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Entry under placement uncertainty

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Keywords

Placement uncertainty, excess entry, cap and trade regulation

Disciplines

Environmental Policy | Industrial Organization

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1 Introduction

Cap-and-trade (CAT) regulation is used worldwide to manage negative production externalities in environmental and natural resource-based industries.[1] CAT places an upper bound on either the aggregate byproduct of a production process, e.g., in the case of pollution emissions, or on the aggregate industry output directly, e.g., in the case of a fishery. Firms operating under these regulations must hold shares of the aggregate cap in quantities that match their own production. When the cap binds, the permits held by one firm displace, one-for-one, the permits available to competing producers. In this setting remaining active requires individual firms to match the productivity of their competitors; if they cannot, they are better off selling their permits to more productive rivals and exiting the industry.

The question we ask in this paper is, under what information structures and beliefs about own-firm and competitor productivity will there be excess entry in CAT-regulated industries? We develop a model where firms simultaneously and noncooperatively choose whether or not to enter an industry that is regulated under CAT. Our inquiry is focused on the unique aspect of CAT regulations where relative productivity determines permit trading prices and residual returns to the firms' capital investments. Firms in our model are heterogeneous in own cost efficiency and uncertain about where they rank along a continuum of cost efficiency levels in the population of potential entrants/competitors. Firms must therefore base their entry decision on a subjective prior of the competition they face post entry. The setting mirrors market entry coordination games used to study entry/investment decisions and, more famously, the phenomenon of excess market entry by exuberant, perhaps overconfident, entrepreneurs (Kahneman, 1988; Rapoport et al., 1998; Camerer and Lovallo, 1999).

Our first main and we believe novel result, is that *uncertainty* over one's productivity rank is sufficient for there to be excess firm entry relative to a full information benchmark. Equilibrium permit prices are determined by the number and productivity of entering firms. The permit price

[1] The European Union Emissions Trading System regulates carbon dioxide emissions from the 28 European Union countries, plus Iceland, Liechtenstein, and Norway. See Narassimhan et al. (2018) for a recent review of cap-and-trade pollution regulations. Marine commercial fisheries have been regulated with tradable quotas since 1983 (in New Zealand). Currently 16 major U.S. fisheries are managed with tradable fishing quota regulations. A full list of world, CAT-regulated fisheries is available in Lynham (2014).

divides industry operating profits between the fixed number of production permits and the entering firms' capital. Underestimating post entry competition means high permit prices and low capital rent. We show that when firms take a modest view of their own productivity, i.e., specifically, they assume they are the *average-cost efficiency firm in the population*, that entry will be excessive and industry wide efficiency will be lower than under full information. The marginal entrant in our model rationally expects there will be some less productive rival firms who enter. This pivotal entrant therefore expects permit prices will be lower and capital rents higher than is objectively warranted. Uncertainty alone can lead to excess and inefficient entry.

Furthermore, we obtain conditions under which this excess entry result can be reversed. We show that potential entrants may under these conditions, overestimate the level of competition for emissions/output permits and underestimate the rent to their own vested capital. Thus firms that would enter under full information, stay out.

A sizable empirical and experimental literature suggest the existence of ”competitive blind spots” (e.g., Camerer and Lovallo (1999); Cain et al. (2015); Koellinger et al. (2007); Schüssler (2018)) or a failure to estimate potential competition that leads to a sub-optimal decision by a firm to enter an industry or invest in a venture. The paper shows the existence of such blind spots as the outcome of a rational expectations market equilibrium. Moreover, we show that they can be attributed to pure uncertainty rather than to overestimation of one's own ability or overconfidence per se.

We extend the model to consider the effects of overconfidence or overplacement *bias* on entry. The second main result of the paper shows that when firms are overconfident and erroneously believe their own productivity is *better than average* the problem of excess entry is, not surprisingly, exacerbated. The paper thus points out the need to differentiate between two different sources of excess entry: pure uncertainty and overconfidence.

It should be pointed out that although the framework adopted in the paper is that of a CAT-regulated industry, our results apply more generally. An endogenously determined permit price can be replaced by an endogenously determined goods price obtained under an inelastic product demand. Firms in this setting must compete for customers; the relative productivity of competing firms continues to play a central role in the returns to capital and thus entry. Thus, market forces that drive our results are not specific to CAT regulations. The framework of a CAT-regulated industry sharpens the role of the relative productivity argument and keeps the model tractable.

Our paper contributes to several strands of literature. First, it contributes to the literature on global games with strategic substitutes. While the theoretical and applied literature on global games with strategic complementarities is substantial (See for example, (Carlsson and Van Damme, 1993; Morris and Shin, 1998, 2001, 2005; Frankel and Pauzner, 2000; Frankel et al., 2003) to mention a few) there are few papers that deal with strategic substitutability (These include (Karp et al., 2007; Harrison and Jara-Moroni, 2015; Morris and Shin, 2009)). The relative paucity is partly explained by the fact that such games in general are inherently less tractable as a Nash equilibrium in pure strategies is not guaranteed. Our basic framework is that of a global game with strategic substitutes and endogenous payoffs. We establish the existence of an unique equilibrium in pure strategies.[2]

A second contribution of our work relates to the recent and fast growing literature on welfare analysis in economies with incomplete private information. This literature highlights the dual nature of prices - as conveyors of information and as determinants of resource allocation. Moreover, various types of inefficiencies - aggregative or distributional - are traced to externalities arising from this dual role (see e.g. Morris and Shin (2002, 2005); Angeletos and Pavan (2007, 2009); Amador and Weill (2010); Vives (1993, 1997, 2017)). The literature primarily focuses on the relative values of private and public information (as conveyed by prices) in a framework of ex-ante (but not ex-post, since they receive different signals) identical agents facing common shocks. The set of agents participating in the market is assumed to be constant - all agents are active. The focus of our paper is the agent's decision itself of whether or not to enter and compete for a fixed quota. The set of active firms is thus endogenous. Within the context of this decision problem, we identify and model a new source of strategic uncertainty that is associated with agent heterogeneity - placement uncertainty and overplacement bias. To the best of our knowledge such an attempt is new to this literature.

The literature on firm dynamics under complete and incomplete information is vast. Jovanovic (1982) studies firm dynamics in a setting where firms are uncertain about their own productivity

[2]Karp et al. (2007) shows the existence of a pure strategy Bayesian Nash equilibrium in a global game that combines strategic complementarity and substitution with exogenous payoffs.

and learn their true productivity after entry has occurred. We reverse the information set up; we envision a relatively mature, perhaps oversized industry in which firms are aware of their own productivity. The CAT regulation, by pricing an externality, forces managers to carefully assess their relative productivity to determine whether or not they can remain active. Our model can therefore be interpreted as one of strategic exit, although exit due to the CAT regulation has not been previously considered. Ghemawat and Nalebuff (1985, 1990) study exit in an exogenously declining industry. The question is, which of the incumbents will exit or reduce their capacity first. The authors find that larger firms either exit, in the all or nothing version of the model (Ghemawat and Nalebuff, 1985), or reduces productive capacity first (in the general version, Ghemawat and Nalebuff (1990)). Fudenberg and Tirole (1986) introduce incomplete information into a dynamic duopoly competition game. As in the model of this paper, firms in Fudenberg and Tirole (1986) know their own productivity (costs) but must choose when if ever to exit based on their beliefs about rival costs. Over time, active firms become more pessimistic about the cost of their rival. In long run equilibrium, an industry shake out occurs with high cost firms exiting and low cost firms remaining active.

Our paper offers new insights for understanding the effects of environmental regulations on industry structure and performance (see reviews in Heyes (2009) and Millimet et al. (2009)). Much of this literature has focused on taxation, and command-and-control type regulations. The determinants of, and the role that permit prices play in industry structure has not been fully vetted. Finally, our analysis of placement uncertainty and overplacement bias on entry decisions and industry performance contributes to the behavioral industrial organization literature that emphasizes the market implications of bounded rationality of firms and consumers (Tremblay et al. (2018); see also Hoffman and Burks (2017)).

The paper is organized as follows. The next section presents the model. Section 3 derives the equilibrium permit price, and entry decisions under a benchmark full information scenario. Section 4 presents the results under imperfect information. Section 5 compares permit prices, industry structure and market performance under full and imperfect information. Section 6 examines the effects of overplacement bias. The final section 7 summarizes our results and discusses extensions.

2 The model

We consider a single production period during which an industry is regulated to produce no more than Q units of output. We assume Q has been pre-determined by a planner to meet a broader social goal.[3] Quota has no value outside the industry, or beyond the single production period under consideration. Units of production must be matched with units of quota. There is no cheating (over quota production). Firm-level production and permit quantities are denoted in common units, q. Industry output is sold at a constant and given unit price, p.[4]

There is a continuum S of firms in the population. Firms decide simultaneously whether to enter the quota regulated industry. Entry implies commitment of a single unit of capital for the full production period at irreversible cost $\delta > 0$, which we assume is common across firms.

Firms are heterogeneous in their productivity, which manifests as differences in variable costs of production. We use θ_i to denote an inverse productivity measure for firm i with larger values corresponding to higher costs. We use $c(q|\theta_i)$ to denote the variable cost given θ_i. $c(q|\theta_i)$ is assumed to be strictly convex in q, with $c(0|\cdot) = 0$. Specifically,

$$c(q|\theta_i) = \theta_i q + \frac{1}{2}\lambda q^2,$$

where $\lambda > 0$ is common across firms. In the sequel, we refer to θ_i as the efficiency or cost efficiency of firm i.

We assume that Q has been allocated *gratis* to a subset of firms with $w_i \geq 0$ denoting the allocation to firm i. All $i \in S$ participate in a post-entry permit trading market. Firms announce a net trade schedule to a market maker who aggregates schedules and determines a market clearing quota price and the purchases/sales for each firm.[5] We will soon show that the initial quota allocation is irrelevant for entry decisions as long as a friction-less quota trading market exists, which we assume.

[3]Our model determines capital entry and thus the total cost of producing a given Q under varying information structures. It does not discuss an *optimal* value for Q.

[4]While our model is fashioned on an output cap regulation, the model and results apply to alternate forms of CAT, e.g., a cap on carbon emissions, or on the by-production of pollution (see Murty et al. (2012) for a discussion of polluting technologies).

[5]All firms are required to transact according to their reported net trade schedule. See Malueg and Yates (2009) for a similar construction.

The initial allocation is of interest insofar as it determines the distribution of quota rent to the initial quota recipients.

Let v_i denote net quota demand for firm i. If v_i is positive (negative), i is a net quota buyer (seller). Because one cannot sell more quota than is held, trade is constrained by $w_i + v_i \geq 0$.

We use $\mathcal{I}_i$ to denote the information available to firm i when the decision to enter is made. In addition to own cost efficiency, θ_i, parameters $(p, Q, \lambda, \delta, w_i) \in \mathcal{I}_i$ under all information scenarios. To reduce notation the inclusion of these five parameters is henceforth suppressed. The scenarios we contrast differ in terms of what firm i knows about the cost efficiency of other firms in the population.

Cost efficiency is distributed uniformly within the firm population around a population mean value which we denote θ. We assume it is common knowledge that cost efficiency follows a uniform distribution with range 2ϵ. Under full information, $\mathcal{I}_i = \{\theta_i, \theta, \epsilon\}$, thus each firm knows its own efficiency,the population mean value and thus knows its efficiency *rank* within the firm population. Under our *incomplete information* scenario, $\mathcal{I}_i = \{\theta_i, \epsilon\}$. In this case, each firm knows its own efficiency and the distribution range but does not know the population mean, and thus by extension does not know its efficiency rank.

Our incomplete information scenario begins with Nature picking θ from an (improper) uniform distribution on $\mathbf{R}$. Firms do not observe Nature's pick. The exact distribution of θ plays no part in the firm's entry decision as we shall see (section 4). The assumption is merely a convenient way to model uncertainty from a firm's point of view. Once picked, θ is fixed and a parameter of the model, although unknown.

Together with the common knowledge assumption a firm's own observed θ_i provides a noisy signal about the true θ. We contrast two assumptions for a firm's beliefs. In the first case we assume that firm i believes that it attains average efficiency in a population. This assumption is usual in the global games literature (Morris and Shin, 2001). Thus conditional on θ_i, firm i believes that θ is uniformly distributed over $[\theta_i - \epsilon,\ \theta_i + \epsilon]$. This implies, in particular, $E[\theta] = \theta_i$.

In the second case we assume that firm i is overconfident and believes that it attains "better than average" cost efficiency. The specific formulation for this biased belief scenario is presented in section 6 but a main implication is that firms believe $E[\theta]$ is greater than the observed θ_i. We describe the two scenarios as one of placement uncertainty with or without *placement bias* on the part of a firm.

The central research question of the paper is, how does rank or placement uncertainty affect a firm's entry decision and the economic performance in the industry? Hereafter, we denote the set of firms that choose to enter the quota-regulated industry as $A \subseteq S$. Non-optimal performance manifests as excess entry and/or entry of a cost inefficient subset of S and thus excessive cost relative to the full information benchmark. Specifically, we compare and contrast the equilibrium mass of entrants under the two assumptions for a firm's belief about its own relative productivity with the full information benchmark. In particular, the paper does not address the question of which of the two beliefs can persist with learning or be part of an "equilibrium belief".

We next present a simple two entrant example to illustrate the determination of post entry quota prices and returns to capital. The insights we develop extend in a straight forward way to the continuum of firms assumed in our model.

2.1 A 2-firm example

Moving left to right on the horizontal axis in Figure 1 denotes the quota and production for firm 1; moving right to left denotes quota and production for firm 2. The length of the horizontal axis is Q. Units on the vertical axis are dollars. Maximum industry revenue is pQ.

Diagonal lines denote marginal costs with intercept θ_i for firm i at $q = 0$; the slope of the marginal cost curves is λ (measured relative to positive production). In the example in the figure, firm 1 is more efficient than firm 2, i.e., $\theta_1 < \theta_2$.

Suppose first that only firm 1 has entered. Profit maximizing production equates price p and marginal cost at b. Firm 1 output is less than Q and thus the aggregate quota constraint is slack. Firm 1's variable profit is the triangle $\theta_1 pb$. Notice that when industry output is less than Q, the equilibrium quota trading price, r^*, is equal to zero. Moreover when the quota is slack, industry profit flows entirely to the physical capital allocated to the industry. In this first example, entry by firm 1 is optimal if and only if area $\theta_1 pb \geq \delta$.

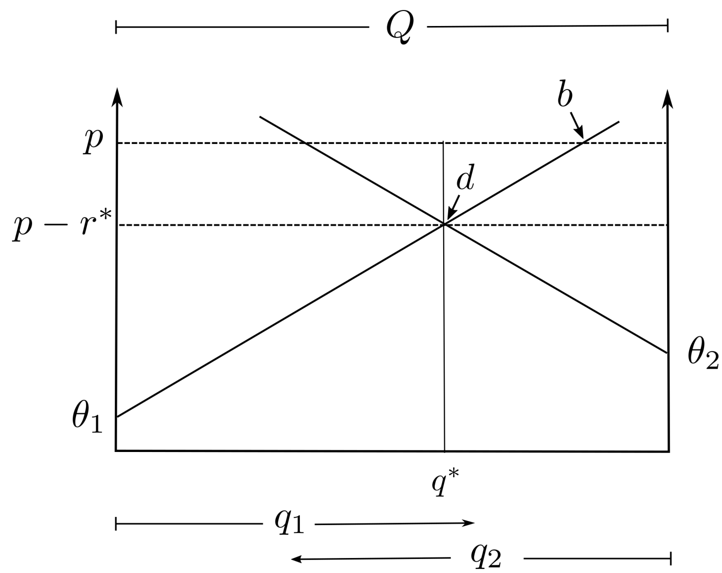

Figure 1: **Quota lease market: two-firm example.**

Next suppose both firm 1 and firm 2 have entered. Now the quota binds; if each firm were to produce the quantity that equates price and marginal cost, aggregate production exceeds Q. Thus, Q must be rationed. The efficient allocation, conditional on the set of entrants (both firms 1 and 2 in this case) occurs at q^* where the marginal profit, $p - r^*$, and marginal cost of producing a unit of the quota, $\theta_i + \lambda q_i$, is equated for each entrant.

Conditional on the set of entrants, the equilibrium quota price is r^* which is denoted by the vertical difference between the two dotted lines, p and $p - r^*$. Industry profit earned by the second fixed factor Q, is positive and equal to r^*Q. When the quota constraint binds, profits flow to two fixed factors, the physical capital and the quota. Firm 1's capital rent is given as triangle $\theta_1(p-r^*)d$; firm 2's capital rent is $\theta_2(p - r^*)d$. Both firms cover capital costs if and only if capital quasi rent evaluated at the net price, $p - r^*$, is greater than δ.

Several insights from the two-entrant example warrant attention.

First, if the quota constraint binds, an increase in p increases quota rent r^*Q, while leaving capital rent unchanged. Entry decisions depend on net price $p - r^*$ rather than on p itself. This is not the case when the quota constraint is slack.

Second, the number (or mass, in case of a continuum of firms) of entering firms depends on the production technology as reflected in the cost parameters $(\theta_i, \lambda, \delta)$, the aggregate quota Q, and when the quota is slack, on price p. As figure 1 shows, more firms *fit* into the industry when cost parameters are low and Q is high.

Third, the example in figure 1 shows that entry decisions are strategically independent variables when the quota constraint is slack. By contrast, when the quota constraint binds, capital allocations are strategic substitutes. Capital quasi rent depends on the number and average productivity of entering firms. Payoff interaction operates through the equilibrium quota price r^* which determines the share of variable profit allocated to the quota and to the vested capital.

2.2 Post entry quota market equilibrium

Our base model is a single period two-stage game. In the first stage firms observe their θ_i-s and choose whether to enter (commit their capital) or stay out. Firms also draw up their quota buy/sell plans. In the second stage, the quotas are traded and equilibrium price determined. Firms execute their production plans and payoffs are realized.

The subgame perfect Nash equilibrium (or perfect Bayesian equilibrium under the incomplete information scenario) is thus also a rational expectations equilibrium. Firms anticipate the stage two quota price and capital rent they will receive when making stage one entry decisions.

