO, PJM, SPP) as Regional Transmission Organizations (RTOs). The essential difference between an ISO and an RTO is that RTOs have larger regional scope. To avoid clumsy notation, this study uses “ISO” in place of “ISO/RTO” to designate the market operator.

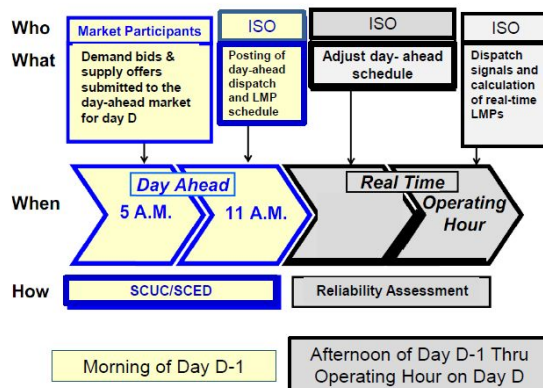


Fig. 1 Illustration of typical ISO activities on successive days D-1 and D, adapted from Ellison et al. (2012).

Why should this proposed DAM reformulation be expected to provide superior DAM outcomes in comparison with existing DAM operations? As argued by Rajagopal et al. (2013, p. 3), “decision-theoretic considerations immediately imply that decoupled dispatch must incur a larger cost than [optimal dispatch].” While we surely do not claim that our proposed DAM reformulation achieves an optimal dispatch, we argue that it offers a practical way to achieve a better approximation to the optimal dispatch by permitting a coupled consideration of past, present, and future energy/reserve procurement.

The key modification made in Section 3 to the standard DAM constraints to achieve a coupling of past, current, and future energy/reserve procurement is a modification of the form of the power balance equation for each bus. This modification entails the inclusion of economically meaningful slack variables under the control of the ISO.<sup>3</sup>

More precisely, MPs in U.S. ISO-managed DAMs are currently permitted to exploit arbitrage opportunities across markets for their own profit advantage by submitting virtual supply offers and virtual demand bids.<sup>4</sup> These virtual offers/bids are then entered into the DAM power balance equations along with physically-backed offers and bids.

In Section 3, the standard DAM power balance equations are modified to permit the inclusion of ISO-determined virtual supply offers and virtual demand bids along with the offers/bids of MPs.<sup>5</sup> The inclusion of ISO-determined virtual supply offers permits the ISO to plan to achieve power balance during each operating hour of the following day by means

<sup>3</sup> Interestingly, up and down slack variables are included in the system power balance equation appearing in the base-case DAM formulation developed by the U.S. Federal Energy Regulatory Commission (FERC 2011, p. 20). However, these slack variables are introduced purely as a computational device, not as economically meaningful decision variables. The goal in FERC (2011) is to have the magnitude of these slack variables be as small as possible, and this is achieved through the introduction of simple penalty functions in the DAM objective function.

<sup>4</sup> A *virtual* supply offer (or increment offer) is an offer to supply energy at a specified location and time in a specified amount that is made by an entity that is not associated with a physical generator. A *virtual* demand bid (or decrement bid) is a bid to buy energy at a specified location and time in a specified amount that is made by an entity that is not associated with a physical load.

<sup>5</sup> The qualifier “virtual” is used to stress that these ISO-determined offers/bids are not backed by physical resources secured through newly cleared DAM contracts.

of a mix of energy/reserve cleared at three different times: prior to the DAM, during the DAM, and subsequent to the DAM. The inclusion of ISO-determined virtual demand bids provides a mechanism by which an ISO could adjust the forecasts of DAM MPs for fixed loads and non-dispatchable generation in situations where these forecasts are less accurate than the forecasts of the ISO, to the detriment of system reliability or system efficiency.

Current U.S. Federal Energy Regulatory Commission (FERC) policies do not permit ISOs this degree of flexibility. Rather, an ISO is required to act in the DAM as though power balance on day D must be ensured by means of financially binding contracts cleared in the DAM on day D-1.<sup>6</sup> Presumably for this reason, previous studies of optimization formulations for energy and reserve procurement in centrally managed wholesale electric power markets have routinely assumed that the pre-contingency and post-contingency power balance equations appearing as constraints in these optimizations must be satisfied as strict equalities via a financially binding scheduling of generation and demand response resources determined at the time the optimization is conducted. See, for example, (Chow et al. 2005; Bouffard et al. 2005; Arroyo et al. 2005; Helman et al. 2008; Ruiz et al. 2009; Morales et al. 2009; Capitanescu et al. 2011; Papavasiliou et al. 2011).

The DAM reformulation proposed in this study can be implemented within any DAM that includes power balance equations among its constraints, which is to say within any conventionally formulated DAM. This is illustrated in Section 3 for both a day-ahead energy market and a co-optimized day-ahead energy/reserve market. Moreover, with its emphasis on the use of physically-covered energy call options as reserve products, the proposed DAM reformulation provides natural support for a stochastic formulation in which ISO uncertainties are handled through contingency planning by means of probability-weighted scenarios. This is demonstrated in Section 4.

In each of the illustrative examples in Sections 3 and 4, it is explicitly shown how the reformulated DAM can be linked to prior and subsequent energy/reserve procurement processes through ISO-determined virtual offers/bids. The ISO can choose to set these linkages to zero, thus permitting standard DAM outcomes to be obtained. However, it is also shown that the linkages provide the ISO a possible way to reduce procurement costs relative to standard DAM outcomes by using prior cleared energy, by exercising prior cleared reserve, or by waiting to clear energy or exercise reserve until post-DAM processes when uncertainties about net loads (hence about actual energy needs) are reduced.

Section 5 addresses several issues regarding the role of the ISO in the proposed DAM reformulation. Concluding remarks are given in Section 6. Nomenclature and definitions for all of the major variables and functional forms appearing in the standard and modified DAM formulations are given in Tables 1 and 2.

## 2 Standard Day-Ahead Market Operations

In a series of notices culminating in (FERC 2003), FERC recommended that U.S. electrical energy regions institute wholesale power markets organized as double auctions (offer/bid-

<sup>6</sup> As noted by Chao et al. (2005, Section II.B), FERC has insisted on this standard since the 1999-2001 crisis in California during which huge imbalances arose in the real-time market. In fact, however, among the many complex causes of this crisis was that California effectively barred its utilities from procuring power via long-term contracts. This prevented MPs from hedging against price volatility, and encouraged them to engage in short-term strategic trading and gaming of system rules that led to large real-time imbalances (Chao et al. 2005; Bush and Mayne 2004). Failure of the California market operator to carry out its fiduciary responsibilities for ensuring system reliability was not the issue.

based exchanges) centrally managed by a nonprofit ISO or RTO. A key feature of this recommended market design was a DAM that permits day-ahead commitment, dispatch and pricing of reserve and/or energy to be determined by the supplies and demands of sellers and buyers, as in ordinary commodity markets, to an extent consistent with reliable operation of the grid. Further, FERC recommended that this DAM be supported by a separately settled real-time market (RTM) functioning as a balancing mechanism for resolving differences between day-ahead scheduled dispatch levels and real-time loads.

Six U.S. energy regions encompassing more than 60% of U.S. generating capacity are currently operating in accordance with FERC's market design: CAISO, ERCOT, ISO-NE, MISO, NYISO, and PJM.<sup>7</sup> Key participants in the DAMs for these energy regions include Generation Companies (GenCos) that own generation units for the physical production of energy and Load-Serving Entities (LSEs) that service the energy needs of retail energy customers.

As detailed by Ellison et al. (2012), ISO-NE and PJM have implemented day-ahead energy markets in which all new financially binding contracts are for the procurement of energy. All financially binding contracts for reserve are determined through prior, concurrent, or subsequent market and/or administrative processes conducted separately from the day-ahead energy market. In contrast, CAISO, ERCOT, MISO, and NYISO have implemented co-optimized day-ahead energy/reserve markets in which GenCos and certain types of Demand Response Resources (DRRs) are permitted to submit both energy and reserve supply offers that then result in new financially binding contracts for both energy and reserve.

As a benchmark of comparison, this section sets out an ISO-managed day-ahead energy market operating under normal conditions. Later sections will then illustrate our proposed DAM reformulation for variously modified versions of this benchmark DAM.

As stressed in Section 1, the primary modification required by our DAM reformulation is the inclusion of economically meaningful slack variables in the power balance equation for each bus. This primary modification does not depend either on the exact forms taken by the remaining DAM constraints or on the precise specification of the DAM MPs. Consequently, for clarity of exposition, various assumptions are made below to simplify the specification of the benchmark DAM constraints and MPs.

Consider an ISO-managed DAM whose MPs include GenCos  $i = 1, \dots, I$  and LSEs  $j = 1, \dots, J$ . GenCos are producers/sellers of bulk power and LSEs are buyers of bulk power for the servicing of retail customers; there are no hybrid (buyer-seller) companies that combine GenCo and LSE functions. The GenCos and LSEs submit physically-backed offers/bids for bulk power into the DAM. Import/export schedules, self-scheduled bilaterals, and virtual offers/bids initiated by MPs are not explicitly considered.

Each GenCo  $i$  operates conventional and/or variable generation (wind, solar) units divided into two categories: units producing non-dispatchable (non-price-sensitive) energy levels; and units producing dispatchable (price-sensitive) energy levels. Each LSE  $j$  services the energy demands of its retail customers under both fixed-price and variable-price retail contracts.<sup>8</sup>

More precisely, during the morning of each day  $D-1$  each GenCo  $i$  submits 24 hourly supply offers to the DAM, together with unit commitment information,<sup>9</sup> for generation

<sup>7</sup> SPP is planning a 2014 launch of an Integrated Market design that conforms with FERC's design.

<sup>8</sup> Under a fixed-price retail contract, the retail price of energy is set at a regulated rate. Under a variable-price retail contract, an LSE is permitted to vary the retail price of energy in response to changes in wholesale energy prices.

<sup>9</sup> This unit commitment information includes start-up costs, no-load costs, ramp-up capabilities, run-time restrictions, ramp-down capabilities, shut-down costs, and down-time restrictions.



scheduling on day D. Each of these supply offers consists of two parts: (i) a fixed generation level  $p_{Gi}^F$  (MW) for GenCo  $i$ 's non-dispatchable generation (with all non-dispatchable units considered together for simplicity of exposition); and (ii) an inverse supply function in the form of a marginal cost function  $MC_i(p_{Gi}^D)$  (\$/MWh), defined over an operating capacity interval  $[Cap_i^L, Cap_i^U]$ , that indicates GenCo  $i$ 's minimum acceptable payment for each successive MW of dispatchable generation it can produce (with all dispatchable generation units considered together for simplicity of exposition).<sup>10</sup>

Also, each LSE  $j$  submits 24 hourly demand bids to the DAM for the scheduling of its retail customer load on day D. Each of these demand bids consists of two parts: (i) a fixed load obligation  $p_{Lj}^F$  (MW) for resale in a downstream retail energy market at an administratively set price; and (ii) an inverse demand function in the form of a marginal benefit function,  $MB_j(p_{Lj}^D)$  (\$/MWh), defined over a bid interval  $[Bid_j^L, Bid_j^U]$ , indicating LSE  $j$ 's maximum willingness to pay for each successive MW it procures beyond its fixed load obligation.

