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**An Exit and Entry Study of Renewable Power Producers:  
A Real Options Approach**

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**Abstract**

In recent years, there has been a substantial increase of renewable power production sites. However, such sites operated in the 1980's were often abandoned in the 1990's in a massive scale with their remnants still visible. Hence, it is highly desirable to understand the exit and entry decisions of such sites. Toward this goal, we formulate and analyze models for such decisions of a single site from a real options perspective when the operation and maintenance (O&M) cost follows the geometric Brownian motion, and derive policy implications. For example, a front-loaded subsidy on initial investment will induce a premature (relative to the physical life) exit while a distributed subsidy on the O&M cost will alleviate such an exit. An extensive numerical example for a wind farm illustrates some of the key features of this study.

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## INTRODUCTION

In recent years, across the U.S. as well as several other regions such as the Western Europe and East Asia, there has been a massive increase in construction and operation of renewable power production sites (in short, “renewable site”). By renewable power, from an economic perspective, we mean electric power generated by energy sources that are without input fuel cost. Currently, such power is primarily wind and solar power. However, it also includes geothermal, tidal, and hydroelectric power (when the purpose of the dam is primarily for power generation).

For example, in 2009, the total capacity was 34,296 MW as compared to 24,651 MW in 2008 (EIA, 2010) in the U.S. The main drivers for this phenomenal growth have not only been the economic efficiency and technology breakthroughs in renewable power production, but also been the favorable government support due to environmental concerns as well as higher oil and natural gas prices. We note that currently such government support typically is in the form of subsidies and incentives that are front-loaded in the construction and early operating years (Wiser *et al.*, 2007a).

This massive increase, however, leads to a few critical questions that are both old and new. One major recurring question is regarding the exit of such renewable sites in a massive scale. That is, in the 1990’s there were numerous cases of abandonment of renewable sites that had been in operation since 1970’s and 1980’s where the renewable site decision makers simply walked away (WebEcoist.com, 2011) as the economic and non-economic conditions deteriorated. For example, there are thousands of abandoned wind turbines still littering the landscape in the areas of Altamont Pass, Tehachapi, and San Gorgonio in California alone (Wilkinson, 2010).

The situation for solar farms is quite similar, and certainly no better (The Center for Land Use Interpretation, 2011).

At this point in time, the “rebirth” of renewable sites is of an order of magnitude larger than the previous manifestation in 1970’s and 1980’s, and a priori there is no basis that one can assume the abandonment in a truly massive scale will not happen in the near future. In fact, currently, in numerous regions, if a renewable site decision maker walks away (i.e., abandons the renewable site), there is no or few consequences or penalties (WebEcoist.com, 2011). Specifically, we observe that, as the operation and maintenance (O&M) cost increases with respect to time (Wiser and Bolinger, 2008), there is a substantial incentive to walk away even when there remains some “physical” life in the renewable site (see e.g., Myers and Majd, 1983). Furthermore, the exact amount of the O&M cost at a given future time is typically stochastic not only because numerous repairs are unpredictable, but also because the current physical age is often quite young relative to the physical life estimated by the site builders (see e.g., Martinze *et al.*, 2009) with few available previous data.

More recently, we do note that, in a few places, there has been some consideration or implementation of a simple and straightforward exit fee as an insurance to enable the proper disposal of the renewable site in case of abandonment (New Hampshire Office of Energy and Planning, 2008). This observation has provided us with a major motivation for this study.

Under these circumstances, it is highly desirable to understand the economically rational decisions on exits and entries of renewable sites, of which understanding includes timely policy implications especially in the areas of government’s incentives and fees. As a first step toward this goal, in this paper, we 1) formulate and analyze mathematical models of exit and entry decisions of a single renewable site from a real options perspective with the assumption that the

O&M cost follows the geometric Brownian motion (GBM) process, 2) show the managerial insights from the sensitivity analyses surrounding the exit and entry decisions that are made by renewable site decision makers, and 3) derive policy implications that are theoretically interesting and practically timely. From this study, we hope to provide specific insights for relevant decisions and policies, and stimulate the critical discussion among renewable site decision makers, government regulators, legislative policy makers, as well as academics.

What distinguishes this paper from the extant literature on the renewable site economics and finance is that 1) this is the first quantitative study that, to our knowledge, explains the economically rational decisions of the exit and entry of renewable sites under the assumption of the GBM based O&M cost, 2) the decision and policy insights are provided concerning critical threshold values such as the optimal O&M cost to exit and the maximum initial O&M cost to enter with respect to various parameters such as the electricity price and the renewable site capacity, and 3) their policy implications on government subsidies and penalties with respect to some key parameters such as the initial investment and exit fee.

For this study, there are several groups of relevant literature. First, in the area of financial and economic applications in the electric power industry, Wang and Min (2006) applied a real options approach to a case of inter-related generation projects. Also, in Wang and Min (2008), a financial portfolio consisting of electric power commodities was managed while, in Wang and Min (2010), financial hedging techniques were developed for electric power producers. We also note that, in the context of nuclear power plants (which is not renewable power), Takashima *et al.* (2007) investigated decommissioning by applying a real options approach. This paper is different from our approach as the exit would be primarily influenced by the stochastic electricity price,

and not by any cost, which was assumed to be deterministic. As for solar power, Lorenz *et al.* (2008) advocated phasing economic incentives out, but only prudently and gradually.

For wind power management, Fleten and Maribu (2004) addressed the investment timing and capacity choice of wind power under uncertainty. Also, Fleten *et al.* (2007) examined the investment strategies when renewable power is generated in a decentralized manner. These papers are different from our approach because they would not be able to explain the exit behavior that can be theoretically predicted or empirically observed. Also, the stochastic components in these papers are not the O&M cost (e.g., electricity price).

As for the stochastic O&M costs, Khoub Bakht *et al.* (2008) investigated statistically repair and maintenance cost models for tractors while Leung and Lai (2003) studied statistically the quality and reliability aspects of bus engines. Almansour and Insley (2011) on the other hand, applied the stochastic cost model for oil sands in Canada. In addition, in Costa Lima and Suslick (2006), both the price and operating costs are modeled as GBM processes for mining projects.

Meanwhile, in the area of equipment replacement, there are numerous examples of more conventional (i.e., non-real options approaches) papers (see e.g., Hartman and Murphy, 2006). In Ye (1990), the replacement strategy was derived under the assumption of a GBM O&M cost while in Zambujal-Oliveira and Duque (2011) considered both O&M cost and salvage value as GBM processes.

The rest of the paper is organized as follows. We first model the exit decision of a currently operating renewable site under the assumption of a GBM based O&M cost. This model leads to the derivation of the threshold O&M cost, above which the renewable site exits the market, as well as to the derivation of the expected remaining life of the renewable site. Next, we conduct the sensitivity analysis of the exit threshold O&M cost and the expected remaining life

with respect to various critical parameters such as the exit fee. We then consider the entry decision of a new renewable site, and derive the threshold O&M cost, below which such a renewable site will enter the market, as well as the expected life of the renewable site. This is also followed by the sensitivity analysis of the entry threshold O&M cost and the expected life. We next present some of the features of our model through an illustrative numerical example based on a wind farm, and discuss the policy implications and the various assumptions defining the scope of our study. Finally, we make concluding remarks and comment on future research.

