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Abstract
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Keywords
Turbulence simulations, Turbulent flows, Fluid equations, Boundary value problems, Homogenous turbulence

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Consistent modeling of interphase turbulent kinetic energy transfer in particle-laden turbulent flows

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The interphase transfer of turbulent kinetic energy (TKE) is an important term that affects the evolution of TKE in fluid and particle phases in particle-laden turbulent flow. This work shows that the interphase TKE transfer terms must obey a mathematical constraint, which in the limiting case of statistically homogeneous flow with zero mean velocity in both phases, requires these terms be equal and opposite. In the single-point statistical approach called the two-fluid theory, the interphase TKE transfer terms are unclosed and need to be modeled. Multiphase turbulence models that satisfy this constraint of conservative interphase TKE transfer admit a term-by-term comparison with true direct numerical simulations (DNS) that enforce the exact velocity boundary condition on each particle's surface. Analysis of three models reveals that not all models satisfy the requirement of conservative interphase TKE transfer. DNS that invoke the point-particle assumption also do not obey this principle of conservative interphase TKE transfer, and this precludes the comparison of model predictions of TKE budgets in each phase with point-particle DNS. This study motivates the development of multiphase turbulence models based on the insights revealed by this analysis, leading to a meaningful comparison of TKE budgets with true DNS. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2756579]

I. INTRODUCTION

Particle-laden turbulent flows are ubiquitous in nature and industrial processes. The conservation of mass, momentum, and energy in the two phases can be described in an average sense using a statistical approach called the two-fluid theory.[1] Since the resulting equations describe both phases in a Eulerian frame, the approach is also referred to as the Eulerian-Eulerian approach. Transport equations for the second moments of the velocity in both phases have also been derived to describe the fluctuations in the particle and fluid velocities. As in statistical models of single-phase turbulence, these equations contain unclosed terms that need to be modeled. Two important unclosed terms are the interphase transfer of turbulent kinetic energy, and the dissipation rate of turbulent kinetic energy in the fluid phase. This paper describes a mathematical constraint that models for these terms must obey, so that they can be meaningfully compared with emerging high-fidelity direct numerical simulations[2–4] and experiments.[5]

For particle-laden flows with non-negligible mass loading, the interphase transfer of momentum must be accounted for, and it manifests itself as the mean interphase momentum transfer term in the averaged equations of the two-fluid theory. A constraint associated with mean momentum equation for particle-laden flows is that the mean interphase momentum transfer is conservative, i.e., equal and opposite in both phases. This constraint is nothing but Newton's third law reflected in the mean momentum equation. It turns out that a similar constraint appears in the velocity second-moment equations for particle-laden turbulence. In this paper we derive this constraint and explore its implications for some existing models.

An important limiting case of turbulent multiphase flows is statistically homogeneous isothermal particle-laden turbulent flow evolving in a zero-gravity environment. If gravity is absent and the mean velocity fields are homogeneous, the mean pressure gradient is zero and the mean momentum equation system results in the trivial solution of zero mean velocity in each phase, which implies a zero mean slip velocity.[1,6] Collisions are assumed to be elastic and therefore there is no energy loss through collisions. In this case, the evolution of second-moments of fluctuating velocity is solely influenced by interphase turbulent kinetic energy (TKE) transfer and viscous dissipation in the fluid phase. The governing equations for this limiting case in the Eulerian-Eulerian approach are[6]

$$\alpha_f \rho_f \frac{dk_f}{dt} = \langle u_i''^{(f)} M_i^{(f)} \rangle + \left\langle u_i''^{(f)} \frac{\partial (I_f \tau_{ki})}{\partial x_k} \right\rangle, \tag{1}$$

$$\alpha_p \rho_p \frac{dk_p}{dt} = \langle u_i''^{(p)} M_i^{(p)} \rangle, \tag{2}$$

where the turbulent kinetic energy (TKE) in the fluid and particle phase are denoted as k_f and k_p, respectively. The volume fraction of the fluid phase and particle phase are denoted as α_f and α_p, respectively, with $\alpha_p = 1 - \alpha_f$. The thermodynamic density in each phase is constant, with ρ_f denoting the fluid phase density, and ρ_p denoting the particle phase density. The fluctuating velocity in phase β ($\beta = f, p$) is defined as the difference between the velocity field at that point

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and the mass-averaged mean velocity in the βth phase,

$$u_i''^{(\beta)} \equiv U_i - \left\langle \tilde{U}_i^{(\beta)} \right\rangle, \tag{3}$$

where the mass-averaged mean velocity in the βth phase is defined as

$$\left\langle \tilde{U}_i^{(\beta)} \right\rangle \equiv \frac{\langle \rho I_\beta U_i \rangle}{\langle \rho I_\beta \rangle}. \tag{4}$$

The interphase momentum transfer terms are denoted as $M_i^{(f)}$ and $M_i^{(p)}$ in the fluid and particle phase, respectively.

The first term on the right-hand side of Eq. (1), $\langle u_i''^{(f)} M_i^{(f)} \rangle$ is the interphase TKE transfer term and the second term $\langle u_i''^{(f)} \partial(I_f \tau_{ki})/\partial x_k \rangle$ is the covariance of the fluctuating velocity in the fluid phase $u_i''^{(f)}$ with the gradient of stress in the fluid phase [where $I_f(\mathbf{x},t)$ is the indicator function of fluid phase at $(\mathbf{x},t)$]. The term $\langle u_i''^{(f)} \partial(I_f \tau_{ki})/\partial x_k \rangle$is usually modeled as the dissipation rate of TKE in the fluid phase. The corresponding term in the solid phase is neglected since collisions between particles are assumed to be elastic.