The general presumption of DAM designs is that GenCo supply offers and LSE demand bids represent objective forecasts for generator marginal operating costs and capacities and for consumer load valuations and amounts. However, in some circumstances these offers/bids could in fact deviate from objective forecasts. For example, they could reflect strategic considerations, such as the exploitation of perceived opportunities to arbitrage differences between DAM and RTM prices. They could also reflect attempts to ensure coverage of fixed costs associated with the production of energy and/or the servicing of load, such as equipment purchase costs.

After the close of the DAM at noon on day D–1, the ISO solves for GenCo unit commitments, dispatch set points for dispatchable generation from committed GenCos, cleared levels for LSE price-sensitive demands, and an LMP at each bus for each hour of the following day D. In practice, such DAM outcomes are found by conducting Security-Constrained Unit Commitment (SCUC) and Security-Constrained Economic Dispatch (SCED) optimizations, either jointly or in coupled succession.<sup>11</sup> For simplicity of exposition, it is assumed for the benchmark DAM that the SCUC optimization is conducted prior to the SCED optimization, that all  $I$  GenCos are committed in this SCUC optimization, and that these committed GenCos are not constrained by cross-hour restrictions (e.g., ramping or minimum run-time/down-time restrictions). The subsequent DAM SCED optimization for each operating hour  $H$  of day D is then assumed to take the following offer/bid-based DC OPF form:

*Minimize net operating cost*

$$\text{NOC}(\mathbf{p}_G^D, \mathbf{p}_L^D) = \sum_{i=1}^I \text{Cost}_i(p_{Gi}^D) - \sum_{j=1}^J \text{Benefit}_j(p_{Lj}^D) \quad (1)$$

*with respect to dispatchable generation levels, dispatchable load levels, and voltage angles*

$$\mathbf{p}_G^D, \mathbf{p}_L^D, \delta \quad (2)$$

<sup>10</sup> ISOs require GenCos to express their offered supply quantities for each hour in MW units, where the MW level is interpreted as an average rate of power injection to be maintained over the hour.

<sup>11</sup> For example, see CAISO (2009) and MISO (2011) for detailed presentations of the DAM SCUC/SCED optimizations conducted in CAISO and MISO. CAISO determines DAM outcomes by means of an integrated optimization combining both SCUC and SCED. In contrast, MISO determines DAM outcomes by means of successive coupled SCUC and SCED optimizations, where the SCED optimization uses as input the SCUC unit commitment outcomes.

subject to power balance at each bus  $k = 1, \dots, K$ :

$$\text{PGen}_k + \text{PDGen}_k = \text{PFLoad}_k + \text{PDLoad}_k + \text{POut}_k \quad (3)$$

where:

$$\text{PGen}_k = \sum_{i \in I_k} p_{Gi}^F \quad (4)$$

$$\text{PDGen}_k = \sum_{i \in I_k} p_{Gi}^D \quad (5)$$

$$\text{PFLoad}_k = \sum_{j \in J_k} p_{Lj}^F \quad (6)$$

$$\text{PDLoad}_k = \sum_{j \in J_k} p_{Lj}^D \quad (7)$$

$$\text{POut}_k = \sum_{km \text{ or } mk \in BR} P_{km} \quad (8)$$

$$P_{km} = S_b B_{km} [\delta_k - \delta_m] \quad (9)$$

Power flow limits for each line  $km \in BR$ :

$$|P_{km}| \leq P_{km}^U \quad (10)$$

Operating capacity limits for dispatchable generation for each GenCo  $i = 1, \dots, I$ :

$$\text{Cap}_i^L \leq p_{Gi}^D \leq \text{Cap}_i^U \quad (11)$$

Bid limits for dispatchable load for each LSE  $j = 1, \dots, J$ :

$$\text{Bid}_j^L \leq p_{Lj}^D \leq \text{Bid}_j^U \quad (12)$$

Voltage angle setting at angle reference bus 1:

$$\delta_1 = 0 \quad (13)$$

The DAM LMP at any bus  $k$ , denoted by  $\text{LMP}_k$  (\$/MWh), is the dual variable solution for the corresponding bus- $k$  power balance constraint (3). Roughly stated,  $\text{LMP}_k$  measures the minimum additional cost to the system if one additional MW of power withdrawal occurs at bus  $k$ .<sup>12</sup> Any GenCo cleared to supply power at bus  $k$  receives from the ISO the price  $\text{LMP}_k$  per MWh of its energy injection, and any LSE cleared to service customer demand at bus  $k$  is required to pay to the ISO the price  $\text{LMP}_k$  per MWh of its customers' energy withdrawal.

<sup>12</sup> More precisely, given certain regularity conditions,  $\text{LMP}_k$  can be expressed as the derivative of the minimized objective function (1) with respect to a change in the constraint constant of the bus- $k$  power balance constraint (3), evaluated at the DC OPF solution point.

### 3 Proposed Reformulations for Energy and Co-Optimized Energy/Reserve DAMs

#### 3.1 Day-Ahead Energy Market Reformulation

In this section we illustrate our proposed DAM reformulation for the benchmark day-ahead energy market set out in Section 2. For clarity of exposition, attention is focused on the reformulation of the benchmark DAM SCED optimization on a particular day  $D-1$ .<sup>13</sup> All pre-existing, new, and planned future resource commitments considered in the DAM SCED optimization are assumed to be for a particular hour  $H$  of the next day  $D$ .

Suppose the ISO managing the DAM SCED optimization on day  $D-1$  is aware of pre-DAM and post-DAM energy/reserve procurement opportunities. In particular, for concreteness of exposition, we assume the ISO can take the following possible actions. The ISO can deploy energy, or exercise reserve for the deployment of energy, using energy/reserve cleared in procurement processes held prior to the DAM. The ISO can also plan to clear additional energy in two post-DAM procurement processes: a Reliability Assessment Process (RAP) held during the afternoon of day  $D-1$ ; and a real-time market (RTM) held just prior to hour  $H$  on day  $D$ .

Eight assumptions are made for reserve, as follows:

1. Fast-start fast-ramp GenCos with dispatchable generation are the only suppliers of reserve, and the ISO is the only entity that procures reserve.
2. All reserve takes the form of available generation capacity to facilitate up/down net load following<sup>14</sup> during hour  $H$  of day  $D$  under normal operating conditions.
3. All reserve is procured through a special form of call option,<sup>15</sup> referred to as a reserve option contract, for potential delivery of energy during hour  $H$  of day  $D$ .
4. A reserve option contract held by the ISO gives the ISO the right, but not the obligation, to exercise the contract any time up to the start of hour  $H$ .
5. The contractual terms of each reserve option contract specify a delivery bus  $k$ , a maximum delivery rate  $r > 0$  (MW), and an exercise price  $\phi$  (\$/MWh).
6. If the ISO exercises a reserve option contract  $(k, r, \phi)$ , the GenCo that issued the contract is obligated to deliver or curtail energy at bus  $k$  during hour  $H$  at a constant rate  $\theta r$ , where  $\theta \in [-1, 1]$  is selected by the ISO at the time of exercise. In turn, the ISO is obligated to pay the GenCo an amount  $\phi \cdot |\theta r|$  for the delivered energy or energy curtailment.
7. A reserve option contract can only be exercised once, for a specific amount of up/down power as permitted under the terms of the contract, even if this exercised up/down power is not provided at the maximum delivery rate.
8. The ISO allocates all costs of reserve procurement to the LSEs based on their load shares.<sup>16</sup>

<sup>13</sup> Our reformulation of a more fully specified DAM SCUC/SCED optimization with GenCo unit commitment costs and constraints, and with 0-1 unit commitment decision variables for the ISO, would proceed along similar lines.

<sup>14</sup> Load-following reserve is the ability to deliver energy or curtail energy to follow general load-pattern trends within a day. It is usually supplied by synchronized or fast-start generation (Ela et al. 2011), but it can also be supplied by DRRs. In the current study, in which all reserve is assumed to be supplied by GenCos, up-reserve refers to energy delivery and down-reserve refers to energy curtailment. Also, net load refers to load adjusted for non-dispatchable generation treated as negative load.

<sup>15</sup> See (Oren 2005a; Oren 2005b; EPRI 2003; Chao and Wilson 2004; Hogan 2005) for detailed discussions of the potential use of call option contracts in power markets.

<sup>16</sup> As detailed in Ellison et al. (2012), ISO-managed energy regions in the U.S. routinely allocate ISO reserve procurement costs to LSEs based on their load shares, with appropriate adjustments for LSE self-provided and self-scheduled reserve.

Given the above assumptions, we propose four modifications of the standard DAM SCED optimization set out in Section 2. Our first modification is that we permit the standard power balance equation (3) for each bus  $k$  to be augmented with an ISO-determined virtual supply offer  $\text{IVOffer}_k$  and an ISO-determined virtual demand bid  $\text{IVBid}_k$ . Let the net fixed load at each bus  $k$  resulting from the offers and bids of the MPs be denoted by

$$\text{MPNetFLoad}_k = \text{PFLoad}_k - \text{PGen}_k \quad (14)$$

Then the modified power balance equation at bus  $k$  takes the form

$$\text{IVOffer}_k + \text{PDGen}_k = \text{IVBid}_k + \text{MPNetFLoad}_k + \text{PDLoad}_k + \text{POut}_k \quad (15)$$

The ISO-determined virtual supply offer  $\text{IVOffer}_k$  in (15) permits the ISO to adjust the timing and amount of the energy clearing and reserve exercise to be used to balance the DAM-forecasted energy withdrawal (load plus outflow) at bus  $k$ . A non-zero  $\text{IVOffer}_k$  indicates that the ISO is planning to meet at least part of this DAM balancing need by energy cleared prior to the DAM, by the exercise of reserve procured prior to the DAM, and/or by energy to be cleared in the subsequent RAP. The remaining DAM balancing need at bus  $k$  is then provided by  $\text{PDGen}_k$  and  $\text{PDLoad}_k$ , the amount of dispatchable energy and dispatchable load that the ISO clears in the DAM at bus  $k$ .

