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THE INFLUENCE OF ASPERITY CONTACT ON THE SCATTERING OF ELASTIC WAVES FROM FATIGUE CRACKS

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ABSTRACT

Theoretical and experimental studies of the effects of contacting asperities on the scattering of elastic waves from fatigue cracks are reported. The analysis is based on a generalization of the representation integral for scattering from a crack to a form explicitly including the transducer radiation patterns. A series of approximations are developed based on various models for the form of the dynamic crack-opening displacement (COD) which appears within the integral. The theory is compared to experimental scattering measurements for fatigue cracks grown in aluminum compact tension specimens. These show that a simple quasistatic model for the COD is adequate to describe measurements of transmission through the crack, but that the discreteness of the contacts must be included if the signals diffracted from the crack tip are to be predicted accurately. Applications of the results to characterize the closure state and the resulting changes in the growth rate of fatigue cracks is presented. This latter point is supported by a discussion of the relationship between the contact parameters and crack tip shielding at the lower loads of the fatigue cycle.

INTRODUCTION

During the growth of a fatigue crack, contact between the crack faces are often developed via a variety of mechanisms, including general plastic deformation, sliding of the two faces with respect to one another, or the collection of debris such as oxide particles. Figure 1 schematically sketches such a situation. The results of these contacts are twofold. First, compressive stresses are created in the material on either side of the partially contacting interface. In reaction, an opening load is applied to the crack tip which acts as a local stress intensity factor.

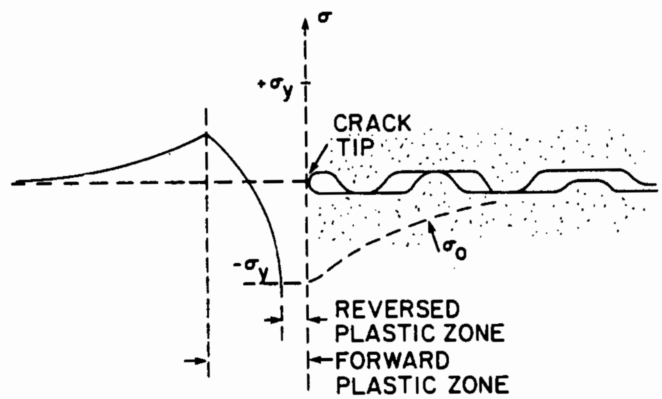

Fig. 1 Schematic illustrations of contacts near crack tip. Included is a sketch of the average stresses, showing the compressive component associated with these contacts.

This reduces the effective stress variation under cyclic loading conditions, thereby slowing the rate of crack growth. The second consequence of these contacts occurs when the crack is illuminated by an elastic wave. When no contacts are present, a singularity exists in the elastodynamic fields at the crack tip, and this leads to a set of diffracted fields emanating from the tip. The presence of the contacts modifies the tip diffracted fields and allows energy to be directly transmitted through the crack.

The study of these two phenomena has several motivations. From the applied perspective, contact induced changes in elastic wave scattering may require modifications in ultrasonic nondestructive evaluation techniques to detect or size the flaws. Furthermore, these scattering changes contain important information regarding the contact state, and this information might be useful in predicting closure induced changes in crack growth rate. From the theoretical mechanics point-of-view, the elastodynamic problems which must be solved to understand these phenomena are very interesting themselves.

EFFECTS OF CONTACTS ON ELASTIC WAVE SCATTERING

General Formalism

The theory for the scattering of elastic waves from fatigue cracks is based on the electromechanical reciprocity theory of Auld [1], which states that the flaw induced change in the signal transmitted from an illuminating to a receiving transducer, $\delta\Gamma$, is given by

$$\delta\Gamma = \frac{j\omega}{4P}\int_A (u_i^R\tau_{ij}^T - u_i^T\ \tau_{ij}^R)\ n_j dA \qquad (1)$$

