

6-2011

Joint Estimation of NDE Inspection Capability and Flawsizes Distribution for in-service Aircraft Inspections

Ming Li

Iowa State University

William Q. Meeker

Iowa State University, wqmeeker@iastate.edu

Peter Hovey

University of Dayton

Follow this and additional works at: https://lib.dr.iastate.edu/stat_las_preprints

 Part of the [Statistics and Probability Commons](#)

Recommended Citation

Li, Ming; Meeker, William Q.; and Hovey, Peter, "Joint Estimation of NDE Inspection Capability and Flawsizes Distribution for in-service Aircraft Inspections" (2011). *Statistics Preprints*. 76.

https://lib.dr.iastate.edu/stat_las_preprints/76

This Article is brought to you for free and open access by the Statistics at Iowa State University Digital Repository. It has been accepted for inclusion in Statistics Preprints by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.

Joint Estimation of NDE Inspection Capability and Flawsize Distribution for in-service Aircraft Inspections

Abstract

Nondestructive evaluation (NDE) is widely used in the aerospace industry during scheduled maintenance inspections to detect cracks or other anomalies in structural and rotating components. Life prediction and inspection interval decisions in aerospace applications require knowledge of the size distribution of unknown existing cracks and the probability of detecting a crack (POD), as a function of crack characteristics (e.g., crack length). The POD for a particular inspection method is usually estimated on the basis of laboratory experiments on a given specimen set. These experiments, however, cannot duplicate the conditions of in-service inspections. Quantifying the size distribution of unknown existing cracks is more difficult. If NDE signal strength is recorded at all inspections and if crack-length information is obtained after “crack find” inspections, it is possible to estimate the joint distribution of crack length, noise response and signal response. This joint distribution can then be used to estimate both the in service POD and the crack-length distribution at a given period of service time. In this paper, we present a statistical model and methodology to do this estimation.

Keywords

Center for Nondestructive Evaluation, Bayesian, bivariate normal distribution, noise interference model, random effects, repeated measures

Disciplines

Statistics and Probability

Comments

This preprint was published as Ming Li, William Q. Meeker & Peter Hovey, " Joint Estimation of NDE Inspection Capability and Flaw-Size Distribution for In-Service Aircraft Inspections" *Research in Nondestructive Evaluation*" (2012): 104-123, doi: [10.1080/09349847.2011.654380](https://doi.org/10.1080/09349847.2011.654380).

Joint Estimation of NDE Inspection Capability and Flaw-size Distribution for in-service Aircraft Inspections

Ming Li and William Q. Meeker

Department of Statistics and Center for Nondestructive Evaluation,

Iowa State University, Ames, IA 50011

Peter Hovey

Department of Mathematics, University of Dayton, Dayton, OH 45469

Abstract:

Nondestructive evaluation (NDE) is widely used in the aerospace industry during scheduled maintenance inspections to detect cracks or other anomalies in structural and rotating components. Life prediction and inspection interval decisions in aerospace applications require knowledge of the size distribution of unknown existing cracks and the probability of detecting a crack (POD), as a function of crack characteristics (e.g., crack length). The POD for a particular inspection method is usually estimated on the basis of laboratory experiments on a given specimen set. These experiments, however, cannot duplicate the conditions of in-service inspections. Quantifying the size distribution of unknown existing cracks is more difficult. If NDE signal strength is recorded at all inspections and if crack-length information is obtained after “crack find” inspections, it is possible to estimate the joint distribution of crack length, noise response and signal response. This joint distribution can then be used to estimate both the in service

POD and the crack-length distribution at a given period of service time. In this paper, we present a statistical model and methodology to do this estimation.

Key Words: Bayesian, Bivariate normal distribution, Longitudinal study, Missing information, Noise interference model, Nondestructive evaluation, Random effects, Repeated measures.

1 INTRODUCTION

1.1 Background

Nondestructive evaluation (NDE) methods that use non-intrusive physical measurements are widely used in aerospace applications to detect flaws or cracks inside structures or parts. Depending on the situation (e.g., a designed laboratory study versus a field study that is based on actual inspection data) and the particular structure of the data that are collected, different statistical models and methods are needed to analyze the NDE data. MIL-HDBK-1823A (2009) describes the standard statistical methods that are used in laboratory studies.

Carefully designed laboratory experiments are expensive, but provide flexibility to study the effect of particular experimental factors. Laboratory experiments are usually based on artificial cracks or other flaws in test specimens (e.g., Li, Meeker and Thompson 2010). The measurement response is modeled as a function of crack length and this model is used to estimate the probability of detection (POD). The laboratory studies are usually for validation and quantification of inspection capability for new NDE methods. After a detection method is developed, tested in the laboratory, and put into use, there is often a desire or a need to do a field study to assess actual field performance and to monitor the inspection process over time to assure that it is being done effectively.

Regularly-scheduled in-service nondestructive inspections look for cracks in aircraft components such as engine fan blades and lap-splice rivet holes. Such inspections are, for example, an integral part of the FAA Aging Aircraft program. The purpose of these inspections is to determine whether there is a crack in a part and if a crack is detected there is usually a need to determine the approximate size of the crack. For a particular inspection method, there is a detection threshold, often based on previous field inspections, laboratory experience, model-based theory, and operator experience. For parts with a signal response above the detection threshold, a crack detection decision is made and that part is either repaired

or removed from future service. The crack-length information could be obtained during repairing or other post detection procedures.

In most applications when an NDE measurement is taken in a place where we know there are no cracks, the reading can still be some value to quantify background and measurement noise. When there are very small flaws, small signals close to the noise level will be obtained from the measurements. Based only on the measurements, we cannot be sure that such measurements were from the crack or some artifact of the part or the test setup that would cause noise. We use a noise-interference model to describe the relationship between signal and noise.

1.2 Motivation

This work was motivated by the need to use information from in-service inspections of lap-splice rivet holes used on aircraft bodies. If a crack signal exceeds a crack-detection threshold, then crack-length information is obtained during the repair process. Although measurements are taken on all holes during each scheduled inspections before the “crack find” inspection, currently there are no data recorded for these measurements. Modern inspection and communications technology will, however, make it possible to record these measurements with little effort, allowing better estimation of POD and crack-length distributions. For small cracks, the measurement response could be signal from the crack or the noise artifact. We use a noise-interference model to describe the rivet-hole measurement data by assuming that the response is the maximum of the signal and the noise.