More precisely,  $\text{IVOffer}_k$  is defined as follows. For each GenCo  $i$ , let  $r_{Gi}^* \geq 0$  (MW) denote the amount of reserve previously procured by the ISO from GenCo  $i$  for potential energy delivery or curtailment in hour H of day D. Suppose, for simplicity, that this reserve is the result of a single reserve option contract. Let  $\theta_i^*$  denote the portion of this reserve that the ISO decides to exercise as up or down reserve prior to hour H to meet balancing needs in hour H, where  $-1 \leq \theta_i^* \leq 1$ . Let  $\text{IVGen}_i$  (MW) denote GenCo  $i$ 's generation obligation for hour H of day D resulting from: (i) energy cleared by the ISO prior to the DAM; (ii) the ISO's exercise  $\theta_i^* r_{Gi}^*$  of the pre-existing reserve  $r_{Gi}^*$ ; and (iii) energy the ISO plans to clear in the subsequent RAP.<sup>17</sup> Then  $\text{IVOffer}_k$  takes the form:

$$\text{IVOffer}_k = \sum_{i \in \mathbf{I}_k} \text{IVGen}_i \quad (16)$$

The ISO-determined virtual demand bid  $\text{IVBid}_k$  in (15) permits the ISO to adjust the MP-forecasted net fixed load (14). A positive  $\text{IVBid}_k$  could indicate a belief by the ISO that the LSEs are underestimating their retail customers' fixed load at bus  $k$  and/or that the GenCos are overestimating their fixed generation at bus  $k$ ; and conversely if  $\text{IVBid}_k$  is negative. Since the  $\text{IVBid}_k$  adjustment can be made at the time of the DAM SCED optimization, after the close of the DAM offer/bid process for MPs, it could give the ISO a chance to improve upon the accuracy of the MPs' forecasts by incorporating additional technical and environmental information.

Our second modification of the DAM SCED optimization is a modification of the operating capacity limits (11) for the dispatchable generation of each GenCo  $i$ . Recall from earlier discussion that the (possibly zero) pre-existing reserve  $r_{Gi}^*$  procured by the ISO from GenCo  $i$  prior to the DAM is assumed to represent a single reserve option contract that can be exercised only once, and that the reserve amount  $\theta_i^* r_{Gi}^*$  resulting from the ISO's planned exercise (if any) of this contract is included in  $\text{IVGen}_i$ . Let  $I(\theta_i^*)$  equal 1 if  $\theta_i^* \neq 0$  (i.e., if exercise is planned) and 0 otherwise.

<sup>17</sup> As will be clarified below, in order to determine  $\text{IVGen}_i$  as part of its DAM decision variables, the ISO has to take into account GenCo  $i$ 's operating capacity limits as well as the comparative costs of exercising the pre-existing reserve  $r_{Gi}^*$ , clearing energy in the DAM, and clearing energy in the subsequent RAP.

The modified DAM operating capacity limits (11) for GenCo  $i$  then take the form

$$\text{Cap}_i^L \leq \text{IVGen}_i + p_{Gi}^D + [1 - I(\theta_i^*)]r_{Gi}^* \leq \text{Cap}_i^U \quad (17)$$

$$\text{Cap}_i^L \leq \text{IVGen}_i + p_{Gi}^D - [1 - I(\theta_i^*)]r_{Gi}^* \leq \text{Cap}_i^U \quad (18)$$

Conditions (17) and (18) ensure that GenCo  $i$ 's operating capacity is sufficient to support its energy obligations as reflected in  $\text{IVGen}_i + p_{Gi}^D$  as well as its potential energy obligations arising from the possible later up/down exercise of  $r_{Gi}^*$  if, at the time of the DAM, the ISO is not planning to exercise this reserve.<sup>18</sup> Note that this modeling assumes, as in practice, that each GenCo  $i$  is required to offer its entire operating capacity into the DAM.

Our third modification of the DAM SCED optimization consists of two augmentations of the objective function (1) that permit linkages to prior and subsequent non-DAM procurement processes. The first augmentation, a contract cost function CC, measures the costs arising from the non-DAM procurement of energy (either directly by energy clearing or indirectly through the exercise of pre-existing reserve) in order to balance the total net load in the DAM; this cost is a function of the ISO-determined virtual supply offers. The second augmentation, a forecast function FC, measures the ISO's anticipated cost of having to procure up/down imbalance energy in the RTM to meet any residual real-time balancing needs; this cost is a function of the ISO-determined virtual demand bids.

More precisely, using previously introduced notation, the total net load in the DAM is given by

$$\text{DAMLoad} = \sum_{k=1}^K (\text{IVBid}_k + \text{MPNetFLoad}_k + \text{PDLload}_k) \quad (19)$$

As seen from (15), the DAMLoad (19) is balanced in the DAM by two means: (a) non-DAM direct/indirect energy procurement, given by the ISO-determined virtual supply offers (16); and (b) DAM energy procurement, given by the DAM-determined variable generation dispatch levels  $\text{PDGen}_k$ ,  $k = 1, \dots, K$ .

The contract cost function CC (\$/h) measures the ISO's anticipated cost of the non-DAM direct/indirect energy procurement (a). This cost consists of two parts: (i) exercise payments arising from any planned exercise by the ISO of pre-existing reserve to ensure the balancing of (19); and (ii) anticipated costs arising from any planned RAP energy procurement by the ISO to ensure the balancing of (19).<sup>19</sup> As illustrated below in (41), the contract cost function can be expressed in the form

$$\text{CC}(\theta^*, \mathbf{IVGen}) = \sum_{k=1}^K \text{CC}_k(\theta^*, \mathbf{IVGen}) \quad (20)$$

where

$$\theta^* = (\theta_1^*, \dots, \theta_I^*) \quad (21)$$

$$\mathbf{IVGen} = (\text{IVGen}_1, \dots, \text{IVGen}_I) \quad (22)$$

<sup>18</sup> An interesting issue here is whether the reserve  $[1 - I(\theta_i^*)]r_{Gi}^*$  belongs in the DAM operating capacity constraints (17) and (18). Including it ensures its availability for possible later exercise; however, the ISO does not anticipate any actual need for this exercise at the time of the DAM. An advantage of the stochastic DAM reformulation proposed below in Section 4 is that this issue can be resolved by introducing DAM operating capacity constraints contingent on scenarios  $s$  for the operating hour  $H$ . Reserve procured by the ISO from a GenCo either prior to or during the DAM would then be included in the GenCo's  $s$ -contingent DAM operating capacity constraint if and only if the ISO plans at the time of the DAM to exercise this reserve for deployment during  $H$  under scenario  $s$ .

<sup>19</sup> Since previously procured energy has already been paid for by the time of the DAM, the cost of this energy procurement is excluded from CC.

are included among the ISO's DAM decision variables.

Let the DAM-forecasted net fixed load at each bus  $k$  be denoted by

$$\text{DNetFLoad}_k = \text{IVBid}_k + \text{MPNetFLoad}_k \quad (23)$$

Note that (23) excludes dispatchable load. Also, let the ISO's own forecast for the net fixed load at each bus  $k$  be denoted by  $\text{ISONetFLoad}_k$ . Finally, let  $\text{FDev}_k(\text{IVBid}_k)$  denote the deviation between these two forecasts as a function of  $\text{IVBid}_k$ :

$$\begin{aligned} \text{FDev}_k(\text{IVBid}_k) &= \text{ISONetFLoad}_k - \text{DNetFLoad}_k \\ &= \text{ISONetFLoad}_k - \text{IVBid}_k - \text{MPNetFLoad}_k \end{aligned} \quad (24)$$

The deviation (24) measures the extent to which the ISO is anticipating a discrepancy during hour  $H$  between actual net fixed load at bus  $k$ , as measured by its own forecast, and DAM-forecasted net fixed load at bus  $k$  as measured by (23).

More precisely, from the vantage point of the ISO in the DAM, the deviation (24) represents the residual up/down load imbalance at bus  $k$  during hour  $H$  that remains after the ISO's DAM decisions. As seen from the DAM power balance equation (15), this imbalance is not covered in the DAM either by the ISO-determined virtual supply offer  $\text{IVOffer}_k$  or by the ISO's dispatch  $\text{PGen}_k$  of variable generation. That is, it is not covered by the ISO's DAM decisions regarding the use of previously secured energy, the exercise of pre-existing reserve, and the procurement of energy in the DAM and the RAP.

The forecast cost function  $\text{FC}$  (\$/h) measures the ISO's anticipated cost, at the time of the DAM, of having to balance the forecasted load deviations (24) by up/down imbalance energy procurement in the RTM. We assume this forecast cost function takes the following form:

$$\text{FC}(\mathbf{IVBid}) = \text{FC}^*(\text{FDev}_1(\text{IVBid}_1), \dots, \text{FDev}_K(\text{IVBid}_K)) \quad (25)$$

where

$$\mathbf{IVBid} = (\text{IVBid}_1, \dots, \text{IVBid}_K) \quad (26)$$

is included among the ISO's DAM decision variables. Note that (25) permits a cost to be assessed for load forecast error either on the basis of separate bus-by-bus load forecast errors or on the basis of the net system-wide load forecast error. Which formulation provides a better cost assessment will depend on anticipated transmission congestion conditions. Examples of both types of formulations are illustrated in subsequent sections.

We then propose that the objective function (1) for the standard DAM SCED be augmented with the contract cost function (20) and the forecast cost function (25), as follows:

$$W(\mathbf{p}_G^D, \mathbf{p}_L^D, \theta^*, \mathbf{IVGen}, \mathbf{IVBid}) = \text{NOC}(\mathbf{p}_G^D, \mathbf{p}_L^D) + \text{CC}(\theta^*, \mathbf{IVGen}) + \text{FC}(\mathbf{IVBid}) \quad (27)$$

The augmented objective function (27) takes into consideration the trade-offs among three types of costs: namely, (i) the net operating cost  $\text{NOC}$  of procuring energy through the DAM to balance the  $\text{DAMLoad}$  (19); (ii) the ISO's anticipated cost  $\text{CC}$  of procuring energy either through pre-DAM energy/reserve contracts or through the RAP in order to balance the  $\text{DAMLoad}$  (19); and (iii) the ISO's anticipated cost  $\text{FC}$  of having to procure up/down imbalance energy in the RTM in order to balance any deviations (24) between ISO-forecasted and DAM-forecasted net fixed loads.