For flows with no interphase mass transfer, the interphase momentum transfer term $M_j^{(\beta)}$ in phase β is given by

$$M_i^{(\beta)} \equiv -\tau_{ji} \frac{\partial I_\beta}{\partial x_j}. \tag{5}$$

The interphase momentum transfer term can be simplified by using the following expression for the gradient of the indicator function[1]

$$\frac{\partial I_\beta}{\partial x_j} = -n_j^{(\beta)} \delta(\mathbf{x} - \mathbf{x}_I), \tag{6}$$

where $\mathbf{n}^{(\beta)}$ is the unit normal at the interface that points outward with respect to phase β, and $\delta(\mathbf{x}-\mathbf{x}_I)$ is the Dirac delta function located at the interface. Substituting Eq. (6) into Eq. (5) results in the following expression for the interphase momentum transfer term

$$M_i^{(\beta)} = \tau_{ji} n_j^{(\beta)} \delta(\mathbf{x} - \mathbf{x}_I), \tag{7}$$

which shows that it represents the contribution to momentum transfer arising from the fact that the interface located at $\mathbf{x}_I$ can support a stress difference. It is clear from Eq. (7) that the interphase momentum transfer term is nonzero only at the interface. Now using Eq. (7) the interphase TKE transfer term can be represented as

$$\left\langle u_i''^{(\beta)} M_i^{(\beta)} \right\rangle = \left\langle u_i''^{(\beta)} \tau_{ji} n_j^{(\beta)} \delta(\mathbf{x} - \mathbf{x}_I) \right\rangle, \tag{8}$$

which reveals that the interphase TKE transfer term is also nonzero only at the interface.

In this article, we show that in the limiting case of zero mean velocity, the interphase TKE transfer between fluid and particle phase is conservative, i.e., equal in magnitude and opposite in sign. This constraint arises because of the interface boundary condition requiring the velocities in both phases to be the same at the interface, and because the instantaneous momentum transfer between the phases is equal and opposite in sign. It follows from this constraint that the *mixture* turbulent kinetic energy is solely determined by the dissipation rate of fluid phase under the condition of zero mean velocity and elastic (nondissipative) particle collisions.

For the limiting case of homogeneous particle-laden turbulent flow, models have the following general form:

$$\alpha_f \rho_f \frac{dk_f}{dt} = \Pi_{k_f} - \varepsilon_f, \tag{9}$$

$$\alpha_p \rho_p \frac{dk_p}{dt} = \Pi_{k_p}, \tag{10}$$

where Π_{k_f} is the model for the interphase TKE transfer in fluid phase $\langle u_i''^{(f)} M_i^{(f)} \rangle$, and Π_{k_p} is the model for $\langle u_i''^{(p)} M_i^{(p)} \rangle$. Typically multiphase turbulence models are validated by comparing model predictions with experimental results or direct numerical simulation (DNS) data. The advantage in using DNS data for model validation is that a term-by-term comparison is possible by examining the budgets of the TKE equations. A meaningful term-by-term comparison of multiphase model predictions with DNS data requires a consistent definition of the interphase TKE transfer terms and the dissipation rate, in the model and DNS. This becomes even more important in light of the fact that there is more than one approach to performing DNS of particle-laden turbulent flows.

Most DNS of particle-laden turbulent flow[7–14] use the point-particle approximation on the basis that the size of particles simulated is usually smaller than the Kolmogorov scale of turbulence. This approximation is convenient because a true direct simulation of particle-laden turbulent flow that imposes the exact boundary conditions at each particle's surface, and resolves the boundary layer around each particle, is computationally expensive. In the point-particle approximation the momentum transfer between the fluid and particle is modeled as a point source. In a two-way coupled DNS of particle-laden turbulent flow using the point-particle approximation, the force exerted by a particle on surrounding flow field needs to be interpolated as the interphase momentum transfer to the fluid momentum equation using kernel averaging. This instantaneous interphase momentum transfer term results in an implied interphase TKE transfer that appears in the evolution equations for the second moments of fluctuating velocity [cf. Eqs. (1) and (2)]. The dissipation rate in the fluid is calculated in these DNS based on the gradients of the fluid velocity field that solves the fluid momentum equation augmented by the modeled interphase momentum transfer term.[9] We denote the fluid-phase dissipation rate in DNS studies that employ the point-particle approximation $\varepsilon_f^{\mathrm{pp}}$. This dissipation rate differs from the single-phase turbulence dissipation rate $\varepsilon_f^{\mathrm{l}}$ due to the modification of the turbulent flow field arising from the modeled momentum transfer from the point-particles. When validating particle-laden turbulent flow models with point-particle DNS data, it is important to interpret the comparison of the model to DNS keeping in mind the definition of the interphase TKE transfer and dissipation terms in both approaches. The choice of initial dissipation rate can dramatically affect model predictions,[15] so it is important to compare the appropriate dissipation.

Direct numerical simulations of particle-laden turbulent flow with no-penetration and no-slip boundary conditions imposed at the surface of each particle are now feasible for single and even multiparticle systems.[2–4] With increasing computational power, using these DNS the boundary layer around each particle can be resolved. We denote these simulations as "true" DNS. Clearly it is desirable that the definition of interphase TKE transfer and dissipation rate in the models be consistent with true DNS. The dissipation rate in true DNS will be different from that obtained from point-particle DNS, since the high velocity gradients found in the boundary layer around each particle can be fully resolved in true DNS. In this study, the dissipation rate from true DNS is denoted as ε_f^t.

