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Microwave mobilities of holes in p-type geranium

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Microwave mobilities of holes in p-type geranium

Abstract

Hall mobilities of an n-type (GN l) and a p-type (GP 2) germanium single crystal were measured at a microwave frequency of 9 Gc/ sec from 80 °K to 300 °K. A bimodal rectangular cavity designed by Nishina was used in the present investigation. The microwave circuit was nearly the same as that described by Nishina except that the microwave signal was modulated by 1000 cycle per second square-wave signal. The microwave mobilities measured (with sample size correction factor of 0. 423 for n-type and 0. 687 for p-type germanium) were compared with the corresponding d. c. Hall mobilities. For n-type germanium, the discrepancy between the d. c. and microwave mobilities was believed to be predominatly due to the E: -1/2 dependence of the relaxation time (acoustical mode scattering). For p-type germanium, a large deviation occurred at low temperatures and was in agreement with the results obtained by Hambleton et al. and by Watanabe. This result might be explained qualitatively as a combined effect of lattice and impurity scattering, particularly the effect of impurity scattering on the light mass holes.

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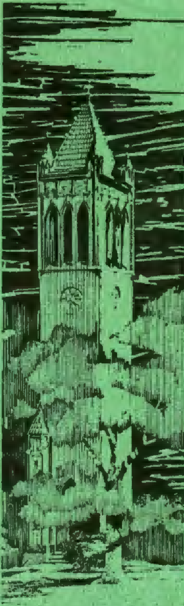

IOWA STATE UNIVERSITY

MICROWAVE MOBILITIES OF HOLES IN P-TYPE GERMANIUM

by

Bou-Loong Ho and G. C. Danielson

AMES LABORATORY

RESEARCH AND DEVELOPMENT REPORT

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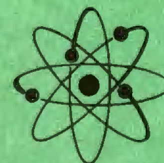

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Research and Development Report

MICROWAVE MOBILITIES OF HOLES IN P-TYPE GERMANIUM

by

Bou-Loong Ho and G. C. Danielson

August, 1964

Ames Laboratory
at
Iowa State University of Science and Technology
F. H. Spedding, Director

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IS-967

MICROWAVE MOBILITIES OF HOLES IN P-TYPE GERMANIUM*

Bou-Loong Ho and G. C. Danielson

ABSTRACT

Hall mobilities of an n-type (GN 1) and a p-type (GP 2) germanium single crystal were measured at a microwave frequency of 9 Gc/sec from 80°K to 300°K. A bimodal rectangular cavity designed by Nishina was used in the present investigation. The microwave circuit was nearly the same as that described by Nishina except that the microwave signal was modulated by 1000 cycle per second square-wave signal. The microwave mobilities measured (with sample size correction factor of 0.423 for n-type and 0.687 for p-type germanium) were compared with the corresponding d.c. Hall mobilities. For n-type germanium, the discrepancy between the d.c. and microwave mobilities was believed to be predominatly due to the $\epsilon^{-1/2}$ dependence of the relaxation time (acoustical mode scattering). For p-type germanium, a large deviation occurred at low temperatures and was in agreement with the results obtained by Hambleton _et al._ and by Watanabe. This result might be explained qualitatively as a combined effect of lattice and impurity scattering, particularly the effect of impurity scattering on the light mass holes.

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*This work is based on an M.S. thesis submitted by Bou-Loong Ho to Iowa State University, Ames, Iowa, August, 1964.

I. INTRODUCTION

A. Problem

Existing information on scattering mechanisms in p-type germanium is very incomplete. The most extensive work in this field has been the measurement of the mobility temperature dependence in the temperature interval 100 to 300°K by several investigators (1, 2), who have reported an empirical relationship $u \propto T^{-2.3}$ in the temperature range 120 to 300°K. The deviation from $T^{-1.5}$ dependence indicates that a simple acoustical mode scattering mechanism is inadequate to describe the mobility. Although the strong magnetic field dependence of the Hall mobility and the magnetoresistance have also been studied intensively (3, 4, 5, 6), these studies have not contributed very much to our understanding of the scattering mechanisms.

Theoretical considerations have shown that any frequency dependence of the Hall mobility is closely related to the energy dependence of the relaxation time. In principle, measurement of the high frequency Hall effect can determine the relaxation time directly and thus contribute to our understanding of the $T^{-2.3}$ dependence of the mobility in p-type germanium. In practice, experimental and theoretical difficulties have not allowed relaxation times to be obtained in this way.

B. Literature Review

Since Cooke (7) observed the Hall effect in some metals at microwave frequencies, several investigators have tried to study this effect in semiconductors by different techniques. Some methods have used a cylindrical waveguide (8); other methods have used a rectangular cavity (9). These investigations can only be considered as preliminary, however, because they lacked a good theoretical analysis of the apparatus used for measurement. As a result a useful formula for analyzing the results could not be obtained. In addition, these experiments were carried out only at room temperature.

Hambleton and Gärtner (10) were the first investigators to extend the measurements to low temperatures. They used a crossed waveguide method and reported the difference between the d.c. and microwave Hall mobility in germanium below 150°K. From an experimental point of view, however, the cavity method has advantages over the waveguide method because the former is much easier for low temperature operation.

Watanabe (11) and Liu, Nishina, and Good (12) derived the Hall power relationship through the analysis of microwave fields in a cavity and its analogy to an equivalent circuit with suitable circuit parameters. Nishina and Danielson (13) obtained the Hall mobility for n-type germanium at 9 Gc over the temperature range of 30 to 300°K by using a bimodal rectangular cavity. The results were analyzed with the formula derived by Liu *et al* (12). They found that the Hall mobility at microwave

frequencies (with sample size correction) agreed with the d.c. Hall mobility over the whole temperature range they studied. Nishina (14) also made some preliminary measurements on p-type germanium. He reported that the d.c. and microwave Hall mobility coincided only at low temperatures. Watanabe (15) used a double mode cylindrical cavity to measure the microwave Hall mobility at 24 Gc between room temperature and 100°K. He found a deviation between the microwave and d.c. Hall mobilities for both n-type and p-type germanium at low temperatures, but the discrepancy in the p-type sample was larger than that in the n-type sample.

C. Purpose of This Investigation

In the present investigation an effort has been made to find out:

(1) if the deviation between the d.c. and the microwave Hall mobilities for the p-type and n-type germanium really exists;

(2) the reason for the discrepancy between Nishina's and Watanabe's work on p-type germanium, and

(3) the mechanisms responsible for any observed differences between d.c. and microwave Hall mobilities in germanium.

II. THEORY

A. Conductivity Tensor and the Hall Mobility in Oscillating Fields

Since the experimental conditions require only a classical theoretical treatment, the physical implication of the Hall mobility in semiconductors at high frequencies can be clarified by calculation of the conduction current from the Boltzmann transport equation:

$$\frac{\partial f}{\partial t} + e\left[\vec{E} + \vec{V} \times \vec{B}\right] \cdot \frac{1}{\hbar}\vec{\nabla}_k f + \vec{V} \cdot \vec{\nabla}_r f = \left[\frac{\partial f}{\partial t}\right]_c \tag{2.1}$$

where f = the distribution function of the carriers,

$\vec{E}$ = applied electric field,

$\vec{B}$ = applied static magnetic field,

$\left[\frac{\partial f}{\partial t}\right]_c$ = the rate of change of f due to the scattering of the carriers.

The general procedure for determining the distribution function to a first-order approximation is to expand f about its equilibrium distribution f_o as follows

$$f = f_o - \vec{V} \cdot \vec{\Psi} \frac{\partial f_o}{\partial \epsilon} . \tag{2.2}$$

The terms of higher order in $\vec{V}$ are neglected. The problem is greatly simplified when a relaxation time can be introduced such that

$$\left[\frac{\partial f}{\partial t}\right]_C = -(f - f_o)/\tau . \tag{2.3}$$

A relaxation time can be used if the energy emitted and absorbed by a charge carrier at collision is small compared with its initial energy (16).

It appears quite justifiable to use a relaxation time for germanium and silicon, except perhaps at very low temperatures.

For a steady state, the term $\frac{\partial f}{\partial t}$ is zero. If the external field changes in a time comparable to the relaxation time, it is then no longer in a steady state. In the case of a sinusoidal external field, the deviation in f from its equilibrium state $f_{\circ}$ must change in the same manner. Thus we have

$$\frac{\partial f}{\partial t} = j\omega\,(f - f_{\circ}), \tag{2.4}$$

where ω is the angular frequency of the applied fields. Equation 2.1 can therefore be written as

$$e\left[\vec{E} + \vec{V} \times \vec{B}\right] \cdot \frac{1}{\hbar}\vec{\nabla}_k f + \vec{V}.\vec{\nabla}_r f = -\,(j\omega + \frac{1}{\tau})\,(f - f_{\circ}) \tag{2.5}$$

When the energy of the charge carriers is a quadratic function of wave number it is possible to obtain an exact solution of equation 2.5 and to determine the distribution function in terms of $\vec{\Psi}$ for small fields. The expression for $\vec{\Psi}$, when τ is a function of energy only, is given in a review article by Fan (17), and may be written in the form

$$\vec{\psi} = \frac{\tau\left(\vec{P} - \frac{e\tau}{m^*}\vec{B} \times \vec{P} + \left(\frac{e}{m^*}\right)^2 \vec{B}\,(\vec{B}.\vec{P})\right.}{1 + \left(\frac{e\tau B}{m^*}\right)^2} \tag{2.6}$$

with

$$\vec{F} = e\vec{E} - \vec{\nabla}\zeta,$$

ζ = Fermi Energy,

m^* = effective mass of carrier, (2.7)

for an isothermal case. If the relaxation time is taken as a function of energy only, it is implied that the relaxation time is isotropic on a surface of constant energy. If the energy surfaces have spherical symmetry, then τ is a function only of $|\vec{k}|$.

