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Demand price sensitivity and market power in a congested fuel and electricity network

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***Abstract*—Fixed demands for electricity are incorporated into a game theoretic model of strategic generators who are supplied by a fuel transportation network and produce power for a congested electricity transportation network governed by an independent system operator. Some counter-intuitive effects on measures of market power result from reducing the proportion of demands that are fixed rather than price-sensitive. Examination of the dual prices found in the complementarity problem's solution reveal how these effects result from either a load pocket created by transmission congestion or the cost structure induced by the fuel supply network. One of the counter-intuitive effects also appears in a competitive benchmark model.**

I. Introduction

In restructured electricity markets, generating companies submit bids to supply electricity at prices based on their marginal costs, which are driven largely by fuel costs. In each regional wholesale power market, an independent system operator (ISO) manages electricity transmission and sets locational marginal prices (LMPs) to match supplies with demands at each location on the constrained grid. To understand the interaction between constraints on fuel supply and constraints on electricity transmission, we recently developed and tested a game theoretic model that combines both sets of constraints [1], [2]. It includes (1) costs of extracting and transporting finite supplies of fuels across routes with limited capacity, (2) strategic decisions of generators at different locations across a congested electricity transmission network, (3) price-sensitive demands for electricity, and (4) matching of electricity supply and demand across the network to maximize total social welfare subject to physical transmission constraints. Given the fuel and electricity network topology and capacities, fuel supplies and costs, transmission line reactances, and demand functions, the model produces LMPs and quantities of electricity generated and consumed at each location on the grid. These represent a static view of, say, a particular hour in some scenario of cost, capacity and demand.

Market equilibrium models typically rely on demand functions with finite negative slopes but, in the absence of clear price signals or the ability to respond to them, most demand for electricity is not actually sensitive to price. For example, according to [3], in MISO "only about 1% of the total bid-in demand for the day-ahead market is price sensitive." If the demand lacks price sensitivity, there is potential for strategic generators to exert market power by withholding production to raise prices. In this paper, we modify the model of [1] to include fixed as well as price-sensitive demands and test the effects on measures of market power of changing the levels of fixed demand. For each level of fixed demand and each hour in a time horizon, an equilibrium is computed by solving a linear mixed complementarity problem. For comparison, equilibria are also computed in a competitive benchmark model in which the ISO dispatches price-taking generators supplied by the same fuel network.

In a numerical example from [2], we examine the behavior of two measures of market power, the Lerner Index and the Relative Market Advantage Index [4], for each generator as amounts of fixed demands are systematically decreased but an invariant linear price-quantity relationship governs demand beyond the fixed quantities. Both measures of market power would be expected to decrease along with the fixed demands as a larger proportion of the demand effectively becomes sensitive to price. Contrary to this intuition, in the strategic-generator model, we observe that these market power measures can increase for some generators as the fixed demands are reduced from their maximum levels. By decomposing the electricity prices and generator marginal costs into the dual prices of different constraints, we find that the Lerner Index for a low-cost generator may increase under congestion because the dual price of either its capacity constraint or its residual demand increases. It may also increase as fixed demands diminish in the absence of congestion due to the cost structure derived from the fuel network. Increasing Lerner Indices due to increasing dual prices of generating capacity also appear in the competitive benchmark case. Moreover, the Relative Market Advantage Index may increase as the drop in fixed demands from their maximum levels reduces the severity of a load pocket.

II. Model and Notation

As in [1] and [2], we model the electricity system along with its fuel supply as a network of electricity nodes and fuel supply nodes with fuel transportation routes as directed arcs from the fuel nodes to the electricity nodes and electricity transmission lines as directed arcs connecting the electricity nodes. Table I specifies the sets of nodes and arcs. Assume each electricity node in N has exactly one load-serving entity (LSE) and one generator. We allow for nodes without generators by setting their generation capacities to zero, while multiple generators at the same node can be modeled by connecting dummy nodes with infinite-capacity transmission lines.

The data for the model include parameters of linear demand functions as well as fixed demand quantities for the LSEs;

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TABLE II
MODEL PARAMETERS

Parameter	Description	Dimension
a	Intercepts of electricity demand prices as linear functions of quantities	$n_E \times 1$
b	Slopes of electricity demand prices as linear functions of quantities	$n_E \times 1$
L	Fixed demands	$n_E \times 1$
V	Generation capacities	$n_E \times 1$
D	Power transfer distribution factors based on a fixed reference node $r \in N$	$m_E \times n_E$
K	Transmission line capacities	$m_E \times 1$
Z	Quantities of fuel available	$n_F \times 1$
c	Costs per MWh-equivalent of fuel transported	$m_F \times 1$
U	Capacities of fuel supply lines	$m_F \times 1$

TABLE I
SETS OF NODES AND ARCS

Set	Description	Indices	Cardinality
N	Electricity nodes	i, j	n_E
B	Electricity transmission lines	l	m_E
F	Fuel supply nodes	g	n_F
A	Fuel supply lines	gj	m_F

capacities of generators, transmission lines, and fuel supply routes; and quantities of fuels available together with their costs. The model parameters are described in Table II. Assume each demand function slope $b_i < 0, i \in N$. Any node i without a load has $L_i = a_i = 0, b_i = -\infty$. Flows on the transmission lines are modeled in terms of a lossless direct current approximation of Kirchhoff's laws using power transfer distribution factors (PTDFs). The element $D_{l,j}$ specifies the change in flow on line l that results from a one-unit injection of electricity at node j accompanied by a corresponding one-unit withdrawal at the fixed reference node. For simplicity, we assume that generation costs are due solely to fuel. Multiple fuel types may be included in the fuel supply network, where all flows are in MWh of energy content. The cost per unit of flow on a fuel arc (f, j) includes all costs for procuring the fuel at the origin f, transporting it, and converting it to electricity at the destination j. A given generator may obtain fuels from multiple supply nodes at different costs and each fuel node may supply multiple generators. The fuel network in this paper ignores the different time horizons that arise from fuel storage capabilities and assumes costs and capacities are on an hourly basis. A non-bipartite network incorporating storage and multiple time scales [5] could be substituted.

