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A mass formula for the baryon mass spectrum

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Abstract
A mass spectrum of the form m~ = M^2 = Ml (M1 + |al |) is derived, where a l is the imaginary part of the l th singularity in the complex energy plane of a T-matrix which describes the interaction of a particle of initial rest mass m with a potential V(x, t), where V(x, t) = 0 in the limit |t| -> oo. This spectrum is found to give, within 1%, the mass of all baryons less than or equal to the mass of the rr if m is taken as 1747 MeV and a1l is a small integral multiple of the K and pi meson masses. Comparison to the Gell-Mann, Okubo formula relates these integers to the hypercharge and the isotopic spin. The analysis proposed by R. L. Lander is used to show that the results can have statistical significance.

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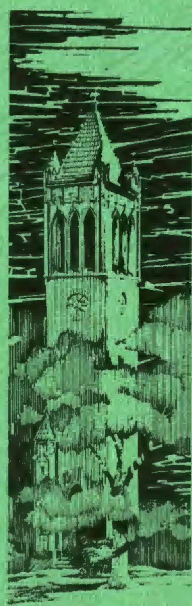

IOWA STATE UNIVERSITY

A MASS FORMULA FOR THE BARYON MASS SPECTRUM

by

Charles L. Hammer and Thomas A. Weber

AMES LABORATORY

RESEARCH AND DEVELOPMENT REPORT

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ABSTRACT

A mass spectrum of the form $m^2 = M_\ell (M_\ell + |\alpha_\ell|)$ is derived, where α_ℓ is the imaginary part of the ℓ^{th} singularity in the complex energy plane of a T-matrix which describes the interaction of a particle of initial rest mass m with a potential V(x, t), where V(x, t) = 0 in the limit $|t| \rightarrow \infty$. This spectrum is found to give, within 1%, the mass of all baryons less than or equal to the mass of the Ω^- if m is taken as 1747 MeV and α_ℓ is a small integral multiple of the K and π meson masses. Comparison to the Gell-Mann, Okubo formula relates these integers to the hypercharge and the isotopic spin. The analysis proposed by R. L. Lander is used to show that the results can have statistical significance.

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RESULTS AND DISCUSSION

A recent approach[1] for obtaining the asymptotic limit of Fourier type integral is applied to potential scattering. As a consequence it appears that the U-matrix formalism can be interpreted to yield a mass spectrum of the form

$$m^2 \quad M_\ell (M_\ell + |\alpha_\ell|) , \tag{1}$$

where m is the rest mass of a particle with incident momentum zero, M_ℓ is the rest mass of the particle after scattering and α_ℓ is the imaginary part of the ℓ^{th} singularity in the complex energy plane of a T-matrix. As will be shown in a later paper, this result also applies to the case of a particle being scattered by its anti-particle. This spectrum can be empirically fitted to the baryon mass spectrum with an accuracy better than 1% by choosing m = 1747 meV and $\alpha_\ell = km_K + nm_\pi$ where k and n are small integers and m_K and m_π are the K° and π° meson masses. In the octet and the decuplet the difference in strangeness of the baryons is accounted for by the number of K mesons and their parity is related to the total number of mesons, k + n.

Consider the Schroedinger wave equation

$$[H_o + V(x, t)]\psi_s = i \frac{\partial}{\partial t} \psi_s , \quad (\hbar = c = 1) \tag{2}$$

where $V(x, t) \to 0$ faster than $|t|^{-1}$ for $|t| \to \infty$ and H_o is the free particle Hamiltonian. A formal solution to this equation is

$$\psi_s = (2\pi)^{-3/2}[e^{i\underset{\sim}{p}_o \cdot \underset{\sim}{x} - E_o t} - i\int d\underset{\sim}{p} e^{i(\underset{\sim}{p}\cdot\underset{\sim}{x} - Et)} \int_{-\infty}^{t} d\xi e^{i\omega_{pp_o}\xi} < p|VU_s|p_o > , \quad (3a)$$

where

$$|p> = (2\pi)^{-3/2} e^{i\underset{\sim}{p}\cdot\underset{\sim}{x}},$$

$$\omega_{pp_o} = (p^2 + m^2)^{\frac{1}{2}} - (p_o{}^2 + m^2)^{\frac{1}{2}}$$

$$= E - E_o \quad (3b)$$

and

$$\psi_s = U_s(t, -\infty) e^{-iE_o t} |p_o> ,$$

$$U_s(-\infty, -\infty) = 1 . \quad (3c)$$

If the substitution

$$<p|VU_s|p_o> = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} d\sigma <p|g(\sigma)|p_o> e^{i\sigma t} , \quad (4)$$