Our fourth and final modification of the DAM SCED optimization is an augmentation of the ISO's decision variables (2). These augmented decision variables are as follows:

$$\mathbf{p}_G^D, \mathbf{p}_L^D, \delta, \theta^*, \mathbf{IVGen}, \mathbf{IVBid} \quad (28)$$

In summary, the ISO selects the decision variables (28) to minimize the objective function (27) subject to the power balance constraints (15), the power flow line limits (10), the GenCo capacity limits (17) and (18), the LSE bid limits (12), and the angle reference bus constraint (13). The energy price  $LMP_k$  (\$/MWh) at each bus  $k$  is determined as the dual variable solution for the bus- $k$  power balance equation (15).

### 3.2 Reformulation of a Co-Optimized Energy/Reserve DAM

The DAM reformulation developed for a day-ahead energy market in Section 3.1 can also be extended to apply to a co-optimized day-ahead energy/reserve market. For concreteness, this section explains how such a reformulation could be implemented for MISO.

As detailed in (MISO 2011), the objective of MISO's co-optimized DAM SCED is to determine optimal cleared amounts of energy and reserve for each hour  $H$  of each day  $D$  using only those resources committed in a prior DAM SCUC optimization. The DAM SCED optimization minimizes net cost (i.e., costs minus benefits) subject to transmission, capacity, and reserve requirement constraints. The costs include operating costs based on GenCo energy supply offers as well as assessments for reserve acquisition based on GenCo and DRR reserve supply offers. The benefits include energy valuations based on LSE energy demand bids as well as valuations for reserve acquisition based on MISO-determined reserve demand functions differing by reserve type (regulating, spinning, and supplemental) and scope (locational versus system-wide). Locational and system-wide reserve requirements are set in accordance with FERC, NERC, and regional reliability entity mandates to ensure the reliability of grid operations.

To see that our DAM reformulation can be applied to a co-optimized day-ahead energy/reserve market such as MISO, consider once again the reformulated day-ahead energy market developed in Section 3.1. As before, we assume this DAM is to be solved on some day  $D-1$  for a particular operating hour  $H$  of day  $D$ .

In a departure from previous assumptions, however, we now suppose that energy and reserve are to be co-optimized in the DAM subject to reserve requirements. For simplicity, we assume these reserve requirements take the form of a minimum locational reserve requirement  $RR_k \geq 0$  (MW) for up/down reserve at each bus  $k$ .

Consequently, one needed modification of our day-ahead energy market reformulation is an expansion of the DAM constraints to include these locational reserve requirements. Let the magnitude of GenCo  $i$ 's DAM-cleared up/down reserve be denoted by  $r_{Gi} \geq 0$  (MW). Using notation introduced in Section 3.1, it follows that the magnitude  $Res_i$  (MW) of the unencumbered reserve available from GenCo  $i$  for possible exercise during hour  $H$  is given by

$$Res_i = [1 - I(\theta_i^*)]r_{Gi}^* + r_{Gi} \quad (29)$$

The locational reserve requirement constraint at each bus  $k$  then takes the form

$$TotRes_k = \sum_{i \in I_k} Res_i \geq RR_k \quad (30)$$

A second needed modification of our day-ahead energy market formulation is an augmentation of the operating capacity limits (17) and (18) for each GenCo  $i$  to include the DAM-cleared reserve  $r_{Gi}$ . These augmented operating capacity limits are as follows:

$$Cap_i^L \leq IVGen_i + p_{Gi}^D + Res_i \leq Cap_i^U \quad (31)$$

$$Cap_i^L \leq IVGen_i + p_{Gi}^D - Res_i \leq Cap_i^U \quad (32)$$

A third needed modification is an augmentation of the reformulated objective function (27) to include a net reserve cost function  $\text{NRC}(\mathbf{r}_G)$  (\$/h) measuring the net cost of the ISO's DAM-procured reserve  $\mathbf{r}_G = (r_{G1}, \dots, r_{GI})$ . This net reserve cost is calculated as reserve costs (determined from GenCo reserve supply offers) net of reserve valuations (determined from ISO reserve demand bids), both expressed as functions of  $\mathbf{r}_G$ . Given this augmentation, the resulting objective function takes the form

$$W^*(\mathbf{p}_G^D, \mathbf{p}_L^D, \mathbf{r}_G, \theta^*, \mathbf{IVGen}, \mathbf{IVBid}) = \text{NOC}(\mathbf{p}_G^D, \mathbf{p}_L^D) + \text{NRC}(\mathbf{r}_G) + \text{CC}(\theta^*, \mathbf{IVGen}) + \text{FC}(\mathbf{IVBid}) \quad (33)$$

Finally, a fourth needed modification is an augmentation of the ISO's DAM decision variables (28) to include  $\mathbf{r}_G$ .

Given these four additional modifications, the resulting DAM SCED reformulation permits the ISO to procure new reserve  $\mathbf{r}_G$  through the DAM. The price  $\text{LMP}_k$  (\$/MWh) for the procurement of energy at each bus  $k$  is determined as the dual variable solution for the bus- $k$  power balance equation (15). The price  $\text{LMP}_k^R$  (\$/MWh) for the procurement of up/down reserve at bus  $k$  is determined as the dual variable solution for the bus- $k$  locational reserve requirement constraint (30).

At the time of the DAM the ISO is not anticipating the exercise of the unencumbered reserve levels (29) for hour H of day D; this is seen from the fact that these reserve levels do not enter into the power balance constraints (15). Rather, these unencumbered reserve levels are procured by the ISO in order to satisfy the locational reserve requirement constraints (30) imposed at each bus  $k$  as insurance against contingencies that could arise during hour H of day D. The possibility of explicitly modeling these contingencies through scenario conditioning is demonstrated below in Section 4.

### 3.3 Illustration of the Reformulated Energy/Reserve DAM

Consider a co-optimized energy/reserve DAM managed by an ISO. As in previous sections, all GenCos participating in this DAM are assumed to be fast-start fast-ramp generating units with no cross-hour restrictions, which reduces each DAM optimization to a sequence of 24 hourly optimization problems.

The discussion below focuses on a DAM SCED optimization held on day D-1 for which all pre-existing, new, and planned energy/reserve contracts are for delivery/exercise for a specific hour H of day D. For concreteness, we continue to assume that the only energy procurement processes available to the ISO subsequent to the DAM for meeting load balancing needs during hour H are a RAP held during the afternoon of day D-1 and a RTM held just prior to hour H on day D.

Suppose the power grid consists of two buses, 1 and 2, connected by a transmission line with a capacity limit  $P_{12}^U$  (MW). Two fast-start fast-ramp GenCos 1 and 2, located at buses 1 and 2 respectively, have operating capacity intervals for dispatchable generation given by  $[\text{Cap}_1^L, \text{Cap}_1^U]$  and  $[\text{Cap}_2^L, \text{Cap}_2^U]$ . The energy supply offer of each GenCo  $i$  consists of a fixed generation level  $p_{Gi}^E$  (MW) for wind and a marginal cost function for dispatchable generation  $p_{Gi}^D$  (MW) given by

$$\text{MC}_i(p_{Gi}^D) = a_i + 2b_i p_{Gi}^D \quad (\$/\text{MWh}) \quad (34)$$

The reserve supply offer of each GenCo  $i$  consists of a marginal cost function for reserve  $r_{Gi}$  (MW) given by

$$\text{RMC}_i(r_{Gi}) = a_i^R + 2b_i^R r_{Gi} \quad (\$/\text{MWh}) \quad (35)$$



The ISO must ensure up/down reserve (MW) in minimum amount  $RR_k \geq 0$  at each bus  $k$  to satisfy NERC reliability standards, where this reserve cannot be encumbered by planned exercise to satisfy future balancing needs. In addition to these locational reserve requirements, the ISO also submits into the DAM a price-sensitive demand bid for reserve at each bus  $k$  assigning a system marginal benefit to each successive MW of DAM-procured reserve  $r$  at bus  $k$ . The ISO reserve demand bid for each bus  $k$  takes the form

$$RMB_k(r) = e_k - 2f_k r \quad (\$/MWh) \quad (36)$$

Two LSEs 1 and 2 are located at buses 1 and 2, respectively. The energy demand bid of each LSE  $j$  consists of a fixed load level  $p_{Lj}^F$  (MW) and a marginal benefit function

$$MB_j(p_{Lj}^D) = c_j - 2d_j p_{Lj}^D \quad (\$/MWh) \quad (37)$$

for dispatchable (price-sensitive) load  $p_{Lj}^D$  (MW) varying over the bid interval

$$0 \leq p_{Lj}^D \leq c_j/2d_j \quad (38)$$

The upper bid limit  $c_j/2d_j$  in (38) is the power level beyond which LSE  $j$ 's marginal benefit from any additional MWs of power is zero.

The net operating cost NOC (\$/h) associated with any DAM-cleared levels for dispatchable generation  $\mathbf{p}_G^D$  and price-sensitive loads  $\mathbf{p}_L^D$  is then measured by

$$NOC(\mathbf{p}_G^D, \mathbf{p}_L^D) = \sum_{i=1}^2 (a_i p_{Gi}^D + b_i [p_{Gi}^D]^2) - \sum_{j=1}^2 (c_j p_{Lj}^D - d_j [p_{Lj}^D]^2) \quad (39)$$

Also, the net reserve cost NRC (\$/h) measuring the net cost of procuring the DAM-cleared reserve levels  $\mathbf{r}_G = (r_{G1}, r_{G2})$  is measured by

$$NRC(\mathbf{r}_G) = \sum_{i=1}^2 (a_i^R r_{Gi} + b_i^R [r_{Gi}]^2) - \sum_{k=1}^2 (e_k r_{Gk} - f_k [r_{Gk}]^2) \quad (40)$$

The constant terms in the NOC and NRC cost functions (39) and (40) are suppressed since they do not affect the DAM optimization.

The ISO-determined virtual supply offers **IVGen**, together with the contract cost function CC measuring the costs of covering these offers, are then specified as follows. Assume there are no energy contracts for hour H that have been cleared prior to the DAM. However, each GenCo  $i$  has sold a reserve option contract  $(i, r_{Gi}^*, \phi_i^*)$  to the ISO in a forward reserve market conducted prior to the DAM that obligates GenCo  $i$  to provide up/down energy during hour H up to a maximum rate  $r_{Gi}^* \geq 0$  at the exercise price  $\phi_i^*$ , if the ISO decides to exercise this reserve option contract prior to the start of hour H. Assume, also, that the ISO anticipates it would be able to clear energy from GenCo  $i$  in the afternoon RAP at a rate not exceeding  $RAP_i^U$  (MW) at a constant marginal cost  $RAPMC_i$  (\$/MWh).

