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Do Social Justice Contexts Matter in Mathematical Modeling?: Modeling Problem Analysis

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Abstract
Since the Common Core State Standards for Mathematics (CCSSM; NGA Center for Best Practices & CCSSO, 2010) include modeling as one of the eight Standards for Mathematical Practices for all grades but also as important conceptual category in high school mathematics, there have been a spotlight on mathematical modeling and demand for teachers to deliberately integrate modeling tasks in mathematics classrooms. One of the efforts to integrate modeling in mathematical curricula is including modeling problems in textbooks. It is necessary to examine if these attempts are adequately made in mathematical curricula (Meyer, 2015).

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Comments
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Do Social Justice Contexts Matter in Mathematical Modeling?: Modeling Problem Analysis

Since the Common Core State Standards for Mathematics (CCSSM; NGA Center for Best Practices & CCSSO, 2010) include modeling as one of the eight Standards for Mathematical Practices for all grades but also as important conceptual category in high school mathematics, there have been a spotlight on mathematical modeling and demand for teachers to deliberately integrate modeling tasks in mathematics classrooms. One of the efforts to integrate modeling in mathematical curricula is including modeling problems in textbooks. It is necessary to examine if these attempts are adequately made in mathematical curricula (Meyer, 2015).

Focusing on the central role of real-world contexts in mathematical modeling, we should note that the real-world students live in involves social justice issues. If teachers truly integrate students' real-worlds in classrooms, mathematics cannot be neutral to the true reality experienced by students (Fasheh, 1997). Teaching mathematics for social justice can engage students in resolving ill-defined problems. Such problems are those that "[address] complex issues and thus cannot easily be described in a concise, complete manner" (Reed, 2002, p. 122). Students are inclined to engage ill-defined problems since they related to conflicts they are likely to encounter in their lives (Gutstein, 2006). We thus argue that to deepen students' understanding of mathematical modeling, students should be provided with rich tasks that involve social justice issues.

In this study, we examine how mathematical modeling problems appear in a widely used US mathematics textbook, *Eureka Math* (www.engageny.org) (Opfer, Kaufman, & Thompson, 2016) and whether social justice is embedded in those modeling problems. In addition, we aim to see how the textbook modeling problems are aligned with the recommended features of modeling problems identified in the literature. Since a textbook is known to be a primary device that teachers often use for instructions (Elsaleh Ilham, 2010; Son & Kim, 2015, 2016), the

findings of this study may inform curriculum designers and teachers about how to enact effectively modeling tasks in textbooks and connect them with social justice.

Mathematical Modeling Problems and Social Justice

Kaiser and Sriraman (2006) classified various perspectives of modeling and one of the modeling classification is socio-critical modeling. In socio-critical modeling, students are given social justice contexts and use mathematics to find a solution to a real-life challenge. Kaiser and Sriraman described the socio-critical perspective as closely related to ethnomathematics—"the mathematics which is practiced among identifiable cultural groups such as national-tribe societies, labour groups, children of certain age brackets and professional classes" (D'Ambrosio, 1985, p. 45). This socio-critical perspective emphasizes the function of mathematical modeling in society and claims the necessity to support critical thinking about the role of mathematics in society. There have been efforts to integrate social justice into mathematics learning and teaching (e.g., Frankenstein, 1990, 2001; Gutstein, 2006; Moses & Cobb, 2002; Stinson, 2017; Wager & Stinson, 2012). As students solve problems that represent injustices in their community, they can use mathematics as a vehicle to deepen their awareness of the social problems (MacLeod, 1991). Hence, curriculum changes are needed to prepare teachers and students to critically *read* (understand) and *write* (change) the world (Gutstein, 2006). Mathematical modeling can be used to develop students' social-critical efficacy (Rosa & Orey, 2013)—students' critical analysis of the specific role of mathematics in the power structures of society—because through mathematical modeling, students can engage in social justice issues in their real-lives and construct mathematical knowledge.