Firms with positive quota endowments $w_i > 0$ that choose not to enter will sell their quota at any positive price. Their profit maximizing trading rule is simple: $v_i = -w_i$ for $r > 0$.

Entrants choose v_i to maximize operating profits (revenue minus cost of production) plus net permit trading receipts. Conditional on the stage two quota price r and the information set, $\mathcal{I}_i$, the entrants' maximum total profit Π is given as,

$$\Pi = \max_{v_i} \Big\{ p(w_i + v_i) - \theta_i(w_i + v_i) - \frac{1}{2}\lambda(w_i + v_i)^2 - rv_i \Big\}, \tag{1}$$

subject to the constraint $w_i + v_i \geq 0$. The Lagrangian for this problem is,

$$L = p(w_i + v_i) - \theta_i(w_i + v_i) - \frac{1}{2}\lambda(w_i + v_i)^2 - rv_i - \mu(-w_i - v_i),$$

where μ is the Lagrangian multiplier. As v_i can be of any sign, the necessary conditions for an optimum are:

$$p - r - \theta_i - \lambda(w_i + v_i) + \mu = 0, \tag{2a}$$

$$w_i + v_i \geq 0, \;\; \mu[w_i + v_i] = 0, \tag{2b}$$

$$\mu \geq 0. \tag{2c}$$

From (2a) and (2b) we derive the net quota demand schedule for the entering firm i:

$$v_i = \begin{cases} \frac{1}{\lambda}[p - r - \theta_i] - w_i & \text{for } 0 \leq r \leq p - \theta_i \\ -w_i & \text{for } p - \theta_i < r. \end{cases} \tag{3}$$

The above emphasizes the possibility that (post-entry) capital rent can be zero for a high cost entrant, i.e., if for firm i, the marginal profit from the first unit produced, $p - \theta_i$, is less than the quota price r, production and post entry variable profit will be zero. The schedule in (3) shows further that if $r > p - \theta_i$ entering firm i will minimize its loss by selling its quota and leaving its capital idle. The quota allocation satisfies the condition whereby marginal profit $(p - r)$ and marginal cost $\lambda q_i + \theta_i = \lambda(v_i + w_i) + \theta_i$ are equalized across all entrants with positive quota/production.

Quota demand schedules are combined to determine the market clearing quota price. Carrying out this derivation obtains,

$$r^* = r^*(A) = \max\left\{p - \frac{\int_{i\in A} \theta_i d\theta_i}{A} - \frac{\lambda Q}{A}, 0\right\}, \tag{4}$$

where A is endogenously determined. The notation $r^*(A)$ emphasizes that the equilibrium permit price depends on the set of entrants. The expression $\frac{\int_{i\in A} \theta_i d\theta_i}{A} + \frac{\lambda Q}{A}$ denotes the mean marginal cost of entering firms and $\int_{i\in A} \theta_i d\theta_i / A$, the mean cost efficiency among entrants. We henceforth denote the mean cost efficiency among entering firms by $\bar{\theta}(A)$ as as opposed to θ which is the population mean. The first argument of the maximand in (4) is the mean marginal profit from producing a unit of quota.

It is clear that mean marginal costs increase, and thus r^* declines, with λ, Q, and the mean efficiency $\bar{\theta}(A)$. The effect of a change in A on r^* operates through two channels; mean efficiency and the share of total Q produced by each entering firm. This in turn implies that both the size of A and the composition of cost efficiency of entering firms matter. Note in particular that the equilibrium quota price is independent of the quota endowments w_i's.

Combining (3) and (4) determines optimal production quantity for entering firm i:

$$q_i = w_i + v_i = \begin{cases} \frac{1}{\lambda}\left(\bar{\theta}(A) - \theta_i + \lambda Q/A\right) & \text{for } r^* > 0 \\ \frac{1}{\lambda}\left(p - \theta_i\right) & \text{for } r^* = 0 \end{cases}$$

The quantity produced by firm i is increasing in its relative cost efficiency, $\overline{\theta}(A) - \theta_i$, and in the total quota relative to the size of the set of entering firms, Q/A. In particular, q_i is independent of w_i.

Substituting (3) into (1), the entrant's total profit can be expressed as,

$$\Pi = \begin{cases} \underbrace{\frac{1}{2\lambda}\left(p - r^* - \theta_i\right)^2}_{\pi(r^*, \mathcal{I}_i)} + r^* w_i & \text{for } 0 \leq r^* \leq p - \theta_i \\ r^* w_i & \text{for } p - \theta_i < r^*. \end{cases} \tag{5}$$

Equation (5) says that an entrant's total profit can be notionally decomposed into two parts. The first part denoted by $\pi(r^*, \mathcal{I}_i) = \frac{1}{2\lambda}\left(p - r^* - \theta_i\right)^2$ may be interpreted as the quasi-rent on capital and is zero if the equilibrium permit price is too high and the entrant chooses to idle its capital. The second part is the value of the quota endowment or quota rent for short which a firm can earn whether or not it chooses to enter.

Note that the capital rent $\pi(r^*, \mathcal{I}_i)$ is function of the parameters making up the firm's information set and equilibrium permit price which in turn depends on the endogenous set of entrants. To keep notations simple however, we henceforth suppress these dependencies and merely point out that they exist.

Equation (5) further shows that π depends on the difference $p - r^*$ rather than on p itself. Substituting the equilibrium quota price from (4) (for the case where $r^* > 0$) obtains the following expression for the sum of the capital and quota rents.

$$\Pi = \frac{1}{2\lambda}\left(\bar{\theta}(A) - \theta_i + \lambda Q/A\right)^2 + \underbrace{\left(p - \bar{\theta}(A) - \lambda Q/A\right)}_{r^*(A)} w_i. \tag{6}$$

Equation (6) shows that the return to firm i's capital increases with its relative cost efficiency, $\bar{\theta}(A) - \theta_i$, and with own production, as reflected in the term $\lambda Q/A$. Firm i's capital rent is highest when A is made up of a small number of higher cost rivals.

2.3 Enter or stay out

We can write the entry strategy for firm i as:

$$\left\{ \begin{array}{c} \text{Enter} \\ \text{Stay out} \end{array} \right\} \text{ as } \pi + r^* w_i \left\{ \begin{array}{c} \geq \\ < \end{array} \right\} \delta + r^* w_i. \tag{7}$$

The above expression shows that firm i earns quota rent $r^* w_i$ regardless of whether it enters or stays out. The entry strategy therefore reduces to a firm entering if its capital rent π is at least as high as the capital cost δ. The implication is that the decision to allocate physical capital to the industry can be made independently of the initial quota endowments.[6]

The next sections present our main results. Entry and efficiency will vary across the full parameter space of the model, i.e., with information, cost structural parameters $(\theta_i, \lambda, \delta)$, and with the demand and regulatory parameters, p and Q, respectively. Space limitations do not allow an exhaustive characterization of all results for all regions of parameter space. We focus instead on parameters recognized as significant determinants of industry structure and performance in CAT-regulated industries. In particular, costly investment reversibility due to specificity of capital is emphasized in Clark et al. (1979); Singh and Weninger (2017); Vestergaard et al. (2005). Correspondingly, we differentiate cases where capital costs are low, moderate, and high.

[6] Montgomery (1972), in a model with cost heterogeneous firms but without capital, shows that when quota trade is frictionless, market efficiency is independent of an initial quota allocation. Although not the main focus of our paper this result, that with frictionless quota trade initial quota allocations do not impact physical capital investment, has not appeared in earlier literature.

A second parameter of interest is Q; due to its role in the determination of the equilibrium quota price r^* and thus strategic entry of capital. Characterizing the effects of Q on industry structure and performance further lays the groundwork for future work to address the question of the optimal choice of Q under placement uncertainty.

3 Entry under full information

This section assumes that firms are fully informed about the mean cost efficiency of the population; that is, $\mathcal{I}_i = (\theta_i, \theta, \epsilon)$ for all $i \in S$. A natural entry rule is a threshold or "switching" strategy where firm i enters if and only if its cost efficiency is less than or equal to a threshold, which we denote θ^*. We are particularly interested in a symmetric equilibrium in pure strategies in which all firms enter (with probability one) if $\theta_i \leq \theta^*$ and stay out otherwise.

Since θ_i is uniformly distributed over $[\theta - \epsilon, \theta + \epsilon]$, under an equilibrium threshold θ^* (if it exists) and for a given θ, the proportion of entering firms, which we denote $\alpha(\theta, \theta^*)$, as follows,

$$\alpha(\theta, \theta^*) = \begin{cases} 0, & \text{if } \theta^* < \theta - \epsilon, \\ \frac{1}{2\epsilon} \int_{\theta-\epsilon}^{\theta^*} d\theta_i = \frac{\theta^* - (\theta - \epsilon)}{2\epsilon}, & \text{if } \theta - \epsilon \leq \theta^* \leq \theta + \epsilon, \\ 1, & \text{if } \theta + \epsilon < \theta^*. \end{cases} \tag{8}$$

If the threshold θ^* is particularly low, below the population lower bound $\theta - \epsilon$, no firms enter; $\alpha(\theta, \theta^*) = 0$ and $A = \emptyset$. If the threshold is particularly high, all firms enter. For intermediate values of θ^*, $A \subset S$.

The corresponding mean cost efficiency of entering firms is,

$$\bar{\theta} = \begin{cases} \text{undefined}, & \text{if } \theta^* < \theta - \epsilon, \\ \frac{\frac{1}{2\epsilon} \int_{\theta-\epsilon}^{\theta^*} \theta_i d\theta_i}{\alpha(\theta, \theta^*)} = \frac{\theta^* + \theta - \epsilon}{2}, & \text{if } \theta - \epsilon \leq \theta^* \leq \theta + \epsilon, \\ \theta, & \text{if } \theta + \epsilon < \theta^*. \end{cases} \tag{9}$$

Then, (4), (8) and (9) can be used to express the equilibrium quota price as a function of the exogenous θ and the endogenous θ^* as follows:

$$r^* = \begin{cases} \text{undefined}, & \text{for } \theta^* < \theta - \epsilon, \\ \max\{p - \frac{\theta^*+\theta-\epsilon}{2} - \frac{2\epsilon\lambda Q}{\theta^*-(\theta-\epsilon)}, 0\}, & \text{for } \theta - \epsilon \leq \theta^* \leq \theta + \epsilon, \\ \max\{p - \theta - \lambda Q, 0\}, & \text{for } \theta + \epsilon < \theta^*. \end{cases} \tag{10}$$

Proposition 1 shows the existence of a Nash equilibrium in pure strategies under full information.

Proposition 1. *For $\mathcal{I}_i = (\theta_i, \theta, \epsilon)$, a threshold θ^{*F} exists such that firm i enters if $\theta_i \leq \theta^{*F}$ and stays out, otherwise. Depending on the parametric configuration, θ^{*F} takes one of the two forms below.*

Case I. When $\delta < \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$, $\theta^{*F} = p - \sqrt{2\lambda\delta}$ and is independent of θ. The equilibrium permit price is 0 for all values of θ.

Case II. When $\delta \geq \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$,

$$\theta^{*F} = \begin{cases} (\theta - \epsilon) + \sqrt{2\lambda\delta + 4\epsilon\lambda Q} - \sqrt{2\lambda\delta} & \textit{for } \theta \leq \hat{\theta} \\ p - \sqrt{2\lambda\delta} & \textit{for } \theta > \hat{\theta} \end{cases}$$

where

$$\hat{\theta} = p + \epsilon - \sqrt{2\lambda\delta + 4\epsilon\lambda Q}.$$

Specifically, θ^{*F} is increasing for $\theta \leq \hat{\theta}$ and constant for $\theta > \hat{\theta}$. Substituting θ^{*F} into (10) obtains the equilibrium quota price under full information.

$$r^F = \begin{cases} p + \epsilon - \sqrt{2\lambda\delta + 4\epsilon\lambda Q} - \theta & \textit{for } \theta < \hat{\theta} \\ 0, & \textit{for } \theta > \hat{\theta} \end{cases}$$

The equilibrium quota price is positive and decreasing for $\theta \leq \hat{\theta}$ and zero for $\theta > \hat{\theta}$.

PROOF: SEE APPENDIX 8.1.

REMARK 1. Proposition 1 separates the multi-dimensional parameter space into regions in which the quota binds and the equilibrium quota price is positive and regions in which the quota price is zero. The cutoff value of $\frac{1}{2\lambda}(\lambda Q - \epsilon)^2$ is interpreted as follows. Consider an equilibrium in switching strategies under which all firms can potentially enter. The highest cost firm in the population attains efficiency, $\theta + \epsilon$. If this least efficient firm enters, all firms in the population enter and $A = S$. Using the relevant form for the quota price, the return to capital for the highest cost firm is $\frac{1}{2\lambda}\Big(\theta - (\theta+\epsilon) + \lambda Q\Big)^2 = \frac{1}{2\lambda}\Big(\lambda Q - \epsilon\Big)^2$. Thus, $\frac{1}{2\lambda}(\lambda Q - \epsilon)^2$, is a *minimum return* to vested capital that any firm can earn by entering.

Case I says that if capital costs are very low and fall below the cutoff, $\frac{1}{2\lambda}(\lambda Q - \epsilon)^2$, there is potentially enough *room* for all firms to enter. The equilibrium quota price is zero even for low values of θ and all firms including the highest cost firm enter. For higher values of θ, some high cost firms do not enter and quotas are in excess supply.

Case II shows a second circumstance with capital costs above the cutoff $\frac{1}{2\lambda}(\lambda Q - \epsilon)^2$, in which the quota market is slack. If θ is particularly high, that is above $\hat{\theta}$, there are too few sufficiently low cost firms in the population to profitably produce Q. Hence there is excess supply of quota.

The next section shows that similar circumstances play out when firms operate under incomplete information about rival costs.

REMARK 2. When the equilibrium quota price is zero, capital rent for firm i depends on its absolute efficiency, θ_i, but not on its relative efficiency. Under these circumstances, entry decisions are *nonstrategic* and depend only on individual firm costs. Proposition I shows that with full information, entry decisions are nonstrategic under two kinds of scenario: a) capital costs of firms are below a cutoff (irrespective of their variable costs) and b) capital costs of firms are above the cutoff but the mean cost is higher than $\hat{\theta}$, so that a sufficiently large number of firms decide not to enter.

REMARK 3. The full information model is a form of Cournot competition with strategic substitutes, in regions where the quota binds. The payoff interaction operates through the equilibrium net price $p - r^F$, rather than an inelastic consumer demand for industry output. As with other types of non-cooperative equilibrium, the full information outcome is not the first best from the point of view of joint profit maximization (Firms fail to internalize the effect of their actions on the payoff function (e.g. Mankiw and Whinston (1986))). Moreover, as we are working with an exogenous Q, the full information outcome may not be in general socially optimal.

REMARK 4. The equilibrium threshold θ^{*F} and quota price r^F are functions of the various parameters. In the region in which entry decisions are strategic, these two are very importantly functions of θ. The quota price r^F is linear and deceasing in θ whereas the entry threshold, θ^{*F} is linear and increasing in θ. Although the entry threshold increases with the mean cost efficiency, some very high cost firms always decide to stay out and sell all of their quota endowments. In other words, the set of entrants A is always a proper subset of S.

On a related but general note, the equilibrium values for thresholds and other variables of interest that we derive in the paper, vary over the full parameter space of the model. To keep discussions short our analysis focuses on specific parameters of interest and accordingly refer to the equilibrium values as functions of these selected parameter(s).

4 Entry under placement uncertainty without placement bias

This section assumes that for all $i \in S$, $\mathcal{I}_i = \{\theta_i, \epsilon\}$. Firm i believes it attains the average efficiency in the firm population. Specifically, conditional on θ_i, firm i believes that the unknown θ is uniformly distributed over $[\theta_i - \epsilon,\ \theta_i + \epsilon]$ and $E[\theta \mid \theta_i] = \theta_i$.

Firms must now consider the *expected* competition, quota price, and capital rent that will be realized post entry. We assume firms to be expected profit maximizers. Thus, conditional on θ_i, firm i enters if

$$E[\pi] = \frac{1}{2\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon} \frac{1}{2\lambda}\left(p - r(\theta,\theta^*) - \theta_i\right)^2 d\theta \geq \delta,$$

and stays out otherwise.

Conditional expected capital rent for firm i takes one of the following forms depending on the mass of entrants and permit price function, $r(\theta, \theta^*)$:

$$\begin{array}{lll} (a) & A = S,\ r(\theta,\theta^*) > 0: & \pi = \frac{1}{2\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon} \frac{1}{2\lambda}\left(\theta + \lambda Q - \theta_i\right)^2 d\theta \\ (b) & A \subset S,\ r(\theta,\theta^*) > 0: & \pi = \frac{1}{2\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon} \frac{1}{2\lambda}\left(\frac{\theta^*+\theta-\epsilon}{2} + \frac{2\epsilon\lambda Q}{\theta^*-(\theta-\epsilon)} - \theta_i\right)^2 d\theta \\ (c) & A \subset S,\ r(\theta,\theta^*) = 0: & \pi = \frac{1}{2\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon} \frac{1}{2\lambda}\left(p - \theta_i\right)^2 d\theta. \end{array} \quad (11)$$

4.1 Equilibrium characterization

We study the existence of a symmetric perfect Bayesian Nash equilibrium in which all firms adopt an identical threshold strategy with common threshold value denoted by θ^{*I}. A firm with efficiency $\theta_i = \theta^{*I}$ is indifferent between entering and staying out. Thus θ^{*I} is the solution to,

$$\frac{1}{2\epsilon}\int_{\theta^{*I}-\epsilon}^{\theta^{*I}+\epsilon}\frac{1}{2\lambda}\left(p - r(\theta,\theta^{*I}) - \theta^{*I}\right)^2 d\theta = \delta$$

We describe the firm with $\theta_i = \theta^{*I}$ as the pivotal firm.

An equilibrium is shown to exist if the noise parameter ϵ is below a certain level. This is specified as Assumption 1. Thus, an equilibrium may not exist for a noise of any size.[7]

Assumption 1: $\epsilon < \lambda Q$.

