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## Comparative Profitability of Product Disclosure Statements

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## Comparative Profitability of Product Disclosure Statements

### Abstract

In insurance industry, product disclosure statements (PDSs) consist of descriptions of uncertain contingencies by the insurance plans (e.g., "hospital coverage", "dental coverage", etc.) and are often very different. In this paper, we model PDSs as information partitions of the state space, which can influence how a consumer perceives the structure of her choice problem and hence her deductible choices. We study a model of an insurance company that aims to promote profit by designing the framing of its PDS. We compare the company's profits under two PDSs, one of which is coarser than the other. Our main results show that under simple conditions, the PDS consisting of finer partitions of the more expensive states is more profitable.

### Keywords

Insurance demand, framing effect, state aggregation, persuasion, behavioral industrial organization

### Disciplines

Behavioral Economics | Economic Theory | Industrial Organization

# Comparative Profitability of Product Disclosure Statements\*

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February 4, 2020

In insurance industry, product disclosure statements (PDSs) consist of descriptions of uncertain contingencies by the insurance plans (e.g., “hospital coverage”, “dental coverage”, etc.) and are often very different. In this paper, we model PDSs as information partitions of the state space, which can influence how a consumer perceives the structure of her choice problem and hence her deductible choices. We study a model of an insurance company that aims to promote profit by designing the framing of its PDS. We compare the company’s profits under two PDSs, one of which is coarser than the other. Our main results show that under simple conditions, the PDS consisting of finer partitions of the more expensive states is more profitable.

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**JEL codes:** D11, D86, D91

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# 1 Introduction

Different ways of framing the same information have an impact on the final consumer decisions (Hershey et al., 1982; Thaler, 1980; Tversky and Kahneman, 1979), implying firms should pay close attention to how the product information is presented in the first place. Indeed, firms can actively use framing to increase their profit (Piccione and Spiegler, 2012; Salant and Siegel, 2018). For an insurance firm, its PDS describes the terms of an insurance plan and reflects the framing of the related contingencies, and the design of the PDS is often up to the insurer. When the consumers are not indifferent to the framing of the PDS, changes to how contingencies are structured in the insurance plan would affect the firm's profit. This paper studies the effect on the insurer's profit that follows from coarsening (refining) its PDS. In our analysis, we propose a decomposition of the total effect into two components: the pure effect from the redistribution of insurance demand inside the newly formed (split) section(s) and the effect from the redistribution of insurance between the sections. This decomposition allows us to identify sufficient conditions for unambiguous prediction of the direction of the total effect, which otherwise would not be possible. Even though firms may often bundle contingencies, our result is clear-cut: Grouping contingencies into expensive sections hurts profit.

Our paper is primarily motivated by the comparisons of different insurance plans across countries, which are illustrated by the examples that follow.

The first example is the private health insurance plans in Australia and Canada. Even though both countries have similar public health care systems, the private health insurance plans group contingencies differently. For example, Blue Cross Ontario offers health insurance plans consisting of five categories: "vision," "dental," "drugs," "hospital," and "massage therapy." At the same time, Australian insurer

NIB offers a coarser version of Blue Cross Ontario’s plan that consists only of two categories: “hospital” and “extras” (vision, dental, physio, pharmacy). In both situations, to buy an insurance plan, the consumer has to visit the company’s website and make her choice inside each category, where the precise details of the plan are specified in fine print.

Another example is dental insurance. For instance, Manulife, a major Canadian insurer, offers a dental insurance policy that groups all relevant contingencies into six categories: “preventive,” “restorative,” “endodontic and periodontics,” “major,” “orthodontia,” and “implants & related.” At the same time, the US insurer Spirit Dental & Vision combines Manulife’s categories “endodontic and periodontics,” “major,” and “implants & related” into its own “major,” and hence offers to the consumers a coarser description of the plan listing only four sections: “preventive,” “restorative,” “major,” and “ortho.”

In both scenarios, we consider a consumer who is sophisticated enough to read the fine print, and yet the initial categorization of events still influences her choices. In particular, her deductible choices are generated in two steps—she first evaluates the policy for each category and then aggregates the categorial values into an overall evaluation, displaying aversion to value variations both within each category and across different categories. Hence, this consumer’s choices depend on how the categories are framed (Burkovskaya, 2019).<sup>1</sup> If the insurer understands the nature of the consumer’s reactions to framing, can he improve profit by administering simple changes to the design of the PDS? In this paper, we aim to find out whether the insurer can benefit from (dis)aggregating categories of an existing PDS.

Formally, the consumer’s behavior in this paper is governed by the State Aggrega-

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<sup>1</sup>Burkovskaya and Teperski (2019) find experimental evidence that consumers are not indifferent to the framing of contingencies in insurance plans.

tion Subjective Expected Utility (SASEU) model (Burkovskaya, 2019). The SASEU consumer perceives the state space as a collection of events/“small worlds” that is imposed on her by the PDS the insurer offers. While the consumer is fully aware of all the states of the world, she also reacts to the framing. The agent evaluates each insurance plan in a two-stage process by first computing the expected utility of the plan in each section and then calculating the expected utility across the sections while applying some aggregation function to the values of the sections.

The insurance firm is a price-taker that offers plans consisting of a bundle of deductibles for each state of the world. Linear pricing of the deductibles determines the insurance premium. Importantly, the company is considering redesigning its current PDS to improve profit. It does so by applying one-step aggregation (or disaggregation) of the PDS, that is, combining several sections of the current PDS into one (or splitting one section into several). We are interested in whether such a change in PDS increases or decreases the firm’s profit.

To study the effect of a change in PDS on the insurer’s profit, we propose a decomposition method reminiscent of the Slutsky decomposition in classic demand analysis. The total impact on profit is decomposed into two parts: the aggregation effect and the event-smoothing effect. The aggregation effect accounts for rearranging insurance demand across states in the newly (dis)aggregated section(s), while keeping the income allocated to the newly (dis)aggregated section(s) unchanged. The event-smoothing effect accounts for redistribution of income across all sections of the new PDS.

We analyze the proposed effect in two cases. First, we consider the case when the utility function is constant absolute risk averse (CARA) and the aggregation function is CARA or decreasing absolute risk averse (DARA). Second, we allow the utility function to be either CARA or DARA, and assume the states and sections

can be aggregated monotonically in prices.

In both cases, our results suggest that for risk-averse and aggregation-averse consumers, the aggregation effect on insurer's profit is negative when aggregated states form an expensive event. The concave aggregation function forces the consumers to care more about the overall utility within each section and the balance across different sections. Consequently, to achieve a greater overall utility within a section, a consumer would redistribute her consumption from the more expensive states to the cheaper ones. As a result, such a consumer would redistribute her insurance demand inside the newly aggregated section from the more expensive states to the cheaper states. Hence, the insurer will be losing money. After this aggregation, the distribution of event values within the aggregated section changes from stochastic to deterministic and experiences a second-order stochastic dominance improvement. Due to the CARA or DARA assumption, the consumer displays positive prudence in the aggregation function and responds to the aggregation by redistributing consumption from the newly aggregated section to the other sections of the new PDS. Whenever the newly aggregated section is expensive enough to dominate the event-smoothing effect, in the second step, the insurance company would be losing money again. To summarize, if the insurance company is dealing with an aggregation-averse and prudent consumer, aggregating sections of its PDS into expensive ones would result in greater loss. Similarly, disaggregating expensive sections of the PDS will imply all the above-mentioned effects in the opposite direction, and lead to higher profit.

Our main findings can shed light on the insurance company's contract design. The results imply that, when consumers are aggregation averse and prudent, an insurance company should avoid aggregating events that are expensive when designing its PDS. Although other small costs could be associated with providing a finer PDS,

such as a slightly higher menu cost or higher cognitive cost borne by the consumers, the change we propose is easy to implement—it only requires rewriting the insurance booklet, breaking down the expensive sections. Importantly, it clearly profits the firm. By contrast, the standard techniques that an insurer normally uses to raise profit—such as coming up with a new product, increasing premiums, or investing in obtaining information that would allow the insurer to better discriminate consumers—can be more costly to implement and may even risk the company’s market share. By comparison, the change we propose is a simple remedy for the firm.

We also provide general characterizations that will help the firm quantify the effect on profit from aggregation, even if neither the monotone pricing nor the CARA utility function and the CARA or DARA aggregation function assumption is convincing (section 4.3). Again the quantitative results can be useful for guiding insurance contract design for a wider family of preferences.

Finally, the decomposition method we develop for the analysis can be useful for demand analysis for preferences beyond the SASEU case. The method applies to any decision problem where optimal decisions depend on how uncertain states are partitioned and the agent evaluates utility recursively in two stages, (e.g., Li, Forthcoming).<sup>2</sup>

## 1.1 Literature review

Our paper builds on the literature that considers individual choices that are sensitive to framing.<sup>3</sup> Salant and Rubinstein (2008) work with extended choice functions

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<sup>2</sup>Li (Forthcoming) considers recursive utilities (under ambiguity) that depend on both the state-contingent outcomes and how uncertainties are gradually resolved in two stage.

<sup>3</sup>The observation that framing could affect choices has been made in the behavioral economics literature for a long time. See, for instance, Tversky and Kahneman (1979), Thaler (1980), and



that depend on frames, and find conditions under which the induced choice correspondences are rationalizable.<sup>4</sup> The authors consider abstract frames, whereas this paper focuses on specific partitional frames. More similar to ours, Ahn and Ergin (2010) see frames as partitions and use frame-dependent preferences over acts to characterize non-additive belief functions. Our paper builds on the utility representation axiomatized by Burkovskaya (2019), where the DM evaluates an act in two stages: first for each event and then in the aggregate.<sup>5</sup> The focus of this paper is on the market implications of frame-sensitive consumer behaviors.

Our paper belongs to the behavioral industrial organization literature, which is surveyed in Spiegler (2011) and Heidhues and Kőszegi (2018).<sup>6</sup> A few papers that analyze market outcomes with frame-sensitive consumers are the most relevant to our paper. Piccione and Spiegler (2012) analyze two firms engaging in price competition, while limiting the comparability of their products by selecting different frames. They find a condition on the comparability structure of frames (called “weighted regularity”) that is necessary and sufficient for the firms to be frame neutral and receive competitive payoff in any symmetric equilibrium. Spiegler (2014) generalizes the analysis and applies this condition to study a firm’s equilibrium strategy in other frame-dependent behavioral models. More recently, Salant and Siegel (2018) consider frames that can either increase a product’s attractiveness or highlight a premium product temporarily,<sup>7</sup> and study the design of adverse-selection contract

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Hershey et al. (1982).

<sup>4</sup>See Bernheim and Rangel (2009) for a related work that focuses on welfare analysis.

<sup>5</sup>Li (Forthcoming) also studies a two-stage evaluation procedure in the context of ambiguity. The utility representation used here is also reminiscent of the smooth ambiguity model (Klibanoff et al., 2005) or the SOEU (Grant et al., 2009). Our method can also be applied to these preference models.

<sup>6</sup>See also Ellison (2006) and Armstrong (2008) for earlier surveys.

<sup>7</sup>A key feature is that such framing often affects the incentive compatibility but not the individual rationality constraints.

with binary types by a monopolist seller. The key observation is that the optimal separating contract often uses framing. Our paper differs from the existing literature in two aspects. First, unlike most of the papers cited above, which follow Salant and Rubinstein (2008) and consider a set of abstract frames, we work with a concrete type of frames—the PDSs in the insurance industry—and explicitly model frame sensitivity as a consequence of the consumer’s two-stage evaluation procedure. Second, we consider an insurer who faces perfect competition (thus a price taker) and yet could influence consumer choices by designing PDSs.

Our paper is also related to Gul et al. (2017), who look at the general equilibrium in a Lucas-tree economy in which a consumer can only choose coarse consumption plans. Different from Gul et al. (2017), we consider a partial equilibrium model with exogenous prices, yet evaluate both consumer demand and the firm’s choice of framing.

Finally, our paper can shed light on the persuasion literature (Bergemann and Morris, 2019; Kamenica and Gentzkow, 2011). It is particularly related to recent papers that analyze the design of information structures with behavioral agents (Lipnowski and Mathevet, 2018; Beauchêne et al., 2019; Lipnowski et al., Forthcoming). In this paper, the sender (insurer) aims to design the PDS, which is an information partition, that is beneficial when the receiver (consumer) is not an EU maximizer. Our findings identify natural conditions on consumers’ preferences under which the insurer benefits from providing a finer information partition of the more expensive states.

The paper proceeds as follows. Section 2 introduces the model and the decomposition method. Section 3 states the main result under the CARA utility assumption and illustrates the model with numerical examples. Section 4 provides a discussion about optimal PDS, delivers the results under monotone pricing and also DARA

utility, and quantifies the aggregation and event-smoothing effects in the general case.

## 2 The model

### 2.1 Notation

There are  $n$  states of the world that might happen at a future date and the state space is  $\Omega = \{s_1, s_2, \dots, s_n\}$ . Let state  $s_1$  be the default state at which no accident occurs. For each state  $s_i$  with  $i > 1$ , a loss  $l_i$  occurs. An insurance company chooses and offers its clients a PDS  $\pi = \{s_1, S_2, \dots, S_N\}$ —an information partition of the state space. A PDS consists of events and a default state of no accident  $s_1$ , where each event is a separate section of the insurance plan. For each section (event)  $S_k$  consisting of states  $s_{k1}, \dots, s_{kt}$ , the insurance plan describes for every state in this section the insurance coverage and the corresponding deductibles  $d_{k1}, \dots, d_{kt}$ . The insurance premium of the plan is  $p = \sum_{i=2}^n (l_i - d_i)p_i$ , where  $l_i - d_i$  is the reimbursement claimed by the consumer in state  $s_i$ , and  $p_i$  is the price of a \$1 deductible decrease in state  $s_i$ .<sup>8</sup>

### 2.2 Consumer

A consumer has a fixed income  $I$  and demands insurance coverage when contingencies are described by PDS  $\pi$ . Our goal is to study the impact on profit, when the frame adopted in the PDS affects the consumer's insurance demand. Hence, we

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<sup>8</sup>Notice  $p_i$  is the price of a unit of Arrow security in state  $s_i$ : Decreasing the deductible by \$1 in some state is equivalent to increasing consumption by \$1 in the same state.

assume the consumer has SASEU<sup>9</sup> preferences:

$$U(s, d_2 \dots d_n; \pi) = P(s_1)\phi(u(s)) + \sum_{i=2}^N P(S_i)\phi\left(\sum_{s_j \in S_i} P(s_j|S_i)u(s - d_j)\right),$$

where  $u : \mathbb{R} \mapsto \mathbb{R}$  is the von Neumann-Morgenstern (vNM) utility index of money,  $\phi : u(\mathbb{R}) \mapsto \mathbb{R}$  is a function that describes the consumer's attitude toward aggregation,  $P(s_i)$  is the probability of state  $s_i$ ,  $s$  is the consumption in state  $s_1$ , and  $d_i$  is the deductible choice in state  $s_i$ . We assume both  $u(\cdot)$  and  $\phi(\cdot)$  are increasing and three-times differentiable.

Preferences for categorization or simplification of the state space guide the non-trivial behavior of the consumer in our paper. The agent is fully aware of the existence of each state; however, she aggregates some of the “similar” states together to create a “small world” relevant for a certain problem. In this paper, the insurer conveniently offers such categorization through the framing of its PDS. Note that even though the consumer groups some states together, she is not restricted to making identical choices in those states.

Formally, an SASEU consumer follows a two-stage procedure in evaluation. She first calculates the expected utility for each section  $S_i$  in PDS  $\pi$  using a standard vNM utility function, and then computes the overall expected utility across various sections in  $\pi$  applying another aggregation function  $\phi$ .

Moreover, the consumer faces budget constraint  $s + \sum_{i=2}^n (l_i - d_i)p_i = I$ . Let  $B(p, I)$  be the set of feasible choices  $(s, d_2 \dots d_n)$  at price  $p$  and income  $I$ . Given

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<sup>9</sup>See Burkovskaya (2019) for the axiomatic foundations for the SASEU model.