is made in the time integral of Eq. (3), then

$$I = \int_{-\infty}^{t} d\xi e^{i\omega_{pp_o}\xi} <p|VU_s|p_o> \quad (5)$$

$$I = -i(2\pi)^{-\frac{1}{2}} \lim_{\epsilon \to 0} \int_{-\infty}^{\infty} d\sigma e^{i(\sigma + \omega_{pp_o} - i\epsilon)t} \frac{<p|g(\sigma - i\epsilon)|p_o>}{\sigma + \omega_{pp_o} - i\epsilon} . \tag{6}$$

The scattering solutions are found by taking the asymptotic limit of $\psi_s(t)$ for large $|\underset{\sim}{x}|$ and t. Thus if $V(\underset{\sim}{x}, t)$ is considered localized in the vicinity of $|\underset{\sim}{x}| = 0$, one obtains in the usual way,

$$\psi_s(t)_{|\underset{\sim}{x}| \to \infty} = \frac{1}{(2\pi)^{3/2}} e^{i(\underset{\sim}{p}_o \cdot \underset{\sim}{x} - E_o t)} - \frac{(2\pi)^{-\frac{1}{2}}}{|\underset{\sim}{x}|} \int_{-\infty}^{\infty} p dp e^{i(p|\underset{\sim}{x}| - Et)} I , \tag{7}$$

where

$$\underset{\sim}{p} = p(\underset{\sim}{x}/|\underset{\sim}{x}|) . \tag{8}$$

The evaluation of I from Eq. (6) for $t > 0$ is easily done by closing the contour in the upper half complex σ-plane with a semicircle the radius of which is allowed to approach infinity and indented about branch points where necessary. For simplicity of presentation $g(\sigma - i\epsilon)$ is assumed to have only simple poles although the results obtained apply equally well for branch points and poles of any order.

Finally, the p integration of Eq. (7) is performed by contour methods. By the methods of reference 1 it is easily shown that the branch point at $p = im$ gives a term of order $|x|^{-3/2} e^{-m|x|}$ which does not contribute to the scattering solution. Also the function I is analytic in the left half p complex plane as it should be in order to insure outgoing spherical waves only. As a result, Eq. (7) can be written as

$$\psi_s(t) \underset{|\underset{\sim}{x}|\to\infty}{=} (2\pi)^{-3/2} e^{i(\underset{\sim}{p}_o \cdot \underset{\sim}{x} - E_o t)} - |\underset{\sim}{x}|^{-1} I_C , \tag{9}$$

where

$$I_C = \int_C p\,dp\,e^{i[p\xi - (p^2+m^2)^{\frac{1}{2}}]t} \langle p|\mathcal{G}(-\omega_{pp_o})|p_o\rangle , \tag{10}$$

$$\xi = (|\underset{\sim}{x}|/t) , \tag{11}$$

and

$\mathcal{G}(-\omega_{pp_o})$ means consider only those singularities of $g(-\omega_{pp_o})$ that are in the first and fourth quadrants of the complex p-plane.

The integral I_C can be evaluated asymptotically for large t considering ξ, which is determined by the position $|\underset{\sim}{x}|$ of the detectors and the time t of observation, to be finite. Thus, consider the contour shown in Fig. 1. The phases shown on the figure are chosen to make $(p^2+m^2)^{\frac{1}{2}}$ positive on the real axis. The contributions from the contours C_1 and C_3 at $|p| = \infty$ are zero because of exponential damping. The contour C_2 is uniquely determined by the requirement that the integral I_{C_2} be a Laplace transform so that it can be directly evaluated asymptotically using the Laplace method without further distortion of the contour. This follows from considering the transformation suggested by the exponent of the exponential in Eq. (10),

$$\zeta = p\xi - (p^2 + m^2)^{\frac{1}{2}} . \tag{12}$$

The contour in the ζ-plane corresponding to C_2 in the p-plane is the contour around the branch point

$$\zeta_o = -m(1-\xi^2)^{\frac{1}{2}} , \tag{13}$$

from $\zeta_o + i\infty$ with phase $\frac{-3\pi}{2}$ to $\zeta_o + i\infty$ with phase $\frac{\pi}{2}$. The asymptotic limit of I_{C_2} is found to be of order $|\underset{\sim}{x}|^{\frac{1}{2}}$ and thus does not contribute to the scattering solution.