With the approximation given by equation 2.2, the current density due to the charge carriers can be written in terms of $\vec{\psi}$ as follows:

$$\vec{j} = \frac{e}{4\pi^3}\int \vec{v} f \, d^3K = -\frac{e}{4\pi^3}\int \vec{v} \, (\vec{v} \cdot \vec{\psi}) \frac{\partial f_o}{\partial \epsilon} \, d^3K. \tag{2.8}$$

Watanabe (11) solved the problem for semiconductors under the assumptions:

(1) the energy surfaces are spherical,

(2) the term $\vec{v} \cdot \vec{\nabla}_r f$ may be neglected, and

(3) the effect of polarization can be taken into account by an effective field in the sample

$$\vec{E}_{eff} = \vec{E}_i - \frac{L_i}{j\omega}\vec{j} \tag{2.9}$$

where

$\vec{E}$ = applied field (2.10)

$$\vec{E}_i = \frac{\vec{E}}{1 + L(\epsilon - \epsilon_o)}$$

$$L_i = \frac{L}{1 + L(\epsilon - \epsilon_o)} \tag{2.11}$$

L = depolarization coefficient

$\vec{j}$ = current density in the sample.

Thus, if a static magnetic field is applied in the z direction, the conductivity matrix will be

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_o & \sigma_1 \\ -\sigma_1 & \sigma_o \end{pmatrix} \begin{pmatrix} E_{ix} \\ E_{iy} \end{pmatrix}, \tag{2.12}$$

where $$\sigma_o = \frac{je^2}{\lambda}\left(1 + \frac{P}{P^2 - Q^2}\right) \tag{2.13}$$

$$\sigma_1 = \frac{e^2}{\lambda} \frac{Q}{P^2 - Q^2}, \tag{2.14}$$

$$\lambda = \frac{eL_i}{\omega}, \tag{2.15}$$

$$P = 1 - j\lambda \sum_i \frac{n_i}{m^*_i} \left\langle \frac{(j\omega\tau + 1)\tau}{(j\omega\tau + 1)^2 + (\omega_c \tau)^2} \right\rangle_i \tag{2.16}$$

$$Q = \lambda \sum_i \frac{n_i}{m^*} \left\langle \frac{\tau^2}{(j\omega\tau + 1)^2 + (\omega_c \tau)^2} \right\rangle_i \omega_{ci}, \tag{2.17}$$

$$\omega_c = \frac{eB}{m^*}$$

n_i = density of the ith carrier (2.18)

Equation (2.13) and (2.14) correspond to the conductivity tensor elements derived by Nishina (14) from the classical equation of motion

$$M \frac{d\vec{V}}{dt} + m \frac{\vec{V}}{\tau} = e \left[\vec{E} + \vec{V} \times \vec{B} \right] \qquad (2.19)$$

and the phenomenological conductivity tensor.

From the definition of d.c. Hall mobility (14),

$$\mu_H = R_h \sigma = \frac{E_y}{J \times B_z} \sigma , \qquad (2.20)$$

with the static magnetic field B_z applied in z direction, the static current density J_x in the x direction, and the static electric field E_y induced by the Hall effect with no current in y direction. By analogy we can also define the Hall mobility in a more general form by the relation:

$$\mu_H = \frac{1}{B} \frac{|\sigma_1|}{Re\ (\sigma_o)} . \qquad (2.21)$$

If the low field approximation is applied, terms higher than second order in B may be neglected. If the relaxation times are assumed to be the same for all bands, we can then obtain the following equation

$$\mu_H = \frac{e \sum_i \frac{n_i}{m_i^{*2}}}{\sum_i \frac{n_i}{m_i^*}} \frac{\left| \left\langle \frac{\tau^2}{(j\omega\tau + 1)^2} \right\rangle_i \right|}{Re \left\langle \frac{\tau}{j\omega\tau + 1} \right\rangle_i} \qquad (2.22)$$

In p-type germanium, the relaxation times of the two kinds of holes are approximately equal as pointed out by Brooks (19). He reasoned that interband scattering is the principal mechanism that determines the mobility of the light holes, while intraband scattering is the predominant mechanism for the heavy holes. The scattering depends essentially on the density of the final states, which in either case is principally in the band having a large density of states. Therefore, Eq. (2.22) are directly applicable to p-type germanium in the extrinsic region, if the warping of the heavy-mass band is neglected.

From Eq. (2.22) it can be concluded that, if any high frequency effect exists in the Hall mobility, it must be caused by the combined effect of the energy dependence of the relaxation time τ and of $\omega\tau \geq 1$. If the relaxation time is independent of energy, no high frequency effect can exist even if $\omega\tau > 1$.

B. Principle of Microwave Measurement

The microwave degenerate cavity system (Fig. 1) and the power relationship for the arrangement have been studied in detail by Nishina _et al._ (13) and by Liu _et al._ (12) respectively.

As shown in Fig. 1, a square semiconductor sample of the linear dimension l is placed at the center of the end wall of the cavity parallel to the xy plane. The cavity can have only the lowest two distinct modes of resonant oscillations at a single microwave frequency, namely the TE_{101} mode and the TE_{011} mode.

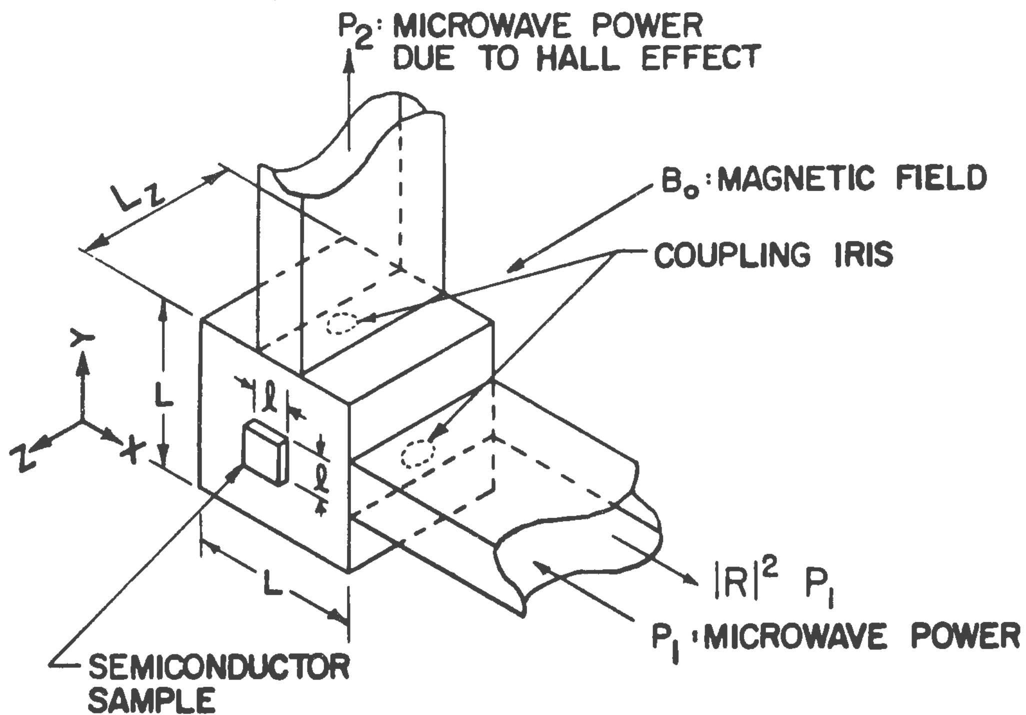

Fig. 1. Principle of microwave Hall mobility measurement

If the microwave power P_1 at resonant frequency is incident on the cavity through one branch of the waveguides (say branch I) and excites the TE_{101} mode of oscillation in the cavity, then when a static magnetic field B is applied in the z direction, the Hall effect in the sample will excite the TE_{011} mode of oscillation in the cavity with power P_2. A part of the power P_2 will be coupled out to the other waveguide. Thus $\frac{P_2}{P_1}$ is a function of the static magnetic field, sample geometry, conductivity, dielectric constant, and the Hall mobility.