The model centers on decisions of the generators. We assume that each is an independent Cournot competitor that decides its own generation quantity, given the corresponding quantities of other generators. In the Nash-Cournot equilibrium model, generators make these decisions simultaneously. We combine these decisions with downstream decisions of an independent system operator (ISO) that determines quantities of electricity supplied and consumed at each node, given the generation quantities, by specifying nodal injections. Rather than assuming a known form for the generation cost, such as linear or quadratic, we explicitly model costs that result from a fuel supply network by including the fuel flows upstream of the generators. We assume these flows are decided by a non-strategic fuel dispatcher that minimizes the total cost of delivering the fuel required by the generators given their decisions. Primal and dual decision variables are summarized in Table III. The dual variable η represents the price of electricity at the reference node while p is the vector of nodal electricity prices. Also define the nodal price premia, $\phi \equiv p - \eta e$, where e is a vector of ones.

Let $F_j \subseteq F$ be the set of fuel supply nodes g such that there exists an arc from g to j and N_g be the set of electricity nodes j supplied by fuel node g. The fuel dispatcher's (primal) decision problem is a linear transportation problem:

$$
\begin{array}{llll}
\min_{x\geq 0} & \sum_{gj\in A} c_{gj}x_{gj} & & \\
\text{s.t.} & \sum_{j\in N_g} x_{gj} \leq Z_g, & \forall g \in F & [\omega_g \geq 0] \\
 & \sum_{g\in F_j} x_{gj} \geq y_j, & \forall j \in N & [\pi_j \geq 0] \\
 & x_{gj} \leq U_{gj}, & \forall gj \in A & [\rho_{gj} \geq 0]
\end{array}
$$

Here, the first set of constraints are on the fuel supplies, the second set ensure that the fuel delivery to each generator meets the demand created by electricity production, and the third enforce the fuel transportation capacities. We assume without loss of generality that $Z_g > 0, \forall g \in F$, and $U_{gj} \geq 0, \forall gj \in A$. The Karush-Kuhn-Tucker (KKT) conditions necessary for optimality are:

$$0 \leq x_{gj} \perp c_{gj} + \omega_g - \pi_j + \rho_{gj} \geq 0, \quad \forall gj \in A \quad (1)$$

$$0 \leq \pi_j \perp \sum_{g\in F_j} x_{gj} - y_j \geq 0, \quad \forall j \in N \quad (2)$$

$$0 \leq \omega_g \perp Z_g - \sum_{j\in N_g} x_{gj} \geq 0, \quad \forall g \in F \quad (3)$$

$$0 \leq \rho_{gj} \perp U_{gj} - x_{gj} \geq 0, \quad \forall gj \in A \quad (4)$$

Also given the decisions of the generators, the ISO's decision problem is to maximize social welfare, which is the total consumer willingness-to-pay less the sum of all the generation costs. It is equivalent to the sum of consumers' surplus, producers' surplus, and transmission rents, where the transmission rent from the flow on a line is the amount of flow multiplied by the difference in the nodal prices at either end. The consumer willingness-to-pay can be evaluated as the total area under the demand functions up to the quantities supplied. On the other hand, the total generation cost is constant with respect to the ISO's decisions and, therefore, can be omitted

TABLE III
DECISION VARIABLES

Variable	Description	Player	Dimension
x	Quantities of fuel delivered	Fuel Dispatcher	$m_F \times 1$
ω	Dual values for supplies at fuel nodes	Fuel Dispatcher	$n_F \times 1$
π	Dual values for demands by generators	Fuel Dispatcher	$n_E \times 1$
ρ	Dual values for fuel line capacities	Fuel Dispatcher	$m_F \times 1$
q	Demand satisfied at electricity nodes	ISO	$n_L \times 1$
r	Net injections of electricity at electricity nodes	ISO	$n_E \times 1$
η	Dual value for total generation equals satisfied demand	ISO	Scalar
p	Dual values for electricity market clearing conditions	ISO	$n_E \times 1$
δ	Dual values for fixed demands for electricity	ISO	$n_E \times 1$
λ^+	Dual values for upper bounds on transmission from lower- to higher-numbered node	ISO	$m_E \times 1$
λ^-	Dual values for upper bounds on transmission from higher- to lower-numbered node	ISO	$m_E \times 1$
y	Generation amounts	Generators	$n_E \times 1$
β	Dual values for aggregated electricity demand	Generators	$n_E \times 1$
μ	Dual values for generation capacities	Generators	$n_E \times 1$

from the primal formulation as a quadratic program:

$$\begin{array}{llll} \max_{q,r} & \sum_{j\in N}\left(a_j q_j + \frac{1}{2} b_j q_j^2\right) & & \\ \text{s.t.} & \sum_{j\in N} r_j = 0 & & [\eta] \\ & q_j - r_j = y_j & \forall j \in N & [p_j] \\ & q_j \geq L_j & \forall j \in N & [\ \delta_j \geq 0] \\ & \sum_{j\in N} D_{lj} r_j \leq K_l & \forall l \in B & [\lambda_l^+ \geq 0] \\ & -\sum_{j\in N} D_{lj} r_j \leq K_l & \forall l \in B & [\lambda_l^- \geq 0] \end{array}$$