4.2 Equilibrium with low capital cost

When capital costs are in the region $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2\right]$, an equilibrium is shown to exist in which the pivotal firm (correctly) expects the post-entry permit price to be zero. The expected payoff of the pivotal firm and the equilibrium threshold is given by the solution to,

$$\pi(\theta^{*I}) = \frac{1}{2\epsilon}\int_{\theta^{*I}-\epsilon}^{\theta^{*I}+\epsilon}\frac{1}{2\lambda}\left(p - \theta^{*I}\right)^2 d\theta = \delta, \tag{12}$$

with equilibrium threshold $\theta^{*I} = p - \sqrt{2\lambda\delta}$.

Proposition 2. *For $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2\right]$, a unique pure strategy perfect Bayesian Nash equilibrium exists under which a firm i enters if $\theta_i \leq \theta^{*I}$ and stays out otherwise.*

PROOF: SEE APPENDIX 8.2.

Proposition 1 Case 1 and Proposition 2 show that entry thresholds are identical under full and incomplete information when the cost of capital is below the cutoff $\frac{1}{2\lambda}(\lambda Q - \epsilon)^2$. In this case placement uncertainty has no effect on entry or on production efficiency.

[7] It is possible for $E[\pi(\theta_i)] \geq \delta$ and yet $p - r(\theta, \theta^*) < \theta_i$ for firm i, for some value of θ, in which case it is optimal for the firm to idle its vested capital. Although this does not nullify any of our conclusions, we can show that such ex-post idleness is ruled out if $\epsilon < \lambda Q$.

4.3 Equilibrium with high capital cost

When $\delta > \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$, there exists an equilibrium in which the pivotal firm expects the permit price to be positive for some values of θ. The permit price function faced by the pivotal firm has the following features.

First, for $\theta \in [\theta^{*I} - \epsilon, \theta^{*I} + \epsilon]$, the pivotal firm expects the permit price function to be either zero or have the form specified in the second expression in (10). Second, the pivotal firm expects the permit price to attain a value of zero for some $\theta < \theta^{*I} + \epsilon$.

Under incomplete information, the level $\hat{\theta}$ at which the permit price hits zero turns out to be a function of the endogenous θ^{*I}. Specifically, $\hat{\theta}(\theta^{*I}) = (p + \epsilon) - \sqrt{(p - \theta^{*I})^2 + 4\epsilon\lambda Q}$. Thus $\hat{\theta}(\theta^{*I})$ lacks a closed form unlike its full information counterpart $\hat{\theta}$.[8]

Combining all observations above, the payoff function of the pivotal firm and consequently the equilibrium threshold, θ^{*I}, is given by the condition,

$$\frac{1}{2\epsilon}\int_{\theta^{*I}-\epsilon}^{\hat{\theta}(\theta^{*I})} \frac{1}{2\lambda}\Big(\frac{\theta^{*I} + \theta - \epsilon}{2} + \frac{2\epsilon\lambda Q}{\theta^{*I} - (\theta - \epsilon)} - \theta^{*I}\Big)^2 d\theta + \frac{1}{2\epsilon}\int_{\hat{\theta}(\theta^{*I})}^{\theta^{*I}+\epsilon} \frac{1}{2\lambda}(p - \theta^{*I})^2 d\theta = \delta \quad (13)$$

Proposition 3. *For $\delta \in \left(\frac{1}{2\lambda}(\lambda Q - \epsilon)^2, \frac{(\lambda Q)^2}{2\lambda} + \frac{\epsilon^2}{6\lambda}\right]$, a symmetric Perfect Bayesian Nash equilibrium in switching strategies exists in which the threshold θ^{*I} satisfies (13).*

PROOF: SEE APPENDIX 8.2.

REMARK 5. The equilibrium thresholds under full and incomplete information differ when the cost of capital is above the critical level, $\frac{1}{2\lambda}(\lambda Q - \epsilon)^2$. Condition (13) reveals that in this case, θ^{*I} does not have a closed form but is independent of θ as required (firms must make entry decisions without knowing θ). More importantly, the condition reveals that the exact *prior* distribution from which Nature picks a specific value for θ plays no role in the determination of θ^{*I}. The critical assumption is that the observed θ_i is uniform around the unobserved θ and that this is common knowledge.

REMARK 6. Although θ^{*I} is independent of θ, the mass of entrants under both the full and incomplete information scenarios, depends not only on the respective thresholds but also on the

[8]Although not critical for our results, it can be shown that under Assumption 1, $\hat{\theta}(\theta^{*I}) < \theta^{*I}$.

realized (but unobserved under the second scenario) value of θ as the next section 5 shows.

REMARK 7. Although θ^{*I} does not have a closed form when the capital cost is above the cutoff, we can show that it has an upper bound, namely, $\theta^{*I} < p - \sqrt{2\lambda\delta}$ (See appendix 8.2). This is essentially because the expected profit of the pivotal firm is lower with a positive permit price compared to a zero permit price under any given value of the entry threshold. This observation allows some interesting comparison of the full and incomplete information scenarios later.

5 Entry and performance: The role of placement uncertainty

In this section we isolate the effects of placement uncertainty by contrasting entry thresholds and the mass of entrants under our two information scenarios.

5.1 Entry thresholds under full and incomplete information

We focus on the case $\delta > \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$ for now, as the entry thresholds differ only under this situation.

The full information entry threshold θ^{*F} is an increasing and piece-wise linear function of θ whereas the threshold under incomplete information, θ^{*I}, is independent of θ.

We begin with a key observation that for low values of θ, the full information threshold is less the incomplete information threshold, and that this inequality is reversed at high values of θ. The relationship between θ^{*I} and θ^{*F} is captured in figure 2. θ^{*I} is represented as a horizontal line. θ^{*F} is linear and increasing up to $\theta = \hat{\theta}$ and is constant and equal to $p - \sqrt{2\lambda\delta}$ for $\theta > \hat{\theta}$.

It is straightforward to check that $\delta > \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$ implies $\sqrt{2\lambda\delta + 4\epsilon\lambda Q} - \sqrt{2\lambda\delta} < 2\epsilon$. The full information threshold evaluated at $\theta = \theta^{*I} - \epsilon$ then turns out to be,

$$\theta^{*F}(\theta^{*I} - \epsilon) = \theta^{*I} - 2\epsilon + \sqrt{2\lambda\delta + 4\epsilon\lambda Q} - \sqrt{2\lambda\delta} < \theta^{*I},$$

as shown in figure 2. Since θ^{*F} is increasing in θ and as this inequality is strict, it is also strictly satisfied for some values of $\theta > \theta^{*I} - \epsilon$. In other words, $\theta^{*F} < \theta^{*I}$ for some low values of θ.

As θ^{*F} increases with θ and attains a maximum value of $p - \sqrt{2\lambda\delta}$ at $\theta = \hat{\theta}$ and as $\theta^{*I} <$

$p - \sqrt{2\lambda\delta}$ (Remark 7), it follows that $\theta^{*F} > \theta^{*I}$ for some values of θ, sufficiently high as indicated in the figure.

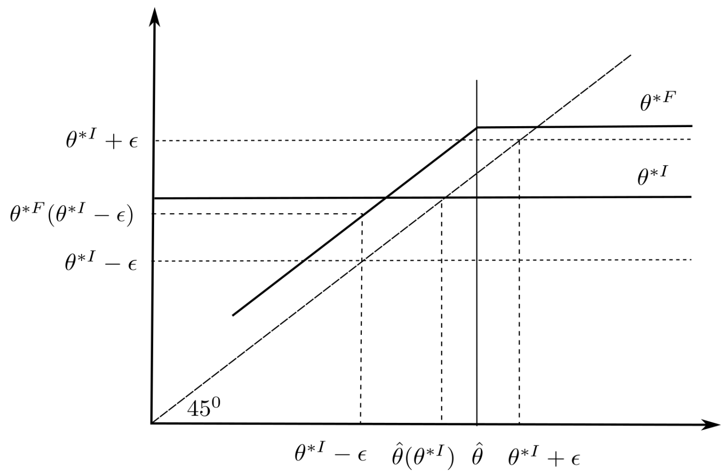

Figure 2: **Entry thresholds with and without placement uncertainty.**

As mentioned earlier, the mass of entrants under either scenario is a function of both its specific threshold and the actual θ. Their specific forms and relative sizes are discussed in the next subsection. It is important to note however that the relationship between θ^{*I} and θ^{*F} for a given θ determines the difference between these masses. Specifically, the fact that θ^{*I} exceeds θ^{*F} for low values of θ, explains why the mass of entrants under incomplete information exceeds that under full information for these values (Figure 4).

As the relationship between θ^{*F} and θ^{*I} for a given θ is key to the main results that follow, it is important to understand its underlying intuition.

5.1.1 Intuition

With full information firms can asses post-entry outcomes and determine, exactly, the capital rent earned with entry. Under incomplete information, post entry outcomes are uncertain and expectations are based on individual firm beliefs about their efficiency rank in the firm population. Figure 3 contrasts pre-entry determination of capital quasi rents without and with placement uncertainty. Figure 3a) shows an example where the mean efficiency θ is low and figure 3b the case where it is

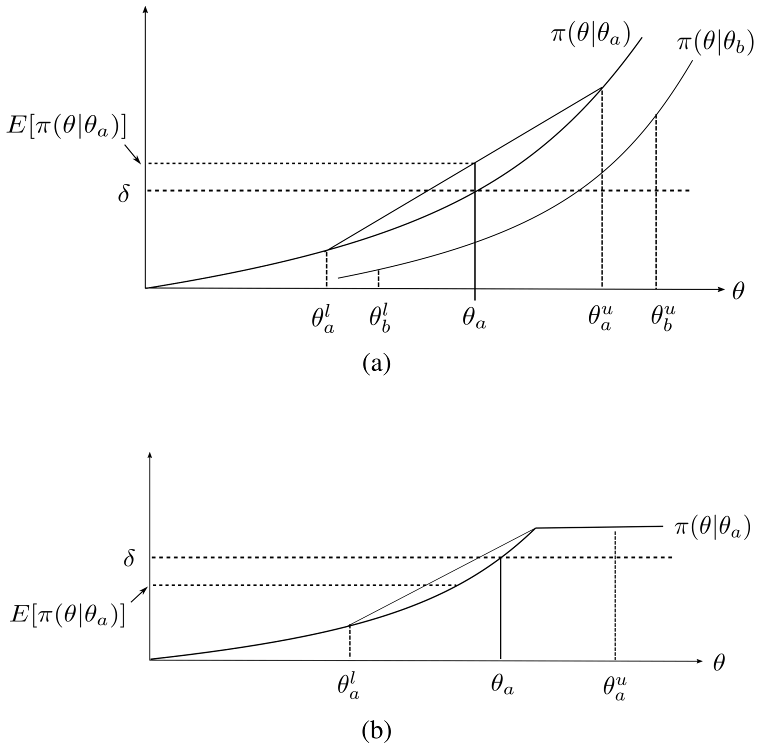

Figure 3: **Placement Uncertainty and capital rent.** Panel (a) shows quasi-convex capital rent for the case where θ is low. In panel (b) θ is high capital rent is bounded above at a quote price of zero.

high. The key difference between these cases is the form that the capital quasi-rent function takes with regard to θ, and firm expectations of post entry outcomes.

Figure 3a shows capital quasi rent, or variable operating profits, as a function of θ. This is an increasing and strictly convex function for low values of θ. Two variable profit functions are shown conditional on two levels of own cost efficiency, θ_a and θ_b.

The example in figure 3a is drawn such that under full information, capital quasi-rent given efficiency θ_a is just equal to capital cost δ. Thus, $\theta_a = \theta^{*F}$. Under uncertainty a firm with cost efficiency θ_a believes that mean efficiency θ is uniformly distributed on the interval shown as $[\theta_a^l, \theta_a^u]$ in the figure. It is easy to see, that by Jensen's inequality, $E[\pi(\theta|\theta_a)] > \delta$. That is, under placement uncertainty, the cutoff threshold (whatever its value is) is greater than $\theta^{*F} = \theta^a$.

For comparison, a second strictly convex capital quasi-rent function $\pi(\theta|\theta_b)$ for a firm that is less efficient - that is $\theta_b > \theta_a$ - is shown in the figure. A firm attaining efficiency θ_b believes that the actual θ is uniformly distributed over an interval $[\theta_b^l, \theta_b^u]$. The curve is drawn such that

$E[\pi(\theta|\theta_b)] \approx \delta$. From the examples in the figure, it is clear that when the capital quasi-rent is strictly convex in θ which happens for low values of θ, the equilibrium threshold under placement uncertainty exceeds its full information counterpart.

Figure 3b illustrates the case where the mean efficiency θ is high. Now the capital quasi rent function $\pi(\theta|\theta_a)$ is truncated from above at the capital rental value that corresponds to a zero quota price (a slack aggregate quota). As above, under full information, a firm that attains cost efficiency θ_a earns capital quasi rent that just offsets the cost of capital δ. That is $\theta^a = \theta^{*F}$. Under placement uncertainty this same firm believes that θ is uniformly distributed on the interval $[\theta_a^l, \theta_a^u]$. Now however, due to the truncation of capital rent the expected rent $E[\pi(\theta|\theta_a)]$ can fall below δ. In other words, under placement uncertainty, a firm will believe that post entry conditions will be such that it will be unable to earn sufficient variable profits to cover its capital costs and choose to stay out. In other words, the cutoff threshold under placement uncertainty is below $\theta^{*F} = \theta^a$.

The next section contrasts entry with and without placement uncertainty at the industry level. The results will show that relative to full information, placement uncertainty can encourage excess entry or may have a mitigating effect depending on the value of θ relative to other model parameters.

5.2 Mass of entrants under full and incomplete information

Expression (8) is used to calculate the mass of entrants under full and incomplete information scenarios. These masses depend on the actual realization of θ (whether observed or unobserved by a firm) as well as on the entry thresholds under both scenarios.

We begin by noting that as the entry thresholds are identical under both scenarios for the parametric configuration $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2\right]$, the mass of entrants are also identical for any θ.

When $\delta > \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$, following Proposition 1, the mass of entrants under full information has the form,

$$\alpha^F = \begin{cases} \frac{\sqrt{2\delta\lambda + 4\epsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\epsilon} < 1 & \text{for } \theta \leq \hat{\theta} \\ \frac{(p - \sqrt{2\delta\lambda}) - (\theta + \epsilon)}{2\epsilon} & \text{for } \theta \in \left(\hat{\theta}, p + \epsilon - \sqrt{2\delta\lambda}\right) \\ 0 & \text{for } \theta \geq p - \sqrt{2\delta\lambda} + \epsilon. \end{cases} \tag{14}$$

where, as was shown previously, $\hat{\theta} = p + \epsilon - \sqrt{2\lambda\delta + 4\epsilon\lambda Q}$.

The mass of entrants in the incomplete information case is given by,

$$\alpha^I = \begin{cases} 1 & \text{for } \theta \leq \theta^{*I} - \epsilon \\ \frac{\theta^{*I} - (\theta - \epsilon)}{2\epsilon} & \text{for } \theta \in \left[\theta^{*I} - \epsilon, \theta^{*I} + \epsilon\right] \\ 0 & \text{for } \theta > \theta^{*I} + \epsilon. \end{cases} \tag{15}$$

Although θ^{*I} lacks a closed form, the difference $(\alpha^I - \alpha^F)$, which defines excess entry (EE for short) under placement uncertainty is a piece-wise linear function of θ. The full form of this function is laid out in Appendix 8.3.

Figure 4 plots EE for positive values of θ and holding other model parameters fixed to show the existence of five distinct regions, which we have labeled A through E.[9]

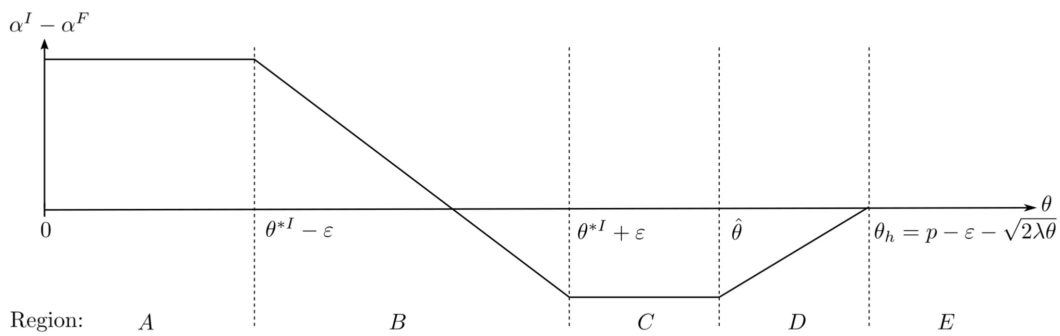

Figure 4: Entry Under Full and Incomplete Information, $\alpha^I - \alpha^F$

As shown in the figure EE is positive for particularly low levels of θ. In region A, where $\theta < \theta^{*I} - \epsilon$, a low and increasing θ^{*F} translates into a constant $\alpha^F < 1$. Whereas, a relatively higher but constant level for θ^{*I} translates into a $\alpha^I = 1$. This accounts for the constant and positive value for EE in region A.

α^I is declining in θ for $\theta > \theta^{*I} - \epsilon$, whereas α^F remains constant for $\theta < \hat{\theta}$ (equation(14)). Hence EE falls and becomes negative at $\tilde{\theta}$ (region B).

The entrant mass α^F decreases until $\theta = p + \epsilon - \sqrt{2\delta\lambda}$; EE is thus constant although negative

[9] Figure 4 is premised on the assumption that $\hat{\theta} \leq \theta^{*I} + \epsilon$ which is irrelevant for results. The alternative case is discussed in Appendix 8.3.)

in region C.

As α^I is zero in region D, whereas α^F continues to decline with θ, the difference in the mass of entrant increases but remains negative in region D.

Finally, for particularly high values of θ in region E, the mass of entrants with and without placement uncertainty is zero and hence their difference is zero.

The next subsection presents our main results on excessive entry on the assumption that a firm uncertain about its own rank believes itself *average* in a population.

5.3 Competitive blind spots

Under parametric configurations satisfying $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2\right]$ the mass of entrants are identical under full and incomplete information. When $\delta > \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$ the excess entry function EE takes positive or negative values depending on the realization of θ. The next Proposition and our first main result lays out the implications of these observations.