where u_i^R, τ_{ij}^R are the displacement and stress patterns that would be produced if the receiving transducer irradiated a flaw free material; u_i^T and τ_{ij}^T are the displacement and stress field produced when the flaw is irradiated by the transmitting transducer, P is the electrical power incident on the transmitting transducer, and integration is performed over the surface of the scatterer, which has a normal n_j. A time dependence of the form $\exp(j\omega t)$ has been assumed. Noting that τ^T, τ^R, and u^R must be continuous in the plane of the crack (assuming the noncontacting regions to have infinitesimal volume), one concludes that

$$\delta\Gamma = \frac{-j\omega}{4P}\int_{A^+} \Delta u_i^T\tau_{ij}^R n_j^+\ dA \qquad (2)$$

where A^+ is the illuminated face of the crack, n_j^+ is its normal, and $\Delta u_i^T = u_i^+ - u_i^-$ is the dynamic crack-opening displacement (COD).

In an experimental situation, one does not measure $\delta\Gamma$ but rather $\Gamma = \Gamma_R + \delta\Gamma$ where Γ_R is the reference signal that would be observed with no crack present. For the experimental geometry illustrated in Fig. 2, the reference signal may be estimated by applying Eq. (2) to a perfect, planar crack (i.e. no contacts). In that case, $\delta\Gamma = -\Gamma_R$ and Eq. (2) becomes

$$\Gamma_R \cong \frac{j\omega}{4P}\int_{A^+} 2u_i^I\tau_{ij}^R n_j^+ dA \qquad (3)$$

where u_i^I is the displacement field of the incident illumination and it is assumed that the displacement is approximately doubled at the stress free crack surface during the reflection of a normally incidence beam. Combining Eqs. (1)-(3) leads to the final result

$$\Gamma = \frac{j\omega}{4P}\int_{A^+} (2u_i^I - \Delta u_i^T)\ \tau_{ij}^R n_i^+\ dA. \qquad (4)$$

Computations of the crack scattering now requires three sets of fields to be known. One must know the stress radiation pattern of the receiver, τ_{ij}^R, the displacement radiation pattern of the transmitter, u_i^I, and the dynamic crack-opening displacement, Δu_i^T. As

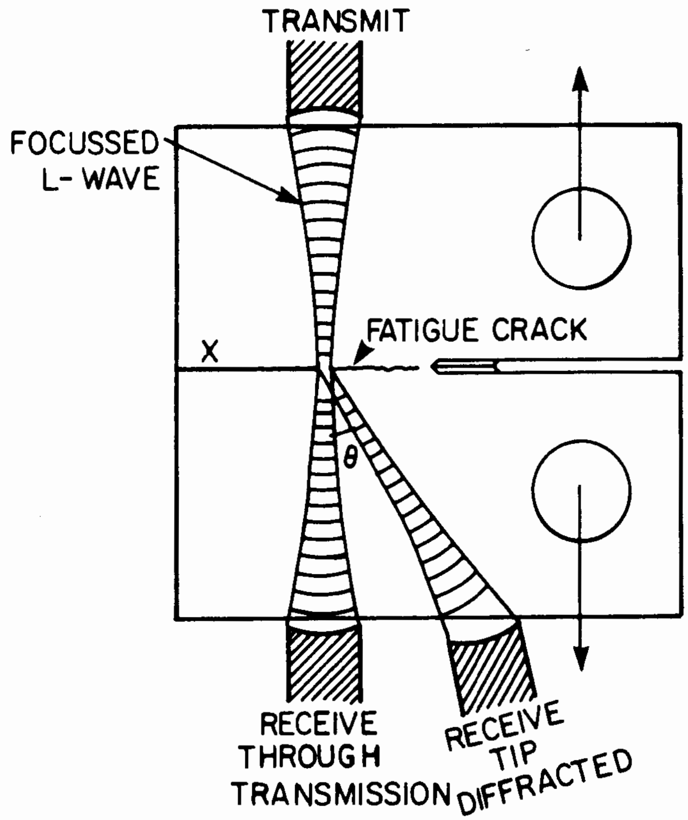

Fig. 2 Experimental geometry for the measurement of through transmission and tip diffracted signals.