The estimation methods proposed here require keeping the repeated measurement response records for all holes with a growing crack. There is one crack-length reading for holes at the “crack find” inspection but no crack-length reading for holes that have not had an above-threshold measurement response. There are no standard statistical methods to analyze such inspection data. In this paper we develop a statistical method to jointly estimate the measurement (i.e., maximum of the signal response and the noise response) and crack length based on assumed knowledge of the crack growth model. The

joint estimation increases the power of the statistical analysis and improves the overall reliability assessment. Although the research in this paper was motivated by the rivet-hole inspection applications, the methods presented here should have broad applicability into other areas of NDE inspections.

1.3 Related Literature

Olin and Meeker (1996) and Spencer (1996) provided an overview of statistical methods for NDE techniques. MIL-HDBK-1823A (2009) described the standard statistical procedures for NDE data analyses and Annis (2009) provided an R package to implement these procedures through maximum likelihood method. Li and Meeker (2009) introduced the noise interference model to extract the signal response from the NDE measurement. Hovey, Meeker and Li (2009) discussed a similar crack growth NDE problem with one fixed crack growth rate. There are a number of books that have discussion about statistical methods for repeated measurement data (e.g. Davidian and Giltinan 1995). Johnson and Wichern (2001) summarize properties of the multivariate normal distribution that is used in our joint estimation model.

1.4 Overview

The rest of this paper is organized as follows. Section 2 describes the inspection procedures and the data structure for scheduled aircraft maintenance inspection. Section 3 presents the standard statistical methods used in NDE and the concept of POD. Section 4 describes the crack growth model and the measurement response model. Section 5 describes the statistical model for the simulated field data. Section 6 describes the Bayesian estimation of the parameters and functions of the parameters of the statistical model. Section 7 contains some concluding remarks and extensions for future research. A summary of the bivariate normal distribution properties used in this paper is presented at the Appendix.

2 IN-SERVICE INSPECTION OF AIRCRAFT LAP-SPLICE RIVET

HOLES

The current procedures for aircraft maintenance require measuring every hole with an eddy current inspection method at each scheduled inspection, but only find or no-find information is recorded. As a result, currently, there are no available field study data sets based on our proposed data recording scheme. Therefore we use a simulated data set for rivet holes used in lap-splices on aircraft bodies to illustrate our proposed inspection procedures and to present the joint estimation statistical methodology. The parameters used in the simulation are based on previous experience with eddy current NDE inspections for rivet holes, as described in Hovey, Meeker, and Li (2009) and Li, Nakagawa, Larson, and Meeker (2010).

The proposed inspection procedures are outlined as follows.

- First, one measurement is taken for each rivet hole at each scheduled inspection. For any rivet holes with measurement below the detection threshold, we will assume the crack is small enough that the rivet hole can, without risk, be continued in service without repair. Thus there is no direct crack-length information for those rivet holes with measurement below the detection threshold.
- Second, at any scheduled inspections, if a rivet hole has a measurement above the detection threshold, the rivet hole is repaired and the crack-length information in unit of inches is obtained during the repair procedure.

This paper considers only the time to a first detectable crack at each hole.

The eddy current measurement from each hole could come from the signal response of a crack or the noise artifact response (e.g., an innocuous scratch). In eddy current inspection, the log signal response is usually described adequately with a linear relationship with the log crack length. The noise response can be described by a log normal distribution and is independent of crack length. For small cracks, the signal responses are usually smaller than noise response (i.e., below the noise floor of the eddy current

inspection output). We therefore model eddy current measurement responses by using the noise interference model (i.e., the maximum of the signal response and noise response). The simulated results from the proposed inspection procedures are illustrated in Figure 1. Figure 1 (top) shows the full data set of signal response (open circles) and noise response (crosses) for each scheduled inspection with service time in thousand hours. Figure 1 (bottom left) shows the relationship between the measurement result (i.e. the maximum of signal response and noise response) and the crack length for the full data set. Figure 1 (bottom right) shows the structure of the actual data that would be observed in real applications based on the proposed inspection procedures. The preset detection threshold ($y_{th} = \log_{10}(1000)$) and noise mean ($\mu_{noise} = \log_{10}(316)$) are also indicated in Figure 1 by horizontal dashed lines and dotted lines respectively. An estimate of the probability of false alarm for this data set is 0.028 (i.e., the proportion of crosses that are above the detection threshold in the top of Figure 1 is 0.028).

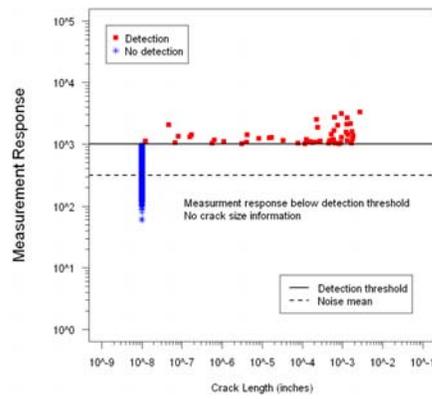
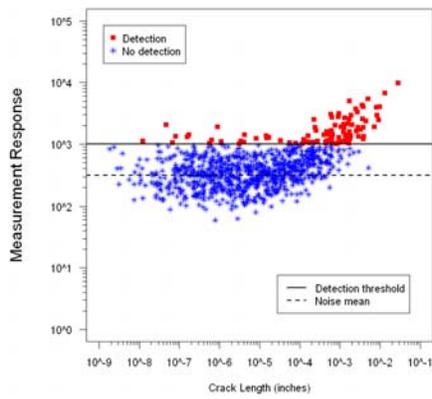
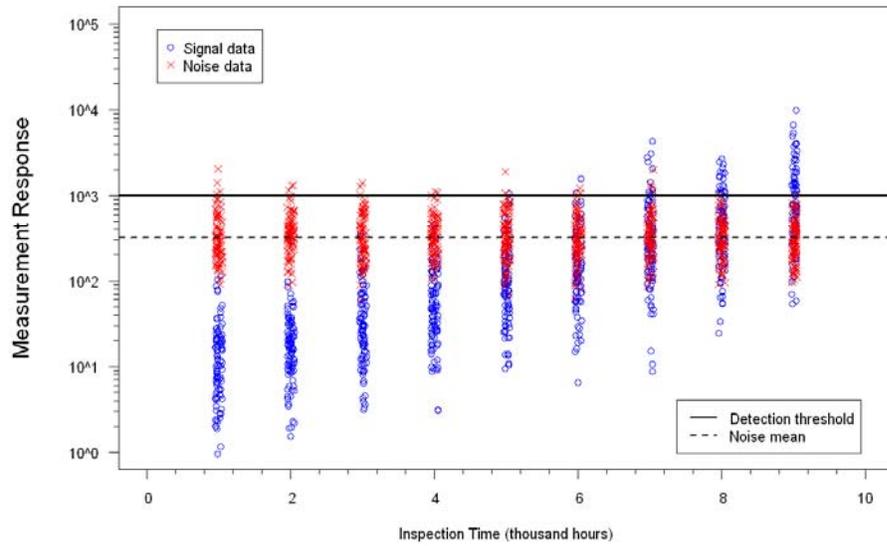


Figure 1. Simulated aircraft rivet hole field data: the full data set of signal and noise response as function of inspection time (top), the full data set of measurement results as function of crack length (bottom left) and the actual observed data structure for proposed field inspection procedures (bottom right).