The ISO's DAM decision variable  $IVGen_i$  therefore consists of two parts: (a) the amount  $\theta_i^* r_{Gi}^*$  of the pre-existing reserve  $r_{Gi}^*$  from GenCo  $i$  that the ISO plans to exercise prior to the start of hour H for up/down energy during hour H; and (b) the amount  $[IVGen_i - \theta_i^* r_{Gi}^*]$  of energy from GenCo  $i$  that the ISO plans to clear in the afternoon RAP for delivery during hour H. The contract cost function CC (\$/h) measuring the cost of this non-DAM energy procurement takes the form

$$CC(\theta^*, \mathbf{IVGen}) = \sum_{i=1}^2 \{ \phi_i^* \cdot |\theta_i^* r_{Gi}^*| + RAPMC_i \cdot [IVGen_i - \theta_i^* r_{Gi}^*] \} \quad (41)$$

For any particular selection  $\mathbf{IVBid} = (\text{IVBid}_1, \text{IVBid}_2)$  for the ISO-determined virtual demand bids, the RTM net (system-wide) imbalance energy expected by the ISO takes the form

$$\text{NetIE}(\mathbf{IVBid}) = \sum_{k=1}^2 \text{FDev}_k(\text{IVBid}_k) \quad (42)$$

where

$$\begin{aligned} \text{FDev}_k(\text{IVBid}_k) &= \text{ISONetFLoad}_k - \text{DNetFLoad}_k \\ &= \text{ISONetFLoad}_k - \text{IVBid}_k - [p_{Lk}^F - p_{Gk}^F] \end{aligned} \quad (43)$$

Suppose the forecast cost function FC (\$/h) measuring the ISO's expected cost of procuring (42) in the RTM takes the form

$$\text{FC}(\mathbf{IVBid}) = w_+ \cdot |\text{NetIE}(\mathbf{IVBid})|_+ + w_- \cdot |\text{NetIE}(\mathbf{IVBid})|_- \quad (44)$$

for some nonnegative values  $w_+$  and  $w_-$ , where

$$|x|_+ = x \text{ if } x > 0, \text{ and } 0 \text{ otherwise} \quad (45)$$

$$|x|_- = |x| \text{ if } x < 0, \text{ and } 0 \text{ otherwise} \quad (46)$$

The formulation (44) assumes that up/down imbalance energy needs can be satisfied in the RTM without violating transmission constraints, hence only the net up/down imbalance energy need (42) has to be considered. However, it takes into consideration that a need for up-imbalance energy arising from actual net fixed load exceeding DAM-forecasted net fixed load will require either the calling up of additional generation or some form of load curtailment. In contrast, a need for down-imbalance energy arising from actual net fixed load being less than DAM-forecasted net fixed load will require either a backing down of generation or some form of load increase. These two situations could entail quite different resource costs.

Given the above assumptions, the ISO's reformulated co-optimized energy/reserve DAM optimization on day D-1 for hour H of day D takes the following form:<sup>20</sup>

*Minimize*

$$\begin{aligned} W^*(\mathbf{p}_G^D, \mathbf{p}_L^D, \mathbf{r}_G, \theta^*, \mathbf{IVGen}, \mathbf{IVBid}) &= \text{NOC}(\mathbf{p}_G^D, \mathbf{p}_L^D) + \\ &+ \text{NRC}(\mathbf{r}_G) + \text{CC}(\theta^*, \mathbf{IVGen}) + \text{FC}(\mathbf{IVBid}) \end{aligned} \quad (47)$$

*with respect to the decision variables*

$$p_{Gi}^D, p_{Lj}^D, r_{Gi}, \theta_i^*, \text{IVGen}_i, \text{IVBid}_k, \delta_k, i, j, k = 1, 2 \quad (48)$$

*subject to the admissibility bounds:*

$$0 \leq p_{Gi}^D, 0 \leq p_{Lj}^D, 0 \leq r_{Gi}, -1 \leq \theta_i^* \leq 1, i = 1, 2 \quad (49)$$

*Power balance at each bus  $k = 1, 2$ :*

$$\text{IVGen}_k + p_{Gk}^D = \text{IVBid}_k + [p_{Lk}^F - p_{Gk}^F] + p_{Lk}^D + \text{POut}_k \quad (50)$$

<sup>20</sup> For the simple case at hand with only two buses, and with only one GenCo and one LSE located at each bus, the indices  $i, j, k$  are essentially interchangeable. Note also, from (16), that  $\text{IVOffer}_1$  and  $\text{IVOffer}_2$  reduce to  $\text{IVGen}_1$  and  $\text{IVGen}_2$ . Similarly, it follows from (30) that  $\text{TotRes}_1$  and  $\text{TotRes}_2$  reduce to  $\text{Res}_1$  and  $\text{Res}_2$ .

where

$$\theta_k^* r_{Gk}^* \leq \text{IVGen}_k \leq \text{RAP}_k^U + \theta_k^* r_{Gk}^*, k = 1, 2 \quad (51)$$

$$\text{POut}_1 = P_{12} = S_b B [\delta_1 - \delta_2] \quad (52)$$

$$\text{POut}_2 = -\text{POut}_1 \quad (53)$$

Power flow line limit:

$$|\text{POut}_1| \leq P_{12}^U \quad (54)$$

Locational reserve requirement constraint at each bus  $k = 1, 2$ :

$$\text{Res}_k = [1 - I(\theta_k^*)] r_{Gk}^* + r_{Gk} \geq \text{RR}_k \quad (55)$$

Operating capacity limits for dispatchable generation for each GenCo  $i = 1, 2$ :

$$\text{Cap}_i^L \leq \text{IVGen}_i + p_{Gi}^D + \text{Res}_i \leq \text{Cap}_i^U \quad (56)$$

$$\text{Cap}_i^L \leq \text{IVGen}_i + p_{Gi}^D - \text{Res}_i \leq \text{Cap}_i^U \quad (57)$$

Bid limits for dispatchable load for each LSE  $j = 1, 2$ :

$$0 \leq p_{Lj}^D \leq c_j / 2d_j \quad (58)$$

Voltage angle setting at angle reference bus 1:

$$\delta_1 = 0 \quad (59)$$

A concern that could be raised about the co-optimized energy/reserve DAM reformulation illustrated in this section is that the exercise payments (contingent upon exercise) for the unencumbered reserve  $\text{Res}_k$  at each bus  $k$  are not represented in the objective function (47). This is because the modeling of this DAM reformulation does not include an explicit specification of the contingencies under which this unencumbered reserve would be exercised. The following Section 4 shows that this problem can be resolved by further extending the DAM reformulation to a stochastic reformulation that explicitly specifies possible future scenarios.

## 4 Stochastic DAM Reformulation

### 4.1 General Stochastic DAM Reformulation

Currently, no ISO-managed DAM in the U.S. is implemented as a stochastic optimization problem. However, research on stochastic DAM formulations is intensifying in view of the increased penetration of variable generation and the increased interest in encouraging price-responsive retail demand. See, for example, (Ruiz et al. 2009, Morales et al. 2009; Capi- tanescu et al. 2011; Papavasiliou et al. 2011).

In this section we develop a stochastic extension of the reformulated co-optimized DAM SCED optimization presented in Section 3. The ISO's uncertainty about net fixed loads, represented in Sections 3.2 and 3.3 by means of point forecasts, is instead modeled here by assuming the ISO knows the probability distribution function (PDF) governing net fixed load realizations. The DAM SCED objective function is then represented as an expected value taken with respect to this PDF, and the DAM SCED constraints are suitably modified.

More precisely, in conformity with the scenario-contingent approach taken by Papavasiliou et al. (2011), we assume that the ISO managing a DAM SCED optimization on any day  $D-1$  for some operating hour  $H$  of day  $D$  formulates scenarios  $s$  for net fixed loads, with associated probabilities  $\pi_s$ . The true scenario  $s^*$  is revealed to the ISO at the start of the RAP during the afternoon of day  $D-1$ . Thus, at the time of the DAM, the ISO conditions its DAM reserve exercise decisions, its planned RAP energy clearing, and its RTM net fixed load forecasts on possible scenario realizations  $s$ .

The modifications of the DAM constraints needed to support the ISO's contingency planning are as follows. For each scenario  $s$ , the DAM SCED power balance equation for each bus  $k$  is modified to include an  $s$ -contingent ISO-determined virtual supply offer  $\text{IVOffer}_{k,s}$ . This offer includes: (i) previously cleared non-contingent energy contracts; (ii) previously procured reserve option contracts that the ISO plans to exercise in scenario  $s$ ; and (iii) energy the ISO plans to clear in the subsequent RAP, contingent on scenario  $s$ .

In addition, for each scenario  $s$  the DAM SCED power balance equation for each bus  $k$  is modified to include an  $s$ -contingent ISO-determined virtual demand bid  $\text{IVBid}_{k,s}$ . This bid permits the ISO to make  $s$ -contingent adjustments in the hour- $H$  forecast for net fixed load implied by the non-dispatchable generation offers and fixed demand bids submitted by the DAM MPs.<sup>21</sup>

Finally, to ensure feasible planning, the DAM SCED operating capacity interval for each GenCo  $i$  is modified to constrain its total potential net energy delivery commitments under each scenario  $s$ . These commitments can arise from a mix of cleared and anticipated energy commitments as well as reserve option contracts to be exercised under scenario  $s$ .

Under this stochastic DAM reformulation, the value of the energy deployed at each bus during the operating hour  $H$  is contingent on the realized scenario. Specifically, the up-energy value (down-energy value) at any bus  $k$  in any scenario  $s$  is the incremental change in the value of the optimized DAM objective function at the solution point with respect to an incremental increase (decrease) in the constraint constant for the  $s$ -contingent bus- $k$  power balance equation.

If market rules of operation require the determination of LMPs for energy/reserve procurement and settlement at the time of the DAM, these ex ante LMPs cannot be  $s$ -contingent. However, they should presumably reflect in some way the expected value of the  $s$ -contingent energy valuations. An alternative approach, explored by Morales et al. (2012), is to develop a "single settlement scheme" in which all settlements are resolved ex post. The appropriate determination of settlement prices for stochastic DAMs is a challenging new research area beyond the scope of the current study.

#### 4.2 Illustration of the Stochastic DAM Reformulation

Consider once again the 2-bus example presented in Section 3.3 for the reformulated co-optimized energy/reserve DAM. A stochastic extension of this example will now be developed that permits the ISO to engage in contingency planning.

