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Three essays in demand analysis

Anuradha Akkaraju Vissa

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Three essays in demand analysis

Vissa, Anuradha Akkaraju, Ph.D.
Iowa State University, 1991
Three essays in demand analysis

by

Anuradha Akkaraju Vissa

A Dissertation Submitted to the
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For the Graduate College

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GENERAL INTRODUCTION</strong></td>
<td>1</td>
</tr>
<tr>
<td>Explanation of Dissertation Format</td>
<td>3</td>
</tr>
<tr>
<td><strong>SECTION I. FLEXIBLE INVERSE DEMAND SYSTEMS</strong></td>
<td>4</td>
</tr>
<tr>
<td>Introduction</td>
<td>4</td>
</tr>
<tr>
<td>Theory of Inverse Demands</td>
<td>4</td>
</tr>
<tr>
<td>Regularity conditions</td>
<td>7</td>
</tr>
<tr>
<td>Derivation on inverse demand functions</td>
<td>8</td>
</tr>
<tr>
<td>Restrictions on the inverse demand system</td>
<td>10</td>
</tr>
<tr>
<td>Flexible Functional Forms</td>
<td>16</td>
</tr>
<tr>
<td>The Aggregation Problem</td>
<td>19</td>
</tr>
<tr>
<td>Homothetic preferences</td>
<td>23</td>
</tr>
<tr>
<td>Quasi-homothetic preferences</td>
<td>24</td>
</tr>
<tr>
<td>Estimation of Systems of Equations</td>
<td>26</td>
</tr>
<tr>
<td>The Linear Inverse Demand System</td>
<td>28</td>
</tr>
<tr>
<td>Simulation results</td>
<td>32</td>
</tr>
<tr>
<td>An Aggregable Flexible Inverse Demand System</td>
<td>35</td>
</tr>
<tr>
<td>Summary and Conclusions</td>
<td>39</td>
</tr>
<tr>
<td>References</td>
<td>40</td>
</tr>
<tr>
<td><strong>SECTION II. MIXED DEMANDS: THE CANADIAN MARKET FOR MEATS</strong></td>
<td>45</td>
</tr>
<tr>
<td>Introduction</td>
<td>45</td>
</tr>
<tr>
<td>Theory of Mixed Demands</td>
<td>48</td>
</tr>
<tr>
<td>Model Specification</td>
<td>55</td>
</tr>
<tr>
<td>Data</td>
<td>60</td>
</tr>
<tr>
<td>Results</td>
<td>61</td>
</tr>
<tr>
<td>The first stage of budget allocation</td>
<td>72</td>
</tr>
<tr>
<td>Summary and Conclusions</td>
<td>75</td>
</tr>
<tr>
<td>References</td>
<td>77</td>
</tr>
<tr>
<td><strong>SECTION III. FLEXIBLE FUNCTIONAL FORMS AND WEAK SEPARABILITY: A MONTE CARLO STUDY</strong></td>
<td>80</td>
</tr>
<tr>
<td>Introduction</td>
<td>80</td>
</tr>
<tr>
<td>Separability</td>
<td>82</td>
</tr>
<tr>
<td>Separability and Flexible Functional Forms</td>
<td>85</td>
</tr>
<tr>
<td>Data Generation</td>
<td>88</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Tests of Separability</td>
<td>92</td>
</tr>
<tr>
<td>Results</td>
<td>95</td>
</tr>
<tr>
<td>Summary and Conclusions</td>
<td>106</td>
</tr>
<tr>
<td>References</td>
<td>108</td>
</tr>
<tr>
<td>GENERAL SUMMARY</td>
<td>111</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>115</td>
</tr>
</tbody>
</table>
LIST OF TABLES

SECTION I

Table 1  A comparison of inverse demand systems  34

SECTION II

Table 1  Estimates of the mixed demand systems  63
Table 2  Compensated and Marshallian mixed elasticities at the mean  64
Table 3  Direct elasticities at the mean retrieved from the mixed system  66
Table 4  Estimates of the direct demand system  68
Table 5  Compensated and Marshallian elasticities at the mean from the direct system  69
Table 6  Estimates of the mixed LES  70
Table 7  Mixed and direct elasticities from the mixed LES  71
Table 8  Unconditional mixed elasticities  75

SECTION III

Table 1  Tests of local separability: Rejection rates of the true and false hypotheses when the elasticities are similar  97
Table 2  Tests of local separability: Rejection rates of the true and false hypotheses when one of the elasticities is low  98
Table 3  Tests of local separability: Rejection rates of the true and false hypotheses when both the elasticities are high  99
Table 4: Tests of global separability: Rejection rates of the true and false hypotheses when the elasticities are similar