reported previously, a scalar Gaussian beam approximation [2] has been employed to estimate the radiation fields τ_{ij}^R and u_i^I. This model includes such effects as diffraction induced beam spread, but of course does not include the full tensor character of the elastic fields. For beams whose widths are several wavelengths in extent, the scalar approximation should be reasonable since the direction of polarization does not substantially vary over the beam cross-section.

The dynamic crack-opening displacement has been represented by a sequence of approximations. The authors have given particular attention to the case in which the contacts are closely spaced with respect to a wavelength. This approximation is motivated by the belief that contact spacing should be on the order of the grain size, whereas much greater ultrasonic wavelengths must be used to avoid excess attenuation in measurements in polycrystals.

Spring Model

The simplest model is a spring model [3], in which the partially contacting interface, in the z=0 plane, is represented by the modified boundary condition

$$\sigma_{3i}^{+} = \sigma_{3i}^{-} \tag{5}$$

$$\sigma_{3i}^{+} = \kappa_{ij}(u_j^{+} - u_j^{-}) \tag{6}$$

where the superscripts "+" and "-" refer to the two sides of the interface. The matrix κ_{ij} may be thought of as representing a set of massless springs joining the two sides of the interface. For simple interface topographies, this matrix will be diagonal, with κ_{11} and κ_{22} representing the contact induced resistance to shear and κ_{33} representing the resistance to compression. The properties of the interface are assumed to be linear. This requires that there be resistance to tension as well as compression, which is true when the dynamic stresses of the ultrasonic wave are small with respect to the static stresses associated with the contact. Baik and Thompson [4] have developed a quasi-static model relating κ_{33}, hereafter abbreviated as κ, to solutions of static deformation problems for a variety of crack topographies. κ is found to be a function of both the contact density and dimensions. For sparse, penney shaped contacts, $\kappa = N\pi E'd/8$, where N is the contact density, d is their diameter, $E' = E/(1-\nu^2)$, E = Young's modulus, and ν = Poisson's ratio. Achenbach and Angel [5,6] have compared plane wave reflection and transmission coefficients, based on Eqs. (5) and (6), to those calculated from exact elastodynamic solutions, for a periodic array of strip cracks. They found good agreement at wavelengths large with respect to the contact spacing. Norris and Achenbach [7] have also analyzed the implications of nonlinear spring constants.

In the spring model, if one assumes normal illumination of the interface by a plane wave having displacement amplitude u_i^{I}, one finds that

$$\Delta u_i = \frac{-2j\pi\rho v f/\kappa}{1+j\pi\rho v f/\kappa}\, u_i^{I}. \tag{7}$$

Consider next the problem of the finite crack illuminated by a bound beam. If the contact density and diameter vary slowly with respect to a wavelength, then one can locally assume that the COD has the value that it would have when illuminated by a plane wave of equal amplitude, as predicted by Eq. (7). This leads to the form

$$\Gamma = \frac{j\omega}{2P} \int_{A^+} [1 + j\alpha]^{-1}\, u_3^{I}\, \tau_{33}^{R}\, dA \tag{8}$$

where $\alpha = \pi\rho v f/\kappa$ and the quantity in brackets can also be shown to be the plane wave transmission coefficient.

This theory has been compared to measurements of the through transmission signal, as a function of the measurement frequency and beam position, for a number of fatigue cracks. In this case, the factor $u_3^{I}\tau_{33}^{R}$ has little phase variation since the beam axes are parallel. Good fits are generally obtained when $\kappa(x)$ is viewed as a variable parameter, adjusted so that the theory best matches the data. The agreement has been achieved for a variety of samples ranging from saw slots (simulating the ideal condition) to fatigue cracks grown in two stages [3,8-10]. Figure 3 presents a comparison of theory to experiment for the latter case [9]. A large increase in transmission is seen at the position of the end of the first stage growth which indicates the presence of closure induced contacts in this region. The interfacial stiffness required to fit this data is also shown.