3 STANDARD STATISTICAL METHODS IN NONDESTRUCTIVE EVALUATION

In this section, we outline the standard statistical procedures in NDE for accessing inspection capability in applications, as described in MIL-HDBK-1823A (2009). There are two types of response in NDE applications: hit and miss binary responses and continuous responses such as voltage. Because the measurements from the rivet-hole field data are continuous, we focus on the statistical model for a continuous response. In subsequent sections, we will present extensions to these existing methods that will allow joint estimation of inspection capability and a flaw size distribution by using data coming from regularly scheduled in-service inspections.

3.1 Statistical Models for NDE

We use Y to denote the NDE measurement response (or its transformation) and x to denote the crack length (or its transformation). Then the statistical model is $Y = \beta_0 + \beta_1 x + \varepsilon$ where β_0 and β_1 are the regression parameters and ε is the measurement error with a normal distribution $N(0, \sigma_\varepsilon^2)$. With the measurement data (possibly censored or truncated), estimates of the parameter vector $(\hat{\beta}_0, \hat{\beta}_1, \hat{\sigma}_\varepsilon^2)$ and the estimated variance covariance matrix of these estimates can be obtained by using standard maximum likelihood (ML) methods described, for example, in Pawitan (2001). MIL-HDBK-1823A (2009) discussed this model and Annis (2009) provided an R package based on the ML method with censored observations. It is common to use a normal distribution to describe the variability in ε , although it is possible to use alternative appropriate distributions when needed.

3.2 Detection Threshold

For rivet holes with cracks that are very small (even newly drilled holes can be considered to have micro-cracks of size on the order of grain boundary sizes), there are still measurement responses due to background noise and other measurement variations. We use Y_{noise} to denote the resulting log noise response which we assume to have a normal distribution of $Y_{noise} \sim N(\mu_{noise}, \sigma_{noise}^2)$. NDE data taken on new rivet holes provide noise data from which ML estimates $(\hat{\mu}_{noise}, \hat{\sigma}_{noise}^2)$ of the noise parameters can be obtained. The detection threshold (y_{th}) is typically set to provide an acceptably small probability (p_f) of false alarm (e.g., $p_f = 0.01$ or 0.05). In particular, the detection threshold can be chosen such that $\Pr(Y_{noise} > y_{th}) = p_f$. Specifically, the detection threshold is chosen as $y_{th} = \hat{\mu}_{noise} + \hat{\sigma}_{noise} \Phi^{-1}(1 - p_f)$ where $\Phi^{-1}(x)$ is the standard normal distribution quantile function.

3.3 Probability of Detection

With a specified detection threshold, POD as a function of crack length can be obtained as follows:

$$\text{POD}(x) = \Pr(Y > y_{th}) = \Phi\left(\frac{\beta_0 + \beta_1 x - y_{th}}{\sigma_y}\right) \quad (1)$$

where $\Phi(x)$ is the standard normal cumulative distribution function. Confidence bounds for the POD can be obtained by using the delta method or by using the likelihood directly (e.g., Meeker and Escobar 1998, Appendix B) where the estimated variance and covariance matrix of the model parameters is needed as an input.

4 CRACK-LENGTH AND MEASUREMENT-RESPONSE MODELS

In this paper, we focus on the fatigue crack growth in aircraft lap-splices rivet holes. The crack growth models can be used to compute reliability properties (see, for example, Chapter 13 of Meeker and Escobar 1998). Crack growth models can be very complicated and are usually function of geometry, materials properties, and usage environmental variables. Many of the more sophisticated models are developed for particular applications and are proprietary. We use a simple fatigue crack growth model that assumes exponential growth over time. This simple model could be extended to more complicated crack growth models when needed.

We denote the crack length by a . We assume that cracks have a random initial size and grow deterministically with rates that are random from aircraft to aircraft. This is a standard model, used in fatigue-fracture aerospace applications.

4.1 Initial Crack Length Distribution

We assume that there is a crack at each rivet hole location at time t_0 . Those cracks are generally very small and the log crack length for rivet hole i in aircraft j is $x_{ij}(t_0) = \log_{10}(a_{ij}(t_0))$ with $i = 1, \dots, I$ and $j = 1, \dots, J$. In our model, the log initial crack length follows a normal distribution $N(\mu_x, \sigma_x^2)$.

4.2 Crack Growth Model

In our model, the size of the crack at rivet hole location i in aircraft j at inspection time t is denoted by $a_{ij}(t) = a_{ij}(t_0) \exp(\lambda_j(t - t_0))$. To take into account the different crack growth rates from aircraft-to-aircraft, a random crack growth model is needed. We assume the log crack growth rates $\log_{10}(\lambda_j)$ for $j = 1, \dots, J$ follow a normal distribution $N(\mu_\lambda, \sigma_\lambda^2)$, although it is possible to use an alternative

appropriate distribution when needed. The inspection time t is the same for all rivet holes in the same aircraft at each scheduled inspection (with index $k = 1, \dots, K$). For rivet holes in different aircraft at the same scheduled inspection, the actual service time may be different because of inspection-scheduling variability. Thus we identify the service time $t = t_{jk}$ by index (j, k) . The log crack length

$x_{ij}(t_{jk}) = \log_{10}(a_{ij}(t_{jk}))$ at time t_{jk} is $x_{ij}(t_{jk}) = x_{ij}(t_0) + \lambda_j(t_{jk} - t_0)$ where λ_j is the crack growth rate for aircraft j .