## **MODELING AND ANALYSIS OF RENEWABLE SITE EXIT DECISION**

In this section, let us consider a firm consisting of a single renewable site currently in operation. For the decision maker of this firm, we assume that there exists an option to abandon the renewable site, and to exit the electricity market. Furthermore, we make the following critical assumptions that enable us to model and analyze the exit decision that is economically rational as well as tractable. We note that the relaxation of the assumptions that would enhance the realism of the models in this paper will be addressed in DISCUSSION Section later.

Assumption 1: There already exists a power purchase contract between the renewable site and a utility company with the appropriate transmission connection at a fixed selling price of  $P$  (\$/MWh) at any time point (see Roques *et al.*, 2008). We assume that the upper bound of the renewable power purchase quantity of the contract will not be reached at this site.

Assumption 2: In making the exit decision, the decision maker will rely on the fixed (expected) production quantity per unit time, which is equal to the quantity demanded by the utility (e.g., via a contract). By this assumption, we are not implying the actual production and demand quantities remain constant over time, but, for *planning purposes* of the firm, we are

simplifying the dynamic aspects of renewable energy source such as wind speed or daylight availability, etc. (see e.g., Fleten and Maribu, 2004). Specifically, we will utilize the fixed annual production quantity,  $K$  (MWh/year). We further assume that, this production quantity is equal to the nameplate capacity times the capacity factor, which is a standard procedure for the generation planning in the electric power industry, alternative procedures notwithstanding (see e.g., Wisner *et al.*, 2007b)

Assumption 3: The operation and maintenance cost (O&M cost; \$/MWh) at any time point,  $C$ , follows the geometric Brownian motion (GBM) process. Specifically,

$$dC = \alpha_c C dt + \sigma_c C dz \quad (1)$$

where  $\alpha_c$  is the instantaneous growth rate of the O&M cost (% per year),  $\sigma_c$  is the instantaneous volatility of  $C$  (% per square root of year), and  $dz$  is the increment of a standard Wiener process  $z$  ( $dz = \varepsilon_t \sqrt{dt}$  where  $\varepsilon_t \sim N(0,1)$ ).

Typically, the O&M cost includes costs associated with repairs, spare parts, maintenances, and consumables necessary for the O&M (see e.g., EWEA, 2004). Also, we note that modeling the O&M cost as a GBM process is not new. For example, Ye (1990), Mauer and Ott (1995), and Zambujal-Oliveira and Duque (2011) all utilized GBM processes to represent the O&M cost in various contexts of equipment replacement policies such as in conjunction with the tax and technology uncertainties. In addition, in Costa Lima and Suslick (2006), the operating cost of a mining company is represented by a GBM process.

Assumption 4: There exists a fixed exit fee of  $W$  ( $W \geq 0$ ) when the option to abandon is exercised, and the decision maker is aware of this a priori. Currently in the U.S., few local, state, and federal rules and regulations exist that require the removal and disposal of renewable sites

when the option to abandon is exercised, and some authors claim that the renewable site decision makers have strong incentive to abandon their renewable sites even if it is physically viable to operate (Wilkinson, 2010). Our model will allow this current situation as a special case of  $W = 0$ .

Assumption 5: The remaining cost components are irrelevant to the decision maker regarding the option to abandon. i.e., the cost components such as non-specific overheads, taxes, etc. are negligible for our planning purposes. We also assume that, there is no salvage value at the time point of exit, and our model does not explicitly factor in specific government programs such as the production tax credit (see e.g., Union of Concerned Scientists, 2011).

Assumption 6: More sophisticated options such as partial shut-down, mothballing, etc. are not available for the renewable site. These options will be beyond the scope of this paper, and will not be considered.

Under these assumptions, our problem can be interpreted as an optimal stopping problem. We observe that, the higher the O&M cost, the stronger the incentive to abandon. Therefore, intuitively, the range of the O&M cost where the optimal decision is to abandon may be characterized by a single scalar threshold of  $C^*$ . That is, if  $C$  is between  $[C^*, \infty)$ , then the optimal decision is to abandon. And, otherwise, the optimal decision is to continue operation. It can be shown (see e.g., Dixit and Pindyck, 1994) that indeed there will be only one threshold  $C^*$  for this problem.

As long as it is not optimal to abandon the renewable site (i.e.,  $C$  is in the continuation region), the value of the renewable site project,  $V(C)$ , must satisfy the following differential equation, which results from Bellman's principle of optimality (Dixit and Pindyck, 1994):

$$\rho V dt = (P - C) K dt + E[dV] \quad (2)$$

where  $\rho$  is the annual discount rate (% per year), which is often called the expected rate of return (Dixit and Pindyck, 1994). The left-hand side of (2) is the total return on the value of the renewable site. The first term of the right-hand side is the immediate profit flow from keeping the renewable site operating while the last term is the expected capital appreciation of the renewable site value function.

By Ito's Lemma, it can be verified that  $dV$  is given by,

$$dV = \frac{\partial V}{\partial C}(\alpha_c C dt + \sigma_c C dz) + \frac{1}{2} \frac{\partial^2 V}{\partial C^2} \sigma_c^2 C^2 dt \quad (3)$$

Substituting (3) into (2), and after rearranging and simplifying terms, we have

$$\frac{1}{2} \frac{\partial^2 V}{\partial C^2} \sigma_c^2 C^2 + \frac{\partial V}{\partial C} \alpha_c C - \rho V + (P - C)K = 0 \quad (4)$$

We note that (4) is a differential equation. To guarantee its convergence, we impose that  $\alpha_c < \rho$  (see e.g., Costa Lima and Suslick, 2006). We also note that the following boundary conditions between the operation and abandonment states are needed to obtain the optimal threshold  $C^*$ .

$$V(C^*) = -W \quad (5)$$

$$V'(C^*) = 0 \quad (6)$$

The first one is the value-matching condition and the second one is the smooth-pasting condition. The value-matching condition (5) requires that at the exit threshold, the value of the renewable site project equals to the value of exit. The smooth-pasting condition (6) assures that  $C^*$  is the optimal exercise point by defining the continuance and smoothness of  $V(C)$  at  $C^*$ .

By solving (4) with (5) and (6), we have the following proposition.

**Proposition 1** Given  $0 < \alpha_c < \rho$ , the value of an operating renewable site is

$$V(C) = A_1 C^{\beta_1} - CK/(\rho - \alpha_c) + PK/\rho \quad (7)$$

$$\text{where } \beta_1 = \left( 1/2 \sigma_c^2 - \alpha_c + \sqrt{(\alpha_c - 1/2 \sigma_c^2)^2 + 2\sigma_c^2 \rho} \right) / \sigma_c^2 \quad (8)$$

$$C^* = \frac{(PK/\rho + W)(\rho - \alpha_c)\beta_1}{(\beta_1 - 1)K} \quad (9)$$

$$A_1 = \frac{K}{(\rho - \alpha_c)\beta_1 (C^*)^{\beta_1 - 1}} \quad (10)$$

The proof is given in Appendix A.1. The economically rational decision is as follows. As soon as the O&M cost is equal to or greater than  $C^*$ , the firm will pay the exit fee and abandon the renewable site with the corresponding renewable site project value of  $-W$ . Otherwise, the firm continues to operate the renewable site with the corresponding value of  $V(C)$ .

We also note that the value function  $V(C)$  given by (7) has the following interpretation. Before the decision maker chooses to abandon (i.e., the decision maker is still holding the option to abandon), the value of the renewable site consists two parts: the value of the operating renewable site and the value of the option to abandon the renewable site. Hence, in (7), the first one term is the value of the option to abandon, while the last two terms represent the expected cost and revenue streams when the initial price and cost are observed as  $P$  and  $C$ .

