Uniaxial three-dimensional shape measurement with projector defocusing

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Abstract
Our recent study showed that the phase error caused by improperly defocused binary structured patterns has a unique relationship with the depth z. Based on this finding, the depth information can be extracted without the need of triangulation. Because the measurement can be performed from the same viewing angle, this uniaxial measurement technique can overcome some limitations of a triangulation-based technique, such as measuring a deep hole. This paper will present the principle of the proposed technique and show some simulation and preliminary experimental results to verify its viability.

Keywords
Uniaxial, binary, three-dimensional, fringe analysis, phase error

Disciplines
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Comments
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Uniaxial 3-D shape measurement with projector defocusing

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ABSTRACT

Our recent study showed that the phase error caused by improperly defocused binary structured patterns has a unique relationship with the depth \( z \). Based on this finding, the depth information can be extracted without the need of triangulation. Because the measurement can be performed from the same viewing angle, this uniaxial measurement technique can overcome some limitations of a triangulation-based technique, such as measuring a deep hole. This paper will present the principle of the proposed technique and show some simulation and preliminary experimental results to verify its viability.

Keywords: Uniaxial; binary defocusing; three-dimensional; fringe analysis; phase error.

1. INTRODUCTION

Due to their speed and flexibility, 3-D shape measurement based on digital sinusoidal fringe projection techniques have been playing an increasingly important role in optical metrology, and have been applied to solve for numerous areas. However, almost all these techniques require to form a triangle for depth recovery. In other words, there must be a certain angle between the projection line and the camera imaging line in order to obtain the depth information for that point. However, for any triangulation-based 3-D shape measurement techniques, the occlusion of the system will be a problem. That is, the depth information of a point cannot be recovered if only one of the two devices (camera or projector) can see it. Therefore, it is very difficult for a triangulation based method to measure a small and deep hole, nor the occlusion of the camera or shadow of the projector.

In contrast, if the depth information can be obtained without triangulation, the limitations of the a triangulation-based method can be significantly alleviated. The technique that do not require the projector and the camera to form a triangle is entitled uniaxial 3-D shape measurement. Otani et al. proposed one of the uniaxial methods based on a fringe analysis technique. This technique analyzes a set of phase-shifted fringe patterns to obtain the depth in formation based on the data modulation (or fringe contrast) changes due to defocusing. This is a great technology to measure a uniform object. However, one of the potential limitations is that its measurement accuracy will be reduced if the object surface has nonuniform reflectivity. Recently, Birch et al. proposed a method to alleviate this problem by measuring the same surface another time with a different amount of defocusing. However, this technique slows down the measurement speed since it measure the object twice, and it is practically difficult to calibrate such a system since it is usually not easy to precisely control the amount of defocusing.

We propose a new uniaxial 3-D shape measurement technique to alleviate the aforementioned problems through a phase analysis. This new technique is based on the characteristics of the binary defocusing technique that we proposed recently. On the study the phase error caused by improperly defocused binary structured patterns, we found that the phase error can be described as a function of wrapped phase, \( \phi(x, y) \), and the depth, \( z \). This finding provides an opportunity to determine the depth from the phase error, which inspires the development of this novel 3-D shape measurement technique. Since it is not necessary to form a triangle to determine the phase error caused by defocusing, this technique can be also be used for uniaxial 3-D shape measurement.

Section 2 explains the principle of the proposed technique. Section 3 shows some simulation results. Preliminary experimental results are shown in Sec. 4 to verify the feasibility of the proposed technique. Section 5 discusses the merits and possible limitations of the proposed technique, and finally Sec. 6 summarizes this paper.

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2. PRINCIPLE

2.1 Three-step phase-shifting technique

Phase-shifting methods are widely used in optical metrology because of their high measurement speed and high achievable accuracy.6 A three-step phase-shifting algorithm with a phase shift of $2\pi/3$ can be described as:

$$I_n(x,y) = I'(x,y) + I''(x,y)\cos[\phi + 2(n-2)\pi/3].$$

(1)

Where $n = 1, 2, 3$, $I'(x,y)$ is the average intensity, $I''(x,y)$ the intensity modulation, and $\phi(x,y)$ the phase to be solved for. From these three equations, the phase can be calculated by

$$\phi(x,y) = \tan^{-1}\left[\sqrt{3}(I_1 - I_3)/(2I_2 - I_1 - I_3)\right].$$

(2)

This equation provides the wrapped phase ranging from $-\pi$ to $+\pi$ with $2\pi$ discontinuities.

