HIGH RESOLUTION ARMA MODEL RECONSTRUCTION FOR

NDE ULTRASONIC IMAGING

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INTRODUCTION

Ultrasonic imaging is of use in a number of important areas, including nondestructive testing and medicine. The field has grown considerably in the past ten years and remains an active and growing area of research. For nondestructive evaluation of materials (NDE), the aim is to provide means for obtaining estimates of the size, shape and orientation of flaws in sufficiently a quantitative manner so that failure of mechanical structural parts can be predicted.\cite{1,2} As an imaging technique, ultrasonic imaging is appropriate whenever the medium considered is opaque to other sources of radiation, such as optical radiation. It is versatile and convenient and may be used in both the transmission and reflection, active and passive modes. The technique is also versatile in another important sense. Use of (nearly) spatial coherent radiation sources and of linear detectors results in the availability of a wide variety of digital signal processing procedures for enhancing signals, improving lateral resolution, range gating and so on.\cite{3}

Specifically, both the Maximum Likelihood and Auto-Regressive signal processing approaches have been used to improve lateral resolution. The Maximum Likelihood method was originally developed for seismic array frequency-wave number analysis,\cite{4} and in that approach one estimates the spatial power spectral density (PSD) by effectively measuring the power out of a set of narrow-band filters.\cite{5} However, the shape of the filters are, in general, different for each (spatial) frequency and the filter coefficients are determined by the very same process for which the power spectral density is sought. This method is therefore considered to be adaptive.
More recently, the Auto-Regressive (AR) technique has also been used to provide improved PSD and therefore improved lateral resolution. In this connection it is worth noting that the ML Spectral Estimate and the AR Spectral Estimate have been related analytically as follows:

\[
\frac{1}{\hat{P}_{ML}(f)} = \frac{1}{\rho} \sum_{m=1}^{\rho} \frac{1}{\hat{P}_{AR}^{(m)}(f)}
\]

where \(\hat{P}_{AR}^{(m)}(f)\) is the AR PSD for an \(m\)th order model and \(\hat{P}_{ML}(f)\) is the ML PSD, both based upon a known autocorrelation matrix of order \(P\). Thus the lower resolution of the ML PSD estimate can be explained by the averaging effect of combining the lower order AR spectra of lower resolution with the higher order AR spectra of higher resolution. Perhaps this is also the reason why in the literature the ML approach is considered to be adaptive, whereas the AR approaches are described in terms of their "super-resolution".

It is known that for a wide range of circumstances, Auto-Regressive Moving-Average (ARMA) modeling yields spectral estimates more accurate than those of AR modeling and is therefore the question of whether ARMA modeling might be used to advantage in acoustic imaging in a computationally effective manner. However, before addressing this matter, it is appropriate to look into the matter of range resolution as well.

In conventional cw acoustic holographic imaging, i.e., with the use of a spatial array of sensors, the attainable lateral angular resolution is at best \(\lambda/D\) where \(\lambda\) is the wavelength of the acoustic radiation and \(D\) is the hologram aperture or, equivalently, the spatial extent of the sensor array. That type of configuration, though good for angular resolution, is inefficient for range resolution since objects in the direction of range are discriminated only through selective focus.

The methods proposed for obtaining both range resolution and angular resolution include (1) the multi-frequency method, (2) the wide band long pulse method, and (3) the focused transmitter approach.

In the multi-frequency method, the ordinary single frequency continuous-wave (cw) imaging is repeated at a large number of contiguous, discrete, and equally spaced frequency values, and the resulting image amplitudes are superimposed to yield a high resolution 3-dimensional image.

In the second method, the wideband long pulse may be obtained in a variety of ways. The basic idea is that using pulse-compression processing, a long, wide-band signal can provide just as good range
resolution as a simple short pulse. Published results\textsuperscript{12} indicate that with appropriate processing, including the use of range gating, substantial increases in range resolution may be obtained without significant loss of image quality. In the investigation reported in Ref. 12, 13-bit Barker-coded 250 kHz pulses were used for imaging. The pulses were 130 μsec long and the time resolution obtained was about 40 μsec (corresponding to a pulse compression by a factor of about 3) or a range resolution of about 3.5 cm.