Theoretical analysis gives the relationship (12)

$$\sqrt{\frac{P_2}{P_1}} \quad \frac{1}{|1 + R|} = \left| \frac{\eta + \alpha BU + j\,(\eta' + \alpha\,BU)}{Y_0 + G + 2\alpha + 2j\,(x + \alpha)} \right| \qquad (2.23)$$

where P_1 = microwave power incident on the cavity to excite the TE_{101} mode,

P_2 = microwave power out of the TE_{011} mode in the cavity,

R = reflection coefficient for the power P_1 at the iris plane,

$= \frac{1}{L_z}\left(\frac{2}{\omega\mu\sigma}\right)^{1/2} \cdot \frac{\bar{\ell}^2}{L^2 + L_z^2}$, $2\alpha = \frac{1}{Q_s}$ where Q_s is the Q

of the cavity due only to the loss in the sample,

u $= \frac{e\tau}{m} \frac{1}{1 + j\omega\tau}$ = mobility with the effect of relaxation time,

= eNu

N = density of carriers,

$\bar{\ell}$ = effective linear dimension of square sample,

μ = permeability of the sample,

$\eta + j\eta'$ = mutual admittance to represent the coupling due to the non-ideality of the cavity,

B = static magnetic field,

$x = \frac{\omega - \omega_o}{\omega_o}$,

ω = microwave frequency,

ω_o = resonant frequency of the TE_{101} and the TE_{011} mode,

Y_o = characteristic admittance of the waveguide in the units of ωc, reciprocal of external Q,

G = conductance representing the loss in the cavity wall in the units of ωc, reciprocal unloaded Q without sample,

C = capacity of the resonant circuit corresponding to the TE_{101} or the TE_{011} mode.

All quantities are expressed in MKS units.

Equation 2.23 was derived with the assumptions:

(1) There is only one kind of carrier with isotropic effective mass m, and relaxation time τ.

(2) The static magnetic field is weak so that $|Bu| \ll 1$.

(3) The sample size is relatively small compared to the dimensions of the cavity.

(4) The field distribution inside the sample is approximated by a plane wave. The microwave electric field is then uniform over

the sample surface except at the boundary to the cavity wall where it suddenly drops down to almost zero.

If Eq. 2.23 is expressed in terms of the conductivity tensor of a sample, the result can then be extended to the cases of the multiple carriers sample and to the arbitrary shape of energy surfaces of the carriers. The result derived by Nishina (14) for two kinds of carriers with isotropic masses in the sample is

$$\sqrt{\frac{P_2}{P_1}}\ \frac{1}{|1+R|} = \frac{\sqrt{2}\,\alpha B u_0}{Y + G + 2\ (q - q')}$$

$$\times \frac{\left[\left\{1 - (\omega\tau_1)^2 \frac{\left(\frac{\tau_2}{\tau_1}\right)^2 + \frac{\sigma_{20}u_{20}}{\sigma_{10}u_{10}}}{1 + \frac{\sigma_{20}u_{20}}{\sigma_{10}u_{10}}}\right\}^2 + 4(\omega\tau_1)^2 \left\{\frac{\frac{\tau_2}{\tau_1} + \frac{\sigma_{20}u_{20}}{\sigma_{10}u_{10}}}{1 + \frac{\sigma_{20}u_{20}}{\sigma_{10}u_{10}}}\right\}^2\right]^{1/2}}{\left[1 + (\omega\tau_1)^2 \left\{\frac{\frac{\tau_2}{\tau_1} + \frac{\sigma_{20}}{\sigma_{10}}}{\frac{\sigma_{20}}{\sigma_{10}}}\right\}^2\right]^{3/4} (1 + (\omega\tau_1)^2)^{1/4} (1 + (\omega\tau_2)^2)^{1/4}} \tag{2.24}$$

where

$$q + jq' = \left[\frac{(1 + (\omega\tau_1)^2)(1 + (\omega\tau_2)^2)}{1 + \omega^2 \left(\frac{\sigma_{10}\tau_2 + \sigma_{20}\tau_1}{\sigma_{10} + \sigma_{20}}\right)^2}\right]^{1/4} e^{j\frac{\beta_1 + \beta_2 + \beta_3}{2}} \tag{2.25}$$

$$\tan\beta_1 = \omega\tau_1 \tag{2.26}$$

$$\tan\beta_2 = \omega\tau_2 \tag{2.27}$$

$$\tan\beta_3 = \omega \frac{\sigma_{10}\tau_2 + \sigma_{20}\tau_1}{\sigma_{10} + \sigma_{20}} \tag{2.28}$$

$u_{10} = \dfrac{e_1\tau_1}{m_1}$ = d.c. mobility for the carrier 1.

$u_{20} = \dfrac{e_2\tau_2}{m_2}$ = d.c. mobility for the carrier 2.

$\sigma_{10} = e_1N_1u_{10}$ = d.c. conductivity contributed by the carrier 1.

$\sigma_{20} = e_1N_1u_{20}$ = d.c. conductivity contributed by carrier 2.

$u_{\circ} = \dfrac{\sigma_{10}u_{10} + \sigma_{20}u_{20}}{\sigma_{10} + \sigma_{20}}$: d.c. mobility for the two carriers.

$$\alpha = \frac{\Gamma^2}{L_z\,(L^2 + L_z^2)} \sqrt{\frac{2}{\omega\mu(\sigma_{10} + \sigma_{20})}} \tag{2.29}$$

Equation 2.24 may be applied to the single carrier case with the condition $\sigma_{20} = 0$.

III. EXPERIMENTAL PROCEDURE

A. d.c. Measurement

The d.c. conductivity and Hall coefficient measurement of n-type and p-type germanium were performed with a cryostat and sample holder similar to those used by Zrudsky (20).

The samples for the d.c. measurement were cut with a diamond saw from the same crystals immediately adjacent to those parts which were cut for microwave measurement. The crystals were cut along their rectangular edges in the <100>, <010>, and <001> directions. The dimensions of the samples were measured by a Gaertner traveling microscope.[1]

Three probes were used for these measurements. The details of the probe arrangement and voltage relations have been discussed by Heller (21). It was found after testing that the values of conductivity and mobility were erratic due to the high contact resistance between the probes and the sample. In order to reduce this contact resistance, the sample surface, where the probes were in contact, was covered with Zn-10[2] by means of an ultrasonic solder gun. To minimize the uncertainty in measuring the distance of the probe separation due to the presence of the soldered spot, the following procedure was followed.

---

[1]The Gaertner Scientific Corp., Chicago, Ill.

[2]10% Zn, 90% Sn.

(1) The resistance of the sample for an arbitary distance of separation was first measured with a Leeds and Northrup Model 7553 type K-3 potentiometer on a special sample holder designed for room temperature measurement. The distance of separation was measured with a traveling microscope before the spots of contact were covered with Zn-10.

(2) The sample with the soldered Z-10 spots was mounted on the low temperature sample holder and the resistivity and Hall coefficient were measured. The resistance obtained at room temperature was used to obtain the distance of separation of the resistance probes by a comparison with the resistance obtained in procedure 1 at the same temperature.

The Hall voltage and the resistivity drop were taken by an integrating digital voltmeter.

B. Microwave Measurement

1. Microwave cavity and temperature control

The microwave cavity system (Fig. 2) used in this investigation is the same system as used by Nishina (13).

The temperature control system is shown in Fig. 4. A resistor heater was dipped directly into the liquid nitrogen tank to evaporate the nitrogen. The vapor pressure was controlled by the voltage applied across the resistor. As the vapor pressure of nitrogen increased some of

the liquid nitrogen was forced to the cavity cryostat through a transfer tube. In this way, the cavity temperature was reduced to that of the liquid nitrogen. Since the level of the liquid nitrogen was below the end of the transfer tube, only nitrogen vapor was forced to the cryostat through the transfer tube. Thus, by controlling the vapor pressure of the nitrogen gas by means of a heated resistor, we were able to maintain the temperature of the cavity at any value from 120 to 300°K. However, it is difficult to reach an equilibrium temperature from 77 to 120°K.

The temperature of the semiconductor sample mounted on the cavity was measured with a copper-constantan thermocouple soldered to the cavity block. The thermocouple voltage was measured with an integrating digital voltmeter.

The temperature of the cavity was raised by reducing the nitrogen vapor pressure to let the cavity warm up automatically. As the temperature approached the desired value, the vapor pressure of nitrogen was increased gradually until equilibrium was reached.

2. Sample preparation

The single crystals of n-type and p-type germanium were obtained from Texas Instrument Co. Both types of crystal were cut to a size of about 10mm x 10mm x 1mm. The square edges of the sample surface were oriented in the <100> and <010> directions so that the crystal could be placed in a position symmetrical to both of the TE_{101} mode and the TE_{011} mode of the microwave fields.

