THE EFFECT OF TEXTURE ON ACOUSTOELASTICITY

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ABSTRACT

The theory for stress determination using acoustoelasticity is most frequently based on the evaluation of the motion of an infinitesimal plane wave propagating through an isotropic, elastic body which is subjected to a homogeneous deformation. The assumption of isotropy in this analysis allows the characterization of the acoustoelastic response to be carried out in terms of two second-order and three third-order elastic constants. Unfortunately, most structural materials do not behave isotropically, but instead have some degree of texture caused by the crystals aligning themselves in certain preferred orientations during the forming process. This paper examines the effect of texture on the acoustoelastic response of polycrystalline materials. In particular, the five second-order and nine third-order elastic constants of bodies exhibiting transverse isotropy are computed in terms of the elastic constants and orientation of the constituent crystals. Two methods of evaluating the constants are presented, the first being a Voigt type procedure in which the elastic stiffnesses are averaged for the chosen crystal orientation distribution, and the second being a Reuss type procedure in which the compliances are averaged. Acoustoelastic constants of the various waves in aluminum and copper are presented for the entire range of ideal textures in which all grains have the same direction cosines between the symmetry axis and the crystallographic axes.
INTRODUCTION

Acoustoelasticity is a nondestructive technique for the evaluation of active and residual stresses within structural components. It is based on the fact that the speeds at which ultrasonic plane waves propagate through a body depend on the state of stress in that body. While acoustoelasticity has been demonstrated to be an effective technique under laboratory conditions, several difficulties have prevented its widespread use in more realistic situations. Among the most serious of these problems is the effect of texture on the acoustoelastic response in a polycrystalline material.

Texture, in the sense of this work, is the arrangement of the grains of an aggregate into certain preferred orientations in such a way as to make the aggregate respond anisotropically on the macroscopic level. The anisotropy caused by texture is present in nearly all structural metals, but until recently has been avoided in studies of the acoustoelastic effect. In the past several years, though, more effort has gone into overcoming the troublesome aspects of texture.

The problems caused by the initial anisotropy of the polycrystalline aggregate are twofold: First, since the material is anisotropic, shear waves propagate with speeds dependent on their polarization thus masking the change in wave speed due to stress; second, material characterization becomes more difficult as there are more elastic constants to identify than in the case of an isotropic body. Most of the previous work involving textured materials has focused on the first problem. The work reported in this paper deals with the second. Specifically, the second-order and third-order elastic constants of a polycrystalline aggregate are computed from the elastic constants of the constituent crystals along with information on the orientation distribution of the crystals. The particular case of a transversely isotropic aggregate made up of cubic crystals in the highest symmetry class is examined in detail. This work follows earlier studies which gave the third-order elastic constants of isotropic polycrystals and also that of Pursey and Cox who evaluated the second-order constants of a transversely isotropic aggregate. Details of the relations between the constants of the crystal and the aggregate may be found in Ref. 14.

In order to arrive at the macroscopic elastic response of the polycrystal, the response of the individual crystals must be averaged in some way. Two methods of averaging are presented here -- an extension of the Voigt procedure in which the aggregate is assumed to be in a state of homogeneous strain, allowing the stress to be discontinuous from grain to grain, and an extension of Reuss averaging in which the stresses are assumed to be uniform from grain to grain, but with compatibility of grain displacements not satisfied.
The results of this procedure are demonstrated by evaluating the second-order, third-order and acoustoeelastic constants for ideal textures of aluminum and copper. In the case of aluminum, a comparison of the predicted results for the acoustoeelastic constant with measurements made on structural alloys (texture unknown) shows good quantitative agreement. This study shows that the acoustoeelastic response of a polycrystal is strongly dependent on the texture, but that reasonable predictions can be made with knowledge of the single crystal response and the degree of texture.