The five sets of constraints represent, respectively, requirements that there is no net injection by the ISO, conservation of energy at each node, satisfaction of fixed demands, and thermal capacity constraints on the transmission flows in either direction. Note that, given the fixed demand constraints, the objective is equivalent to maximizing $\int_{L_j}^{q_j}(a_j + b_j s)\,ds$, which represents the total consumer willingness-to-pay for price-sensitive quantities beyond the fixed demands. The ISO KKT conditions are:

$$a_j + b_j q_j - p_j + \delta_j = 0 \quad \forall j \in N \tag{5}$$
$$-\eta + p_j - \phi_j = 0 \quad \forall j \in N \tag{6}$$
$$\sum_{j\in N} r_j = 0 \tag{7}$$
$$q_j - r_j = y_j \quad \forall j \in N \tag{8}$$
$$0 \leq \delta_j \perp -L_j + q_j \geq 0 \quad \forall j \in N \tag{9}$$
$$0 \leq \lambda_l^+ \perp K_l - \sum_{j\in N} D_{lj} r_j \geq 0 \quad \forall l \in B \tag{10}$$
$$0 \leq \lambda_l^- \perp K_l + \sum_{j\in N} D_{lj} r_j \geq 0 \quad \forall l \in B, \tag{11}$$

where $\phi_j = \sum_l D_{lj}(\lambda_l^+ - \lambda_l^-), \quad \forall j \in N$.

Next we consider generator i's problem of maximizing profit, which is the difference of revenue and generation cost, subject to its capacity constraint. Revenue depends on the ISO decisions while cost is determined by the fuel dispatch. We will set the fuel charges such that each generator pays for just the fuel that it uses (there is no congestion charge); this means that generator i is charged an amount $\sum_{g|gi\in A} c_{gi} x_{gi}$, determined from the optimal solution to the fuel dispatch problem. Ryan et al. [1] proved that in the case of unlimited fuel supply ($Z = \infty$) it is sufficient for generator i to see only π_i, the marginal cost of fuel associated with its current generation level.

Generator i sells its production at the price p_i determined by the ISO. For tractability, we adopt the bounded rationality assumption of [6], in which each generator observes the price premia at each node relative to the reference node, and does not anticipate the effect of its decisions on the transmission quantities. Specifically, the ISO's dual decision variables, ϕ, are constants in each generator's decision problem (as are the fuel dispatcher's dual decision variables, π). In [1] and [2], generator i's decision problem is to maximize its profit, given by $(\eta + \phi_i - \pi_i) y_i$, subject to its capacity constraint $y_i \leq V_i$ as well as a constraint equating total market supply with total market demand. All of the generator problems are solved simultaneously with the ISO and the fuel dispatch problems. Upon inclusion of the fixed demand constraints in the ISO problem, its KKT conditions (5) - (8) together imply that

$$\sum_j y_j = \sum_j \frac{\eta + \phi_j - \delta_j - a_j}{b_j}.$$

For consistency, therefore, we assume that generator i accounts for the price increases δ induced by the fixed demands as well as the nodal price premia ϕ induced by the transmission constraints. It strategically chooses a generation amount y_i to set the reference node price according to the residual market demand given the other generation amounts. Generator i's decision problem is:

$$\begin{array}{lll} \max_{y_i \geq 0, \eta} & (\eta + \phi_i - \pi_i) y_i & \\ \text{s.t.} & y_i - \left(\sum_{j=1}^{n_E} \frac{1}{b_j}\right)\eta = & \\ & \sum_{j=1}^{n_E} \frac{\phi_j - \delta_j - a_j}{b_j} - \sum_{j\neq i} y_j & [\beta_i] \\ & y_i \leq V_i & [\mu_i \geq 0] \end{array}$$

The whole set of generator KKT conditions is given by:

$$0 \leq y_j \perp -\eta - \phi_j + \pi_j + \beta_j + \mu_j \geq 0, \quad \forall j \in N \tag{12}$$
$$0 \leq \mu_j \perp V_j - y_j \geq 0, \quad \forall j \in N \tag{13}$$
$$y_j + \left(\sum_{i\in N} \frac{1}{b_i}\right)\beta_j = 0, \quad \forall j \in N \tag{14}$$

The generator residual demand constraints are omitted due to redundancy with the ISO conditions.

If an equilibrium exists, it may be identified by solving the linear mixed complementarity problem given by (1) - (14). The existence of an equilibrium for this game was established in [1] for the case where $L = 0$. The proof has not yet been extended to $L > 0$ but we conjecture that existence will hold under suitable conditions on the fuel transportation, generation, and electricity transmission capacities.

III. Competitive Benchmark

Assume henceforth that $Z = \infty$, in which case the fuel dispatch problem can be decomposed into separate problems for each generator. For this case, Ryan et al. [1] showed that an equilibrium for the problem given in Section II is also an equilibrium in a modified problem where each generator optimizes its own fuel supply from the network. A competitive benchmark can be constructed by omitting the separate generator optimization problems and instead including their decisions (including fuel procurement) in the ISO optimization problem. In this case, the ISO dispatches the generators to maximize consumer willingness-to-pay less the true generation costs derived from the fuel network. The KKT conditions are identical to (1) - (13) with $\beta_j \equiv 0, \forall j$.

In constrast, the economic interpretation of β_j in a strategic-generator equilibrium (found by solving (1) - (14)) can be seen by substituting conditions (12) into generator i's residual demand constraint and simplifying the latter to

$$y_i = \sum_j q_j - \sum_{j \neq i} y_j.$$

Because the demand function slopes b_i are negative, equations (14) imply that $\beta_j \geq 0$ for all j. This implies that the residual demand constraint in each generator decision problem is acting as a "less than or equal to" inequality (see [7], p. 165). Therefore, β_j represents the marginal benefit that would accrue to generator j if either total demand increased or total production by the other generators decreased.