Thus far, we have derived the value function of the renewable site project, and the threshold value of  $C^*$  in terms of aforementioned critical parameters. We now proceed to derive the expected remaining life of the renewable site under the technical assumption of  $\alpha_c > 1/2 \sigma_c^2$ ,

as shown in the relevant literature (see e.g., Mauer and Ott, 1995). Let  $F(C) = \ln C$ . Then,  $dF(C)$  can be expanded by Ito's Lemma as  $dF(C) = (\alpha_c - 1/2 \sigma_c^2)dt + \sigma_c dz$  and therefore for any finite time period  $T$ , the change in  $F(C)$  is distributed with mean  $(\alpha_c - 1/2 \sigma_c^2)T$  and variance  $\sigma_c^2 T$ . Hence the expected first passage time of  $C^*$ , measured from  $C_c$ , the current level of the O&M cost, can be calculated as  $(\ln C^* - \ln C_c) / (\alpha_c - 1/2 \sigma_c^2)$ .

Hence, the expected remaining operating life is given by,

$$T_{EX}^* = (\ln C^* - \ln C_c) / (\alpha_c - 1/2 \sigma_c^2) \quad (11)$$

where  $C^*$  is the exit threshold as in (9).

## SENSITIVITY ANALYSIS ON THE EXIT DECISION

Given the expression for  $C^*$  shown in (9), the sensitivity analysis can be performed in a straightforward manner with respect to the key parameters of  $\sigma_c$ ,  $\alpha_c$ ,  $W$ ,  $P$ , and  $K$ , and the results are summarized in the following proposition.

**Proposition 2** Given  $0 < \alpha_c < \rho$ ,  $\frac{\partial C^*}{\partial \sigma_c} > 0$ ,  $\frac{\partial C^*}{\partial \alpha_c} < 0$ ,  $\frac{\partial C^*}{\partial W} > 0$ ,  $\frac{\partial C^*}{\partial P} > 0$ , and  $\frac{\partial C^*}{\partial K} < 0$ .

The outline of the proof is given in Appendix A.2, and the economic interpretation of the results is as follows.  $\frac{\partial C^*}{\partial \sigma_c} > 0$  indicates that the increase in the volatility leads to the increase in the threshold value. This is so because with more volatility, there is more chance of deeper reduction in the O&M cost in the near future, and it turns out to be more beneficial to wait a little longer (hence,  $C^*$  becomes higher). As for  $\frac{\partial C^*}{\partial \alpha_c} < 0$ , we note that as the O&M cost growth rate

increases, it is more beneficial to exit earlier (hence,  $C^*$  becomes lower).  $\frac{\partial C^*}{\partial W} > 0$  indicates that the increase in the exit fee leads to the increase in the threshold value. This is so because, with a higher level of the exit fee, the decision maker has an incentive to wait a little longer for a possible reduction in the O&M cost in the near future (hence,  $C^*$  becomes higher). Now, as for  $\frac{\partial C^*}{\partial P} > 0$ , if the contract power price is increasing, we note that the revenue and the value of the renewable site project should increase, and the decision maker has an incentive to continue to operate a little longer (hence,  $C^*$  becomes higher). Finally,  $\frac{\partial C^*}{\partial K} < 0$  indicates that as the production quantity increases, the threshold value decreases. This is so because the total O&M cost increases even more as it is proportional to the production quantity. Therefore, it is more beneficial to exit earlier (hence,  $C^*$  becomes lower). We note that, due to our Assumption 2 (this production quantity is equal to the nameplate capacity times the capacity factor), this sensitivity analysis can be applied to the capacity in place of production quantity interchangeably.

If we assume that preventing premature exit (relative to the physical life) is environmentally desirable (so that we can delay exploiting new resources), increases in  $W$  and  $P$  are desirable while an increase in  $K$  is not. Hence, any government policies through incentives and fees to increase the contract power price or the exit fee are desirable while to increase the capacity is not. For example, if a community is recruiting a single renewable site, the smaller capacity is more desirable than the larger capacity assuming that preventing premature exit is the primary criterion. We caution that this interpretation is strictly focusing on the exit decision of the existing renewable sites under the aforementioned criterion, and will be revisited numerous times in the succeeding sections of this paper.

Finally, we note that the sign of  $\frac{\partial C^*}{\partial \rho}$  is ambiguous as some components of  $C^*$  increase while others decrease in a way that the total effect is unwieldy to interpret. We also note that, with the expected remaining operating life of (11), we have  $\frac{\partial T_{EX}^*}{\partial C_c} < 0$ ,  $\frac{\partial T_{EX}^*}{\partial \sigma_C} > 0$ ,  $\frac{\partial T_{EX}^*}{\partial \alpha_C} < 0$ ,  $\frac{\partial T_{EX}^*}{\partial W} > 0$ ,  $\frac{\partial T_{EX}^*}{\partial P} > 0$ , and  $\frac{\partial T_{EX}^*}{\partial K} < 0$ . The interpretation of these results is analogous to the case with respect to  $C^*$ .

## MODELING AND ANALYSIS OF RENEWABLE SITE ENTRY DECISION

In this section, we extend the previous model by considering the entry decision for the renewable site. For this extension, we assume that the aforementioned power purchase contract is available for a new renewable site. Two additional assumptions are made as follows.

Assumption 7: The construction period of the renewable site is assumed to be negligible in our model. This simplifying assumption is made so as to focus on the entry decision without diluting our attention on how best to make economic decisions during the construction period. This type of simplifying assumptions can be found in numerous papers (see e.g., Fleten and Maribu, 2004).

Assumption 8: Once the construction occurs, then there is a lump sum investment cost of  $I$  (\$), which includes the cost of materials, labor, land, etc. This cost is treated as an irreversible sunk cost, which cannot be recovered later.

We note that the firm makes an entry decision by evaluating the direct net revenue (i.e., revenue minus cost) from the renewable site plus the value of the option to exit. We recall that

the option value critically depends on the O&M cost, and we will denote  $C_0$  as the initial O&M cost at time point at which the renewable site starts to operate. We further note that there is no O&M cost prior to the start of the renewable site.

We also note that, under the additional assumption of the contract power price following a GBM process, the option value for waiting to enter can be incorporated. In our model construction, however, such an option is excluded by design because 1) there are numerous fixed price contracts already in practice and 2) parallel GBM processes of the price and cost typically make analytical studies infeasible and often make numerical results difficult to sort out (see more in the Discussion section).

Under our model framework, if the value of the potential renewable site project is greater than or equal to the irreversible investment  $I$ , the firm will enter the market. Therefore, the condition under which the firm decides to enter becomes:

$$V(C_0) = A_1 C_0^{\beta_1} - C_0 K / (\rho - \alpha_c) + PK / \rho \geq I \quad (12)$$

For the boundary “marginal” firm without a strictly positive net benefit, we define another type of the O&M cost threshold,  $\bar{C}_0$ . Namely,

$$A_1 \bar{C}_0^{\beta_1} - \bar{C}_0 K / (\rho - \alpha_c) + PK / \rho - I = 0 \quad (13)$$

This  $\bar{C}_0$  of (13) can be viewed as the upper bound of the initial O&M cost at which the firm will decide to enter the market.

More formally, even though there is no explicit closed-form solution for  $\bar{C}_0$  in (13), we have the following proposition that proves the existence and uniqueness of  $\bar{C}_0$  under two fairly undemanding conditions.