Suppose the ISO correctly believes at the start of the DAM on day  $D-1$  that the fixed loads and wind generation levels  $(p_{L1}^F, p_{L2}^F, p_{G1}^F, p_{G2}^F)$  at buses 1 and 2 for hour  $H$  of day  $D$  are random variables taking on the values  $(p_{L1,s}^F, p_{L2,s}^F, p_{G1,s}^F, p_{G2,s}^F)$  with probability  $\pi_s$  for each load scenario  $s$  in some discrete scenario set  $\mathcal{S}$ . Suppose also, for simplicity of

<sup>21</sup> Another alternative would be to permit MPs to submit scenario-contingent offers/bids. However, it is difficult to imagine that MPs would be willing to cope with this added complexity.

exposition, that variability in these fixed loads and wind generation levels is the only source of uncertainty regarding system conditions that the ISO explicitly takes into consideration in its contingency planning. Thus, the ISO treats the scenario set  $\mathcal{S}$  as if it were a complete representation for all possible contingencies that could occur.

As will be seen below, this assumption of scenario completeness, commonly made in stochastic DAM studies, has important ramifications for the formulation of the stochastic DAM optimization. Since the ISO is able to predict perfectly the need for reserve in each scenario  $s$ , there is no reason to provide for any reserve buffer. Hence, the only reserve included in a GenCo's  $s$ -contingent operating capacity interval is reserve that the ISO plans to exercise under scenario  $s$ ; there is no need to consider unencumbered reserve. Moreover, there is no need to include non-contingent reserve requirements among the system constraints, either in the form of locational reserve requirements at individual buses or in the form of system-wide reserve requirements.<sup>22</sup>

Finally, suppose the ISO will learn the true scenario  $s^*$  just before the start of the afternoon RAP. Consequently, the ISO in the DAM can condition its reserve exercise decisions, its planned RAP energy procurement, and its RTM net fixed load forecasts on possible scenario realizations.

In particular, as part of its DAM decisions, the ISO can decide for each scenario  $s \in \mathcal{S}$  the amount (if any) of up/down energy it will ensure through the up/down exercise of its pre-existing reserve  $\{r_{G1}^*, r_{G2}^*\}$  at exercise prices  $\phi_1^*$  and  $\phi_2^*$ , where this reserve was procured prior to the DAM from GenCos 1 and 2. Let these exercise decisions be represented by variables  $\{\theta_{1,s}^*, \theta_{2,s}^*\}$ . In addition, the ISO can decide for each  $s \in \mathcal{S}$  the amount (if any) of up/down energy it will ensure through the up/down exercise of its new reserve  $\{r_{G1}, r_{G2}\}$  at exercise prices  $\phi_1$  and  $\phi_2$ , where this reserve is procured in the DAM from GenCos 1 and 2. Let these exercise decisions be represented by variables  $\{\theta_{1,s}, \theta_{2,s}\}$ .

As in Section 3.3, let the net reserve cost function  $\text{NRC}(\mathbf{r}_G)$  measuring the ISO's net cost of procuring the reserve  $\mathbf{r}_G$  in the DAM be given by (40). The reserve exercise cost function  $\text{REC}(\mathbf{r}_G, \theta)$  measuring the ISO's anticipated cost arising from the  $s$ -contingent exercise of  $\mathbf{r}_G$  takes the form

$$\text{REC}(\mathbf{r}_G, \theta) = \sum_{s \in \mathcal{S}} \pi_s \cdot \text{REC}_s(\mathbf{r}_G, \theta_s) \quad (60)$$

where

$$\theta = (\theta_s)_{s \in \mathcal{S}} = (\theta_{1,s}, \theta_{2,s})_{s \in \mathcal{S}} \quad (61)$$

$$\text{REC}_s(\mathbf{r}_G, \theta_s) = \sum_{i=1}^2 \phi_i \cdot |\theta_{i,s} r_{Gi}| \quad (62)$$

The contract cost function  $\text{CC}(\theta^*, \mathbf{IVGen})$  measuring the costs anticipated by the ISO from its  $s$ -contingent exercise of pre-existing reserve and its  $s$ -contingent plans for RAP energy procurement takes the form

$$\text{CC}(\theta^*, \mathbf{IVGen}) = \sum_{s \in \mathcal{S}} \pi_s \cdot \text{CC}_s(\theta_s^*, \mathbf{IVGen}_s) \quad (63)$$

<sup>22</sup> If, more realistically, the ISO were to believe the scenario set  $\mathcal{S}$  to be only a rough approximation for possible scenarios, then presumably the ISO would want to impose some form of non-contingent reserve requirement constraints to ensure system reliability. It would then need to account for these non-contingent reserve requirements in the GenCo capacity constraints.

where

$$\theta^* = (\theta_s^*)_{s \in \mathcal{S}} = (\theta_{1,s}^*, \theta_{2,s}^*)_{s \in \mathcal{S}} \quad (64)$$

$$\mathbf{IVGen} = (\mathbf{IVGen}_s)_{s \in \mathcal{S}} = (\mathbf{IVGen}_{1,s}, \mathbf{IVGen}_{2,s})_{s \in \mathcal{S}} \quad (65)$$

and

$$\text{CC}_s(\theta_s^*, \mathbf{IVGen}_s) = \sum_{i=1}^2 \{ \phi_i^* \cdot |\theta_{i,s}^* r_{Gi}^*| + \text{RAPMC}_i \cdot [\mathbf{IVGen}_{i,s} - \theta_{i,s}^* r_{Gi}^*] \} \quad (66)$$

Also, the ISO can decide for each scenario  $s \in \mathcal{S}$  how much (if any) to adjust the DAM MPs' non-contingent net fixed load forecasts  $\text{MPNetFLoad}_1 = p_{L1}^F - p_{G1}^F$  and  $\text{MPNetFLoad}_2 = p_{L2}^F - p_{G2}^F$  towards its own correct  $s$ -contingent forecasts, given by  $\text{ISONetFLoad}_{1,s} = p_{L1,s}^F - p_{G1,s}^F$  and  $\text{ISONetFLoad}_{2,s} = p_{L2,s}^F - p_{G2,s}^F$ . These adjustments are determined by the ISO's choice of

$$\mathbf{IVBid} = (\mathbf{IVBid}_s)_{s \in \mathcal{S}} = (\mathbf{IVBid}_{1,s}, \mathbf{IVBid}_{2,s})_{s \in \mathcal{S}} \quad (67)$$

For example, at one extreme the ISO could act now, in the DAM, to ensure that sufficient energy will be available in each scenario  $s$  to cover actual net fixed load at each bus  $k$ . It can do this by setting

$$\mathbf{IVBid}_{k,s} = \text{ISONetFLoad}_{k,s} - \text{MPNetFLoad}_k \quad (68)$$

for each  $k = 1, 2$  and  $s \in \mathcal{S}$ . At the other extreme, the ISO could set  $\mathbf{IVBid}_{k,s} = 0$  for each  $k = 1, 2$  and  $s \in \mathcal{S}$ . In the latter case the ISO is making the decision to rely fully on the procurement of up/down imbalance energy in the RTM to cover any deviation that arises between actual and MP-forecasted net fixed load during hour H.

The ISO's actual choice of its DAM decision variables  $\mathbf{IVBid}$  will depend on the expected cost of procuring up/down imbalance energy in the RTM. This expected cost is assumed to be measured by the forecast cost function

$$\text{FC}(\mathbf{IVBid}) = \sum_{s \in \mathcal{S}} \pi_s \cdot \left\{ \sum_{k=1}^K \text{FC}_{k,s}(\mathbf{IVBid}_{k,s}) \right\} \quad (69)$$

where

$$\text{FC}_{k,s}(\mathbf{IVBid}_{k,s}) = w_{+,k,s} \cdot |\text{FDev}_{k,s}(\mathbf{IVBid}_{k,s})|_+ + w_{-,k,s} \cdot |\text{FDev}_{k,s}(\mathbf{IVBid}_{k,s})|_- \quad (70)$$

and

$$\begin{aligned} \text{FDev}_{k,s}(\mathbf{IVBid}_{k,s}) &= \text{ISONetFLoad}_{k,s} - \text{DNetFLoad}_{k,s} \\ &= [p_{Lk,s}^F - p_{Gk,s}^F] - \mathbf{IVBid}_{k,s} - [p_{Lk}^F - p_{Gk}^F] \end{aligned} \quad (71)$$

The terms  $w_{+,k,s}$  and  $w_{-,k,s}$  in (70) are the ISO's expectations for the RTM prices of up/down imbalance energy at bus  $k$  in scenario  $s$ . If the ISO anticipates there will be no RTM transmission grid congestion in scenario  $s$ , the expected RTM price  $w_{+,k,s}$  for up-imbalance energy should be the same at each bus  $k$ , and similarly for the RTM price  $w_{-,k,s}$  for down-imbalance energy. Otherwise, however, these expected prices could be bus dependent.

Given the above assumptions and function specifications, the ISO's stochastic co-optimized energy/reserve DAM SCED optimization on day D-1 for hour H of day D takes the following form:

Minimize

$$W^{**}(\mathbf{p}_G^D, \mathbf{p}_L^D, \mathbf{r}_G, \theta, \theta^*, \mathbf{IVGen}, \mathbf{IVBid}) = \text{NOC}(\mathbf{p}_G^D, \mathbf{p}_L^D) + \text{NRC}(\mathbf{r}_G) + \text{REC}(\mathbf{r}_G, \theta) + \text{CC}(\theta^*, \mathbf{IVGen}) + \text{FC}(\mathbf{IVBid}) \quad (72)$$

with respect to the decision variables:

$$p_{Gi}^D, p_{Lj}^D, r_{Gi}, \theta_{i,s}, \theta_{i,s}^*, \text{IVGen}_{i,s}, \text{IVBid}_{k,s}, \delta_{k,s} \quad (73)$$

for  $i, j, k = 1, 2$  and  $s \in \mathcal{S}$ , subject to the admissibility bounds:

$$0 \leq p_{Gi}^D, 0 \leq p_{Lj}^D, 0 \leq r_{Gi}, i, j = 1, 2 \quad (74)$$

$$-1 \leq \theta_{i,s}^*, \theta_{i,s} \leq 1, i = 1, 2, s \in \mathcal{S} \quad (75)$$

Power balance at each bus  $k = 1, 2$  for each  $s \in \mathcal{S}$ :

$$\text{IVGen}_{k,s} + p_{Gk}^D + \theta_{k,s} r_{Gk} = \text{IVBid}_{k,s} + [p_{Lk}^F - p_{Gk}^F] + \text{POut}_{k,s} \quad (76)$$

where

$$\theta_{k,s}^* r_{Gk} \leq \text{IVGen}_{k,s} \leq \text{RAP}_k^U + \theta_{k,s}^* r_{Gk} \quad (77)$$

$$\text{POut}_{1,s} = P_{12,s} = S_b B[\delta_{1,s} - \delta_{2,s}] \quad (78)$$

$$\text{POut}_{2,s} = -\text{POut}_{1,s} \quad (79)$$

Power flow line limit for each  $s \in \mathcal{S}$ :

$$|\text{POut}_{1,s}| \leq P_{12}^U \quad (80)$$

Operating capacity limits for dispatchable generation for each GenCo  $i = 1, 2$  and for each  $s \in \mathcal{S}$ :

$$\text{Cap}_i^L \leq \text{IVGen}_{i,s} + p_{Gi}^D + \theta_{i,s} r_{Gi} \leq \text{Cap}_i^U \quad (81)$$

Bid limits for dispatchable load for each LSE  $j = 1, 2$ :

$$0 \leq p_{Lj}^D \leq c_j/2d_j \quad (82)$$

Voltage angle setting at angle reference bus 1 for each  $s \in \mathcal{S}$ :

$$\delta_{1,s} = 0 \quad (83)$$

## 5 General DAM Reformulation Issues

This study recommends that ISOs be permitted to submit virtual supply offers and virtual demand bids into DAMs as a way to provide linkages among past, current, and future energy/reserve procurement. In effect, we are proposing that the slack variables often inserted into DAM power balance equations as purely computational devices instead be given economic meaning as ISO-determined virtual offers/bids.