To teach mathematical modeling or use modeling to teach mathematics, Barbosa (2003) proposed three potential cases to implement modeling in school curriculum. The first case is that

teachers choose a problem/situation and present the problem with data. In the second case, teachers pose a question in which students are responsible for collecting data and presenting their solutions. A third case involves a non-mathematical topic selected by either teachers and students. Compared to the first two cases, students have more control in the third case because students formulate their problems, collect information, and solve the problems. Teachers may use any of these cases with consideration of students' grade level, knowledge, preferences and experiences. Among the three cases, the first case is related to using existing curriculum tasks, which can often be a stepping stone for teachers in developing and modifying problems and facilitating student learning of mathematics based on their interests.

Criteria of Social Justice Modeling Problems

The CCSSM define modeling as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions." (NGA & CCSSO, 2010, p. 73). This definition implies that modeling is problem solving in nature because problem solving is the process of choosing and using appropriate method or strategy to solve a problem. Baroody (1993) distinguishes a problem from an exercise using one criteria of Charles and Lester (1982): the lack of an obvious way to find a solution. Hence, mathematical modeling problem is a unique form of a problem that requires to involve a real-world situation, such as physical, social, or scientific phenomena (Anhalt & Cortez, 2015), which is not always required in general problem-solving.

Author et al. (2017) proposed the criteria of problems, modeling problems, and social justice modeling problems based on the hypothesis that these categories have an inclusive relationship in the order of problems, modeling problems, and social justice modeling problems from broader to more specific. This is aligned with two main features that Barbosa (2006)

proposed: the boundaries of a modeling activity are (1) the activity has to be a problem (not an exercise); and (2) the activity has to be extracted from the everyday or other sciences that are not pure mathematics. The first feature satisfies the condition of being problems and the second one illustrates the uniqueness of modeling problems. We further refined this framework for curriculum analysis. Table 1 describes our inclusive framework and the features required or related to each category.

Table 1. *Criteria of Problems, Modeling Problems, and Social Justice Modeling Problems*

Category	Code	Code Description
Social Justice Modeling Problem	SJ	A task includes a context related to social justice or requires to discuss social justice issues with the context or solution.
Modeling Problem	RC	A task embeds an authentic real-life context
	IP	A task includes an iterative process of testing mathematical solution(s) in the real-life context to make reconciliation between math and real-world
	R	A task includes a process of developing representational descriptions for specific purpose to mathematize (e.g., quantify, symbolize) the real-life situation. Or an interpretation of the embedded representation(s) (e.g., tables, graphs, symbols, words) is used to describe the problem situation.
	NP	Not all parameter is given in a task.
	GT	A task includes a process of developing generalizable/transferable knowledge.
	AE	A task includes a process of performing approximation/estimation.
Problem	FM	A task requires mathematics to solve the problem.
	UM	A task requires unpredictable methods and no particular algorithm is defined
	MS	A task requires multiple solution pathways.

To find the essential features of mathematical modeling problems, we reviewed prior studies related to modeling (e.g., Anhalt, 2014; Barbosa, 2006; Cirillo, Pelesko, Felton-Koestler, & Rubel, 2016; Galbraith, 2007; Lesh & Lehrer, 2003; Pollak, 2003) and identified commonalities among definitions and features of modeling stated in the studies. We shaded the features that have high commonalities and use them to determine each category but still included unshaded features to see how the problems involve those features. For example, the tasks that focus on

mathematics (FM) and have unpredictable method (UM) are considered as problems, and the problems that satisfy the features of RC, IP, and R are coded as mathematical modeling problems. Some problems may also have multiple solution pathways (MS) and some modeling problems may include other features (NP, GT, and AE). The modeling problems with social justice context are categorized as social justice modeling problems.

Methods

We chose *Eureka Math* designed by Engage NY because this online textbook is most widely used in the U.S. in response to the CCSSM, according to a study released by the RAND Corporation (Opfer, Kaufman, & Thompson, 2016). Yet, how this curriculum presents mathematical modeling problems or social justice modeling problems has not been studied yet. *Eureka Math* is downloadable online and structured with three layers: modules, topics, and lessons. Each subject (mathematics or English) consists of *modules* (i.e., the largest unit) that is divided into several *topic*s; and each topic includes multiple *lessons*. Algebra 1 modules were used for this study because CCSSM High School content standards include modeling as one domain; and all high school students in the U.S. are required to take Algebra 1.