Therefore applying the residue theorem to Eq. (9) shows that for each pole, given by

$$-\omega_{pp_o} = \beta_\ell + i\alpha_\ell ; \quad \beta_\ell < E_o \tag{14}$$

within the contour, one obtains an outgoing spherical wave of the form

$$|\underset{\sim}{x}|^{-1} e^{i[q_{R\ell}|\underset{\sim}{x}| - (E_o - \beta_\ell)]t} e^{-\alpha_\ell t'} f_q(\theta, \phi) \tag{15}$$

where

$$p = q_{R\ell} + iq_{I\ell} \tag{16}$$

is the location of the pole and

$$t' = t + (q_{I\ell}/\alpha_\ell)|\underset{\sim}{x}| . \tag{17}$$

The quantity $(q_{I\ell}/\alpha_\ell)$ is obtained from the imaginary part of Eq. (14) as

$$q_{R\ell} q_{I\ell} = -\alpha_\ell (E_o - \beta_\ell) . \tag{18}$$

Thus Eq. (17) becomes

$$t' = t - (E_o - \beta_\ell)(|\underset{\sim}{x}|/q_{R\ell}) \,. \tag{19}$$

The interpretation of the results expressed by Eqs. (18) and (19) follows from the fact that the singularities $(\beta_\ell + i\alpha_\ell)$ can be related to the singularities which give rise to the "decay" states of S-matrix theory.[4] For example, for spinless particles, I_C (see Eq. (10)) can be expanded in the form

$$I_C = \sum_\ell \left(\frac{2\ell + 1}{4\pi q_{R\ell}}\right) T_\ell \, P_\ell(\cos\theta) e^{i(q_\ell |\underset{\sim}{x}| - E_\ell t)}$$

where

$$E_\ell = E_o - \beta_\ell - i\alpha_\ell = \sqrt{q_\ell^2 + m^2}$$

are those singularities of $\langle p | \mathcal{G}(-\omega_{pp_o}) | p_o \rangle$ corresponding to a given angular momentum ℓ of the initial or final state. Substitution for I_C from this equation into Eq. (9) then shows that the expansion coefficients T_ℓ are the off-diagonal elements of the S-matrix.

Therefore, Eqs. (15) and (19) are interpreted as giving the probability at the time t' of a state of energy E_o located at $|\underset{\sim}{x}| = 0$ for the decay into a state of energy $E_o - \beta_\ell$, momentum $q_{R\ell}$ and speed $[q_{R\ell}/(E_o - \beta_\ell)]$ which is observed at the time t at the position $(|\underset{\sim}{x}|, \theta, \varphi)$.

If $\beta_\ell = 0$ both incident and scattered waves have the same energy E_o so that if $p_o = 0$ then

$$E_o^2 = m^2 = q_{R\ell}^2 + M_\ell^2 , \tag{20}$$

$$q_{R\ell}^2 = \tfrac{1}{2}\left[|\alpha_\ell| (\alpha_\ell^2 + 4m^2)^{\frac{1}{2}} - \alpha_\ell^2 \right] . \tag{21}$$

Substitution for $q_{R\ell}^2$ in Eq. (20) from Eq. (21) gives Eq. (1) upon simplification.

It should be emphasized that the pole characterized by α_ℓ can arise either from the time dependence of the U-matrix or from the time dependence of the interaction $V(\underset{\sim}{x}, t)$. Thus, in the usual "adiabatic" limit, the interaction becomes independent of time, the singularity, if present, must come from the U-matrix alone.

Examination of Eq. (1) shows that $m > M_\ell$ so that the largest observed mass M_ℓ occurs for the smallest, non-trivial value of α_ℓ. In the baryon mass spectrum a reasonable choice for the most massive baryon is the mass of the Ω^-, M_Ω, and for the smallest "intrinsic" time, the mass of the π meson, m_π. Substitution for these quantities in Eq. (1) gives for

$$M_\Omega = 1681 \text{ MeV} .$$

$$m_\pi = 134.97 \; , \qquad (22)$$

the result

$$m^2 = M_\Omega(M_\Omega + m_\pi) \; ,$$

$$= 3.053355 \times 10^6 \, \mathrm{MeV}^2 \; . \qquad (23)$$

The "intrinsic" times for the observed baryon masses less than m can now be determined. As seen from the quantities in parentheses in Table I the surprising result is that α_ℓ is either an integral number of K meson masses (m_K = 497.9 MeV) or an integral number of π meson masses.