Proposition 4. *I. If $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2\right]$, there is no excess entry of firms due to placement uncertainty.*

II. When $\delta > \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$, placement uncertainty: (a) causes excess entry over and above the full information benchmark, for $\theta \leq \theta^{*I} - \epsilon$ where the endogenous θ^{*I} depends on the model parameters but has a least upper bound of $p - \lambda Q$ (b) mitigates entry that otherwise happens under full information for $\theta \geq \hat{\theta}$. The effect of placement uncertainty on entry can work in either direction for $\theta \in (\theta^{*I} - \epsilon, \hat{\theta})$.

PROOF: Straightforward, follows from the expression of $\alpha^I - \alpha^F$.

Part II of Proposition 4 says that placement uncertainty causes excess entry for low values of θ. This is because the full information threshold is less than the incomplete information threshold and at the low values of θ specified in Part IIa) all firms in the population enter under incomplete information ($\alpha^I = 1$) whereas only a subset of the firms enter under full information ($\alpha^F < 1$). This is consistent with results reported in the experimental literature (Camerer ...) which seem to suggest that participants under-estimate the extent of the competition they may face if a task is easy (costs are low). Part IIb) also provides condition under which agents may *over-estimate* the amount of rival competition and stay out of the market because of placement uncertainty whereas they would have entered if they had full information. In other words, placement uncertainty may also mitigate competition, a possibility not pointed out before, to the best of our knowledge.

5.4 Comparative statics

Among the parameters that determine the value of the function $\alpha^I - \alpha^F$ and the length of the regions $A - D$ in Figure 4, of special interest are the capital cost δ and the size of the quota Q.

It is straightforward to check that as $\frac{\partial \theta^{*I}}{\partial \delta} < 0$ the length of the region A diminishes as δ increases. The height of $\alpha^I - \alpha^F$ however increases over this region due to a constant α^I and a lower α^F. Thus an increase in capital costs increases excess entry due to placement uncertainty but over a smaller range of θ values.

As the length of the region C equals $\sqrt{2\delta\lambda + 4\epsilon\lambda Q} - \sqrt{2\delta\lambda}$, a rise in δ diminishes this length. The sign of $\frac{\partial(\alpha^I - \alpha^F)}{\partial \delta}$ is unclear over this region although the value of $\alpha^I - \alpha^F$ stays negative. This implies that the mitigating effect of placement uncertainty prevails over a narrower range of θ values.

A rise in the size of the quota Q increases the length of the region A as $\frac{\partial \theta^{*I}}{\partial Q} > 0$. As $\alpha^I = 1$ and α^F diminishes because of a rise in Q over the region A, the extent of excess entry due to placement uncertainty is lower but the phenomenon is prevalent over a wider range of θ values.

Similarly it is easy to check that a rise in Q increases $\sqrt{2\delta\lambda + 4\epsilon\lambda Q} - \sqrt{2\delta\lambda}$, the length of the region C. Moreover, $\frac{\partial(\alpha^I - \alpha^F)}{\partial Q} > 0$ in the region C although the value stays negative. Thus because of a rise in Q, the mitigating effect of placement uncertainty is stronger and over a wider range.

6 Entry under placement uncertainty with placement bias

This section considers the effect of overconfidence on a firm's entry decision, market structure, and industry performance. We assume that a firm believes that it attains a better than average cost efficiency. Thus, when entry decisions are made, a firm with cost parameter θ_i believes that the unobserved θ follows the distribution, $\theta \sim U[\theta_i + \beta - \epsilon, \theta_i + \beta + \epsilon]$ where $\beta > 0$ represents the overconfidence or over-placement bias. This implies that from the firm's point of view, $E[\theta] = \theta_i + \beta > \theta_i$. The model allows for firms to have different degrees of bias hence different β's. As a matter of convenience however, we assume that β is identical across firms. We also assume $\beta \leq \epsilon$ for consistency with the assumption that $\theta_i \in [\theta - epsilon, \theta + \epsilon]$.[10]

We first show that a rational expectations (perfect Bayesian) equilibrium in switching strategies exists. Denote by $\theta^{*I,\beta}$ the common entry threshold. Then, equilibrium $\theta^{*I,\beta}$ is characterized by,

$$\frac{1}{2\epsilon}\int_{\theta^{*I,\beta}+\beta-\epsilon}^{\theta^{*I,\beta}+\beta+\epsilon} \pi(\theta, \theta^{*I,\beta})d\theta = \delta.$$

Note that the form of the key expressions (8), (9) and (10) do not change with the introduction of the bias factor. The equilibrium threshold is a function of all the model parameters including β.

6.1 Equilibrium with placement bias

Proposition 5. *1. Suppose $\delta \leq \frac{1}{2\lambda}\left(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2}\right)^2$. A unique pure strategy Bayesian Nash equilibrium in switching strategies exist. The equilibrium $\theta^{*I,\beta}$, such that a firm chooses to enter if $\theta_i \leq \theta^{*I,\beta}$ and to stay out otherwise, is the unique solution to the following equation:*

$$\frac{1}{4\lambda\epsilon}\int_{\theta^{*I,\beta}+\beta-\epsilon}^{\theta^{*I,\beta}+\beta+\epsilon} (p - \theta^{*I,\beta})^2 = \delta \tag{16}$$

Equilibrium $\theta^{*I,\beta} = p - \sqrt{2\lambda\delta}$ and the quota price is zero for all θ.

2. Suppose $\delta \in (\frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2})^2, \frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\epsilon^2}{6\lambda}]$. A unique pure strategy Bayesian Nash equilibrium in switching strategies exist. The equilibrium threshold $\theta^{*I,\beta}$ such that a firm chooses to enter if $\theta_i \leq \theta^{*I,\beta}$ and to stay out otherwise, is the unique solution to the following equation:

[10] When $\beta = \epsilon$, the distribution $\theta \sim U[\theta_i + \beta - \epsilon, \theta_i + \beta + \epsilon]$ is equivalent to $\theta \sim U[\theta_i, \theta_i + 2\epsilon]$, implying $E[\theta] = \theta_i + \epsilon$ or $\theta_i = E[\theta - \epsilon]$. In other words, firm i believes it has the lowest cost parameter in a population that is uniformly distributed around an unknown mean θ. Whereas, if $\beta > \epsilon$, $E[\theta - \epsilon] = \theta_i + \beta - \epsilon > \theta_i$. Under such a case firm i believes its cost efficiency lies outside the distribution which is inconsistent with our common knowledge assumption.

$$\frac{1}{4\lambda\epsilon}[\int_{\theta^{*I,\beta}+\beta-\epsilon}^{\hat{\theta}(\theta^{*I,\beta})}(\frac{\theta^{*I,\beta}+\theta-\epsilon}{2}+\frac{2\epsilon\lambda Q}{\theta^{*I,\beta}-(\theta-\epsilon)}-\theta^{*I,\beta})^2 d\theta+\int_{\hat{\theta}(\theta^{*I,\beta})}^{\theta^{*I,\beta}+\beta+\epsilon}(p-\theta^{*I,\beta})^2 d\theta)]=\delta \tag{17}$$

where $\hat{\theta}(\theta^{*I,\beta}) = (p+\epsilon) - \sqrt{(p-\theta^{*I,\beta})^2 + 4\epsilon\lambda Q}$*. The quota price,* $r(\theta, \theta^{*I,\beta})$*, is strictly positive for some values of* $\theta \in [\theta^{*I,\beta} + \beta - \epsilon, \theta^{*I,\beta} + \beta + \epsilon]$.

PROOF: SEE APPENDIX 8.4.

REMARK 9: The notable difference between Propositions 1-3 on the one hand and Proposition 5 on the other is that a zero quota price equilibrium is now guaranteed for a larger set of δ values compared to the full information case and to the case when placement uncertainty exists but firms consider themselves average. This is because, $\frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2})^2 > \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$ for $0 < \beta \leq \epsilon$ implying $[0, \frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2})^2] \supset [0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2]$. Furthermore, the interval $[0, \frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2})^2]$ is increasing in the set order in β implying that as the overconfidence bias goes up, a larger and larger set of δ values sustain a zero quota price equilibrium. The implications for excess entry are discussed in the next subsections.

6.2 Comparing thresholds

Consider the parametric region $\delta \in (\frac{1}{2\lambda}(\lambda Q - \epsilon)^2, \frac{(\lambda Q)^2}{2\lambda} + \frac{\epsilon^2}{6\lambda}]$ under which equilibrium exists under both scenarios - firms place themselves as average and firms place themselves as above average. We begin by noting that overconfidence raises the entry threshold for any given value of δ in this region.

Proposition 6. $\theta^{*I,\beta}(\delta, \beta) > \theta^{*I}(\delta)$ *for* $\beta > 0$ *for* $\delta \in (\frac{1}{2\lambda}(\lambda Q - \epsilon)^2, \frac{(\lambda Q)^2}{2\lambda} + \frac{\epsilon^2}{6\lambda}]$.

PROOF: SEE APPENDIX 10.4.3.

We compare the thresholds, θ^{*F}, θ^{*I} and $\theta^{*I,\beta}$, in three regions over which all three are defined.

Region I: $\delta \in [0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2]$

All three are identical and equal to $p - \sqrt{2\lambda\delta}$ for a given δ in this region and are independent of θ. Thus placement uncertainty does not matter. Figure 5a) depicts this case.

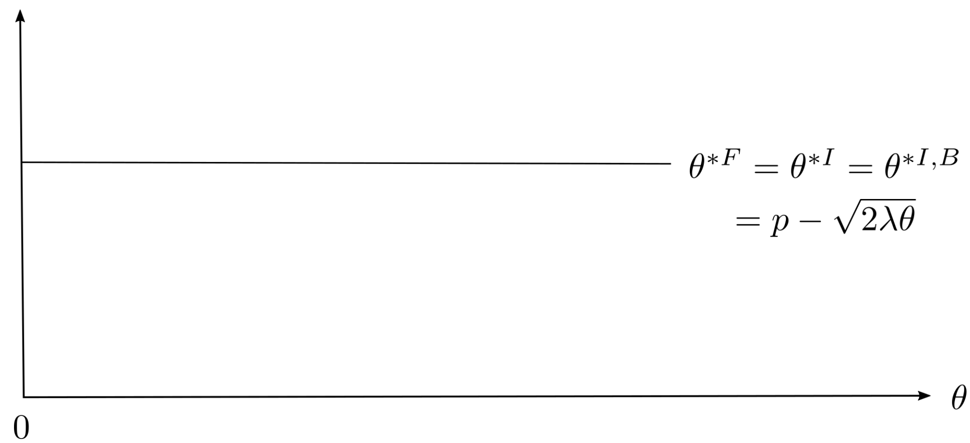

Figure 5: $\delta \in \left(0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2\right)$

Region II: $\delta \in (\frac{1}{2\lambda}(\lambda Q - \epsilon)^2, \frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2})^2]$

For a give value of δ in this region, $\theta^{*I,\beta} = p - \sqrt{2\lambda\delta}$ and is independent of θ. The threshold θ^{*F} depends on θ and has the form and appearance discussed in Section 3 and Figure 2. The threshold θ^{*I} is independent of θ and less than $\theta^{*I,\beta}$ for any given δ. The relationship between all three is as depicted in Figure 5b).

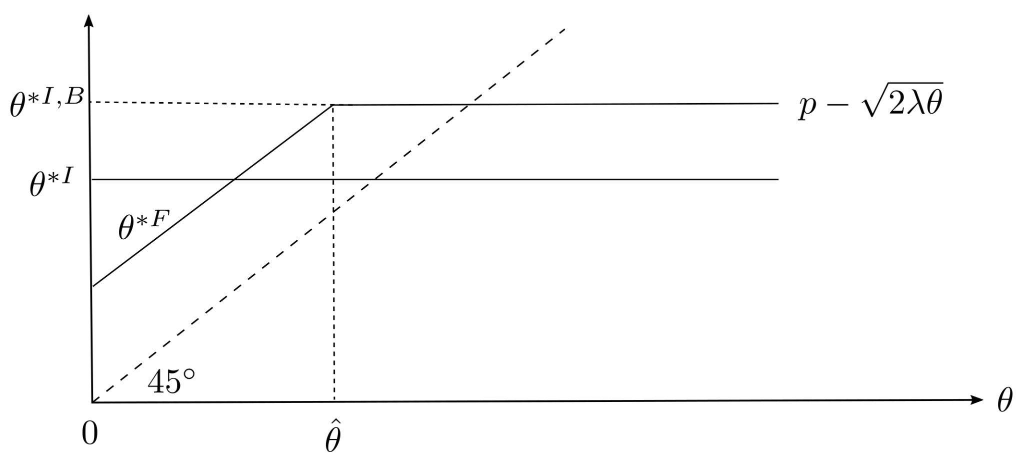

Figure 6: $\delta \in \left(\frac{1}{2\lambda}(\lambda Q - \epsilon)^2, \frac{1}{2\lambda}\left(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2}\right)^2\right)$

Region III: $\delta \in (\frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2})^2, \frac{(\lambda Q)^2}{2\lambda} + \frac{\epsilon^2}{6\lambda}]$

In this region, for any given δ, $\theta^{*I,\beta} < p - \sqrt{2\lambda Q}$ but greater than θ^{*I}. The threshold θ^{*F} has the previous form. These are depicted in Figure 5c).

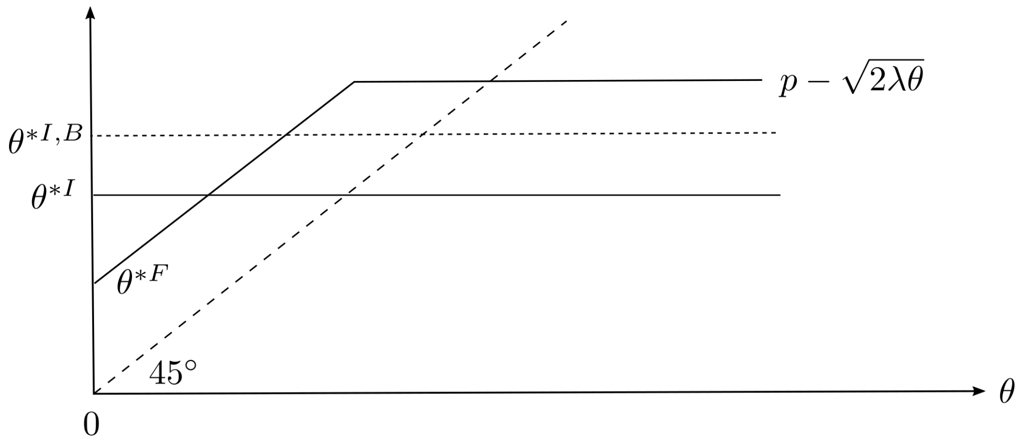

Figure 7: $\delta \in \left(\frac{1}{2\lambda}\left(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2}\right)^2, \left(\frac{(\lambda Q)^2}{2\lambda} + \frac{\epsilon^2}{6\lambda}\right)\right)$

6.3 Mass of entrants under placement bias

Denote by, $\alpha^{I,\beta}(\theta)$, the mass of entrants under incomplete information with a placement bias. Expected excess entry relative to the full information benchmark is given by,

$$\alpha^{I,\beta}(\theta) - \alpha^F(\theta) = (\alpha^{I,\beta}(\theta) - \alpha^I(\theta)) + (\alpha^I(\theta) - \alpha^F(\theta)) \tag{18}$$

The difference $\alpha^{I,\beta}(\theta) - \alpha^I(\theta)$ is non-zero only over the range $\theta \in (\theta^{*I} - \epsilon, \theta^{*I,\beta} + \epsilon)$ and turns out to be,

$$\alpha^{I,\beta}(\theta) - \alpha^I(\theta) = \begin{cases} \frac{\theta-(\theta^{*I}-\epsilon)}{2\epsilon} & \text{for } \theta^{*I} - \epsilon \leq \theta \leq \theta^{*I,\beta} - \epsilon \\ \frac{\theta^{*I,\beta}-\theta^{*I}}{2\epsilon} & \text{for } \theta^{*I,\beta} - \epsilon \leq \theta \leq \theta^{*I} + \epsilon \\ \frac{\theta^{*I,\beta}-(\theta-\epsilon)}{2\epsilon} & \text{for } \theta^{*I} + \epsilon \leq \theta \leq \theta^{*I,\beta} + \epsilon \end{cases} \tag{19}$$

It is straightforward to check that $\alpha^{I,\beta}(\theta) - \alpha^I(\theta) \geq 0$ for $\theta \in (\theta^{*I} - \epsilon, \theta^{*I,\beta} + \epsilon)$ and an average value that has a particularly simple form, namely, $Av(\alpha^{I,\beta} - \alpha^I) = (\theta^{*I,\beta} - \theta^{*I}) \geq 0$ for given values of the model parameters.

The discussions in the previous subsection combined with expressions (18) and (19) provide our second main result.

Proposition 7. *I. When* $\delta \in [0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2]$*, placement uncertainty does not matter and in particular overconfidence or over-placement bias does not matter. The mass of entrants are the same as under full information.*

II. When $\delta \in (\frac{1}{2\lambda}(\lambda Q - \epsilon)^2, \frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2})^2]$*, overconfidence or over-placement bias causes excess entry. This set is empty if firms consider themselves as average in the population and* $\beta = 0$*.*

III. When $\delta \in [\frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2})^2, \frac{(\lambda Q)^2}{2\lambda} + \frac{\epsilon^2}{6\lambda}]$*, overconfidence or over-placement bias exacerbates the effect of placement uncertainty. ...*

PROOF: Part I follows from the fact that $\theta^{*F} = \theta^{*I} = \theta^{*I,\beta}$ for all parametric configuration in this region. Part II follows from the fact that in this region, $\theta^{*I,\beta} > \theta^{*F}$ resulting in strictly positive values of $\alpha^{*I,\beta}(\theta) - \alpha^{*F}(\theta)$ irrespective of the sign of $(\alpha^{*I}(\theta) - \alpha^{*F}(\theta))$ which can be positive or negative. So far as the region of Part III is concerned, as $(\alpha^{*I,\beta}(\theta) - \alpha^{*I}(\theta)) \geq 0$, it either mitigates a negative $(\alpha^{*I}(\theta) - \alpha^{*F}(\theta))$ or reinforces a positive $(\alpha^{*I}(\theta) - \alpha^{*F}(\theta))$. Hence Part III is true.