4.3 Eddy Current Response Model

4.3.1 Signal Response

In our model, the log signal response (open circles in top of Figure 1) for the rivet hole at location i in aircraft j at scheduled inspection k is $Y_{ij}(t_{jk}) = \beta_0 + \beta_1 x_{ij}(t_{jk}) + \varepsilon_{ijk}$ with $\varepsilon_{ijk} \sim N(0, \sigma_y^2)$. Here we assume that the signal response errors ε_{ijk} are independently and identically distributed. Finally, recalling that the log initial crack length follows a normal distribution $N(\mu_x, \sigma_x^2)$ and using the bivariate normal distribution results (A-1) and (A-2) in the Appendix, the crack-size/NDE signal process can be modeled through a random vector $(Y(t_{jk}), X(t_{jk}))^T$ with a bivariate normal distribution:

$$\begin{pmatrix} Y(t_{jk}) \\ X(t_{jk}) \end{pmatrix} \sim \text{BVN} \left(\begin{pmatrix} \beta_0 + \beta_1 (\mu_x + \lambda_j (t_{jk} - t_0)) \\ \mu_x + \lambda_j (t_{jk} - t_0) \end{pmatrix}, \begin{pmatrix} \sigma_y^2 + \beta_1^2 \sigma_x^2 & \beta_1 \sigma_x^2 \\ \beta_1 \sigma_x^2 & \sigma_x^2 \end{pmatrix} \right). \quad (2)$$

4.3.2 Noise Response

The log noise response for rivet-hole inspections at any inspection time can be described by a normal distribution

$$Y_{\text{noise}}(t_{jk}) \sim N(\mu_{\text{noise}}, \sigma_{\text{noise}}^2) \quad (3)$$

with mean μ_{noise} and variance σ_{noise}^2 . The log noise response is independent of crack length and service time. The proportion of noise data above the detection threshold is 0.028 (i.e. the PFA for our simulated data is 0.028).

4.3.3 Noise Interference Model

The actual eddy current NDE response is the maximum of the signal response and the noise response.

That is,

$$Y_{\text{actual}}(t_{jk}) = \max(Y(t_{jk}), Y_{\text{noise}}(t_{jk})). \quad (4)$$

Li and Meeker (2009) and Li, Nakagawa, Larson, and Meeker (2010) used the noise interference model to describe NDE measurement responses in other applications.

4.4 Simulation Parameters

Based on available expert knowledge and previous experience with the crack growth in lap-splice rivet holes in aircraft bodies (e.g., Li, Nakagawa, Larson, and Meeker 2010), our simulation parameters were chosen as follows:

- Initial log crack length distribution: $N(\mu_x, \sigma_x^2)$ with $\mu_x = -7.39$ and $\sigma_x^2 = 0.51$.
- Linear model for the log measurement response: $y_{ij}(t_{jk}) = \beta_0 + \beta_1 x_{ij}(t_{jk}) + \varepsilon_{ijk}$ and $\varepsilon_{ijk} \sim N(0, \sigma_y^2)$ with $\beta_0 = 4.50$, $\beta_1 = 0.50$ and $\sigma_y^2 = 0.065$.
- Log random crack growth rate distribution: $N(\mu_\lambda, \sigma_\lambda^2)$ with $\mu_\lambda = -0.35$ and $\sigma_\lambda^2 = 4.0 \times 10^{-4}$.
- Log noise response distribution: $N(\mu_{\text{noise}}, \sigma_{\text{noise}}^2)$ with $\mu_{\text{noise}} = 2.50$ and $\sigma_{\text{noise}}^2 = 0.064$.

We assume there are $I = 5$ particular rivet holes under study in each aircraft (i.e., group) and $J = 20$ aircraft in the fleet (actual numbers could be expected to be much larger, but we use these smaller numbers for our illustrative example so that we can present informative plots using all of our data). All rivet holes from the same aircraft have the same crack growth rate which is sampled from the random crack growth distribution. We further assume there are $K = 9$ scheduled inspections for service at nominal times 1000, 2000, ..., and 9000 operating hours. The actual inspection time for each aircraft at each nominal inspection time is determined by the sum of the nominal scheduled inspection time and a random number generated from a uniform $(-50, 50)$ distribution to account for inspection-scheduling variability. The simulated data (including parts of the data that are not observable) are shown in Figure 1.

5 STATISTICAL MODEL

When measurements are available on the noise response, signal response, and crack length, a bivariate normal distribution can be used to model the data. In this section we illustrate the details of our joint bivariate normal statistical model.

In order to explain our estimation procedure, we separated the rivet holes into two categories, based on whether a crack is eventually found or not in the sequence of scheduled inspections. The first category includes rivet holes for which a crack existence decision was made at one of the scheduled inspections. For these rivet holes, both the NDE measurements and crack length measurements are available at the “crack find” inspection. The second category includes rivet holes that have a measurement below the detection threshold at every scheduled inspection. No crack-length information is available for these rivet holes. For both categories, the actual measurement results are described by the noise interference model (4) with the signal response from (2) and noise response from (3). The relationship between the signal response and crack length follows from (2) and we assume that the noise response is independent of crack length.

5.1 Rivet Holes with a “Crack Find” Inspection

5.1.1 Statistical model for the “crack find” inspection data

Some locations within an aircraft will eventually have a crack-find event. Suppose the measurement for rivet hole i in aircraft j is above the detection threshold at the scheduled inspection $\kappa(i, j)$

$(1 \leq \kappa(i, j) \leq K)$ with service time $t_{j\kappa(i,j)}$. The log crack length

$$x_{ij}(t_{j\kappa(i,j)}) = x_{ij}(t_0) + \lambda_j(t_{j\kappa(i,j)} - t_0)$$

is measured at the time of repair where $x_{ij}(t_0)$ is from the $N(\mu_x, \sigma_x^2)$ log initial crack length distribution and $\log_{10}(\lambda_j)$ is from the log crack growth rate distribution $N(\mu_\lambda, \sigma_\lambda^2)$. We assume that measurement error is negligible, although this would be easy to generalize if the measurement error has a known distribution.