PDS  $\pi$ , the consumer’s insurance demand problem, which we will call (DP- $\pi$ ), is<sup>10</sup>

$$\max_{(s, d_2, \dots, d_n) \in B(p, I)} U(s, d_2 \dots d_n; \pi).$$

Note that if  $\phi(\cdot)$  is linear, the model reduces to the classical subjective expected utility (SEU) model and the consumer is indifferent to the framing of PDS. Similar to risk-aversion for  $u(\cdot)$ , the curvature in  $\phi(\cdot)$  brings about the attitude toward aggregation. In particular, a concave  $\phi(\cdot)$  delivers aggregation aversion—a type of behavior characterized by more “spread-out” consumption choices in each event as a reaction to the aggregation of states. As  $\Delta c_i = \Delta s - \Delta d_i$ , the effect on deductible choices goes in the opposite direction of that on consumption.

To illustrate different attitudes toward aggregation, consider the following numerical example. Suppose there are three states of the world,  $\Omega = \{s_1, s_2, s_3\}$ , with probabilities  $P(s_1) = 0.4$ ,  $P(s_2) = 0.2$ , and  $P(s_3) = 0.4$ . The consumer can buy an insurance plan consisting of two deductibles  $(d_2, d_3)$  corresponding to risky states  $s_2$  and  $s_3$ , whereas state  $s_1$  is the default state with no losses. In addition, the Arrow prices of the risky states are  $p_2 = 0.3$  and  $p_3 = 0.4$ . The consumer has income  $I = 100$ , and her utility is  $u(x) = \ln x$ . Consider two types of aggregation function: (1)  $\phi(u) = u$  is linear; and (2)  $\phi(u) = 1 - e^{-u}$  is concave. Table 1 compares the deductible choices for aggregated PDS  $\pi = \{s_1, \{s_2, s_3\}\}$  and finest PDS  $\Omega = \{\{s_1\}, \{s_2\}, \{s_3\}\}$  for the two cases of aggregation attitudes. Observe that the aggregation-averse consumers choose more “spread-out” deductibles when facing the aggregated frame  $\pi$ . We hereafter focus on behaviors of the aggregation-averse consumers.

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<sup>10</sup>Note we allow the deductibles to take on negative values, hence permitting overinsurance in the states with very low prices. This assumption is equivalent to free access to the financial credit

| $\phi$  | $d_2^\Omega$ | $d_3^\Omega$ | $d_2^\pi$ | $d_3^\pi$ |
|---------|--------------|--------------|-----------|-----------|
| linear  | 66.7         | 33.3         | 66.7      | 33.3      |
| concave | 34.1         | 15.6         | 44.7      | 8.6       |

Table 1: Deductible choices for different aggregation attitudes.

Throughout this paper,  $A_u(x) = -\frac{u''(x)}{u'(x)}$  is the measure of absolute risk aversion, and  $A_\phi(u) = -\frac{\phi''(u)}{\phi'(u)}$  is the analogous measure of absolute aggregation aversion, whereas  $P_u(x) = -\frac{u'''(x)}{u''(x)}$  and  $P_\phi(u) = -\frac{\phi'''(u)}{\phi''(u)}$  are the measures of absolute prudence and absolute aggregation prudence, respectively.

## 2.3 Insurance company

We assume the insurance company is risk neutral and takes asset prices  $p_i$  as given. The company aims to maximize expected profit of the following form:

$$\begin{aligned}
profit(\pi) &= premium(\pi) - \sum_{i=2}^n P(s_i)(l_i - d_i^\pi) = \alpha - \sum_{i=2}^n d_i^\pi p_i - \sum_{i=2}^n P(s_i)(l_i - d_i^\pi) \\
&= const - \sum_{i=2}^n d_i^\pi (p_i - P(s_i)) = const - L^\pi,
\end{aligned}$$

where  $\alpha$  is a fixed premium for full coverage,  $const = \alpha - \sum_{i=2}^n l_i P(s_i)$ <sup>11</sup> is a constant part of the profit that does not depend on consumer choices, and  $L^\pi = \sum_{i=2}^n d_i^\pi (p_i - P(s_i))$  is the expected loss function of the insurer under PDS  $\pi$ . In the analysis that follows, we work with the loss  $L^\pi$ , implying its direct relationship with the profit.

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markets.

<sup>11</sup>Note that generally the insurance company would choose  $\alpha$  to maximize profit, or it might come from the market if the company is a price-taker. In the latter case,  $\alpha = \sum_{i=2}^n l_i p_i$ . Nevertheless, the choice of  $\alpha$  is irrelevant to the problem of the choice of PDS framing in this model.

## 2.4 One-step aggregation of PDS

Our goal is to analyze the changes in insurance demand and insurer's loss resulting from aggregating some events described in a PDS into a coarser event.<sup>12</sup>

Denote the original PDS by  $\pi = \{s_1, S_2, \dots, S_N\}$  and the aggregated PDS by  $\rho = \{s_1, S_2, \dots, S_{k-1}, B, S_{t+1}, \dots, S_N\}$ , where event  $B$  is the union of  $S_k, \dots, S_t$  ( $k < t$ ). We assume the default state  $s_1$  cannot be aggregated with any other states, because  $s_1$  represents the distinctive case when no loss occurs. Throughout the paper, we assume the insurer observes the insurance demand under the current PDS  $\pi$ , state prices, probabilities, and income of its clients. For the quantitative analysis in section 4.3, we also suppose the insurance company knows or is able to estimate the vNM utility function  $u(\cdot)$  and the aggregation function  $\phi(\cdot)$ . To simplify notation, we normalize  $p_1 = 1 - \sum_{i=2}^n p_i$  and denote  $r_i = \frac{p_i}{P(s_i)}$ . And we will treat the state prices and probabilities as fixed, and hence we can ignore how consumer choices are dependent on them in our notation.

In the beginning, the consumer with the PDS  $\pi$  chooses  $(s, d_2, \dots, d_n)$  to solve the insurance demand problem (DP- $\pi$ ). For simplicity, we denote the solution to this problem as a consumption bundle  $c = (s, c_2, \dots, c_n)$ , the optimal deductibles as  $d_i = s - c_i$  (for  $i = 2, \dots, n$ ), and the corresponding loss as  $L = \sum_{i=2}^n d_i(p_i - P(s_i))$ . The following FOCs characterize solutions to this  $\pi$ -optimization problem:

$$\phi'(V_{S_k}(s))u'(s - d_i) = \lambda_\pi r_i \text{ for all } S_k \in \pi, s_i \in S_k, \quad (1)$$

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<sup>12</sup>Characterizing the optimal PDS depends on many details and is ambiguous overall. Instead, in this paper, we concentrate on a relatively "small" change in the structure of the current PDS—the case that produces clear-cut predictions. Even though we are not able to provide a complete characterization about the general structure of the optimal PDS, we show some of its necessary features. See section 4.1 for further discussion.

where  $\lambda_\pi \geq 0$  is the Lagrange multiplier of optimization problem with PDS  $\pi$ , and  $V_{S_k}(s) = \sum_{s_i \in S_k} P(s_i|S_k)u(s - d_i)$  is the expected utility conditional on event  $S_k$ , which we call the value of event  $S_k$ .

By dividing the FOCs for any two events  $S_k, S_l$ , we have

$$\frac{r_i}{r_j} = \frac{\phi'(V_{S_k}(s))}{\phi'(V_{S_l}(s))} \frac{u'(s - d_i)}{u'(s - d_j)} \text{ for all } s_i \in S_k, s_j \in S_l.$$

After aggregation, events  $\{S_k, \dots, S_t\}$  are lumped into a single event  $B$ . The consumer facing PDS  $\rho = \{s_1, S_2, \dots, S_{k-1}, B, S_{t+1}, \dots, S_N\}$  solves the problem (DP- $\rho$ ):

$$\max_{(s, d_2, \dots, d_n) \in B(p, I)} U(s, d_2, \dots, d_n; \rho) = P(s_1)\phi(u(s)) + \sum_{S_i \in \rho \setminus \{s_1\}} P(S_i)\phi\left(\sum_{s_j \in S_i} P(s_j|S_i)u(s - d_j)\right).$$

We denote the optimal solution to (DP- $\rho$ ) as a consumption bundle  $\tilde{c} = (\tilde{s}, \tilde{c}_2, \dots, \tilde{c}_n)$  and optimal deductibles as  $\tilde{d}_i = \tilde{s} - \tilde{c}_i$ . The corresponding loss is denoted  $\tilde{L} = \sum_{i=2}^n \tilde{d}_i(p_i - P(s_i))$ . The FOCs for the  $\rho$ -optimization problem are

$$\phi'(V_{S_k}(\tilde{s}))u'(\tilde{c}_i) = \lambda_\rho r_i \text{ for all } S_k \in \rho \text{ and } s_i \in S_k. \quad (2)$$

Our main interest in this paper is the changes in insurance demand  $\Delta d_i = \tilde{d}_i - d_i$  and the expected loss of the insurer  $\Delta L = \tilde{L} - L$ , following an aggregation of PDS from  $\pi$  to  $\rho$ . Note our analysis applies directly to the impact following a disaggregation of PDS from  $\rho$  to  $\pi$ .



## 2.5 Decomposition of the effect

We call the effect of a change from  $\pi$  to  $\rho$  on consumption and deductibles a total one-step aggregation effect. We decompose this total effect into two parts—an *aggregation effect* and an *event-smoothing effect*. The aggregation effect accounts for the agent’s desire to rearrange consumption according to the newly aggregated event. The event-smoothing effect refers to the agent’s desire to balance the overall values of different sections of the PDS.

The decomposition we use is close in spirit to the classic Slutsky decomposition of the effect on consumption from a change in price. Our aggregation effect is in line with the classic substitution effect, which requires consumption to adjust according to the new level of the marginal rate of substitution dictated by the new price ratio. The aggregation effect requires the consumption to adjust according to the new level of marginal rate of substitution inside the aggregated event dictated by the change in aggregation. In the same fashion, our event-smoothing effect is in line with the classic income effect, which requires the adjustment of income to satisfy the budget constraint. Similarly, the event-smoothing effect requires readjustment of a fixed amount of income across different events. Nevertheless, it differs from the income effect in that the changes in income on the newly aggregated event and those on the unaffected events are in opposite directions.

To decompose the total effect on consumption in each state  $s_i$ ,  $\Delta c_i = \tilde{c}_i - c_i$ , we define an intermediate insurance bundle  $(s^*, d_2^*, \dots, d_n^*)$  that is reminiscent of the Hicksian demand, which solves the dual expenditure-minimization problem at a fixed indirect utility level and new prices. In a similar spirit, our intermediate consumption solves the utility-maximization problem with the same fixed expenditure on the aggregated event  $B$  and the newly aggregated conditional preferences on

$B$ . By construction, this change applies only to the states in  $B$ , and hence for any  $s_i \notin B$ ,  $d_i^* = d_i$ ,  $s^* = s$ . Formally, for any  $s_i \in B$ ,  $d^*$  is the solution to the following intermediate problem, denoted (DP-B):

$$\begin{aligned} \max_{\{d_i^*\}_{s_i \in B}} & P(B)\phi\left(\sum_{s_i \in B} P(s_i|B)u(s^* - d_i^*)\right) \\ \text{s.t.} & \sum_{s_i \in B} p_i(s - d_i^*) = \sum_{s_i \in B} p_i(s - d_i). \end{aligned}$$

We can also define an intermediate consumption bundle  $c_i^* = s - d_i^*$  and let  $I_B = \sum_{s_i \in B} p_i(s - d_i)$ .

The FOCs for the solution of the intermediate bundle  $(s^*, d_2^*, \dots, d_n^*)$  are

$$\phi'(V_B(s^*))u'(s - d_i^*) = \lambda_B r_i, \quad (3)$$

where  $\lambda_B$  is the Lagrange multiplier of the intermediate optimization problem at event  $B$ .

Our decomposition first considers the change from  $c$  to  $c^*$ ,  $\Delta c^* = c^* - c$ , which is the pure aggregation effect because consumption in all the states unaffected by the one-step aggregation is kept fixed. We then examine the change from  $c^*$  to  $\tilde{c}$ ,  $\Delta \tilde{c} = \tilde{c} - c^*$ , which is the event-smoothing effect because it requires only redistribution of available income across different events in the PDS. We can define the corresponding intermediate changes in the expected loss,  $\Delta L^* = \sum_{i=2}^n \Delta d_i^*(p_i - P(s_i))$  and  $\Delta \tilde{L} = \sum_{i=2}^n \Delta \tilde{d}_i(p_i - P(s_i))$ , the sum of which is the total change in loss; that is,  $\Delta L = \Delta \tilde{L} + \Delta L^*$ .

### 3 Comparative statics of one-step aggregation

In this section, we consider the case when the analyst does not observe consumer preferences directly; however, she is willing to make certain assumptions about the utility and aggregation functions. In addition, the analyst has access to insurance demand and price data under the current PDS  $\pi$ . We are interested in analyzing the impact of one-step aggregation of PDS on profit and deductible choices under the following assumptions on  $u(\cdot)$  and  $\phi(\cdot)$ .

**Assumption 1.**  $u(\cdot)$  is CARA.

**Assumption 2.**  $\phi(\cdot)$  is CARA or DARA.

Assumption 1 implies  $u(c) = \frac{1}{\gamma}(1 - e^{-\gamma c})$ , where  $\gamma > 0$  is the coefficient of absolute risk aversion. Assumption 2 guarantees non-decreasing absolute aggregation aversion. In section 4, we discuss the comparative statics when these assumptions are relaxed.

For each event  $S_k \in \pi$ , denote by  $p_{S_k} = \sum_{s_i \in S_k} p_i$  the price of a security that delivers \$1 in every state in event  $S_k$ , and by  $r_{S_k} = \frac{p_{S_k}}{P(S_k)}$  the probability-adjusted price of \$1 consumption in event  $S_k$ .

Our main result states that under the two assumptions, aggregating states into more expensive events (relative to the default state  $s_1$ ) hurts profit.

**Theorem 1.** *Suppose  $u(\cdot)$  and  $\phi(\cdot)$  satisfy Assumptions 1 and 2. If  $r_B \geq r_1$ , aggregating the PDS from  $\pi$  to  $\rho$  strictly increases the expected loss. Moreover, both the aggregation effect and the event-smoothing effect lead to a strict increase in expected loss.*

Under simple assumptions on preferences, Theorem 1 offers a straightforward method for the insurance firm to raise profit. To do so, the insurer only needs to slightly tweak its PDS by separating events that are more expensive than the default state  $s_1$ . Standard methods to promote profit, such as higher premiums, more obfuscated contract terms, or more intricate price-discrimination schemes, can cost the firm heavily in market share or administrative expenses. Unlike these methods, the change proposed here is easy to implement. The firm only needs to redesign the presentation of its policy booklet or website, making sure the expensive states are listed separately. For example, the category “heart diseases” would normally include a number of very expensive medical conditions such as stroke and heart attack.<sup>13</sup> However, our result suggests the insurer would benefit if these conditions were listed separately.

The following example illustrates the findings in our Theorem 1.

**Example 1.** *Consider a consumer who has wealth  $I = \$15$  and would like to purchase an insurance plan. There are four states of the world  $\Omega = \{s_1, s_2, s_3, s_4\}$ , which may occur with probabilities  $P(s_1)$ ,  $P(s_2)$ ,  $P(s_3)$ , and  $P(s_4)$ .*

*The consumer’s preferences are represented by the SASEU discussed above, with an vNM utility index  $u(\cdot)$  and an event-aggregation index  $\phi(\cdot)$  that captures sensitivity to framing. To illustrate, we consider a CARA  $u$  and a logarithmic  $\phi$  functions as follows:*

$$u(x) = \frac{1}{\gamma}(1 - e^{-\gamma x}), \quad \phi(u) = \ln(u).$$

*Consider the consumer first chooses insurance and faces the finest PDS  $\pi =$*

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<sup>13</sup>Note that even after normalizing the price by the probability, the price is likely to be very high for these conditions.