One issue of possible concern is that the introduction of ISO-determined virtual offers/bids could result in the ISO taking a financial position in the DAM. However, this is not the case. Current ISO-managed DAMs in the U.S. are organized as double auctions in which all payments for cleared energy/reserve demands are paid to the ISO and all payments for cleared energy/reserve sales are paid by the ISO. However, to retain its nonprofit status as

mandated by FERC, the ISO must allocate back to the MPs any net gains or losses incurred through these settlements. This same policy is retained for our proposed DAM reformulation.

A second issue of possible concern is that permitting the ISO to submit virtual offers/bids into a DAM could affect the resulting DAM commitment, dispatch, and pricing solutions, as well as the amount of ISO net surplus.<sup>23</sup> Although this is possible, we would argue that this should not be viewed with concern.

Ideally, for the determination of energy and reserve for each operating hour  $H$  of each operating day  $D$ , the ISO should be solving a sequence of continually updated intertemporal optimization problems spanning all the way from years in advance to intrahour. Since this is not computationally practical, the ISO must instead resort to approximate optimization processes. The choice of these approximate optimization processes will naturally affect commitment, dispatch, and pricing solutions, whatever form these approximations take; but the goal should be to obtain as close an approximation as possible to the underlying intertemporal optimization problem.

Relative to existing DAM practices, the introduction of linkages in the form of ISO-determined virtual offers/bids should result in a better approximation to this underlying intertemporal optimization problem because it permits the DAM to be more coherently connected to prior and subsequent procurement processes. In particular, the availability of these linkages should result in an increase in the average total net benefit<sup>24</sup> realized through DAM, RAP, and RTM optimizations. Conversely, since the ISO can obtain standard DAM outcomes by setting these linkages to zero, there should be no degradation in system performance.

A third issue of possible concern is whether there is truly any need to have ISO virtual demand bidding in the DAM to adjust the net fixed load forecasts of MPs. The short answer is that there might not be a need for such adjustments at the current time, with largely conventional loads; but the need could dramatically increase in the future with a higher penetration of renewables.

More precisely, ISO-determined virtual demand bidding simply provides a mechanism to the ISO that could potentially be used to adjust the net fixed load forecasts of DAM MPs. Such adjustments might be justifiable if historical data reveal that these MP forecasts are less accurate than the net fixed load forecasts of the ISO, to the detriment of system reliability or system efficiency.

No claim is made in this study that this is currently the case. However, it is important to keep in mind that net fixed load, as defined in this study, is not simply conventional load; it includes non-dispatchable variable generation (wind, solar) treated as negative fixed load. As the penetration of wind and solar power continues to increase, the forecasting of net fixed load will presumably become more difficult. If an ISO has access to more comprehensive and/or more up-to-date information than DAM MPs regarding the conditions affecting wind and solar generation, then the ISO might be in a better position to forecast net fixed loads.

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<sup>23</sup> As detailed in Li and Tesfatsion (2011), ISO net surplus (“congestion rent”) in a DAM is the difference between the payments made to the ISO by DAM MPs for energy purchases minus the payments made by the ISO to DAM MPs for energy/reserve sales. Under standard day-ahead energy market designs, ISO net surplus is positive in the presence of grid congestion, due to LMP separation across the buses of the grid, and zero in the absence of grid congestion. To retain its nonprofit status, mandated by FERC, an ISO must allocate any nonzero ISO net surplus back to the MPs in some form. We retain this policy for our reformulated DAM.

<sup>24</sup> The total net benefit for any ISO-managed procurement process is the sum of the net benefits received by process participants. This includes both immediate net benefits received through process settlements and subsequent net benefits received through later distributions of ISO net surplus.



Consequently, at some point it might become desirable for an ISO, with a fiduciary responsibility to maintain system reliability and efficiency, to adjust the net fixed load forecasts of DAM MPs.

Another important point to consider is that DAM MPs can and do hedge their price risks by entering into a variety of private financial contracts. These contracts often take the form of bilateral contracts-for-difference in which each counterparty agrees to make whole the other counterparty to ensure an agreed upon “strike price” for a desired energy trade subsequent to the DAM clearing and settlement of this energy trade at DAM LMPs. If the counterparties are located at different grid pricing locations (hence subject to different DAM LMPs), these contracts-for-difference can be supported by suitable purchases of financial transmission rights (FTRs) to effectively place each counterparty at the same bus subject to the same DAM LMP. The DAM reformulation proposed in this study envisions that MPs will continue to hedge their price risks through these and other forms of private financial contracting. However, the ability of the ISO to improve upon the accuracy of the DAM net fixed load forecasts could help to reduce price volatility and hence the need for price-risk hedging by MPs.

On the other hand, if regulators and market operators are wary about inserting ISO forecasts into market processes, another possibility would be to only allow an ISO to submit virtual demand bids into DAM SCUC optimization processes for the improved determination of unit commitments. MP demand bids could then be fully relied on in DAM SCED optimization processes for the determination of DAM dispatch and price levels.

Interestingly, as discussed in Ellison et al. (2012) and Chao et al. (2005, Section II.B), MPs in current U.S. ISO-managed energy regions are not permitted to submit energy demand bids (load forecasts) into the RTM or into reliability assessment processes (RAPs). Rather, the ISO schedules further supply-side resources in these RAP/RTM processes to meet the load it predicts rather than the load predicted by MPs. The rationale for this reliance on ISO-determined load forecasts is that, as operating points near, system reliability must take precedence over market considerations; and the ISO is in a better position to ensure this system reliability if it makes use of its own load forecasts based on up-to-date technical and environmental information.

A fourth issue of possible concern is whether a RAP held subsequent to a DAM is a sufficient vehicle for handling changed system conditions between the DAM and the RAP, without any need to include DAM linkages that explicitly take the subsequent RAP into account. We would argue that it is not efficient to decouple DAM decisions from RAP decisions in this manner.

As detailed in Ellison et al. (2012), RAPs for any given hour  $H$  on an operating day  $D$  typically take place between the close of the DAM offer/bid process on day  $D-1$  up to the start of hour  $H$ . Thus, by the time of a RAP, flexibility can be very limited. Adequate reserve must already be available for possible contingent dispatch in hour  $H$  to ensure system reliability. Slower-start generators must already have been committed for hour  $H$ . Moreover, by strong custom if not by rule, ISOs are extremely reluctant to decommit generators in a RAP that have earlier been committed in the DAM.

In short, as a practical matter, by the time of a RAP the bulk of the needed energy/reserve procurement must already have been secured in earlier processes. By focusing on a reformulation of the DAM, we are able to address the orderly procurement of energy and reserve over a longer time horizon, potentially lessening the need to adjust commitments and dispatch levels in subsequent RAPs.

A fifth issue of interest is the extent to which the DAM reformulation proposed in this study would require substantial changes in current DAM optimization procedures. As ex-

plained in Section 3, the key modification required by our proposed DAM reformulation is the inclusion of slack variables (ISO-determined virtual offers/bids) in the DAM power balance equations together with corresponding penalty terms in the DAM objective function. As seen in FERC (2011), slack variables are commonly included in DAM power balance equations, with corresponding penalty terms then inserted into the DAM objective function to enable the approximate satisfaction of the power balance equations through successive reductions in the magnitudes of the slack variables. Consequently, in terms of analytical modeling, the reformulation proposed in this study for non-stochastic DAMs does not represent a substantial departure from current practice.<sup>25</sup>

On the other hand, in our proposed DAM reformulation the slack variables and penalty terms are economically meaningful concepts. The penalty term included in the DAM objective function for the ISO-determined virtual supply offers measures the cost of procuring energy through non-DAM processes to help in balancing the DAM-Load (19). The penalty term inserted into the DAM objective function for the ISO-determined virtual demand bids measures the ISO's anticipated cost of having to procure imbalance energy in the RTM to eliminate discrepancies (if any) between DAM-forecasted net fixed loads and "actual" net fixed loads as measured by the ISO's own net fixed load forecasts. Consequently, in our DAM reformulation the optimal determination of the slack terms to be inserted into the power balance equations depends on a careful consideration of the risks and costs associated with DAM versus non-DAM energy/reserve procurement.

## 6 Concluding Remarks

The key idea motivating the DAM reformulation proposed in this study is that a DAM is a forward market whose outcomes are financial contracts; it is not a physical spot market. More precisely, a DAM is a day-ahead financial planning process situated within a cascade of financial planning processes that span all the way from years to seconds in advance of each operating instant.

If an ISO under-schedules generation in today's DAM for some next-day operating hour, there is no immediate *physical* harm since the scheduled outcomes of the DAM are financially binding but not physically binding. Moreover, there is no longer-term physical harm if the ISO can make up the difference by procuring additional energy (or energy curtailment) in the reliability assessment processes undertaken between the close of the DAM and the start of the operating hour, or in the real-time processes conducted during the operating hour.

Conversely, if the ISO over-schedules generation in today's DAM for some next-day operating hour, then unit commitments might be inefficient in the sense that the generation committed for this operating hour is excessive relative to actual need. However, the reliability of the power system is not compromised.

Consequently, to economize on overall energy/reserve procurement costs, we recommend that the DAM power balance equations not be enforced as hard constraints that must be met entirely through energy secured through the DAM. Rather, ISOs charged with the fiduciary responsibility of maintaining reliable and efficient system operations should be permitted to augment the DAM power balance equations with ISO-determined virtual supply offers that permit load-balancing resources to be secured through an efficient mix of pre-DAM, DAM, and post-DAM procurement.