Table 5a: Tests of global separability: Rejection rates of the true and false hypotheses for the $P_o$ test when one of the elasticities is low

Table 5b: Tests of global separability: Rejection rates of the true and false hypotheses for the $P_o$ test when one of the elasticities is low

Test 6a: Tests of global separability: Rejection rates of the true and false hypotheses for the $P_o$ test when both the elasticities are high

Test 6b: Tests of global separability: Rejection rates of the true and false hypotheses for the $P_1$ test when both the elasticities are high
LIST OF FIGURES

SECTION I

Figure 1  Scale and Slutsky effects for inverse demands 11
Figure 2  Aggregation in demand analysis 20
GENERAL INTRODUCTION

Demand systems obtained from the utility maximization process are a tractable representation of preferences. The variables in these demand systems (prices, quantities and income) can be observed and the estimation of these demand systems yield empirical measures of elasticities that throw light on consumer behavior and have obvious implications for policy formation. It is therefore not surprising that the area of applied demand analysis has forged an active interaction of consumer theory and econometrics.

Empirical demand studies often examine demand systems that are specified and estimated in apparent isolation of the supply side. To obtain consistent estimates of the underlying preferences, one of two assumptions are made. Either prices are assumed predetermined or quantities are assumed to be predetermined. The first of these assumptions leads to quantity dependent demand systems which is the usual representation of preferences for the individual consumer. At the aggregate level they would imply that prices are given in the market as in the case of a small trading economy or when prices are fixed as in the case of some public utilities.

The second assumption leads to price dependent demands which imply that supplies are fixed at the market level and prices must adjust for markets to clear. Such 'inverse' demand functions are useful, for example, when analyzing the demands for perishable products over a short period of time. Demand systems, whether direct or inverse, are estimated using
Flexible Functional Forms (FFFs), a concept that is now widely used in applied econometrics.

In section I of this dissertation, a new flexible linear inverse demand system is proposed. Unlike other flexible inverse demands that are nonlinear, the new demand system is easy to estimate, an advantage that might be significant for large demand systems. The performance of this demand system is examined in a simulation exercise. The existence of such market demands, however implies that preferences belong to a certain class that satisfies conditions necessary for aggregating over consumers. A flexible inverse demand system that satisfies aggregation conditions is also proposed in this section.

In addition to the two polar cases of direct and inverse demand systems, demand systems can reflect yet another market situation where optimal quantity decisions are made for some (say, group A) commodities, given their prices, while prices adjust to clear the market for the other (group B) commodities, given their supplies. The Canadian market for meats is a clear example of the above market situation since Canada trades freely in beef and pork with the U.S., while the market for chicken is subject to supply management. Under the small country assumption, prices of beef and pork are thus given while for chicken it is the supply that is given.

However, the existing theoretical framework that examines mixed demands does not help in specifying an empirical model that is amenable to estimation. In this study, the concept of virtual or shadow prices is used in deriving a dual representation of preferences for the mixed demand case. The equivalent of the Slutsky relations are derived, which are used in the specification of a differential approximation of mixed demands. The
proposed model is then used to analyze the Canadian situation.

Separability of preference structure is a frequently invoked assumption (as in the case of mixed demands for meat) in a lot of empirical work. It is, however, a strong assumption and ideally should be tested before it is maintained. Testing separability using flexible functional forms is, however, problematic. This is because imposing separability globally on most FFFs renders them inflexible. As an alternative, separability has been tested locally at a point, recognizing that a FFF, after all, provides only a local approximation at a point, to some unknown function.

In Section III, the ability of a recently proposed globally separable functional form is compared in a Monte Carlo study to locally imposed separability. Since the globally separable model is not nested in the general non-separable model, non-nested hypothesis tests are used to test for separability in this case. The study reveals the performance of the new functional form and the performance of the non-nested tests used.

Explanation of dissertation format

This dissertation consists of three complete, self-contained essays. Each essay contains its own introduction, sections on theory and applications, results and references. The axioms of consumer preference, and estimation of systems of equations are discussed in Section I although they apply to all the sections of the dissertation. The concept of flexibility of functional forms is also introduced in Section I.
SECTION I. FLEXIBLE INVERSE DEMAND SYSTEMS

Introduction

Studies in applied demand analysis commonly deal with quantity dependent demands where prices are assumed to be exogenous. This is usually the case for a single consumer who makes optimal quantity decisions given prices and income. At the market level, however, quantity dependent or 'direct' demands are valid for a small trading economy that faces world prices or in the case of public utilities where prices are fixed.