6.4 Comparative statics

The average excess entry attributed to placement bias $Av(\alpha^{*I,\beta} - \alpha^{*I}) = (\theta^{*I,\beta} - \theta^{*I})$ is zero for values of δ in region I and strictly positive for values of δ in regions II and III for given values of the model parameters. We discuss the comparative statics of this average with respect to some parameters of interest.

7 Conclusion

Placement uncertainty should be distinguished from common cost uncertainty, a scenario more widely considered in the literature. Under common cost uncertainty, firms are assumed to be ex-ante identical in terms of their cost efficiencies. However they do not observe their own costs when entry decisions are made. Instead each firm receives a noisy signal which causes (ex-post) differences in the actions they take. Although we do not study common cost uncertainty here, a working paper version shows that on the issue of excessive entry, placement uncertainty is actually a mitigating factor under certain conditions. Fewer inefficient firms enter the CAT industry when they are certain about their own costs but uncertain about their rank, than when they are uncertain about costs broadly.

We study a two-stage entry game in a CAT-regulated industry. Atomistic firms choose whether or not to commit capital to the industry under uncertainty over their productivity rank among a population of potential entrants. Firms form subjective prior beliefs about the distribution of productivity and thus the competition for the fixed production permits they will face if they choose to enter. Post entry competition for permits determines entry payoffs, which are also the returns to vested capital. We derive rational expectations, Bayesian Nash equilibrium entry rules, and compare industry structure and performance when firms face placement uncertainty, overplacement bias, and absolute uncertainty, to the case where firms have full information about their relative productivity.

Placement uncertainty can lead to excess entry; across relevant regions of our model parameter space, firms underestimate post-entry permit prices and capital rent. This results in excess entry and cost inefficient production relative to full information. Excess entry is exacerbated by overplacement bias, i.e., subjective beliefs that firms possess better-than-average productivity. Placement uncertainty and placement bias cause similar outcomes for industry structure and performance and therefore should be addressed separately in the behavioral industrial organization literature. In particular, our results raise new questions about prevailing views on the role of overconfidence in market entry, capital investment, failed mergers, and overtrading of stocks. Overoptimism or overplacement bias are commonly put forth at the sole mechanism behind over-exuberant and failed entrepreneurial activity. Our results show that uncertainty is sufficient to generate excess entry into a CAT-regulated industry. The extent to which this mechanism extends to other settings is a worthy topic of research.[11]

To this point we have not discussed policies to restore efficiency in market settings where placement uncertainty is a factor. Interventions to reduce placement uncertainty, e.g., exposing relative firm productivity is intrusive, data intensive, may require release of confidential information, and

[11] Experimental research may be able to isolate/seprate the effects of uncertainty and bias on decision making in entrepreneurial settings.

thus an unlikely policy solution. Nudges may help firms combat decision biases and help decision makers to expose their competitive blind spots. It should be noted that we study a one shot entry game. The equilibrium CAT permit price that obtains in our model, post entry, is a function of the mean population productivity and thus firms learn population mean productivity in stage two of our model. Real world CAT permit trading markets may however be thin and permit trading prices can be noisy. Learning the true productivity of competitors can therefore be slowed allowing inefficiency to persist over multiple production periods.[12] In these settings, policies that reduce trading frictions and improve transparency in CAT-permit markets may help with price discovery and improve market performance.

[12] Asche et al. (2014) and Turner and Weninger (2005) report 10-20 year delays in the transition from overcapitalized to efficient fleet structures in fisheries that implement CAT regulations.

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8 Appendices

8.1 Appendix I: (Proposition 1) Equilibrium under complete information

The profit function for a firm with efficiency θ_i assumes two different forms, depending on whether equilibrium permit price is zero or non-zero:

$$\pi(\theta_i, \theta) = \begin{cases} \frac{1}{2\lambda}(p - \theta_i)^2, & \text{if for all } \theta \in [\theta_i - \epsilon, \theta_i + \epsilon], r(\theta, \theta^*) = 0 \\ \frac{1}{2\lambda}(p - r(\theta, \theta^*) - \theta_i)^2, & \text{if for some } \theta \in [\theta_i - \epsilon, \theta_i + \epsilon], r(\theta, \theta^*) > 0 \end{cases}$$

Depending on the parametric configuration, there are two equilibria in a common threshold strategy.

8.1.1 Equilibrium with zero permit price: $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2\right)$

The solution to the equation $\frac{1}{2\lambda}(p - \theta^{*F})^2 = \delta$ yields $\theta^{*F} = p - \sqrt{2\lambda\delta}$.

The permit price is zero for all values of θ and in particular for $\theta < \theta^* - \epsilon$ at which all firms enter and $A = S$. Given the form of the permit price function, 10, $r = 0$ for $\hat{\theta} = p - \lambda Q$ when $A = S$. Hence $\theta^* = p - \sqrt{2\lambda\delta}$ is equilibrium if the following conditions hold.

1. $\frac{1}{2\lambda}(\theta + \lambda Q - \theta_i)^2 > \delta$ for $\theta < p - \lambda Q$ and $\theta_i \in [\theta - \epsilon, \theta + \epsilon]$.
2. $\frac{1}{2\lambda}(p - \theta_i)^2 > \delta$ for $\theta \geq p - \lambda Q$, $\theta_i \in [\theta - \epsilon, \theta + \epsilon]$ and $\theta_i \leq p - \sqrt{2\lambda\delta}$.
3. $\frac{1}{2\lambda}(p - \theta_i)^2 < \delta$ for $\theta \geq p - \lambda Q$, $\theta_i \in [\theta - \epsilon, \theta + \epsilon]$ and $\theta_i > p - \sqrt{2\lambda\delta}$.

Condition (1) implies, $\frac{1}{2\lambda}(\theta + \lambda Q - \theta - \epsilon)^2 = \frac{1}{2\lambda}(\lambda Q - \epsilon)^2 > \delta$. Since $\frac{1}{2\lambda}(p - \theta_i)^2$ is monotone decreasing in θ_i and has a zero at $\theta_i = p - \sqrt{2\lambda\delta}$, (2) and (3) are true.

Hence $\theta^{*F} = p - \sqrt{2\lambda\delta}$ is equilibrium for $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2\right)$.

8.1.2 Equilibrium with positive permit price: $\delta \geq \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$

The candidate threshold θ^{*F} is determined by $\frac{1}{2\lambda}\left(\frac{\theta^{*F} + \theta - \epsilon}{2} - \theta^{*F} + \frac{2\epsilon\lambda Q}{\theta^{*F} - (\theta - \epsilon)}\right)^2 = \delta$ which yields

$$\theta^{*F}(\theta) = (\theta - \epsilon) \pm \sqrt{2\delta\lambda + 4\epsilon\lambda Q} - \sqrt{2\delta\lambda}$$

Given that the equilibrium solution must satisfy, $\theta - \epsilon \leq \theta^{*F}(\theta) \leq \theta + \epsilon$, the first half of the inequality implies that the admissible solution is

$$\theta^{*F}(\theta) = (\theta - \epsilon) + \sqrt{2\delta\lambda + 4\epsilon\lambda Q} - \sqrt{2\delta\lambda}$$

The second half of the inequality, $\theta^{*F}(\theta) \leq \theta + \epsilon$, gives us the admissible parametric configuration, $\delta \geq \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$ upon simplification.

Given the admissible form of $\theta^*(\theta)$, the admissible form for $r(\theta, \theta^*)$ is,

$$r(\theta, \theta^*) = \begin{cases} p + \epsilon - \frac{\sqrt{2\lambda\delta + 4\epsilon\lambda Q} - \sqrt{2\lambda\delta}}{2} - \frac{2\epsilon\lambda Q}{\sqrt{2\lambda\delta + 4\epsilon\lambda Q} - \sqrt{2\lambda\delta}} - \theta & \text{for } \theta < \hat{\theta} \\ 0, & \text{for } \theta > \hat{\theta} \end{cases}$$

where,

$$\begin{aligned}\hat{\theta} &= p+\epsilon-\tfrac{\sqrt{2\lambda\delta+4\epsilon\lambda Q}-\sqrt{2\lambda\delta}}{2}-\tfrac{2\epsilon\lambda Q}{\sqrt{2\lambda\delta+4\epsilon\lambda Q}-\sqrt{2\lambda\delta}} \\ &= p+\epsilon-\sqrt{2\lambda\delta+4\epsilon\lambda Q}\end{aligned}$$

upon simplification.

Combining, the admissible threshold function is

$$\theta^*(\theta)=\begin{cases}(\theta-\epsilon)+\sqrt{2\delta\lambda+4\epsilon\lambda Q}-\sqrt{2\delta\lambda} & \text{for } \theta<\hat{\theta} \\ p-\sqrt{2\lambda\delta} & \text{for } \theta\geq\hat{\theta}\end{cases}$$

For θ^{*F} to be equilibrium, the following conditions must be satisfied.

1. For $\theta<\hat{\theta}$

$$\begin{aligned}&\tfrac{1}{2\lambda}\left(\tfrac{\theta^*(\theta)+\theta-\epsilon}{2}+\tfrac{2\epsilon\lambda Q}{\theta^*(\theta)-(\theta-\epsilon)}-\theta_i\right)^2>\delta \quad \text{for } \theta_i\leq\theta^*(\theta) \\ &\tfrac{1}{2\lambda}\left(\tfrac{\theta^*(\theta)+\theta-\epsilon}{2}+\tfrac{2\epsilon\lambda Q}{\theta^*(\theta)-(\theta-\epsilon)}-\theta_i\right)^2<\delta \quad \text{for } \theta_i>\theta^*(\theta)\end{aligned}$$

2. For $\theta>\hat{\theta}$

$$\begin{aligned}&\tfrac{1}{2\lambda}\left(p-\theta_i\right)^2>\delta \quad \text{for } \theta_i\leq\theta^*(\theta) \\ &\tfrac{1}{2\lambda}\left(p-\theta_i\right)^2<\delta \quad \text{for } \theta_i>\theta^*(\theta)\end{aligned}$$

These conditions are satisfied because the profit functions are monotone decreasing in θ_i and has zeros at $\theta^*(\theta)$ for the relevant regions.

8.2 Appendix II: (Propositions 2 and 3) Equilibrium under incomplete information with no bias

8.2.1 Proposition 2: Equilibrium with zero expected permit price

The admissible threshold is given by the solution of

$$\pi(\theta^{*I}) = \frac{1}{2\epsilon}\int_{\theta^{*I}-\epsilon}^{\theta^{*I}+\epsilon}\frac{1}{2\lambda}\left(p-\theta^{*I}\right)^2 d\theta = \delta$$

or $\theta^{*I}(\delta) = p - \sqrt{2\lambda\delta}$.

To see why $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \epsilon)^2\right]$, note that when $\theta = \theta^{*I} - \epsilon$, the pivotal firm expects itself to be the highest cost firm in the population and assuming that it chooses to enter, expects all the other firms to enter as well. Hence the permit price expected by the pivotal firm is $r(\theta, \theta^{*I}) = p - \theta - \lambda Q \leq 0$. Substituting these expressions into the expected payoff function and simplifying, gives us

$$\delta \leq \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$$

To show $\theta^* = p - \sqrt{2\lambda\delta}$ is equilibrium we need to show that the expected profit, $\pi(\theta_i, \theta^*)$, is greater than δ when $\theta_i < \theta^{*I}$ and less than δ when $\theta_i > \theta^{*I}$. The expected profit function takes different forms depending on the zones in which θ_i lie.

1. For $\theta_i < \theta^{*I} - 2\epsilon = p - \sqrt{2\lambda\delta} - 2\epsilon$, all firms are expected to participate for all possible values of θ. Hence individual profit function must satisfy the condition,

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon}(\theta - \theta_i + \lambda Q)^2 d\theta > \delta$$

Note that

$$\begin{aligned}
\pi(\theta_i, \theta^*) &= \frac{1}{4\lambda\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon}(\theta - \theta_i + \lambda Q)^2 d\theta \\
&= \frac{1}{12\lambda\epsilon}\left[(\lambda Q + \epsilon)^3 - (\lambda Q - \epsilon)^3\right] \\
&= \frac{1}{12\lambda\epsilon}(2\epsilon)\left((\lambda Q)^2 - \epsilon^2 + 2(\lambda Q)^2 + 2\epsilon^2\right) \\
&= \frac{\lambda Q^2}{2} + \frac{\epsilon^2}{6\lambda} > \frac{1}{2\lambda}(\lambda Q - \epsilon)^2 \geq \delta
\end{aligned}$$

by Assumption 1. Hence the condition is satisfied.

2. In an equilibrium in which the pivotal firm expects $r(\theta, \theta^{*I}) = 0$ for $\theta \in [\theta^{*I} - \epsilon, \theta^{*I} + \epsilon]$, the value of θ at which the equilibrium permit price is expected to equal zero is less than $\theta^{*I} - \epsilon$. In other words, at the value of θ at which the equilibrium permit price is expected to equal zero, an individual firm expects all firms to enter, that is $A = S$. Denote by $\hat{\theta} = p - \lambda Q$, the value of θ at which permit price is zero for $A = S$.

   Then, for $p - \sqrt{2\lambda\delta} - 2\epsilon = \theta^{*I} - 2\epsilon \leq \theta_i \leq p - \lambda Q + \epsilon$, the individual profit function must satisfy,

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\epsilon} \int_{\theta_i - \epsilon}^{p - \lambda Q} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\epsilon} \int_{p - \lambda Q}^{\theta_i + \epsilon} (p - \theta_i)^2 \, d\theta > \delta$$

   Note that, $\theta_i \leq p - \lambda Q + \epsilon$ implies that $\theta_i - \epsilon \leq p - \lambda Q$. Moreover, $p - \lambda Q \leq \theta^{*I} - \epsilon$ implies $p - \lambda Q + \epsilon \leq \theta^{*I}$ and hence $\theta_i \leq \theta^{*I}$. The individual profit function simplifies to,

$$\begin{aligned}
\pi(\theta_i, \theta^*) &= \frac{1}{4\lambda\epsilon} \int_{\theta_i - \epsilon}^{p - \lambda Q} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\epsilon} \int_{p - \lambda Q}^{\theta_i + \epsilon} (p - \theta_i)^2 \, d\theta \\
&= \frac{1}{4\lambda\epsilon} \left[\frac{(p - \theta_i)^3}{3} - \frac{(\lambda Q - \epsilon)^3}{3} + (p - \theta_i)^2 (\theta_i + \epsilon - p + \lambda Q) \right]
\end{aligned}$$

   The right hand side equals $\frac{\lambda Q^2}{2} + \frac{\epsilon^2}{6\lambda}$ when $\theta_i = p - \lambda Q - \epsilon$, and $\frac{1}{2\lambda} (\lambda Q - \epsilon)^2 \geq \delta$ when $\theta_i = p - \lambda Q + \epsilon$. Thus the value of the function is higher than δ at one of the end points and as the following derivative shows, the function is strictly declining.

   The derivative of $\pi(\theta_i, I_{\theta^*})$ with respect to θ_i equals

$$\frac{1}{4\lambda\epsilon}\left[-\left(\lambda Q-\epsilon\right)^{2}+\left(p-\theta_{i}\right)^{2}-2\int_{\theta_{i}-\epsilon}^{p-\lambda Q}\left(\theta-\theta_{i}+\lambda Q\right)d\theta-2\int_{p-\lambda Q}^{\theta_{i}+\epsilon}\left(p-\theta_{i}\right)d\theta\right]$$
$$=\frac{1}{4\lambda\epsilon}\left[-\left(\lambda Q-\epsilon\right)^{2}+\left(p-\theta_{i}\right)^{2}-\left(p-\theta_{i}\right)^{2}+\left(\lambda Q-\epsilon\right)^{2}-2\int_{p-\lambda Q}^{\theta_{i}+\epsilon}\left(p-\theta_{i}\right)d\theta\right]$$
$$=\frac{1}{4\lambda\epsilon}\left[-2\int_{p-\lambda Q}^{\theta_{i}+\epsilon}\left(p-\theta_{i}\right)d\theta\right]<0$$

The condition is therefore satisfied.

3. When $p-\lambda Q<\theta_i-\epsilon<\theta^*-\epsilon$, the individual profit function must satisfy,

$$\pi(\theta_i,\theta^*)=\frac{1}{2\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon}\frac{1}{2\lambda}\left(p-\theta_i\right)^2 d\theta=\frac{1}{2\lambda}\left(p-\theta_i\right)^2\geq\delta$$

The right hand side is strictly declining and greater than δ for $\theta_i<\theta^{*I}$, because at $\theta_i=\theta^*$, $\frac{1}{2\lambda}\left(p-\theta_i\right)^2=\delta$. Hence the condition is satisfied

4. When$\theta_i>\theta^*$, the individual profit function must satifsfy, have

$$\pi(\theta_i,\theta^*)=\frac{1}{2\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon}\frac{1}{2\lambda}\left(p-\theta_i\right)^2 d\theta<\delta$$

This is true because the right hand side is strictly declining and has a value of δ at $\theta_i=\theta^*$.

Thus $\theta^*=p-\sqrt{2\lambda\delta}$ is equilibrium.

8.2.2 Propostion 3: Equilibrium with positive expectation of permit price

For any given θ^*, the quota price function, $r(\theta,\theta^*)=p-\frac{\theta^*+\theta-\epsilon}{2}-\frac{2\epsilon\lambda Q}{\theta^*-(\theta-\epsilon)}$ attains zero at a value of $\theta<\theta^*+\epsilon$, since p is finite and last term tends to infinity as $\theta\longrightarrow\theta^*+\epsilon$. Given θ^*, the roots of $p-\frac{\theta^*+\theta-\epsilon}{2}-\frac{2\epsilon\lambda Q}{\theta^*-(\theta-\epsilon)}=0$ are given by $\hat{\theta}(\theta^*)=(p+\epsilon)\pm\sqrt{(p-\theta^*)^2+4\epsilon\lambda Q}$. The restriction

$\hat{\theta}(\theta^*) < \theta^* + \epsilon$ implies that only the root $\hat{\theta}(\theta^*) = (p+\epsilon) - \sqrt{(p-\theta^*)^2 + 4\epsilon\lambda Q}$ need be considered. It is also clear from previous discussions that if an equlibrium exists for $\delta > \frac{1}{2\lambda}\left(\lambda Q - \epsilon\right)^2$, then the permit price is strictly positive at $\theta = \theta^* - \epsilon$, implying, $\hat{\theta}(\theta^*) > \theta^* - \epsilon$.