Thus from the result of (2) and the bivariate normal distribution properties given in (A-1), and (A-2) from the appendix, the conditional distribution of the signal response for a rivet hole of a given crack length $x_{ij}(t_{j\kappa(i,j)})$ at the “crack find” inspection can be modeled with a random variable $Y(t_{j\kappa(i,j)})$ through the normal distribution:

$$Y(t_{j\kappa(i,j)}) \sim N\left(\beta_0 + \beta_1\left(\mu_x + \lambda_j(t_{j\kappa(i,j)} - t_0)\right), \sigma_y^2\right). \quad (5)$$

The eddy current NDE signal response for rivet hole i in aircraft j at scheduled inspection $\kappa(i, j)$ is modeled with the random variable $Y_{\text{actual}}(t_{j\kappa(i,j)})$ through the noise interference model:

$$Y_{\text{actual}}\left(t_{j\kappa(i,j)}\right) = \max\left(Y\left(t_{j\kappa(i,j)}\right), Y_{\text{noise}}\left(t_{j\kappa(i,j)}\right)\right) \quad (6)$$

where $Y\left(t_{j\kappa(i,j)}\right)$ is defined in (5) and $Y_{\text{noise}}\left(t_{j\kappa(i,j)}\right)$ is defined in (3).

5.1.2 Statistical model before the “crack find” inspection

The crack length at any scheduled inspections before a “crack find” inspection is

$x_{ij}\left(t_{jk}\right) = x_{ij}\left(t_{j\kappa(i,j)}\right) + \lambda_j\left(t_{jk} - t_{j\kappa(i,j)}\right)$ for $k = 1, \dots, \kappa(i, j)$. The log signal responses for these

scheduled inspections are $Y_{ij}\left(t_{jk}\right) = \beta_0 + \beta_1\left[x_{ij}\left(t_{j\kappa(i,j)}\right) + \lambda_j\left(t_{jk} - t_{j\kappa(i,j)}\right)\right] + \varepsilon_{ijk}$ and are modeled by the

normal distribution:

$$Y\left(t_{jk}\right) \sim N\left(\beta_0 + \beta_1\left[x_{ij}\left(t_{j\kappa(i,j)}\right) + \lambda_j\left(t_{jk} - t_{j\kappa(i,j)}\right)\right], \sigma_y^2\right). \quad (7)$$

The actual measurement result $y_{\text{actual},ij}\left(t_{jk}\right)$ for rivet hole i in aircraft j at scheduled inspection k with

$k = 1, \dots, \kappa(i, j)$ can be modeled through a random variable $Y_{\text{actual},ij}\left(t_{jk}\right)$ in the noise interference model

$$Y_{\text{actual}}\left(t_{jk}\right) = \max\left(Y\left(t_{jk}\right), Y_{\text{noise}}\left(t_{jk}\right)\right) \quad (8)$$

where $Y\left(t_{jk}\right)$ is defined in (7) and $Y_{\text{noise}}\left(t_{j\kappa(i,j)}\right)$ is defined in (3).

5.2 Rivet Holes without a “Crack Find” Inspection

The inspection results for rivet holes in the second category are below the detection threshold at all

scheduled inspections. Thus no direct crack-length information is available for these rivet holes. What we

know about these cracks is that they follow the crack growth model that says that the crack length at each

scheduled inspection is $x_{ij}(t_{jk}) = x_{ij}(t_0) + \lambda_j(t_{jk} - t_0)$ for $k = 1, \dots, K$ where $x_{ij}(t_0)$ is the unknown initial log crack length having a normal distribution $N(\mu_x, \sigma_x^2)$. The log signal response at each scheduled inspection is then modeled by a normal random variable

$$Y(t_{jk}) \sim N\left(\beta_0 + \beta_1 \left[x_{ij}(t_0) + \lambda_j(t_{jk} - t_0) \right], \sigma_y^2\right). \quad (9)$$

The actual eddy current log response is thus modeled with the noise interference model random variable

$$Y_{\text{actual}}(t_{jk}) = \max\left(Y(t_{jk}), Y_{\text{noise}}(t_{jk})\right) \quad (10)$$

where $Y(t_{jk})$ is defined in (9) and $Y_{\text{noise}}(t_{jk(i,j)})$ is defined in (3).

5.3 WinBUGS Implementation

There is no available commercial software that will be able to directly estimate the parameters of our proposed model. The highly-regarded, freely available WinBUGs (2007) software, described by Lunn, Thomas, Best, and Spiegelhalter (2000) makes it easy to specify the model and complicated data structure and to do the needed analysis. A copy of the WinBUGS codes is given in the [supplementary material](#) of this paper and complete details of the analysis are provided in the next section.

Deleted: appendix

6 BAYESIAN ESTIMATION

Likelihood based methods could be developed to estimate the model parameters

$(\mu_x, \sigma_x^2, \beta_0, \beta_1, \sigma_y^2, \mu_\lambda, \sigma_\lambda^2)$ with the likelihood contributions corresponding to the two different types of observations described in Section 5. Computation of the likelihood for this model is, however, difficult because of the multiple-fold integrals needed to represent the random effects. Bayesian methods (e.g., Gelman, Carlin, Stern and Rubin 2003), which are closely related to likelihood methods, provide an easy-

to-use and versatile alternative approach to do the estimation for the field data from the proposed rivet-hole inspection procedures. Bayesian methods also provide a formal way to incorporate useful prior information such as physics-based theory, information from previous studies, or expert opinion into the statistical analysis. In our analysis, however, we use diffuse (approximately non-informative) prior distributions. We have used WinBUGs (2007) to do the Bayesian analysis.

6.1 Model Specification

We use the statistical model described at Section 5 and diffuse prior distributions. The WinBUGs Markov Chain Monte Carlo (MCMC) algorithm is used to generate a large number of sampling draws from the joint posterior distribution for the model parameters $(\mu_{\text{noise}}, \sigma_{\text{noise}}^2, \mu_x, \sigma_x^2, \beta_0, \beta_1, \sigma_y^2, \mu_\lambda, \sigma_\lambda^2)$.

After the MCMC algorithm has converged, we have M sampling draws for each model parameter from the joint posterior distribution. Based on the M sampling draws, we can calculate statistics of interest such as the mean, standard deviation, and the 5% and 95% posterior quantiles for each model parameter or functions of the model parameters (e.g., POD and PFA). The 5% and 95% posterior quantiles also determine the 90% credible bounds for the model parameter. Summary results for all model parameters, comparing estimates with the true parameters used in the simulation are given in Table 1.