In the focused transmitter approach,\textsuperscript{13} a line array is used to focus the transmitted beam and to scan the focus point longitudinally so that the return signal from the focus point, received with appropriate time gating, forms an image point with good lateral and longitudinal resolution.

Against this background of known results, the excellent results which we had been obtaining with the use of ARMA modeling for spectrum estimation\textsuperscript{9} suggested that we should investigate the benefits achievable in three dimensional ultrasonic imaging with ARMA processing. This was indeed done and the results of that investigation are reported in this paper.

In that investigation, both the multi-frequency and pseudo-noise coded pulse techniques for improving range resolution were explored as a side issue in connection with the use of ARMA modeling. The results indicate that neither the multi-frequency nor the PN coding schemes interfere with the use of ARMA modeling for obtaining high lateral resolution and for interpolating between sparsely spaced detectors. In addition, for the multi-frequency method, ARMA modeling may also be used for frequency extrapolation and interpolation purposes. Therefore, for the multi-frequency case, it might be possible to substitute processing for many additional scans at different frequencies, and the ability to make this substitution may be very useful in the case of towed arrays and moving objects. As is the case in lateral resolution, ARMA modeling can yield improved range resolution and remove range "ghosts" due to aliasing.

THEORETICAL MATTERS

A geometry typical of the ultrasonic imaging systems investigated is shown in Fig. 1. For a cw transmitted signal of frequency \( f \), i.e., \( S_T = \exp(j2\pi ft) \), the back-scattered signal at point \((x,y)\) is heterodyned with a reference signal, \( S_R = \exp(j2\pi ft) \), to yield the detector output
Fig. 1. Geometry of Imaging System.

\[
S_H(x, y, f) = \iiint_{\text{obj}} 0(x_n, y_n, z_n) \frac{f}{z_n^2 v} \exp[-j2\pi f(2z_n/v)] \\
\exp[-j2\pi f((x-x_n)^2 + (y-y_n)^2 + x_n^2 + y_n^2)/2z_n] dx_n dy_n dz_n
\]

(1)

where \( v \) is the ultrasonic velocity, and \( 0(x_n, y_n, z_n) \) is the object distribution function describing the objects in terms of the reflection coefficients. The transmitter is considered to be at zero location \((0,0,0)\). In Eq. (1), the Fresnel approximation was used to provide a "propagator" for the radiation.

Using the same propagator in an inverse manner, the image distribution on any plane at any specific distance, \( z_n = z_{no} \), can then be reconstructed to be

\[
\hat{0}(x_n, y_n, z_{no}) = \iint_{xy} S_H(x, y, f) \frac{z_{no}^2 v}{f} \exp[j2\pi f(2z_{no}/v)] \\
\exp[j2\pi f((x-x_n)^2 + (y-y_n)^2 + x_n^2 + y_n^2)/2z_{no} v] dx dy
\]

(2)
The fields $S_H$ and $\hat{0}$ can be rewritten in forms more convenient for subsequent manipulations,

$$S'_H(x',y',f) = S_H(x',y',f) \frac{z^2 f}{\nu} \exp\left[j2\pi f(2z_{no}/\nu)\right] \exp\left[j2\pi z_{no}\nu(x'^2 + y'^2)/2\nu\right]$$

and

$$\hat{0}'(x_n, y_n, z_{no}) = \hat{0}(x_n, y_n, z_{no}) \exp[-j2\pi f(x_n^2 + y_n^2)/z_{no} \nu]$$

where $x' = fx/z_{no}^2$ and $y' = fy/z_{no}^2$. Equation (2) can then be re-arranged as follows:

$$\hat{0}'(x_n, y_n, z_{no}) = \int \int S'_H(x',y',f) \exp[-j2\pi f(x_n x' + y_n y')] dx'dy'$$

From this equation, it is seen that $\hat{0}'(x_n, y_n, z_{no})$ is the Fourier transform of $S'_H(x', y')$. It is also clear that any method of reconstruction which ultimately yields the equivalent of the Fourier transform of $S'_H(x', y')$ would do just as well.

It is known that the use of an ARMA model of a time sequence can result in a robust description of that data. One form of that description is a $z$-transform of that sequence; the $z$-transform can be economically specified in terms of a few parameters and the values of that transform on the unit circle can provide a good approximation to the Fourier transform of the time data. This Fourier transform is continuous and does not suffer from the deficiencies associated with the corresponding discrete Fourier transform.