**Proposition 3** Let us assume that  $\bar{C}_0 < C^*$  and  $PK/\rho - I > 0$ , then there exists a unique solution for  $\bar{C}_0$  in  $A_1 \bar{C}_0^{\beta_1} - \bar{C}_0 K / (\rho - \alpha_c) + PK/\rho - I = 0$ .

The proof is given in Appendix A.3. The condition  $\bar{C}_0 < C^*$  is not stringent as, if  $\bar{C}_0 \geq C^*$ , a firm will enter and exit the market instantaneously, which leads to no practical use nor sense (not unlike assuming  $\alpha_c \geq \rho$ ). The condition  $PK/\rho - I > 0$  indicates that the cumulative revenue ( $\int_0^\infty PK e^{-\rho s} ds = PK/\rho$ ) is greater than the initial investment  $I$ . It is a reasonable assumption that any firm considering the entry has a tangible revenue stream that will at least cover the initial investment.

From Proposition 3 and the monotonicity of  $V(C_0)$  shown in the proof, we claim that any firm with  $C_0 \leq \bar{C}_0$  will enter the market while any other firm with  $C_0 \geq \bar{C}_0$  will not.

Finally, we note that, as in the case of the exit decision, we derive the expected economic life of a new renewable site to be

$$T^* = (\ln C^* - \ln C_0) / (\alpha_c - 1/2\sigma_c^2) \quad (14)$$

and that of a marginal renewable site with  $C_0 = \bar{C}_0$  to be

$$\bar{T} = (\ln C^* - \ln \bar{C}_0) / (\alpha_c - 1/2\sigma_c^2) \quad (15)$$

## SENSITIVITY ANALYSIS ON THE ENTRY DECISION

Given the implicit function for  $\bar{C}_0$  in (13), the sensitivity analysis can be performed with respect to the key parameters of  $\sigma_c$ ,  $\alpha_c$ ,  $W$ ,  $P$ , and  $I$ , and the results are summarized in the following proposition.

**Proposition 4** Given  $0 < \alpha_c < \rho$ ,  $\frac{\partial \bar{C}_0}{\partial \sigma_c} > 0$ ,  $\frac{\partial \bar{C}_0}{\partial \alpha_c} < 0$ ,  $\frac{\partial \bar{C}_0}{\partial W} < 0$ ,  $\frac{\partial \bar{C}_0}{\partial P} > 0$ , and  $\frac{\partial \bar{C}_0}{\partial I} < 0$ .

The outline of the proof is given in Appendix A.4. The interpretation for  $\frac{\partial \bar{C}_0}{\partial \sigma_c} > 0$ ,

$\frac{\partial \bar{C}_0}{\partial P} > 0$ , and  $\frac{\partial \bar{C}_0}{\partial \alpha_c} < 0$  is straightforward. i.e., as the volatility and the contract power price

increase, the entry threshold O&M cost increases. i.e., more (marginal) firms will enter the market. On the other hand, as the growth rate in the O&M cost increases, less (marginal) firms will enter the market.

As for  $\frac{\partial \bar{C}_0}{\partial W} < 0$ , the increase in the exit fee will lead to the decrease in the threshold

O&M cost. This implies that less firms will enter, resulting in a lower power production quantity from the renewable energy. At the same time, this will lead to a lower number of marginal firms, reducing the number of premature exit (relative to the physical life). Therefore, the economic and environmental consequences of a government policy for a higher exit fee on the entry (not exit) of renewable sites are far from simple and straightforward.

As for  $\frac{\partial \bar{C}_0}{\partial I} < 0$ , the increase in the initial investment will decrease the threshold O&M

cost. This implies that any initial subsidy provided by the government will lead to more (marginal) firms entering the market resulting in a higher power production quantity from the renewable energy. On the other hand, this will lead to a higher number of marginal firms, increasing the number of premature exit (relative to the physical life). Once again, the consequence of a government policy for a higher level of initial subsidy on the entry (not exit) is complex.

In addition, we note that the signs of  $\frac{\partial \bar{C}_0}{\partial \rho}$  and  $\frac{\partial \bar{C}_0}{\partial K}$  are ambiguous with the reason that is

similar to the one for the case of  $\rho$  in the exit sensitivity analysis.

Let us now turn our attention to the expected life of a new renewable site,  $T^*$ . As in the case of the exit decision, we can obtain  $\frac{\partial T^*}{\partial C_0} < 0$ ,  $\frac{\partial T^*}{\partial \sigma_c} > 0$ ,  $\frac{\partial T^*}{\partial \alpha_c} < 0$ ,  $\frac{\partial T^*}{\partial W} > 0$ ,  $\frac{\partial T^*}{\partial P} > 0$ , and

$\frac{\partial T^*}{\partial K} < 0$  with straightforward and intuitive interpretation.

As for the expected life of a new renewable site from a marginal firm,  $\bar{T}$ , generally, the signs from the sensitivity analysis are ambiguous and the corresponding interpretation unwieldy.

The only exceptions are:  $\frac{\partial \bar{T}}{\partial W} > 0$  and  $\frac{\partial \bar{T}}{\partial I} > 0$ , i.e., as the exit fee or the initial investment increases, the corresponding expected life increases. This is so because, as the exit fee or the initial investment increases, the threshold O&M cost to enter will decrease, which result in longer expected life.

## NUMERICAL ANALYSIS: THE CASE OF A WIND FARM

In this section, we numerically illustrate some of the key features of our models as follows.

### 1. Parameter values

Let us first present the parameter values used in this section. Even though these values are hypothetical, to be realistic numbers, we have consulted U.S. Energy Information Administration's *Updated Capital Costs Estimates for Electricity Generation Plants* (EIA, 2010) as well as others (e.g., Kjarland, 2007; Takashima *et al.*, 2007). They are summarized in Table 1.

**Table 1** Parameters and corresponding values

Contract power price $P$	48 \$/MWh
Nameplate capacity/Capacity factor	3 MW/33.33%
Production quantity $K$	8,760 MWh/yr
Investment cost $I$	\$1,000,000
Exit fee $W$	\$300,000
Annual discount rate $\rho$	0.05
Annualized growth rate of O&M cost $\alpha_C$	0.04
Annualized volatility of O&M cost $\sigma_C$	0.10

As explained in Assumption 2, the linkage between the capacity and production quantity is as follows. Let us assume that the total number of hours of operations per year is given by 8,760 hours. Then, as the production quantity per year as well as the nameplate capacity and capacity factor are assumed to be constant,  $K = 3 * 0.3333 * 1 * 8,760 = 8,760$  (MWh/yr).

## 2. The entry/exit decisions

By applying the parameter values to (7), (8), (9), (10), and (13), the threshold values of  $C^*$  and  $\bar{C}_0$  (\$/MWh) as well as the function of  $V(C)$  can be calculated. At the same time with  $\bar{C}_0$ , we can use (15) to calculate  $\bar{T}$  as well. The numerical results are summarized in Table 2.