Particle image velocimetry (PIV) measurements of homogeneous turbulence[5] laden with small particles $d_p \sim \eta$ (Kolmogorov length scale) reveal higher turbulent kinetic energy reduction (as compared to the unladen single-phase turbulence) than predicted by the point-particle DNS studies in Refs. 7–9. The discrepancy in attenuation of fluid-phase TKE is attributed to the particle point-force coupling scheme used in these simulations, and it is suggested that this approach may not be capturing all the physics. The experimental study also reports the dissipation rate, but the dissipation rate is not directly measured, but rather it is indirectly calculated using a dynamic equilibrium assumption. The experimental estimates of dissipation rate are also quite different from those of point-particle DNS (up to 40% in some cases), although neither experiment nor point-particle DNS resolve the small-scale motions in the vicinity of the particles and flow conditions are not identical. Nevertheless, the experimental results point to the importance of distinguishing between ε^t and ε^{pp}. They also suggest that the current discrepancy between experiments and point-particle DNS could be explained by calculating dissipation and TKE from true DNS, although the qualitative trends of dissipation rate and TKE with nondimensional parameters such as Stokes number and mass loading are still reasonably predicted using point-particle DNS.

As true DNS becomes commonplace due to rapid increase in computational power, and with high-resolution PIV results for particle-laden turbulence, detailed budgets for particle-laden turbulence will soon be readily available. Hence, multiphase turbulence models will be required to satisfy more stringent tests arising from comparison of budgets with true DNS and high-resolution PIV. Those models that obey the inherent mathematical constraints arising from the exact equations will compare more favorably with true DNS or high-resolution experimental data.

This paper derives one such mathematical constraint, the conservation of interphase TKE transfer in homogeneous particle-laden turbulence. In the following section, the conservation principle is derived using the Eulerian-Eulerian approach. In Sec. III the implications of this principle for multiphase turbulence models are examined. Section IV addresses the issue of comparing the interphase TKE transfer and dissipation rate terms in models to DNS. The principal conclusions of the study are summarized in the final section.

II. THE PRINCIPLE OF CONSERVATIVE INTERPHASE TKE TRANSFER

The TKE of the two-phase mixture e_m is defined as

$$e_m \equiv \alpha_f \rho_f k_f + \alpha_p \rho_p k_p. \tag{11}$$

For constant-density homogeneous particle-laden turbulence, the mixture TKE e_m evolution is obtained by adding Eqs. (1) and (2) to obtain

$$\frac{de_m}{dt} = \left\langle u_i''^{(f)} \frac{\partial (I_f \tau_{ki})}{\partial x_k} \right\rangle + \left\langle \left(u_i''^{(p)} - u_i''^{(f)}\right) M_i^{(p)} \right\rangle, \tag{12}$$

where the fact that the instantaneous interphase momentum source in each phase is equal and opposite $M_i^{(f)} = -M_i^{(p)}$ has been used. At the fluid-solid interface the instantaneous velocity in each phase is equal because of the boundary conditions of no-slip and zero normal relative velocity.

For the zero mean velocity case considered here, the difference in the fluctuating velocity in each phase is the same as the difference in the instantaneous velocity $\mathbf{U}$,

$$u_i''^{(p)} - u_i''^{(f)} = \left(U_i - \left\langle \tilde{U}_i^{(p)} \right\rangle\right) - \left(U_i - \left\langle \tilde{U}_i^{(f)} \right\rangle\right) = 0 \tag{13}$$

which is zero at the interface. Hence, it follows that the interphase TKE transfer terms should be conservative,

$$\begin{aligned} \left\langle u_i''^{(p)} M_i^{(p)} \right\rangle + \left\langle u_i''^{(f)} M_i^{(f)} \right\rangle &= \left\langle \left(u_i''^{(p)} - u_i''^{(f)}\right) M_i^{(p)} \right\rangle \\ &= \left\langle \left(U_i - U_i\right) M_i^{(p)} \right\rangle = 0. \end{aligned} \tag{14}$$

From this principle of conservative interphase TKE transfer it follows that if there is zero dissipation in the particle phase, then the evolution of the mixture TKE is determined by the fluid phase dissipation rate only,

$$\frac{de_m}{dt} = \left\langle u_i''^{(f)} \frac{\partial (I_f \tau_{ki})}{\partial x_k} \right\rangle. \tag{15}$$

Although Eq. (13) is intuitive and essentially correct, the simple derivation presented above does not reveal certain assumptions that are needed to establish the conservation principle in Eq. (14). A detailed derivation is presented in the following subsection for completeness, but the reader may proceed to the following section without loss of continuity.

Detailed derivation of the conservation principle

There are two important aspects of this problem that the simple derivation in Eqs. (13) and (14) does not explicitly account for. The first is that the instantaneous velocity U_i and the fluctuating velocities in each phase $u_i''^{(f)}$ and $u_i''^{(p)}$ are *random fields* that are parametrized in space $\mathbf{x}$ and time t. Yet the equality in Eq. (13) holds only at the boundary surfaces of the fluid-solid interface. Secondly, the expectation symbol $\langle \cdot \rangle$ needs to be interpreted properly depending on whether the quantity within the angle brackets is surface-measurable or volume-measurable. As noted earlier, the terms $u_i''^{(\beta)} M_i^{(\beta)} = u_i''^{(\beta)} \tau_{ji} n_j^{(\beta)} \delta(\mathbf{x} - \mathbf{x}_I)$ are nonzero only at the fluid-solid interface. As we shall see, such terms are only surface-measurable, and they have zero volume measure.

