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Abstract

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Abstract— This article examines the hybrid traffic control problem to minimize total travel time (TTT) of a highway network through traffic management infrastructures, including dynamic speed limit signs, ramp metering, and information board.We first build the traffic flow model based on the Moskowitz function for each highway link to predict traffic status within a control horizon. The traffic density is predicted based on the flow dynamic model and corrected periodically by measured traffic flow data. The minimum TTT traffic control problem is then formulated as a mixed-integer quadratic programming problem with quadratic constraints. Numerical simulation of a real world highway network is provided to demonstrate significant reduction of TTT and alleviation of traffic congestion compared to results obtained from ALINEA and PI-ALINEA methods.

Index Terms— Macroscopic Traffic Control; Travel Time Minimization; Hybrid Optimal Control

I. INTRODUCTION

Highway network plays a critical role in today's transportation system. However, a great number of automobile travelers are suffering from traffic congestion, extended traveling time, and pollution emission due to huge amount of transportation requests, especially during peak hours of working days [1]. The fundamental solution to alleviate traffic congestion is to extend and improve the highway networks. For example, constructing new highway sections will improve the traffic volume capacity, decrease travel time, and address the safety issues. However, highway construction and improvement requires a great amount of public investments and resources [2]. Moreover, it takes considerable time and efforts to complete a highway construction project [3]. Therefore, intelligent transportation technologies that aim to improve highway transportation efficiency in short-term become more appealing.

Advanced traffic control methods have been studied in recent years for congestion management [4]–[7], throughput improvement [8], [9], fuel and emission reduction [10], [11], and safety issues [12], [13]. One frequently used strategy is to model the traffic control problem as an optimization problem with pre-defined objective and constraints describing traffic dynamics. For example, work in [14] formulates a linear programming optimization problem for traffic density estimation. Due to relative error of measured traffic flow, solving

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such type of problem generates the range of estimated density. Another example in [15] constructs a quadratic programming problem for traffic flow maximization.

To incorporate the traffic flow dynamics as constraints in the optimal traffic control problem, the Lighthill-Whitham-Richards (LWR) model is employed when developing the analytical solution to predict the traffic flow status [16], [17]. The cumulative vehicle count is introduced to formulate the Moskowitz function that yields an identical solution to the one from Hamilton-Jacobi (HJ) Partial Differential Equations (PDEs) [18], [19]. Based on the compatibility conditions [20], the semi-analytical solutions to the Moskowitz function proposed by Barron-Jensen/Frankowska (B-J/F) [21] are transformed into a finite number of linear model constraints and integrated into the optimal control problem. In this paper, we apply B-J/F solutions to each highway link and establish a unified optimization problem for travel time minimization of the entire highway network through hybrid infrastructures.

Although traffic flow can be predicted using the semi-analytical solution to the Moskowitz function, there is prediction error between the predicted value and the real one. Over the time, the prediction value from the LWR model is not reliable due to the uncertainties in the fundamental diagram [22]. In order to prevent prediction error, the traffic flow data from volume sensors is measured and used to correct the real world density status.

A single type traffic management infrastructure, such as dynamic speed limit signs or ramp metering, has been widely used in today's transportation system. However, there is no investigation on control of hybrid infrastructures that work simultaneously to contribute to improved transportation efficiency. For example, as a popular local responsive feedback ramp metering strategy, ALINEA is verified to be effective in throughput maximization, congestion alleviation, and risk reduction in both field test and simulation. Recent years, researchers pay their attention to advanced tuning approaches for feedback gains and operational parameters in ALINEA [23], [24], integration of ALINEA with Iterative Learning Control (ILC) [25], and the derivatives of ALINEA, e.g. PI-ALINEA [26]. In addition to ramp metering control, work in [27] presented an integration of local ramp metering with dynamic speed limits to reduce the Total Time Spent (TTS). Proposed split-range-like scheme separated the control period for ramp metering and variable speed limits such that the integrated controller remains simple in field implementation. Another example is in work [28], where an optimal coordination of dynamic speed limits and ramp

metering is proposed based on Model Predictive Control (MPC). The effectiveness is verified for TTS reduction. Different from previous work on optimal traffic control using ALINEA-based ramp metering or integrated strategy in [27] and [28], we present an integrated controller simultaneously designing the optimal control strategies for dynamic speed limit sign, ramp metering, and highway information board. The major contribution of this paper include:

1. A hybrid traffic control infrastructure is proposed that integrates dynamic speed limit sign, ramp metering and highway information board.

2. We formulate the hybrid traffic control problem as a mixed-integer quadratic programming problem with quadratic constraints, named MIQQ.

3. The traffic flow dynamics are transformed as linear constraints in the optimal control problem and the traffic density is updated by the optimization results and also corrected periodically using measured volume data to improve precision.