The spring model predictions have also been compared to measurements of the tip diffracted signals [8]. In this case, the factor $u_3^{I}\tau_{33}^{R}$ varies as $\exp[-j\kappa x \sin\theta]$ where θ is the angle of diffraction. It has been found that the experimental signals can be considerably greater than the predictions of the model and this deficiency has been suggested to be a consequence of the absence of discrete contacts in the model.

Discrete Contact Model

To test this hypothesis, a model has been developed which explicitly includes the discreteness of contacts [8,9]. Figure 4 illustrates graphically the spatial variation of the dynamic COD as it would be expected to exist near the crack tip [11]. The spring model essentially averages this function, with the spatial variation of κ representing the slow change in the average COD as one moves along the crack. The discrete contact model adds to this spring contribution a series of delta functions representing the rapid change in the COD in the vicinity of the contacts.

In the initial form of the model [8,9], the strengths of these delta functions were chosen based on certain ad hoc assumptions regarding local contributions to Eq. (4). These were essentially dimensional arguments with no detailed guidance from elasticity solutions. Given these assumptions, it was concluded that, for a given form of $\kappa(x)$, the number density of contacts strongly influenced the strengths of the diffracted signals. Figure 5 illustrates this with the example of the predicted magnitude of the 45°, 4 MHZ diffracted shear signal as a function N for a crack with fixed $\kappa(x)$.

A more rigorous solution of this problem has more recently been considered for the case of circular contacts whose separation is large with respect to their diameter but much less than a wavelength. For this case, an approximation can be built up from the static solution for the deformation of two half-spaces joined by a circular contact, i.e. the solution for the infinitely deep outer notch of axial symmetry [12]. This solution can be related by a simple transformation to the deformation of a rigid punch on a half-space [13]. The COD per unit force, $\Delta u_3/F$, at an isolated contact is then found to be given by

$$\Delta u_3/F = \frac{(1-\nu)}{\pi\mu a} \arcsin\left(1 - \frac{a^2}{r^2}\right)^{1/2}, \quad r > a. \tag{9}$$

In the approximation developed, the force per contact, F, is estimated from the spring model, with the result