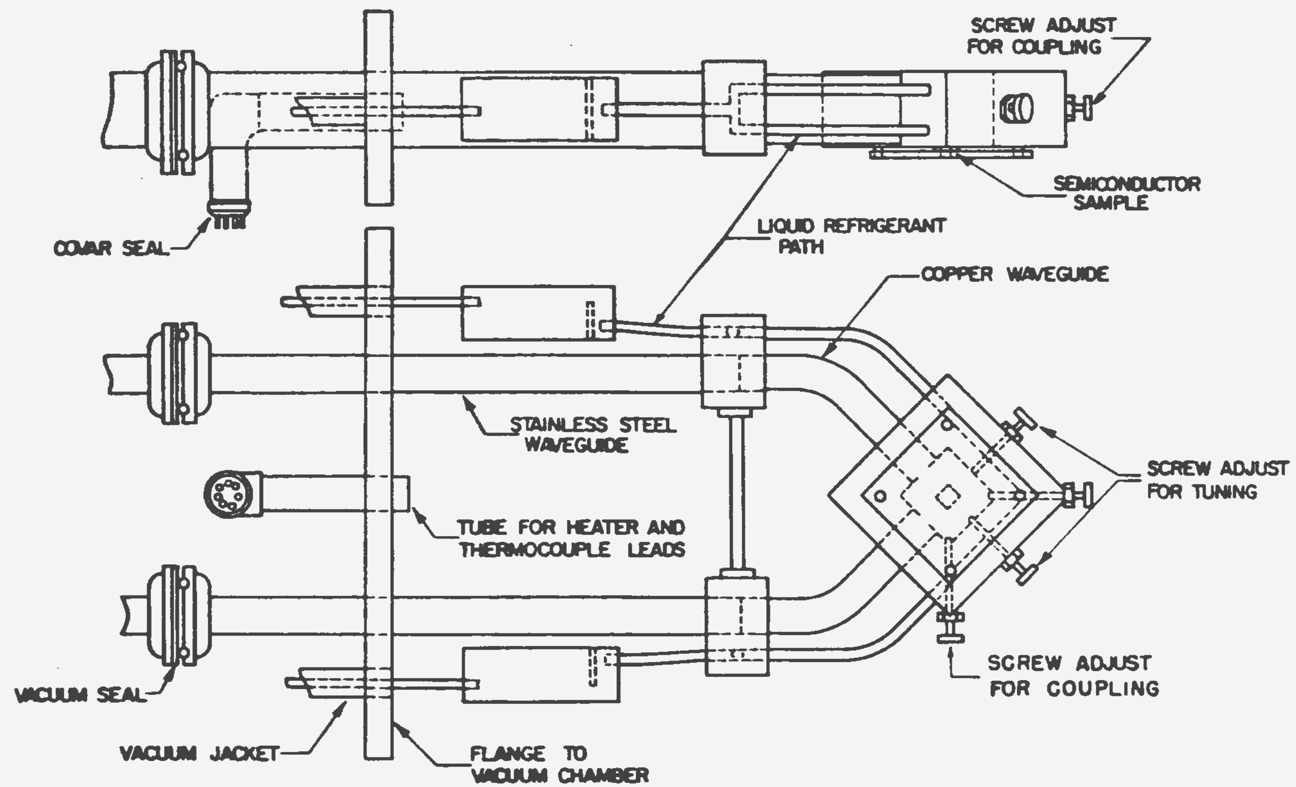

Fig. 2. Cavity construction for low temperature measurement

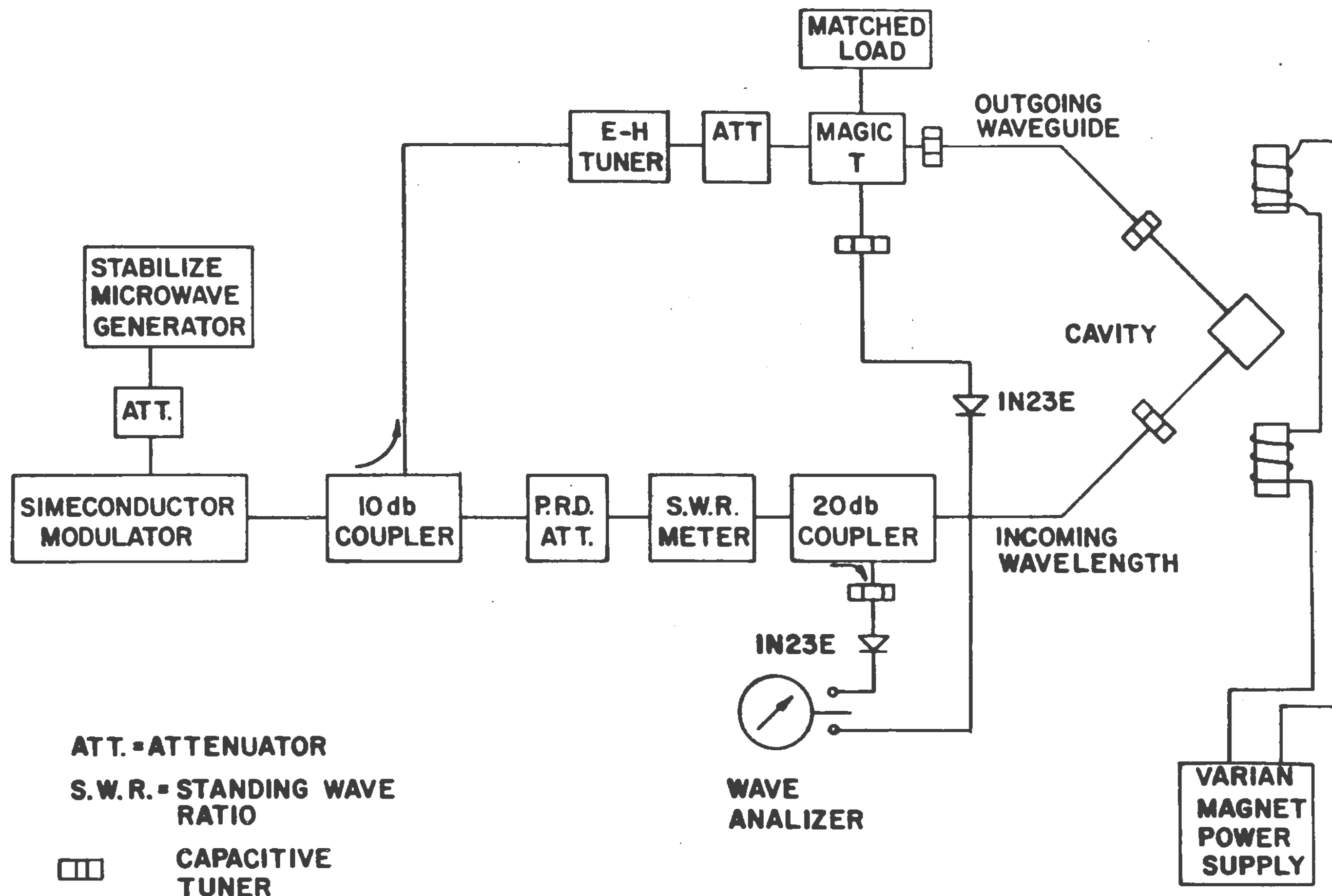

Fig. 3. Schematic diagram of microwave circuit

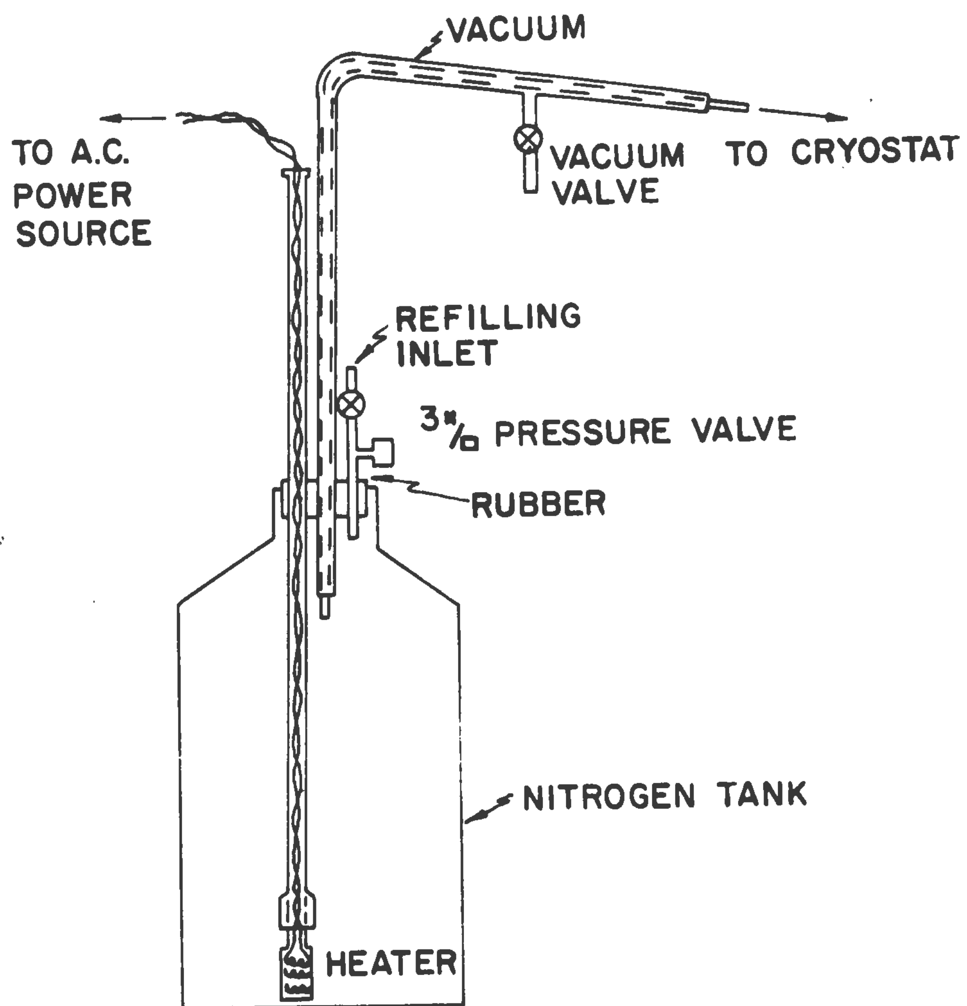

Fig. 4. Schematic diagram of temperature control

The sample was mounted on one side of the cavity walls with silver paint as shown in Fig. 1 and was further pressed against the frame with another brass plate.