Price dependent or 'inverse' demands may be more appropriate for certain other market situations where the quantities (supplies) are given at any point in time and prices must adjust to clear the markets. Arguably, most agricultural commodities at the market level provide a good illustration of this situation. Also, from the perspective of the planner (or price forecaster) the relevant demand concept is the inverse demand system explaining prices in terms of (expected) output. Often, therefore, it may be natural to think of price as a function of quantities. From a theoretical point of view there is no problem with this, as the demand relation can be expressed in either the direct or indirect sense. As Hurwicz (1971) points out, there is no contradiction between the view of the consumer as the price taker and the use of price as the dependent variable.

However, for market demands to exist preferences must satisfy certain aggregation conditions. Aggregating demands over individuals is legitimate when Engel curves are linear (i.e., preferences are homothetic or quasi-
homothetic), in which case the cost or indirect utility functions are of the Gorman Polar Form (GPF), or if the Engel curves are generalized linear, in which case the cost or indirect utility functions are of the Generalized Gorman Polar Form (GGPF) (Deaton and Muellbauer 1980a).

A concept that has gained wide acceptance in empirical economics is that of a Flexible Functional Form. Typically, flexible inverse demands are derived by applying Wold's identity to a flexible specification of the direct utility function. In general, these inverse demands do not satisfy aggregation conditions, and are non-linear in form which may pose a problem for demand systems involving large number of commodities. For example, in the case of the Translog specification (Christensen and Manser 1977), inverse demands satisfy aggregation conditions and have an easily estimable linear form when homotheticity is imposed. However, homotheticity is a strong and implausible assumption in most cases. Barnett (1977) estimates an inverse demand system that satisfies linear aggregation conditions but a very restrictive utility function is used.¹ More recently, Salvas-Bronsard, Leblanc and Bronsard (1977), Huang (1988) and Barten and Bettendorf (1989) estimated Rotterdam type differential inverse demand systems, that satisfy an alternate but meaningful criteria of flexibility.

In this section, two flexible inverse demand systems are proposed. An inverse demand system that satisfies the flexibility criteria and offers the advantage of being linear is proposed. This demand system is derived from the symmetric dual of the cost function underlying the Almost Ideal

¹Barnett estimates the inverse demands derived from a separable utility function (the W-S branch discussed in section III) that is not flexible for the 3-group (with 10 goods) case that he considers.
Demand System of Deaton and Muellbauer (1980b), but does not itself claim any aggregation properties. A simulation exercise studies its performance in relation to the older Translog inverse demand system. A second flexible inverse demand system that satisfies aggregation is also proposed. This demand system belongs to the class of quasi-homothetic preferences which is less restrictive than the class of homothetic preferences.

This section of the dissertation is organized as follows. First, the theory of inverse demands is discussed. This includes the regularity conditions, the derivation of the inverse demand functions and the restrictions on the inverse system. For the sake of clarity in what is to follow, the concept of flexible functional forms is introduced and the aggregation problem is discussed. This is followed by a brief discussion of the estimation of demand systems. The linear inverse demand system is then presented along with the simulation results after which the aggregable flexible model is specified. Summary and conclusions are presented in the final sub-section.

Theory of Inverse Demands

Inverse demands can be derived directly from the direct utility function or from its implied representation, the distance or transformation function. In either case it is the duality between the direct and the indirect representation of preferences that is exploited.
Regularity conditions

The following axioms of choice guarantee the existence of a quasi-concave utility function. The symbol $\geq$ is used to mean "at least as good as". Subscripts indicate different bundles of commodities.

Axiom 1: Reflexivity. For any bundle $x$, $x \geq x$.

Axiom 2: Completeness. For any two bundles in the choice set $x_A$ and $x_B$, either $x_A \geq x_B$ or $x_B \geq x_A$; if both $x_A \geq x_B$ and $x_B \geq x_A$ then $x_A = x_B$ i.e., $x_A$ is indifferent to $x_B$.

Axiom 3: Transitivity. If $x_A \geq x_B$ and $x_B \geq x_C$ then $x_A \geq x_C$.

Axiom 4: Continuity. For any bundle $x_A$ define $a(x_A)$ the "at least as good as $x_A$ set" and $b(x_A)$ the "no better than $x_A$ set" by

$$a(x_A) = \{ x \mid x \geq x_A \}$$

$$b(x_A) = \{ x \mid x_A \geq x \}$$

Then $a(x_A)$ and $b(x_A)$ are closed.