2.2 Binary defocusing technique

Our recent study showed that by properly defocusing a binary structured pattern, a pseudo-sinusoidal one can be generated,4 which is similar to the Ronchi grating defocusing method proposed by Su et al.7 However, it is difficult for a Ronchi grating method to generate precise phase shift due to the requirement of mechanical adjustment. Therefore, the measurement error could be dominated by the phase shift error. In contrast, the digital fringe projection technique does not have the phase shift error because of its digital fringe generation nature. Therefore, for a fringe projection system using the binary defocusing technique, the single dominant error source is the nonsinusoidal structure of the defocused binary patterns. Our previous study has already showed that good quality sinusoidal fringe patterns can be generated by properly defocusing binary structured patterns. However, if the projector is not properly defocused, systematic phase error will be introduced, which will be addressed next.

2.3 Phase error determination

If the fringe patterns contains nonsinusoidal waveform, the phase error will show up in the recovered phase. For the technique based on defocusing, if the projector is not properly defocused, there are still binary structures on the resultant fringe patterns, and the phase error is significant. The phase error is a noise that needs to be reduced for a conventional 3-D shape measurement system. To alleviate the noise, we found that the phase error can be described as a function of wrapped phase, $\phi(x,y)$, and the depth distance, $z$.5 This finding provides an opportunity to determine the depth from the phase error, which inspires the development of this new 3-D shape measurement technique. Interestingly, the phase error becomes signal for this proposed technique.

It is important to notice that it is very difficult to obtain the phase error directly from three phase-shifted fringe images if the object is not uniform and flat, or the camera and the projector have ideal lenses. To circumvent this problem, three ideal sinusoidal fringe patterns with exactly the same fringe pitch, number of pixels per fringe period, are utilized. The phase error is determined by finding the difference between the phase obtained from the binary defocused pattern and that obtained from the ideal sinusoidal patterns. In other words, we use six fringe patterns to determine the phase error. Three phase-shifted binary patterns gives the phase $\phi^b(x,y)$ with errors, and three phase-shifted ideal sinusoidal fringe patterns provides the phase $\phi^s(x,y)$ with negligible errors. The phase error is then computed point by point by

$$\Delta\phi(x,y) = \phi^b(x,y) - \phi^s(x,y) \mod 2\pi.$$  

(3)

It is important to note that the phase error is calculated from wrapped phase with $2\pi$ discontinuities. This requires the $2\pi$ discontinuous positions occur at the same locations for the binary and sinusoidal patterns. However, practically, there may be a such-pixel phase shift due to camera sampling, the $2\pi$ modulus operation is to resolve this problem.

Figure 1 shows an example of the defocusing effect on the binary structured patterns. The first row images of Fig. 1 show some typical results when the projector is defocused to different degrees while the camera is in focus. This figure show that as the projector becomes increasingly defocused, the binary structured pattern becomes increasingly distorted. Fig. 1(a) shows the result when the projector is in focus: clear binary structures of the image. As the degree of defocusing increases, the binary structures become less and less clear, and the sinusoidal ones become more and more apparent, as shown in Fig. 1(e). The phase errors can be calculated by comparing with the ideal sinusoidal ones at the same degree of defocusing. Figures 1(f)-1(j) show 240th column of phase error map. It clearly shows that when the projector is nearly focused, the phase error is very large; and with the degree of defocusing increases, the phase error magnitude reduces, and its structures diminish and eventually become more random.
Fig. 1. Example of sinusoidal fringe generation by defocusing a binary structured pattern. (a) shows the result when the projector is in focus; (b)–(e) show the result when the projector is increasingly defocused. (f)–(j) show the 240th column cross sections of the phase error when the projector is increasingly defocused.

3. SIMULATIONS

To illustrate how the different amounts of defocusing affect the phase error, a simulation was carried. To simplify the analysis, only one cross-section of the fringe pattern is used. A Gaussian filter is utilized to smooth the curve to emulate the defocusing effect of a projection lens, and the degree of defocusing is realized by applying different number times of the filter, which is similar to use different breadths of a single Gaussian filter. Figure 2(a) plots the phase error against the wrapped phase. For this simulation, the phase error fringe pitch used is 256 pixels, and the Gaussian filter used is 61 pixels in size with and standard deviation of 10.1 pixels. In this figure, levels 1-5 are realized by applying the Gaussian filter 1-5 times, respectively. It is interesting to find that the zero points of the error function is evenly distributed within $-\pi$ and $+\pi$ region. The error patterns look similarly whilst having different amplitude. This figure also indicates the amplitude of the error increases with the increase of degree of defocusing. This figure also indicates that the peak and valley points of the phase error occur at approximately $\phi(x,y) = \pm(2k+1)\pi/12$, where $k = 0, 1, \ldots, \pm 5$.