The rest of paper is organized as follows. The problem statement and traffic flow dynamics model are introduced in §II. §III describes the highway network and the hybrid traffic control infrastructures. We formulate the hybrid traffic control problem as a MIQQ problem in §IV. §V verifies the efficiency of the proposed MIQQ method by comparing with ALINEA and PI-ALINEA methods using a real world highway network. We address the concluding remarks in §VI.

II. PROBLEM STATEMENT AND TRAFFIC FLOW DYNAMICS

A. Problem Statement

A typical example of highway network consists of N conjunctions and L links. A specific link l, $l = 1, \ldots, L$, with small distance represents a highway link or roadway link connecting different highways, on-ramp or off-ramp. For each highway link, it is simplified as a uniform highway section. Without loss of generality, we focus on modeling traffic flow dynamics along a single direction on each link with x and t denoting location and time and $[\xi_l, \chi_l]$ denoting upstream and downstream location of link l. Assuming the inflow and outflow of link l, denoted as $Q(t, \xi_l)$ and $Q(t, \chi_l)$, as well as traffic density $\rho(t, x)$, remain constant during a short time interval $t \in [t_p, t_{p+1}]$, $p = 0, ..., P - 1$, at any location $x \in [\xi_l, \chi_l]$ on link l, $l = 1, ..., L$. Our purpose is to minimize total traveling time and queue waiting time of on-ramps in a highway network over time $[t_1, t_P]$ through hybrid control infrastructures, including dynamic speed limit sign, ramp metering, and highway information board, as shown in Fig. 1.

B. Cauchy Problem on single highway link

The LWR model describes the conservation of traffic flow on the highway sections and is expressed as [16]

$$\frac{\partial \rho(t, x)}{\partial t} + \frac{\partial Q(\rho(t, x))}{\partial x} = 0. \tag{2.1}$$
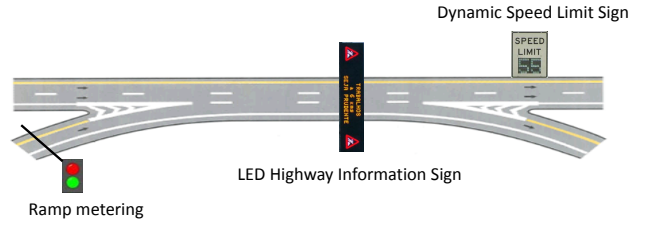

Fig. 1. Example of Hybrid Control Infrastructures.

By introducing the cumulative vehicle count $N(t, x)$, the LWR PDE is converted to the Moskowitz function as follows,

$$\frac{\partial N(t, x)}{\partial t} - Q\left(-\frac{\partial N(t, x)}{\partial x}\right) = 0, \tag{2.2}$$

which yields the Hamilton-Jacobi PDE. The solutions to (2.2) need to satisfy additional equality constraints associated with initial, upstream, and downstream boundary conditions, denoted as $c_{ini}(x)$, $c_{up}(t)$ and $c_{down}(t)$, respectively. Work in [29] presents the Cauchy problem in the form of

$$\begin{cases} (2.2) \\ N(0, x) = c_{ini}(x) \\ N(t, \xi_l) = c_{up}(t) \\ N(t, \chi_l) = c_{down}(t), \end{cases} \tag{2.3}$$

which indicates the cumulative number of vehicles, denoted as $N(0, x)$, at initial time on highway section $[\xi_l, \chi_l]$ is equivalent to $c_{ini}(x)$. Similarly, the number of vehicles entering and flowing out of the highway section, denoted as $N(t, \xi_l)$ and $N(t, \chi_l)$, respectively, should be $c_{up}(t)$ and $c_{down}(t)$. The boundary conditions, including initial, upstream and downstream conditions, are defined as,

$$c_{ini}(t, x) = \begin{cases} -\rho_{ini}^{(t_p, l)} x, & \text{if } t = t_p \ \& \ x \in [\xi_l, \chi_l] \\ +\infty & \text{otherwise,} \end{cases} \tag{2.4}$$

$$c_{up}(t, x) = \begin{cases} \sum_{i=0}^{p-1} Q_{up}^{(t_i, l)} \Delta t + Q_{up}^{(t_p, l)}(t - p\Delta t), \\ \qquad \text{if } x = \xi_l \ \& \ t \in [t_p, t_{p+1}] \\ +\infty \quad \text{otherwise,} \end{cases} \tag{2.5}$$

$$c_{down}(t, x) = \begin{cases} \sum_{i=0}^{p-1} Q_{down}^{(t_i, l)} \Delta t + Q_{down}^{(t_p, l)}(t - p\Delta t) - \rho_{ini}^{(t_p, l)} X_l, \\ \qquad \text{if } x = \chi_l \ \& \ t \in [t_p, t_{p+1}] \\ +\infty \quad \text{otherwise,} \end{cases} \tag{2.6}$$

where $\rho_{ini}^{(t_p, l)}$, $Q_{up}^{(t_p, l)}$ and $Q_{down}^{(t_p, l)}$ are the initial traffic density, upstream and down stream traffic flow, respectively, during the time interval $[t_p, t_{p+1}]$ on link l. X_l is the length of link l and Δt is the uniform time duration for each time interval.