Axiom 5: Non-satiation. The utility function $U(x)$ is non-decreasing in each of its arguments for all $x$ in the choice set and is increasing in at least one of its arguments.

Axiom 6: Convexity. If $x_A \geq x_B$, then for $0 \leq \lambda \leq 1$, $\lambda x_A + (1-\lambda)x_B \geq x_B$.

Axioms 1–3 define a preference ordering but axioms 4 and 5 are needed for the existence of a utility function that represents the preference ordering. Axiom 6 implies a quasi-concave utility function (the utility function is strictly quasi-concave for strictly convex preferences; in terms of Axiom 6, the last weak inequality is changed to a strict inequality).

In this analysis, attention is restricted to cases where the demand functions (direct, inverse or mixed) are unique solutions to their
respective optimization processes. Strong quasi-concavity is assumed, and in addition, it is assumed that the utility function (and all its dual representations) are twice continuously differentiable in their respective arguments.\footnote{In the absence of differentiability, monotonicity of the primal and dual representations is necessary for most duality results. Alternatively, one can restrict the domain to the interior, assume local non-satiation or assume indifference curves do not intersect the axis (Blackorby, Primont and Russel 1977).}

**Derivation of inverse demand functions**

The Lagrangian for the primal problem of utility maximization subject to the budget constraint is

$$\max L = U(x) - \lambda [p \cdot x - y]$$

The first order conditions are

$$U_i(x) = \lambda p_i \quad i=1,2,...,n$$

and

$$p \cdot x = y$$

where $U_i(x)$ is the partial differential of $U$ with respect to $x_i$. Solving for $x_i$ we get the direct Marshallian demands,

$$x_i = x_i(p, y) \quad \text{or} \quad x_i = x_i(\pi)$$

where $\pi_i = p_i/y$ is the normalized price of $x_i$. Substituting for the optimal quantities in the direct utility function, we obtain the indirect utility function,
\[ V(x(\pi)) = V(\pi) \]  

where \( V(.) \) is continuous, decreasing and quasi-convex in \( \pi \). The expenditure or cost function \( C(u,p) \) can be obtained by inverting the indirect utility function. Multiplying both sides of (2) by \( x_i \) and summing over \( i \) and solving for \( \lambda \) we have

\[ \lambda = \sum_i U_i(x) / y \]

Substituting for \( \lambda \) in (2) and solving for \( \pi_i \) we have,

\[ \pi_i = U_i(x) / \sum_i U_i(x) x_i \]

for all \( i = 1, 2, \ldots, n \). This is the Wold-Hotelling identity which defines the inverse demand system from a differentiable direct utility function. In general we can re-write (7) as

\[ \pi_i = f_i(x) \]

The same solution can be obtained by substituting for \( x_i \) from (4) into the direct utility function. Minimizing this indirect utility function (5) over the normalized prices subject to the budget constraint \( \pi \cdot x = 1 \) yields the normalized prices as functions of quantities.

The distance or transformation function which is an implicit and equivalent representation of the direct utility function is defined (Deaton and Muellbauer 1980a) as \( D(x,u) \) which satisfies

\[ U[x/D(x,u)] = u \]

If \( U(x) = u, D(x,u) = 1 \). Its duality to the cost function is evident from
$$D(x,u) = \min \limits_{p} \{ p \cdot x : C(u,p) = 1 \}$$ and
$$C(u,p) = \min \limits_{x} \{ p \cdot x : D(x,u) = 1 \}$$

$D(x,u)$ is increasing, linear homogeneous and concave in $x$ and decreasing in $u$. There also exists the following derivative property which is analogous to Shephard's lemma:³

$$\pi_i = f^{i*}(x,u) = D_i(x,u)$$ (9)

The partial of $D(.)$ with respect to $x_i$ gives the level of normalized prices that induce the consumer to consume a bundle that lies on a ray passing through $x$, which gives utility 'u'. These (starred) inverse demands are thus compensated demands. If $u = U(x)$, we have the following identity which gives us the Marshallian inverse demands.

$$\pi_i = f^{i*}(x,U(x)) = f^i(x)$$

Restrictions on the inverse demand system

Restrictions on inverse demands are analogous to those on the direct demands (Anderson 1980). Just as in the case of the compensated and uncompensated elasticities, the income elasticity also has its counterpart for the inverse demand system.