$\{\{s_1\}, \{s_2\}, \{s_3\}, \{s_4\}\}$ . Then, suppose the consumer is provided with a coarser PDS  $\rho = \{\{s_1\}, B, \{s_4\}\}$ , where event  $B = \{s_2, s_3\}$  is the aggregate of states  $s_2$  and  $s_3$ .

We set  $\gamma = 0.3$ ,  $I = 15$ ,  $\alpha = 5$ ,  $P(s_1) = 0.2$ ,  $P(s_2) = 0.4$ ,  $P(s_3) = 0.1$ , and  $P(s_4) = 0.3$ . In addition, we consider prices  $p_1^1 = 0.2$ ,  $p_2^1 = 0.5$ ,  $p_3^1 = 0.2$ , and  $p_4^1 = 0.1$ . Table 2 shows the choices of insurance plans and the decomposition of the difference in loss for frames  $\pi$  and  $\rho$ .

|                           | $d_2$  | $d_3$  | $d_4$   | $s$     | $\Delta L^*$ | $\Delta \tilde{L}$ | $\Delta L$ |
|---------------------------|--------|--------|---------|---------|--------------|--------------------|------------|
| $\pi^1$                   | 0.7076 | 2.1678 | -3.5634 | 10.4310 | 0.0046       | 0.0018             | 0.0064     |
| intermediate <sup>1</sup> | 0.6771 | 2.2438 |         |         |              |                    |            |
| $\rho^1$                  | 0.6861 | 2.2528 | -3.5636 | 10.4372 |              |                    |            |

Table 2: Insurance choices and decomposition changes for different frames.

Observe that in Example 1, the insurer achieves a higher profit under the finer PDS that separates states  $s_2$  and  $s_3$  than that under the coarser PDS  $\rho$ . Moreover, in the one-step aggregation, both aggregation and event-smoothing effects reduce profit. Note also that  $r_1^1 = 1 < 1.4 = r_B^1$ .

The next example illustrates the role of the price condition  $r_1 \leq r_B$ .

**Example 2.** Suppose all else is the same as Example 1 but the prices are  $p_1^2 = 0.2$ ,  $p_2^2 = 0.2$ ,  $p_3^2 = 0.2$ , and  $p_4^2 = 0.4$  instead. So Assumptions 1 and 2 still hold, but  $r_1^2 = 1 > 0.8 = r_B^2$ . Then, the choices of insurance plans and the decomposition of the difference in loss for frames  $\pi$  and  $\rho$  become as shown in Table 3.

Observe that in Example 2, the aggregation effect is still in the same direction as before, but relaxing the price condition might change the sign of the event-smoothing effect.

|                           | $d_2$   | $d_3$  | $d_4$  | $s$     | $\Delta L^*$ | $\Delta \tilde{L}$ | $\Delta L$ |
|---------------------------|---------|--------|--------|---------|--------------|--------------------|------------|
| $\pi^2$                   | -2.2349 | 2.1643 | 0.9095 | 10.3497 |              |                    |            |
| intermediate <sup>2</sup> | -2.3458 | 2.2752 |        |         | 0.0333       | -0.0065            | 0.0268     |
| $\rho^2$                  | -2.2807 | 2.3403 | 0.9099 | 10.3759 |              |                    |            |

Table 3: Insurance choices and decomposition changes for different frames.

Together, Example 1 and Example 2 satisfy Assumptions 1 and 2. However, Example 1 satisfies condition  $r_B \geq r_1 \geq 1$ , whereas Example 2 does not. This condition allows us to predict the direction of the loss change for both effects in Example 1, and hence to know unambiguously the direction of the total loss change. At the same time, we are still able to establish the direction of the aggregation effect in Example 2, but we cannot say the same about the event-smoothing effect. Thus, we cannot predict the direction of the loss change in Example 2.

In what follows, we prove Theorem 1 in three steps. First, we show Assumptions 1 and 2 imply monotonic ordering of events with respect to event prices, which allows us to rank events without additional knowledge about the consumer's utility and aggregation functions. Second, we analyze the aggregation effect and show the consumer redistributes insurance from the more expensive sub-events to the cheaper ones inside the newly aggregated event  $B$ , which reduces the firm's profit. Finally, we analyze the event-smoothing effect. We verify that the consumer redistributes insurance from the newly aggregated event  $B$  to all events unaffected by the aggregation. If  $B$  is also expensive enough, this effect further reduces profit.

### 3.1 Monotonicity

The first and most important implication of Assumptions 1 and 2 is that the conditional expected utility of an event is negatively related to its probability-adjusted

price, which is stated in the following lemma.

**Lemma 1** (Monotonicity). *Under Assumptions 1 and 2, for any two events  $S_i, S_j \in \pi$ ,  $V_{S_i}(s) > V_{S_j}(s)$  if and only if  $\frac{p_{S_i}}{P(S_i)} < \frac{p_{S_j}}{P(S_j)}$ .*

The monotonicity property indicated in Lemma 1 allows us to reorder the events according to their prices in the following way. For each event  $S_k \in \pi$ ,  $r_{S_k} = \frac{p_{S_k}}{P(S_k)}$  is the probability-adjusted price of \$1 consumption in event  $S_k$ . Let the events in  $\pi$  be ranked from the cheapest to the most expensive; that is,  $r_{S_2} < \dots < r_{S_N}$ . By Lemma 1, we have  $V_{S_2}(s) > \dots > V_{S_N}(s)$ . Upon aggregation to PDS  $\rho$ , the price of the aggregated event  $B = \{S_k, \dots, S_t\}$  can be calculated and ranked with events in  $\rho \setminus B$ . Furthermore, the events inside  $B = \{S_k, \dots, S_t\}$  can be ranked similarly. An attraction of Lemma 1 is that the order of the events can be computed based on observable variables only (i.e., prices and probabilities).

**Example 3.** *According to Lemma 1, we can rank events under each frame in the illustrating examples that we give earlier. For the prices in Example 1, we have the following ranking:*

$$\pi^1 : r_1 = 1; r_2 = 1.25; r_3 = 2; r_4 = 0.33 \Rightarrow V_{s_3}(s) > V_{s_2}(s) > V_{s_1}(s) > V_{s_4}(s)$$

$$\rho^1 : r_1 = 1; r_B = 1.4; r_4 = 0.33 \Rightarrow V_B(s) > V_{s_1}(s) > V_{s_4}(s).$$

*At the same time, for the prices in Example 2, we have*

$$\pi^2 : r_1 = 1; r_2 = 0.5; r_3 = 2; r_4 = 1.33 \Rightarrow V_{s_3}(s) > V_{s_4}(s) > V_{s_1}(s) > V_{s_2}(s)$$

$$\rho^2 : r_1 = 1; r_B = 0.8; r_4 = 1.33 \Rightarrow V_{s_4}(s) > V_{s_1}(s) > V_B(s).$$

### 3.2 Aggregation effect

In this subsection, we consider the aggregation effect— $\Delta c^*$  and  $\Delta L^*$ —under the assumptions on  $u$  and  $\phi$ . We start with Lemma 2, which establishes the aggregation effect on consumption.

**Lemma 2.** *If Assumptions 1 and 2 hold, then for any state  $s_j \in S_i$  and event  $S_i \in B$ , the aggregation effect on consumption is*

$$\Delta c_j^* = \frac{1}{\gamma} \left( 1 - \frac{\phi'(V_{S_i}(s))}{\sum_{S_k \in B} \frac{p_{S_k}}{p_B} \phi'(V_{S_k}(s))} \right).$$

Note Lemma 2 implies the effect on consumption in event  $S_i$  depends on the relationship between the original marginal value in this event,  $\phi'(V_{S_i}(s))$ , and the relative-price-weighted average of marginal values of sub-events inside  $B$ . This fact also suggests the aggregation effect on consumption is the same for all states in the same original event, which is the result of CARA  $u$ . In addition, the aggregation effect on consumption in the cheaper events (with the greatest values and the lowest marginal values) will be positive, pushing the already high consumption and event value up even higher. On the other hand, the aggregation effect on the more expensive events (with the smallest values and the greatest marginal values) will be negative, pushing the already low consumption and the event value further down.

For the aggregation effect on deductibles, because consumption in default state  $s_1$  does not change, we have  $\Delta d_i^* = -\Delta c_i^*$ . Thus, deductibles will increase for the more expensive events, and they will decrease for the cheaper events. Such behavior suggests the desire to redistribute insurance from the more expensive events in  $B$  to the cheaper ones.

The next proposition characterizes the aggregation effect on the insurer's losses.



**Proposition 1.** *If Assumptions 1 and 2 hold, then*

$$\Delta L^* = \sum_{s_i \in B} P(s_i) \Delta c_i^* = \frac{P(B)}{\gamma} \left( 1 - \frac{\sum_{S_k \in B} P(S_k|B) \phi'(V_{S_k}(s))}{\sum_{S_k \in B} \frac{p_{S_k}}{p_B} \phi'(V_{S_k}(s))} \right) > 0.$$

First, note the change in losses corresponds to the change in premium,  $(-\sum_{s_i \in B} p_i \Delta d_i^*)$ , less the change in the expected reimbursement,  $(\sum_{s_i \in B} P(s_i) \Delta d_i^*)$ . For the aggregation effect, the premium is fixed; hence, the change in loss depends only on the redistribution of insurance between the states in  $B$ , and it equals the change in expected consumption.

Second, the direction of the effect is defined by the relationship between the relative price of event  $S_k$  in  $B$ ,  $\frac{p_{S_k}}{p_B}$ , and the conditional probability of event  $S_k$  given  $B$ ,  $P(S_k|B)$ . For the more expensive events, the relative prices  $\frac{p_{S_k}}{p_B}$  will be greater than the corresponding conditional probabilities  $P(S_k|B)$ . In this case, the consumer redistributes insurance from the more expensive events toward the cheaper events, while increasing the loss of the insurer.

**Example 4.** *The aggregation effect is always positive for CARA  $u$  and CARA or DARA  $\phi$ , which is the case for both Example 1 and Example 2 (see Tables 2 and 3 above). However, when Assumptions 1 and 2 do not hold, the aggregation effect might become negative. Consider a case with  $u(x) = \ln x$  and  $\phi(u) = e^{0.1u}$  and the rest of the parameters from Example 1. Table 4 demonstrates the corresponding solutions to the  $\pi$ -,  $\rho$ - and intermediate problems together with the aggregation and event-smoothing effects on losses. In this particular scenario, the aggregation effect is negative.*

|              | $d_2$ | $d_3$ | $d_4$   | $s$   | $\Delta L^*$ | $\Delta \tilde{L}$ | $\Delta L$ |
|--------------|-------|-------|---------|-------|--------------|--------------------|------------|
| $\pi$        | 2.149 | 5.256 | -23.386 | 9.787 |              |                    |            |
| intermediate | 2.227 | 5.062 |         |       | -0.0117      | 0.0008             | -0.0108    |
| $\rho$       | 2.229 | 5.064 | -23.389 | 9.788 |              |                    |            |

Table 4: Insurance choices and decomposition changes for different frames.

### 3.3 Event-smoothing effect

In this subsection, we turn to the event-smoothing effect that takes care of redistributing the income across events in the eventual PDS  $\rho$ . We start with establishing an important relationship among the Lagrange multipliers of the three problems we analyze.

**Lemma 3.** *If Assumptions 1 and 2 hold,  $\lambda_B < \lambda_\rho < \lambda_\pi$ ,  $\Delta \tilde{c}_i < 0$  for all  $s_i \in B$  and  $\Delta \tilde{c}_j > 0$  for all  $s_j \in B^c$ .*

In the case of CARA  $u$  and CARA or DARA  $\phi$ , the value of event  $B$  in the intermediate problem always dominates the weighted average value of events in  $B$  before the aggregation. Because the marginal event values have the opposite relationship (because  $\phi$  is concave), the above-mentioned effect implies  $\lambda_B < \lambda_\pi$ , so the shadow value of income in (DP- $B$ ) is higher than that in (DP- $\pi$ ). Consequently, post-aggregation income should be redistributed from the aggregated event  $B$  to other events unaffected by the aggregation to mitigate this gap and equalize the marginal event values again in (DP- $\rho$ ). This income redistribution implies  $\lambda_\rho < \lambda_\pi$ , and consumption in states in  $B$  will drop, whereas those in states outside  $B$  will go up in the event-smoothing stage. Intuitively, after the aggregation stage, the value of event  $B$  goes up more than necessary, so the event-smoothing stage brings it slightly down to balance with the values of the unaffected events. Hence, the consumer

redistributes income from event  $B$  onto other events.

Furthermore, observe that Assumption 2— $\phi$  is CARA or DARA—implies  $\phi''' > 0$ , which generates an income-redistribution effect. The condition is analogous to the notion of positive absolute prudence and its application in the “precautionary saving” problem in classic risk analysis (Kimball, 1990). In our context, the event-attitude index  $\phi$  has positive absolute prudence, which implies a similar “precautionary saving motive” for expenditure redistribution between events  $B$  and  $B^c$ —the consumer will save more for consumption in  $B$  if more variations are present in event values  $V_{S_k}$  for  $S_k \in B$ . In particular, under the original PDS  $\pi$ , she experiences variations in conditional utilities from events  $S_k \in B$ , and these event values are lumped into a constant utility  $V_B(s^*)$  after the event-aggregation stage.<sup>14</sup> Hence, the aggregation effect leads to a lower variation in the event values in  $B$  that are aggregated by the  $\phi$  function, whereas event values outside  $B$  remain unchanged. Therefore, at the event-smoothing stage, when the consumer is redistributing expenditure/“savings” from event  $B^c$  to event  $B$ , she perceives lower variation in the event values in  $B$ , and hence will “save” less for consumption in  $B$ . Consequently, together with other standard assumptions,  $\phi''' > 0$  implies  $\Delta I_B < 0$  (i.e.,  $\lambda_B < \lambda_\pi$ ) in the event-smoothing stage.

Similar to the aggregation effect, because  $u$  is CARA, the changes in consumption are the same for all the states in an event.

**Lemma 4.** *If Assumption 1 holds, then for any event  $S_k \in \rho$  and  $s_i, s_j \in S_k$ ,  $\Delta \tilde{c}_i = \Delta \tilde{c}_j = \Delta \tilde{c}_{S_k}$ .*

Lemma 5 below suggests changes in consumption outside event  $B$  can be monotonically ranked by the corresponding event prices. Hence, the cheaper the event, the greater the change in consumption in the event-smoothing stage.

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<sup>14</sup>In fact,  $V_B(s^*)$  second-order stochastically dominates  $\{V_{S_k}\}_{S_k \in B}$ , as  $V_B(s^*) > V_B(s)$ .

**Lemma 5.** *If Assumptions 1 and 2 hold, then for  $S_k, S_l \subseteq B^c$ ,  $V_{S_k} > V_{S_l}$  if and only if  $\Delta\tilde{c}_{S_k} > \Delta\tilde{c}_{S_l}$ .*

Given that  $\Delta\tilde{d}_i = \Delta\tilde{s} - \Delta\tilde{c}_i$ , Lemmas 4 and 5 suggest the sequence of deductible changes in states unaffected by aggregation,  $\{\Delta\tilde{d}_i : s_i \in B^c \setminus s_1\}$ , is non-decreasing in  $i$ . Moreover, the deductible sequence crosses zero from negative to positive values exactly once between events  $S_k$  and  $S_m$ :  $r_{S_k} < r_1 < r_{S_m}$ .

**Proposition 2.** *Under Assumptions 1 and 2, if  $r_B \geq r_1 \geq 1$ , then  $\Delta\tilde{L} > 0$ .*

The result  $\Delta\tilde{L} > 0$  depends closely on the price condition  $1 \leq r_1 \leq r_B$ . To see this claim, observe that inside event  $B$ , the deductible changes  $\{\Delta\tilde{d}_i : s_i \in B\}$  are always positive. Hence, if the prices of the states in  $B$  are not too cheap compared to the price of  $s_1$ —that is,  $r_B \geq r_1$ —the full sequence of deductible changes  $\{\Delta\tilde{d}_i\}_{i=2}^n$  would turn positive at a state before event  $B$ . In addition, due to Lemma 1,  $\Delta\tilde{L}$ —the change in loss from event smoothing—is a weighted average of all deductible changes with increasing weights, which leads to the conclusion. In other words, whenever  $B$  is expensive enough, the event-smoothing effect on losses is dominated by the redistribution of the insurance from expensive event  $B$  to cheaper events, which results in greater losses for the insurer.