If a Born approximation representation of the scattering were used, $S'_H(x', y')$ would be different, but the approach would still be valid inasmuch as it depends on a Fourier transform operation.

If the frequency of the transmitted continuous wave is varied sequentially through a range of discrete values, the image field can be obtained by taking an average over the frequency domain, i.e.,

$$\hat{0}(x_n, y_n, z_{no}) = \int \hat{0}'(x_n, y_n, z_{no}) \exp[j2\pi f(x_n^2 + y_n^2)/z_{no} \nu] df$$

For computer simulation, the discrete form of Eq. (6) can be written as follows:
\[ 0(m \Delta x_n, l \Delta y_n, z_{no}) = \sum_{p=1}^{P} \sum_{l=1}^{N} \sum_{k=1}^{K} S'_H(n \Delta x', k \Delta y', p \Delta f) \]

\[ \exp[-j2\pi(n \Delta x_n m + k \Delta y_n l)] \]

\[ \exp[j2\pi p \Delta f(m^2 \Delta x_n^2 + l^2 \Delta y_n^2)/z_{no}] \]  

where \( P \) is the total number of frequency points, \( N \) is the total number of receiving sensors along the \( x \) axis, \( K \) is the total number of receiving sensors along the \( y \) axis, \( \Delta x' = f \Delta x/z_{no} v \) and \( \Delta y' = f \Delta y/z_{no} v \); \( \Delta x \) and \( \Delta y \) are sensor spacings along the \( x \) and \( y \) axes, respectively, and \( \Delta x_n \) and \( \Delta y_n \) are the corresponding spacings used for reconstructing the object along the \( x_n \) and \( y_n \) axes, respectively.

The ARMA processing approach yields higher resolution in reconstruction because it provides a continuous transform. It can be shown that this corresponds to an extrapolation of the aperture of the detectors. But once this is realized, further advantage of these characteristics of the ARMA approach can be realized to yield in effect interpolation of detector outputs. The objective of extrapolation is to increase resolution and that of interpolation to increase sampling rate and to avoid aliasing.

For interpolation, let \( \{S_H(n \Delta x), (n=1, \ldots N)\} \) be the received signal (after coherent detection), and obtain from Eq. (3) \( \{S_H(n \Delta x') (n=1, \ldots N)\} \). We take the inverse discrete Fourier transform of \( \{S_H(n \Delta x'), (n=1, \ldots N)\} \) to obtain \( \{F^{-1}[S_H(n \Delta x')], (n=1, \ldots N)\} \), and then, use an ARMA model

\[ a(n) = \sum_{k=1}^{p} a_k a(n-k) + \sum_{k=0}^{q} b_k \delta(n-k) \]  

(8)

to model \( \{F^{-1}[S_H(n \Delta x')], (n=1, \ldots N)\} \). In the original domain, which is the domain of \( x' \) and also the transform of (8), we obtain

\[ S'_H(x') \equiv X(\omega) = B(\omega)/A(\omega) = \sum_{k=0}^{q} b_k e^{-j\omega k} / (1- \sum_{k=1}^{p} a_k e^{-j\omega k}) \]  

(9)

where \( \omega \equiv x' \) and use of \( \omega \) is merely to remind ourselves that we have evaluated the "z transform" of expression (8) on the unit circle. Given expression (9), it is possible to obtain as many values as desired of \( S'_H \). A less sparse discrete sequence of \( S' \) might now be written as \( \{S'_H(n \Delta x^i), (n=1, \ldots N_1)\} \) where \( \Delta x^i < \Delta x \) and \( N_1 > N \). This interpolation procedure serves to fill in the sparse array and reconstruction can then be carried out without "ghosts" due to aliasing.
Similarly, extrapolation is achieved in the following manner. An ARMA model can be used, as in Eq. (8), to model \( S_H(n\Delta x') \), \( (n=1,...,N) \). It is also clear that given the values of \( x(n-p) \), \( x(n-p+1) \), ..., \( x(n) \), it is possible to predict the value of \( x(n+1) \). In this manner it is possible to obtain \( \{S_H(n\Delta x'),(n=1,...,N_2)\} \), where \( N_2 > N \).