Table 1. Posterior mean and standard derivation for all model parameters

Model Parameter	μ_x	σ_x^2	β_0	β_1	σ_y^2
True Value	-7.39	0.51	4.50	0.50	0.065
Posterior Mean	-7.19	0.46	4.68	0.55	0.057
95% Credible Bounds	(-7.65,-6.76)	(0.30,0.68)	(4.31,5.11)	(0.44,0.67)	(0.037,0.079)

Model Parameter	μ_λ	σ_λ^2	μ_{noise}	σ_{noise}^2
True Value	-0.35	0.00040	2.50	0.064
Posterior Mean	-0.38	0.00145	2.51	0.069
95% Credible Bounds	(-0.44,-0.32)	(0.00035,0.00409)	(2.49,2.53)	(0.061,0.078)

6.2 Estimate of the Response Function and POD

The mean log signal response function $\mu(x)$ for rivet holes with log crack length x can be expressed as:

$$\mu(x) = \beta_0 + \beta_1 x. \quad (11)$$

The noise interference model POD, as described by Li and Meeker (2009), as a function of crack length, is

$$\text{POD}(x) = 1 - \Phi\left(\frac{y_{\text{th}} - \mu(x)}{\sigma_y}\right) \Phi\left(\frac{y_{\text{th}} - \mu_{\text{noise}}}{\sigma_{\text{noise}}}\right). \quad (12)$$

By substituting the M sampling draws of $\beta_0, \beta_1, \sigma_y^2, \mu_{\text{noise}}$ and σ_{noise}^2 into (11) and (12), we can get the M sampling draws of $\mu(x)$ and $\text{POD}(x)$ respectively, for any specified log crack length x .

The posterior mean, standard deviation and 90% credible bound for $\mu(x)$ and $\text{POD}(x)$ can be obtained through their respective M sampling draws. The estimated relationship between the posterior mean signal response and crack length and corresponding pointwise two-sided 90% credible bounds are shown in Figure 2 (left) along with the detection threshold (horizontal dashed line) and the posterior noise mean (horizontal dotted line). The posterior mean POD estimate, as a function of crack length, and corresponding 95% lower credible bounds are shown in Figure 2 (right). The estimated asymptotic POD as crack size approaches zero is 0.031 and it is close to the actual (unobserved) PFA of 0.028.

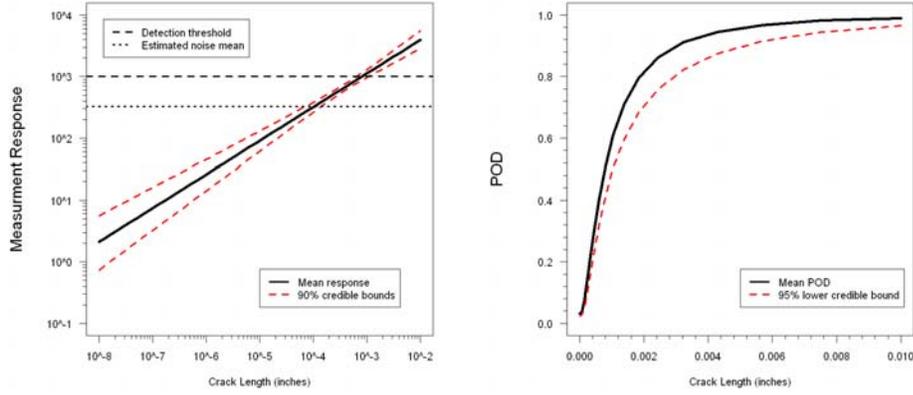


Figure 2. Posterior mean signal response and two-sided pointwise 90% credible bounds as a function of crack length (left) with detection threshold (horizontal dashed line) and posterior noise mean (horizontal dotted line), and posterior mean POD and 95% lower credible bound as functions of crack length (right).

6.3 Estimate of the Crack-Length Distribution and the Crack-Growth Model

One of the main advantages of our proposed inspection procedures is that it provides the information needed to estimate the noise distribution, the crack growth rates and the crack-length distribution at any point in time. In our model, the log noise response distribution has the normal distribution

$N(\mu_{\text{noise}}, \sigma_{\text{noise}}^2)$. The initial log crack length has a normal distribution of $N(\mu_x, \sigma_x^2)$ and the log crack growth rates have a normal distribution of $N(\mu_\lambda, \sigma_\lambda^2)$. Given the measurement data from the proposed

inspection procedures, we can accurately estimate $\mu_{\text{noise}}, \sigma_{\text{noise}}^2, \mu_x, \sigma_x^2, \mu_\lambda$ and σ_λ^2 as shown at Table 1.

With the estimates of the crack-length distribution and crack growth rates, we can also predict the expected number of rivet holes needed to be replaced for a future inspection. The estimated log crack-length distribution at the last scheduled inspection in our data set (i.e., 9000 hours in service) and its 90% credible bounds are shown in Figure 3. Such information provides not only guide lines for spare parts

inventory but also a criterion to detect any unusual behaviors (such as extreme larger numbers of rivet hole replacement at certain period of services), and in turn improves the overall aircraft reliability.

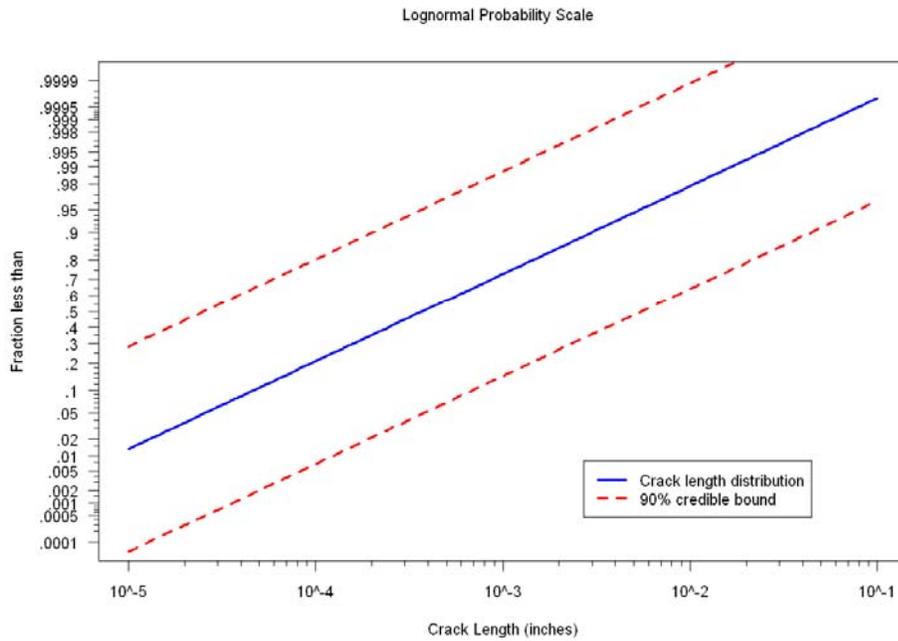


Figure 3. Crack length distribution at last scheduled inspection (9000 hours in service).