IV. Results in Numerical Tests

We tested the effects of adding fixed demands in the 6-node system from [2], depicted in Figure 1. Tables IV and V, respectively, contain the transmission line and demand data. The slope of the demand function is $b_j = -0.08, j = 4, 5, 6$, in each hour. The fuel network and generation data shown in Table VI are the same as in [2] except that the capacities of the higher-cost arcs into each generation node are inflated to render those constraints non-binding. We obtained results for all 24 hours but focus our analysis in this paper on four of them: Hour 4 has the lowest demand of the day and hour 17 represents the peak, while hours 11 and 0 represent medium-high and medium-low demands, respectively.

A qualitative description of the network is as follows: Generators at nodes 1 - 3 are supplied with fuel at comparatively low cost from "coal" nodes C1 and C2, but they potentially are isolated from the demand nodes 4 - 6 by thin transmission lines $(1, 4)$, $(3, 4)$ and $(3, 6)$. Generator 1 has high capacity but there is a significant jump in its fuel cost when the fuel supply arc $(\text{C1}, 1)$ is saturated. On the demand side of the network, generation from "gas" supplied by G1 and G2 is much more expensive. Generator 5 has high capacity, and can obtain a limited amount of fuel on arc $(\text{G2}, 5)$ at relatively low cost. Generator 4 is a "peaker" generator with low capacity and high cost. Demand is highest at node 6 (where no generation is located), followed by node 5 and then node 4.

Different levels of fixed demand are adjusted using a factor R to represent the degree of price-sensitivity of demand, similarly to [8]. For a given value of R and a given hour, h, the fixed demand is $L_j(h) = (1 - R)\bar{L}_j(h)$. Thus, $R = 1$ represents the case when demand is fully price-sensitive, as in [2], and $R = 0$ corresponds to the highest levels of fixed demand. We computed an equilibrium in each hour for each value of R from 0 to 1 in increments of 0.1.

TABLE IV
Transmission line data

Line, l	K_l	x_l
(1,3)	400.0	0.0064
(1,4)	240.0	0.0297
(2,3)	1000.0	negl.
(3,4)	150.0	0.0304
(3,6)	250.0	0.0281
(4,5)	240.0	0.0297
(5,6)	350.0	0.0108

TABLE V
Demand data

	Node 4		Node 5		Node 6	
Hour(s), h	$a_4(h)$	$\bar{L}_4(h)$	$a_5(h)$	$\bar{L}_5(h)$	$a_6(h)$	$\bar{L}_6(h)$
0	21.05	250.00	31.65	300.00	35.50	350.00
1	20.60	230.66	30.39	276.80	33.95	322.93
2	20.30	217.89	29.55	261.47	32.92	305.04
3,6	20.15	211.44	29.13	253.73	32.40	296.02
4	20.00	205.11	28.72	246.13	31.89	287.16
5	20.07	208.28	28.93	249.93	32.15	291.59
7	20.45	224.33	29.97	269.20	33.44	314.07
8	21.20	256.33	32.06	307.60	36.01	358.86
9,13	21.81	282.00	33.74	338.40	38.08	394.80
10,12	21.96	288.44	34.16	346.13	38.60	403.82
11,16	22.03	291.61	34.37	349.93	38.85	408.25
14,15,22	21.73	278.83	33.53	334.60	37.82	390.37
17	32.61	320.44	66.07	384.53	78.24	448.62
18	23.90	307.67	39.78	369.20	45.55	430.73
19	22.33	304.39	35.20	365.26	39.88	426.14
20	22.26	301.22	35.00	361.47	39.63	421.71
21	22.11	294.78	34.57	353.73	39.11	412.69
23	21.28	259.61	32.28	311.53	36.28	363.46

A. Strategic Generator Results

The results for each hour and each value of R include the nodal prices, the quantities of electricity generated and consumed at each node, electricity transmission flows, amounts of fuel transported on each supply arc, and marginal cost of fuel to each generator. In this model, the marginal fuel cost is the same as the marginal generation cost. If the generation quantity of generator j, $y_j > 0$ in an equilibrium, its Lerner Index (LI) is its marginal profit as a proportion of the nodal price:

$$LI_j = \frac{p_j - \pi_j}{p_j}.$$

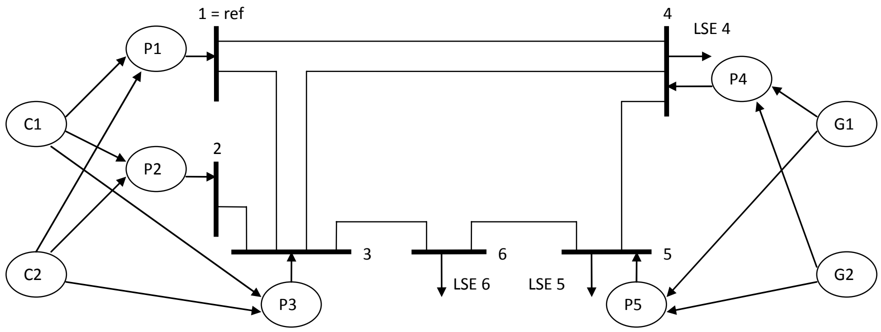

Fig. 1. Six node transmission grid with fuel network

TABLE VI
FUEL AND GENERATION COST AND CAPACITY DATA

$j =$	1			2			3			4			5		
f	c_{fj}	U_{fj}	V_j	c_{fj}	U_{fj}	V_j	c_{fj}	U_{fj}	V_j	c_{fj}	U_{fj}	V_j	c_{fj}	U_{fj}	V_j
C1	10.00	200.0		15.90	166.7		14.83	173.3							
C2	16.30	500.0		15.00	33.3		14.00	36.7							
G1										30.00	66.7		32.80	446.7	
G2										33.60	233.3		25.00	173.3	
Total		700.0	600.0		200.0	100.0		210.0	110.0		300.0	200.0		620.0	520.0