3. Microwave measurement

The microwave system is shown in Fig. 3. A Strand Labs., Inc.[1] X-band microwave generator with an average power output of about 20 milliwatts was used. This oscillator consisted of a klystron, a ferrite isolator, a stabilization discriminator, and a reference cavity. The ferrite isolator isolated the klystron from the outside circuit so that the klystron could operate without being affected by changes in the impedance of the microwave load. The frequency was stabilized to the tunable reference cavity.

The measuring system was the same as that described by Nishina (14) except for a modification in the method of modulation. The microwave signal was modulated by a 1000 cycles per second square-wave signal. The modulation was performed by a section of a semiconductor switch/modulator/attenuator.[2] This modulator was made in the form of a section of wave-guide inserted in the microwave circuit before the 10 db coupler. The modulating action depended upon the increase in energy absorption resulting from the increase in conductivity caused by the

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[1] Strand Labs., Inc., 294 Center St., Newton 58, Mass.

[2] Somerset Radiation Lab., Inc., 192 Central Ave., Stirling, N. J.

injection of excess minority carriers. By modulating the conductivity of the semiconductor crystal one can obtain a change in the microwave energy transmitted through the system. This method of modulation had two advantages:

(1) The signal was nearly free from phase or frequency modulation.

(2) The modulation did not affect the r. f. oscillator, so fluctuation of the r. f. was kept to a minimum.

The microwave power detected by the diodes was then amplified and measured by a wave analyzer.[1]

The procedure for the microwave measurements was the following. At first, the sample-loaded cavity was adjusted to be degenerate. The non-ideal coupling between the TE_{101} and the TE_{011} mode was reduced to a minimum by adjustment of the screws at the corners of the cavity. The residual part of the non-ideal power out of the cavity was then canceled out at the Magic T detector by adjustment of the E-H tuner and the attenuator in the bridge circuit between the 10 db directional coupler and the magic T. This adjustment allowed the coherent wave out of the 10 db directional coupler to have the proper magnitude and phase as it reached the magic T. After the cancellation of the non-ideal power, a static magnetic field was applied.

The reading of the P. R. D. attenuator was set to zero at the beginning of the measurement. As the static magnetic field was applied, the output detector detected a signal which was related to the microwave

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[1]Hewlett Packard, Palo Alto, Calif.

Hall power P_z shown in equation 2.23. This signal was read on the wave analyzer in an arbitrary scale. The signal corresponding to the input power was then measured from the detector at the 20 db directional coupler by turning the P. R. D. attenuator to such a value (say α_1) that the wave analyzer gave the same reading as that for the output signal. In order to get the actual output and input power ratio, the output of the magic T detector must be calibrated with respect to the output of the detector at the 20 db coupler. This calibration was performed by removal of the cavity and by connecting a piece of waveguide in half-circled shape directly between the E-arm of the magic T and the main guide of the 20 db coupler. The attenuation of the P. R. D. (say α_2) with the same reading at the two detectors was recorded. The power ratio was then given by the relation

$$10 \text{ Log } \frac{P_2}{P_1} = \alpha_1 + \alpha_2 .$$

IV. EXPERIMENT RESULTS

A. Field Dependence Measurement

In order that the Hall measurement can be operated in the linear field region so the low field approximation of equation 2.23 can be applied properly and the system can still give a detectable signal, the field dependence of the Hall power ($\sqrt{\frac{P_2}{P_1}}$) of both n-type and p-type germanium single crystals was measured at room temperature and at liquid nitrogen temperature.

Fig. 5 shows that for n-type sample the Hall power started to saturate at about 3.5 kG at liquid nitrogen temperature and at 7.5 kG at room temperature. Fig. 6 shows that for a p-type sample the Hall power started to saturate at about 3.5 kG at the same temperatures. The curves for the p-type sample deviate more from a straight line than those for the n-type. This phenomenon is probably due to light holes in a p-type crystal as explained by Willardson et al.(3). These data were used to choose the magnetic field strength in the study of the temperature dependence of the Hall mobility for these same crystals. The static magnetic field was chosen as 0.2107 Wb/m^2 for both types of germanium crystals. This magnetic field intensity was considered to be satisfactory throughout the whole range of temperature under study. The use of equation 2.23 as a justifiable approximation in this investigation was thus assured.