It is straightforward to check that under Assumption 1, $\frac{1}{2\lambda}(\lambda Q - \epsilon)^2 < \frac{\lambda Q^2}{2} + \frac{\epsilon^2}{6\lambda}$ and the set $\delta \in \left(\frac{1}{2\lambda}(\lambda Q - \epsilon)^2, \frac{\lambda Q^2}{2} + \frac{\epsilon^2}{6\lambda}\right]$ is non-empty. We now proceed to the main steps of the proof of Proposition 3.

STEP I. We show that there is a unique solution θ^{*I} that satisfies condition (13).

Consider the function,

$$\pi(k) = \frac{1}{2\epsilon}\int_{k-\epsilon}^{\hat{\theta}(k)} \frac{1}{2\lambda}\left(\frac{k+\theta-\epsilon}{2} + \frac{2\epsilon\lambda Q}{k-(\theta-\epsilon)} - k\right)^2 d\theta + \frac{1}{2\epsilon}\int_{\hat{\theta}(k)}^{k+\epsilon} \frac{1}{2\lambda}(p-k)^2 d\theta$$

where $\hat{\theta}(k) = p + \epsilon - \sqrt{(p-k)^2 + 4\epsilon\lambda Q}$.

A change of variable $z = \theta - (k+\epsilon)$ allows us to evaluate the first integral. With this change of variable, the upper and lower limits of the integration are, respectively, $\hat{\theta}\,(k) - k - \epsilon = (p-k) - \sqrt{(p-k)^2 + 4\epsilon\lambda Q}$ and -2ϵ. Evaluating the integral using the new variable and then substituting the new variable back and simplifying, we have,

$$\pi\,(k) = \frac{1}{4\lambda\epsilon}\left[\begin{array}{c} -\frac{4(\epsilon\lambda Q)^2}{(p-k)-\sqrt{(p-k)^2+4\epsilon\lambda Q}} - \frac{4(\epsilon\lambda Q)^2}{2\epsilon} + \frac{\left((p-k)-\sqrt{(p-k)^2+4\epsilon\lambda Q}\right)^3}{12} \\ +\frac{8\epsilon^3}{12} - 2\epsilon\lambda Q\left[(p-k) - \sqrt{(p-k)^2 + 4\epsilon\lambda Q} + 2\epsilon\right] \\ -\left[(p-k) - \sqrt{(p-k)^2 + 4\epsilon\lambda Q}\right](p-k)^2 \end{array}\right] \tag{20}$$

We try to show next that $\frac{d\pi(k)}{dk} < 0$. A second change of variable helps us to do that. Define

$$x \equiv \sqrt{(p-k)^2 + 4\epsilon\lambda Q} - (p-k) > 0$$

and note that

$$\frac{dx}{dk} = 1 - \frac{p-k}{\sqrt{(p-k)^2 + 4\epsilon\lambda Q}} > 0$$

Further note that $(p-k)^2 = \frac{x^2}{4} + \frac{(2\epsilon\lambda Q)^2}{x^2} - 2\epsilon\lambda Q$.

With the second change of variable, $\pi(k)$ can be rewritten as

$$\pi(k, I_k) = \frac{1}{4\lambda\epsilon}\left[\begin{array}{c} -\frac{4(\epsilon\lambda Q)^2}{2\epsilon} + \frac{8\epsilon^3}{12} - 2\epsilon\lambda Q\,(2\epsilon) \\ +\frac{x^3}{6} + \frac{8(\epsilon\lambda Q)^2}{x} \end{array}\right]$$

Thus, whether $\pi(k)$ is increasing or decreasing in k depends on whether it decreases or increases in x.

$$\begin{aligned}\frac{d\pi}{dx} &= \frac{1}{4\lambda\epsilon}\left[\frac{x^2}{2} - \frac{8(\epsilon\lambda Q)^2}{x^2}\right] \\ &= \frac{1}{4\lambda\epsilon}\left(\frac{x}{\sqrt{2}} + \frac{2\sqrt{2}\epsilon\lambda Q}{x}\right)\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\epsilon\lambda Q}{x}\right)\end{aligned}$$

Thus the sign of $\frac{d\pi(k)}{dk}$ depends on the sign of $\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\epsilon\lambda Q}{x}\right)$. Substituting the expression for x back and simplifying, it is straightforward to check that so long as $(p-k) > 0$ (true for values of k we are interested in), $\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\epsilon\lambda Q}{x}\right) < 0$.

Thus $\frac{d\pi(k)}{dk} < 0$.

We next show that the function $\pi(k)$ is greater than δ for some k and less than δ for some k.

We begin by asserting that for any given k and $\theta \in [k-\epsilon, k+\epsilon]$, the following inequality is true.

$$(\theta + \lambda Q - k) \leq \left(\frac{k+\theta-\epsilon}{2} + \frac{2\epsilon\lambda Q}{k-(\theta-\epsilon)} - k\right) \leq (p-k) \tag{21}$$

The first inequality holds for the following reason. For any given k and $\theta \in [k-\epsilon, k+\epsilon]$, the expression,

$$\left(\frac{k+\theta-\epsilon}{2}+\frac{2\epsilon\lambda Q}{k-(\theta-\epsilon)}\right)=\theta+\lambda Q \text{ when } \theta=k-\epsilon.$$

Both functions are increasing in θ. The slope of $\theta+\lambda Q$ is 1. The slope of $\left(\frac{k+\theta-\epsilon}{2}+\frac{2\epsilon\lambda Q}{k-(\theta-\epsilon)}\right)$ is $\left(\frac{1}{2}+\frac{2\epsilon\lambda Q}{(k-(\theta-\epsilon))^2}\right)$. As $\frac{2\epsilon\lambda Q}{(k-(\theta-\epsilon))^2}>\frac{4\epsilon^2}{(k-(\theta-\epsilon))^2}$ by Assumption 1 and $k-(\theta-\epsilon)\leq 2\epsilon$, the ratio $\frac{2\epsilon\lambda Q}{(k-(\theta-\epsilon))^2}>1$. Hence $(\theta+\lambda Q-k)\leq\left(\frac{k+\theta-\epsilon}{2}+\frac{2\epsilon\lambda Q}{k-(\theta-\epsilon)}-k\right)$ for $\theta\in[k-\epsilon,k+\epsilon]$, for any given k.

The second equality follows from the fact that the expression, $\frac{k+\theta-\epsilon}{2}+\frac{2\epsilon\lambda Q}{k-(\theta-\epsilon)}$, represents quasi rent on capital and is less than or equal to p.

We therefore have,

$$\frac{1}{2\epsilon}\int_{k-\epsilon}^{k+\epsilon}\frac{1}{2\lambda}\left(\theta+\lambda Q-k\right)^2 d\theta\leq\pi(k)\leq\frac{1}{2\epsilon}\int_{k-\epsilon}^{k+\epsilon}\frac{1}{2\lambda}\left(p-k\right)^2 d\theta \tag{22}$$

Note that for $k=p-\lambda Q+\epsilon$, $\frac{1}{2\lambda}(p-k)^2=\frac{1}{2\lambda}\left(\lambda Q-\epsilon\right)^2$. Hence, $\pi(k)\leq\frac{1}{2\lambda}\left(\lambda Q-\epsilon\right)^2<\delta$ for some value of k.

Similarly note that for $k=p-\lambda Q-\epsilon$,

$$\frac{1}{2\epsilon}\int_{k-\epsilon}^{k+\epsilon}\frac{1}{2\lambda}\left(\theta+\lambda Q-k\right)^2 d\theta=\frac{1}{12\lambda\epsilon}\left[(\lambda Q+\epsilon)^3-\lambda Q-\epsilon)^3\right]=\frac{\lambda Q^2}{2}+\frac{\epsilon^2}{6\lambda}>\delta$$

Thus the function $\pi(k)$ is greater than δ for some k. Combining all observations, it has a unique intersection $k=\theta^{*I}$ with δ. Furthermore as $\pi(k)$ is decreasing in k, the intersection θ^{*I} is decreasing in δ.

STEP II. We next show that the switching strategy with the threshold θ^{*I} is an equilibrium. We need to show that for a firm with efficiency θ_i, $\pi(\theta_i,\theta^{*I})>\delta$ for $\theta_i<\theta^{*I}$ and $\pi(\theta_i,\theta^{*I})<\delta$ for $\theta_i>\theta^{*I}$.

The fact that θ^{*I} has no closed form makes the analysis relatively more complicated compared to what it is under Proposition 2. We begin by characterizing the function $\pi(\theta_i,\theta^{*I})$ for different zones in which θ_i may lie, given the solution θ^{*I} and the condition it must satisfy.

1. For $\theta_i < \theta^{*I} - 2\epsilon$ or $\theta_i + 2\epsilon < \theta^{*I}$.

$$\pi(\theta_i, \theta^{*I}) = \frac{1}{4\lambda\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta > \delta$$

   From the perspective of the θ_i-type, all possible types for any $\theta \in [\theta_i - \epsilon, \theta_i - \epsilon]$ are below the threshold θ^*. Thus, all firms enter for any $\theta \in [\theta_i - \epsilon, \theta_i - \epsilon]$ and the individual profit function takes the above form. The expected profit of a firm with efficiency θ_i must be higher than δ.

2. For $\theta^{*I} - 2\epsilon < \theta_i < \hat{\theta}(\theta^{*I}) - \epsilon < \theta^{*I}$,

$$\begin{aligned}\pi(\theta_i, \theta^{*I}) \quad = \quad & \frac{1}{4\lambda\epsilon}\int_{\theta_i-\epsilon}^{\theta^{*I}-\epsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta \\ & + \frac{1}{4\lambda\epsilon}\int_{\theta^*-\epsilon}^{\theta_i+\epsilon} \left(\frac{\theta^{*I} + \theta - \epsilon}{2} - \theta_i + \frac{2\epsilon\lambda Q}{\theta^{*I} - (\theta - \epsilon)}\right)^2 d\theta > \delta\end{aligned}$$

   A firm with efficiency θ_i does not expect all firms to be active for all $\theta \in [\theta_i - \epsilon, \theta_i + \epsilon]$. For that range, $\theta \in [\theta_i - \epsilon, \theta^{*I} - \epsilon]$, the firm expects all firms to enter, because the highest cost firm in this scenario has a cost parameter less than the threshold. For values of θ above $\theta^{*I} - \epsilon$, some of the high cost firms are expected to have parameter values above the threshold and will not enter. The expected profit function therefore has two parts and must have a value higher than δ.

3. For $\hat{\theta}(\theta^*) - \epsilon < \theta_i < \theta^*$, the profit function has three parts.

$$\pi(\theta_i, \theta^{*I}) = \frac{1}{4\lambda\epsilon}\int_{\theta_i-\epsilon}^{\theta^{*I}-\epsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta$$
$$+ \frac{1}{4\lambda\epsilon}\int_{\theta^{*I}-\epsilon}^{\hat{\theta}(\theta^{*I})} \left(\frac{\theta^{*I} + \theta - \epsilon}{2} - \theta_i + \frac{2\epsilon\lambda Q}{\theta^{*I} - (\theta - \epsilon)}\right)^2 d\theta$$
$$+ \frac{1}{4\lambda\epsilon}\int_{\hat{\theta}(\theta^{*I})}^{\theta_i+\epsilon} (p - \theta_i)^2 \, d\theta > \delta$$

Similar arguments as in (2) explain the first two components of the function. The third component is explained by the fact that if $\theta > \hat{\theta}(\theta^*)$, the expected permit price is zero.

4. For, $\theta^{*I} < \theta_i < \hat{\theta}(\theta^{*I}) + \epsilon < \theta^{*I} + 2\epsilon$, the profit function has two parts.

$$\pi(\theta_i, \theta^{*I}) = \frac{1}{4\lambda\epsilon}\int_{\theta_i-\epsilon}^{\hat{\theta}(\theta^{*I})} \left(\frac{\theta^{*I} + \theta - \epsilon}{2} - \theta_i + \frac{2\lambda\epsilon Q}{\theta^{*I} - (\theta - \epsilon)}\right)^2 d\theta$$
$$+ \frac{1}{4\lambda\epsilon}\int_{\hat{\theta}(\theta^{*I})}^{\theta_i+\epsilon} (p - \theta_i)^2 \, d\theta < \delta$$

The quota price is positive for some values of θ and zero for others, accounting for the two components. As $\theta_i > \theta^*$, production must be less profitable for the θ_i-type, compared to the outside option.

5. For $\hat{\theta}(\theta^{*I}) + \epsilon < \theta_i < \theta^{*I} + 2\epsilon$,

$$\pi(\theta_i, \theta^{*I}) = \frac{1}{4\lambda\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon} (p - \theta_i)^2 \, d\theta < \delta$$

It is straightforward to check that $\pi(\theta_i, \theta^*)$ is continuous in θ_i.

The rest of the proof, shows that the required inequality is satisfied for each zone.

1. When $\theta_i < \theta^* - 2\epsilon$,

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta = \frac{1}{12\lambda\epsilon}\left[(\lambda Q + \epsilon)^3 - \lambda Q - \epsilon)^3\right] = \frac{\lambda Q^2}{2} + \frac{\epsilon^2}{6\lambda} > \delta$$

by Assumption 1 and the first inequality is satisfied.

2. When $\theta^* - 2\epsilon < \theta_i < \hat{\theta}(\theta^*) - \epsilon < \theta^*$, the condition (22) shows that for any θ_i and given θ^*,

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\epsilon}\int_{\theta_i-\epsilon}^{\theta^*-\epsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\epsilon}\int_{\theta^*-\epsilon}^{\theta_i+\epsilon} \left(\frac{\theta^* + \theta - \epsilon}{2} - \theta_i + \lambda \frac{2\epsilon Q}{\theta^* - (\theta - \epsilon)}\right)^2 d\theta$$

$$\geq \frac{1}{4\lambda\epsilon}\int_{\theta_i-\epsilon}^{\theta_i+\epsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta > \delta$$

Thus the second inequality is also true.

3. Note that $\pi(\theta_i, \theta^{*I})$ is not only continuous but differentiable (in fact, twice differentiable) over the interval $\hat{\theta}(\theta^{*I}) - \epsilon < \theta_i < \theta^{*I}$ as well. From item 2 above, at $\theta_i = \hat{\theta}(\theta^{*I}) - \epsilon$, $\pi(\theta_i, \theta^{*I}) > \delta$. By continuity, as $\theta_i \longrightarrow \theta^{*I}$, the function $\pi(\theta_i, \theta^{*I})$ converges to the function $\pi(\theta^{*I})$ pointwise. Hence, $\pi(\theta_i, \theta^{*I}) \longrightarrow \delta$ as as $\theta_i \longrightarrow \theta^{*I}$.

   Moreover, STEP I shows that $\pi(\theta^{*I})$ is declining at $\theta_i = \theta^{*I}$. Hence the slopes of the two functions must also converge as $\theta_i \longrightarrow \theta^{*I}$ and in particular $\pi(\theta_i, \theta^{*I})$ must be declining at $\theta_i = \theta^{*I}$. Thus, $\pi(\theta_i, \theta^{*I})$ must have at least one stationary point that is a maximum in this interval.

   We therefore check the roots of the derivative of $\pi(\theta_i, \theta^{*I})$ with respect to θ_i.

   The derivative of the first term of $\pi(\theta_i, \theta^{*I})$ with respect to θ_i is given by (using Leibnitz's rule),

$$\frac{-1}{2\lambda\epsilon}\int_{\theta_i-\epsilon}^{\theta^{*I}-\epsilon}(\theta-\theta_i+\lambda Q)\,d\theta-(\lambda Q-\epsilon)^2$$
$$=-\left(\theta^{*I}-\theta_i+\lambda Q-\epsilon\right)^2+(\lambda Q-\epsilon)^2-(\lambda Q-\epsilon)^2$$
$$=-\left(\theta^{*I}-\theta_i+\lambda Q-\epsilon\right)^2$$

The derivative of the second term is given by,

$$-\frac{1}{2\lambda\epsilon}\int_{\theta^{*I}-\epsilon}^{\hat{\theta}(\theta^{*I})}\left(\frac{\theta^{*I}+\theta-\epsilon}{2}-\theta_i+\lambda\frac{2\epsilon Q}{\theta^{*I}-(\theta-\epsilon)}\right)d\theta$$
$$=2\left(\theta^{*I}-\epsilon\right)^2-\frac{\left(\theta^{*I}+\hat{\theta}-\epsilon\right)^2}{2}+4\epsilon\lambda Q\left(\ln\left[\theta^{*I}+\epsilon-\hat{\theta}\right]-\ln\left[2\epsilon\right]\right)+2\theta_i\left(\hat{\theta}\left(\theta^{*I}\right)-\left(\theta^{*I}-\epsilon\right)\right)$$

The derivative of the third term is given by

$$\frac{1}{4\lambda\epsilon}\left[3\left(p-\theta_i\right)^2-2\left(p-\theta_i\right)\sqrt{\left(p-\theta^{*I}\right)^2+4\epsilon\lambda Q}\right]$$

These terms of the derivatives can be combined to get

$$\frac{1}{2\lambda\epsilon}\left[\theta_i^2-2\left(p-\frac{\lambda Q+\epsilon}{2}\right)\theta_i+\Omega\left[\theta^{*I}\right]\right] \tag{23}$$

where

$$\Omega\left[\theta^{*I}\right]\equiv\left(\theta^{*I}-\epsilon\right)^2-\frac{\left(\hat{\theta}\left(\theta^{*I}\right)+\theta^{*I}-\epsilon\right)^2}{4}+2\epsilon\lambda Q\ln\left[1-\frac{\hat{\theta}\left(\theta^{*I}\right)-\left(\theta^{*I}-\epsilon\right)}{2\epsilon}\right]$$
$$+\frac{p^2-\left(\lambda Q+\theta^{*I}-\epsilon\right)^2}{2}+p\left(\hat{\theta}\left(\theta^{*I}\right)-\epsilon\right)$$

Derivative (23) has two roots given by

$$p - \frac{\lambda Q + \epsilon}{2} \pm \sqrt{\left(p - \frac{\lambda Q + \epsilon}{2}\right)^2 - \Omega\left[\theta^{*I}\right]}$$

Note that both roots cannot be less than θ^{*I} because of the following contradiction.

$$\theta_1^R = p - \frac{\lambda Q + \epsilon}{2} - \sqrt{\left(p - \frac{\lambda Q + \epsilon}{2}\right)^2 - \Omega\left[\theta^{*I}\right]} < \theta^{*I} \Longrightarrow 2\left(p - \frac{\lambda Q + \epsilon}{2}\right)\theta^{*I} - (\theta^{*I})^2 > \Omega\left[\theta^{*I}\right]$$

$$\theta_2^R = p - \frac{\lambda Q + \epsilon}{2} + \sqrt{\left(p - \frac{\lambda Q + \epsilon}{2}\right)^2 - \Omega\left[\theta^{*I}\right]} < \theta^{*I} \Longrightarrow 2\left(p - \frac{\lambda Q + \epsilon}{2}\right)\theta^{*I} - (\theta^{*I})^2 < \Omega\left[\theta^{*I}\right]$$

As one of the roots must be less than θ^{*I} for $\pi(\theta_i, \theta^{*I})$ to be declining at $\theta_i = \theta^{*I}$ and as $\theta_2^R > \theta_1^R$, the root $\theta_1^R < \theta^{*I}$. Thus $\pi(\theta_i, \theta^{*I})$ has only one stationary point in the interval which is a maximum and assumes a value of δ at $\theta_i = \theta^{*I}$. Hence the value of the function is greater than δ over the interval.