This index is one measure of market power in a monopoly. Intuitively, we would expect it to decrease as R increases because, when more of the demand is price-sensitive then strategic generators would lose more profit if they attempted to raise prices by withholding generation. However, this effect could be muted or even reversed in an oligopolistic market subject to network congestion. The equilibrium results show that, for the higher-cost generators 4 and 5 co-located with demands, the LI does decrease with R. However, for the lower-cost generators separated from the load by a congested transmission line, the LI may increase with R. Figures 2, 3, 4, and 5 show the LIs computed for each generator for different R in the low, medium-low, medium-high, and peak hour, respectively. In all hours, LI_2 and LI_3 increase with small values of R. This is also true for generator 1 in all but the lowest-demand hours [1]. The LI for generator 1 also increases with R approaching 1 in all hours. We focus on the results in the moderately low-demand hour 0, where both of these effects are most sustained. Table VII shows the generation quantities, prices and amounts of demand satisfied at each node for each R. Table VIII shows the corresponding electricity transmission quantities. As R increases from 0 to 0.1; i.e., the fixed demands decrease from $\bar{L}_j$ to $0.9\bar{L}_j$, expensive generation at nodes 4 and 5 is reduced and less-expensive generation at node 1 actually increases. The nodal price separation also decreases.

Some insight into the increase of LI with small values of R for generators 1-3 is gained by examining the decomposition of price and marginal cost into the dual prices of constraints, as shown in Table IX for generators 1 and 3 (corresponding values for generator 2, qualitatively similar to those for generator 3, are omitted to save space). From

[1]A similar effect was reported in a computational agent simulation of the same example with quadratic generation costs [9] but vanished when the generator reinforcement learning parameters were calibrated [8].

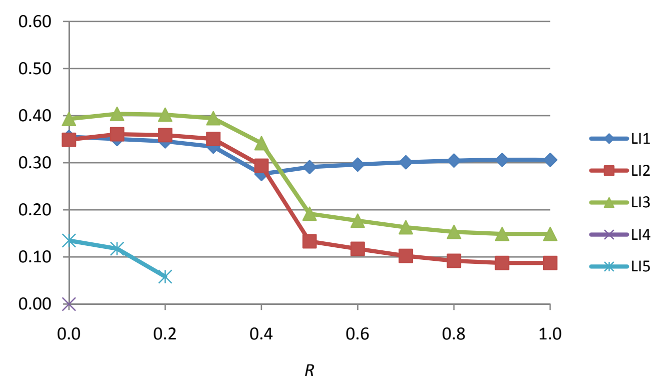

Fig. 2. Lerner Indices for strategic generators vs. R in low-demand hour 4.

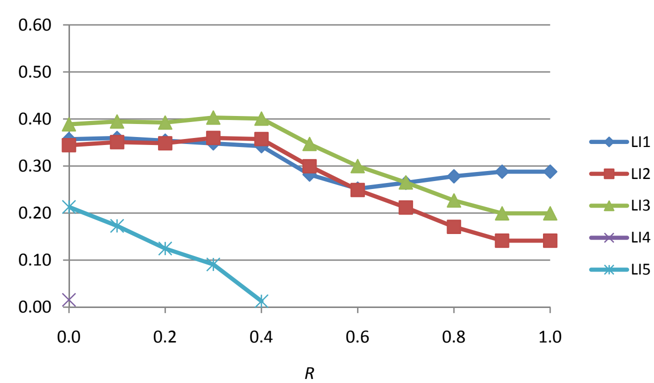

Fig. 3. Lerner Indices for strategic generators vs. R in moderately low-demand hour 0.

TABLE VII
STRATEGIC-GENERATOR EQUILIBRIUM PRICES AND QUANTITIES IN HOUR 0. DEMAND SATISFIED SHOWN IN ITALICS IF IT INCLUDES SOME PRICE-SENSITIVE DEMAND.

	Node 1		Node 2		Node 3		Node 4			Node 5			Node 6	
R	y_1	p_1	y_2	p_2	y_3	p_3	y_4	p_4	q_4	y_5	p_5	q_5	p_6	q_6
0.0	339.47	25.35	100.00	24.25	110.00	24.25	17.74	30.47	250.00	332.79	41.67	300.00	45.75	350.00
0.1	343.22	25.45	100.00	24.49	110.00	24.49	0.00	29.91	225.00	256.78	39.65	270.00	43.19	315.00
0.2	334.80	25.23	100.00	24.40	110.00	24.40	0.00	29.07	200.00	175.20	37.47	240.00	40.53	280.00
0.3	326.38	25.00	100.00	24.83	110.00	24.83	0.00	25.79	175.00	93.62	27.50	210.00	28.12	245.00
0.4	317.95	24.78	100.00	24.74	110.00	24.74	0.00	24.95	150.00	12.05	25.32	180.00	25.46	210.00
0.5	240.00	22.70	100.00	22.70	110.00	22.70	0.00	22.70	125.00	0.00	22.70	150.00	22.70	175.00
0.6	200.00	21.18	100.00	21.18	110.00	21.18	0.00	21.18	100.00	0.00	21.18	*130.94*	21.18	*179.06*
0.7	200.00	20.18	100.00	20.18	110.00	20.18	0.00	20.18	75.00	0.00	20.18	*143.44*	20.18	*191.56*
0.8	200.00	19.18	100.00	19.18	110.00	19.18	0.00	19.18	50.00	0.00	19.18	*155.94*	19.18	*204.06*
0.9	200.00	18.52	98.12	18.52	110.00	18.52	0.00	18.52	*31.67*	0.00	18.52	*164.17*	18.52	*212.29*
1.0	200.00	18.52	98.12	18.52	110.00	18.52	0.00	18.52	*31.67*	0.00	18.52	*164.17*	18.52	*212.29*

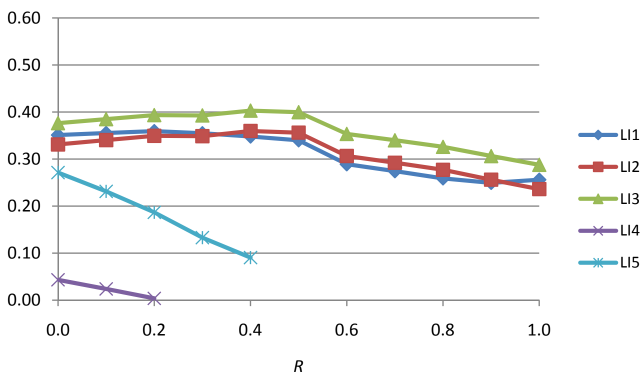

Fig. 4. Lerner Indices for strategic generators vs. R in moderately high-demand hour 11.