The analysis proposed by Richard L. Lander[5] can be used to show that this observation is statistically significant. The results of this analysis are summarized in Fig. 2(a). Each point represents the percentage error for a given mass. This percentage error represents the absolute value of the ratio of the difference between the observed mass and the nearest calculated mass to the difference between the nearest calculated mass and the next nearest calculated mass that brackets the observed mass. For example, the abscissa for the cascade particle Ξ is

$$x = \left| \frac{M_{2m_K} - 1321}{M_{2m_K} - M_{7m_\pi}} \right| .$$

Consequently if there is no or little correlation, the points should be evenly spread between x = 0 and x = 0.50.

It is also clear from Table I that the baryon masses could have been obtained if the values for $|\alpha_\ell|$ not shown in parentheses were used. This suggests that

$$\alpha_\ell = km_K + nm_\pi , \qquad (24)$$

where k and n are certain small integers.

The high degree of correlation shown in Fig. 2 can still be maintained if the difference in strangeness between the baryons is accounted for by the number of K-meson masses used in Eq. (24). Consequently in calculating the percentage error $|\alpha_\ell|$ can at most be changed by $\pm m_\pi$.

TABLE I. Observed and calculated baryon masses.[a]

Octet

Particle	Obs. mass MeV	$\lvert\alpha_\ell\rvert$		Calc. mass MeV
Ξ	1321	$2m_K$	$(2m_K)$	1319
Σ	1192	$3m_K - m_\pi$	$(10m_\pi)$	1195 (1198)
Λ	1115	$3m_K + m_\pi$	$(12m_\pi)$	1115 (1118)
N	939	$4m_K + 2m_\pi$	$(17m_\pi)$	951 (943)

TABLE I. (Cont.)

Decuplet

Particle	Obs. mass MeV	$\lvert\alpha_\ell\rvert$		Calc. mass MeV
Ω	1675	m_π	(m_π)	1681
Ξ_1^*	1529	m_K	(m_K)	1516
Y_{13}^*	1382	$2m_K - m_\pi$	$(6m_\pi)$	1369 (1389)
N_{33}^*	1236	$3m_K - 2m_\pi$	$(9m_\pi)$	1242 (1243)

Negative parity baryons

Particle	Obs. mass MeV	$\lvert\alpha_\ell\rvert$		Calc. mass MeV
Y_{03}^*	1519	m_K	(m_K)	1516
N_{13}^*	1518	m_K	(m_K)	1516

Other baryons

Particle	Obs. mass MeV	$\lvert\alpha_\ell\rvert$		Calc. mass MeV
N_{15}^*	1688	m_π	(m_π)	1681
Y_1^*	1660	m_π	(m_π)	1681
Y_o^*	1405	$m_K + 2m_\pi$	$(6m_\pi)$	1405 (1389)

[a] The observed masses and particle designation are taken from the Rosenfeld et al. Review article[3].

The calculated masses in parentheses are obtained using the values for $\lvert\alpha_\ell\rvert$ which are in parentheses.

The integers k and n can be obtained as functions of the hypercharge Y and the isospin I by linearizing Eq. (1) and comparing the result to the Gell-Mann, Okubo formula.[2] The result is

$$k = 3 + Y - (L/2)(L + 1) ,$$

$$n = 1 + (1 - L)(Y/2)(Y + 1) + (Y/2)[(Y/2) + 1] - I(I + 1) , \quad (25)$$

where Y is the hypercharge and I is the isotopic spin of the baryon and L is an empirical quantum number. The octet is obtained for L = 0 and the decuplet is obtained for L = 1. If L = 2, a negative parity octet seems to be generated. As shown in Table II, this accurately gives the negative parity resonance N_{13}^* and is consistent with the negative parity

TABLE II. Negative parity octet.[a]

Particle	Parity	Y	I	J	Obs. mass	$\lvert\alpha_\ell\rvert$	Calc. mass
Y_o^*	(-)	(0)	(0)	(3/2)		m_π	1681
Y_1^*	(-)	0	1	(3/2)	1660	m_π	1681
Ξ^*	(-)	(-1)	($\frac{1}{2}$)	(3/2)		m_K	1516
N_{13}^*	-	1	$\frac{1}{2}$	3/2	1518	m_K	1516

[a]The particle designations are taken from the Rosenfeld et al. Review article.[3] The quantum numbers in brackets are the predicted values.

assignments[6] by Chew, Gell-Mann and Rosenfeld for the Y_1^* and the negative parity Ξ^* (although they tentatively assigned the Ξ^* mass at

1600 Mev). However, Eq. (25) has a Y_o^* of mass 1681 as the negative parity Λ_0 resonance rather than the Chew, Gell-Mann, Rosenfeld assignment, Y_{03}^* with mass 1520 MeV. Thus Eq. (25) gives rise to the prediction of a Ξ^* resonance with quantum numbers S = -2, I = 1/2, J = 3/2 (angular momentum) and negative parity with mass 1516 MeV and also a Y_o^* resonance with quantum numbers S = -1, I = 0, J = 3/2 and negative parity with mass of 1681 MeV.