C. B-J/F Solutions and Model Constraints

According to the B-J/F solution to HJ PDE (2.2), which is a semi-continuous solution proposed by Frankowska [30] and Barron-Jensen [31], solutions to the HJ PDE associated with initial and boundary conditions is expressed as [32]

$$N_{\mathbf{c}}(t,x) = \inf_{(u,\Delta t) \in [w^l, v_f^l] \times \mathbb{R}_+} [\mathbf{c}(t - \Delta t, x - \Delta t u) + \Delta t R(u)],$$

(2.7)

where $\mathbf{c}$ is a set satisfying $c_{ini}(x)$, $c_{up}(t)$ and $c_{down}(t)$. $R(u)$ is a convex transform, expressed as

$$R(u) = \sup_{\rho \in [0, \rho_j^l]} (Q(\rho) - u\rho), \ \forall u \in [w^l, v_f^l],$$

(2.8)

where $w^l = \frac{dQ}{d\rho}|_{\rho=\rho_j^l} < 0$, v_f^l and ρ_j^l are obtained from the fundamental diagram of traffic flow. This work considers the triangular fundamental diagram, which is defined by

$$Q(\rho) = \begin{cases} v_f^l \rho, & \text{if for link } l : 0 \le \rho \le \rho_c^l, \\ w^l(\rho - \rho_j^l) & \text{if for link } l : \rho_c^l < \rho \le \rho_j^l, \end{cases}$$

(2.9)

where ρ_c^l and ρ_j^l are critical density and jam density of link l. To save space, we omit the explicit expressions of (2.7). More details can be found in [29]. However, equalities in Cauchy problem (2.3) cannot be incorporated into the traffic flow dynamic model due to the continuous time t and unknown variable x. To handle this issue, we introduce the compatibility conditions to obtain finite number of equalities.

Lemma 2.1: Compatibility Conditions [21]: The solution to HJ PDE is characterized by the Inf-morphism property, i.e. $\mathbf{c}(t,x) = \min_{i \in I} c_i(t,x)$, where I is the index number of value condition, the solution $N_{\mathbf{c}}(t,x) = \min_{i \in I} N_{c_i}(t,x)$ for $(t,x) \in [t_0, t_P] \times [\xi_l, \chi_l]$. The B-J/F solution to (2.2) satisfies the boundary conditions if and only if

$$N_{c_i}(t,x) \ge c_j(t,x), \ \forall (t,x) \in Dom(c_j), \ (i,j) \in I^2. \quad (2.10)$$

Work in [14] proved the affinity of $N_{c_i}(t,x)$ and $c_j(t,x)$, as well as the convex feasible region defined in (2.10). Intuitively, affine solution $N_{c_i}(t,x)$ is greater than or equal to the affine value condition $c_j(t,x)$ defined in (2.4)-(2.6) on all points of a segment, as long as it holds at two extremity points of the segment. Therefore condition in (2.10) can be reduced to finite number of inequalities. In the following sections, we first describe the highway network and then implement the model constraint in each highway link.

III. HIGHWAY NETWORK

A. Components of a Highway Network

A highway network is composed of nodes and edges that connect two distinct nodes, as illustrated in Fig. 2. Each node $n \in \mathcal{N}$ represents one of the three types of location:

1. The conjunction of different highway mainstreams, specifically, the conjunction node $n_c^i \in \mathcal{N}_{c_{out}}$, e.g., n_c^2 and n_c^4 in Fig. 2, that allows for outgoing traffics, or $n_c^j \in \mathcal{N}_{c_{in}}$, e.g., n_c^1 and n_c^5, that have incoming traffic flow from other highway sections, where $\mathcal{N}_{c_{out}}$ and $\mathcal{N}_{c_{in}}$ are two subsets of $\mathcal{N}$.
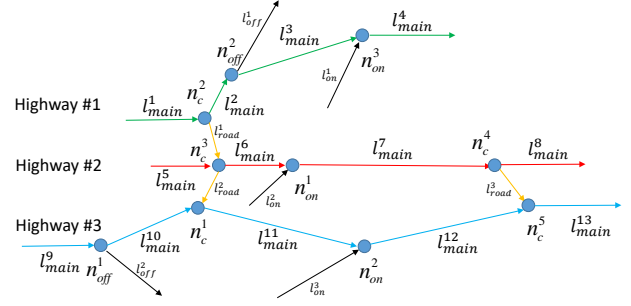

Fig. 2. Example of a highway network. Mainstream links are marked as green for Highway #1, red for Highway #2, and blue for Highway #3. *Black arrows*: on-ramps or off-ramps connecting urban/rural roadway to highway. *Yellow arrows*: roadway links connecting different highways links.