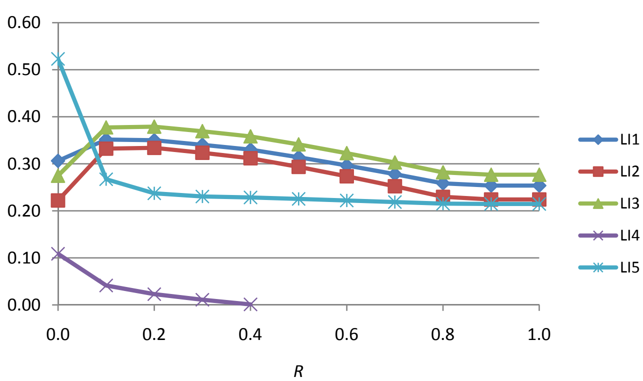

Fig. 5. Lerner Indices for strategic generators vs. R in peak-demand hour 17.

equations (6) and (12), $y_j > 0$ implies that $p_j = \pi_j + \beta_j + \mu_j$. Therefore, $LI_j = \frac{\beta_j+\mu_j}{\pi_j+\beta_j+\mu_j}$, where from equation (14), $\beta_j = -\left(\sum_{i\in N}\frac{1}{b_i}\right)^{-1} y_j$. If generator j produces more electricity at the same marginal cost while remaining below its generating capacity, then its LI increases due to the increase in β_j with both π_j and μ_j remaining constant. This is the case for generator 1 in hour 0 as R increases from 0.0 to 0.1. The increasing values of β_1 indicate that relaxing the fixed demand constraints causes the high-cost generators 4 and 5

TABLE VIII
STRATEGIC-GENERATOR EQUILIBRIUM TRANSMISSION FLOWS IN HOUR 0, BOLD IF AT CAPACITY.

R	1,3	1,4	2,3	3,4	3,6	4,5	5,6
0.0	169.9	169.6	100.0	129.9	**250.0**	67.2	100.0
0.1	171.6	171.7	100.0	131.6	**250.0**	78.2	65.0
0.2	167.8	167.0	100.0	127.8	**250.0**	94.8	30.0
0.3	164.1	162.3	100.0	124.1	**250.0**	111.4	-5.0
0.4	160.3	157.7	100.0	120.3	**250.0**	128.0	-40.0
0.5	109.3	130.7	100.0	104.7	214.6	110.4	-39.6
0.6	86.4	113.6	100.0	92.8	203.5	106.5	-24.5
0.7	91.1	108.9	100.0	87.2	213.9	121.2	-22.3
0.8	95.8	104.2	100.0	81.6	224.2	135.8	-20.1
0.9	99.6	100.4	98.1	77.1	230.6	145.9	-18.3
1.0	99.6	100.4	98.1	77.1	230.6	145.9	-18.3

located near demands to back off generation to an extent that the residual demand seen by generator 1 actually increases. Similarly, if generator j is producing at capacity but there is pressure on it to increase, as is the case for generators 2 and 3 in hour 0 as R increases from 0.0 to 0.1 and from 0.2 to 0.3, then LI_j increases because μ_j increases and forms a larger component of the price. This first type of effect occurs when R is small enough that only fixed demand is met.

The increase of LI_1 with large R occurs where the low-cost fuel arc directed into generator 1 is fully saturated but no high-cost fuel is used. From equation (1), if $x_{gj} > 0$ then the marginal cost $\pi_j = c_{gj} + \rho_{gj}$ (recall that $w_g = 0$ for all g here because Z_g is unlimited). As R increases from 0.6 to 0.9; i.e., the fixed demands decrease, the dual price of additional low-cost fuel decreases. This results in the numerator of LI_1 remaining constant (equal to β_1, proportional to y_1) while the price at node 1 decreases. This second type of effect occurs when R is large enough that the fixed demand constraints are not binding. Note that it is a direct result of the piecewise-linear cost structure derived from the fuel transportation network. It results from congestion in the fuel supply network rather than the electricity transmission system.

Economic measures, totaled over market participants and averaged over all hours, are shown in Table X. Generator cost and revenue along with consumer payments for energy consumed all decrease with R. ISO surplus, equal to the difference between consumer payments and generator revenue, decreases with small values of R but increases again as R approaches 1. The ISO surplus can also be computed by summing over transmission lines the product of the line flow and the price differential. The increase in average hourly ISO surplus is attributable to the peak hours 17 and 18, in which line (3,6) is congested for each R and significant amounts of price-sensitive demand are satisfied for large R.

TABLE IX
DUAL PRICES RELEVANT TO LIS FOR STRATEGIC GENERATORS 1 AND 3 IN HOUR 0. COST c_{*j} AND DUAL VALUE ρ_{*j} ARE FOR THE MARGINAL FUEL ARC DIRECTED INTO NODE j.