ELASTIC CONSTANTS OF THE AGGREGATE

In discussing the computation of quantities in the polycrystalline aggregate, it is convenient to consider two reference frames in the undeformed state, \( X \) and \( X^* \), called the body axes and crystalline axes, respectively. We choose \( X_3 \) to be the symmetry axis of the aggregate. For any particular grain these sets of axes are related as

\[
X^* = t X
\]

where \( t \) is a proper orthogonal transformation matrix.

If the body is deformed, the positions in the body and crystalline representations are \( x \) and \( x^* \), respectively with

\[
x^* = t x
\]

The deformation gradients of this motion are then

\[
F = \frac{\partial x}{\partial \tilde{x}} , \quad F^* = \frac{\partial x^*}{\partial \tilde{x}^*} = t F T
\]

while the associated Lagrangian strains are

\[
2E = \tilde{F}^T \tilde{F} - \tilde{I} , \quad 2E^* = \tilde{F}^{*T} \tilde{F}^* - \tilde{I} = 2t E t^T
\]

The constitutive relation between stresses and strains will be contained within strain energy functions \( \Phi \) and \( \Phi^* \) for the aggregate and crystal respectively. The symmetric Piola-Kirchoff stresses are then

\[
S = \rho_0 \frac{\partial \Phi}{\partial E} , \quad S^* = \rho_0 \frac{\partial \Phi^*}{\partial E^*}
\]
The strain energy of the aggregate may be expressed in terms of second- and third-order elastic constants (Brugger's convention) as

$$\rho_o \Phi = \frac{1}{2} C_{ijkl} E_{ij} E_{kl} + \frac{1}{6} C_{ijklmn} E_{ij} E_{kl} E_{mn}$$

(6)

Similarly, the strain energy of an individual grain may be written

$$\rho_o \Phi^* = \frac{1}{2} C^*_{ijkl} E^*_{ij} E^*_{kl} + \frac{1}{6} C^*_{ijklmn} E^*_{ij} E^*_{kl} E^*_{mn}$$

(7)

Using Eq. (4) we can also write

$$\rho_o \Phi^* = \frac{1}{2} \bar{C}_{ijkl} E_{ij} E_{kl} + \frac{1}{6} \bar{C}_{ijklmn} E_{ij} E_{kl} E_{mn}$$

(8)

where

$$\bar{C}_{ijkl} = t_{pi} t_{qj} t_{rk} t_{sl} C^*_{pqrs}$$

(9)

and

$$\bar{C}_{ijklmn} = t_{pi} t_{qj} t_{rk} t_{sl} t_{um} t_{vn} C^*_{pqrsuv}$$

(10)

Since a "continuum point" in the aggregate is assumed to contain many individual grains, the strain energy of the aggregate will be the average strain energy of these grains for a given crystalline orientation distribution. Thus, the stiffness tensors for the aggregate in Eq. (6) are the averages of the stiffness tensors for the individual grains, Eqs. (9) and (10), with the average taken over the particular crystalline orientation distribution for that aggregate.

In performing this averaging in the case of a transversely isotropic material, it is convenient to break the orientation tensor into two parts as

$$\bar{t} = t \bar{t}$$

(11)

where $\bar{t}$ represents the rotation about the $x_3$ axis

$$\bar{t} = \begin{bmatrix}
\cos \phi & \sin \phi & 0 \\
-sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}$$

(12)

while $\bar{t}$ gives the remaining rotation needed to bring the crystal to any arbitrary orientation,
The parameters $\psi, \theta,$ and $\phi$ are the Euler angles for the crystal orientation.