$$F = \left(\frac{-2j\alpha}{1+j\alpha}\right) u_3^{I}/N\kappa. \tag{10}$$

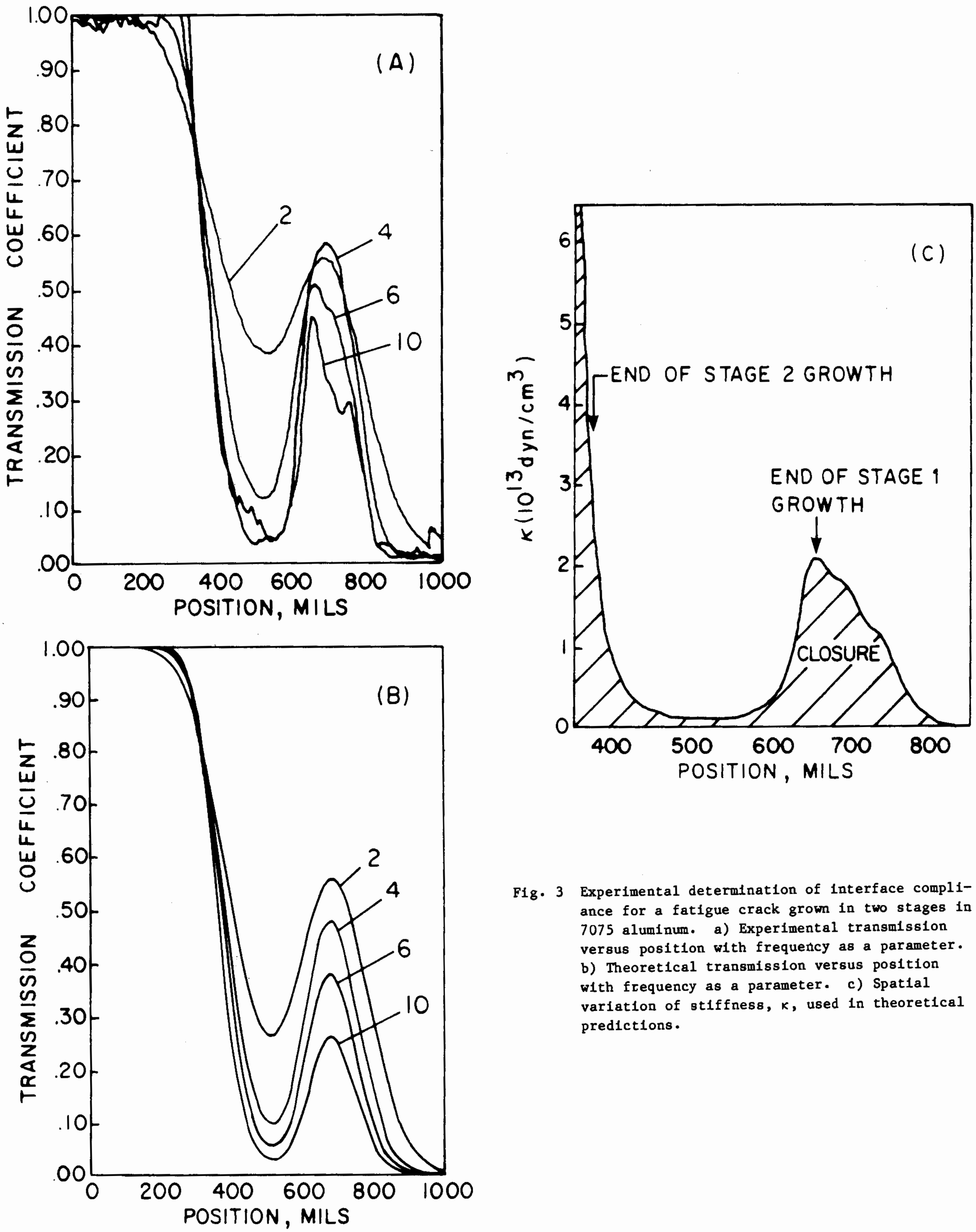

Fig. 3 Experimental determination of interface compliance for a fatigue crack grown in two stages in 7075 aluminum. a) Experimental transmission versus position with frequency as a parameter. b) Theoretical transmission versus position with frequency as a parameter. c) Spatial variation of stiffness, κ, used in theoretical predictions.