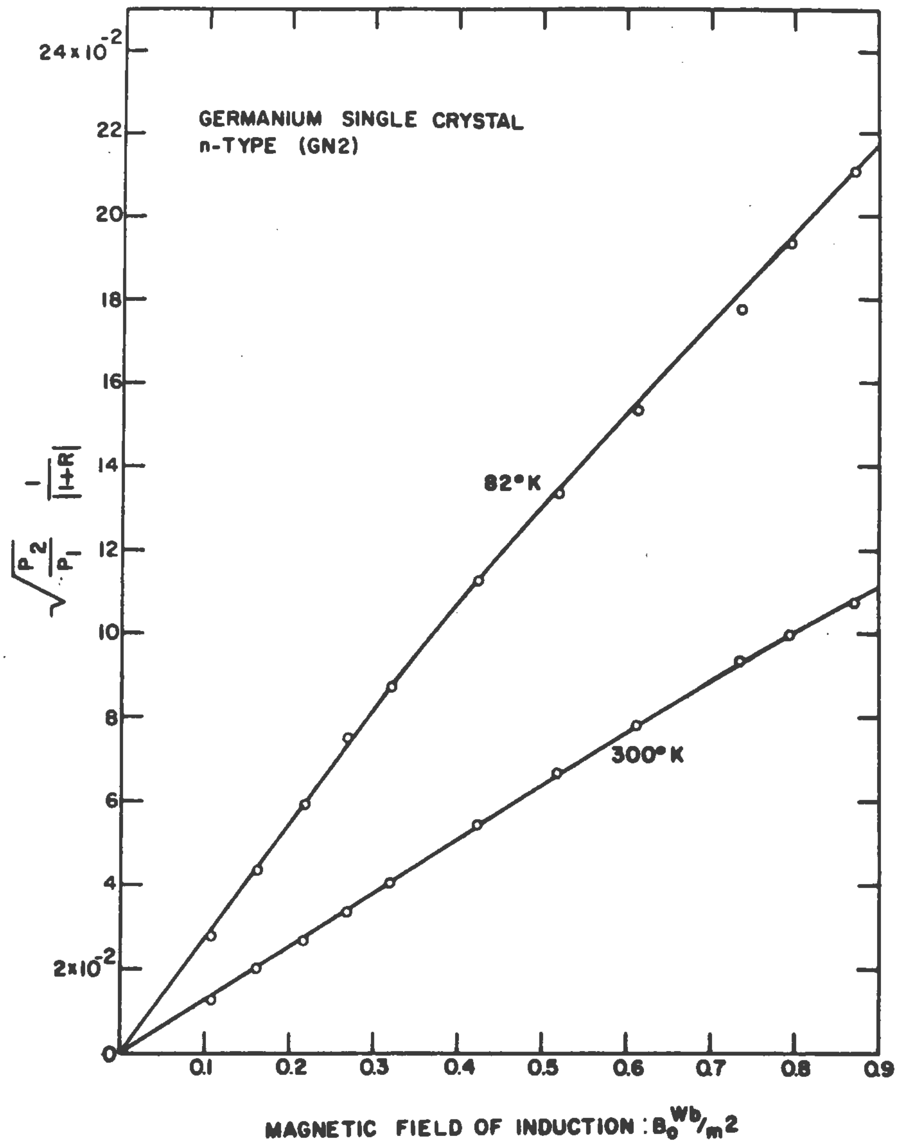

Fig. 5. Magnetic field dependence of $\frac{P_2}{P_1}$ for n-type germanium

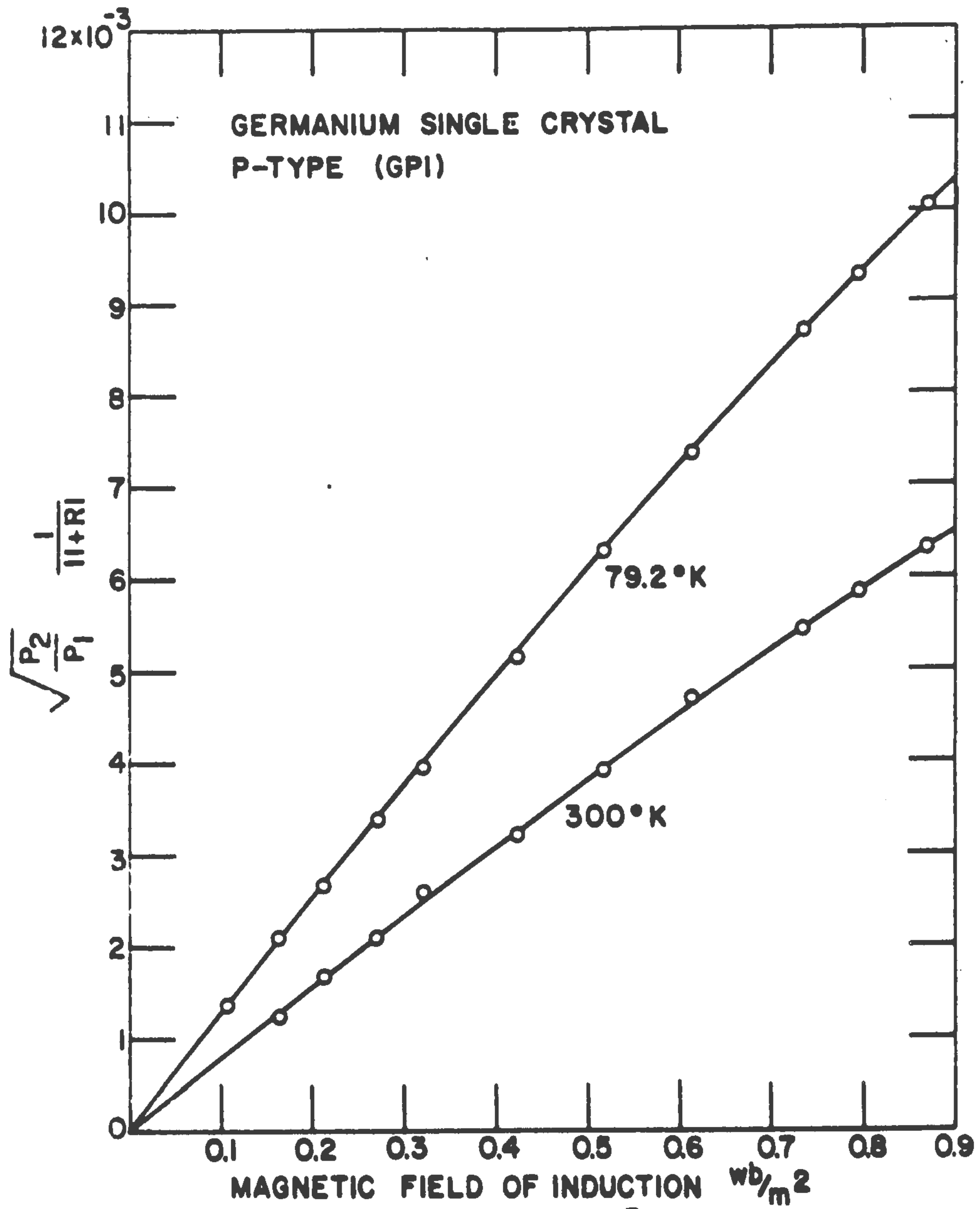

Fig. 6. Magnetic field dependence of $\frac{P_2}{P_1}$ for p-type germanium

B. Temperature Dependence Measurement

1. N-type germanium

The temperature dependence of the microwave Hall mobility of a n-type germanium single crystal (GN2) was calculated according to equation 2.24 with $\sigma_{20} = 0$ and as suggested by Nishina et al.(13). First an α ($2\alpha =$ the reciprocal of the cavity Q given by the losses in the sample) value of 0.534×10^{-4} was chosen in such a way as to give the best agreement between the d.c. and microwave mobility values at room temperature. This value of α corresponded to an effective sample size($\bar{l} = 0.423\ l = (0.423)(9.5\text{mm}) = 4.02\text{mm}$). The temperature dependence of α was assumed to be inversely proportional to the square root of the conductivity of the same sample. The characteristic admittance of the waveguide Y_0 and the conductance G which represented the loss in the cavity wall were assumed to be the same as those used by Nishina, since the construction of the apparatus used was identical with the one used by him. Nishina assumed G was proportional to $\sqrt{\rho_s}$ where ρ_s was the resistivity of the silver wall of the cavity.

The comparison between the microwave and the d.c. Hall mobility $R\sigma$ vs. temperature is shown in Fig. 7. It can be seen that a discrepancy of about 9% occurs at 79.2°K.

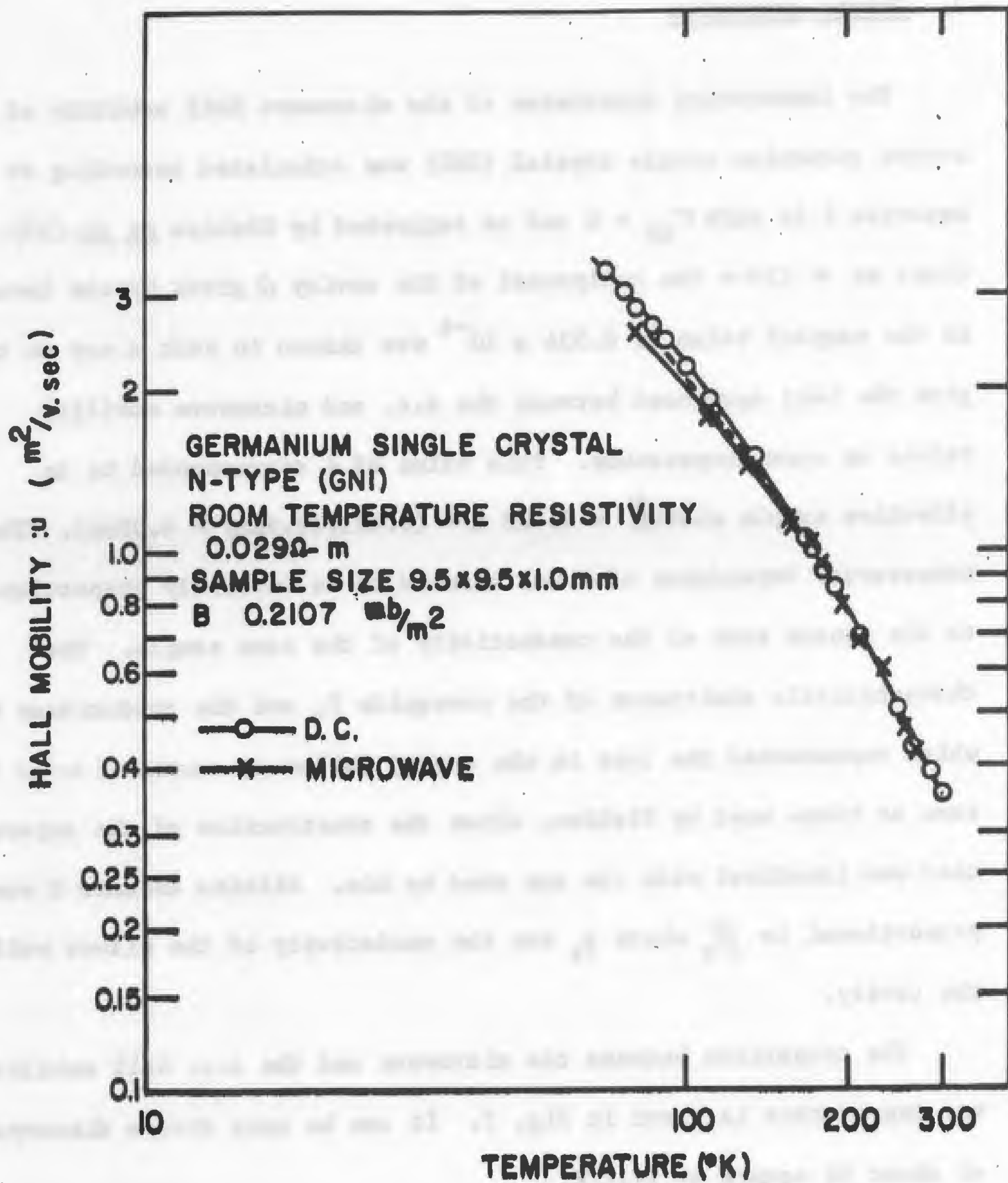

Fig. 7. Hall mobility of n-type germanium vs. temperature

2. P-type germanium

Fig. 8 illustrates the temperature dependence of the Hall mobilities of p-type germanium. The microwave Hall mobility was calculated from equation 2.22 with the assumptions that $\omega\tau_1$ and $\omega\tau_2$ were much less than unity and the higher order terms Bu_1 and Bu_2 were negligible. The value at room temperature here was calculated the same way as that mentioned above. This value of α corresponded to an effective sample size $\bar{l}$ = 0.687 l = (0.687) (9.7mm) = 6.665mm. The relationship between the temperature dependence of α and the conductivity of sample, and values of Y, and G were also the same as those stated in the last paragraph.

The variation in sample size correction factor (0.423 for n-type, 0.687 for p-type) for these two samples resulted from the difference in physical contact with the cavity wall when the sample was mounted with silver paint. Therefore, a correction factor for each sample must be determined independently.

In the present investigation the sample thickness might cause some deviation between the d.c. and the microwave mobilities. The skin depth of the n-type germanium sample was approximately equal to the sample thickness at room temperature (skin depth was 0.85mm and sample thickness was 1mm). This finite sample thickness will then introduce a reflection in the electromagnetic wave from the brass plate on the outside surface of the sample. The resulting field in the sample will usually be smaller than the field corresponding to no reflection. This reduction in field in the sample corresponds to a reduction in energy loss and in α as well.