We perform the averaging for ideal textures (i.e., for particular values of $\theta$ and $\psi$) in which the direction cosines between crystal axes and the symmetry axis of the aggregate are the same for all crystals. In this case the averaging then takes place only about the symmetry axis so that

$$C_{ijkl} = \frac{1}{4\pi} \int_0^{2\pi} \bar{C}_{ijkl} \, d\phi.$$  

A similar expression holds for $C_{ijklmn}$. This integral is evaluated by expanding $C$ from Eqs. (9) and (10) in a Fourier series in $\phi$,

$$\bar{C} = \bar{C}^0 + \sum_{n=1}^{\infty} \left[ \bar{c}^n \cos n\phi + \bar{c}^s \sin n\phi \right].$$  

Using Eq. (15) in Eq. (14) we find that the only nonzero term of the integral is the constant term, $\bar{C}^0$, and that this constant term is itself the stiffness of the aggregate.

The procedure outlined above is a method for averaging stiffnesses assuming that the strain within each grain is the same and is that of the aggregate. It is an extension of Voigt averaging for determining the second-order stiffness of isotropic aggregates. Another method of averaging which assumes that the stress is constant throughout the aggregate is an extension of the Reuss averaging of elastic compliances. In order to use this, we need the definition of third-order compliances given by Brugger and Barsch. In terms of the stiffnesses, the compliance tensors are

$$S_{ijkl} = \frac{1}{2} \left( \delta_{im} \delta_{jn} + \delta_{in} \delta_{jm} \right)$$  

and

$$S_{ijklmn} = -S_{ijpq} S_{klrs} S_{mnuv} C_{pqrsuv}$$  

The analysis to this point is valid for transversely isotropic aggregates of any crystal. The results of this procedure for the Voigt average in the case of cubic crystals of the highest symmetry (m3m, 432 and 43m) have been presented in Ref. 14. It is shown in
Ref. 14 that the five second-order and nine third-order constants of the aggregate may be written in terms of the three second-order and six third-order constants of the crystal, along with two orientation factors

$$A = \frac{-2}{13} - \frac{2}{23} + \frac{2}{23} - \frac{2}{33} + \frac{2}{33} - \frac{2}{13}$$

$$= \sin^2 \psi \cos^2 \psi + \sin^2 \theta \cos^2 \theta \cos^4 \psi$$

(18)

and

$$B = \frac{-2}{13} - \frac{2}{23} - \frac{2}{33} = \cos^2 \theta \sin^2 \theta \cos^4 \psi \sin^2 \psi$$

(19)

These orientation factors are combinations of the direction cosines between the crystal axes and the symmetry axis and are the only way in which the texture appears in the solution.

For cubic crystals, an aggregate which is isotropic in its second- and third-order constants can be formed by choosing $\theta$ and $\psi$ such that $A = 1/5$ and $B = 1/105$ ($\theta = 26.01^\circ$, $\psi = 15.46^\circ$). In this case the results of the analysis in Ref. 14 (and as outlined above) agree with those of previous work on evaluating third-order elastic constants in isotropic polycrystals9-12. Additionally, the second-order elastic constants of the transversely isotropic aggregate are equivalent to those found by Pursey and Cox13.

RESULTS FOR ALUMINUM AND COPPER

As an example of the utility of the procedure given in the previous section, the second- and third-order elastic constants, along with the acoustoelastic constants for various waves are computed as functions of texture for polycrystalline aggregates of aluminum and copper. Acoustoelastic constants in this paper are given as the relative velocity change of a certain wave per unit applied stress. Single crystal data for these materials are taken from the literature16,17 and are summarized in Table 1.

The particular loading case chosen for examination of the acoustoelastic constancs is uniaxial tension along the symmetry axis, with waves propagating either along or perpendicular to the loading direction. This choice assures that all waves considered will be pure mode.

Results for the various constants are presented for the full range of ideal textures (for which every grain has the same direction cosines between its axes and the symmetry axis of the aggregate). This presentation is accomplished by use of the technique of
stereographic projection familiar to metallurgists in which contours of the constants are plotted and the position on the plot defines the ideal texture. A brief description of this display is included here. More detailed information is available in a number of texts (Barrett and Massalski for example).