The price condition in Proposition 2 accounts for the difference between Example 1 and Example 2. In Example 1,  $r_B = 1.4 > r_1 = 1$  and the deductible changes are:  $\Delta\tilde{d}_2 = \Delta\tilde{d}_3 = 0.009$  and  $\Delta\tilde{d}_4 = -0.0002$ . In this example, the events prices are ranked as follows:  $r_4 < r_1 < r_B$ . We have already established that  $\Delta\tilde{d}_2 = \Delta\tilde{d}_3$  because  $u$  is CARA and the changes are positive because both states belong to  $B$ . The fact that  $r_4 < r_1$  implies negative value for  $\Delta\tilde{d}_4$ . And, finally, because  $r_1 < r_B$ , the sequence of deductibles turns non-negative at state  $s_1$ , which is right before  $B$ .

in terms of pricing. This fact guarantees the dominance of event  $B$  in the event-smoothing effect.

By contrast, in Example 2,  $r_B = 0.8 < r_1 = 1 < r_4 = 1.33$  and the deductible changes are  $\Delta\tilde{d}_2 = \Delta\tilde{d}_3 = 0.0651$  and  $\Delta\tilde{d}_4 = 0.0004$ . Even though all deductibles are positive, the price condition is violated as  $r_B < 1$ , which implies the deductible change will be multiplied by a negative number when calculating the change in loss. Hence, the event-smoothing effect might take any direction in this case.

Propositions 1 and 2 directly imply Theorem 1: When  $u$  is CARA and  $\phi$  is either CARA or DARA,  $\Delta L = \Delta L^* + \Delta\tilde{L} > 0$  as long as  $r_B \geq r_1 \geq 1$ . The aggregation effect redistributes the insurance from the expensive sections to the cheaper ones inside  $B$ , and the event-smoothing effect redistributes the insurance from section  $B$  to the other sections, implying that whenever  $B$  is expensive enough, both effects have the same direction and increase the total loss. An immediate implication is that disaggregating expensive events would lead to a decrease in losses. Hence, the insurer should keep the expensive sections separate from one another.

## 4 Discussion

### 4.1 The optimal PDS

So far, we have focused on the comparative statics analysis from a relatively “small” change in the structure of the current PDS. Although a natural question, the optimal PDS depends on more details, and a complete characterization of it is beyond the scope of this paper. Nevertheless, our comparative statics results can shed light on some of its necessary features.

We still focus on the case with CARA  $u(\cdot)$  and CARA or DARA  $\phi(\cdot)$ . For

any PDS  $\pi$ , if some non-singleton event  $B$  is more expensive than state  $s_1$ , that is,  $r_B \geq r_1$ , then by Proposition 1 and Proposition 2, disaggregating event  $B$  will increase the profit. The insurer can repeat this process until no room remains to improve profit. This procedure leads to a monotone PDS that better approximates the optimal monotone PDS than the original PDS  $\pi$ .

**Corollary 1.** *Suppose  $u(\cdot)$  satisfies CARA and  $\phi(\cdot)$  satisfies CARA or DARA. In addition,  $r_1 \geq 1$ . Let  $\pi^*$  be an optimal PDS. Then, the optimal PDS does not contain a non-singleton event  $B$  such that  $r_B \geq r_1$ .*

*Proof.* Suppose the optimal PDS  $\pi^*$  has such an event  $B$  and  $r_B \geq r_1 \geq 1$ . In this case,  $\pi^*$  cannot be optimal, because by Proposition 1 and Proposition 2, disaggregating event  $B$  would increase the profit.  $\square$

**Example 5.** *Consider Example 1 again. Now we are interested in which PDS potentially could be optimal. The first step is to calculate  $r_B$  for all non-singleton events  $B$ :*

$$r_{s_2 \& s_3} = \frac{7}{5}; \quad r_{s_2 \& s_4} = \frac{6}{7}; \quad r_{s_3 \& s_4} = \frac{3}{4}; \quad r_{s_2 \& s_3 \& s_4} = 1.$$

*In this problem,  $r_1 = 1$ ; hence, an optimal PDS cannot include events  $\{s_2, s_3\}$  and  $\{s_2, s_3, s_4\}$ . In addition, our results allow us to compare some of the PDSs. For example, the frame  $\{s_1, \{s_2, s_3, s_4\}\}$  will be worse than any other frame because no matter which events we are aggregating into  $\{s_2, s_3, s_4\}$ ,  $r_{s_2 \& s_3 \& s_4} \geq r_1$ . Thus, the insurer should consider only  $\{s_1, s_2, \{s_3, s_4\}\}$ ,  $\{s_1, s_3, \{s_2, s_4\}\}$ , and  $\{s_1, s_2, s_3, s_4\}$  as reasonable PDSs.*

*Similarly, for the prices in Example 2, we have*

$$r_{s_2 \& s_3} = \frac{4}{5}; \quad r_{s_2 \& s_4} = \frac{6}{7}; \quad r_{s_3 \& s_4} = \frac{3}{2}; \quad r_{s_2 \& s_3 \& s_4} = 1.$$

In this case, the optimal PDS should not contain events  $\{s_3, s_4\}$  and  $\{s_2, s_3, s_4\}$ .

## 4.2 Aggregation of monotone PDS

In this subsection, we relax the assumption that  $u$  is CARA, but focus on the special class of monotone PDSs, which are PDSs that only contain events lumped by states with adjacently ranked prices. Again, our goal is to find conditions on preferences ( $u$  and  $\phi$  functions) and observable variables (e.g., consumption, prices) under which a one-step aggregation of the PDS will lead to a clear prediction of the direction of the profit change.

**Assumption 3** (Monotonicity). *Suppose the states are ranked according to  $r_i$ ; that is,  $r_2 < r_3 < \dots < r_n$ . Then, for any event  $A$  either in  $\pi$  or  $\rho$ , there exists  $i, k \in \mathbb{N}$  such that  $i > 1, k > 0$ , and  $A = \cup_{j=i}^{i+k} s_j$ .*

Under monotonicity,  $S_1 = \{s_1\}$ , and all other events are obtained by monotonic aggregation of the states. Without loss of generality, we can always relabel states in any PDS to follow the order of the prices, ranging from the cheapest to the most expensive. Monotonicity requires that only states with adjacently ranked prices are aggregated. This assumption is reasonable in situations when similarly priced states are lumped together. For instance, consider Manulife’s dental insurance described in the introduction. A typical contract groups all covered services into six categories: “preventive,” “restorative,” “endodontic and periodontic,” “major,” “implant and related services,” and “orthodontia,” with services becoming more expensive as one moves from the first to the sixth category.<sup>15</sup> In the same fashion, a standardized government health insurance contract in Australia consists of two parts: “hospital”

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<sup>15</sup>In this anecdote, the expenditure for a service is a proxy for the “expensiveness” of the state when it is needed, which should be proportional to its (unobserved) price of the Arrow security.

and “extras.” The first part includes the more expensive states that require hospitalization and the second part covers the cheaper and less serious states such as “physio” or “dental.”

Similar to section 3, we decompose the total effect of one-step aggregation on the consumer choice into the aggregation and event-smoothing effects, using the intermediate bundle  $(s^*, d_2^* \dots, d_n^*)$ . Again, we are interested in the signs of the changes in deductibles  $\Delta d_i = \tilde{d}_i - d_i$  and the change in expected loss  $\Delta L = \tilde{L} - L$ .

To begin with, we need to rank events and consumption monotonically (by their indices). As Lemma 6 below shows, Assumption 3 allows us to order the events monotonically when  $u$  and  $\phi$  are concave; however, this assumption is not enough to guarantee that the consumption sequence is also monotonically ordered. Hence, we impose the following assumption, which excludes  $\phi \circ u$  to be a lot more concave than  $u$ .

**Assumption 4.** *For all  $S_k \in \pi$  and all  $x \in \mathbb{R}$ , we have*

$$(A_{\phi \circ u}(x) - A_u(x)) \left( \max_{s_i \in S_k} c_i - \min_{s_j \in S_{k+1}} c_j \right) \leq 1 - \frac{\max_{s_i \in S_k} r_i}{\min_{s_j \in S_{k+1}} r_j}.^{16}$$

The next lemma shows that when the normalized state price  $r_i$  is monotonically increasing in  $i$ , under concavity of  $u$  and  $\phi$ , the event value  $V_{S_k}$  is decreasing in the event index  $k$ . If, in addition,  $\phi \circ u$  is not too concave compared to  $u$ , the state consumption  $c_i$  is also decreasing in  $i$ .

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<sup>16</sup>Let  $c_{S_k} = u^{-1}(V_{S_k})$  be the *certainty equivalent* to consumption on event  $S_k$ . When information about certainty equivalent to an event is also available, Assumption 4 can be replaced by the weaker requirement,

$$(A_{\phi \circ u}(c_{S_{k+1}}) - A_u(c_{S_{k+1}}))(c_{S_k} - c_{S_{k+1}}) \leq 1 - \frac{\max_{s_i \in S_k} r_i}{\min_{s_j \in S_{k+1}} r_j}.$$



**Lemma 6** (Monotonicity). *If Assumption 3 holds, then for any  $k < l$ ,  $V_{S_k}(s) > V_{S_l}(s)$ . Moreover, if Assumption 4 also holds, then for all  $i < j$ ,  $c_i > c_j$ .*

The first part of Lemma 6 follows from the standard arguments given that  $\phi \circ u$  is a concave utility function. However, without Assumption 4, consumption belonging to different events may not be monotonically decreasing. To see this claim, let  $s_i$  be the last state in  $S_k$  and let  $s_j$  be the first state in  $S_{k+1}$ ; the FOC yields

$$\frac{\phi'(V_{S_k}(s))}{\phi'(V_{S_{k+1}}(s))} \cdot \frac{u'(c_i)}{u'(c_j)} = \frac{r_i}{r_j}.$$

While  $\frac{r_i}{r_j} < 1$ ,  $s_i$  and  $s_j$  are adjacent states, and hence the ratio can be close to 1. The marginal rate of substitution (in  $\phi$ ) between events  $S_k$  and  $S_{k+1}$  is also less than 1, due to decreasing event utility and concavity of  $\phi$ . If  $\phi$  is sufficiently concave or the overall consumption decline in the adjacent events is sufficiently large, the marginal rate of substitution (in  $\phi$ ) between events  $S_k$  and  $S_{k+1}$  can be so small that the marginal rate of substitution (in  $u$ ) between  $c_i$  and  $c_j$  must be greater than 1. In this case, consumption at the border of two adjacent events can jump upward, whereas consumption within an event is still decreasing. Assumption 4 rules out this possibility.

#### 4.2.1 Aggregation effect

In this subsection, we consider the aggregation effect— $\Delta c^*$  and  $\Delta L^*$ —under monotone pricing.

**Assumption 5.**  $u(\cdot)$  is *CARA* or *DARA*.

The assumption relaxes Assumption 1. Under Assumption 3, the price-probability ratio  $r_i = \frac{p_i}{P(S_i)}$  is monotonically increasing. Assumption 5 implies the risk-adjustment

factor  $\frac{1}{A_u(c_i)}$  is non-increasing in  $i$ , because  $c_i$  is decreasing in  $i$ . Hence,  $\sum_{s_i \in S_k} \frac{P(s_i)}{A_u(c_i)}$  first-order stochastically dominates  $\sum_{s_i \in S_k} \frac{p_i}{A_u(c_i)}$  in event index  $k$ .

Together with Assumptions 3 and 4, Assumption 5 also allows us to rank changes in consumption according to the prices of states as well. The following proposition suggests monotone pricing could ensure lumping events generates a negative aggregation effect on the insurer's profit.

**Proposition 3.** *Suppose Assumptions 3-5 hold and  $\phi$  is concave. Then,  $\Delta d_i^*$  is non-decreasing in  $i$  for all  $s_i \in B$ , and*

$$\Delta L^* = \sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)} \left( 1 - \frac{\sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)} \sum_{S_k \in B} \left( \sum_{s_i \in S_k} \frac{P(s_i)}{A_u(c_i)} \right) \phi'(V_{S_k}(s))}{\sum_{s_i \in B} \frac{p_i}{A_u(c_i)} \sum_{S_k \in B} \left( \sum_{s_i \in S_k} \frac{p_i}{A_u(c_i)} \right) \phi'(V_{S_k}(s))} \right) > 0.$$

Similar to the CARA- $u$  case, the aggregation effect pushes cheaper consumption up and more expensive consumption further down. The only difference from the CARA  $u$  case is that consumption on each event is monotonically decreasing and no longer constant. The same fact applies to the changes in consumption as well. Intuitively speaking, given that  $s$  and  $I_B$  remain unchanged, the consumer redistributes insurance from more expensive states to cheaper ones, keeping insurance premium the same and hence hurting the insurer. Formally, this result comes from the following observation: When  $u$  is CARA or DARA,  $\sum_{s_i \in S_k} \frac{P(s_i)}{A_u(c_i)}$  (the risk-adjusted conditional probability) puts heavier weights on the events with the lower consumption and higher prices than  $\sum_{s_i \in S_k} \frac{p_i}{A_u(c_i)}$  (the risk-adjusted relative price).

#### 4.2.2 Event-smoothing effect

Next, we turn to the event-smoothing effect that takes care of redistributing income across events in the final PDS  $\rho$ .

Observe that the event-smoothing effect is closely related to whether  $\frac{\lambda_\pi}{\lambda_\rho}$  is smaller or greater than 1. Intuitively, when  $\lambda_\pi > \lambda_\rho$ , the marginal utility of wealth in the (DP- $\rho$ ) problem is smaller than that from the (DP- $\pi$ ) problem for events outside  $B$ . Hence, the consumer is richer in those events under  $\rho$  than under  $\pi$ . Consequently, the event-smoothing effect should result in an increase in income allocated to states outside  $B$  and a decrease in income allocated to the states inside event  $B$ ; that is,  $\Delta I_B < 0$  and  $\Delta I_{B^c} > 0$ .<sup>17</sup>

**Assumption 6.**  $(1/A_u(x))' \leq 1$  for all  $x \in \mathbb{R}_+$ .

This assumption states that although the absolute risk tolerance (the reciprocal of  $A_u(x)$ ) is non-decreasing in wealth (by Assumption 5), it cannot increase faster than the speed of the wealth increase. Technically, Assumption 6 is equivalent to either of the following two conditions: (i)  $2A_u(x) \geq P_u(x)$  or (ii)  $g(x) = \frac{1}{u'(x)}$  is convex. In our model, Assumption 6 is necessary for clear predictions of the event-smoothing effect. For states inside the aggregated event  $B$ , Assumption 6 ensures the risk-adjusted price for each state  $\frac{r_i}{A_u(c_i)}$  is non-decreasing in  $i$  and consequently shifts the distribution of event values in  $B$  to the right after aggregation. This stochastic shift implies that the marginal value of event  $B$  decreases, and hence the consumer still wants to redistribute income away from event  $B$  after aggregation, even though  $u$  is no longer CARA. Consequently, inside the aggregated event, consumption changes in the event-smoothing step are always negative (Lemma 15 in the Appendix). For states outside  $B$ , which are unaffected by the one-step aggregation, consumptions increase in the event-smoothing stage due to the positive income transfer. In this case, Lemmas 17 and 18 in the Appendix show that Assumption 6 is a technical condition needed to ensure the sequence of consumption changes on  $B^c$  is non-increasing.<sup>18</sup>

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<sup>17</sup>For more details, see Lemma 11 in the Appendix.

<sup>18</sup>Assumption 6 has also been used in the classic risk analysis as a necessary and sufficient condi-

The next proposition says Assumptions 2-6 together with a price condition, which requires states in the aggregated event  $B$  must be expensive enough, are sufficient for the event-smoothing effect on loss to be positive. We use  $S_{l+1}$  to denote the event in  $\rho$  right after  $B$  in its statement.