2. The joint where incoming traffic contributes to the highway mainstream from on-ramp, denoted as $n_{on}^i \in \mathcal{N}_{on}$.

3. The joint where traffic exits the mainstream via off-ramp, denoted as $n_{off}^i \in \mathcal{N}_{off}$.

Based on the above assumptions, the set $\mathcal{N}$ contains four subsets, $\mathcal{N} = \{\mathcal{N}_{c_{in}}, \mathcal{N}_{c_{out}}, \mathcal{N}_{on}, \mathcal{N}_{off}\}$. Similarly, three types of edges are defined below:

1. Mainstream link (highway link), denoted by $l_{main}^m \in \mathcal{L}_{main}$, $m = 1, ..., L_{main}$, that connects two adjacent nodes to form the mainstream of traffic on highway section, e.g. l_{main}^7 represent highway link (n_{on}^1, n_c^4) in Fig. 2.

2. On-ramp or off-ramp, denoted as $l_{on}^h \in \mathcal{L}_{on}$, $h = 1, ..., L_{on}$ and $l_{off}^g \in \mathcal{L}_{off}$, $g = 1, ..., L_{off}$, respectively, that allows the traffic entering or exiting the highway.

3. Roadway, denoted by $l_{road}^r \in \mathcal{L}_{road}$, $r = 1, ..., L_{road}$, represents the link for which traffic flow changes the route from one highway section to another, e.g. l_{road}^3 represents roadway link (n_c^4, n_c^5) in Fig. 2.

The total number of links is determined by $L = L_{main} + L_{road} + L_{on} + L_{off}$. Furthermore, we make the following assumptions to simplify the problem.

Assumption 3.1: The on-ramp traffic volume $Q_{on}^{(t_p, l_{on}^h)}$, is constant and controlled by ramp metering.

Assumption 3.2: The off-ramp vehicle volume is proportional to the corresponding downstream volume at mainstream link l_{main}^m with the constant ratio $R_{off}^{(t_p, l_{off}^g)}$ during $[t_p, t_{p+1}]$.

Assumption 3.3: The traffic flow exiting a highway section via roadway link l_{road}^r remains a constant ratio $R_{off}^{(t_p, l_{road}^r)}$ with respect to the corresponding downstream volume at mainstream link l_{main}^m during $[t_p, t_{p+1}]$. The downstream flow of link l_{road}^r, denoted by $Q_{on}^{(t_p, l_{road}^r)}$, is controlled by a ramp meter.

B. A Hybrid Traffic Control Infrastructure

A hybrid traffic control infrastructure, consisting dynamic speed limit sign, ramp metering, and highway information board, is expected to improve efficiency of the traffic management than a single control method. As illustrated in Fig. 1, the dynamic speed limit sign is employed as one of the traffic management infrastructures to control flow on each highway link. The desired vol-

ume can be obtained by displaying an appropriate speed $v(t_p, l_{main}^m)$ on the speed limit sign. According to the time-varying traffic states, the speed limit on each mainstream link l_{main}^m is adjusted periodically via the control variable set $\mathbf{v} = [v(t_0, l_{main}^1), ..., v(t_0, l_{main}^{L_{main}}), ..., v(t_{P-1}, l_{main}^1), ..., v(t_{P-1}, l_{main}^{L_{main}})]$.

The ramp metering controls the outflow of on-ramp traffic, $Q_{on}^{(t_p, l)}$ for all $l \in \{\mathcal{L}_{road}, \mathcal{L}_{on}\}$. The one vehicle per green principle is adopted in meter control, where one vehicle is allowed to pass the meter during a short green light cycle. To obtain the desired on-ramp volume, the meter cycle length $\mathbf{T}$ is designed for each time interval at downstream of l_{road}^r and l_{on}^h. The control variable set for ramp metering is $\mathbf{T} = [T(t_0, l_{on}^1), ..., T(t_0, l_{on}^{L_{on}}), T(t_{P-1}, l_{on}^1), ..., T(t_{P-1}, l_{on}^{L_{on}}), T(t_0, l_{road}^1), ..., T(t_0, l_{road}^{L_{road}}), T(t_{P-1}, l_{road}^1), ..., T(t_{P-1}, l_{road}^{L_{road}})]$.

The highway information board guides the traffic to their destination by selecting the optimal routes. For example, as shown in Fig. 2, traffic from Highway #2 with destination n_c^5 is guided to travel via l_{road}^2 or l_{road}^3. The highway information board is located at link l_{main}^5 in this case. The control variable set determining the route selection is set as $\mathbf{b} = [b^{(t_0, l_{road}^1)}, ..., b^{(t_0, l_{road}^{L_{road}})}, ..., b^{(t_{P-1}, l_{road}^1)}, ..., b^{(t_{P-1}, l_{road}^{L_{road}})}]$. The element in $\mathbf{b}$ is defined as a binary variable according to

$$b^{(t_p, l_{road}^r)} = \begin{cases} 1, & \text{if } l_{road}^r \text{ is allowed} \\ 0, & \text{if } l_{road}^r \text{ is not allowed.} \end{cases} \quad (3.11)$$

In practice, the highway information board closes or activates the links between highway sections. If a link is closed, alternative route information will be displayed on the information board. For example, if l_{road}^2 is closed, then traffic with destination n_c^5 will be guided to travel trough l_{road}^3.