Since we set the d.c. and the microwave mobilities equal at room temperature, the α value calculated was larger than the actual value. The over-estimated α value will in turn result in an over-estimated effective sample size, $\bar{l}$, according to equation 2.29. At temperatures above liquid nitrogen, the skin depth decreased because of the increasing conductivity of the sample (the skin depth was 0.301 at 80°K). In this case, the sample thickness may be considered as nearly infinite owing to the fact that the intensity of the field decreases exponentially with penetration into the sample. Therefore, the larger effective sample size at low temperatures results in larger α and a smaller microwave mobility compared to the values of these quantities at room temperatures.

The skin depth of p-type germanium was 0.60mm at room temperature and the sample thickness was 1.4mm. Hence, the ratio of skin depth to sample thickness was smaller than this ratio for the n-type sample. If the finite sample thickness for p-type germanium had any appreciable effect on the deviation between the d.c. and the microwave mobilities, the magnitude of this effect must have been much smaller than the magnitude of the effect in the n-type sample. However, as Fig. 7 and Fig. 8 show, the difference between the d.c. and the microwave mobilities for p-type germanium was much larger than this difference for the n-type sample. Therefore, the effect of finite sample thickness alone might explain the small discrepancy between the d.c. and the microwave mobilities of the n-type sample at low temperatures, but it cannot explain the large discrepancy between the d.c. and microwave mobilities of the p-type sample at low temperatures.

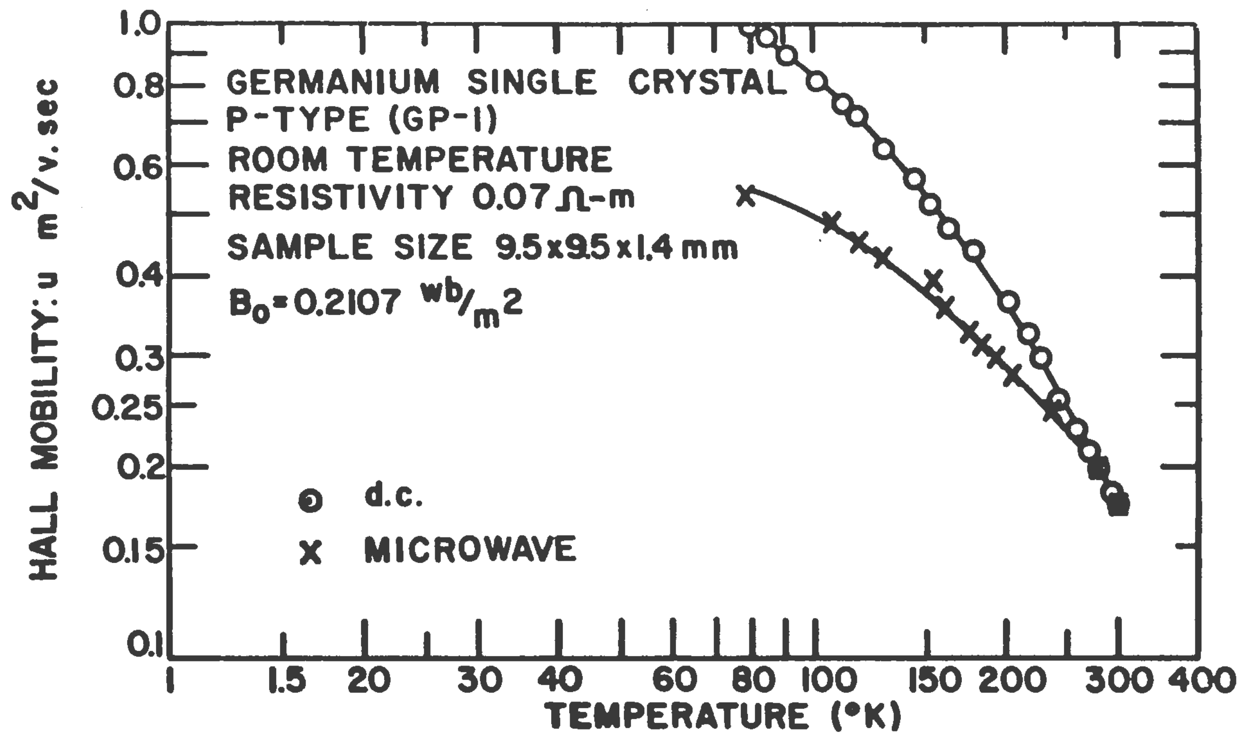

Fig. 8. Hall mobility of p-type germanium vs. temperature

V. DISCUSSION

A. N-type Germanium

According to Nishina et al.(13), the microwave Hall mobility fits fairly well with the d.c. values over the whole temperature range studied if the α used was chosen in such a way as to give the best agreement between d.c. and microwave mobility at room temperature. However, the present results indicated that a small discrepancy occurred between d.c. and microwave Hall mobilities at low temperatures even if the method of selecting α at room temperature as suggested by Nishina and Danielson (13) was followed. This disagreement may be explained as follows. The experimental carrier mobility of n-type germanium varies approximately as $T^{-1.66}$ in the temperature range 100 to 300°K (2), but the theoretical carrier mobility varies as $T^{-1.5}$ in the temperature range where the scattering by acoustic modes predominates (equivalent to $\epsilon^{-1/2}$ dependence of the relaxation time). It seems that acoustic-mode lattice scattering may be the dominant mechanism of scattering for n-type germanium in the temperature range 100 to 300°K. Then $\tau \sim \epsilon^{-1/2}$ and, as shown in equation 2.22, if $\omega\tau$ has the order of unity, a high frequency effect for the mobility can exist. Watanabe (15) also reported that the microwave Hall mobilities of n-type germanium were lower than the d.c. values at low temperatures. He assumed that $\tau = \tau_0 \left(\frac{\epsilon}{kT}\right)^{-1/2}$, where τ_0 was obtained from the d.c. Hall mobility, and that the effect of ellipsoidal energy surfaces of the conduction band could be taken into

account by replacing $\Sigma \frac{n_i}{m_i^*}$ and $\Sigma \frac{n_i}{m_i^{*2}}$ by $(\frac{n}{3})(\frac{2}{m_\perp} + \frac{1}{m_\parallel})$ and $(\frac{n}{3})$ $(\frac{2}{m_\perp m_\parallel} + \frac{1}{m_\perp^2})$ respectively in equation 2.22, where $m_\perp$ and $m_\parallel$ were transverse and longitudinal masses respectively. Watanabe then found that the theoretical microwave Hall mobility fitted the experimental results quite well. In the case of Nishina <u>et al</u>.,the deviation between the theoretical microwave Hall mobility and the corresponding d.c. values was much smaller than Watanabe's deviation because the samples contained more impurities and $\omega\tau \ll 1$. The maximum deviation was less than 1% of the measured value at liquid nitrogen temperature. Our samples were purer than Nishina's samples and the deviation (first observed by Watanabe) is clearly evident in Fig. 7. The theoretical high frequency Hall mobility of the sample used in the present investigation is shown by the dashed line in Fig. 7. The experimental results are shown to be slightly lower than the theoretical values. The maximum deviation between the theoretical and experimental values is about 4.5% at liquid nitrogen temperature. We could obtain further quantitative correlation between the theoretical and experimental data by studying additional scattering mechanisms.

B. P-type Germanium

The discrepancy between the Hall mobilities of p-type germanium measured at d.c. and microwave frequency (Fig. 8) was different from the results obtained by Nishina (14). Nishina obtained no agreement

between d.c. and microwave mobilities for p-type germanium. If Nishina had matched the d.c. and microwave mobilities at room temperature, he would have found the microwave mobility higher (not lower) than the d.c. mobility at 80°K.

The present investigation shows that the microwave and the d.c. Hall mobilities agree only at high temperatures. The same results were reported by Hambleton and Gärtner (10) and by Watanabe (15) for p-type germanium. Hambleton and Gärtner reported that at low temperatures the Hall mobilities at microwave frequency were lower than the d.c. values and showed saturation at temperatures below 100°K in p-type germanium. They attributed this discrepancy to a collision frequency comparable to the microwave frequency used. In other words, they believed that this discrepancy resulted from the high frequency limitation of the charged carrier response in the microwave field due to the inertia of the carriers. However, as pointed out by Watanabe (15) the formula used by Hambleton _et al._ would not hold owing to the high magnetic field that they applied (i.e., uB has the order of unity). Consequently, the saturation phenomenon mentioned above appeared.

As shown by equation 2.22, the mobility should be independent of frequency if the relaxation time is constant. Therefore, if any frequency dependence of the Hall mobility exists, the relaxation time must be a function of energy.

Watanabe's results also indicate that the microwave Hall mobility of p-type germanium was lower than the corresponding d.c. value at temperatures below 200°K. But he did not obtain any saturation phenomena as Hambleton and Gartner did. This may be a good indication that he carried out the experiment in the proper low field region as used in the present investigation. The same investigator made a theoretical calculation for the microwave Hall mobility using the d.c. Hall mobilities as basis and assuming the $\epsilon^{-1/2}$ dependence of the relaxation time (i.e., he considered the acoustical mode scattering only). The result, however, failed to explain the discrepancy. The failure is expected due to the fact that his assumption of the acoustical mode scattering alone is not adequate to account for the actual scattering mechanisms occurring in p-type germanium. But his conclusion that the energy dependence of the relaxation time should be steeper than $\epsilon^{-1/2}$ for p-type germanium is a reasonable guide for further investigation.