We now present a more rigorous derivation of the conservation principle that fully accounts for these important aspects of the problem. This derivation reveals the assump-

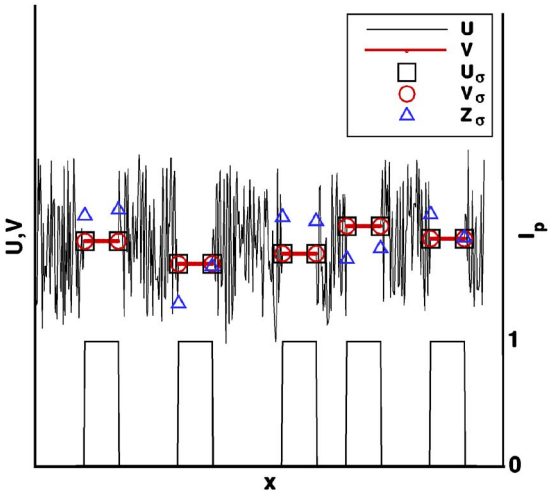

FIG. 1. Sketch showing a realization of random processes corresponding to the 1-component of fluid velocity U, and the 1-component of particle velocity V in a one-dimensional parameter space x. There are five solid particles in this realization, corresponding to the indicator function $I_p=1$ (see right vertical axis). The particle-fluid boundaries define the ten fluid-solid interface locations. These interfaces induce the surface processes U_σ and V_σ. The interphase momentum transfer term is a pure surface process denoted as Z_σ.

tions inherent in the simple derivation presented earlier in the section. Since dimensionality of the parameter space is not important, for simplicity we consider random fields indexed by only one variable x, i.e., random *processes*. Also we consider only one component of the vector velocity field for simplicity.

Let $U(x)$ be a random process that represents the 1-component of the instantaneous velocity field U_i in the fluid phase, as depicted in Fig. 1. Solid particles are located randomly in space x, with their indicator function $I_p(x)$ as shown in Fig. 1. The presence of the particles defines the interface locations $\delta(x-x_I)$ that induce a surface process $U_\sigma(x_I)$, which is a new random process defined by the value that $U(x)$ takes at the interface locations $x=x_I$.

Similarly the instantaneous velocity field in the particle phase is denoted as $V(x)$. Again the interface locations induce a surface process $V_\sigma(x_I)$, which is defined by the value $V(x)$ takes at the interface locations $x=x_I$. Because the particles are rigid, it follows that V and V_σ are identical, and so are their probability distributions, which we write as

$$V \overset{D}{=} V_\sigma. \tag{16}$$

Note that in general U and U_σ will not be identically distributed. The interphase momentum transfer term $M_i^{(p)}$ is a pure surface process (i.e., it is not defined anywhere else on x other than x_I), and its 1-component is denoted as $Z_\sigma(x_I)$.

We distinguish between the two types of expectation that appear in the Eulerian-Eulerian averaged equations for two-phase flow. Details concerning the definitions of these measures and expectations are given in the Appendix. For any volume–measurable flow quantity Q in phase β, we define its phase-volume mean as

$$\langle Q_\beta \rangle_v(x) \equiv \frac{\langle Q_\beta^v(x) \rangle}{\alpha_\beta(x)}. \tag{17}$$

Similarly for any surface-measurable flow quantity Q in phase β we define its phase-surface mean as

$$\langle Q_\beta \rangle_s(x) \equiv \frac{\langle Q_\beta^s(x) \rangle}{\sigma_\beta(x)}. \tag{18}$$

Clearly U_σ, V_σ, and Z_σ are not volume-measurable. We first consider statistically homogeneous flows where both phase-volume means, as well as phase-surface means are independent of x. Note that this is a stronger requirement than simply requiring α_p to be independent of x. Specifically, this stronger homogeneity requires both the particle number density and the particle size distribution to be homogeneous in physical space.

The interphase TKE transfer term is the covariance of fluctuating velocity with the interphase momentum transfer term, and the latter is nonzero only at the fluid-solid interface. We define the velocity fluctuation in fluid phase U' with respect to the phase-volume mean velocity as

$$U'(x) \equiv U(x) - \langle U \rangle_v(x). \tag{19}$$

For the limiting case of zero mean velocity in both phases considered in this study, we have $\langle U \rangle_v = 0$, and hence $U'(x) = U(x)$. We now evaluate the term corresponding to $\langle u_i''^{(f)} M_i^{(f)} \rangle$ as

$$-\langle U' Z_\sigma \rangle_s = -\langle U Z_\sigma \rangle_s = -\langle U_\sigma Z_\sigma \rangle_s, \tag{20}$$

where we have used $M_1^{(f)} = -M_1^{(p)} = -Z_\sigma$, and the definition that $U = U_\sigma$ at $x = x_I$.

The fluctuating particle velocity at the interface V_σ' is identical to the instantaneous particle velocity at the interface V_σ, if the mean particle velocity $\langle V \rangle_v = 0$ is zero. In order to establish

$$\langle u_i''^{(f)} M_i^{(f)} \rangle = -\langle u_i''^{(p)} M_i^{(p)} \rangle$$

we evaluate the term corresponding to $\langle u_i''^{(p)} M_i^{(p)} \rangle$ as

$$\langle V' Z_\sigma \rangle_s = \langle V Z_\sigma \rangle_s = \langle V_\sigma Z_\sigma \rangle_s. \tag{21}$$

Since at the interface the velocity boundary condition requires instantaneous velocities to be equal, we have

$$U_\sigma = V_\sigma, \tag{22}$$

which establishes

$$\langle u_i''^{(f)} M_i^{(f)} \rangle = -\langle U_\sigma Z_\sigma \rangle_s = -\langle V_\sigma Z_\sigma \rangle_s = -\langle u_i''^{(p)} M_i^{(p)} \rangle. \tag{23}$$

The exact interphase TKE terms for this limiting case are equal in magnitude, and opposite in sign. Thus, the principle of conservative interphase TKE transfer in Eq. (14) is rigorously established here.