IV. PROBLEM FORMULATION

A. Intermediate Control Variables and The Objective Function

To minimize the TTT of the highway network, the minimum time traffic management problem is formulated as a MIQQ problem. The intermediate control variables include upstream, $Q_{up}^{(t_p, l_{main}^m)}$, and downstream traffic flow, $Q_{down}^{(t_p, l_{main}^m)}$, of all highway links, the outflow at the end of all on-ramps, $Q_{on}^{(t_p, l_{road}^r)}$ and $Q_{on}^{(t_p, l_{on}^h)}$, as well as the binary variables for route selection, $b^{(t_p, l_{road}^r)}$, during each time interval $[t_p, t_{p+1}]$ for all $p = 0, ..., P-1$. Therefore, the intermediate control variables include both continuous and binary variable sets.

The TTT for all vehicles in the highway network over the duration $[t_1, t_P]$ consists of traversing time J_m along all highway link l_{main}^m, waiting time J_c in the queue on all roadway link l_{road}^r, as well as the waiting time J_{on} on all

on-ramp l_{on}^h. Accordingly, the objective is expressed as

$$J = J_m + J_c + J_{on}$$
$$= \sum_{p=0}^{P-1} \Delta t \Big\{ \sum_{l_{main}^m \in \mathcal{L}_{main}} [(Q_{up}^{(t_p, l_{main}^m)} - Q_{down}^{(t_p, l_{main}^m)}) \Delta t + \rho_{ini}^{(t_p, l_{main}^m)} X_{l_{main}^m}] + \sum_{l_{road}^r \in \mathcal{L}_{road}} [(Q_{up}^{(t_p, l_{road}^r)} - Q_{on}^{(t_p, l_{road}^r)}) \Delta t + \rho_{ini}^{(t_p, l_{road}^r)} X_{l_{road}^r}] + \sum_{l_{on}^h \in \mathcal{L}_{on}} [(Q_{up}^{(t_p, l_{on}^h)} - Q_{on}^{(t_p, l_{on}^h)}) \Delta t + \rho_{ini}^{(t_p, l_{on}^h)} X_{l_{on}^h}] \Big\},$$
$$(4.12)$$

where $X_{l_{main}^m}$, $X_{l_{road}^r}$, and $X_{l_{on}^h}$ are the segment length of link l_{main}^m, l_{road}^r, and l_{on}^h respectively, ρ_{ini} is the initial density and periodically updated by the new measurements from the volume sensors, and $Q_{up}^{(t_p, l_{on}^h)}$ is obtained from the volume sensors. $Q_{up}^{(t_p, l_{road}^r)}$ is a quadratic function of the intermediate variables, expressed as

$$Q_{up}^{(t_p, l_{road}^r)} = Q_{down}^{(t_p, l_{main}^m)} b^{(t_p, l_{road}^r)} R_{off}^{(t_p, l_{road}^r)} + Q_{other}$$

where traffic on link l_{main}^m flows into other highway links via l_{road}^r and Q_{other}, measured by volume sensors, is the volume contributed from other resources excluding l_{main}^m.

B. MIQQ Problem Formulation

By incorporating the linear model constraints describing the traffic dynamics of each highway into the objective function, the hybrid traffic control problem to minimize the TTT of the highway network is formulated as

$$\min_{\mathbf{y}} \ J = (4.12)$$
$$s.t. \ (a) \ A_{model} \mathbf{y} \leq b_{model}$$
$$(b) \ 0 \leq (Q_{up}^{(t_p, l_{road}^r)} - Q_{on}^{(t_p, l_{road}^r)}) \Delta t + \rho_{ini}^{(t_p, l_{road}^r)} X_{l_{road}^r}$$
$$< \mathcal{MAX}_{l_{road}^r}, \forall \ l_{road}^r \in \mathcal{L}_{road}, p = 0, ..., P-1,$$
$$(c) \ 0 \leq (Q_{up}^{(t_p, l_{on}^h)} - Q_{on}^{(t_p, l_{on}^h)}) \Delta t + \rho_{ini}^{(t_p, l_{on}^h)} X_{l_{on}^h}$$
$$< \mathcal{MAX}_{l_{on}^h}, \forall \ l_{on}^h \in \mathcal{L}_{on}, p = 0, ..., P-1,$$
$$(d) \ (1 - R_{off}^{(t_p, l)}) Q_{down}^{(t_p, l_{main}^j)} = Q_{up}^{(t_p, l_{main}^k)}, \ p = 0, ..., P-1$$
$$\text{if } l_{main}^j \ l_{main}^k \text{and } l \text{ are separated at } n_{off}^i \in \mathcal{N}_{off}$$
$$\text{or } n_c^q \in \mathcal{N}_{c_{out}},$$
$$l_{main}^j, l_{main}^k \in \mathcal{L}_{main}, \ l \in \{\mathcal{L}_{off}, \mathcal{L}_{road}\},$$
$$(e) \ Q_{down}^{(t_p, l_{main}^j)} + Q_{on}^{(t_p, l)} = Q_{up}^{(t_p, l_{main}^k)}, \ p = 0, ..., P-1$$
$$\text{if } l_{main}^j \ l_{main}^k \text{and } l \text{ are separated at } n_{on}^i \in \mathcal{N}_{on}$$
$$\text{or } n_c^q \in \mathcal{N}_{c_{in}},$$
$$l_{main}^j, l_{main}^k \in \mathcal{L}_{main}, \ l \in \{\mathcal{L}_{on}, \mathcal{L}_{road}\},$$
$$(4.13)$$