Fig. 2. Simulation result for the phase error with different amounts of defocusing. (a) Phase errors as a function of wrapped phase; (b) Number of Gaussian filtering as a function of peak phase error shown in (a).

Figure 2(b) shows a plot the relates the degree of defocusing with the magnitude of the phase errors at those peak and
valley points. It clearly shows that the magnitude can be described as a function of the degree of defocusing. Because the degree of defocusing has a unique relationship with the depth distance \( z \) of the object stays, the depth information can then be retrieved from the degree of defocusing. In order words, the depth \( z \) of the object points at those peak and valley phase errors can be written as functions of the phase error,

\[
z = f(\Delta \phi(x,y)).
\]

This error function can be determined by calibration. Once this function is known, within each period of fringe patterns, 12 points can be measured.

4. EXPERIMENTAL RESULTS

The proposed method was tested under the a system illustrated in Fig. 3. The system includes a DLP projector (Model: Samsung SP-P310MEMX) and a digital CCD camera (Model: Jai Pulnix TM-6740CL). The camera is attached with a 16 mm focal length Mega-pixel lens (Model: Computar M1614-MP) at F/1.4 to 16C. The resolution of the camera is 640 × 480, with a maximum frame rate of 200 frames/sec. The pixel size of the camera sensor is 7.4 × 7.4 μm². The projector has a resolution of 800 × 600 with a projection distance of 0.49-2.80 m. Figure 3 shows the projector sits below the camera and project horizontal stripes to make sure that the optical axis of the projector and that of the camera are approximately in parallel. It is important to note that it is practically impossible to configure the commercial video projector with the camera as a parallax system, and the current setup was found to be adequate to verify the proposed technique.

A linear translation stage was used to provide the desired motion backward and forward for calibration. In this research, we used the TECHSPEC Metric long travel linear translation stage. This stage is 250 mm long with a traveling accuracy of 0.1 mm. An uniform white flat object is mounted on top of the translation stage and travels with the stage for this study.

![Fig. 3. System setup.](image)

To calibrate the phase error function in terms of the wrapped phase \( (\phi(x,y)) \) and the depth \( (z) \), we set up the system in a manner so that the projected image is focused at a plane, and the camera is also focused at the same plane. This plane was chosen as \( z = 0 \), we then move the plane towards the system with an increment of \( \Delta z = 5 \) mm. For each plane, we recorded three phase-shifted binary patterns, and three phased-shifted sinusoidal patterns with exactly the same fringe pitch, and computed the phase error using Eq. (3). To reduce the random noise caused by the camera and the projector, each pattern was an average of 15 patterns. Once the phase error is calculated, a 1024-element error look-up-table (LUT) was created by evenly quantize the wrapped phase within \([-\phi, +\phi] \) with a 2π/1024 rad phase interval. Within each interval, the phase error is determined by averaging all points fall within that interval.

For example, Figures 4(a)-4(b) shows three phase-shifted binary patterns, from which the wrapped phase was obtained as shown in Fig. 4(d). At each position, three sinusoidal fringe patterns with the same fringe pitch were also captured as...
shown in Figs. 4(e)-4(g), and the wrapped phase (shown in Fig. 4(h)) was calculated as the reference to determine the phase error point by point. Figure 4(i) shows the 240th column cross section of the phase error map. It clearly shows periodical error structure whilst embracing randomness. The phase error is dependent of the camera sampling pixel position. In contrast, if we plot the phase error map as a function of wrapped phase, as shown in Fig. 4(j), the 6X frequency error structure becomes very apparent, and is independent of sampling pixel position. From the phase error obtained as a function of wrapped phase, the LUT can be created. Figure 4(k) shows the 1024-element LUT created for this calibration plane.