Extensions

This derivation shows that the result holds for statistically homogeneous flows (where both phase-volume means and phase-surface means are homogeneous) with zero mean velocity in both phases. The result holds for both monodisperse as well as polydisperse particle-laden turbulent flows,

provided they satisfy the statistical homogeneity requirement on the number density and size distribution. If there is nonzero mean slip, then the conservation principle is modified to read

$$\langle u_i''^{(f)} M_i^{(f)} \rangle + \langle u_i''^{(p)} M_i^{(p)} \rangle = (\langle U \rangle_v - \langle V \rangle_v) \langle Z_\sigma \rangle_s, \tag{24}$$

which holds provided

$$\langle \langle U \rangle_v \rangle_s = \langle U \rangle_v, \tag{25}$$

$$\langle \langle V \rangle_v \rangle_s = \langle V \rangle_v. \tag{26}$$

These relationships hold if the phase-volume means $\langle U \rangle_v(x)$ and $\langle V \rangle_v(x)$ are statistically homogeneous in x, and if the phase-surface measure $\sigma_\beta(x)$ is also statistically homogeneous in x. The result can be extended to general inhomogeneous flows provided the velocity statistics are locally homogeneous on the scale of the particle size.

III. IMPLICATIONS FOR MULTIPHASE TURBULENCE MODELS

The implications of conservative interphase TKE transfer for some multiphase turbulence models are now examined. If Π_{k_f} is a model for $\langle u_i''^{(f)} M_i^{(f)} \rangle$ and Π_{k_p} is a model for $\langle u_i''^{(p)} M_i^{(p)} \rangle$, then the principle of conservative interphase TKE transfer demands that in the limiting case of statistically homogeneous flow with zero mean velocity

$$\Pi_{k_f} = -\Pi_{k_p}.$$

We now test some models to see if they satisfy this principle. The multiphase turbulence models considered are: (i) Ahmadi's model,[16,17] (ii) equilibration of energy model (EEM),[6] and (iii) Simonin's model.[18]

In Ahmadi's model, the governing equations for k_f and k_p in homogeneous particle-laden turbulent flows simplify to

$$\alpha_f \rho_f \frac{dk_f}{dt} = 2D_0(k_p - ck_f) - \alpha_f \rho_f \varepsilon_f, \tag{27}$$

$$\alpha_p \rho_p \frac{dk_p}{dt} = 2D_0(ck_f - k_p). \tag{28}$$

The evolution equation for mixture energy results from adding Eqs. (27) and (28)

$$\frac{de_m}{dt} = 2D_0(ck_f - k_p) + 2D_0(k_p - ck_f) - \alpha_f \rho_f \varepsilon_f = -\alpha_f \rho_f \varepsilon_f. \tag{29}$$

This shows that Ahmadi's model satisfies the principle of conservative interphase TKE transfer.

For the EEM,[6] the simplified equations for homogeneous particle-laden turbulent flow are

$$\frac{dk_f}{dt} = -\frac{1}{\tau_\pi}\left[C_2 k_f - (1 - C_2)\phi k_p\right] - \varepsilon_f, \tag{30}$$

$$\frac{dk_p}{dt} = -\frac{1}{\tau_\pi}\left[(1 - C_2)k_p - \frac{C_2}{\phi}k_f\right]. \tag{31}$$

The time scale τ_π is the interphase TKE transfer time scale and C_2 is a model constant. Adding Eqs. (30) and (31), the governing equation for the specific mixture energy e_m is obtained:

$$\frac{de_m}{dt} = -\alpha_f \rho_f \varepsilon_f. \tag{32}$$

The model for interphase TKE transfer in EEM also obeys the principle of conservative interphase TKE transfer.

Simonin's model for k_f and k_p evolution in homogeneous particle-laden turbulence is

$$\alpha_f \rho_f \frac{dk_f}{dt} = \alpha_p \rho_p \frac{1}{\tau_{12}^F}(k_{fp} - 2k_f) - \alpha_f \rho_f \varepsilon_f, \tag{33}$$

$$\alpha_p \rho_p \frac{dk_p}{dt} = -\alpha_p \rho_p \frac{1}{\tau_{12}^F}(2k_p - k_{fp}). \tag{34}$$

The resulting evolution equation for the specific mixture energy e_m is

$$\frac{de_m}{dt} = 2\alpha_p \rho_p \frac{1}{\tau_{12}^F}(k_{fp} - k_f - k_p) - \alpha_f \rho_f \varepsilon_f. \tag{35}$$

Here the interphase TKE transfer terms in fluid and particle phase are not conservative. Of the three multiphase turbulence models considered here, Ahmadi's model and EEM respect the principle of conservative interphase TKE transfer, whereas Simonin's model does not. We now examine the consequence of this observation in terms of meaningful comparison of these models to DNS.

IV. COMPARISON OF MODELS TO DNS

Since Ahmadi's model and EEM satisfy the principle of conservative interphase TKE transfer, their model expressions for Π_{k_β} can be directly compared with true DNS data for interphase TKE transfer. It is also consistent to then compare the Ahmadi and EEM models for $\alpha_f \rho_f \varepsilon_f$, as they appear in Eqs. (29) and (32), with true DNS data for the exact evolution of e_m [Eq. (15)]. Since Simonin's model does not satisfy the principle of conservative interphase TKE transfer, its model expression for Π_{k_β} cannot legitimately be compared with true DNS data for interphase TKE transfer. It is also not clear how Simonin's expression for ε_f, as it appears in Eq. (35), should be validated using true DNS data.