It is of interest to note that the difference in strangeness between the baryons of a given family is given by the number of K meson masses and that the parity of each family is given by $(-1)^{k+n+L}$. With this interpretation the correlation is considerably improved, as shown in Fig. 2(b), since the smallest change in $|\alpha_\ell|$ used to compute the percentage error must be $2m_\pi$ (rather than m_π) in order to preserve the parity and the strangeness of the state. Also, it appears that the empirical quantum number L could be interpreted as an "intrinsic orbital angular momentum" since it is involved in describing the parity and the angular momentum J of the state.

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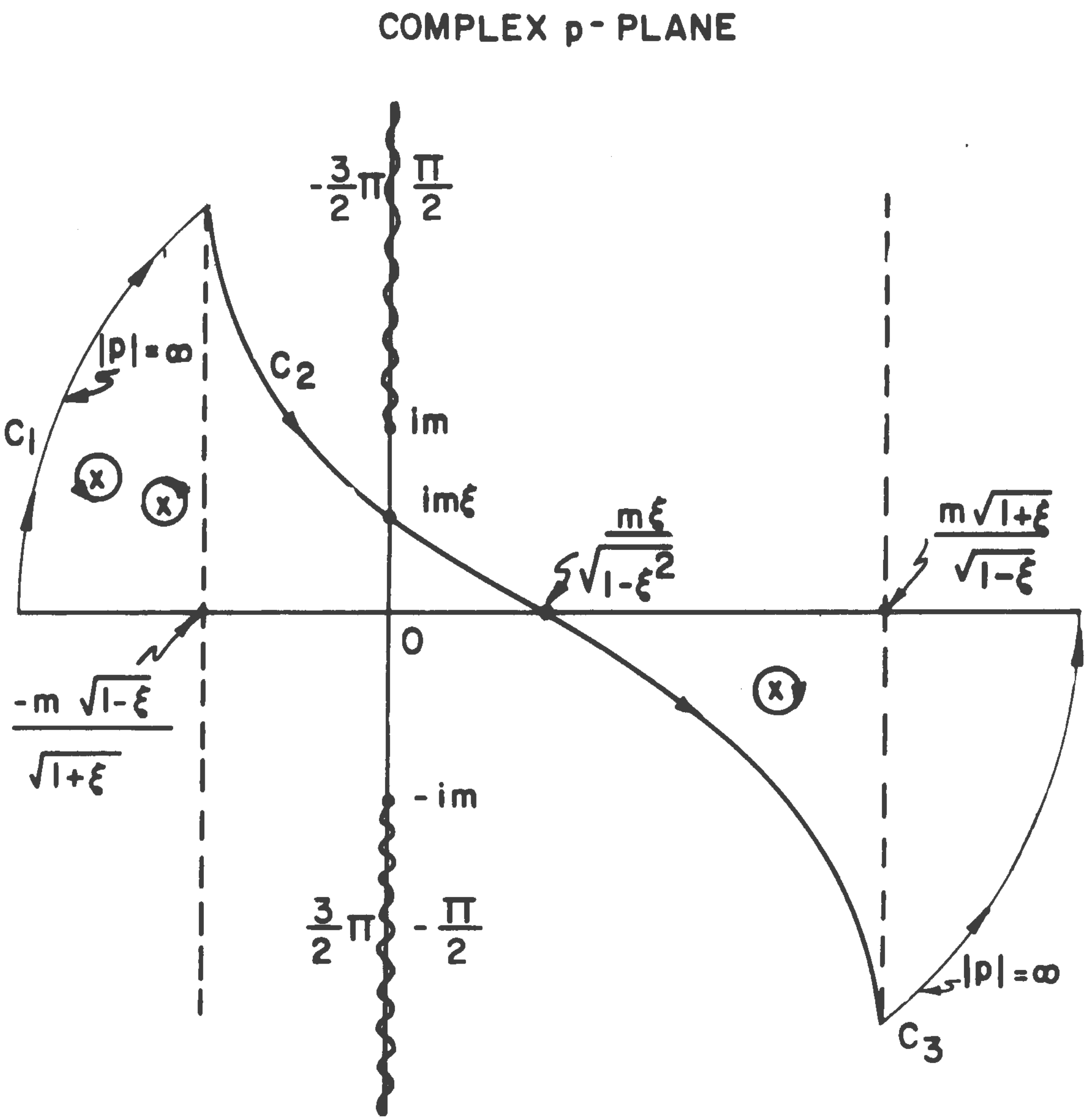
FIGURE CAPTIONS