Our further experiments confirmed that the peak and valley points of the phase error function for an arbitrary plane occur at approximately 
\[ \phi(x, y) = \pm (2k + 1) \pi / 12, \quad (k = 0, 1, \ldots, \pm 5) \].
For each of these extreme points, the depth \( z \) can be approximated as a polynomial function of the phase error \( \Delta \phi(x, y) \),
\[ z_i = f(\Delta \phi; \phi_i) \]  
(5)
Here \( i = 1, 2, \ldots, 12 \). In our experiments, we used 28 planes to fit the polynomial functions. We found that 3rd-order polynomials are sufficient to represent these functions. Figure 5(a) shows these calibrated 8 LUT’s, which clearly show that the error structure looks similar to that obtained in simulation. Furthermore, Figure 5(b) shows the 562th element (one of the peaks) in each LUT along the depth \( z \). This figure indicates that the peak phase error changes monotonically as a function of depth \( z \). However, the trend of this function of depth \( z \) is slightly different from that simulated function. This might be caused by (1) the \( z \) change for the simulation is not linear whilst the experimental stage moves with an fixed increment (5 mm); and (2) the phase error changes for the simulation are only induced by the defocusing of the projector (or the pattern), whilst those for real experiments are also influenced by the camera defocusing. Nevertheless, both functions are monotonic, and thus the inverse function can be determined.

Once the system is calibrated, we measured a step-height object with a known depth of approximately 53 mm. Figure 6(a) shows one of the binary fringe patterns. There is a large area without any fringe stripes, which was the shadow of the projector due to the use of a non-collimated projection lens system. Figure 6(b) shows 240th column cross section of the phase error. It clearly shows the magnitude of the phase error has two distinct levels that represent the top and bottom surface of the object.

From the phase error, the 3D shape can be determined for those peak and value points. Figure 7(a) shows the 3D result which was smoothed by a \( 7 \times 7 \) Gaussian filter. The plot was generated by remeshing the sparse measurement points for
Fig. 5. Experimental result for the phase error with different amounts of defocusing. (a) Phase errors as a function of wrapped phase; (b) Peak phase error shown in (a) as a function of depth $z$.

Fig. 6. Experimental result of a step-height object measurement (a) One of the binary fringe patterns; (b) One column cross section of phase error map.

In the sake of visualization, Figure 7(b) shows one of the cross sections of the 3D result. This preliminary data shows that the height of the 3D object can be recovered from the phase error. However, even after applying a Gaussian smoothing filter, the measurement accuracy is not high due to the sensitivity to the noise of the camera/projector.

5. DISCUSSIONS

By analyzing the phase error, the depth information can be retrieved from the same view as the projection. This uniaxial 3D shape measurement technique has the following merits:

- **Deep hole measurement.** This is an advantage of uniaxial 3D shape measurement technique since it does not require to form a triangle to obtain depth.

- **High sensitivity.** Its depth sensitivity could be very high depending on the projection lens system used. If the depth of focus for the projector is very small, the degree of defocusing will change rapidly with depth changes, and the phase error will change proportionally. Therefore, this measurement technique could be very sensitive to depth changes.
Fig. 7. 3D shape reconstruction of the step-height object shown in Fig. 6. (a) The 3D plot; (b) One column cross section of the 3D shape.

- **Less sensitivity to surface reflectivity variation.** Unlike other uniaxial method that retrieves depth by analyzing data modulation, this technique obtain depth from the phase which is naturally less sensitive to surface reflectivity variations.

- **No phase unwrapping.** Since the phase error is obtained by taking the phase difference obtained from the binary defocused patterns and the sinusoidal patterns, it does not make any difference whether the error is obtained from the wrapped phase or the unwrapped phase. Therefore, a single-wavelength phase-shifting algorithm can be used to measure arbitrary step-height object.

However, despite its many advantages, it has the following limitations:

- **Lower spatial resolution.** The current technique can only measure 12 points per period of fringe patterns, which is relatively low comparing other uniaxial methods when the camera-pixel spatial resolution can be achieved.

- **Slower measurement speed.** This technique requires three phase-shifted fringe patterns and three binary patterns to obtain one 3D shape, its measurement speed is lower than a standard triangulation based method that only requires three fringe patterns.

- **Better hardware requirement.** This technique requires precisely generated sinusoidal fringe patterns as a reference to obtain phase errors thus subsequently 3D shape. Any error in the sinusoidal fringe patterns will be coupled into the final measurement. In addition, even for those peak points of the phase error, their actual values are relatively small, making them more sensitive to noise than a conventional 3D shape measurement system. Therefore, better hardware (camera and projector) is needed to improve the measurement quality.

6. CONCLUSIONS

This paper has presented a new technique for uniaxial displacement measurement based the analyzing the phase error caused by projector defocusing. Principle of this proposed technique has been explained, and both simulation and preliminary experimental results have verified the feasibility of the proposed approach. In the future, we plan to explore methodologies to improve the spatial resolution, and to reduce the noise effect of this proposed uniaxial 3D shape measurement technique.

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