However, it was found that greater improvements in lateral resolution could be obtained by working directly on \( \hat{S}(m\Delta x_n,l\Delta y_n,z_{no}) \) as expressed in Eq. (7). Once an ARMA model is available for \( S_H(n\Delta x',k\Delta y',p\Delta f) \), the value of the z transform of that model evaluated on the unit circle provides a "super-resolution" reconstruction of the image.

Similar considerations apply to the interpolation and extrapolation of frequencies.

It might be mentioned in passing that the "reconstruction" procedure considered in the present case is quite different from that of acoustic tomography or x-ray tomography. But there are interesting connections between the two schemes.

RESULTS FOR MULTI-FREQUENCY IMAGING AND ARMA MODELING

Computer simulations of objects and sensor arrays provided means for exploring the effectiveness and processing requirements of the various reconstruction procedures. In the absence of multi-frequency, the angular resolution is good but range resolution is less satisfactory. This is illustrated by results exhibited in Figs. 2 and 3. In Fig. 2(a) we see that the angular positions of two objects can be resolved but the true ranges cannot be discerned equally well. As illustrated in Fig. 3(b), this deficiency can be overcome with the use of the multi-frequency scheme.

In a sense it is quite easy to understand why the multi-frequency method works so well. In general the reconstructed image amplitude is a complex quantity. The real part of that reconstructed amplitude, at the correct range, does not vary with frequency, whereas at all other (incorrect) ranges, the estimated value oscillates with frequency. Clearly, averaging over a range of frequencies does not change the value of the estimated amplitude at the correct range, but yields a value close to zero for all other ranges.

Very interestingly, for the formulation used in this present investigation, the reconstructed amplitude is always a real quantity at the correct range. Therefore, the imaginary part is consistently zero for the correct range but oscillates at all other frequencies. This behavior suggests a simplification in the reconstruction procedure; namely, that the imaginary component can be consistently
Fig. 2(a). Reconstructed image of a two-point object using single frequency continuous wave holography, illustrating lack of range resolution. ($X_{n1} = 30$, $Z_{n1} = 205$, $X_{n2} = 40$, $Z_{n2} = 200$ mm., $N = 80$, $\Delta X = 1$ mm., $f = 1$ MHz.)

Fig. 2(b). Image of the same two-point object using multifrequency reconstruction. Frequency range 0.85-1.35 MHz, $\Delta f = 10$ kHz, $N = 80$, $\Delta X = 2$ mm.
Fig. 3. Full view of the reconstructed image of a 9-point object, arranged as a circle with radius = 30 mm. and centered at $X_n = 50$ mm., $Z_n = 200$ mm. $N = 32$, sensor spacing = 3 mm. (a) when a single frequency, $f = 1$ MHz, is used. (b) when multifrequency scheme is used (frequency range 0.95 - 1.05 MHz, frequency step = 10 KHz).
ignored. However, this practice is not followed in practice by most researchers. The more general procedure allows for inclusion of various phase factors in specifying the reflection coefficient and could be followed without being inconsistent with the practice adopted by us.

The ARMA modeling procedure is illustrated schematically in Fig. 4. Some matters need to be clarified at this point. Given a complicated time series, the intent behind the use of ARMA modeling is to try to ascribe that time series to a rather simple process. That is, there is a "black box" with a simple linear relationship between input and output. The relationship can be specified in terms of a few $a_k$ coefficients and $b_k$ coefficients. When that "black box" is excited by a specific input, the originally observed time series is obtained as output. (Actually not exactly the observed time series but an optimum approximation to it in a last mean squared error sense.)

Usually the input is specified to be white noise. However, there is no basic theoretical reason why this has to be so and the input is in fact entirely a fictitious matter used merely to provide a convenient analytical model for the time series. In principle, it is therefore possible and permissible to specify any other convenient input and in this investigation it is chosen to be $q$ evenly spaced unit pulses. With this approach it is possible to use an ARMA model and still have linear rather than nonlinear equations for the $a_k$ and $b_k$ coefficients.