7 CONCLUDING REMARKS AND AREAS FOR FUTURE RESEARCH

In this paper, we have proposed modified scheduled maintenance inspection procedures for crack detection in aircraft components through nondestructive evaluation techniques and show how to properly analyze the resulting data. We developed a joint estimation statistical method to model the data obtained from the procedures. We used the Bayesian analysis software WinBUGs to model jointly, crack growth rates, a crack length distribution, and the probability of detection. The proposed inspection procedures

and the joint statistical analysis would provide much better understanding for the cracks inside aircraft components and improve the overall reliability assessment.

There are a number of extensions for the methodology presented in this article that suggest future research directions. These include the following:

1. Crack growth rates vary within an aircraft, perhaps with several types of locations within an aircraft type. The hierarchical model used in this paper could be extended in a straight-forward manner to allow for this.
2. It would also be possible to extend the hierarchical model in this paper to pool data across different types of aircraft.
3. In our presentation we have assumed that, at the time of a detection event, crack length is measured precisely. Actually, when a crack is detected, crack-size information is obtained by drilling the rivet hole with successively larger drill bits until the crack can no longer be detected. Thus the crack-size observation is actually interval censored. Such interval-censored data can be easily accommodated in either a likelihood or a Bayesian estimation framework.
4. It would be possible to use our approach to model NDE-signal/crack-growth data with a more complicated crack-growth model.

ACKNOWLEDGEMENTS

This material is based upon work supported by the Federal Aviation Administration under Contract #DTFA03-98-D-00008, Delivery Order #0058 and performed at Iowa State University's Center for Nondestructive Evaluation as part of the Center for Aviation Systems Reliability program.

APPENDIX: BIVARIATE NORMAL DISTRIBUTION

A.1 Density Function

The multivariate normal distribution is widely used to model the joint distribution of more than two random variables. The multivariate normal distribution has nice mathematical properties, described, for example, in Johnson and Wichern (2001). The bivariate normal is a special case of multivariate normal with dimension two. For a random vector $(Y, X)^T$ following a bivariate normal distribution

$\text{BVN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, if we denote $\boldsymbol{\mu} = (\mu_1, \mu_2)^T$ as the mean vector and denote

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}$$

as the variance-covariance matrix, then the density function for the random vector $(Y, X)^T$ is:

$$f(y, x; \mu_1, \mu_2, \sigma_{11}, \sigma_{22}, \rho) = \frac{1}{2\pi\sqrt{\sigma_{11}\sigma_{22}}\sqrt{1-\rho^2}} \times \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(y-\mu_1)^2}{\sigma_{11}} - 2\rho\frac{(y-\mu_1)(x-\mu_2)}{\sqrt{\sigma_{11}\sigma_{22}}} + \frac{(x-\mu_2)^2}{\sigma_{22}}\right]\right\} \quad (\text{A-1})$$

where $\rho = \sigma_{12} / \sqrt{\sigma_{11}\sigma_{22}}$ is the correlation between Y and X . Given data in form of $(y, x)^T$ pairs, the estimate of parameters $(\mu_1, \mu_2, \sigma_{11}, \sigma_{12}, \sigma_{22})$ can be obtained through likelihood or Bayesian methods.

A.2 Relationship to Linear Regression

An important property of the bivariate normal distribution used in this paper is that the distribution of one of the random variables, conditional on a fixed value of the other random variable, is a univariate normal distribution. For example, conditional on a fixed value of $X = x$, the distribution of Y is normal with

mean $\mu_{Y|X=x} = (\mu_1 - (\sigma_{12}/\sigma_{22})\mu_2) + (\sigma_{12}/\sigma_{22})x$ and variance $\sigma_{Y|X=x}^2 = \sigma_{11} - \sigma_{12}^2/\sigma_{22}$ (see for example, Chapter 4 of Johnson and Wichern 2001).

Suppose we have observations in the form of $(y, x)^T$ pairs. We can model the relationship between Y and X with linear regression as $Y = \beta_0 + \beta_1 x + \varepsilon$ with x the observation from $X \sim N(\mu_x, \sigma_x^2)$ and $\varepsilon \sim N(0, \sigma_y^2)$. Thus traditional ML and linear regression methods can be applied to estimate the regression model parameters (μ_x, σ_x^2) and $(\beta_0, \beta_1, \sigma_y^2)$. Equivalently we can treat $(Y, X)^T$ as following a bivariate normal distribution $\text{BVN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ of (A-1) with parameters $(\mu_1, \mu_2, \sigma_{11}, \sigma_{12}, \sigma_{22})$. For complete observations of $(y, x)^T$, the linear regression model and the bivariate normal model return the same estimates. However when there are censoring or when one or the other of the elements from the pair $(y, x)^T$ is missing, the bivariate normal model has the important advantage of using all the available data. The relationship between the two sets of parameters is summarized as follows:

Deleted: as illustrated in this paper

$$\begin{cases} \beta_0 = \mu_1 - \mu_2 \sigma_{12} / \sigma_{22} \\ \beta_1 = \sigma_{12} / \sigma_{22} \\ \mu_x = \mu_2 \\ \sigma_x^2 = \sigma_{22} \\ \sigma_y^2 = \sigma_{11} - \sigma_{12}^2 / \sigma_{22} \end{cases} \quad \text{and} \quad \begin{cases} \mu_1 = \beta_0 + \beta_1 \mu_x \\ \mu_2 = \mu_x \\ \sigma_{11} = \sigma_y^2 + \beta_1^2 \sigma_x^2 \\ \sigma_{12} = \beta_1 \sigma_x^2 \\ \sigma_{22} = \sigma_x^2 \end{cases} \quad (\text{A-2})$$