where $\mathbf{y}$ represent the intermediate control variable set. The linear model constraints describing the traffic dynamics in (2.10) are represented in a compact form in (a) of (4.13). To prevent traffic congestion on ramps due to ramp metering, additional quadratic and linear inequality constraints are

introduced in (b) and (c). The left hand side of (b) and (c) compute the number of vehicles on links l_{road}^r and l_{on}^h for every time interval and are assumed to be less than the pre-determined maximum allowed vehicle number $\mathcal{MAX}_{l_{road}^r}$ and $\mathcal{MAX}_{l_{on}^h}$, respectively. According to assumptions 3.1-3.2, equalities (d) and (e) hold in (4.13). Since the above formulation includes mixed continuous and binary variables and the objective is a quadratic function subject to quadratic and linear constraints, problem (4.13) is classified as a MIQQ problem.

C. Conversion from Intermediate Control Variable Set to Final Set

Solution from problem (4.13) generates intermediate control variables. To convert the intermediate control variables to the final ones, including $\mathbf{v}$, $\mathbf{T}$, and $\mathbf{b}$, to display on the traffic infrastructures, the following transformations are required.

First, based on the triangular fundamental diagram defined in (2.9), elements in $\mathbf{v}$ are determined by

$$v(t_p, l_{main}^m) = \begin{cases} v_f^{l_{main}^m}, \text{ if } 0 \le \rho \le \rho_c^{l_{main}^m} \\ w^{l_{main}^m}(1 - \dfrac{\rho_j^{l_{main}^m}}{\rho}), \text{ if } \rho_c^{l_{main}^m} < \rho \le \rho_j^{l_{main}^m} \end{cases} \tag{4.14}$$

where

$$\rho = \frac{(Q_{up}^{(t_{p-1}, l_{main}^m)} - Q_{down}^{(t_{p-1}, l_{main}^m)})\Delta t}{X_{l_{main}^m}} + \rho_{ini}^{(t_{p-1}, l_{main}^m)} \tag{4.15}$$

Hence, the parameters in the fundamental diagram of highway link l_{main}^m are represented by $v_f^{l_{main}^m}$, $w^{l_{main}^m}$, $\rho_c^{l_{main}^m}$ and $\rho_j^{l_{main}^m}$ in (4.14)-(4.15).

Second, the meter cycle length $\mathbf{T}$ is set in unit of second and calculated based on the outflow of on-ramp in form of. Given a constant green phase T_g that allows one vehicle passing the meter during that cycle, the controlled meter cycle length is determined by

$$T(t_p, l) = \frac{T_g * 3600}{Q_{on}^{(t_p, l)}}, \ l \in \{\mathcal{L}_{road}, \mathcal{L}_{on}\}. \tag{4.16}$$

As the final control variable set $\mathbf{b}$ is consistent with the corresponding ones in the intermediate control variables, no conversion is required for this set.

D. Implementation of the Hybrid Traffic Control Strategy

When determining control variables, the traffic flow and density are predicted based on current density and the traffic flow dynamics model. However, the traffic dynamic model is not an ideal one due to uncertainties of the fundamental diagram. In order to improve prediction accuracy, the real world traffic flow data is measured to update real-time density periodically. During each updating period, the control system integrates the sensing-optimizing-displaying (SOD) procedure which is illustrated in Fig.3.