**Proposition 4.** *Suppose Assumptions 2-6 hold. If  $1 \leq r_1 < \min_{s_j \in S_{l+1}} r_j$ , then  $\{\Delta \tilde{d}_i\}_{s_i \in B^c \setminus s_1}$  is a non-decreasing sequence that crosses 0 and  $\Delta \tilde{L} > 0$ .*

First, similar to the CARA- $u$  case, aggregation of the events pushes marginal values down in the (DP- $\rho$ ), implying  $\lambda_\rho < \lambda_\pi$  and redistribution of consumption from  $B$  to the events outside  $B$ . Moreover, by the discussion above for the states outside  $B$ , the sequence of consumption changes  $\Delta \tilde{c}_i$  is non-increasing in  $i$ , implying  $\Delta \tilde{d}_i$  is non-decreasing in  $i$  outside  $B$ . Thus, the consumer not only distributes consumption from  $B$  to the events outside  $B$ , but also distributes more in the cheaper states; that is, she chooses even lower deductibles in the cheaper states. The redistribution obviously means consumption in the default state goes up as well. In other words, the consumer redistributes insurance not only from  $B$  to outside  $B$ , but also from the more expensive events outside  $B$  to the cheaper ones, implying a reduction in the overall premium paid for the insurance.

Second, when event  $B$  is so cheap that the price condition  $1 \leq r_1 < \min_{s_j \in S_{l+1}} r_j$  fails, the event-smoothing effect on loss cannot be predicted because there are two opposite effects at play: (1) redistribution of insurance from  $B$  to other events; and

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tion for an EU agent to consume more in the face of a new favorable risky investment opportunity tomorrow (Gollier and Kimball, 2018). Consider an EU agent with vNM index  $u$ , initial wealth  $x_0$ , and who faces a favorable risky investment opportunity with return  $\tilde{x}$  in the second period. Then, an agent with  $u$  satisfying Assumption 6 displays the following behavior: For all distributions of  $\tilde{x}$  and all  $x_0$ ,

$$\mathbb{E}[\tilde{x}u'(x_0 + \tilde{x})] = 0 \Rightarrow \mathbb{E}[u'(x_0 + \tilde{x})] \leq \mathbb{E}[u'(x_0)].$$

(2) redistribution of insurance from the more expensive events to the cheaper ones. Alternatively, when event  $B$  is expensive enough for the price condition to hold, the consumer redistributes insurance only from the more expensive events to the cheaper ones, implying the increase in loss.

To summarize, we have the analogue of Theorem 1 in the monotone PDS case: When the aggregated event is expensive enough, the total effect of a one-step aggregation on the insurer loss  $\Delta L = \Delta L^* + \Delta \tilde{L}$  is positive under Assumptions 2-6.

### 4.3 Quantitative analysis

In this section, we provide general characterizations of the aggregation and event-smoothing effects that rely solely on the concavity and differentiability of  $u$  and  $\phi$ .

For that purpose, first, we define the risk-adjusted prices and probabilities that are building blocks of our analysis.

**Definition 1.** For any event  $S_j \in B$  and  $S_k \in \rho$ , the risk-adjusted relative price and conditional probability at  $S_j$  are

$$\alpha(S_j) = \frac{\sum_{s_i \in S_j} \frac{p_i}{A_u(c_i)}}{\sum_{s_i \in B} \frac{p_i}{A_u(c_i)}} \text{ and } \beta(S_j) = \frac{\sum_{s_i \in S_j} \frac{P(s_i)}{A_u(c_i)}}{\sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)}},$$

whereas the post-aggregation risk-adjusted relative price and conditional probability at  $S_k$  are

$$\alpha^*(S_k) = \frac{\sum_{s_i \in S_k} \frac{p_i}{A_u(c_i^*)}}{\sum_{s_i \in \Omega} \frac{p_i}{A_u(c_i^*)}} \text{ and } \beta^*(S_k) = \frac{\sum_{s_i \in S_k} \frac{P(s_i)}{A_u(c_i^*)}}{\sum_{s_i \in \Omega} \frac{P(s_i)}{A_u(c_i^*)}}.$$

**Example 6.** If  $u(\cdot)$  is CARA, then  $\alpha(S_i) = \frac{\sum_{s_j \in S_i} p_j}{\sum_{s_j \in B} p_j} = \frac{p_{S_i}}{p_B}$  is the price of \$1 consumption in event  $S_i$  relative to that in event  $B$  and  $\beta(S_i) = \frac{\sum_{s_j \in S_i} P(s_j)}{\sum_{s_j \in B} P(s_j)} = \frac{P(S_i)}{P(B)}$  is the probability of event  $S_i$  conditional on event  $B$ . Alternatively, if  $u(\cdot)$  is constant

relative risk averse (CRRA); that is,  $u(x) = \ln x$  or  $u(x) = x^\rho$ , then  $\alpha(S_i) = \frac{I_{S_i}}{I_B}$  is the share of event  $S_i$ 's income in  $I_B$  and  $\beta(S_i) = \frac{\sum_{s_j \in S_i} P(s_j)c_j}{\sum_{s_j \in B} P(s_j)c_j}$  is the ratio of expected consumption on  $S_i$  to expected consumption on  $B$ .

### 4.3.1 General characterization of the aggregation effect

We start with the aggregation effects on  $\Delta c^*$  and  $\Delta L^*$ —change from the original consumption  $c$  to the intermediate bundle  $c^*$  and the corresponding change in the firm's loss.

Define  $\Gamma_{S_i} = 1 - \frac{u'(c_j^*)}{u'(c_j)}$ , which is the change in marginal (state) utility in state  $s_j \in S_i \in B$ . Note the first-order conditions require that such change is constant for all the states in an event. The following lemmas provide the exact formula for the change in marginal (state) utility together with the aggregation effects for consumption and loss in terms of consumer's utility  $u$ , aggregation function  $\phi$ , consumptions at the original frame  $\pi$ , the risk-adjusted relative prices  $\alpha(S_k)$ , and the risk-adjusted conditional probabilities  $\beta(S_k)$ .

**Lemma 7.** *If  $u$  and  $\phi$  are concave and three-times differentiable, then for any state  $s_j \in S_i$  and event  $S_i \in B$ , the aggregation effect on consumption is*

$$\begin{aligned}\Delta c_j^* &= \frac{\Gamma_{S_i}}{A_u(c_j)}, \\ \Gamma_{S_i} &= 1 - \frac{\phi'(V_{S_i}(s))}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))}.\end{aligned}$$

**Lemma 8.** *If  $u$  and  $\phi$  are concave and three-times differentiable, then*

$$\Delta L^* = \sum_{s_i \in B} P(s_i) \Delta c_i^* = \left( \sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)} \right) \left( 1 - \frac{\sum_{S_k \in B} \beta(S_k) \phi'(V_{S_k}(s))}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))} \right).$$

The direction of the effect is defined by the relationship between the risk-adjusted relative prices  $\alpha(S_k)$  and the risk-adjusted conditional probabilities  $\beta(S_k)$ : If the events with the lower values have “greater” risk-adjusted relative prices than their risk-adjusted conditional probabilities, the aggregation effect on losses will be positive. The losses would increase because the consumer would redistribute insurance from the relatively more expensive events with lower values toward relatively cheaper events with higher values.

### 4.3.2 General characterization of the event-smoothing effect

Now we move on to the event-smoothing effects on  $\Delta\tilde{c}$  and  $\Delta\tilde{L}$ —change from the intermediate bundle  $c^*$  to the consumption bundle  $\tilde{c}$  with the PDS  $\rho$  and the corresponding change in loss.

Denote by  $E_{S_k} = \sum_{s_i \in S_k} P(s_i|S_k) \frac{u'(c_i^*)}{A_u(c_i^*)}$  the risk-adjusted average marginal (state) utility on event  $S_k \in \rho$ . By analogy to the aggregation effect, define  $\tilde{\Gamma}_{S_i} = 1 - \frac{u'(\tilde{c}_j)}{u'(c_j^*)}$  as the change in marginal (state) utility in state  $s_j \in S_i \in \rho$ . As before, the first-order conditions require that such change is constant for all the states in an event. Lemma 16 in the Appendix provides a system of equations on  $\tilde{\Gamma}_{S_i}$  that allows us to pin down the event-smoothing effect using information (input) on consumer’s utility  $u$ , aggregation function  $\phi$ , prices, probabilities, consumption choices at (DP- $\pi$ ) and (DP-B), as well as the ratio of the Lagrange multipliers  $\frac{\lambda_B}{\lambda_\pi}$ . Note that for state  $s_i \in B$ ,  $c_i^*$  can be pinned down by  $u$ ,  $\phi$ , and  $c_i$  from the aggregation effect (Lemma 7), and thus the ratio  $\frac{\lambda_B}{\lambda_\pi}$  can be computed. For states  $s_i \notin B$ , we have  $c_i^* = c_i$ . Hence, the last two input variables can be obtained from the aggregation effect, and the other input variables are observable. Therefore, the proposed system of equations can be solved computationally in every specific case; unfortunately, obtaining the

explicit expression for the solution is not feasible.

Finally, the event-smoothing effect on the insurer losses is as follows.

**Lemma 9.** *Suppose  $u$  and  $\phi$  are three-times differentiable and concave; then,*

$$\Delta \tilde{L} = \sum_{s_i \in \Omega} P(s_i) \Delta \tilde{c}_i = \left( \sum_{s_i \in \Omega} \frac{P(s_i)}{A_u(c_i^*)} \right) \sum_{S_k \in \rho} \beta^*(S_k) \tilde{\Gamma}_{S_k}.$$

Analogous to the aggregation effect, the event-smoothing effect on loss is equal to the expected change in consumption. Lemma 16 requires  $\sum_{S_k \in \rho} \alpha^*(S_k) \tilde{\Gamma}_{S_k} = 0$ , which simply says that the total income does not change. On the other hand, the direction of the event-smoothing effect on losses depends on the sign of  $\sum_{S_k \in \rho} \beta^*(S_k) \tilde{\Gamma}_{S_k}$ . Thus, the direction of the effect again depends on the relationship between the post-aggregation risk-adjusted relative prices  $\alpha^*(S_k)$  and the post-aggregation risk-adjusted conditional probabilities  $\beta^*(S_k)$ ; however, this relationship is less straightforward than that in the aggregation effect. Even so, we are still able to see the direction of the effect is determined by whether event  $B$  is relatively more expensive than the rest of the events. For example, in the case of  $\lambda_B < \lambda_\pi$ , if event  $B$  is relatively more expensive than the other events, the event-smoothing effect on losses will be dominated by the changes in the unaffected states, which are relatively cheaper and hence would increase the loss of the insurer. On the other hand, if  $B$  is relatively cheap, then  $B$  will dominate the direction of  $\Delta \tilde{L}$ , and the consumer would redistribute the insurance from “cheap”  $B$  to “more expensive” other events, which would reduce the insurer’s loss.

Finally, the total effect on the insurer’s loss is simply the sum of the two effects; that is,  $\Delta L = \Delta L^* + \Delta \tilde{L}$ . The exact quantity of change would depend on different relative prices and conditional probabilities. Hence, even though we cannot predict



clearly the direction of profit change for preferences outside the families considered in section 3 or 4.2, the general results in this subsection can educate a firm on how to make quantitative predictions of consumption and profit changes following a one-step aggregation using numerical methods. The method applies to any one-step aggregation (not necessarily restricted to expensive events) and any concave and smooth utility and aggregation functions, as long as the firm can observe current prices, income, and consumption, and is willing to make functional-form assumptions about the consumer's preferences.<sup>19</sup>

## A Appendix

### A.1 Notation

To simplify working with the proofs, we list all the notation used below:

$$\alpha(S_j) = \frac{\sum_{s_i \in S_j} \frac{p_i}{A_u(c_i)}}{\sum_{s_i \in B} \frac{p_i}{A_u(c_i)}} \text{ and } \beta(S_j) = \frac{\sum_{s_i \in S_j} \frac{P(s_i)}{A_u(c_i)}}{\sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)}} \text{ for any } S_j \in B$$

$$\alpha^*(S_k) = \frac{\sum_{s_i \in S_k} \frac{p_i}{A_u(c_i^*)}}{\sum_{s_i \in \Omega} \frac{p_i}{A_u(c_i^*)}} \text{ and } \beta^*(S_k) = \frac{\sum_{s_i \in S_k} \frac{P(s_i)}{A_u(c_i^*)}}{\sum_{s_i \in \Omega} \frac{P(s_i)}{A_u(c_i^*)}} \text{ for any } S_k \in \rho$$

$$\Gamma_{S_i} = 1 - \frac{u'(c_j^*)}{u'(c_j)} \text{ for any } S_i \in B$$

$$\tilde{\Gamma}_{S_i} = 1 - \frac{u'(\tilde{c}_j)}{u'(c_j^*)} \text{ for any } S_i \in \rho$$

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<sup>19</sup>In applied work (e.g., empirical industrial organization), making functional form assumptions about consumers' utility with some unknown taste shocks is common. The firm can estimate the key parameters of the utility and aggregation functions with consumption data, and then use the estimated utility and aggregation functions as inputs to our quantitative analysis.

$$E_{S_k} = \sum_{s_i \in S_k} P(s_i|S_k) \frac{u'(c_i^*)}{A_u(c_i^*)} \text{ for any } S_k \in \rho.$$

## A.2 Special case

### A.2.1 Aggregation effect

**Proof of Lemma 1.** The FOC suggests

$$P(s_i)\phi'(V_{S_k}(s))e^{-\gamma c_i} = \lambda_\pi p_i,$$

which implies

$$P(s_i)\phi'(V_{S_k}(s))\frac{1}{\gamma}(1 - e^{-\gamma c_i}) = \frac{1}{\gamma}(P(s_i)\phi'(V_{S_k}(s)) - \lambda_\pi p_i)$$

$$P(S_k)\phi'(V_{S_k}(s))P(s_i|S_k)u(c_i) = \frac{1}{\gamma}(P(s_i)\phi'(V_{S_k}(s)) - \lambda_\pi p_i).$$

By adding up the above expression for all states  $s_i \in S_k$ , we obtain

$$P(S_k)\phi'(V_{S_k}(s)) \sum_{s_i \in S_k} P(s_i|S_k)u(c_i) = \frac{1}{\gamma}(P(S_k)\phi'(V_{S_k}(s)) - \lambda_\pi p_{S_k})$$

$$P(S_k)\phi'(V_{S_k}(s))V_{S_k}(s) = \frac{1}{\gamma}(P(S_k)(\phi'(V_{S_k}(s)) - \lambda_\pi p_{S_k}))$$

$$\phi'(V_{S_k}(s))(1 - \gamma V_{S_k}(s)) = \lambda_\pi \frac{p_{S_k}}{P(S_k)}.$$

Now note the left-hand side is decreasing in  $V_{S_k}(s)$  (as  $V_{S_k}(s) \in (0, \frac{1}{\gamma})$  and  $\phi(\cdot)$  is strictly concave); hence, the greater the ratio  $\frac{p_{S_k}}{P(S_k)}$ , the smaller the value  $V_{S_k}(s)$ .  $\square$

**Proof of Lemma 7.** The ratio of the FOCs between the  $\pi$ -aggregation problem

and the intermediate bundle suggests that for any  $s_j \in S_i$  and  $S_i \in B$ ,

$$\frac{u'(c_j^*)}{u'(c_j)} = \frac{\lambda_B \phi'(V_{S_i}(s))}{\lambda_\pi \phi'(V_B(s^*))}.$$

Then, by the Taylor expansion, we get

$$\begin{aligned} 1 - A_u(c_j)\Delta c_j^* &= \frac{\lambda_B \phi'(V_{S_i}(s))}{\lambda_\pi \phi'(V_B(s^*))} \\ \Rightarrow A_u(c_j)\Delta c_j^* &= 1 - \frac{\lambda_B \phi'(V_{S_i}(s))}{\lambda_\pi \phi'(V_B(s^*))} = \Gamma_{S_i}; \end{aligned}$$

hence,  $\Delta c_j^* = \frac{\Gamma_{S_i}}{A_u(c_j)}$ . Now denote  $\gamma = \frac{\lambda_B}{\lambda_\pi \phi'(V_B(s^*))}$  and also notice  $\gamma$  does not depend on specific states or events inside  $B$ . In addition,  $\Gamma_{S_i} = 1 - \gamma \phi'(V_{S_i}(s))$ .