Most of the DNS data available for particle-laden turbulent flow are based on simulations that use the point-particle approximation, and these are widely used for model validation. Since the exact velocity boundary condition at each particle surface is not imposed in the point-particle approximation, it is useful to check if the implied DNS model for interphase TKE transfer satisfies the conservation principle. Note that true DNS satisfies the principle of conservative interphase TKE transfer automatically, since the exact velocity boundary condition is imposed at each particle surface.

Here we use the governing equations from Sundaram and Collins[9] for homogeneous particle-laden turbulence (with zero mean velocity in both phases) as an example of a DNS that uses the point particle approximation. The governing equations for total TKE in fluid phase T_f, particle phase T_p, and mixture energy T_t are

$$\frac{dT_f}{dt} = -\sum_{n=1}^{N_p} \frac{m_p \mathbf{u}(\mathbf{x}^n) \cdot [\mathbf{u}(\mathbf{x}^n) - \mathbf{v}^n]}{\tau_p} - \phi_v, \tag{36}$$

$$\frac{dT_p}{dt} = \sum_{n=1}^{N_p} \frac{m_p \mathbf{v}^n \cdot [\mathbf{u}(\mathbf{x}^n) - \mathbf{v}^n]}{\tau_p}, \tag{37}$$

$$\frac{dT_t}{dt} = -\phi_v - \phi_p, \tag{38}$$

where ϕ_v and ϕ_p are defined as

$$\phi_v = \int_{\mathcal{V}} \rho_f \varepsilon_f dV, \tag{39}$$

$$\phi_p = \sum_{n=1}^{N_p} \frac{m_p [\mathbf{u}(\mathbf{x}^n) - \mathbf{v}^n]^2}{\tau_p}. \tag{40}$$

In the above equations the total energy (T_p, T_f, T_t) is an extensive property since it is integrated over the entire flow domain $\mathcal{V}$. Here $\mathbf{x}^n$ and $\mathbf{v}^n$ denote the instantaneous position and velocity of the nth particle center, and $\mathbf{u}(\mathbf{x}^n)$ represents the fluid velocity at position $\mathbf{x}^n$. The total number of particles is N_p, and τ_p is the particle momentum response time.

The first terms on right-hand side of Eqs. (36) and (37), which represent the interphase TKE transfer between fluid and particle phase, do not sum to zero. Therefore, the interphase TKE transfer terms in point particle DNS are not conservative. The two terms that contribute to the evolution of the mixture energy T_t are: (i) the fluid phase dissipation rate ϕ_v, where $\varepsilon_f = 2\nu\langle s_{ij} s_{ij} \rangle$ is obtained from the gradients of the fluid velocity field, and (ii) the energy losses due to drag at the particle interfaces, ϕ_p. The dissipation rate ε_f in ϕ_v is the same as $\varepsilon_f^{\mathrm{pp}}$ introduced in Sec. I. The dissipation rate in true DNS ε_f^t, which determines the evolution of mixture energy, is different from $\varepsilon_f^{\mathrm{pp}}$, and this difference is presumably accounted for in point-particle DNS by the quantity ϕ_p, which scales as the square of the instantaneous relative velocity between the two phases, in Eq. (40). Therefore, comparing Eq. (38) with Eq. (15) reveals that $-(\phi_v + \phi_p)$ is a model for the right-hand side of Eq. (15) integrated over the flow domain. Whether the point particle DNS model for evolution of mixture energy that is described by Eq. (38) is accurate has yet to be verified by true DNS.

Clearly it is not meaningful to compare conservative models of interphase TKE transfer, such as Ahmadi's or EEM, with point-particle DNS data for the interphase TKE transfer term. Also in light of Eqs. (29) and (32), the Ahmadi and EEM models for ε_f should not be compared directly with the point-particle DNS dissipation rate $\varepsilon_f^{\mathrm{pp}}$. Instead, the integrated modeled mixture energy evolution equations [Eqs. (29) and (32)] should be directly compared to Eq. (38). In other words, the EEM and Ahmadi ε_f are models for the sum of $\varepsilon_f^{\mathrm{pp}}$ and the dissipation due to the relative velocity difference at the location of each particle [that appears in ϕ_p, which is defined in Eq. (40)]. One plausible identification of the terms in Simonin's model for comparison with point-particle DNS is ε_f should be compared with $\varepsilon_f^{\mathrm{pp}}$, and the Simonin model for $\Pi_{k_f} - \Pi_{k_p}$ corresponds to the term in Eq. (39) that scales as the square of the instantaneous relative velocity difference.

V. SUMMARY AND CONCLUSIONS

This study shows that for the limiting case of statistically homogeneous particle-laden turbulent flow with zero mean velocity in both phases, the interphase TKE transfer terms in the evolution of TKE in fluid and particle phases are equal and opposite in sign. The result holds under the following conditions:

1. the particle phase consists of rigid particles with constant thermodynamic density,
2. the flow is statistically homogeneous and the phase-volume mean and phase-surface mean of all flow quantities are statistically homogeneous,
3. the mean velocity in both phases is zero.

This conservative nature of the interphase TKE transfer term implies that the *exact* evolution equation for the mixture TKE does not depend on the interphase TKE transfer terms [cf. Eq. (15)]. The result can be extended in a slightly modified form to flows with nonzero mean slip velocity if the additional conditions expressed in Eqs. (25) and (26) hold. The result can also be extended to general inhomogeneous flows provided the velocity statistics are locally homogeneous on the scale of particle size.

This principle of conservative interphase TKE transfer has implications for single-point Eulerian second-moment closure models of particle-laden turbulent flow. Three models, Simonin's,[18] Ahmadi's,[16,17] and the Equilibrium of Energy model,[6] are examined to see if they obey this principle. Ahmadi's model and EEM satisfy the principle of conservative interphase TKE transfer, but Simonin's model does not.