The results exhibited in Figs. 5(a) and (b) indicate that ARMA extrapolation of a short array of sensors can result in increased angular resolution. In Figs. 6(a) and (b) it is seen that ARMA interpolation of a sparse linear array of detectors indeed results in the removal of an aliasing "ghost" image.

Similarly, frequency interpolation through ARMA modeling results in the removal of "ghost" images in range. Extrapolation of the frequency range results in increased range resolution but very little was done along those lines in this investigation. Additional results are available in Ref. 14.

PSEUDO-RANDOM CODING AND ARMA MODELING

Qualitatively it is clear that use of a wideband long pulse should be equivalent in many ways to the use of many such long pulses, each of a single but different frequency. As mentioned previously, the results of an analysis by Keating, et al.$^{12}$ indicate that Barker code bi-level phase modulated long pulses yield good range resolution and lateral resolution. In this investigation, the range resolution and lateral resolution attainable in imaging
IN 'TIME' DOMAIN:

\[ x(n) = \sum_{k=1}^{P} q_k x(n-k) + \sum_{k=0}^{Q} b_k \delta(n-k) \]

IN 'TRANSFORM' DOMAIN:

\[ X(\omega) = \frac{B(\omega)}{A(\omega)} = \frac{\sum_{k=0}^{Q} b_k e^{-j\omega k}}{1 - \sum_{k=1}^{P} q_k e^{-j\omega k}} \]

Fig. 4. Schematic illustration of ARMA modeling.
Fig. 5(a). Reconstructed Image of a 2-Point Object Located at $x_1 = 20, 30$ mm, $z = 200$ mm, $f = 1$ MHz, $DS = 2$ mm. (The 2 points are not resolved.)

Fig. 5(b). ARMA (8,8) Reconstructed Image of the 2-Point Object. (The 2 points are resolved.)
Fig. 6(a). Reconstructed Image of a 2-Point Objected Located at $x = 20, 40; z = 200$ mm. $f = 1$ MHz, $DS = 6$ mm (the images at 70 and 90 mm are ghost images due to sparse sampling).

Fig. 6(b). ARMA (15,15) Reconstructed Image of the Same Object, after ARMA (12,12) Spatial Interpolation. $N = 31$, $N = 31$, $N = 31$, $N = 31$.
Fig. 7. Reconstructed objects.
(a) using multifrequency,
(b) using phase-modulated pseudo-noise coded signal, with
the same bandwidth in both cases.
Fig. 8. (a) Ball-type object,
(b) reconstructed ball-type image using PN coded signal,
with 2-D sensor array (32x32). Transmitter location
at (16,16,0). Noise free.
with PN coded pulses were also explored. The details of the PN coding scheme and the manner in which bi-phase PN coded pulses are utilized in acoustic imaging are described in Ref. 15.

Two types of results were of interest to us. One category dealt with the characteristics of the PN coded pulse imaging system and an evaluation of its performance relative to that of a multi-frequency system. In this connection, details may be obtained from Ref. 15, but some typical results are exhibited in Figs. 7-8. The reconstructions exhibited in Figs. 7(a) and (b) indicate that comparable range resolution can be attained with either method if the overall bandwidth is the same for both cases. However, the PN method is approximately more efficient computationally by a factor of 40. Three dimensional reconstruction for a ball-shaped object is shown in Fig. 8. However, it seems that the PN code approach is superior to the multi-frequency approach in noisy environments.

A second category of results dealt with the use of PN coding and ARMA processing. The results agreed with those obtained for combined use of multi-frequency and ARMA processing. It would seem that ARMA processing can be quite effective in increasing spatial resolution and in allowing the use of sparse arrays. In the PN coded pulse case, there is no need for ARMA processing to remove range "ghosts."

SUMMARIZING REMARKS

In reviewing the results of this investigation, it would seem that the demonstration of the ability of ARMA model processing to increase lateral resolution and to accommodate the use of sparse arrays is new, interesting, and of potential practical value. The exploration of the use of PN coded long pulses in acoustic imaging yielded interesting results which are new insofar as published unclassified work is concerned. It is not known whether similar work had been reported, perhaps several years ago in classified documents. The fact that the PN code pulses and ARMA processing can be used in combination to provide the basis for a high resolution three dimensional imaging system is indeed a significant result of practical value.

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REFERENCES