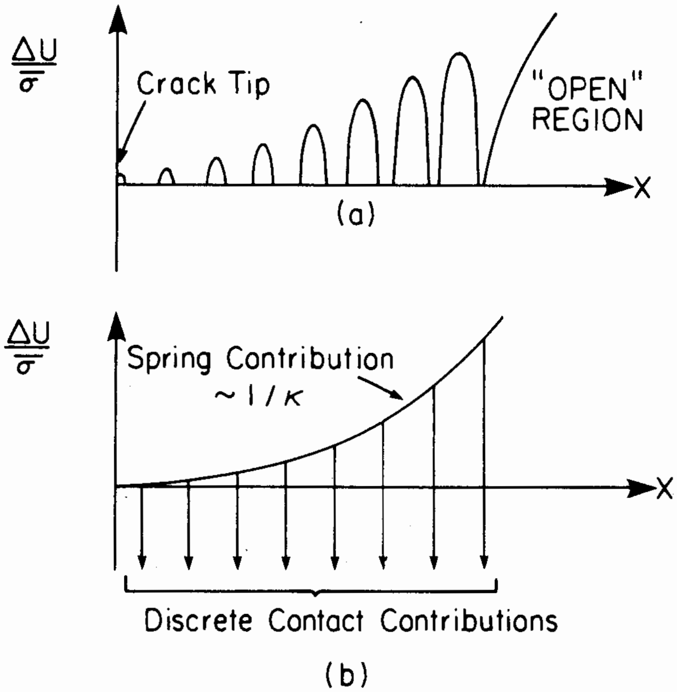

Fig. 4 Dynamic crack opening displacement of a fatigue crack. a) Conceptual sketch of true COD. b) Spring plus delta function model.

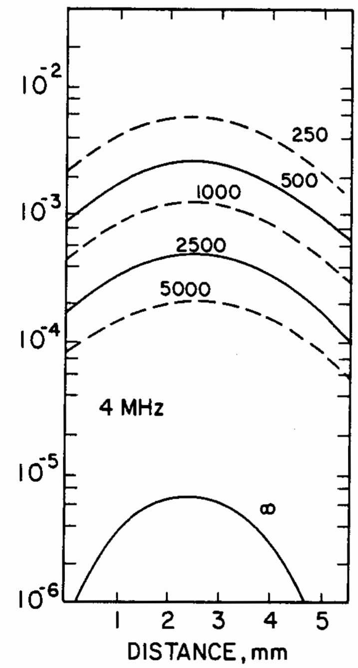

Fig. 5 Theoretical tip diffracted shear wave signal at 4MHZ versus contact density, as computed using the discrete contact model for a fatigue crack in 7075-T651 aluminum.

As noted before, τ_{ij}^{R} will vary primarily as $\exp(-j\kappa x \sin\theta)$, where θ is the angle of the diffracted wave. When these results are substituted into Eq. (4), the physical basis for the delta function model is clarified. The domain of integration can be broken up into separate elements in the vicinity of the individual contacts. In each of these, the COD, as given locally by Eq. (9), is by far the most rapidly varying function. Expansion of the factor $\exp(-jkx \sin\theta)$ as a Taylor series leads to a contribution to $\delta\Gamma$ which has the form of the value of the integrand at the contact times a series of terms involving the moments of the local COD. This has the same form that arises from the ad hoc delta function model. Furthermore, if the domain of integration is a circle, the lower moments can be integrated in closed form.

This result formally relates the previous ad hoc delta functions model to a more rigorous approach. However, numerical results have not yet been investigated in detail because of two remaining questions. First, the effects on the COD of the uncracked region, into which the tip would be expected to subsequently propagate, have not yet been incorporated in the model. Second, the results are quite sensitive to the symmetry of the local COD. For example, when Eq. (9) is employed in the vicinity of a contact, the zeroth moment can be associated with the spring contribution while the first moment vanishes by symmetry. The second moment then determines the strength of the delta function. Modifications of this symmetry by other contacts or the uncracked edge could strongly change the numerical results. Further work is needed to assess the relative importance of these contributions.

EFFECTS OF CONTACTS ON CRACK GROWTH RATE

It is well known [14] that the surfaces of a fatigue crack contact each other during unloading, that this occurs at individual contact points (asperities) caused by a mismatch of the fracture surfaces [15]. As this contact occurs, the stresses ahead of the crack will be redistributed such that the driving force for crack propagation, ΔK_{eff}, becomes smaller than would be expected from a simple calculation of the stress intensity range, $\Delta K = K_{Imax}-K_{Imin}$. The standard procedure for describing this phenomena has been to define an effective driving force

$$\Delta K_{eff} = K_{Imax}-K_{Iclosure} \tag{11}$$

where $K_{Iclosure}$ has been operationally defined in several related ways associated with establishment of the last contact.