<sup>25</sup> As previously noted, implementation of a stochastic DAM would represent a considerable departure from current DAM practices, whatever the details of this implementation.

In addition, we recommend that ISOs be permitted to augment the DAM power balance equations with ISO-determined virtual demand bids to adjust the net fixed load forecasts of market participants in situations where errors in these participant forecasts pose serious problems for system reliability or system efficiency. Recalling that net fixed load includes non-dispatchable variable generation treated as negative load, such situations could arise in the future as the increased penetration of wind and solar power requires ever more sophisticated system-wide forecasting techniques.

In future work the performance of our proposed DAM reformulation will be carefully investigated by means of systematic simulation studies.

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**Table 1** Exogenous Variables and Functions in the Day D–1 DAM Optimization for Hour H of Day D

Variable/Function	Description
$B_{km}$	$[1/X_{km}]$ (pu) for line $km \in BR$
$\text{Benefit}_j(\cdot)$	LSE $j$ 's benefit function (\$/h) for dispatchable load
$\text{Bid}_j^L, \text{Bid}_j^U$	LSE $j$ 's dispatchable-load bid limits (MW), $0 \leq \text{Bid}_j^L \leq \text{Bid}_j^U$
$BR$	Connected set of distinct symmetric three-phase lines $km$
$\text{Cap}_i^L, \text{Cap}_i^U$	GenCo $i$ 's dispatchable-generation capacity limits (MW), $0 \leq \text{Cap}_i^L \leq \text{Cap}_i^U$
$\text{CC}(\cdot)$	ISO contract cost function (\$/h) for non-DAM energy to meet DAM load
$\text{Cost}_i(\cdot)$	GenCo $i$ 's operating cost function (\$/h) for dispatchable generation
$\text{FC}(\cdot)$	ISO forecast cost function (\$/h) for RTM up/down imbalance energy
$\text{FC}_s(\cdot)$	$s$ -Contingent ISO forecast cost fct (\$/h) for RTM up/down imbalance energy
$I$	Total number of DAM SCUC-committed GenCos
$I_k$	Subset of DAM SCUC-committed GenCos located at bus $k$ , $\text{Card}(\cup_{k=1}^K I_k) = I$
$\text{ISONetFLoad}_k$	DAM ISO forecast (MW) for RTM net fixed load at bus $k$
$\text{ISONetFLoad}_{k,s}$	$s$ -Contingent DAM ISO forecast (MW) for RTM net fixed load at bus $k$
$J$	Total number of LSEs
$J_k$	Subset of LSEs located at bus $k$ , $\text{Card}(\cup_{k=1}^K J_k) = J$
$K$	Total number of buses
$km$	Line connecting buses $k$ and $m$
$\text{MB}_j(\cdot)$	LSE $j$ 's marginal benefit (demand) function (\$/MWh), $\text{MB}_j = d\text{Benefit}_j/dp_{L_j}^D$ , $\text{MB}_j \geq 0$ , $\text{MB}'_j \leq 0$
$\text{MC}_i(\cdot)$	GenCo $i$ 's marginal cost (supply) function, (\$/MWh), $\text{MC}_i = d\text{Cost}_i/dp_{G_i}^D$ , $\text{MC}_i \geq 0$ , $\text{MC}'_i \geq 0$
$\text{MPNetFLoad}_k$	Net fixed load (MW) at bus $k$ implied by DAM market participant offers and bids
$\text{NOC}(\cdot)$	Net operating cost fct (\$/h) measuring energy operating cost net of energy benefit
$\text{NRC}(\cdot)$	Net reserve cost function (\$/h) measuring reserve cost net of reserve benefit
$p_{G_i}^F$	Fixed (non-dispatchable) generation (MW) offered by GenCo $i$
$p_{G_i,s}^F$	$s$ -Contingent ISO forecast for GenCo $i$ 's fixed (non-dispatchable) generation (MW)
$p_{L_j}^F$	Fixed (non-dispatchable) load (MW) bid by LSE $j$
$p_{L_j,s}^F$	$s$ -Contingent ISO forecast for LSE $j$ 's fixed (non-dispatchable) load (MW)
$p_{km}^U$	Thermal limit (MW) for power flow on line $km \in BR$
$\text{PFGen}_k$	Total fixed generation (MW) offered by GenCos at bus $k$
$\text{PFLoad}_k$	Total fixed load (MW) bid by LSEs at bus $k$
$r_{G_i}^*$	Magnitude (MW) of GenCo $i$ 's pre-existing up/down reserve commitment
$\text{RAPMC}_i$	GenCo $i$ 's marginal cost (\$/MWh) in the Reliability Assessment Process (RAP)
$\text{RAP}_i^U$	GenCo $i$ 's upper generation capacity limit (MW) in the RAP
$\text{REC}(\cdot)$	ISO expected cost function (\$/h) for exercise of DAM-procured up/down reserve
$\text{RMB}_k(\cdot)$	ISO marginal benefit (demand) function (\$/MWh) for reserve at bus $k$ , $\text{RMB}_k \geq 0$ , $\text{RMB}'_k \leq 0$
$\text{RMC}_i(\cdot)$	GenCo $i$ 's marginal cost (supply) function (\$/MWh) for reserve, $\text{RMC}_i \geq 0$ , $\text{RMC}'_i \geq 0$
$\text{RR}_k$	Locational reserve requirement (MW) for up/down reserve at bus $k$ , $\text{RR}_k \geq 0$
$\mathcal{S}$	Discrete set of scenarios $s$ for the stochastic DAM optimization
$S_b$	Base power (in three-phase MVA), $S_b > 0$
$X_{km}$	Positive-sequence per-phase reactance (pu) of line $km$ , $X_{km} = X_{mk} > 0$
$W(\cdot)$	Objective function (\$/h) for reformulated energy-only DAM
$W^*(\cdot)$	Objective function (\$/h) for reformulated energy/reserve DAM
$W^{**}(\cdot)$	Objective function (\$/h) for reformulated stochastic energy/reserve DAM
$w_+$	ISO expected price (\$/MWh) for RTM net (system-wide) up-imbalance energy
$w_-$	ISO expected price (\$/MWh) for RTM net (system-wide) down-imbalance energy
$w_{+,k,s}$	$s$ -Contingent ISO expected price (\$/MWh) for RTM up-imbalance energy at bus $k$
$w_{-,k,s}$	$s$ -Contingent ISO expected price (\$/MWh) for RTM down-imbalance energy at bus $k$
$\delta_1$	Voltage angle (radians) at angle reference bus 1, $\delta_1 = 0$
$\pi_s$	Probability that scenario $s$ will be realized, $\pi_s \geq 0$ , $\sum_s \pi_s = 1$
$\phi_i^*, \phi_i$	Exercise prices (\$/MWh) for pre-existing and DAM-cleared reserve from GenCo $i$

**Table 2** Endogenous Variables Determined by the Day D–1 DAM Optimization for Hour H of Day D

Variable	Description
DAMLoad	Total net load (MW) to be balanced in the DAM, including dispatchable load
DNetFLoad <sub>k</sub>	Total net fixed load (MW) at bus $k$
DNetFLoad <sub>k,s</sub>	Total net fixed load (MW) at bus $k$ in scenario $s$
FDev <sub>k</sub>	Deviation (MW) between ISONetFLoad <sub>k</sub> and DNetFLoad <sub>k</sub>
FDev <sub>k,s</sub>	Deviation (MW) between ISONetFLoad <sub>k,s</sub> and DNetFLoad <sub>k,s</sub>
IVBid <sub>k</sub>	ISO's virtual demand bid (MW) at bus $k$
IVBid <sub>k,s</sub>	ISO's virtual demand bid (MW) at bus $k$ in scenario $s$
IVGen <sub>i</sub>	Generation obligation (MW) of GenCo $i$ resulting from pre-DAM and RAP energy/reserve contracts
IVGen <sub>i,s</sub>	Generation obligation (MW) of GenCo $i$ in scenario $s$ resulting from pre-DAM and RAP energy/reserve contracts
IVOffer <sub>k</sub>	ISO's virtual supply offer (MW) at bus $k$ ( $\text{IVOffer}_k = \sum_{i \in I_k} \text{IVGen}_i$ )
IVOffer <sub>k,s</sub>	ISO's virtual supply offer (MW) at bus $k$ in scenario $s$ ( $\text{IVOffer}_{k,s} = \sum_{i \in I_k} \text{IVGen}_{i,s}$ )
LMP <sub>k</sub>	Locational marginal price (\$/MWh) of energy at bus $k$
LMP <sub>k</sub> <sup>R</sup>	Locational marginal price (\$/MWh) of up/down reserve at bus $k$
NetIE	ISO's anticipated need for net up/down imbalance energy (MW) in the RTM
$P_{km}$	Power (MW) flowing in line $km \in \text{BR}$
$P_{km,s}$	$s$ -Contingent power (MW) flowing in line $km \in \text{BR}$
$p_{Gi}^D$	GenCo $i$ 's cleared dispatchable generation (MW)
$p_{Lj}^D$	LSE $j$ 's cleared dispatchable load (MW)
PDGen <sub>k</sub>	Total dispatchable generation (MW) cleared at bus $k$
PDLoad <sub>k</sub>	Total dispatchable load (MW) cleared at bus $k$
POut <sub>k</sub>	Net power outflow (MW) at bus $k$
POut <sub>k,s</sub>	$s$ -Contingent net power outflow (MW) at bus $k$
$r_{Gi}$	Magnitude (MW) of GenCo $i$ 's DAM-cleared up/down reserve
Res <sub>i</sub>	Unencumbered up/down reserve (MW) available from GenCo $i$ for hour-H exercise
TotRes <sub>k</sub>	Total unencumbered up/down reserve (MW) available at bus $k$ for hour-H exercise
$\delta_k$	Voltage angle (radians) at bus $k \neq 1$
$\tilde{\delta}_{k,s}$	$s$ -Contingent voltage angle (radians) at bus $k \neq 1$
$\theta_i^*$	Up/down portion of GenCo $i$ 's pre-existing reserve $r_{Gi}^*$ that the ISO plans to exercise for hour H
$\theta_{i,s}^*$	Up/down portion of GenCo $i$ 's pre-existing reserve $r_{Gi}^*$ that the ISO plans to exercise for hour H in scenario $s$
$\theta_{i,s}$	Up/down portion of GenCo $i$ 's DAM-cleared reserve $r_{Gi}$ that the ISO plans to exercise for hour H in scenario $s$