We first identified modeling-related lessons using two inclusive criteria: (1) a lesson's title includes *modeling* and (2) a lesson whose topic overview (teacher edition) designated it as a modeling cycle lesson. In each modeling-related lesson, we individually coded each task in the student worksheet using our framework. Each lesson includes both teacher guideline and student worksheet (e.g., classwork, exercises, and problem set). We included multiple sub-questions in one task if they shared the same context or story data. When multiple tasks were connected like one task, we counted them as one task. After identifying the modeling related lessons, we coded each task in the lessons using the criteria (see Table 1) individually, categorized each task into

one of the four categories: exercise, problems, modeling problems, social justice modeling problems, and refined the coding result through discussions until a consensus was made.

Findings and Discussion

In five modules of Algebra 1, we found 22 lessons whose objectives were teaching the modeling cycle or whose titles included "modeling," and identified 186 tasks in those lessons. Among 186 tasks, there was no single modeling problem that addressed social (in)justice in the real-life context, and only about 30 percent of the tasks (n = 57) were coded as mathematical modeling problems (see Figure 1). Among the remaining 129 problems, 26 tasks were neither problems nor modeling problems, which were coded as exercises that provided an algorithm solution pathway. These findings suggest that a large portion of the "modeling" tasks in *Eureka Math* does not provide students with an opportunity to experience mathematical modeling that meets the criteria for mathematical modeling problems recommended by researchers (e.g., Cirillo et al., 2016; Galbraith, 2007; Lesh & Lehrer, 2003; Pollak, 2003). This reveals that students may not have an opportunity to develop their social-critical perspectives through mathematical modeling.

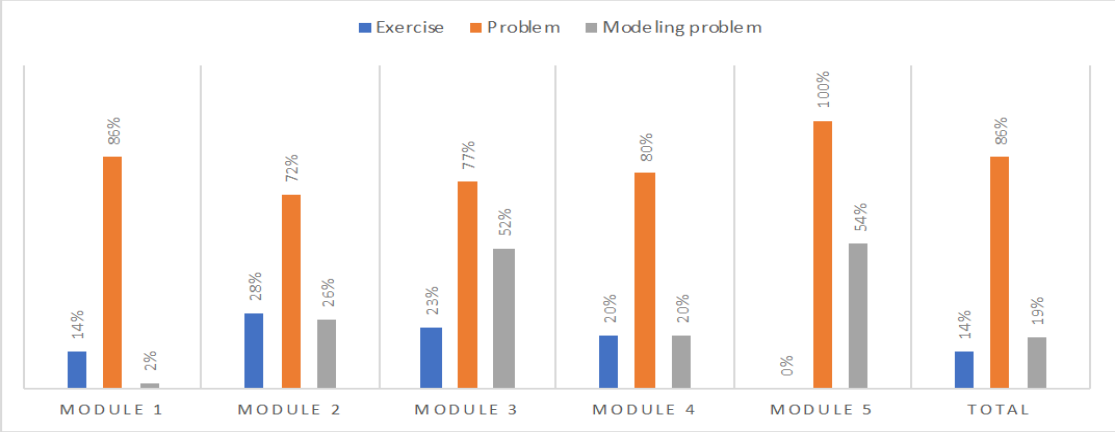

Figure 1. Percentage of Exercise, Problem and Modeling Problem in Algebra Modules

Figure 1 shows the percentage of exercises, problems and modeling problems in each module of Algebra 1. We did not include social justice modeling problems because it was zero

percent in all modules. More than 50 percent of tasks in Modules 3 and 5 were considered as modeling problems but only 2 percent of tasks in Module 1 turned out to be modeling problems. Module 1 addresses the relationships between quantities and reasoning with equations and their graphs. The titles of Modules 3 and 5 are "Linear and Exponential Functions" and "A Synthesis of Modeling with Equations and Functions" respectively. We further analyzed 186 tasks with respect to other features shown in Table 1 that we did not consider as essential requirements to determine problems or modeling problems. Note that around 75 percent of the tasks require students to use multiple solutions; approximately 30 percent of the tasks ask students to use approximation, estimation, or predication. Furthermore, 19 percent of the tasks demand students to come up with generalizable pattern and product. 10 percent of the tasks require students to identify necessary parameters or to make assumptions. Again, none of the tasks involved a context of social justice such as racism, sexism, or other social unequal phenomena.