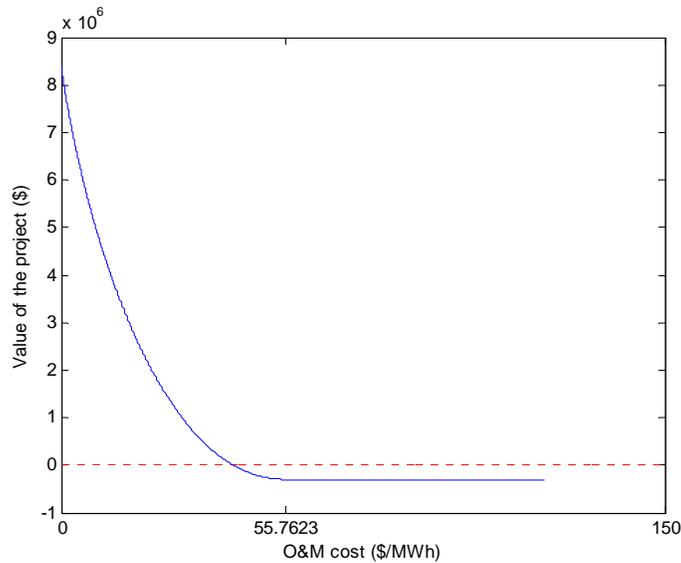
**Table 2** Numerical results of the entry/exit decisions

$\beta_1$	1.2170
$A_1$	300, 800
$C^*$	55.7623
$\bar{C}_0$	30.0549
$V(C)$	$300,800C^{1.2170} - 876,000C + 8,409,600$
$\bar{T}$	17.6592

We do note that, the first term of  $V(C)$  is the option value to exit, the second term is the O&M cost, and the third term is the revenue. Also, given  $\bar{C}_0$  value of 30.0549, it can be verified that  $V(\bar{C}_0)$  is 1,000,000, which is the initial investment. Furthermore, if a firm is considering an

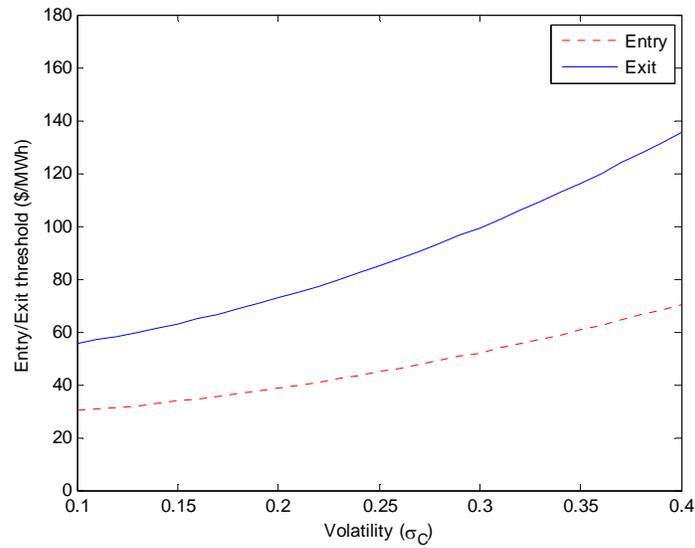
entry, given the initial O&M cost, we can calculate the expected life of a new wind farm by (14). Likewise, if a firm is currently operating the wind farm, then given the current O&M cost, we can calculate the remaining life of the existing wind farm by (11).

Figure 1 shows the value of the project with respect to the O&M cost when the wind farm is in operation. As indicated in the graph, the value of the project decreases as the O&M cost increases until it reaches  $C^*$ . Once the cost reaches the exit threshold, the firm will pay the exit fee to abandon the wind farm and thus  $V_{EX}(C) = -W$ .



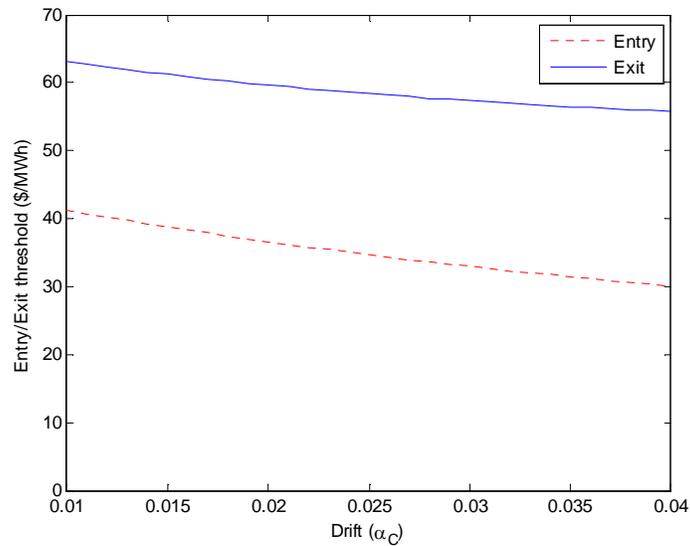
**Figure 1** Value of the project vs. the O&M cost

In Figure 2, the thresholds of the O&M cost  $C^*$  and  $\bar{C}_0$  are depicted with respect to the O&M cost volatility. Both threshold values increase as the O&M cost becomes more volatile, which indicates that a higher degree of volatility will delay the exit and allow more (O&M cost-wise) marginal firms to enter. It is interesting to note that the slope is much steeper for the exit threshold than that for the entry threshold as the volatility increases. i.e., it seems that the exit threshold is more sensitive to the volatility than the entry threshold is.



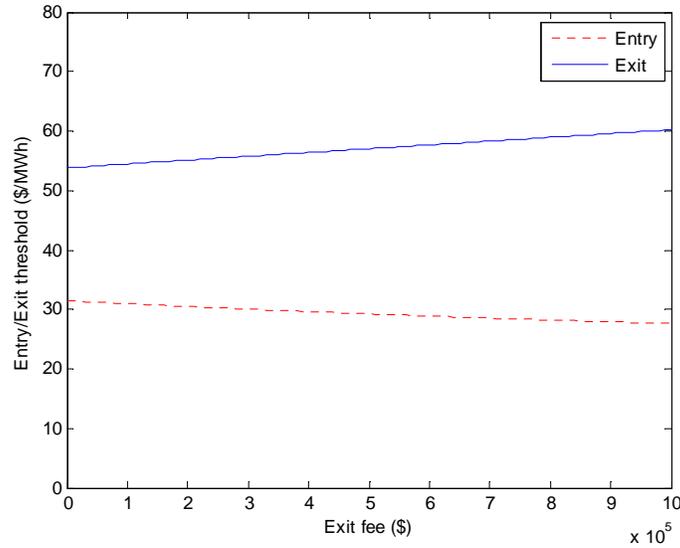
**Figure 2** Entry/exit thresholds vs. the volatility of O&M cost

In Figure 3, the thresholds of the O&M cost  $C^*$  and  $\bar{C}_0$  are depicted with respect to the O&M cost growth rate. Both threshold values decrease as the growth rate increases. This implies that a higher growth rate of the O&M cost induces early exit and allows less marginal firms to enter the market.