### TABLE 1. ELASTIC CONSTANTS FOR SINGLE CRYSTALS (GPa)

<table>
<thead>
<tr>
<th></th>
<th>Aluminum</th>
<th>Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{1111}$</td>
<td>106.7</td>
<td>166.1</td>
</tr>
<tr>
<td>$C_{1122}$</td>
<td>60.4</td>
<td>119.9</td>
</tr>
<tr>
<td>$C_{1212}$</td>
<td>28.3</td>
<td>75.6</td>
</tr>
<tr>
<td>$C_{111111}$</td>
<td>-1076</td>
<td>-1271</td>
</tr>
<tr>
<td>$C_{111122}$</td>
<td>-315</td>
<td>-814</td>
</tr>
<tr>
<td>$C_{112233}$</td>
<td>36</td>
<td>-50</td>
</tr>
<tr>
<td>$C_{111212}$</td>
<td>-340</td>
<td>-780</td>
</tr>
<tr>
<td>$C_{112323}$</td>
<td>-23</td>
<td>-3</td>
</tr>
<tr>
<td>$C_{122331}$</td>
<td>-30</td>
<td>-95</td>
</tr>
</tbody>
</table>

Consider a typical crystallite with an imaginary reference sphere around it. The symmetry axis, which makes certain angles with the grain axes, intersects this sphere at a particular point. A stereographic projection of the points on the sphere onto a plane then allows a two-dimensional representation of the position at which the symmetry axis intersects the sphere. To envision the stereographic projection, imagine that the reference sphere is transparent and that the points to be projected are solid dots on the surface. If a light source is placed on the surface of the sphere and a projection screen is placed tangent to the sphere, diametrically opposite the light, the pattern on the screen is the stereographic projection of the points on the sphere. Note that every point on the sphere has a unique projection point on the plane. In this work we choose the location of the illumination point to be along one of the crystalline axes; that is, in a crystalline direction of the type $<100>$. Since, for the case of cubic crystals, the crystal axes are indistinguishable from one another we need only consider a small region of the projection called the unit stereographic triangle. Positions within this triangle then define all possible ideal textures.

A few of the constants for the aluminum aggregate are found in Figures 1-3. For this particular material only the Voigt aver-
ages are presented since the results for the Reuss procedure are nearly identical to those shown. That the two methods give such similar results for aluminum is not unexpected given the small degree of anisotropy of the second-order constants found in the single crystal. Contours for the second-order constants $C_{1111}$ and $C_{2323}$ are found in Fig. 1. We note the value of the Voigt average for an aggregate in which all crystals have a $<100>$ type direction in the symmetry direction is found at the left corner of the stereographic triangle, while textures with $<110>$ and $<111>$ directions in the symmetry direction give values found in the lower and upper right corners, respectively. Values for $C_{1111}$ vary from 109.3 GPa for a $<100>$ texture to 111.8 GPa for a $<111>$ texture, a difference of only 2.2%. Values for $C_{2323}$ vary somewhat more with the minimum and maximum differing by 12%.

Third-order elastic constants $C_{111111}$ and $C_{332323}$ are shown in Fig. 2. In this figure we see a significant increase in the effect of texture on the value of the constant, with $C_{111111}$ and $C_{332323}$ varying by 9.3% and 40.2%, respectively. In addition, there is a marked difference in the overall shape of the contours for $C_{332323}$ as compared to the other constants. Specifically, $C_{1111}$, $C_{2323}$ and $C_{111111}$ increase or decrease monotonically as the texture goes from $<100>$ to $<111>$, while for $C_{332323}$ the maximum occurs at $<110>$ and the value decreases toward both other extremes.

This same characteristic shape is found in the contours for the acoustoelastic constants shown in Fig. 3. The constant for a longitudinal wave propagating perpendicular to the symmetry axis (change in $V_{11}$) varies from 5.7 TPa$^{-1}$ at $<110>$ to 14.8 TPa$^{-1}$ at $<111>$, a 62% difference. The effect of texture on the acoustoelastic constant for a shear wave propagating along the symmetry axis ($V_{31}$)
is even more severe. In fact, the predicted value of this constant actually changes sign, being positive near \(<110>\) textures and negative elsewhere.