4. The profit function $\pi(\theta_i, \theta^{*I})$ is twice differentiable for $\theta^{*I} < \theta_i < \hat{\theta}(\theta^{*I}) + \epsilon < \theta^{*I} + 2\epsilon$. The derivative of the profit function with respect to θ_i is $\frac{1}{4\lambda\epsilon}$ times the expression,

$$(p - \theta_i)^2 - \left[\begin{array}{c} \left(\frac{\theta^{*I} - \epsilon + \theta_i - \epsilon}{2} - \theta_i + \frac{2\epsilon\lambda Q}{\theta^{*I} + \epsilon - (\theta_i - \epsilon)}\right)^2 \\ +2\left(\int_{\theta_i - \epsilon}^{\hat{\theta}(\theta^{*I})}\left(\frac{\theta^{*I} + \theta - \epsilon}{2} - \theta_i + \frac{2\lambda\epsilon Q}{\theta^{*I} - (\theta - \epsilon)}\right) d\theta + 2\int_{\hat{\theta}(\theta^{*I})}^{\theta_i + \epsilon}(p - \theta_i)\, d\theta\right) \end{array}\right] \quad (24)$$

where the term within square brackets is positive. In particular, note that

$$\frac{\theta^{*I} - \epsilon + \theta_i - \epsilon}{2} - \theta_i + \frac{2\epsilon\lambda Q}{\theta^{*I} + \epsilon - (\theta_i - \epsilon)} > 0$$

The second order cross partial derivative, $\frac{\partial^2 \pi(\theta_i,\theta^{*I})}{\partial \theta^{*I} \partial \theta_i}$, is given by $\frac{1}{4\lambda\epsilon}$ times the expression,

$$-\left[\begin{array}{c} \left(\frac{\theta^{*I}-\epsilon+\theta_i-\epsilon}{2}-\theta_i+\frac{2\epsilon\lambda Q}{\theta^{*I}+\epsilon-(\theta_i-\epsilon)}\right)\left(1-\frac{4\epsilon\lambda Q}{\left(\theta^{*I}+2\epsilon-\theta_i\right)^2}\right) \\ \int_{\theta_i-\epsilon}^{\hat{\theta}(\theta^{*I})}\left(1-\frac{4\epsilon\lambda Q}{\left(\theta^{*I}+\epsilon-\theta\right)^2}\right) d\theta \end{array}\right]$$

By Assumption 1, $4\epsilon\lambda Q > 4\epsilon^2 \geq \left(\theta^{*I} + 2\epsilon - \theta_i\right)^2$ which implies that both terms within the square brackets is negative. Hence $\frac{\partial^2 \pi(\theta_i,\theta^{*I})}{\partial \theta^{*I} \partial \theta_i} \geq 0$ for this region and the function $\pi(\theta_i, \theta^{*I})$ has the single crossing property.

Together with the fact that as $\theta_i \longrightarrow \theta^{*I}$, the slope of the function $\pi(\theta_i, \theta^{*I})$ converges to the slope of the function $\pi(\theta^{*I})$ which is negative for any given θ^{*I}, the single crossing property implies that the slope of $\pi(\theta_i, \theta^{*I})$ cannot be positive for higher values of θ_i for which $\theta^{*I} < \theta_i$. Hence, the required inequality is satisfied.

5. For, $\hat{\theta}(\theta^{*I}) + \epsilon < \theta_i < \theta^{*I} + 2\epsilon$, as the profit function equals

$$\pi(\theta_i, \theta^{*I}) = \frac{1}{4\lambda\epsilon} \int_{\theta_i-\epsilon}^{\theta_i+\epsilon} (p - \theta_i)^2 \, d\theta = \frac{1}{4\lambda\epsilon} (p - \theta_i)^2$$

the derivative is given by

$$-\frac{1}{2\lambda} (p - \theta_i)$$

which is negative for admissible values of p. Hence the condition is satisfied and this proves the proposition.

8.3 Appendix III: Mass of entrants under full and incomplete information

The function is defined under two different scenarios - (1) $\hat{\theta} \leq \theta^{*I}(\delta) + \epsilon$ which holds when $2\lambda\delta < (p-\theta^{*I}(\delta))^2 \leq 2\lambda\delta+4\epsilon\lambda Q$ and (2) $\theta^{*I}(\delta)+\epsilon \leq \hat{\theta}$ which holds when $2\lambda\delta < 2\lambda\delta+4\epsilon\lambda Q < (p - \theta^{*I}(\delta))^2$. As the results are identical for both, we lay out the form of the function under the first one only. The interested reader is referred to the working paper version for details of the second case.

8.3.1 Case 1: $\hat{\theta} \leq \theta^{*I}(\delta) + \epsilon$

Figure 4 depicts the function $(\alpha^I(\delta) - \alpha^F(\delta))$ for this scenario. The functional form is as follows:

A. For $\theta^l \leq \theta \leq \theta^{*I} - \epsilon$, as $\alpha^I(\delta) = 1$,

$$(\alpha^I(\delta) - \alpha^F(\delta)) = 1 - \frac{\sqrt{2\delta\lambda + 4\epsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\epsilon} \tag{25}$$

which is constant in θ.

B. For $\theta^{*I} - \epsilon \leq \theta \leq \hat{\theta} \leq \theta^{*I} + \epsilon$,

$$(\alpha^I(\delta) - \alpha^F(\delta)) = \frac{\theta^{*I} - (\theta - \epsilon)}{2\epsilon} - \frac{\sqrt{2\delta\lambda + 4\epsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\epsilon} \tag{26}$$

which is linear and decreasing in θ.

C. For $\hat{\theta} \leq \theta \leq \theta^{*I} + \epsilon$,

$$(\alpha^I(\delta) - \alpha^F(\delta)) = \frac{\theta^{*I} - (p - \sqrt{2\lambda\delta})}{2\epsilon} \tag{27}$$

after due simplification and is a negative constant in θ.

D. For $\theta^{*I} + \epsilon \leq \theta \leq \theta^h$, as $\alpha^I(\delta) = 0$,

$$(\alpha^I(\delta) - \alpha^F(\delta)) = \frac{(\theta - \epsilon) - (p - \sqrt{2\lambda\delta})}{2\epsilon} \tag{28}$$

after due simplification and is linear and increasing in θ.

E. For $\theta > \theta^h$, the active mass is zero under both incomplete and full information. Hence, $(\alpha^I(\delta) - \alpha^F(\delta)) = 0$

8.4 Appendix IV: Placement bias

8.4.1 Proof of Proposition 5, Part 1

As the permit price function has the same functional form with or without bias, $r(\theta, \theta^{*I,\beta}) = 0$ for all $\theta \in [\theta^{*I,\beta} + \beta - \epsilon, \theta^{*I,\beta} + \beta + \epsilon]$, iff

$$r(\theta^{*I,\beta} + \beta - \epsilon, \theta^{*I,\beta}) = 0$$

which on substitution is equivalent to the condition, $p - \frac{2\theta^{*I,\beta}+\beta-2\epsilon}{2} - \frac{2\epsilon\lambda Q}{2\epsilon-\beta} \leq 0$. This simplifies to $p - \theta^{*I,\beta} \leq \frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2}$.

When $r(\theta, \theta^{*I,\beta}) = 0$ for all $\theta \in [\theta^{*I,\beta} + \beta - \epsilon, \theta^{*I,\beta} + \beta + \epsilon]$, the equilibrium $\theta^{*I,\beta}$ is given by the solution of

$$\frac{1}{2\epsilon} \int_{\theta^{*I,\beta}+\beta-\epsilon}^{\theta^{*I,\beta}+\beta+\epsilon} \frac{1}{2\lambda} (p - \theta^{*I,\beta})^2 d\theta = \delta$$

or $\theta^{*I,\beta} = p - \sqrt{2\lambda\delta}$.

Substituting the expression for $\theta^{*I,\beta}$ in the previous expression, we obtain the following necessary condition on the parameters for the type (1) equilibrium.

$$\delta \leq \frac{1}{2\lambda} (\frac{2\epsilon\lambda Q}{2\epsilon - \beta} - \frac{2\epsilon - \beta}{2})^2$$

We need to show that the solution $\theta^{*I,\beta} = p - \sqrt{2\lambda\delta}$ is an equilibrium.

As we are looking for an equilibrium under which $r(\theta, \theta^{*I,\beta}) = 0$ for all $\theta \in [\theta^{*I,\beta} + \beta - \epsilon, \theta^{*I,\beta} + \beta + \epsilon]$, at the value of θ at which the equilibrium permit price falls to zero, all firms are active. Hence the value of θ at which the equilibrium permit price equals zero is given by $\hat{\theta} = p - \lambda Q < \theta^{*I,\beta} + \beta - \epsilon$.

The individual profit function takes on three different forms depending on three zones in which θ_i may lie. We discuss the condition that each must satisfy and show that these are met.

1. For $\theta_i < \mu^* - \beta - 2\epsilon$, we require

$$\pi(\theta_i, \theta^{*I,\beta}) = \frac{1}{4\lambda\epsilon}\int_{\theta_i+\beta-\epsilon}^{\theta_i+\beta+\epsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta > \delta$$

The condition is true for $\beta = 0$ as shown in Proposition 2. Since the integrand is increasing and convex in θ, the above is true for $\beta > 0$.

2. For $\theta^{*I,\beta} - \beta - 2\epsilon \leq \theta_i \leq \theta^{*I,\beta} - \beta$, the profit function and the required condition are

$$\pi(\theta_i, \theta^{*I,\beta}) = \frac{1}{4\lambda\epsilon}\int_{\theta_i+\beta-\epsilon}^{p-\lambda Q} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\epsilon}\int_{p-\lambda Q}^{\theta_i+\beta+\epsilon} (p - \theta_i)^2 \, d\theta > \delta$$

Upon simplification,

$$\pi(\theta_i, \theta^{*I,\beta}) = \frac{1}{4\lambda\epsilon}\left[\frac{(p-\theta_i)^3}{3} - \frac{(\lambda Q + \beta - \epsilon)^3}{3} + (p-\theta_i)^2(\theta_i + \beta + \epsilon - p + \lambda Q)\right]$$

The expression equals $\frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\epsilon^2}{6\lambda}$ when $\theta_i + \beta + \epsilon = p - \lambda Q$, implying that the profit function is continuous at this value of θ_i.

It is easy to show that the derivative of the function with respect to θ_i is negative. Hence, $\pi(\theta_i, \theta^{*I,\beta})$ is strictly declining in θ_i through the range under consideration. Since $\pi(\theta_i, \theta^{*I,\beta}) \longrightarrow \delta$ as $\theta_i \longrightarrow \theta^{*I,\beta}$, the condition is satisfied.

3. When$\theta^{*I,\beta} - \beta < \theta_i < \theta^{*I,\beta}$, we must have

$$\pi(\theta_i, \theta^{*I,\beta}) = \frac{1}{2\epsilon}\int_{\theta_i+\beta-\epsilon}^{\theta_i+\beta+\epsilon} \frac{1}{2\lambda}(p-\theta_i)^2 \geq \delta$$

Since $\frac{1}{2\epsilon}\int_{\theta_i+\beta-\epsilon}^{\theta_i+\beta+\epsilon} \frac{1}{2\lambda}(p-\theta_i)^2 = \frac{1}{2\lambda}(p-\theta_i)^2$, and the latter is strictly declining in θ_i, the condition is satisfied, because at $\theta_i = \theta^{*I,\beta}$, $\frac{1}{2\lambda}(p-\theta_i)^2 = \delta$.

4. When $\theta^{*I,\beta} < \theta_i$, we must have

$$\pi(\theta_i, \mu^*) = \frac{1}{2\epsilon}\int_{\theta_i+\beta-\epsilon}^{\theta_i+\beta+\epsilon} \frac{1}{2\lambda}(p-\theta_i)^2 < \delta$$

By the same arguments as in the previous step, the condition is satisfied.

8.4.2 Proof of Proposition 5, Part 2

As in the case of $\beta = 0$, the Proposition will be proved through multiple steps.

STEP 1.: To show that the interval $(\frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2})^2, \frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\epsilon^2}{6\lambda}]$, it suffices to show that under appropriate restrictions on the parameters,

$$\left(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2}\right)^2 < (\lambda Q+\beta)^2, \text{ or, } \left(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2}\right) < (\lambda Q+\beta)$$

which on simplification turns out to be

$$\lambda Q < \frac{(2\epsilon-\beta)(2\epsilon+\beta)}{2\beta}$$

For any value of $\beta \in (0, \epsilon]$, there exists an upper bound on λQ, for which the above inequality is satisfied. When $\beta = \epsilon$, the desired interval is non-empty if $\lambda Q < \frac{3}{2}\epsilon$. The upper bound is higher for lower β.

STEP II: We next show that there is a unique solution $\theta^{*I,\beta}$ that satisfy (17).

Consider the function,

$$\pi(k) = \frac{1}{2\epsilon}\int_{k+\beta-\epsilon}^{\hat{\theta}(k)} \frac{1}{2\lambda}\left(\frac{k+\theta-\epsilon}{2} + \frac{2\epsilon\lambda Q}{k-(\theta-\epsilon)} - k\right)^2 d\theta + \frac{1}{2\epsilon}\int_{\hat{\theta}(k)}^{k+\beta+\epsilon} \frac{1}{2\lambda}(p-k)^2 d\theta$$

where $\hat{\theta}(k) = p + \epsilon - \sqrt{(p-k)^2 + 4\epsilon\lambda Q}$. On simplification,

$$\pi(k) = \frac{1}{4\lambda\epsilon}\int_{k+\beta-\epsilon}^{\hat{\theta}(k)}\left(\frac{4\left(\epsilon\lambda Q\right)^2}{\left(\theta-(k+\epsilon)\right)^2}+\frac{\left(\theta-(k+\epsilon)\right)^2}{4}-2\epsilon\lambda Q\right)d\theta$$
$$+\frac{1}{4\lambda\epsilon}\left(p-k\right)^2\left[(k+\beta-p)+\sqrt{\left(p-k\right)^2+4\epsilon\lambda Q}\right]$$

As before, a change of variable $z=\theta-(k+\epsilon)$ allows us to evaluate the first integral. With this change of variable, the upper and lower limits of the integration are, respectively, $\hat{\theta}\left(k\right)-k-\epsilon=(p-k)-\sqrt{\left(p-k\right)^2+4\epsilon\lambda Q}$ and $\beta-2\epsilon$. Evaluating the integral using the new variable and then substituting the new variable back and simplifying, we have,

$$\pi\left(k\right)=\frac{1}{4\lambda\epsilon}\left[\begin{array}{c} -\frac{4(\epsilon\lambda Q)^2}{(p-k)-\sqrt{(p-k)^2+4\epsilon\lambda Q}}-\frac{4(\epsilon\lambda Q)^2}{2\epsilon}+\frac{\left((p-k)-\sqrt{(p-k)^2+4\epsilon\lambda Q}\right)^3}{12} \\ +\frac{(2\epsilon-\beta)^3}{12}-2\epsilon\lambda Q\left[(p-k)-\sqrt{\left(p-k\right)^2+4\epsilon\lambda Q}+2\epsilon-\beta\right] \\ -\left[(p-k)-\beta-\sqrt{\left(p-k\right)^2+4\epsilon\lambda Q}\right]\left(p-k\right)^2 \end{array}\right] \tag{29}$$

We try to show next that $\frac{d\pi(k)}{dk}<0$. A second change of variable helps us to do that. Define

$$x\equiv\sqrt{\left(p-k\right)^2+4\epsilon\lambda Q}-(p-k)>0$$

and note that

$$\frac{dx}{dk}=1-\frac{p-k}{\sqrt{\left(p-k\right)^2+4\epsilon\lambda Q}}>0$$

Further note that $\left(p-k\right)^2=\frac{x^2}{4}+\frac{(2\epsilon\lambda Q)^2}{x^2}-2\epsilon\lambda Q$.