The significance of the constraint expressed by the principle of conservative interphase TKE transfer manifests itself when performing term-by-term comparison of models with DNS data. We distinguish between true DNS, where the exact boundary conditions on velocity are imposed at each particle's surface, and point-particle DNS where the particles are point sources of momentum. Models for the interphase TKE transfer term that obey the conservation principle can be legitimately compared with data from true DNS. For these models, the modeled fluid dissipation rate solely determines the mixture TKE evolution [cf. Eqs. (29) and (32)], and it can be consistently compared with true DNS data for the exact mixture TKE evolution equation [Eq. (15)]. It is found that point-particle DNS do not satisfy the principle of conservative interphase TKE transfer. Therefore, it is not meaningful to compare conservative models for the interphase TKE transfer term with point-particle DNS data. Rather, when comparing predictions of a multiphase turbulence

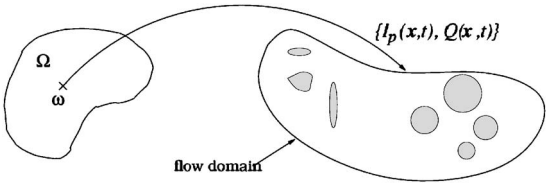

FIG. 2. The ensemble of realizations is defined by the event space Ω. Each realization of the two-phase flow corresponds to a mapping of an element ω in the event space Ω to a particular instance of the phase indicator function $I_\beta(\mathbf{x},t)$ and flow property $Q(\mathbf{x},t)$, which are defined in the flow domain $\mathcal{D}$.

model that satisfies conservative interphase TKE transfer with point-particle DNS data, the mixture TKE equations should be matched. In other words, the term $\alpha_f\rho_f\varepsilon_f$ in Eqs. (29) and (32) that contains the modeled dissipation rate should be compared (after integration over the flow domain) with $-(\phi_v+\phi_p)$ in Eq. (38), which represents the sum of ε_f^{pp} and the additional dissipation (assumed to scale as the square of the relative velocity between fluid and particle phases).

APPENDIX: MEASURES FOR TWO-PHASE FLOWS

As discussed in Sec. II, there are two types of expectations that arise when deriving averaged equations in the Eulerian-Eulerian approach: the phase-volume mean and the phase-surface mean. This is because quantities like the interphase momentum transfer are not volume-measurable because they are defined only on the particle surface and have zero volume measure. In other words, because these interface quantities have zero volume measure one cannot construct their expectation with respect to the Lebesgue measure in $\mathbb{R}^3$. On the other hand it is clear that the mean interphase momentum transfer is not zero. The resolution lies in the fact that the measure and expected value of interface quantities is different from those flow quantities defined in each phase, and this phase-surface measure and expectation needs to be unambiguously defined. Having identified the need to clearly define and distinguish between the phase-volume and phase-surface measure and expectation, we now describe the mathematical foundations needed to define these quantities.

We define a probability triple $(\Omega,\mathcal{F},P)$,[19] where Ω is the set of all events, $\mathcal{F}$ is a σ-field and P is a probability measure that is defined on this σ-field. Define a mapping from Ω space to a flow domain $\mathcal{D}$ in Euclidean space-time $\mathcal{D}\subset\mathbb{R}^4$ which takes every event $\omega\in\Omega$ to a realization of a two-phase flow in space-time that is described by the phase indicator function $I_\beta(\mathbf{x},t)$ and any flow property $Q(\mathbf{x},t)$ (for example, Q could be the velocity field, as shown in Fig. 2). This unambiguously defines the ensemble of realizations.

1. Phase-volume measure

Consider a set A in the flow domain $\mathcal{D}$. The phase volume measure of phase β denoted as $\mu_\beta(A;\omega)$, is defined as

$$\mu_\beta(A;\omega)\equiv\int_A I_\beta(\mathbf{x},t;\omega)\,dA. \tag{A1}$$

Note that this is a random measure because it depends on the realization of the two-phase flow corresponding to ω. The expected value of this random measure is

$$\langle\mu_\beta(A)\rangle=\int_\Omega\mu_\beta(A;\omega)\,dP_\omega=\int_\Omega\int_A I_\beta(\mathbf{x},t;\omega)\,dA\,dP_\omega, \tag{A2}$$

which is obtained by integrating with respect to the probability measure defined on Ω.

Because the integrations commute,

$$\langle\mu_\beta(A)\rangle=\int_A\int_\Omega I_\beta(\mathbf{x},t;\omega)\,dP_\omega\,dA=\int_A\langle I_\beta\rangle\,dA, \tag{A3}$$

where $\langle I_\beta\rangle$ is the expected value of the indicator function that is defined as

$$\langle I_\beta\rangle(\mathbf{x},t)\equiv\int_\Omega I_\beta(\mathbf{x},t)\,dP_\omega. \tag{A4}$$

If $\langle I_\beta\rangle(\mathbf{x},t)$ exists, then $\langle\mu_\beta(A)\rangle$ is absolutely continuous with respect to the Lebesgue measure (this is reasonable for particle two-phase flows where this measure is nonatomic except in pathological cases) and it can be written as the integral of a density over the set A. This density is nothing but the volume fraction $\alpha_\beta(\mathbf{x},t)$ of phase β. If the volume occupied by the phase β is denoted by V_β, and is given by

$$V_\beta\equiv\int_{\mathcal{D}} I_\beta(\mathbf{x},t)\,d\mathbf{x},$$

then it follows that its expected value is

$$\langle V_\beta(t)\rangle=\int_{\mathcal{D}}\langle I_\beta\rangle(\mathbf{x},t)d\mathbf{x}=\int_{\mathcal{D}}\alpha_\beta(\mathbf{x},t)\,d\mathbf{x},$$

where $\alpha_\beta(\mathbf{x},t)$ is the volume fraction occupied by phase β at location $\mathbf{x}$ and time t.