The volume sensors record the amount of vehicles entering and fluxing out of all highway links l_{main}^m, roadway links l_{road}^r and on-ramp links l_{on}^h during the updating period

$[t_{p-p'}, t_p]$. p' is defined as time steps in between two consecutive updating times. After receiving the measured volume data, the traffic density is updated via

$$\rho_{ini}^{(t_p, l)} = \frac{N_{up}^{(t_{p-p'}, l)} - N_{down}^{(t_{p-p'}, l)}}{X_l} + \rho_{ini}^{(t_{p-p'}, l)}, \tag{4.17}$$

where $l \in \{\mathcal{L}_{main}, \mathcal{L}_{road}, \mathcal{L}_{on}\}$, $N_{up}^{(t_{p-p'}, l)}$ and $N_{down}^{(t_{p-p'}, l)}$ are measured number of vehicles during $[t_{p-p'}, t_p]$ at upstream and downstream of link l. Before new measurements from the volume sensors become available at the next updating time $t_{p+p'}$, problem (4.13) will be solved at each time interval t_{p+i}, t_{p+i+1} for index i and $0 \le i < p'$ to determine the new intermediate control variables which are converted into the final control variables based on (4.14)-(4.16). Values of the final control variables at each time interval are then sent to corresponding dynamic speed limit signs, ramp metering and highway information boards. The density will be updated by new measured data at $t_{p+p'}$, which initiates the next SOD procedure.
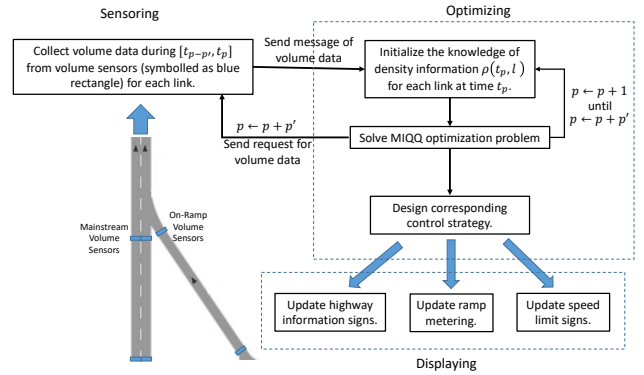

Fig. 3. A Sensoring-Optimizing-Displaying (SOD) Procedure for Real-Time Highway Traffic Control

V. SIMULATION EXAMPLE

A. A Highway Network Example and VISSIM Settings

In this section, a real world scenario is considered as a test highway network. We extract two sections with 6.08 km length of each one from two major highways, I-35 and US-69, which are located between the cities Ames and Des Moines in Iowa, as illustrated in Fig. 4. Two on-ramps divide the test highway sections into four segments. Two roadways, with one located in north and another one in south, allows traffic traveling between US-69 and I-35.

Volume sensors are installed at the starting and ending point of each highway segment, roadway link and on-ramp. Each of them records the number of vehicle entering and fluxing out of the corresponding link. For every 8 mins, the volume sensors send the sensor data to the computation center update real time density on highway segments and on-ramps.

VISSIM is connected to the MIQQ solver [33] through the Component Object Model (COM) interface in MATLAB. The COM interface provides access to dynamically change
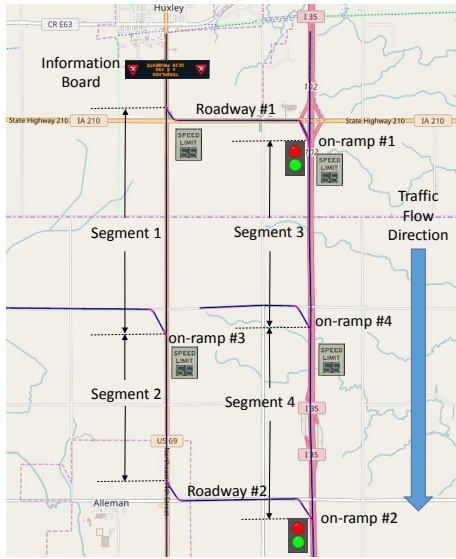
Fig. 4.   A Test Highway Network

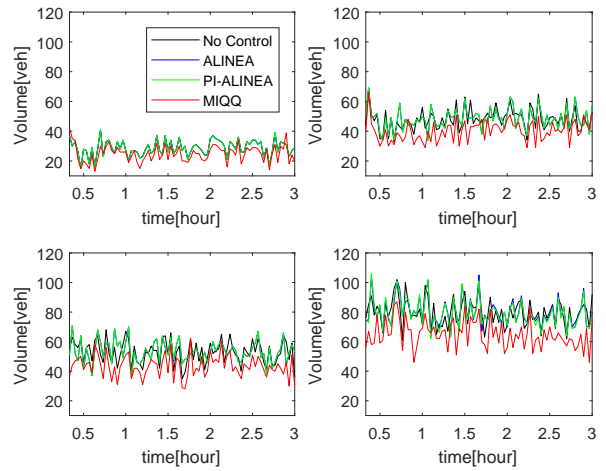|	No Control	ALINEA	PI-ALINEA	MIQQ
Speed Limit Signs	90km/h	90km/h	90km/h	Dynamic
Ramp Metering	Unavailable	Dynamic	Dynamic	Dynamic
Information Board	Unavailable	Unavailable	Unavailable	Available
TTT $[veh * h]$	633.26	634.53	624.53	542.63

Fig. 5.   Time history of vehicle conservation at highway segment 1 (*upper left*), highway segment 2 (*upper right*), highway segment 3 (*lower left*) and highway segment 4 (*lower right*).

simulation parameters of VISSIM in real time [34], [35]. To control speed limit in this study, the desired speed limits are dynamically adjusted at upstream point of each highway segment for every 2 mins. Moreover, ramp metering cycle lengths are updated per 2 mins based on the optimal solution from MIQQ. For ramps that are supposed to be closed from the control results, traffic volume of the corresponding ramp is assumed to be zero in the simulation.