Denote  $\omega(S_k) = \sum_{s_i \in S_k} \frac{p_i}{A_u(c_i)}$ . Finally, note that income in event  $B$  is fixed, so  $\sum_{s_j \in B} \Delta c_j^* p_j = 0$ , which implies

$$\begin{aligned} \sum_{s_j \in B} \Delta c_j^* p_j &= \sum_{s_j \in B} \frac{p_j}{A_u(c_j)} \Gamma_{S_i} = \sum_{S_k \in B} \Gamma_{S_k} \sum_{s_j \in S_k} \frac{p_j}{A_u(c_j)} = \sum_{S_k \in B} \Gamma_{S_k} \omega(S_k) \\ &= \sum_{S_k \in B} (1 - \gamma \phi'(V_{S_k}(s))) \omega(S_k) = \omega(B) - \gamma \sum_{S_k \in B} \omega(S_k) \phi'(V_{S_k}(s)) = 0, \end{aligned}$$

implying  $\gamma = \frac{1}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))}$ , and all the results follow.  $\square$

**Proof of Lemma 2.** Note Lemma 2 is a corollary of Lemma 7, where we take into account that  $A_u(c_i) = \gamma$  and  $\alpha(S_k) = \frac{p_{S_k}}{p_B}$  for the case of CARA  $u$ .  $\square$

**Proof of Lemma 8.** Because  $\sum_{s_i \in B} p_i \Delta c_i^* = 0$  and by taking into account Lemma 7,

we have

$$\begin{aligned}
\Delta L^* &= \sum_{s_i \in B} (p_i - P(s_i)) (-\Delta c_i^*) = - \sum_{s_i \in B} p_i \Delta c_i^* + \sum_{s_i \in B} P(s_i) \Delta c_i^* \\
&= \sum_{s_i \in B} P(s_i) \Delta c_i^* = \sum_{S_j \in B} \Gamma_{S_j} \sum_{s_i \in S_j} \frac{P(s_i)}{A_u(c_i)} = \sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)} \sum_{S_j \in B} \Gamma_{S_j} \beta(S_j) \\
&= \sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)} \sum_{S_j \in B} \beta(S_j) \left( 1 - \frac{\phi'(V_{S_j}(s))}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))} \right) \\
&= \sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)} \left( 1 - \frac{\sum_{S_k \in B} \beta(S_k) \phi'(V_{S_k}(s))}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))} \right).
\end{aligned}$$

□

**Lemma 10.** Under Assumption 1,  $\frac{\alpha(S_k)}{\beta(S_k)} < \dots < \frac{\alpha(S_t)}{\beta(S_t)}$ .

*Proof.* Note

$$\begin{aligned}
\frac{p_{S_2}}{P(S_2)} &< \dots < \frac{p_{S_N}}{P(S_N)} \\
\frac{p_{S_2}}{P(S_2)} \frac{P(B)}{p_B} &< \dots < \frac{p_{S_N}}{P(S_N)} \frac{P(B)}{p_B}
\end{aligned}$$

implies

$$\frac{\alpha(S_k)}{\beta(S_k)} < \dots < \frac{\alpha(S_t)}{\beta(S_t)}.$$

□

**Proof of Proposition 1.** By Lemma 8, the aggregation effect on losses is

$$\Delta L^* = \frac{P(B)}{\gamma} \left( 1 - \frac{\sum_{S_k \in B} \beta(S_k) \phi'(V_{S_k}(s))}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))} \right).$$

Now note that by Lemma 10, we have  $\frac{\alpha(S_k)}{\beta(S_k)} < \dots < \frac{\alpha(S_t)}{\beta(S_t)}$ . Thus,  $\{\alpha(S_k) : S_k \in B\}$  first-order stochastically dominates  $\{\beta(S_k) : S_k \in B\}$  in index  $k$ . In addition,  $V_{S_k}(s)$

is decreasing in  $k$  and  $\phi$  is a strictly concave function; hence,

$$\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s)) > \sum_{S_k \in B} \beta(S_k) \phi'(V_{S_k}(s)),$$

and the result follows.  $\square$

## A.2.2 Event-smoothing effect

### Lemma 11

The next lemma shows the direction of income redistribution between the aggregated event  $B$  and its complement  $B^c$  is pinned down by the comparison of the Lagrange multipliers of the two problems  $\lambda_\pi$  and  $\lambda_\rho$ .

**Lemma 11.** *If  $u$  and  $\phi$  are concave and three-times differentiable, one of the following cases holds:*

- $\lambda_\pi > \lambda_\rho \Leftrightarrow \lambda_\rho > \lambda_B \Leftrightarrow \lambda_\pi > \lambda_B \Leftrightarrow \Delta I_B < 0$  and  $\Delta I_{B^c} > 0$ , then  $\Delta \tilde{c}_i < 0$  for all  $s_i \in B$  and  $\Delta \tilde{c}_j > 0$  for all  $s_j \in B^c$ ;
- $\lambda_\pi = \lambda_\rho \Leftrightarrow \lambda_\rho = \lambda_B \Leftrightarrow \lambda_\pi = \lambda_B \Leftrightarrow \Delta I_B = 0$  and  $\Delta I_{B^c} = 0$ , then  $\Delta \tilde{c}_i = 0$  for all  $s_i \in B$  and  $\Delta \tilde{c}_j = 0$  for all  $s_j \in B^c$ ;
- $\lambda_\pi < \lambda_\rho \Leftrightarrow \lambda_\rho < \lambda_B \Leftrightarrow \lambda_\pi < \lambda_B \Leftrightarrow \Delta I_B > 0$  and  $\Delta I_{B^c} < 0$ , then  $\Delta \tilde{c}_i > 0$  for all  $s_i \in B$  and  $\Delta \tilde{c}_j < 0$  for all  $s_j \in B^c$ .

*Proof.* For all  $s_i, s_j \in B$ , the marginal rate of substitution satisfies

$$\frac{u'(\tilde{c}_i)}{u'(\tilde{c}_j)} = \frac{r_i}{r_j} = \frac{u'(c_i^*)}{u'(c_j^*)}.$$

Hence,  $\text{sgn}(\Delta \tilde{c}_i) = \text{sgn}(\Delta \tilde{c}_j)$  for all  $s_i, s_j \in B$ . By the same argument, for arbitrary  $s_i, s_j \in S_k \subseteq B^c$ ,  $\text{sgn}(\Delta \tilde{c}_i) = \text{sgn}(\Delta \tilde{c}_j) = \text{sgn}(\Delta V_{S_k}(\tilde{s}))$ .

Then in addition for any other event  $S_l \subseteq B^c$  and any state  $s_j \in S_l$ , we have

$$\frac{\phi'(V_{S_k}(\tilde{s})) u'(\tilde{c}_i)}{\phi'(V_{S_l}(\tilde{s})) u'(\tilde{c}_j)} = \frac{r_i}{r_j} = \frac{\phi'(V_{S_k}(s)) u'(c_i)}{\phi'(V_{S_l}(s)) u'(c_j)}, \quad \forall s_i \in S_k, s_j \in S_l.$$

By the same argument,  $\text{sgn}(\Delta\tilde{c}_i) = \text{sgn}(\Delta V_{S_k}(\tilde{s}))$  for any  $s_i \in S_k$ . Hence, we get that for any  $S_k, S_l \in B^c$  and  $s_i \in S_k, s_j \in S_l$ :  $\text{sgn}(\Delta\tilde{c}_i) = \text{sgn}(\Delta\tilde{c}_j) = \text{sgn}(\Delta V_{S_k}(\tilde{s})) = \text{sgn}(\Delta V_{S_l}(\tilde{s}))$ . Hence, we obtain that event-smoothing effect has the same direction in the events outside  $B$  and  $\text{sgn}(\Delta V_{S_k}(\tilde{s})) = \text{sgn}(\Delta I_{B^c})$  for any  $S_k \in B^c$ .

Moreover, note  $\Delta I_B + \Delta I_{B^c} = 0$ . Thus, for any  $s_i \in S_k \in B^c$  and  $s_j \in B$ , we have

$$\begin{aligned} \text{sgn}(\Delta\tilde{c}_i) &= \text{sgn}(\Delta V_{S_k}(\tilde{s})) = \text{sgn}(\Delta I_{B^c}) = -\text{sgn}(\Delta\tilde{c}_j) \\ &= -\text{sgn}(\Delta V_B(\tilde{s})) = -\text{sgn}(\Delta I_B). \end{aligned}$$

Next, consider the ratio of the FOCs for  $\pi$ - and  $\rho$ -aggregation problems for any  $S_k \in B^c$ :

$$\frac{\lambda_\pi}{\lambda_\rho} = \frac{\phi'(V_{S_k}(s)) u'(c_i)}{\phi'(V_{S_k}(\tilde{s})) u'(\tilde{c}_i)}.$$

Due to  $\text{sgn}(\Delta\tilde{c}_i) = \text{sgn}(\Delta V_{S_k}(\tilde{s}))$  for all  $s_i \in S_k \in B^c$  and  $s_j \in B$ , we get that

$$\frac{\lambda_\pi}{\lambda_\rho} > 1 \Leftrightarrow \Delta\tilde{c}_i > 0, \Delta V_{S_k} > 0, \Delta I_{B^c} > 0, \Delta I_B < 0, \Delta\tilde{c}_j < 0.$$

For any  $s_i \in S_k \subseteq B$ , dividing the FOCs (3) and  $\rho$ -aggregation problems yields

$$\frac{\lambda_B}{\lambda_\rho} = \frac{\phi'(V_B(s^*)) u'(c_i^*)}{\phi'(V_B(\tilde{s})) u'(\tilde{c}_i)}.$$

Hence, when  $\Delta I_B < 0$ , we also obtain  $\frac{\lambda_B}{\lambda_\rho} < 1$ . The other two cases follow by

analogy.

□

**Proof of Lemma 3.** Note

$$V_B^*(s^*) \geq \sum_{S_k \in B} P(S_k|B) V_{S_k}(s) \quad (4)$$

because  $V_B^*(s^*)$  is the optimal value at the intermediate bundle  $c^*$ . Also note  $P(S_k|B) = \beta(S_k)$ ; hence,

$$V_B^*(s^*) \geq \sum_{S_k \in B} \beta(S_k) V_{S_k}(s) > \sum_{S_k \in B} \alpha(S_k) V_{S_k}(s).$$

The last expression follows from  $\frac{\alpha(S_k)}{\beta(S_k)} < \dots < \frac{\alpha(S_t)}{\beta(S_t)}$  and  $V_{S_k}(s) > \dots > V_{S_t}(s)$ .

Now notice  $\phi$  is concave and  $\phi'$  is convex in our case; thus, we obtain

$$\phi'(V_B^*(s^*)) < \phi' \left( \sum_{S_k \in B} \alpha(S_k) V_{S_k}(s) \right) < \sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s)),$$

where the first  $<$  uses (4) and  $\phi'$  is decreasing, and the second  $<$  follows from Jensen's inequality.

Finally,

$$\frac{\lambda_B}{\lambda_\pi} = \frac{\phi'(V_B^*(s^*))}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))} < 1.$$

The rest follows from Lemma 11.

□

**Proof of Lemma 4.** Take any  $s_i \in S_k \in \rho \setminus B$ ,  $c_i = c_i^*$  and the ratio of the FOCs for  $\pi$ - and  $\rho$ -aggregation problems is

$$\frac{\lambda_\rho}{\lambda_\pi} = \frac{\phi'(V_{S_k}(\tilde{s})) u'(\tilde{c}_i)}{\phi'(V_{S_k}(s)) u'(c_i)}.$$

By Taylor approximation,

$$\frac{\lambda_\rho}{\lambda_\pi} \approx \frac{\phi'(V_{S_k}(\tilde{s}))}{\phi'(V_{S_k}(s))} [1 - A_u(c_i)\Delta\tilde{c}_i] = \frac{\phi'(V_{S_k}(\tilde{s}))}{\phi'(V_{S_k}(s))} [1 - \gamma\Delta\tilde{c}_i].$$

Clearly,  $\Delta\tilde{c}_i$  remains constant for all the states in  $S_k$ .

The proof for  $s_i \in B$  goes by analogy, except that we take the ratio of FOCs for  $B$ - and  $\rho$ -aggregation problems, implying instead  $\frac{\lambda_\rho}{\lambda_B}$  on the left-hand side. It follows that  $\Delta\tilde{c}_i$  is constant on event  $B$ .  $\square$

**Lemma 12.**  $\Delta\tilde{V}_{S_k} \approx \Delta\tilde{c}_{S_k}(1 - \gamma V_{S_k})$ .

*Proof.* By definition,

$$V_{S_k}(s^*) = \sum_{s_i \in S_k} P(s_i|S_k)u(c_i^*) = \sum_{s_i \in S_k} P(s_i|S_k)\frac{1}{\gamma}(1 - e^{-\gamma c_i^*}).$$

Hence,

$$\begin{aligned} \Delta\tilde{V}_{S_k} &= \sum_{s_i \in S_k} P(s_i|S_k)\frac{1}{\gamma}(1 - e^{-\gamma c_i^*}) - \sum_{s_i \in S_k} P(s_i|S_k)\frac{1}{\gamma}(1 - e^{-\gamma \tilde{c}_i}) \\ &= \sum_{s_i \in S_k} P(s_i|S_k)\frac{1}{\gamma}(e^{-\gamma \tilde{c}_i} - e^{-\gamma c_i^*}) \\ &\approx \sum_{s_i \in S_k} P(s_i|S_k)\frac{1}{\gamma}(e^{-\gamma c_i^*} \gamma \Delta\tilde{c}_i) \end{aligned} \tag{5}$$

$$= \left[ \sum_{s_i \in S_k} P(s_i|S_k)\frac{1}{\gamma}(e^{-\gamma c_i^*}) \right] \gamma \Delta\tilde{c}_{S_k}, \tag{6}$$

where (5) uses Taylor approximation and (6) uses Lemma 4.