Recently, several attempts have been made to obtain more rigorous solutions for ΔK_{eff}. Beevers et al. [16,18] have developed a two-dimensional model, in which the contacts are assumed to be strips, while Buck et al. [19] have considered the effects of arrays of point contacts. In each case, the work leads to the concept of a parameter K_I(local) which represents the stress intensity at the crack tip produced by the local contacts. For the strip case [16]

$$K_I(\text{local}) = \left(\frac{2}{\pi}\right)^{1/2} \frac{P}{BC_1^{1/2}} \qquad (12)$$

where P is the contact force, B is the specimen width, and C_1 is the distance from the asperity to the crack tip. The array of point contacts have also been shown to reduce to this result in appropriate limits [19].

When the cyclic load is sufficiently great to overcome the contact stress, then K_{Imax} is determined only by the global geometry using standard relationships. This has been called K_I(global). Then, as sketched in Fig. 6, the effective intensity variation at the crack tip is

$$\Delta K_{eff} = K_{Imax}(\text{global}) - K_{Imax}(\text{local}) \qquad (13)$$

where K_{Imax}(local) is the maximum value of the local stress intensity factor, which usually occurs in the absence of the applied load.

USE OF SCATTERING INFORMATION TO PREDICT K_I(local)

An important application of the understanding of the effects of contacts on ultrasonic scattering would be a technique to experimentally determine K_I(local). A preliminary demonstration of such a technique has been made by Buck et al. [19]. The sample was a compact tension specimen of 7075-T651 aluminum. From through transmission movements, $\kappa(x) = N\pi E'd(x)/8$ was estimated. By using the delta function model for the tip diffracted signal, the value for N was then determined, which then allowed an independent computation of d. From these parameters, it was possible to predict values of K_{Imax}(local) and σ_o, the average static stress across the partially closed fracture surface. These values appeared physically reasonable.

CONCLUSIONS

The results clearly indicate the importance played by contacting asperities in influencing both ultrasonic scattering and fatigue crack growth rates. Refinements in the theories discussed above are essential to complete our understanding of these phenomena and their interrelationships.

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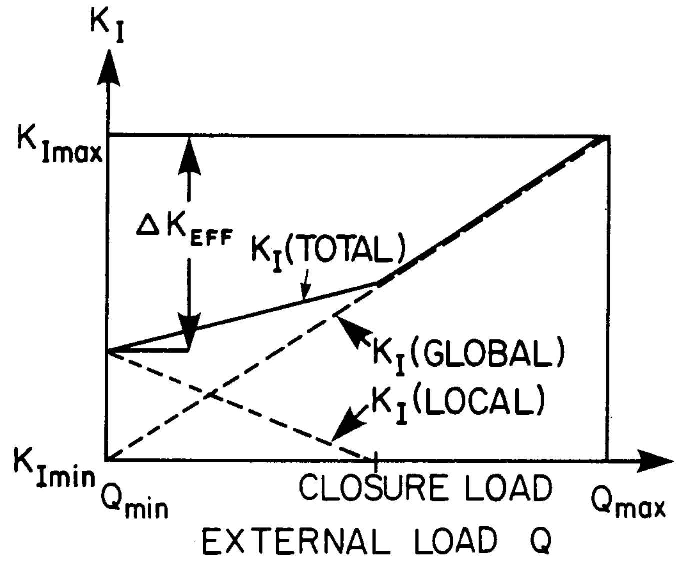

Fig. 6 The stress intensity factors K_I(global) and K_I(local) as a function of external load Q.

REFERENCES

1. Auld, B. A., "General Electromechanical Reciprocity Relations Applied to the Calculation of Elastic Wave Scattering Coefficients," Wave Motion, Vol. 1, 1979, p. 3.

2. Thompson, R. B. and Lopes, G. F., "The Effects of Focusing and Refraction on Gaussian Ultrasonic Beams," J. Nondestr. Eval., Vol. 4, 1984, p. 107.

3. Thompson, R. B. and Fielder, C. J., "The Effects of Crack Closure on Ultrasonic Scattering Measurements," Review of Progress in Quantitative Nondestructive Evaluation, Vol. 3A, D. O. Thompson and D. E. Chimenti, Eds., Plenum Press, New York, 1984, p. 207.