B. Simulation Results

In order to compare the performance of MIQQ with other control strategies, four typles of simulation results are provided, including cases without control, ALINEA strategy, PI-ALINEA ramp metering method, and the proposed MIQQ method. Relative setting from the four methods are shown in Table I. For each case, the simulation lasts for 3 hours, where simulation from the 1st 20 mins is ignored due to the unstable traffic status at the beginning time.

ALINEA is a popular local responsive feedback ramp metering strategy, and has been verified to be an effective strategy in both field tests and simulation [36], [37]. ALINEA determines the metering rates based on the downstream mainline occupancy from the meter. Its objective is to maximize the mainline throughout by maintaining occupancy values below the preset threshold. Since it focuses on preventing merging congestion, ALINEA requires the real-time occupancy measurements around the merging areas to achieve efficiency. However, bottlenecks may be further away from the merging areas in real world scenarios, where ALINEA cannot lead to high efficiency. Thus, PI-ALINEA, a Proportional-Integral (PI) extension of ALINEA has been proposed and proved to be an efficient ramp-metering algorithm in the presence of far-downstream bottlenecks [26].

To verify the feasibility of the proposed MIQQ strategy in high traffic demands, north-to-south traffic flow are set to

1600 veh per hour (vph) and 1200 vph at source location of highway I-35 and US-69, respectively. Traffic volumes is 600 vph for on-ramp #3 and #4. It is assumed that 20% of traffic flow on US-69 coming from north will transfer to I-35. During each time interval, they are guided to travel through roadway #1 or #2 by the highway information board. For every 8 mins, densities are updated by the measured data on each highway link and on-ramp. The control variables are regenerated and displayed through the hybrid infrastructures every 2 mins.

Simulation results are shown in Table I and Fig. 5. The proposed MIQQ leads to further reduced TTT compared to the other three methods. The TTT reduction percentages are 14.31%, 14.48% and 13.11% compared to cases with no control, ALINEA, and PI-ALINEA, respectively. Furthermore, less vehicles are observed in each test highway link for every time interval. The comparative results verify that the proposed MIQQ strategy has improved efficiency in congestion alleviation during rush hours.

Since not all on ramps have ramp metering in a real world highway network, we assume only a subset of on-
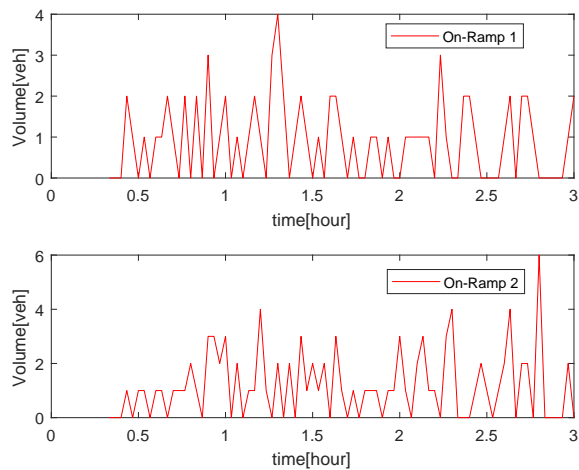
Fig. 6. Time history of vehicle conservation at on-ramp 1 (*upper*) and 2 (*lower*).

ramps is controlled by ramp metering. In this case, on-ramp 1 and 2 are controlled while on-ramp 3 are 4 have no ramp metering. Time history of vehicle count of on-ramps 1 and 2 are illustrated in Fig. 6. A threshold is considered to restrict queue length at on-ramps, i.e. maximum 6 vehicles in the waiting queue. Figure 6 demonstrates the queue length restriction is satisfied on each of the two controlled on-ramps.

VI. CONCLUSION

This article presents a time efficient traffic control strategy using hybrid highway infrastructures, including dynamic limit signs, ramp metering, and highway information boards. To predict the highway traffic status, the Barron-Jensen/Frankowska explicit solutions to the Cauchy problem is introduced based on the triangular fundamental diagram. The Lighthill-Whitham-Richards model is applied to each highway link to construct a finite number of linear constraints to describe the traffic dynamics. The minimum time transportation problem for the entire highway network is formulated as a mixed-integer quadratic programming problem with quadratic constraints, named MIQQ. Performance of the proposed MIQQ method is verified in a real world simulation example using VISSIM. Compared to existing methods, ALINEA and PI-ALINEA, MIQQ lead to more reduced travel time and alleviation of congestion during busy hours.

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