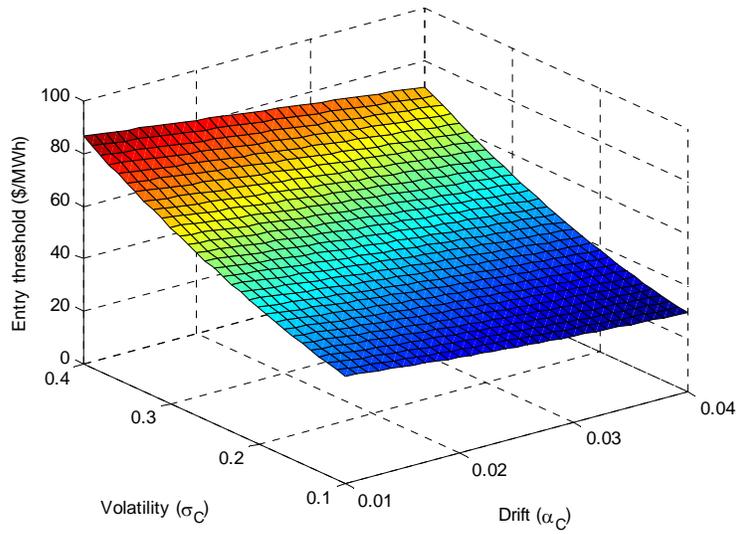
**Figure 3** Entry/exit thresholds vs. the growth rate of O&M cost

In Figure 4, the thresholds of the O&M cost  $C^*$  and  $\bar{C}_0$  are depicted with respect to the exit fee. As we can see from the graph, as the exit fee increases, the exit threshold increases, which leads the existing firm to delay its abandonment. At the same time, it will allow less marginal firms to enter the market, reducing the total power production from wind energy.



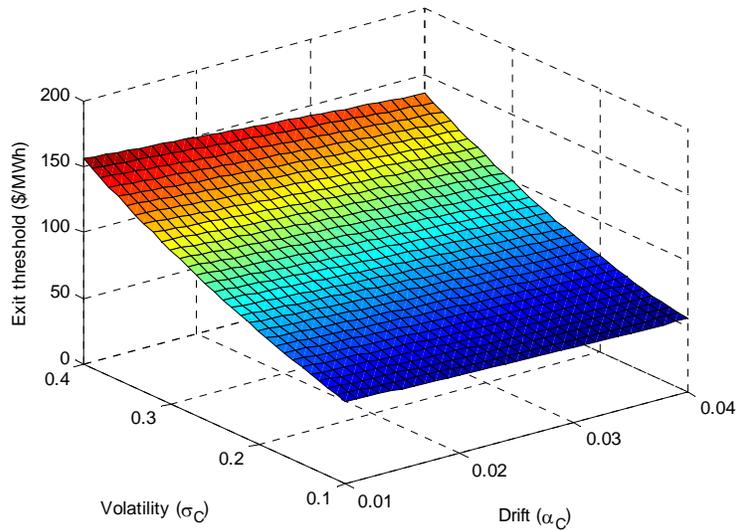
**Figure 4** Entry/exit thresholds vs. the exit fee

We note that similar graphs can be produced for the remaining parameters such as the contract power price  $P$ , the power production quantity  $K$ , and the initial investment  $I$ , among others. We now proceed to examine the impact of bi-directional changes on the entry and exit thresholds. First in Figure 5, we are varying the volatility and growth rate, and observe that the entry threshold is behaving as predicted. Furthermore, the degree of changes seems more linear than nonlinear.



**Figure 5** Entry threshold vs. the growth rate and volatility of O&M cost

In Figure 6, we are varying the volatility and growth rate, and observe that the exit threshold is behaving as predicted. Furthermore, the degree of changes here also seems more linear than nonlinear.



**Figure 6** Exit threshold vs. the growth rate and volatility of O&M cost

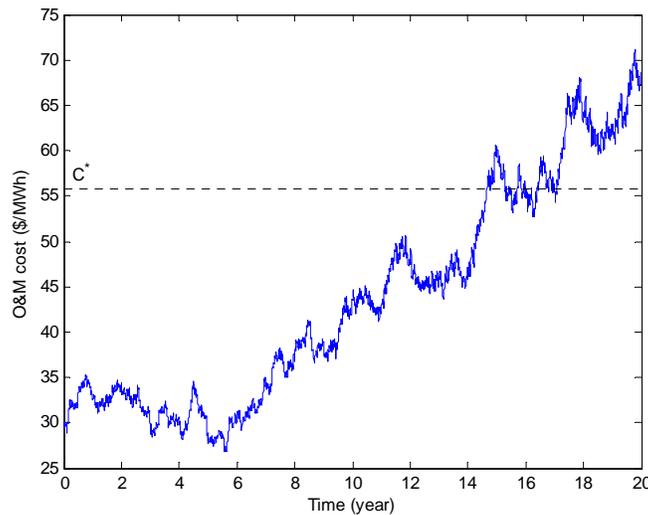
We note that this type of bi-directional analyses can be conducted for the remaining parameters

### 3. Monte Carlo simulation

In the previous subsections, the value of a wind farm project, the entry/exit thresholds of the O&M cost, and the expected economic life are calculated based on the fixed hypothetical data. In this section, in contrast, Monte Carlo simulation are used to simulate sample paths given  $C_0 = \bar{C}_0 = 30.0549$  \$/MWh as the starting point and the GBM process as

$$dC = 0.04Cdt + 0.10Cdz$$

A typical sample path is shown below in Figure 7 with the horizontal axis as the time (year) and the vertical axis as the O&M cost (\$/MWh).



**Figure 7** Sample path of a wind farm's O&M cost in 20 years

In our study, we used MATLAB (version: R2010a) on a 64 bit operation system with Intel Pentium D processor (3.00GHZ) to generate such GBM paths for 1,000 times, given all parameters and time horizons as we indicated before. The algorithm of computing the expected marginal economic life  $T^*$  is simply presented as follows:

Initiation: Set  $\text{index}_0 = \emptyset$  and  $j = 1$ .

Step 1: Generate a GBM process and find the first time it hits  $C^*$  as  $t_j$ . If it does not exit, let  $t_j = 60$ .  $\text{index}_j = \{t_j, \text{index}_{j-1}\}$ . Go to step 2.

Step 2:  $j = j + 1$ . If  $j > 1,000$ , go to step 3; otherwise go to step 1.

Step 3: Calculate the mean value of all elements in  $\text{index}_{j-1}$  then STOP.

By executing this simple algorithm, we compute the mean value of the economic life to be 17.4115 years, which, is fairly close to the analytical expected value of 17.6592 years that we obtained in the previous subsection. The difference will become more negligible once more simulations are run. Finally, we note that in this algorithm, there is one minor “adjustment” as follows. We implicitly assumed that the maximum physical life with all the repair and maintenance to be 60 years. Hence, any simulation run that was not reaching  $C^*$  in 60 years was terminated and we entered its age as 60 years. By running the Monte Carlo simulation with 60 years for 1,000 times, we have the result showing that the possibility of a wind farm not to retire within 60 years is less than 1%. Hence, we believe that this adjustment is indeed minor.

## **DISCUSSION ON POLICY IMPLICATIONS AND ASSUMPTIONS**

This section consists of two discussion items: policy implications and assumptions. Let us first address the policy implications. As shown in the previous two sensitivity analysis sections on exit and entry decisions, as the exit fee  $W$  increases, the exit threshold  $C^*$  increases. This implies that the government could increase the length of the economic life of the renewable site by imposing or increasing the exit fee. At the same time, as  $W$  increases, the entry threshold  $\bar{C}_0$  decreases. That is, the number of firms that would enter into the renewable power market

will decrease as the government imposes or increases the exit fee. Interestingly, the firms that would be able to enter the renewable power market will have a lower level of  $\bar{C}_0$ , which will increase the economic life of the renewable site.

Hence, the government's exit fee can be viewed as a policy tool to increase the economic life of a renewable site in two different ways. First, it achieves this objective by increasing the exit threshold  $C^*$ . Concurrently, it reduces the entry threshold  $\bar{C}_0$ , contributing positively to the same objective.