REFERENCES

1. Annis, C. (2009), R package: mh1823, Version 2.5.4.2, <http://www.statisticalengineering.com/>, downloaded March 2010.
2. Davidian, M. and Giltinan, D. M. (1995), *Nonlinear Models for Repeated Measurement Data*, London: Chapman Hall.
3. Gelman, A., Carlin, J. B., Stern, H. S. and Rubin, D. B. (2003), *Bayesian Data Analysis (Second Edition)*, Florida: Chapman and Hall.
4. Johnson, R. A. and Wichern, D. W. (2001), *Applied Multivariate Statistical Analysis (5th Edition)*, New Jersey: Prentice Hall.
5. Hovey, P., Meeker, W. Q. and Li, M. (2009), "Joint estimation of the flaw size distribution and POD function," *Review of Progress in Quantitative Nondestructive Evaluation*, 28, 1832-1839.
6. Li, M. and Meeker, W. Q. (2009), "A noise interference model for estimating probability of detection for nondestructive evaluations," *Review of Progress in Quantitative Nondestructive Evaluation*, 28, 1769-1776.
7. Li, M., Meeker, W. Q. and Thompson, R. B. (2010), "Physical model assisted probability of detection in nondestructive evaluation," to be submitted.
8. Li, M., Nakagawa, N., Larson, B. F. and Meeker, W. Q. (2010), "Statistical assessment of probability of detection for automatic eddy current inspection nondestructive evaluation data," to be submitted.
9. Lunn, D.J., Thomas, A., Best, N., and Spiegelhalter, D. (2000), "WinBUGS -- a Bayesian modelling framework: concepts, structure, and extensibility," *Statistics and Computing*, 10, 325-337.
10. Meeker, W. Q. and Escobar, L. A. (1998), *Statistical Methods for Reliability Data*, New York: Wiley.

11. MIL-HDBK-1823A (2009), *Nondestructive Evaluation System Reliability Assessment*, Standardization Order Desk, Building 4D, 700 Roberts Avenue, Philadelphia, PA 19111-5094.
12. Olin, D. B. and Meeker, W. Q. (1996), "Application of statistical methods to nondestructive evaluation," *Technometrics*, 38, 95-112.
13. Pawitan, Y. (2001), *In All Likelihood: Statistical Modelling and Inference Using Likelihood*, New York: Oxford University Press.
14. Spencer, F. W. (1996), "Discussion: application of statistical methods to nondestructive evaluation," *Technometrics*, 38, 122-124.
15. WinBUGs (2007), Version 1.4.2, <http://www.mrc-bsu.cam.ac.uk/bugs>. Downloaded March 2010.

Formatted: German (Germany)

Field Code Changed

Formatted: German (Germany)

SUPPLEMENTARY MATERIAL: WINBUGS CODE

```

### This is the WinBUGs code used in the paper.
model {

  ### Define diffuse priors distribution for all model parameters.
  a0~dnorm(0,0.001)          ### mean of initial crack size distribution
  tau.x~dgamma(0.001,0.001)
  sigma.x <- 1/tau.x          ### variance of initial crack size distribution

  for (j in 1:20) {
    lamda[j]~dnorm(mu.lamda,tau.lamda)
  }
  mu.lamda~dnorm(0,0.001)    ### mean of crack growth rate distribution
  tau.lamda~dgamma(0.001,0.001)
  sigma.lamda <- 1/tau.lamda  ### variance of crack growth rate distribution

  for (j in 1:20) {
    lamda.10[j] <- pow(10,lamda[j])  ### crack growth rate follow a log normal
  }

  b0~dnorm(0,0.001)          ### intercept of signal response function
  b1 ~ dnorm(0,0.001)        ### slope of signal response function
  tau.y ~ dgamma(0.001,0.001)
  sigma.y <- 1/tau.y         ### variance of signal response

  noise.mean ~ dnorm(0,0.001)  ### mean of noise response
  noise.tau ~ dgamma(0.001,0.001)
  noise.sigma <- 1/noise.tau  ### variance of noise response

  ### Noise interference model for specimens at "crack find" inspections
  ### The "zero" tricks is used to find likelihood contribution of max(y1,y2)
  for (iii in 1:53) {
    mu[iii] <- a0 + lamda.10[lamda.bvn[iii]]*t.bvn[iii]
    yx.bvn[iii,2] ~dnorm(mu[iii],tau.x)
    aaa[iii] <- b0 + b1*yx.bvn[iii,2]
    signal[iii] <- (yx.bvn[iii,1]-aaa[iii])/sqrt(sigma.y)
    noise[iii] <- (yx.bvn[iii,1]-noise.mean)/sqrt(noise.sigma)

    zeros[iii] <- 0
    zeros[iii] ~ dpois(tmp[iii])
    tmp[iii] <- -log( 1/sqrt(sigma.y)*exp(-0.5*pow(signal[iii],2))*phi(noise[iii])
      +1/sqrt(noise.sigma)*exp(-0.5*pow(noise[iii],2))*phi(signal[iii]) )
      +10000
  }
  ### NIM for the "crack find" rivet holes before the "crack find" inspection

  for (M in 1:290) {
    mu.cat1.y[M] <- b0 + b1*(yx.bvn[bvn.index[M],2] +
      lamda.10[lamda.cat1.slr[M]]*(t.cat1.slr[M]-t.bvn[bvn.index[M]]))
    signal.cat1[M] <- (y.cat1.slr[M]-mu.cat1.y[M])/sqrt(sigma.y)
    noise.cat1[M] <- (y.cat1.slr[M]-noise.mean)/sqrt(noise.sigma)

    zeros.cat1[M] <- 0
    zeros.cat1[M] ~ dpois(tmp.cat1[M])
    tmp.cat1[M] <- -log( 1/sqrt(sigma.y)*exp(-0.5*pow(signal.cat1[M],2))
      *phi(noise.cat1[M])+1/sqrt(noise.sigma)*exp(-
      0.5*pow(noise.cat1[M],2))*phi(signal.cat1[M]) )+10000
  }

  ### Noise interference model for the inspections for rivet hole without "crack find" inspections

  for (jjj in 1:47) {

```

```

x.last[jjj] ~ dnorm(mu.last[jjj],tau.x)
mu.last[jjj] <- a0 + lamda.10[lamda.cat2.main[jjj]]*t.last[jjj]
}

for (M in 1:423) {
  mu.cat2.y[M] <- b0 + b1*(x.last[last.idx[M]] + lamda.10[lamda.cat2.slr[M]]
    *(t.cat2.slr[M]-t.last[last.idx[M]]))
  signal.cat2[M] <- (y.cat2.slr[M]-mu.cat2.y[M])/sqrt(sigma.y)
  noise.cat2[M] <- (y.cat2.slr[M]-noise.mean)/sqrt(noise.sigma)

  zeros.cat2[M] <- 0
  zeros.cat2[M] ~ dpois(tmp.cat2[M])
  tmp.cat2[M] <- -log( 1/sqrt(sigma.y)*exp(-0.5*pow(signal.cat2[M],2))
    *phi(noise.cat2[M])+1/sqrt(noise.sigma)*
    exp(-0.5*pow(noise.cat2[M],2))*phi(signal.cat2[M]) )+10000
}
}

### WinBUGs code for Chapter 6 end here.

```