4. Baik, J.-M. and Thompson, R. B., "Ultrasonic Scattering from Imperfect Interfaces: A Quasi-static Model," J. Nondestr. Eval., Vol. 4, 1984, p. 177.

5. Angel, Y. C. and Achenbach, J. D., "Reflection and Transmission of Elastic Waves by a Periodic Array of Cracks," J. Appl. Mech., Vol. 107, 1985, p. 33.

6. Angel, Y. C. and Achenbach, J. D., "Reflection and Transmission of Elastic Waves by a Periodic Array of Cracks: Oblique Incidence," Wave Motion, Vol. 7, 1985, p. 375.

7. Achenbach, J. D. and Norris, A. N., "Loss of Specular Reflection Due to Nonlinear Crack-Face Interaction," J. Nondestr. Eval., Vol. 3, 1982, p. 229.

8. Rehbein, D. K., Thompson, R. B., and Buck, O., "Interaction of Ultrasonic Waves with Simulated and Real Fatigue Cracks," Review of Progress in Quantitative Nondestructive Evaluation, Vol. 4A, D. O. Thompson and D. E. Chimenti, Eds., Plenum Press, New York, 1985, p. 61.

9. Buck, O., Thompson, R. B., and Rehbein, D. K., "The Interaction of Ultrasound with Contacting Asperities: Applications to Crack Closure and Fatigue Crack Growth," J. Nondestr. Eval., Vol. 4, 1984, p. 203.

10. Rehbein, D. K., Thompson, R. B., and Buck, O., "Ultrasonic Studies of Load Induced Changes in Fatigue Crack Closure," Review of Progress in Quantitative Nondestructive Evaluation, Vol. 5, D. O. Thompson and D. E. Chimenti, Eds., Plenum Press, New York, 1986, in press.
11. Thompson, R. B., Buck, O., and Rehbein, D. K., "Elastic Wave Interactions with Partially Contacting Interfaces: Statistics of Theory and Application to Fatigue Crack Closure Characterization," Proceedings of the ONR Symposium on Solids Mechanics Research For QNDE, J. D. Achenbach and Y. Rajapakse, Eds., Martinus Nijhoff, Dordrecht, in press.
12. Bueckner, H. F., "Field Singularities and Related Integral Representations," Mechanics of Fracture: Vol. 1, Methods of Analysis and Solution of Crack Problems, G. C. Sih, Ed., Noordhoff International, Leyden, 1973, p. 292.
13. Lure', A. I., Three-Dimensional Problems in the Theory of Elasticity, Interscience, London, 1964, p. 271.
14. Elber, W., Damage Tolerance in Aircraft Structures, ASTM STP 486, 1971, p. 230.
15. Bowles, C. Q. and Schijve, J., Fatigue Mechanism: Advances in Quantitative Measurements of Physical Damage, ASTM STP 811, 1983, p. 400.
16. Beevers, C. J., Bell, K., Carlson, R. L., and Storke, E. A., "A Model for Fatigue Crack Closure," Eng. Fract. Mechanics, Vol. 19, 1984, p. 93.
17. Carlson, R. L. and Beevers, C. J., "A Multiple Asperity Fatigue Crack Closure Model," Eng. Fract. Mechanics, Vol. 20, 1984, p. 687.
18. Carlson, R. L. and Beevers, C. I., "A Mixed Mode Fatigue Crack Closure Model," Eng. Fract. Mechanics, Vol. 22, 1985, p. 651.
19. Buck, O., Rehbein, D. K., and Thompson, R. B., "Determining Crack Tip Shielding by Means of Acoustic Transmission and Diffraction Measurements," Review of Progress in Quantitative Nondestructive Evaluation, Vol. 6, D. O. Thompson and D. E. Chimenti, Eds., Plenum Press, New York, in press.