Moreover, we recall that, as the initial investment  $I$  decreases, the entry threshold  $\bar{C}_0$  increases. Hence, if the government provides an initial subsidy, it can be interpreted that more firms will be able to enter. On the other hand, the marginal firms that enter will have a higher level of the initial O&M cost, which will only increase expected value-wise. In this sense, the initial subsidy encourages the premature exit of the (marginal) renewable sites. In view of this observation, let us consider the following scenario.

If the government subsidy is provided as a constant matching fund to  $C$  throughout the duration of the renewable site operation, this will directly extend the economic life of many renewable sites through an increase in pre-subsidy  $C^*$ . At the same time, more marginal renewable sites will enter through an increase in pre-subsidy  $\bar{C}_0$ . In this case, both expected economic life and production of electric power from the renewable energy increase.

From our observation, it is certainly worthwhile to investigate the conversion of the initial subsidy to the O&M subsidy. We note that, the O&M cost subsidy is different from the production tax credit (PTC) currently in use in the U.S. because the PTC is front-loaded for early

years only, and is under the assumption that a sufficient amount of tax has already been paid to the government (see e.g., Wiser *et al.*, 2007a).

Finally, we caution that, for a thorough investigation, the total benefit vs. cost will have to be quantified, which will critically depend on the distribution of firms over key parameters such as  $\bar{C}_0$ . For example, even in a simpler case of imposing a scalar  $W$ , it is recommended that the government strike a careful balance between one environmentally desirable goal of preventing premature exit (relative to the physical life of the renewable site) and another environmentally desirable goal of encouraging the entry of firms to the renewable power market, and increase the production of electric power from the renewable energy as the distribution of such firms over  $\bar{C}_0$  values may be far from certain.

So far, we have discussed the policy implications. Now we proceed to discuss the assumptions. The most fundamental assumption in our model is that the O&M cost (\$/MWh) follows GBM process and some cases have been referred earlier in this paper to support this assumption. We note that, in these referred cases, the cost is random over a period, and the expected cost as well as the volatility of the cost are increasing.

In the case of the wind farm, we can make similar observations on the O&M cost. In EWEA (2004, pp. 101), some of the O&M cost paths over time of wind turbines meet these characteristics. Qualitatively, Wiser and Bolinger (2008) reported that the average O&M cost of a wind turbine (\$/MWh) ‘appears to increase with project age’, while some other authors claimed that the operating and maintenance costs escalate over time (Wilkinson, 2010).

In practice, we envision that first the raw data on the daily production as well as O&M cost of the renewable site will be recorded over a period. From these data, we will obtain the O&M cost per MWh data. We note that once enough of these O&M data become available in the

future, more rigorous tests to see the degree of fit can be administered using various statistical tools (e.g. Postali and Picchetti, 2006).

In terms of the relaxation of simplifying assumptions, the power production quantity, capacity, and contract power price seem all worthwhile. In the case of power production quantity, a simulation model can accommodate the daily (perhaps hourly) fluctuation of the power production, taking the specific technological characteristics (wind speed, daylight availability, etc.) into account. Such a simulation will add numerical and computational insights to more tactical decisions of temporary contraction or expansion such as partial shut-down due to seasonality.

In the case of capacity, the granularity will be a central question. As the renewable site is subdivided into smaller groups (until an individual renewable generator such as a wind turbine or a solar panel), there will be an explosive number of options for partial shut-downs and gradual entries. We acknowledge that such features will add realism to the models studied here. However, the analytic tractability of any such extension remains to be seen.

Finally, for the contract power price, as mentioned previously, certainly a power price following a separate GBM will allow us to model the option to wait before any entry. With this parallel GBM process, whether the numerical and computational analyses (analytic solutions are not likely) can exploit the interaction of the price and O&M cost GBM's and yield unambiguous insights will be a significant future challenge.

## **CONCLUDING REMARKS**

In this study, we modeled and analyzed how economically rational decision maker of a renewable site will exit and enter when the O&M cost is represented by a GBM process where

the renewable site is without any input fuel cost. For such a site, we obtained the threshold level of the O&M cost above which a currently operating renewable site will exit. We also obtained the threshold level of the O&M cost below which a new renewable site will enter.

Based on these two findings, we conducted extensive sensitivity analyses with respect to various critical parameters with major policy implications. For example, the exit fee by the government will help in preventing premature exit relative to the physical life. At the same time, such a fee will prevent O&M cost-wise marginal firms from entering the market, which reduces the total production amount of electric power from the renewable energy. Moreover, the government subsidy for the initial investment is shown to allow the O&M cost-wise marginal firms to enter the market, of which renewable sites have a shorter expected economic life relative to the physical life. At the same time, such entries will increase the total production amount of electric power from the renewable energy.

As an alternative, it is desirable to investigate diverting the subsidy on the initial investment to the subsidy on the O&M cost as the O&M cost subsidy will extend the expected economic life. At the same time, more O&M cost-wise marginal firms will enter the market, which will increase the total production amount of electric power from the renewable energy.

We note that for conclusive results over competing policies, the total benefit and cost must be quantified, which is beyond the scope of this paper. However, this paper has discovered a much plausible alternative subsidy to the current government policy of front-loading grants and incentives in the initial and early years of renewable site operations.

As this paper can be seen as an initial exploration, there are numerous worthwhile future studies. Specifically, it may be worthwhile to relax each simplifying assumption, and examine the ramifications of such relaxation. For example, by assuming no prior power purchase contract,

the electric power price can be modeled as a separate GBM process. This type of endeavor will enhance the realism of our study, and widen the applicability of our models.

In addition, as the data on O&M costs across renewable sites accumulate, it is worthwhile to measure the degree of the fitness for the GBM assumption. As such a degree is typically far from being binary, we do anticipate differing degree of fitness across renewable sites. However, such an examination will enhance our ability to fine-tune the exact GBM process (out of so many GBM inspired processes) and their corresponding parameter values.

Moreover, one could consider a single firm with multiple sites of the same or different renewable energies whose O&M costs may be positively/negatively correlated. Other expansions could include a competitive model of single-site firms of the same or different renewable energies as well as the management of changing technology with respect to time.

Finally, we note that, the massive abandonment of 1990's has already happened in the renewable power industries including the wind and solar power producers. Given the current expansion of these industries across the U.S. and other countries, we believe that the questions raised and addressed (to a varying degree) in this paper are timely (if not urgent). Furthermore, we hope that this paper contributes positively to the resolution of the upcoming massive scale problem for the aging and retiring renewable sites as well as alternative energy facilities of similar economic and environmental characteristics (to a degree, biomass and waste energy).