	Node 1				Node 3			
R	c_{*1}	ρ_{*1}	β_1	μ_1	c_{*3}	ρ_{*3}	β_3	μ_3
0.0	16.30	0.00	9.05	0.00	14.83	0.00	2.93	6.49
0.1	16.30	0.00	9.15	0.00	14.83	0.00	2.93	6.73
0.2	16.30	0.00	8.93	0.00	14.83	0.00	2.93	6.64
0.3	16.30	0.00	8.70	0.00	14.83	0.00	2.93	7.08
0.4	16.30	0.00	8.48	0.00	14.83	0.00	2.93	6.98
0.5	16.30	0.00	6.40	0.00	14.83	0.00	2.93	4.94
0.6	10.00	5.84	5.33	0.00	14.83	0.00	2.93	3.42
0.7	10.00	4.84	5.33	0.00	14.83	0.00	2.93	2.42
0.8	10.00	3.84	5.33	0.00	14.83	0.00	2.93	1.42
0.9	10.00	3.18	5.33	0.00	14.83	0.00	2.93	0.76
1.0	10.00	3.18	5.33	0.00	14.83	0.00	2.93	0.76

TABLE X
STRATEGIC GENERATOR AVERAGE HOURLY TOTAL COST, GENERATOR REVENUE, CONSUMER PAYMENTS AND ISO SURPLUS.

R	Gen. Cost	Gen. Revenue	Cons. Pmts	ISO Surplus
0.0	19400.14	8355.97	40595.06	9145.12
0.1	16290.83	8502.86	32988.44	6781.52
0.2	13383.57	8497.65	27476.82	5401.83
0.3	10732.93	8225.87	21221.64	3143.19
0.4	8581.35	7431.84	15648.63	1062.90
0.5	6775.81	6195.30	12125.20	447.76
0.6	6044.71	4955.57	10481.46	426.41
0.7	5804.95	4594.64	9951.62	443.06
0.8	5557.23	4260.47	9434.70	459.72
0.9	5425.07	4023.39	9039.52	469.89
1.0	5398.97	3947.31	8915.45	475.20

B. Competitive Benchmark Results

Table XI shows the nodal prices as well as quantities of electricity produced and demand satisfied in hour 0 for the competitive benchmark and Table XII shows the corresponding transmission amounts. As expected, prices are lower for each value of R. The satisfaction of higher amounts of price-sensitive demand results in congestion and price separation for all values of R. For the most part, more of the generation is by lower-cost generators 1-3 than in the strategic-generator model. The effect of the fuel supply network is seen in the generation quantities, y_2, of generator 2. It produces less in the competitive benchmark case than in the strategic case for R values less than 0.5 as all of the low-cost fuel available but none of its high-cost fuel is used.

In the competitive results for hour 0, the Lerner Index for generator 1 is 0 but those for generators 2 and 3 are positive and increase with R (see Figure 6). With $\beta_j = 0$, the Lerner Index is simply $LI_j = \frac{\mu_j}{\pi_j + \mu_j}$, where $\pi_j = c_{fj} + \rho_{fj}$ for the marginal fuel arc (f, j). Table XIII shows the components of price and marginal cost in hour 0 for these two nodes. The increases in a generator's LI occur when its capacity constraint

TABLE XII
COMPETITIVE BENCHMARK EQUILIBRIUM TRANSMISSION FLOWS IN HOUR 0, BOLD IF AT CAPACITY.

R	1,3	1,4	2,3	3,4	3,6	4,5	5,6
0.0	237.6	185.2	33.3	130.9	**250.0**	66.2	100.0
0.1	233.8	180.6	33.3	127.2	**250.0**	82.7	65.0
0.2	230.1	175.9	33.3	123.4	**250.0**	99.3	30.0
0.3	226.3	171.2	33.3	119.6	**250.0**	115.9	-5.0
0.4	222.6	166.6	33.3	115.9	**250.0**	132.5	-40.0
0.5	152.2	147.6	100.0	112.2	**250.0**	134.8	-36.1
0.6	144.1	137.6	100.0	104.1	**250.0**	141.7	-32.3
0.7	136.0	127.6	100.0	96.0	**250.0**	148.6	-28.5
0.8	129.7	119.8	100.0	89.7	**250.0**	154.0	-25.5
0.9	129.7	119.8	100.0	89.7	**250.0**	154.0	-25.5
1.0	129.7	119.8	100.0	89.7	**250.0**	154.0	-25.5

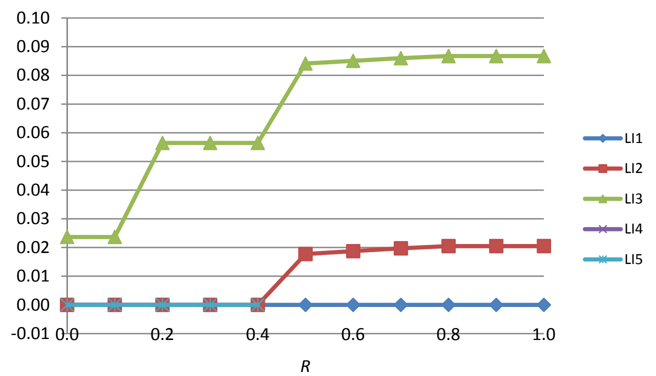

Fig. 6. Lerner Indices for competitive generators vs. R in moderately low-demand hour 0.

more tightly binds the solution to the ISO's total welfare maximization problem. Here, μ_j represents the marginal increase in the total welfare that would accrue if generation capacity could be increased at node j.

The Relative Market Advantage Index, which measures the increase in profits earned by a strategic generator compared to the competitive benchmark, has been proposed as a necessary indicator of market power [4]. With s and c denoting the strategic and competitive cases, respectively, it is computed as:

$$\text{RMAI}_i \equiv \frac{\text{PS}_i^s - \text{PS}_i^c}{\text{PS}_i^c},$$

where the producer surplus is computed as

$$\text{PS}_i = p_i y_i - \sum_{f \in F_i} c_{fi} x_{fi}.$$

Table XIV displays the RMAI values for each generator in hour 0. It is consistent with the counterintuitive LI results for the strategic model in that the RMAI values for generators 1-3 increase as R increases from 0.0 to 0.1. At the highest level of fixed demand, the high-cost generators co-located with demand exploit the opportunity provided by the load pocket created by transmission congestion and reduce the profits of the low-cost generators separated from the demand. When the fixed demand constraints are relaxed slightly, the low-cost generators can make more profit relative to the competitive benchmark.