For an impurity semiconductor, the lattice scattering generally decreases with decreasing temperature because of the T^{-1} dependence of the mean free path (22) and the impurity ion scattering increases as temperature decreases (23). As pointed out by Becker (24) and Willardson and Bear (25) impurity scattering should have a very strong effect, particularly on the light-mass band, reducing its mobility even at low ionized impurity concentrations. Therefore, for p-type

germanium at low temperatures, it may be necessary not only to treat the energy dependence of the relaxation time by a mixture of acoustical mode and ionized impurity scattering, but it may also be necessary to treat the impurity scattering on the light- and heavy-mass band in different weight. It is interesting to note that Hambleton and Gärtner (10) found that there was no discrepancy between the microwave and the d.c. Hall mobilities of p-type silicon single crystal above liquid nitrogen temperatures. This difference in the effect of high frequencies on the Hall mobilities of germanium and silicon further suggests that the effect of impurity scattering on light- and heavy-mass band should be considered differently in germanium. Since p-type germanium has a higher mobility ratio of light hole to heavy hole (u_ℓ / u_h = 8) than silicon (u_ℓ / u_h = 3), it should be expected that the contribution of the light-mass band of germanium to the discrepancy between the d.c. and the microwave Hall mobility would be greater than that for silicon. It may also be worthwhile to remeasure the Hall mobilities in p-type silicon single crystals because the formula used by Hambleton _et al_.(10) might be too simple to account for the difference between the d.c. and the microwave Hall mobilities in this crystal. According to previous discussion, the difference in the Hall mobilities by d.c. and microwave frequency measurements in p-type silicon is expected to be smaller than that of p-type germanium. By comparing the degree of discrepancy between the d.c. and the microwave Hall mobilities in germanium and silicon,one may separate the effect of the relative weight of the impurity scattering on light and heavy holes in germanium.

C. Future Work

As shown by the results of some independent investigators, it is now believed that the microwave Hall mobility of p-type germanium is lower than the corresponding d.c. value at temperatures below 200°K. The next step will be to study both qualitatively and quantitatively the reason for this discrepancy. This detailed investigation probably would help to understand the effect of different scattering mechanisms on the relaxation time and, possibly, to contribute some explanation to the $T^{-2.3}$ dependence of the mobility of p-type germanium as shown by other investigators.

In order to follow this line, the immediate experimental work is to measure the Hall mobilities of p-type germanium samples at different impurity concentrations and at lower temperatures. Therefore, scattering predominantly from the lattice vibrations to scattering predominantly from ionized impurities can be studied.

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VII. APPENDIX

Table 1. d.c. electrical measurement of n-type germanium

Temperature (°K)	Hall coefficient R_h (m^3/coulomb) (x 10^{-3})	Conductivity (mho/m) (x 10^3)	Hall mobility (m^2/V sec)
76.8	11.20	0.2696	3.014
80.7	11.15	0.2554	2.848
86.7	11.11	0.2364	2.624
91.2	11.09	0.2223	2.465
96.7	11.07	0.2086	2.309
100.0	11.06	0.1993	2.204
103.7	11.05	0.1907	2.107
106.7	11.05	0.1836	2.029
111.0	11.04	0.1738	1.919
114.3	11.04	0.1681	1.856
166.3	11.10	.09571	1.062
171.0	11.24	.08919	1.003
179.4	11.20	.08234	0.9222
187.4	11.17	.07672	0.8570
197.0	11.14	.07095	0.7904
211.3	11.02	.06291	0.6933
246.3	10.72	.04855	0.5205
249.8	10.65	.04597	0.4896
262.0	10.55	.04374	0.4615
300.0	9.935	.03515	0.3492

Table 2. d.c. electrical measurement of p-type germanium

Temperature (°K)	Hall coefficient R_h (m^3/coulomb) (x 10^{-3})	Conductivity (mho/m) (x 10^3)	Hall mobility (m^2/V sec)
75.9	1.118	0.9129	1.020
79.7	1.103	0.8985	0.992
84.3	1.087	0.8774	0.953
91.4	1.070	0.8398	0.898
102.6	1.059	0.7630	0.807
111.9	1.056	0.7055	0.745
113.5	1.053	0.6949	0.733
117.4	1.055	0.6713	0.709
122.5	1.056	0.6395	0.675
129.7	1.059	0.5986	0.634
143.0	1.070	0.5276	0.565
153.0	1.080	0.4763	0.515
155.3	1.095	0.4638	0.5075
162.3	1.103	0.4310	0.476
167.5	1.108	0.4094	0.453
169.8	1.108	0.4000	0.443
172.3	1.114	0.3897	0.435
174.0	1.120	0.3839	0.430
179.5	1.197	0.3655	0.437
201.0	1.195	0.3043	0.364
217.8	1.218	0.2653	0.323
227.8	1.216	0.2454	0.298
242.3	1.182	0.2159	0.255
258.0	1.197	0.1912	0.229
270.7	1.206	0.1745	0.210
280.0	1.208	0.1636	0.198
291.8	1.208	0.1509	0.182
299.0	1.221	0.1429	0.174

Table 3. Magnetic field dependence of $\sqrt{\frac{P_2}{P_1}}\ \frac{1}{|1 + R|}$ for n-type germanium (GN2)

Magnetic field B (Wb/m^2)	$\sqrt{\frac{P_2}{P_1}}\ \frac{1}{\lvert 1 + R\rvert}$ (x 10^{-2})	$\sqrt{\frac{P_2}{P_1}}\ \frac{1}{\lvert 1 + R\rvert}$ (x 10^{-3})
	Temperature 300°K	Temperature 82°K
0.1067	1.283	6.65
0.1628	2.006	10.36
0.2107	2.693	14.05
0.2694	3.390	17.88
0.3199	4.057	20.75
0.4218	5.414	26.85
0.5154	6.695	31.77
0.6123	7.837	36.52
0.7343	9.349	42.22
0.7945	9.983	46.13
0.8695	10.723	50.23

Table 4. Magnetic field dependence of $\sqrt{\frac{P_2}{P_1}}\ \frac{1}{|1 + R|}$ for p-type germanium (GP1)

Magnetic field B (Wb/m^2)	$\sqrt{\frac{P_2}{P_1}}\ \frac{1}{\lvert 1 + R\rvert}$ (x 10^{-3})	$\sqrt{\frac{P_2}{P_1}}\ \frac{1}{\lvert 1 + R\rvert}$ (x 10^{-3})
	Temperature 300°K	Temperature 82°K
0.1628	1.228	2.098
0.2107	1.670	2.660
0.2694	2.091	3.395
0.3199	2.580	3.942
0.4218	3.220	5.140
0.5154	3.907	6.280
0.6123	4.686	7.351
0.7343	5.420	8.680
0.7945	5.830	9.274
0.8695	6.315	10.053

Table 5. Microwave Hall mobility of n-type germanium (GN2) at low temperatures

Temperature (°K)	G (x 10^{-4})	$\frac{2\alpha + G + Y_0}{2\alpha}$	$\sqrt{\frac{P_2}{P_1}} \frac{1}{\lvert 1 + R \rvert} \frac{2}{B_0}$	Hall mobility: u m^2/V sec
80.3	2.30	11.573	22.03	2.55
110.4	2.74	10.734	16.49	1.77
113.7	2.79	10.632	16.55	1.76
131.5	3.03	10.060	14.31	1.44
156.0	3.32	9.310	12.35	1.15
158.8	3.35	9.248	12.00	1.11
168.5	3.47	9.080	11.65	1.06
180.8	3.61	8.858	10.61	0.94
182.9	3.63	8.842	10.52	0.93
197.0	3.79	8.609	9.07	0.78
213.2	3.96	8.364	8.13	0.68
231.2	4.14	8.069	7.34	0.60
257.3	4.39	7.681	6.11	0.45
271.5	4.53	7.531	5.56	0.42
299.2	4.80	7.248	4.90	0.36

Table 6. Microwave Hall mobility of p-type germanium (GP1) at low temperature

Temperature (°K)	G ($\times 10^{-4}$)	α ($\times 10^{-4}$)	$\frac{2\alpha + G + Y_o}{2\alpha}$	$\sqrt{\frac{P_2}{P_1}} \frac{2}{\lvert 1 + R \rvert}$ ($\times 10^{-2}$)	Hall mobility: u (m^2/sec)
78.5	2.28	0.0909	23.83	2.208	0.526
78.0	2.27	0.0908	23.80	2.234	0.532
106.5	2.68	0.1001	23.73	2.030	0.482
118.2	2.83	0.1055	23.27	1.922	0.447
128.4	2.97	0.1100	23.00	1.854	0.426
154.0	3.30	0.1257	21.56	1.810	0.390
160.3	3.37	0.1297	21.20	1.674	0.355
178.0	3.57	0.1416	20.20	1.624	0.328
183.2	3.64	0.1449	20.01	1.544	0.309
194.0	3.76	0.1522	19.49	1.525	0.297
205.5	3.88	0.1597	19.00	1.452	0.276
237.8	4.22	0.1820	17.73	1.371	0.243
255.0	4.39	0.1948	17.06	1.213	0.207
276.6	4.58	0.2113	16.263	1.212	0.197
300.0	4.8	0.2275	15.659	1.111	0.174