With the second change of variable, $\pi(k)$ can be rewritten as

$$\pi\left(k\right)=\frac{1}{4\lambda\epsilon}\left[\begin{array}{c} -\frac{4(\epsilon\lambda Q)^2}{2\epsilon-\beta}+\frac{(2\epsilon-\beta)^3}{12}-2\epsilon\lambda Q\left(2\epsilon-\beta\right) \\ +\frac{x^3}{6}+\frac{8(\epsilon\lambda Q)^2}{x}+\beta(\frac{x}{2}-\frac{2\epsilon\lambda Q}{x})^2 \end{array}\right]$$

Thus, whether $\pi(k)$ is increasing or decreasing in k depends on whether it decreases or increases in x.

$$\frac{d\pi}{dx} = \frac{1}{4\lambda\epsilon}\left[\frac{x^2}{2} - \frac{8(\epsilon\lambda Q)^2}{x^2}\right] + \beta\left(\frac{x}{2} - \frac{2\epsilon\lambda Q}{x}\right)\left(\frac{1}{2} + \frac{2\epsilon\lambda Q}{x}\right)$$
$$= \frac{1}{4\lambda\epsilon}\left(\frac{x}{\sqrt{2}} + \frac{2\sqrt{2}\epsilon\lambda Q}{x}\right)\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\epsilon\lambda Q}{x}\right) + \beta\left(\frac{x}{2} - \frac{2\epsilon\lambda Q}{x}\right)\left(\frac{1}{2} + \frac{2\epsilon\lambda Q}{x}\right)$$

Thus the sign of $\frac{d\pi(k)}{dk}$ depends on the signs of the terms, $\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\epsilon\lambda Q}{x}\right)$ and $\left(\frac{x}{2} - \frac{2\epsilon\lambda Q}{x}\right)$.

Substituting the expression for x back and simplifying, it is straightforward to check that so long as $(p-k) > 0$ (true for values of k we are interested in),

$$\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\epsilon\lambda Q}{x}\right) = -\sqrt{2}(p-k) < 0$$

and

$$\left(\frac{x}{2} - \frac{2\epsilon\lambda Q}{x}\right) = -(p-k) < 0$$

Thus $\frac{d\pi(k)}{dk} < 0$.

We next need to show that the function $\pi(k)$ is greater than δ for some k and less than δ for some k.

As before, for any given k, the following inequality is true.

$$\pi(k) \leq \frac{1}{2\epsilon}\int_{k+\beta-\epsilon}^{k+\beta+\epsilon} \frac{1}{2\lambda}(p-k)^2\, d\theta = \frac{1}{2\lambda}(p-k)^2 \tag{30}$$

Moreover, for $k = p - (\lambda Q - \epsilon)$, $\pi(k) \leq \frac{1}{2\lambda}(p-k)^2 = \frac{1}{2\lambda}(\lambda Q - \epsilon)^2$. Since, $\frac{1}{2\lambda}(\lambda Q - \epsilon)^2 < (\frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2})^2 < \delta$ for $0 < \beta \leq 2\epsilon$, $\pi(k) < \delta$ for some value of k.

Similarly, for any given k, the following inequality is true for $\theta \leq p - \lambda Q$.

$$\frac{1}{2\epsilon}\int_{k+\beta-\epsilon}^{k+\beta+\epsilon} \frac{1}{2\lambda}(\theta + \lambda Q - k)^2\, d\theta \leq \pi(k) \tag{31}$$

The inequality is true by the following arguments. At $\theta = k + \beta - \epsilon$, the expressions within integral signs have following values:

$$\theta + \lambda Q - k = \lambda Q + \beta - \epsilon$$

$$\left(\frac{k+\theta-\epsilon}{2} + \frac{2\epsilon\lambda Q}{k-(\theta-\epsilon)} - k\right) = \left(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2}\right)$$

Note that,

$$\left(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - \frac{2\epsilon-\beta}{2}\right) - (\lambda Q + \beta - \epsilon) = \frac{\beta}{2}\left(\frac{2\epsilon\lambda Q}{2\epsilon-\beta} - 1\right) > 0,$$

since $\frac{2\epsilon\lambda Q}{2\epsilon-\beta} > 1$.

Hence, for $\theta = k + \beta - \epsilon$,

$$\theta + \lambda Q - k < \left(\frac{k+\theta-\epsilon}{2} + \frac{2\epsilon\lambda Q}{k-(\theta-\epsilon)}\right)$$

Both functions are increasing in θ, but the slope of $\theta + \lambda Q$ is 1 and following the same steps as in Proposition 2, we can show that the slope of the RHS expression is greater than 1.

Hence, for any given k and $\theta \leq p - \lambda Q$, inequality (31) is true.

Thus, for $k = p - \lambda Q - \epsilon$,

$$\pi(k) \geq \frac{1}{2\epsilon}\int_{k-\epsilon}^{k+\epsilon} \frac{1}{2\lambda}(\theta + \lambda Q - k)^2\, d\theta = \frac{1}{12\lambda\epsilon}\left[(\lambda Q + \beta + \epsilon)^3 - (\lambda Q + \beta - \epsilon)^3\right] = \frac{(\lambda Q + \beta)^2}{2\lambda} + \frac{\epsilon^2}{6\lambda} \geq \delta$$

Hence $\pi(k)$ has a unique intersection $k = \theta^{*I,\beta}$ with δ.

STEP III: We next show that the switching strategy with the threshold μ^* is an equilibrium. As before, we need to show that for any firm of type θ_i, $\pi(\theta_i, \theta^{*I,\beta}) > \delta$ for $\theta_i < \theta^{*I,\beta}$ and $\pi(\theta_i, \theta^{*I,\beta}) < \delta$ for $\theta_i > \theta^{*I,\beta}$.

The following list characterizes the individual profit function $\pi(\theta_i, \theta^{*I,\beta})$, for each zone in which θ_i may lie and provides the condition that the profit function must satisfy. The rational for the form of the profit function for each zone is identical to that for the no-bias ($\beta = 0$) case and is therefore omitted. The only exception is zone 4 below which is new to $\beta > 0$.

1. $\theta_i < \theta^{*I,\beta} - \beta - 2\epsilon$

$$\pi(\theta_i, \theta^{*I,\beta}) = \frac{1}{4\lambda\epsilon}\int_{\theta_i+\beta-\epsilon}^{\theta_i+\beta+\epsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta > \delta$$

2. $\theta^{*I,\beta} - \beta - 2\epsilon < \theta_i < \hat{\theta}(\theta^{*I,\beta}) - \beta - \epsilon < \theta^{*I,\beta} - \beta$.

$$\pi(\theta_i, \theta^{*I,\beta}) = \frac{1}{4\lambda\epsilon}\left[\int_{\theta_i+\beta-\epsilon}^{\theta^{*I,\beta}-\epsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta \right.$$
$$\left. + \int_{\theta^{*I,\beta}-\epsilon}^{\theta_i+\beta+\epsilon} \left(\frac{\theta^{*I,\beta} + \theta - \epsilon}{2} - \theta_i + \frac{2\epsilon\lambda Q}{\theta^{*I,\beta} - (\theta - \epsilon)}\right)^2 d\theta\right] > \delta$$

3. $\hat{\theta}(\theta^{*I,\beta}) - \beta - \epsilon < \theta_i < \theta^{*I,\beta} - \beta$

$$\pi(\theta_i, \theta^{*I,\beta}) = \frac{1}{4\lambda\epsilon}\left[\int_{\theta_i+\beta-\epsilon}^{\theta^{*I,\beta}-\epsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta \right.$$
$$+ \int_{\theta^{*I,\beta}-\epsilon}^{\hat{\theta}(\theta^{*I,\beta})} \left(\frac{\theta^{*I,\beta} + \theta - \epsilon}{2} - \theta_i + \frac{2\epsilon\lambda Q}{\theta^{*I,\beta} - (\theta - \epsilon)}\right)^2 d\theta$$
$$\left. + \int_{\hat{\theta}(\theta^{*I,\beta})}^{\theta_i+\beta+\epsilon} (p - \theta_i)^2 \, d\theta\right] > \delta$$

4. $\theta^{*I,\beta} - \beta < \theta_i < \theta^{*I,\beta}$.

$$\pi(\theta_i, \theta^{*I,\beta}) = \int_{\theta_i+\beta-\epsilon}^{\hat{\theta}(\theta^{*I,\beta})} \left(\frac{\theta^{*I,\beta} + \theta - \epsilon}{2} - \theta_i + \frac{2\epsilon\lambda Q}{\theta^{*I,\beta} - (\theta - \epsilon)}\right)^2 d\theta + \int_{\hat{\theta}(\theta^{*I,\beta})}^{\theta_i+\beta+\epsilon} (p - \theta_i)^2 \, d\theta > \delta$$

Since $\theta \in [\theta_i + \beta - \epsilon, \theta_i + \beta + \epsilon]$, $\theta^{*I,\beta} - \beta < \theta_i \implies \theta^{*I,\beta} - \epsilon < \theta$ for all possible values of θ. Hence, it is never the case that all firms are active and this explains the form of the profit function.

5. $\theta^{*I,\beta} < \theta_i < \hat{\theta}(\theta^{*I,\beta}) - \beta + \epsilon$

$$\pi(\theta_i, \theta^{*I,\beta}) = \int_{\theta_i+\beta-\epsilon}^{\hat{\theta}(\theta^{*I,\beta})} \left(\frac{\theta^{*I,\beta} + \theta - \epsilon}{2} - \theta_i + \frac{2\epsilon\lambda Q}{\theta^{*I,\beta} - (\theta - \epsilon)} \right)^2 d\theta + \int_{\hat{\theta}(\theta^{*I,\beta})}^{\theta_i+\beta+\epsilon} (p - \theta_i)^2 \, d\theta < \delta$$

6. $\hat{\theta}(\theta^{*I,\beta}) - \beta + \epsilon < \theta_i$

$$\pi(\theta_i, \theta^{*I,\beta}) = \frac{1}{4\lambda\epsilon} \int_{\theta_i+\beta-\epsilon}^{\theta_i+\beta+\epsilon} (p - \theta_i)^2 \, d\theta < \delta$$

To prove the rest of the proposition, we need to show that the required condition for each zone is satisfied.

1. For $\theta_i < \theta^{*I,\beta} - 2\epsilon$,

$$\pi(\theta_i, \theta^{*I,\beta}) = \frac{1}{4\lambda\epsilon} \int_{\theta_i+\beta-\epsilon}^{\theta_i+\beta+\epsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta = \frac{1}{12\lambda\epsilon} \left[(\lambda Q + \beta + \epsilon)^3 - (\lambda Q + \beta - \epsilon)^3 \right]$$

   The last expression equals $\frac{(\lambda Q + \beta)^2}{2\lambda} + \frac{\epsilon^2}{6\lambda} \geq \delta$.

2. The proof for this region is identical to the one for the case $\beta = 0$ and is hence omitted.

3. We shall verify the inequalities for the next three regions together.

   Using the same arguments as in Step III of Proposition 2, we note that the slopes of the two functions, $\pi(\theta_i, \theta^{*I,\beta})$ and $\pi(\theta^{*I,\beta})$ must converge as $\theta_i \longrightarrow \theta^{*I,\beta}$ and in particular $\pi(\theta_i, \theta^{*I,\beta})$ must be declining at $\theta_i = \theta^{*I,\beta}$. These statements taken together imply that $\pi(\theta_i, \theta^{*I,\beta})$ must have at least one stationary point that is a maximum in the interval, $[\hat{\theta}(\theta^{*I,\beta}) - \beta - \epsilon, \theta^{*I,\beta}]$ which includes the region $[\hat{\theta}(\theta^{*I,\beta}) - \beta - \epsilon, \theta^{*I,\beta} - \beta]$.

We therefore check the roots of the derivative of $\pi(\theta_i, \theta^{*I,\beta})$ with respect to θ_i.

The derivatives of the first and the second term of $\pi(\theta_i, \theta^{*I,\beta})$ with respect to θ_i are the same as the derivatives of the first and second terms of $\pi(\theta_i, \theta^{*I})$ in Proposition 2.

The derivative of the third term is given by

$$\frac{1}{4\lambda\epsilon}\left[3\left(p-\theta_i\right)^2-2\left(p-\theta_i\right)\left(\sqrt{\left(p-\theta^{*I,\beta}\right)^2+4\epsilon\lambda Q}+\beta\right)\right]$$

which is more conveniently written as,

$$\frac{1}{4\lambda\epsilon}\left[3\left(p-\theta_i\right)^2-2\left(p-\theta_i\right)\left(p+\epsilon-\hat{\theta}(\theta^{*I,\beta})+\beta\right)\right]$$

As before, these terms can be combined to get

$$\frac{d\pi(\theta_i, \theta^{*I,\beta})}{d\theta_i}=\frac{1}{2\lambda\epsilon}\left[\theta_i^2-2\left(p-\frac{\lambda Q+\epsilon+\beta}{2}\right)\theta_i+\Omega\left[\theta^{*I,\beta},\beta\right]\right] \tag{32}$$

where

$$\Omega\left[\theta^{*I,\beta},\beta\right]\equiv\left(\theta^{*I,\beta}-\epsilon\right)^2-\frac{\left(\hat{\theta}(\theta^{*I,\beta})+\mu^*-\epsilon\right)^2}{4}+2\epsilon\lambda Q\log\left[1-\frac{\hat{\theta}(\theta^{*I,\beta})-\left(\theta^{*I,\beta}-\epsilon\right)}{2\epsilon}\right]$$
$$+\frac{p^2-\left(\lambda Q+\theta^{*I,\beta}-\epsilon\right)^2}{2}+p\left(\hat{\theta}(\theta^{*I,\beta})-\epsilon-\beta\right)$$

Derivative (32) has two roots given by

$$\theta_{1,2}^R\left(\theta^{*I,\beta}\right)\equiv\left(p-\frac{\lambda Q+\epsilon+\beta}{2}\right)\pm\sqrt{\left(p-\frac{\lambda Q+\epsilon+\beta}{2}\right)^2-\Omega\left[\theta^{*I,\beta},\beta\right]},$$

Using the same steps as in the case of $\beta=0$ in Proposition 2, we show that both roots cannot be less than $\theta^{*I,\beta}-\beta$ because of a contradiction.

We next show that $\frac{d\pi(\theta_i,\theta^{*I,\beta})}{d\theta_i} < 0$ for $\theta_i \in [\theta^{*I,\beta}-\beta, \hat{\theta}(\theta^{*I,\beta})-\beta+\epsilon]$. If the last statement is true, then the necessary maxima of $\pi(\theta_i, \theta^{*I,\beta})$ lies in the interval, $[\hat{\theta}(\theta^{*I,\beta})-\beta-\epsilon, \theta^{*I,\beta}-\beta]$ and is unique.

As the form of the function, $\pi(\theta_i, \theta^{*I,\beta})$, is identical over the sub-intervals $[\theta^{*I,\beta}-\beta, \mu^*]$ and $[\theta^{*I,\beta}, \hat{\theta}(\theta^{*I,\beta})-\beta+\epsilon]$, the arguments put forth for the function, $\pi(\theta_i, \theta^{*I})$ for region 4 in Proposition 2 apply and $\frac{d\pi(\theta_i,\theta^{*I,\beta})}{d\theta_i} < 0$ for the entire interval $\theta_i \in [\theta^{*I,\beta}-\beta, \hat{\theta}(\theta^{*I,\beta})-\beta+\epsilon]$. Thus $\pi(\theta_i, \theta^{*I,\beta})$ has a unique maxima in $[\hat{\theta}(\theta^{*I,\beta})-\beta-\epsilon, \theta^{*I,\beta}-\beta]$. Hence all the three required inequalities for regions 3, 4 and 5 are satisfied.

6. The required inequality follows from the same argument provided for region 5 in Proposition 2.

8.4.3 Proof of Proposition 6

PROOF: For $\delta \in (\frac{1}{2\lambda}(\lambda Q-\epsilon)^2, \frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta}-\frac{2\epsilon-\beta}{2})^2]$, the threshold $\theta^{*I,\beta}=p-\sqrt{2\delta\lambda}$ whereas the threshold $\theta^{*I}<p-\sqrt{2\delta\lambda}$, as Propositions 3 and 5 show. Hence the result is true.

For $\delta \in (\frac{1}{2\lambda}(\frac{2\epsilon\lambda Q}{2\epsilon-\beta}-\frac{2\epsilon-\beta}{2})^2, \frac{(\lambda Q)^2}{2\lambda}+\frac{\epsilon^2}{6\lambda}]$ the entry threshold $\theta^{*I,\beta}$ is determined by the solution of the following equation, for a given β and δ

$$\pi(k,\beta)=\frac{1}{2\epsilon}\int_{k+\beta-\epsilon}^{\hat{\theta}(k)}\frac{1}{2\lambda}\left(\frac{k+\theta-\epsilon}{2}+\frac{2\epsilon\lambda Q}{k-(\theta-\epsilon)}-k\right)^2 d\theta+\frac{1}{2\epsilon}\int_{\hat{\theta}(k)}^{k+\beta+\epsilon}\frac{1}{2\lambda}(p-k)^2 d\theta=\delta$$

Note that the expression within brackets in the first term is increasing and convex in θ. Hence, as the limits of the integrals are increasing in β, $\pi(k,\beta)$ is increasing in β. Hence, the solution is increasing in β. Δ

8.4.4 Comparative statics of $Av(\alpha^{*I,\beta}-\alpha^{*I})$

It is straightforward to show that $Av(\alpha^{*I,\beta}(\delta)-\alpha^{*I}(\delta))=(\theta^{*I,\beta}(\delta)-\theta^{*I}(\delta))\geq 0$ from equation (19).

Denoting the expected payoff to a firm with placement bias by $\pi(k, \beta)$ and using implicit function theorem, we have

$$\begin{aligned}\frac{dE(\alpha^{*I,\beta}(\delta) - \alpha^{*I}(\delta))}{d\delta} &= \frac{d(\theta^{*I,\beta}(\delta) - \theta^{*I}(\delta))}{d\delta} \\ &= \frac{1}{\pi'(k) - 2\beta(p-k)} - \frac{1}{\pi'(k)} \\ &= \frac{2\beta(p-k)}{\pi'(k)(\pi'(k) - 2\beta(p-k))} > 0\end{aligned}$$

since $\pi'(k)$ and $\pi'(k) - 2\beta(p-k)$ are negative.