We note that the effect of texture is stronger for the shear constants and that it becomes increasingly pronounced as we go from second-order to third-order to acoustoelastic constants. Again, given the slight anisotropy of the second-order constants, this observation is not in itself too surprising.
In order to compare the results of this work with available experimental data, we examine the acoustoelastic constant for a wave propagating normal to the loading direction. In a recent report, Hunter\(^{19}\) shows acoustoelastic results for several specimens of nominally identical aluminum alloys, one from a 6061-T6 bar and the other from 6061-T651 plate. Hunter shows that the acoustoelastic constants are 13.4 TPa\(^{-1}\) for the bar and 5.7 TPa\(^{-1}\) for the plate (see Table 2 in Reference 19). It is also shown\(^{19}\) that the grains in the bar are highly elongated while those in the plate are more nearly equiaxed. Figure 4 gives Hunter's results along with the extremes obtained in this work (<110> and <111> textures). It is seen that the experimental data, while showing a large difference, falls within the predicted extremes. Thus, it is suggested that much of the variability in the acoustoelastic constants reported in the literature may be due to differing textures, not necessarily differences in the underlying material constants.

![Graph](image)

Figure 4. Relative velocity change versus stress - a comparison of experiment with prediction for polycrystalline aluminum. (Experimental data after Hunter\(^{19}\).)
Values for the constants in polycrystalline copper show the same basic characteristics as those for aluminum. It is interesting, though, that copper single crystals exhibit significant anisotropy in their second-order constants. The result of this anisotropy is that the Voigt and Reuss averaging procedures give different predictions for the various constants. Figure 5 shows the results of the two procedures for the second-order constant $C_{1111}$. The Voigt averages range from 192 GPa to 219 GPa, while the Reuss averages range from 178 GPa to 203 GPa, both indicating a 12.3% variability. A point-by-point comparison of the two techniques shows that the Reuss average gives value for $C_{1111}$ which are about 7% below those for the Voigt average.

![Voigt](image1.png)  
Voigt

![Reuss](image2.png)  
Reuss

Figure 5. Aggregate Values for $C_{1111}$ in Copper (GPa).

Results for $C_{111111}$ are found in Fig. 6 where we see a 25% variation for the Voigt procedure and a 43% variation for the Reuss procedure. The absolute value of the constants from the Reuss method are between 7% and 30% below for the Voigt method.

The acoustoelastic constant for $V_{11}$ in copper is given in Fig. 7. In this figure, we find a major difference in the responses predicted by the two averaging procedures. In particular, near the $<100>$ texture the Voigt average predicts a positive constant while the Reuss procedure predicts a negative value. Such a large difference between the two averaging techniques is an indication of how sensitive the acoustoelastic response is to the actual stress and strain distribution within the aggregate.

In order for the averaging to be of quantitative value, more information must be known about which mode of deformation (uniform stress or uniform strain) is the appropriate approximation.
CONCLUSIONS

A procedure has been outlined which allows the second-order, third-order and acoustoelastic constants of a textured polycrystalline aggregate to be computed from the elastic constants of the constituent crystals. This work has focused on ideal textures which exhibit transverse isotropy and are made up of cubic crystals. Examples of the results for aggregates of aluminum and copper have been presented, and for the case of aluminum, the acoustoelastic constants have been compared with experimental data.

The study shows that texture plays a significant role in the acoustoelastic response of polycrystalline bodies. It is therefore
suggested that the large variability in reported acoustoelastic constants may be caused mainly by differing textures. Although quantitative measurements of texture are not easily obtained, such data along with acoustoelastic response would help to clear up the great variability in acoustoelastic constants.

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REFERENCES