Thus,

$$\begin{aligned}\Delta\tilde{c}_{S_k} - \Delta\tilde{V}_{S_k} &= \left[ \sum_{s_i \in S_k} P(s_i|S_k) \frac{1}{\gamma} (1 - e^{-\gamma c_i^*}) \right] \gamma \Delta\tilde{c}_{S_k} = V_{S_k}(s^*) \gamma \Delta\tilde{c}_{S_k} \\ \Delta\tilde{V}_{S_k} &= (1 - \gamma V_{S_k}(s^*)) \Delta\tilde{c}_{S_k}.\end{aligned}$$

□

**Proof of Lemma 5.** For any  $S_k \subseteq B^c$ , no consumption change occurs under the aggregation effect, and hence  $V_{S_k}^* = V_{S_k}$ . The ratio of FOC from PDS  $\pi$  to PDS  $\rho$  implies

$$\frac{\phi'(\tilde{V}_{S_k}) u'(\tilde{c}_i)}{\phi'(V_{S_k}) u'(c_i)} = \frac{\lambda_\rho}{\lambda_\pi}.$$

Substituting the CARA functional forms of  $u$  implies

$$\begin{aligned}\left( \frac{\phi'(\tilde{V}_{S_k})}{\phi'(V_{S_k})} \right) \frac{e^{-\gamma \tilde{c}_i}}{e^{-\gamma c_i}} &= \frac{\lambda_\rho}{\lambda_\pi} \\ \left( 1 + \frac{\phi''(V_{S_k}) \Delta\tilde{V}_{S_k}}{\phi'(V_{S_k})} \right) e^{-\gamma \Delta\tilde{c}_i} &\approx \frac{\lambda_\rho}{\lambda_\pi}\end{aligned}\tag{7}$$

$$\left( 1 - A_\phi(V_{S_k}) \Delta\tilde{V}_{S_k} \right) e^{-\gamma \Delta\tilde{c}_{S_k}} \approx \frac{\lambda_\rho}{\lambda_\pi},\tag{8}$$

where (7) follows from Taylor approximation and (8) follows from Lemma 4. And substituting the expression from Lemma 12 yields

$$\left[ 1 - A_\phi(V_{S_k}) (1 - \gamma V_{S_k}) \Delta\tilde{c}_{S_k} \right] e^{-\gamma \Delta\tilde{c}_{S_k}} \approx \frac{\lambda_\rho}{\lambda_\pi}.\tag{9}$$

Note for a given value of  $V_{S_k}$ ,  $\Delta\tilde{c}_{S_k}$  is determined by  $V_{S_k}$  via equation (9), whereas

$V_{S_k}$  does not depend on  $\Delta\tilde{c}_{S_k}$ . For simplicity, use notation  $x$  for  $V_{S_k}$ ,  $y$  for  $\Delta\tilde{c}_{S_k}$ , and  $F(x, y)$  for the left-hand side of equation (9). Let  $y(x)$  be the implicit function that determines the value of  $y$  for each  $x$  that solves equation (9), which is  $F(x, y(x)) = \frac{\lambda_\rho}{\lambda_\pi}$ . As the expression below shows, by the implicit function theorem, because the regularity condition  $\partial F/\partial y \neq 0$  clearly holds, the function  $y(x)$  is differentiable and

$$\begin{aligned} \frac{dy(x)}{dx} &= -\frac{\partial F/\partial x}{\partial F/\partial y} = -\frac{-A'_\phi(x)(1-\gamma x)ye^{-\gamma y} + A_\phi(x)\gamma ye^{-\gamma y}}{e^{-\gamma y}[-\gamma(1-A_\phi(x)(1-\gamma x)y) - A_\phi(x)(1-\gamma x)]} \\ &= -y \cdot \frac{-A'_\phi(x)(1-\gamma x) + A_\phi(x)\gamma}{-\gamma[1-A_\phi(x)(1-\gamma x)y] - A_\phi(x)(1-\gamma x)}. \end{aligned}$$

Note  $\gamma > 0$ ,  $y = \Delta\tilde{c}_{S_k} > 0$  for  $S_k \in \rho \setminus B$  (by Lemma 3 and Lemma 11), and  $1-\gamma x > 0$  as  $x = V_{S_k} \in (0, \frac{1}{\gamma})$ . By equation (9),  $1-A_\phi(x)(1-\gamma x)y \approx (\lambda_\rho/\lambda_\pi) \cdot e^{\gamma y} > 0$ . Also,  $A_\phi > 0$  and  $A'_\phi \leq 0$  (by Assumption 2). Hence, the above derivative is positive; that is,  $\Delta\tilde{c}_{S_k}$  is increasing in  $V_{S_k}$ . □

**Proof of Proposition 2.**  $r_{S_k}$  is increasing in  $k$ . Because  $\lambda_B < \lambda_\pi$ , we have (i)  $\Delta I_B < 0$ , and hence  $\Delta\tilde{c}_B < 0$  for all  $s_i \in B$ ; (ii)  $\Delta I_{B^c} > 0$ , and hence  $\Delta\tilde{c}_{S_k} > 0$  for all  $S_k \subseteq B^c$ . Hence,  $\Delta\tilde{s} > 0$  and  $\Delta\tilde{d}_B = \Delta\tilde{s} - \Delta\tilde{c}_B > 0$  for all  $s_i \in B$ .

And for all  $S_k \subseteq B^c$ , the following statements are equivalent: (i)  $r_{S_k} \geq r_1$ ; (ii)  $\tilde{V}_{S_k} \leq \tilde{V}_{s_1}$ ; (iii)  $\Delta\tilde{c}_{S_k} \leq \Delta\tilde{s}$ ; and (iv)  $\Delta\tilde{d}_{S_k} \geq 0$ .

Again, we can define  $S_+ = \{S_k \in \rho : \Delta\tilde{d}_{S_k} \geq 0\}$  and  $S_- = \{S_k \in \rho : \Delta\tilde{d}_{S_k} < 0\}$ .

Because  $r_B \geq r_1$ ,  $B \in S_+$ . Hence,  $r_B \geq \min_{S_+} r_{S_k} \geq r_1 > \max_{S_-} r_{S_k}$ . Thus,

$$\begin{aligned}
\Delta \tilde{L} &= \sum_{S_k \in \rho} p_{S_k} \Delta \tilde{d}_{S_k} \left(1 - \frac{1}{r_{S_k}}\right) \\
&= \sum_{S_k \in S_+} p_{S_k} \Delta \tilde{d}_{S_k} \left(1 - \frac{1}{r_{S_k}}\right) + \sum_{S_k \in S_-} p_{S_k} \Delta \tilde{d}_{S_k} \left(1 - \frac{1}{r_{S_k}}\right) \\
&> \left( \sum_{S_k \in S_+} p_{S_k} \Delta \tilde{d}_{S_k} \right) \left(1 - \frac{1}{\min_{S_+} r_{S_k}}\right) + \left( \sum_{S_k \in S_-} p_{S_k} \Delta \tilde{d}_{S_k} \right) \left(1 - \frac{1}{\max_{S_-} r_{S_k}}\right) \\
&\geq \left(1 - \frac{1}{\min_{S_+} r_{S_k}}\right) \sum_{S_k \in \rho} p_{S_k} \Delta \tilde{d}_{S_k} = \left(1 - \frac{1}{\min_{S_+} r_{S_k}}\right) \Delta \tilde{s} \geq \left(1 - \frac{1}{r_1}\right) \Delta \tilde{s} \geq 0,
\end{aligned}$$

where the last  $\geq$  follows from  $\Delta \tilde{s} > 0$  and  $r_1 \geq 1$ .

□

## A.3 Generalization

### A.3.1 Aggregation effect

**Proof of Lemma 6.** First, note consumption inside each event is monotone, due to the FOCs

$$\frac{u'(c_i)}{u'(c_j)} = \frac{r_i}{r_j}.$$

Because  $u$  is concave,  $r_i < r_j$  implies  $c_i > c_j$  for  $s_i, s_j \in S_k$ .

Now consider concave  $\phi$ . Suppose not and  $V_{S_k} \leq V_{S_l}$ . By concavity of  $\phi$  and by monotonicity  $\frac{r_i}{r_j} < 1$ , we have

$$\begin{aligned}
\frac{\phi'(V_{S_k})}{\phi'(V_{S_l})} \geq 1 &\Rightarrow \frac{u'(c_i)}{u'(c_j)} < 1 \text{ for all } s_i \in S_k, s_j \in S_l \\
&\Rightarrow c_i > c_j \text{ for all } s_i \in S_k, s_j \in S_l.
\end{aligned}$$

Suppose not and  $V_{S_k} \leq V_{S_l}$ . Because  $\phi(u(\cdot))$  is concave,

$$\frac{\phi'(V_{S_k}(s)) u'(c_i)}{\phi'(V_{S_l}(s)) u'(c_j)} = \frac{r_i}{r_j} < 1 \Rightarrow c_i > c_j \text{ for all } s_i \in S_k, s_j \in S_l.$$

Hence,

$$\begin{aligned} V_{S_k} &= \sum_{s_i \in S_k} u(c_i) P(s_i | S_k) \\ &\geq \min_{s_i \in S_k} u(c_i) > \max_{s_j \in S_l} u(c_j) \\ &\geq \sum_{s_j \in S_l} u(c_j) P(s_j | S_l) = V_{S_l}, \end{aligned}$$

which leads to a contradiction.

For the consumption sequence to be monotonically decreasing, note

$$\begin{aligned} (\phi(u(x)))' &= \phi'(u(x))u'(x) \\ (\phi(u(x)))'' &= \phi''(u(x))(u'(x))^2 + \phi'(u(x))u''(x) \\ -\frac{(\phi(u(x)))''}{(\phi(u(x)))'} &= -\frac{\phi''(u(x))u'(x)}{\phi'(u(x))} - \frac{u''(x)}{u'(x)} \end{aligned}$$

Hence,

$$A_{\phi \circ u}(x) - A_u(x) = A_\phi(u(x))u'(x).$$

The FOCs are

$$[1 - A_\phi(V_{k+1})(V_k - V_{k+1})] \frac{\min_{s_j \in S_{k+1}} r_j}{\max_{s_i \in S_k} r_i} = \frac{u'(c_{\min S_{k+1}})}{u'(c_{\max S_k})}, \quad (10)$$

where  $\max S_k$  and  $\min S_{k+1}$  are the last and the first state in  $S_k$  and  $S_{k+1}$ , respectively. It suffices to show the left-hand side is greater than 1 for all  $S_k \in \pi$ .

$$V_k - V_{k+1} = u(c_{S_k}) - u(c_{S_{k+1}}) \leq u'(c_{S_{k+1}}) [c_{S_k} - c_{S_{k+1}}] \leq u'(c_{S_{k+1}}) \left[ \max_{s_i \in S_k} c_i - \min_{s_j \in S_{k+1}} c_j \right],$$

where  $c_{S_k}$  and  $c_{S_{k+1}}$  are the certainty equivalents to consumptions in  $S_k$  and  $S_{k+1}$ . Let  $A_\phi(V_{k+1}) = A_\phi(u(c_{S_{k+1}}))$ ; then Assumption 4 implies the left-hand side of equation (10) is greater than 1.  $\square$

**Proof of Proposition 3.** By the monotonicity assumption,  $r_i = \frac{p_i}{P(s_i)}$  is increasing in  $i$ ; thus,  $\frac{\frac{p_i}{A_u(c_i)}}{\frac{P(s_i)}{A_u(c_i)}}$  is also increasing in  $i$ . The last statement implies the normalized sequence

$$\frac{\frac{p_i}{A_u(c_i)}}{\sum_{s_i \in B} \frac{p_i}{A_u(c_i)}} = \frac{\frac{P(s_i)}{A_u(c_i)}}{\sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)}}$$

is increasing in  $i$ . Finally, by monotone aggregation of the states, we get that

$$\frac{\alpha(S_k)}{\beta(S_k)} = \frac{\sum_{s_i \in S_k} \frac{p_i}{A_u(c_i)}}{\sum_{s_i \in B} \frac{p_i}{A_u(c_i)}} = \frac{\sum_{s_i \in S_k} \frac{P(s_i)}{A_u(c_i)}}{\sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)}}$$

is increasing in  $k$ . Now note that  $V_{S_k}(s)$  is decreasing in  $k$  by Lemma 6 and  $\phi$  is concave, so  $\phi'(V_{S_k}(s))$  is increasing in  $k$ , implying

$$\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s)) > \sum_{S_k \in B} \beta(S_k) \phi'(V_{S_k}(s)).$$

By Lemma 8, the aggregation effect on losses is

$$\Delta L^* = \sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)} \left( 1 - \frac{\sum_{S_k \in B} \beta(S_k) \phi'(V_{S_k}(s))}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))} \right) > 0.$$

By Lemma 6,  $\Gamma_{S_k}$  is decreasing in  $k$ . By Assumption 4,  $A_u(c_i)$  is non-decreasing in  $i$ . Lemma 7 implies  $\Delta c_i^*$  is non-increasing.

□

### A.3.2 Event-smoothing effect

**Lemma 13.** *Given  $u'(x) > 0$ , Assumption 6 is equivalent to  $g(x) = \frac{(u'(x))^2}{u''(x)}$  being non-decreasing.*

*Proof.* Given  $u'(x) > 0$ , note

$$g'(x) = 2u'(x) - \frac{(u'(x))^2 u'''(x)}{(u''(x))^2} \geq 0 \Leftrightarrow 2 \geq \frac{(u'(x)) u'''(x)}{(u''(x))^2} \Leftrightarrow 2A_u(x) \geq P_u(x).$$

Also,

$$1 \geq T_u(x)' = \left( -\frac{u'(x)}{u''(x)} \right)' = \frac{-(u''(x))^2 + u'''(x)u'(x)}{(u''(x))^2},$$

which holds if and only if

$$(u''(x))^2 \geq -(u''(x))^2 + u'''(x)u'(x) \Leftrightarrow 2(u''(x))^2 \geq u'''(x)u'(x) \Leftrightarrow 2A_u(x) \geq P_u(x).$$

□

**Lemma 14.** *If  $u$  and  $\phi$  are concave and three-times differentiable, and Assumptions 3, 4, and 6 hold,  $\frac{r_j}{A_u(c_j)}$  is non-decreasing in  $j$ .*

*Proof.* Consider state  $s_j \in S_k$  and its FOC:

$$\phi'(V_{S_k}(s))u'(c_i) = \lambda_\pi r_i \Rightarrow \phi'(V_{S_k}(s)) \frac{u'(c_i)}{A_u(c_i)} = \lambda_\pi \frac{r_i}{A_u(c_i)}.$$

First, note  $\phi$  is concave and  $V_{S_k}(s)$  is decreasing in  $k$  by Lemma 6, so  $\phi'(V_{S_k}(s))$  is increasing in  $k$ .

Second, note that by Lemma 13, Assumption 6 is equivalent to  $g(x) = \frac{(u'(x))^2}{u''(x)}$  being non-decreasing. In addition,  $\frac{u'(x)}{A_u(x)} = -\frac{(u'(x))^2}{u''(x)}$ , implying  $\frac{u'(x)}{A_u(x)}$  is a non-increasing function; hence,  $\frac{u'(c_i)}{A_u(c_i)}$  is non-decreasing in  $i$  because  $c_i$  decreases in  $i$  by Lemma 6 and the result follows. □

**Lemma 15.** *If  $u$  is concave and three-times differentiable and Assumptions 2–4 and 6 hold,  $\lambda_\rho < \lambda_\pi$ .*

*Proof.* CARA or DARA  $\phi$  implies  $\phi''' > 0$ . By Lemma 14, Assumptions 2–4 and 6 imply  $\frac{r_j}{A_u(c_j)}$  is non-decreasing in  $j$ .