The phase-volume measure of flow quantity Q in phase β is denoted as

$$Q_\beta^v(A;\omega)\equiv\int_A I_\beta Q\,dA. \tag{A5}$$

Just as μ_β is a random measure, so is $Q_\beta^v(A;\omega)$. Its expectation is simply,

$$\langle Q_\beta^v(A)\rangle=\int_\Omega Q_\beta^v(A;\omega)\,dP_\omega=\int_\Omega\int_A I_\beta Q\,dA\,dP_\omega. \tag{A6}$$

Again, because the integrations commute we have

$$\langle Q_\beta^v(A)\rangle=\int_A\int_\Omega I_\beta Q\,dP_\omega\,dA=\int_A\langle I_\beta Q\rangle\,dA. \tag{A7}$$

If we assume that $\langle Q_\beta^v(A)\rangle$ is absolutely continuous with respect to the Lebesgue measure, then it can be written as the integral of a density function over the set A as

$$\langle Q_\beta^v(A)\rangle = \int_A \langle Q_\beta^v\rangle(\mathbf{x},t)\, d\mathbf{x}\, dt. \tag{A8}$$

Rewriting this density of the phase-volume measure of Q as a conditional density with respect to the density of expected phase-volume measure leads to the usual "phase average" in two-fluid theory,

$$\langle Q_\beta\rangle_v(\mathbf{x},t) = \frac{\langle Q_\beta^v(\mathbf{x},t)\rangle}{\alpha_\beta(\mathbf{x},t)}. \tag{A9}$$

2. Phase-surface measure

Consider a set A in the flow domain $\mathcal{D}$ and let S_β be the phase-surface indicator function for the β-phase side of the interface,

$$S_\beta(\mathbf{x},t) = \delta(\mathbf{x}-\mathbf{x}_I) I_\beta(\mathbf{x},t),$$

such that $S_\beta(A)$ selects the β-phase boundary of all I_β surfaces contained within set A.

The phase-surface measure of phase β is denoted as $\Sigma_\beta(A;\omega)$, and is defined as

$$\Sigma_\beta(A;\omega) \equiv \int_A S_\beta(\mathbf{x},t)\, dA. \tag{A10}$$

This quantity also is a random measure since it depends on the realization of the two-phase flow corresponding to ω. The expected value of this random measure is

$$\langle \Sigma_\beta(A)\rangle = \int_\Omega \Sigma_\beta(A;\omega)\, dP_\omega = \int_\Omega \int_A S_\beta(\mathbf{x},t)\, dA\, dP_\omega, \tag{A11}$$

which is obtained by integrating with respect to the probability measure defined on Ω.

Since the integrations commute,

$$\langle \Sigma_\beta(A)\rangle = \int_A \int_\Omega S_\beta\, dP_\omega\, dA = \int_A \langle S_\beta\rangle\, dA,$$

where $\langle S_\beta\rangle$ is the expected value of the phase-surface indicator that is defined as

$$\langle S_\beta\rangle(\mathbf{x},t) \equiv \int_\Omega S_\beta(\mathbf{x},t)\, dP_\omega. \tag{A12}$$

If $\langle S_\beta\rangle(\mathbf{x},t)$ exists, then $\langle \Sigma_\beta(A)\rangle$ is absolutely continuous with respect to Lebesgue measure and it can be written as the integral of a density over the set A. This density is nothing but the interfacial area density of phase β introduced in Drew and Passman,[1] which is denoted as $\sigma_\beta(\mathbf{x},t)$ in this work.

The phase-surface measure of flow quantity Q in phase β is denoted as $Q_\beta^s(A)$, and is defined as

$$Q_\beta^s(A;\omega) \equiv \int_A S_\beta Q\, dA. \tag{A13}$$

Note that this random measure picks out the value of Q on the β side of the interface. The value of a flow variable Q can be discontinuous at the interface, and this definition ensures that the correct value of $Q_\beta^s(A;\omega)$ is calculated for each phase on the appropriate side of the interface. Its expectation is simply

$$\langle Q_\beta^s(A)\rangle = \int_\Omega Q_\beta^s(A;\omega) dP_\omega = \int_\Omega \int_A S_\beta Q\, dA\, dP_\omega. \tag{A14}$$

Again, because the integrations commute we have

$$\langle Q_\beta^s(A)\rangle = \int_A \int_\Omega S_\beta Q\, dP_\omega\, dA = \int_A \langle S_\beta Q\rangle\, dA. \tag{A15}$$

If $\langle Q_\beta^s(A)\rangle$ is absolutely continuous with respect to the Lebesgue measure, then it can be written as the integral of a density which can be defined over the set A as

$$\langle Q_\beta^s(A)\rangle = \int_A \langle Q_\beta^s\rangle(\mathbf{x},t) d\mathbf{x}\, dt. \tag{A16}$$

Rewriting this density of the phase-surface measure of Q as a conditional density with respect to the density of expected phase-surface measure leads to the correct definition of the phase-surface mean,

$$\langle Q_\beta\rangle_s(\mathbf{x},t) = \frac{\langle Q_\beta^s(\mathbf{x},t)\rangle}{\sigma_\beta(\mathbf{x},t)}. \tag{A17}$$

This quantity has not yet been introduced in the two-fluid theory, and to the authors' knowledge, this is the first time it has been defined.

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