TABLE XI
COMPETITIVE BENCHMARK EQUILIBRIUM PRICES AND QUANTITIES IN HOUR 0. DEMAND SATISFIED SHOWN IN ITALICS IF IT INCLUDES SOME PRICE-SENSITIVE DEMAND.

	Node 1		Node 2		Node 3		Node 4			Node 5			Node 6	
R	y_1	p_1	y_2	p_2	y_3	p_3	y_4	p_4	q_4	y_5	p_5	q_5	p_6	q_6
0.0	422.82	16.30	33.33	15.18	110.00	15.18	0.00	21.48	250.00	333.85	32.80	300.00	36.92	350.00
0.1	414.39	16.30	33.33	15.18	110.00	15.18	0.00	21.48	225.00	252.27	32.80	270.00	36.92	315.00
0.2	405.97	16.30	33.33	15.71	110.00	15.71	0.00	19.03	200.00	170.69	25.00	240.00	27.17	280.00
0.3	397.55	16.30	33.33	15.71	110.00	15.71	0.00	19.03	175.00	89.12	25.00	210.00	27.17	245.00
0.4	389.13	16.30	33.33	15.71	110.00	15.71	0.00	19.03	150.00	7.54	25.00	180.00	27.17	210.00
0.5	299.83	16.30	100.00	16.19	110.00	16.19	0.00	16.82	125.00	0.00	17.97	*170.96*	18.39	*213.87*
0.6	281.72	16.30	100.00	16.20	110.00	16.20	0.00	16.75	100.00	0.00	17.73	*174.02*	18.08	*217.69*
0.7	263.61	16.30	100.00	16.22	110.00	16.22	0.00	16.67	75.00	0.00	17.48	*177.09*	17.78	*221.52*
0.8	249.47	16.30	100.00	16.23	110.00	16.23	0.00	16.61	*55.49*	0.00	17.29	*179.48*	17.54	*224.51*
0.9	249.47	16.30	100.00	16.23	110.00	16.23	0.00	16.61	*55.49*	0.00	17.29	*179.48*	17.54	*224.51*
1.0	249.47	16.30	100.00	16.23	110.00	16.23	0.00	16.61	*55.49*	0.00	17.29	*179.48*	17.54	*224.51*

TABLE XIII
DUAL PRICES RELEVANT TO LIS FOR GENERATORS 2 AND 3 IN HOUR 0 IN THE COMPETITIVE BENCHMARK. COST c_{*j} AND DUAL VALUE ρ_{*j} ARE FOR THE MARGINAL FUEL ARC DIRECTED INTO NODE j.

	Node 2			Node 3		
R	c_{*2}	ρ_{*2}	μ_2	c_{*3}	ρ_{*3}	μ_3
0.0	15.00	0.18	0.00	14.83	0.00	0.36
0.1	15.00	0.18	0.00	14.83	0.00	0.36
0.2	15.00	0.71	0.00	14.83	0.00	0.89
0.3	15.00	0.71	0.00	14.83	0.00	0.89
0.4	15.00	0.71	0.00	14.83	0.00	0.89
0.5	15.90	0.00	0.29	14.83	0.00	1.36
0.6	15.90	0.00	0.30	14.83	0.00	1.38
0.7	15.90	0.00	0.32	14.83	0.00	1.40
0.8	15.90	0.00	0.33	14.83	0.00	1.41
0.9	15.90	0.00	0.33	14.83	0.00	1.41
1.0	15.90	0.00	0.33	14.83	0.00	1.41

TABLE XIV
RELATIVE MARKET ADVANTAGE INDICES IN HOUR 0.

R	Gen 1	Gen 2	Gen 3	Gen 4	Gen 5
0.0	2.44	139.61	14.29		2.18
0.1	2.49	143.58	14.67		1.30
0.2	2.37	36.09	7.48		
0.3	2.25	37.92	7.85		
0.4	2.14	37.53	7.77		
0.5	1.22	11.10	3.98		
0.6	0.77	8.24	3.01		
0.7	0.62	6.38	2.37		
0.8	0.46	4.65	1.75		
0.9	0.35	3.53	1.36		
1.0	0.35	3.53	1.36		

V. CONCLUSION

Numerical results obtained here indicate the feasibility of including fixed demands in this equilibrium model of a fuel and electricity system. In the strategic-generator model, Lerner Indices for generators counter-intuitively move in opposition to amounts of fixed demands for three reasons. The first two occur because high amounts of fixed demands may not increase electricity prices at every node in the network. In the numerical example studied here, low-cost generators are located at "supply" nodes that are separated from the demand by a congested transmission line when the demands are high while higher-cost generation is co-located with the demands. When the fixed demands are reduced, the prices at the supply nodes climb slightly, so that their generator Lerner Indices increase. Examination of the dual prices obtained from solving the complementarity problem that describes an equilibrium reveals that LMPs at the supply nodes rise either because the value of additional generating capacity increases there or the residual demands seen by those strategic generators increase as fixed demands decrease. As the generator capacity impact also appears in the competitive benchmark, it is not a consequence of the Cournot or bounded rationality assumptions on the generators. The third reason generator Lerner Indices increase when fixed demands decrease results from the piecewise-linear cost function derived from the fuel supply network incorporated here. Trends in the Relative Market Advantage Indices of generators echo some of the Lerner Index results.

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