## **ACKNOWLEDGMENT**

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## APPENDIX

### A.1 Proof of Proposition 1

The structure of (4)'s solution contains the general solution of the homogeneous part of it as well as a particular solution to the full equation, which is in the form of

$$V(C) = A_1 C^{\beta_1} + A_2 C^{\beta_2} - CK/(\rho - \alpha_c) + PK/\rho$$

where  $\beta_1$  and  $\beta_2$  are the roots of the characteristic quadratic equation as

$$\frac{1}{2}\sigma_c^2\beta^2 + \left(\alpha_c - \frac{1}{2}\sigma_c^2\right)\beta - \rho = 0$$

Solving the quadratic equation we have

$$\beta_1 = \left(1/2\sigma_c^2 - \alpha_c + \sqrt{(\alpha_c - 1/2\sigma_c^2)^2 + 2\sigma_c^2\rho}\right) / \sigma_c^2 > 1$$

$$\beta_2 = \left(1/2\sigma_c^2 - \alpha_c - \sqrt{(\alpha_c - 1/2\sigma_c^2)^2 + 2\sigma_c^2\rho}\right) / \sigma_c^2 < 0$$

We also notice that when  $C \rightarrow 0$ , i.e., the O&M cost becomes negligible, the renewable site will not be abandoned, which indicates that the value of the option to abandon approaches zero, therefore  $A_2 = 0$ . After eliminating this speculative bubble, the general solution then becomes

$$V(C) = A_1 C^{\beta_1} - CK/(\rho - \alpha_c) + PK/\rho$$

Solving the above solution with boundary conditions (5) and (6) we have the following,

$$C^* = \frac{(PK/\rho + W)(\rho - \alpha_c)\beta_1}{(\beta_1 - 1)K}$$

$$A_1 = \frac{K}{(\rho - \alpha_c)\beta_1(C^*)^{\beta_1 - 1}}$$

## A.2 Proof of Proposition 2

To prove  $\frac{\partial C^*}{\partial \sigma_c} > 0$ , we transform it into an equivalent problem of proving

$$\frac{\partial C^*}{\partial \sigma_c} = \frac{\partial C^*}{\partial \beta_1} \cdot \frac{\partial \beta_1}{\partial \sigma_c} > 0 \text{ by the chain rule.}$$

For simplicity we denote  $\sqrt{(\alpha_c - 1/2\sigma_c^2)^2 + 2\sigma_c^2\rho}$  as  $\sqrt{\Delta}$ , and

$$\begin{aligned} \frac{\partial \beta_1}{\partial \sigma_c} &= \left[ \left( \sigma_c + \frac{\sigma_c^3 - 2\alpha_c\sigma_c + 4\rho\sigma_c}{2\sqrt{\Delta}} \right) \sigma_c^2 - 2 \left( \frac{1}{2}\sigma_c^2 - \alpha_c + \sqrt{\Delta} \right) \sigma_c \right] / (\sigma_c^2)^2 \\ &= \left( \frac{\sigma_c^4 - 2\alpha_c\sigma_c^2 + 4\rho\sigma_c^2}{2\sqrt{\Delta}} + 2\alpha_c - 2\sqrt{\Delta} \right) / \sigma_c^2 \\ &= \frac{\sigma_c^4 - 2\alpha_c\sigma_c^2 + 4\rho\sigma_c^2 + 4\alpha_c\sqrt{\Delta} - 4\Delta}{2\sigma_c^2\sqrt{\Delta}} \end{aligned}$$

Since the denominator is positive, we only need to check the sign of the numerator to determine the sign of the whole equation. For the numerator, if  $4\alpha_c\sqrt{\Delta} > -\sigma_c^4 + 2\alpha_c\sigma_c^2 - 4\rho\sigma_c^2 + 4\Delta$ , it is positive. Thus we square both sides of the inequality and derive the difference between them as

$$(4\alpha_c\sqrt{\Delta})^2 - (-\sigma_c^4 + 2\alpha_c\sigma_c^2 - 4\rho\sigma_c^2 + 4\Delta)^2 = -4\rho\sigma_c^4(\rho - \alpha_c) < 0$$

Therefore we have  $\frac{\partial \beta_1}{\partial \sigma_c} < 0$ . For the sign of  $\frac{\partial C^*}{\partial \beta_1}$ ,  $\frac{\partial C^*}{\partial \beta_1} = -\frac{(PK/\rho + W)(\rho - \alpha_c)}{(\beta_1 - 1)^2 K} < 0$ .

The other remaining properties can be proved similarly.

### A.3 Proof of Proposition 3

First we use (10) to reform (13) into a function of the entry threshold  $\bar{C}_0$  as

$$F(\bar{C}_0) = \frac{(\bar{C}_0)^{\beta_1} K}{(\rho - \alpha_c) \beta_1 (C^*)^{\beta_1 - 1}} - \bar{C}_0 K / (\rho - \alpha_c) + PK / \rho - I$$

When  $\bar{C}_0 = 0$  and  $C^*$ , the value of  $F(\bar{C}_0)$  can be calculated as

$$F(\bar{C}_0 = 0) = PK / \rho - I > 0$$

$$F(\bar{C}_0 = C^*) = -W - I < 0$$

By taking the partial derivative of  $F(\bar{C}_0)$  with respect to  $\bar{C}_0$  we also have that  $F(\bar{C}_0)$  is monotonically decreasing, i.e.,

$$\frac{\partial F(\bar{C}_0)}{\partial \bar{C}_0} = K \frac{(\bar{C}_0 / C^*)^{\beta_1 - 1} - 1}{\rho - \alpha_c} < 0 \text{ given } \bar{C}_0 < C^*$$

Hence, there exists a unique solution of  $\bar{C}_0$ .

### A.4 Proof of Proposition 4

For Proposition 4 we briefly present the proof for  $\frac{\partial \bar{C}_0}{\partial W} < 0$  here.

Recall  $F(\bar{C}_0)$  in Appendix A.3, it can be further (fully) expanded by using (9) and get

$$F(\bar{C}_0) = \frac{(\bar{C}_0)^{\beta_1} K^{\beta_1} (\beta_1 - 1)^{\beta_1 - 1}}{(\rho - \alpha_c)^{\beta_1} \beta_1^{\beta_1} (PK / \rho + W)^{\beta_1 - 1}} - \bar{C}_0 K / (\rho - \alpha_c) + PK / \rho - I$$

As shown in A.3,  $\frac{\partial F(\bar{C}_0)}{\partial \bar{C}_0} \neq 0$  given  $\bar{C}_0 < C^*$ , which indicates that implicit function

theorem can be applied to (13). We then differentiate (13) with respect to  $W$  into the following form

$$\frac{\partial A_1}{\partial W} (\bar{C}_0)^{\beta_1} + \left( A_1 \beta_1 (\bar{C}_0)^{\beta_1-1} - \frac{K}{\rho - \alpha_c} \right) \frac{\partial \bar{C}_0}{\partial W} = 0$$

$$\frac{\partial \bar{C}_0}{\partial W} = - \frac{\frac{\partial A_1}{\partial W} (\bar{C}_0)^{\beta_1}}{A_1 \beta_1 (\bar{C}_0)^{\beta_1-1} - \frac{K}{\rho - \alpha_c}}$$

As in (10),  $A_1 = \frac{K}{(\rho - \alpha_c) \beta_1 (C^*)^{\beta_1-1}}$ , and after substitution, we have

$$\frac{\partial \bar{C}_0}{\partial W} = - \frac{\frac{\partial A_1}{\partial W} (\bar{C}_0)^{\beta_1}}{\frac{K}{\rho - \alpha_c} \left[ \left( \frac{\bar{C}_0}{C^*} \right)^{\beta_1-1} - 1 \right]}$$

Since  $\bar{C}_0 < C^*$ , the denominator above as  $\frac{K}{\rho - \alpha_c} \left[ \left( \frac{\bar{C}_0}{C^*} \right)^{\beta_1-1} - 1 \right] < 0$ .

Also, it could be mathematically proved that the numerator,  $\frac{\partial A_1}{\partial W} (\bar{C}_0)^{\beta_1} < 0$  because

$$\frac{\partial A_1}{\partial W} < 0. \text{ Therefore, } \frac{\partial \bar{C}_0}{\partial W} < 0.$$

The other remaining properties can be proved similarly.

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