For all  $s_j \in S_i \in B$ , dividing the FOCs from the problem with  $\pi$  and the aggregation problem on  $B$  implies

$$\frac{\phi'(V_B(s^*))u'(c_j^*)}{\phi'(V_{S_i}(s))u'(c_j)} = \frac{\lambda_B}{\lambda_\pi}.$$

By construction,

$$V_B(s^*) = \sum_{s_i \in B} P(s_i|B)u(c_i^*) \geq V_B(s) = \sum_{s_i \in B} P(s_i|B)u(c_i).$$

By first-order Taylor approximation of  $u'(c_j^*)$  at  $c_j$ ,

$$\frac{\phi'(V_B(s^*))}{\phi'(V_{S_i}(s))} [1 - A_u(c_j) \Delta c_j^*] = \frac{\lambda_B}{\lambda_\pi}.$$

That is,

$$\Delta c_j^* = \frac{1}{A_u(c_j)} \left( 1 - \frac{\lambda_B}{\lambda_\pi} \cdot \frac{\phi'(V_{S_i}(s))}{\phi'(V_B(s^*))} \right).$$

Because total expenditure on  $B$  remains unchanged, we have

$$\begin{aligned} 0 &= \sum_{s_j \in B} p_j \Delta c_j^* = \sum_{s_j \in B} \frac{p_j}{A_u(c_j)} \left( 1 - \frac{\lambda_B}{\lambda_\pi} \cdot \frac{\phi'(V_{S_i}(s))}{\phi'(V_B(s^*))} \right), \\ \frac{\lambda_B}{\lambda_\pi} &= \frac{\sum_{s_j \in B} \frac{p_j}{A_u(c_j)}}{\sum_{S_i \subseteq B} \frac{\phi'(V_{S_i}(s))}{\phi'(V_B(s^*))} \left( \sum_{s_j \in S_i} \frac{p_j}{A_u(c_j)} \right)} = \frac{\phi'(V_B(s^*))}{\sum_{S_i \subseteq B} \alpha(S_i) \phi'(V_{S_i}(s))}. \end{aligned}$$

For each  $S_i \subseteq B$ ,  $S_i \in \pi$ , observe that

$$\alpha(S_i) = \frac{\sum_{s_j \in S_i} \frac{p_j}{A_u(c_j)}}{\sum_{s_j \in B} \frac{p_j}{A_u(c_j)}} = \frac{\sum_{s_j \in S_i} \frac{r_j}{A_u(c_j)} P(s_j)}{\sum_{s_j \in B} \frac{r_j}{A_u(c_j)} P(s_j)}$$

is a conditional probability on event partition  $\pi_B = \{S_k, \dots, S_l\}$ , transformed from probability  $P(\cdot|B)$  with Radon-Nikodym derivative  $\frac{r_j}{A_u(c_j)}$ . Because  $\frac{r_j}{A_u(c_j)}$  is non-decreasing,  $\frac{\alpha(S_k)}{P(S_k|B)}$  is non-decreasing in the index  $k$ . By Lemma 6,  $V_{S_k}$  is decreasing in  $k$ . Because  $\phi'(\cdot)$  is decreasing and convex,

$$\sum_{i=k}^t \alpha(S_i) \phi'(V_{S_i}(s)) \geq \sum_{i=k}^l P(S_i|B) \phi'(V_{S_i}(s)) > \phi'(V_B(s)) \geq \phi'(V_B(s^*)). \quad (11)$$

Hence,  $\lambda_B < \lambda_\pi$ . By lemma 11,  $\lambda_\rho < \lambda_\pi \Leftrightarrow \lambda_B < \lambda_\pi$  and the result follows. □



**Lemma 16** (Event-smoothing effect). *If  $u$  and  $\phi$  are concave and three-times differentiable, then for any  $s_i \in S_k \in \rho$ , the event-smoothing effect can be calculated as  $\Delta\tilde{c}_i = \frac{\tilde{\Gamma}_{S_k}}{A_u(c_i^*)}$ , where  $\frac{\lambda_\rho}{\lambda_\pi}$  and  $\tilde{\Gamma}_{S_k}$  solve the following system of equations:*

$$\begin{aligned} \frac{\lambda_\rho}{\lambda_\pi} &= (1 - A_\phi(V_{S_k}(s))\tilde{\Gamma}_{S_k}E_{S_k})(1 - \tilde{\Gamma}_{S_k}) \text{ for any } S_k \in \rho \setminus B & (12) \\ \frac{\lambda_\rho}{\lambda_\pi} &= \frac{\lambda_B}{\lambda_\pi}(1 - A_\phi(V_B(s^*))\tilde{\Gamma}_B E_B)(1 - \tilde{\Gamma}_B) \\ 0 &= \sum_{S_k \in \rho} \alpha^*(S_k)\tilde{\Gamma}_{S_k}, \end{aligned}$$

where

$$\frac{\lambda_B}{\lambda_\pi} = \frac{\phi'(V_B(s^*))}{\sum_{S_k \in B} \alpha(S_k)\phi'(V_{S_k}(s))}.$$

*Proof.* Consider first  $s_i \in S_k \in \rho \setminus B$ , the ratio of the FOCs for  $\pi$ - and  $\rho$ -aggregation problems is

$$\frac{\lambda_\rho}{\lambda_\pi} = \frac{\phi'(V_{S_k}(\tilde{s}))}{\phi'(V_{S_k}(s))} \frac{u'(\tilde{c}_i)}{u'(c_i)},$$

however, note

$$\frac{\phi'(V_{S_k}(\tilde{s}))}{\phi'(V_{S_k}(s))} = 1 + \frac{\phi''(V_{S_k}(s))}{\phi'(V_{S_k}(s))} \Delta\tilde{V}_{S_k}(s) = 1 - A_\phi(V_{S_k}(s))\Delta\tilde{V}_{S_k}(s)$$

$$\frac{u'(\tilde{c}_i)}{u'(c_i)} = 1 + \frac{u''(c_i)}{u'(c_i)} \Delta\tilde{c}_i = 1 - A_u(c_i)\Delta\tilde{c}_i,$$

implying

$$\frac{\lambda_\rho}{\lambda_\pi} = (1 - A_\phi(V_{S_k}(s))\Delta\tilde{V}_{S_k}(s))(1 - A_u(c_i)\Delta\tilde{c}_i).$$

Denote  $\tilde{\Gamma}_{S_k} = A_u(c_i)\Delta\tilde{c}_i = 1 - \frac{\lambda_\rho}{\lambda_\pi} \frac{\phi'(V_{S_k}(s))}{\phi'(V_{S_k}(\tilde{s}))}$ . Then,  $\Delta\tilde{c}_i = \frac{\tilde{\Gamma}_{S_k}}{A_u(c_i)} = \frac{\tilde{\Gamma}_{S_k}}{A_u(c_i^*)}$  since the

events not in  $B$  are not affected by the aggregation effect. Also, note

$$\Delta \tilde{V}_{S_k}(s) = \sum_{s_i \in S_k} P(s_i | S_k) (u(\tilde{c}_i) - u(c_i)) = \sum_{s_i \in S_k} P(s_i | S_k) u'(c_i) \Delta \tilde{c}_i = \tilde{\Gamma}_{S_k} E_{S_k}.$$

Hence,

$$\frac{\lambda_\rho}{\lambda_\pi} = (1 - A_\phi(V_{S_k}(s)) \tilde{\Gamma}_{S_k} E_{S_k}) (1 - \tilde{\Gamma}_{S_k}).$$

And so we have obtained a quadratic equation for  $\tilde{\Gamma}_{S_k}$ .

The proof for event  $B$  goes by analogy, with the exception that we should take the ratio of the FOCs from the  $\rho$ -aggregation problem and  $B$ -intermediate bundle problem, implying  $\frac{\lambda_\rho}{\lambda_B}$  would appear instead of  $\frac{\lambda_\rho}{\lambda_\pi}$ . However, note  $\frac{\lambda_\rho}{\lambda_B} = \frac{\lambda_\rho}{\lambda_\pi} \frac{\lambda_\pi}{\lambda_B}$ . And the second equation follows.

Finally, note the total income does not change, so the sum of the change in income must be zero:

$$\sum_{s_i \in \Omega} p_i \Delta \tilde{c}_i = \sum_{s_i \in \Omega} \frac{p_i}{A_u(c_i^*)} \tilde{\Gamma}_{S_k} = \sum_{S_k \in \rho} \tilde{\Gamma}_{S_k} \sum_{s_i \in S_k} \frac{p_i}{A_u(c_i^*)} = \sum_{S_k \in \rho} \alpha^*(S_k) \tilde{\Gamma}_{S_k} = 0.$$

□

**Lemma 17.** *If  $u$  is concave and three-times differentiable, and Assumptions 2–4 and 6 hold,  $E_{S_k} A_\phi(V_{S_k}(s))$  does not decrease in  $k$ .*

*Proof.* First, consider

$$E_{S_k} = \sum_{s_i \in S_k} P(s_i | S_k) \frac{u'(c_i)}{A_u(c_i)} = - \sum_{s_i \in S_k} P(s_i | S_k) \frac{(u'(c_i))^2}{u''(c_i)}.$$

By Lemma 6,  $c_i$  decreases in  $i$ , and by Lemma 13,  $\frac{(u'(x))^2}{u''(x)}$  is non-increasing; hence,  $\frac{(u'(c_i))^2}{u''(c_i)}$  does not increase in  $i$ . Due to monotonic aggregation of the states, we get

that  $E_{S_k}$  does not decrease in  $k$ .

Now consider  $A_\phi(V_{S_k}(s))$ .  $V_{S_k}(s)$  decreases in  $k$  by Lemma 6. In addition,  $\phi$  is CARA or DARA, implying  $A_\phi(\cdot)$  is a non-increasing function. Hence,  $A_\phi(V_{S_k}(s))$  does not decrease in  $k$ . Then, the result follows.  $\square$

**Lemma 18.** *If  $u$  and  $\phi$  are concave and three-times differentiable,  $\tilde{\Gamma}_{S_k} > (<)0$  and  $A_\phi(V_{S_k}(s))E_{S_k}$  does not decrease in  $k$  for any  $S_k \in \rho \setminus B$ , then  $\tilde{\Gamma}_{S_k}$  does not increase (decrease) in  $k$ .*

*Proof.* To simplify notation of this proof, we denote  $A_\phi(V_{S_k}(s))E_{S_k} = a_k$  inside this proof only. First, consider the quadratic equation from Lemma 16:

$$\frac{\lambda_\rho}{\lambda_\pi} = (1 - a_k \tilde{\Gamma}_{S_k})(1 - \tilde{\Gamma}_{S_k}) \text{ for any } S_k \in \rho \setminus B,$$

which can be rewritten as

$$a_k \tilde{\Gamma}_{S_k}^2 - (a_k + 1)\tilde{\Gamma}_{S_k} + 1 - \frac{\lambda_\rho}{\lambda_\pi} = 0.$$

Now we take the derivative of the above equation with respect to  $a_k$  and get the following:

$$\begin{aligned} \tilde{\Gamma}_{S_k}^2 + 2a_k \tilde{\Gamma}_{S_k} \frac{d\tilde{\Gamma}_{S_k}}{da_k} - \tilde{\Gamma}_{S_k} - (a_k + 1) \frac{d\tilde{\Gamma}_{S_k}}{da_k} &= 0 \\ \Rightarrow \frac{d\tilde{\Gamma}_{S_k}}{da_k} &= \frac{\tilde{\Gamma}_{S_k}(1 - \tilde{\Gamma}_{S_k})}{2a_k \tilde{\Gamma}_{S_k} - a_k - 1}. \end{aligned}$$

We are interested in the sign of  $\frac{d\tilde{\Gamma}_{S_k}}{da_k}$ .

The actual solutions to the equation are

$$\tilde{\Gamma}_{S_k} = \frac{a_k + 1}{2a_k} \pm 0.5 \sqrt{\left(1 + \frac{1}{a_k}\right)^2 - \frac{4}{a_k} \left(1 - \frac{\lambda_\rho}{\lambda_\pi}\right)}.$$

Note that when  $\frac{\lambda_\rho}{\lambda_\pi} = 1$ ,  $\tilde{\Gamma}_{S_k} = 0$ , because the event-smoothing effect is 0 in this case; hence we can throw away the solution with “+”. Thus,  $\tilde{\Gamma}_{S_k} \leq \frac{a_k+1}{2a_k}$ .

In addition,  $(1 - a_k\tilde{\Gamma}_{S_k})(1 - \tilde{\Gamma}_{S_k}) > 0$ , implying both elements  $(1 - a_k\tilde{\Gamma}_{S_k})$  and  $(1 - \tilde{\Gamma}_{S_k})$  must be of the same sign. The greater of the roots of the equation produces negative elements, whereas the smaller produces positive elements, because  $1 - \tilde{\Gamma}_{S_k}$  is smaller when  $\tilde{\Gamma}_{S_k}$  is greater. Thus,  $\tilde{\Gamma}_{S_k} < 1$  because we are dealing with the smallest of the two roots. Hence, we obtain  $\text{sgn}\left(\frac{d\tilde{\Gamma}_{S_k}}{da_k}\right) = -\text{sgn}\tilde{\Gamma}_{S_k}$  and the result follows.  $\square$

**Proof of Proposition 4.** Because Assumptions 2–6 hold, by Lemma 15,  $\lambda_\pi > \lambda_\rho$ . Then, Lemma 11 implies  $\Delta I_{B^c} > 0$ ,  $\Delta\tilde{s} > 0$ , and  $\Delta\tilde{c}_i > 0$  for all  $s_i \in S_k \in \rho \setminus (B \cup \{s_1\})$ . By Lemma 17,  $E_{S_k}A_\phi(V_{S_k}(s))$  does not decrease in  $k$  for all  $S_k \in \rho \setminus (B \cup \{s_1\})$ .

For all  $s_i \notin B \cup \{s_1\}$ , Lemma 16 implies  $\Delta\tilde{c}_i = \frac{\tilde{\Gamma}_{S_k}}{A_u(c_i)}$ . Hence, for all  $S_k \in \rho \setminus (B \cup \{s_1\})$ ,  $\tilde{\Gamma}_{S_k} > 0$ . By Lemma 18, we have that  $\tilde{\Gamma}_{S_k}$  does not increase in  $k$ . By Assumption 5,  $u$  is CARA or DARA, and  $c_i$  is decreasing in  $i$ ,  $\Delta\tilde{c}_i$  is non-increasing in  $i$ . Therefore, we have that  $\Delta\tilde{d}_i = \Delta\tilde{s} - \Delta\tilde{c}_i$  is non-decreasing in  $i$  for all  $s_i \notin B \cup \{s_1\}$ .

Define  $S_+ = \{S_k \in \rho : \Delta\tilde{d}_i \geq 0, \forall s_i \in S_k\}$  and  $S_- = \{S_l \in \rho : \Delta\tilde{d}_j < 0, \forall s_j \in S_l\}$ . By Lemma 18,  $S_-$  contains all the events  $S_k \in \rho \setminus B$  such that  $r_i < r_1$  for all  $s_i \in S_k$ . And  $S_+$  contains all the events  $S_l \in \rho \setminus B$  such that  $r_1 < r_j$  for all  $s_j \in S_l$ . Because  $\Delta\tilde{d}_i > 0$  for all  $s_i \in B$ ,  $B \in S_+$ . Then, the condition  $r_1 < \min_{s_j \in S_{l+1}} r_j$  implies  $S_+$  contains event  $B$  and all the events  $S_k \in \rho$  after  $B$ .<sup>20</sup> Therefore,  $\min_{S_+} > \max_{S_-}$ .

<sup>20</sup>In other words, the sequence  $\{\Delta\tilde{d}_i : i \geq 2\}$  crosses zero from below exactly once.

Thus,

$$\begin{aligned}
\Delta \tilde{L} &= \sum_{s_i \in S} p_i \Delta \tilde{d}_i \left(1 - \frac{1}{r_i}\right) \\
&= \sum_{S_k \in S_+} \sum_{s_i \in S_k} p_i \Delta \tilde{d}_i \left(1 - \frac{1}{r_i}\right) + \sum_{S_l \in S_-} \sum_{s_j \in S_l} p_j \Delta \tilde{d}_j \left(1 - \frac{1}{r_j}\right) \\
&> \left( \sum_{S_k \in S_+} \sum_{s_i \in S_k} p_i \Delta \tilde{d}_i \right) \left(1 - \frac{1}{\min_{S_+} r_i}\right) + \left( \sum_{S_l \in S_-} \sum_{s_j \in S_l} p_j \Delta \tilde{d}_j \right) \left(1 - \frac{1}{\max_{S_-} r_j}\right) \\
&\geq \left(1 - \frac{1}{\min_{S_+} r_i}\right) \sum_{s_i \in S} p_i \Delta \tilde{d}_i = \left(1 - \frac{1}{\min_{S_+} r_i}\right) \Delta \tilde{s} \geq 0,
\end{aligned}$$

where the last  $\geq$  follows from  $\min_{S_+} r_i \geq r_1 \geq 1$  and  $\Delta \tilde{s} > 0$ .

□

### Proof of Lemma 9.

$$\begin{aligned}
\Delta \tilde{L} &= \sum_{s_i \in \Omega \setminus \{s_1\}} (p_i - P(s_i)) (\Delta \tilde{s} - \Delta \tilde{c}_i) = -(p_1 - P(s_1)) \Delta \tilde{s} - \sum_{s_i \in \Omega \setminus \{s_1\}} (p_i - P(s_i)) \Delta \tilde{c}_i \\
&= - \sum_{s_i \in \Omega} p_i \Delta \tilde{c}_i + \sum_{s_i \in \Omega} P(s_i) \Delta \tilde{c}_i = \sum_{s_i \in \Omega} P(s_i) \Delta \tilde{c}_i = \sum_{S_k \in \rho} \tilde{\Gamma}_{S_k} \sum_{s_i \in S_k} \frac{P(s_i)}{A_u(c_i^*)} \\
&= \left( \sum_{s_i \in \Omega} \frac{P(s_i)}{A_u(c_i^*)} \right) \sum_{S_k \in \rho} \beta^*(S_k) \tilde{\Gamma}_{S_k}.
\end{aligned}$$

□

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