Our findings reveal some concerns about modeling problems presented in *Eureka Math,* a popular U.S. mathematical curriculum in use, but we also found potential to alter the curricula resources for effective social justice mathematical modeling teaching. Overall the ways in which mathematical modeling was treated were procedure-oriented, which means that the instructional focus with mathematical modeling was mostly on teaching *process of each part of the modeling cycle*, not on helping students make sense of what mathematical modeling entails. In particular, some tasks in the modeling lessons did not include any real-life context, which deprives of student's opportunity to explore connections between mathematics and real-life situations. For example, one task in Module 1, "If two numbers represented by $(2m+1)$ and $(2m+5)$ have a sum of 74, find m," was provided students with practice of building equations. Although *Eureka Math* presented this problem with an intention to address mathematical modeling as stated in

their teacher guide, we coded this task as a problem but not a modeling problem because it did not satisfy the required features of modeling in our criteria such as realistic context, iterative process, and representations. Although social justice contexts provide students with the opportunity to deepen their awareness of their identities and injustice in their community and the roles of mathematics as a tool to solve problems in society (Gutstein, 2006; Wager & Stinson, 2012), no problem of social justice contexts existed in *Eureka Math*.

Recommendation for Teachers

Given the lack of modeling problems whose contexts are social justice, it is crucial for teachers to modify textbook modeling problems to include a social justice context to develop students' critical thinking and deepen their understanding of mathematics. Lopez (2017) showed a way to bring a conversation about social justice matters into the mathematics classroom while teaching the mathematical content of their courses (see *Figure 2*).

[Original] A group of friends rented a place for a camping trip. A week before the trip 4 friends had to withdraw from the trip. The remaining participants will then have to pay $5 more per person. Then two days before the trip 2 more friends realized they cannot make the trip and let the group know that they will not pay. As a result, the remaining friends have to pay $3 extra each. How many friends went to the trip? How much money did they pay?
[Modified] A company hired temporary workers to complete a building project and assigned a given number of hours to each worker. Two days before the day the project was expected to start, the manager fired 4 workers and assigned 5 more hours to each of the remaining workers. Then, the night before the job was to begin, the manager decided to fire 2 more workers and assign three more hours to the remaining ones. How many workers worked on the project? How many hours did each of them work?

Figure 2. An example of modifying a problem to social justice modeling problem (Lopez, 2017)

The original problem uses a context of camping, and Lopez modified it to include a social issue, firing workers. This context implies cutting the number of workers would result in the increasing working hours. While mathematizing the given problem is important, teachers can use the social justice context to provide an opportunity for students to have reflective discussions, connecting the problem with their lives (Barbosa, 2006). We also found some modeling tasks in *Eureka*

Math that could potentially be converted to a social justice modeling problem. The following task included in Module 3 can be an example.

> The Albany International Airport wants to increase the average daily parking revenue by 10%. Make a recommendation to management of one or more parking rates to change to increase daily parking revenue by 10%. Then, use the data Helena collected [from the previous task] to show that revenue would increase by 10% if they implement the recommended change.

Using Lopez's example, teachers can modify this task by adding a social justice context while maintaining the mathematical concept. One possible modification is to use a context of a company's profit and reasonable (and just) salary increase based on the profit. In addition, as Barbosa (2006) emphasized, teachers can guide students to have meaningful discussions before and after the modeling process.

What distinguishes mathematical modeling from other forms of applications of mathematics is: (1) explicit attention at the beginning of the process of getting from the problem outside of mathematics to its mathematical formulation, and (2) an explicit reconciliation between the mathematics and the real-world situation at the end by determining reasonableness of their mathematical answer(s) in the real-world context (Pollak, 2003, p. 649). Holding up Pollak, we highlight the importance of varying real-world contexts to use for mathematical modeling. Integrating social justice into modeling problems has a potential to address critical social issues while keeping the mathematical rigor in the problem (Gutstein, 2006; MacLeod, 1991; Wager & Stinson, 2012). Given the gap reported between intended curriculum (i.e., CCSSM) and available textbooks (i.e., *Eureka Math*) concerning mathematical modeling tasks, this study confirms the important role of teachers. Teachers need to be aware of what is intended and what is presented in textbooks for teaching mathematical modeling and then work toward helping students make sense of modeling through social justice issues embedded in their lives.

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