Fig. 1. Contour in the complex p-plane for Eq. (10).

Fig. 2. Distribution of the absolute values of the percentage differences between observed and predicted masses.

(a) a point plot assuming $|\alpha_\ell| = km_K$ or nm_π.

(b) a point plot assuming $|\alpha_\ell| = km_K + nm_\pi$.



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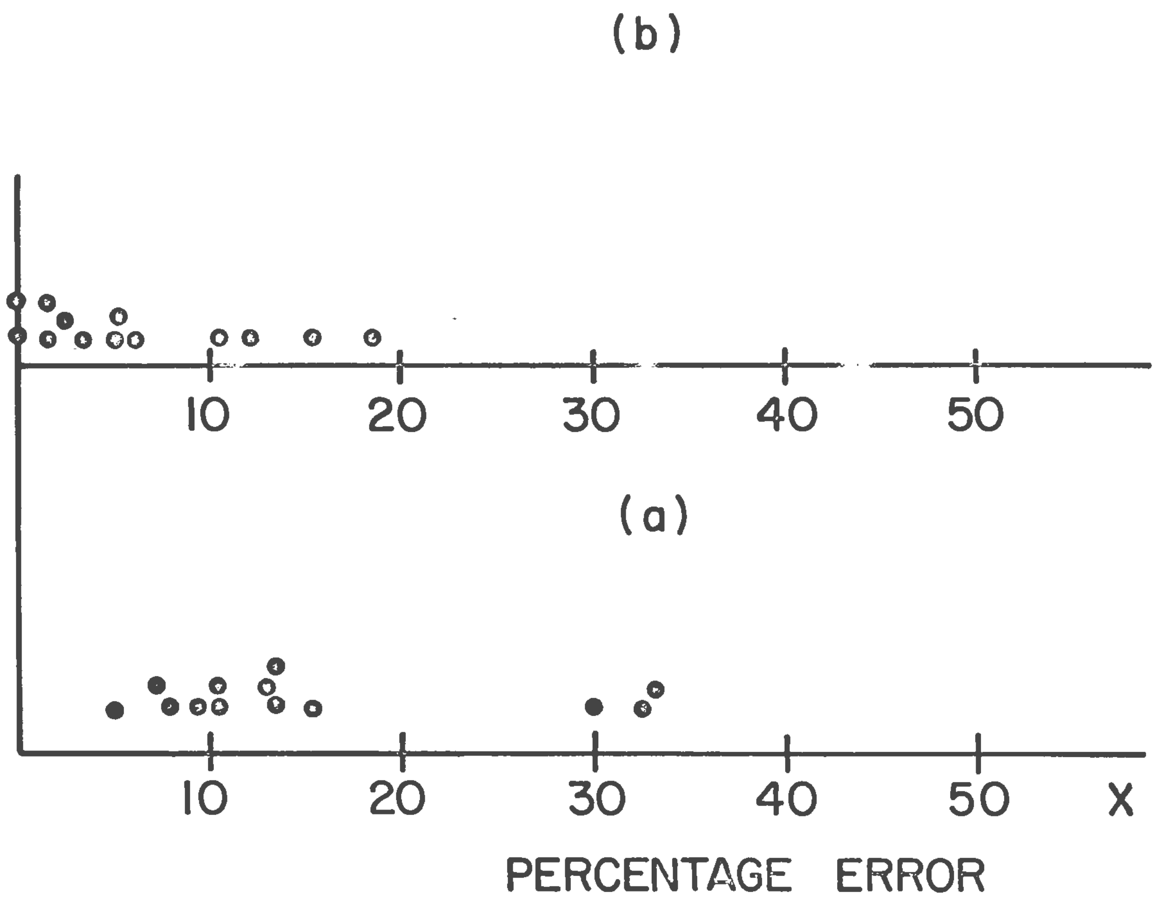

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