The analogue to the income elasticity is the scale elasticity. The question raised is, if the quantity of all commodities is changed proportionately, how would the normalized prices change? In terms of Figure 1, the scale elasticity would tell us how prices would change if

³ The distance function has been used to define quantity and utility indices (e.g., Deaton 1979).
Table 1. Scale and Slutsky effects for inverse demands
quantities increased from \( x_C \) to \( kx_C \), i.e. along the ray radiating from the origin.

Formally, let \( x^o \) be the reference vector and let \( x=kx^o \) be the vector of interest where \( k \) is a scaler. The inverse demand can then be expressed as

\[
\pi^i = \frac{f^i(kx^o)}{g^i(k)} = g^i(k, x^o)
\]

(10)

The scale elasticity of good \( i \) is defined as

\[
\mu^i = \frac{g^i_0(k/g^i)}
\]

where \( g^i_0 \) is the derivative of \( g^i \) with respect to \( k \). When preferences are homothetic, the scale elasticities are identically equal to \(-1\). In general, for normal goods, the scale elasticities can be positive or negative. A positive scale elasticity implies a luxury good, and a negative scale elasticity close to zero implies a necessity. Scale elasticities that are negative and large in absolute value imply inferior commodities.\(^4\) The proportionate change in the price of good \( i \) with respect to good \( j \) or 'flexibility' is defined as

\[
\delta_{ij} = (\partial \pi_i/\partial x_j)(x_j/\pi_i) = f^i_j(x)(x_j/f^i(x))
\]

From the definition of \( g^i \) in (10),

\(^4\)Kohli (1985) shows that in a two good model when one of the goods is a Giffen good (i.e., has a negative income elasticity and a positively sloped direct demand), the inverse demand of the other good has a positively sloped inverse demand and a positive scale elasticity while the inverse demand of the Giffen good itself is downward sloping. The other good is called an 'anti-Giffen' good. This observation of course does not generalize for the more than two goods case.
\[
\mu_i = \sum_j \frac{f_j(x_j/f_i)}{f_i} \quad \text{or} \quad \mu_i = \sum_j \delta_{ij} \tag{11}
\]

This is analogous to the homogeneity restriction in the case of direct demands. The analogues to the Cournot and Engel aggregation conditions are shown below. Using the definition of the expenditure share \( \omega_i = \pi_i x_i \), we can write the budget constraint as

\[
\sum_i f_i(x) x_i = 1
\]

Differentiating with respect to \( x_j \),

\[
\sum_i \frac{f_j}{f_i} x_i + f_j = \sum_i f_j(x_j/f_i) f_i x_i + f_j x_j = 0 \quad \text{or} \quad \sum_i \delta_{ij} \omega_i = -\omega_j \tag{12}
\]

which is analogous to Cournot aggregation. The Engel aggregation is obtained by summing (12) over \( j \) and using (11),

\[
\sum_i \mu_i \omega_i = -1
\]

Just as the Slutsky equation breaks up the price effect into the substitution and the income effects, its analogue in the case of inverse demand breaks up the change in normalized prices into the substitution and scale effects. The second partial differentials of the distance function give the Antonelli substitution effects, which state the amounts by which normalized prices change with respect to a marginal change of the reference consumption \( x_j \), keeping the consumer on the same indifference level. From (9) we can derive the compensated flexibilities
Since $D(.)$ is homogeneous of degree one in $x$, $f^i_*$ is homogeneous of degree zero in $x$. Hence, using Euler's theorem,

$$
\sum_j \delta^*_i j = 0 \tag{14}
$$

From the properties of $D(.)$, the matrix of Antonelli effects is negative-semi-definite. Therefore the 'own' flexibilities are always negative, i.e. $\delta^*_i i < 0$. Anderson (1980) calls this the 'law of inverse demand'.

In Figure 1, to induce the consumer to change the consumption bundle from $x_A$ to $x_B$, the normalized prices have to change from $\pi_A$ to $\pi_B$. This change can be broken up into the substitution effect, where prices change from $\pi_A$ to $\pi_C$ so that the consumer is allowed to consume the bundle $x_C$ which leaves her utility unchanged; and the scale effect, where the prices change from $\pi_C$ to $\pi_B$ so that the consumer is able to consume more of all the commodities ($x_B$), leaving their proportions unchanged.

Thus the total change in normalized prices can be decomposed into substitution and scale effects. To derive formally the analogue to the Slutsky equation, let $x^0$ be the reference vector as before, and $x$ the vector of interest such that $x = kx^0$. The utility function is $U(kx^0)$ and as in (10), the normalized prices are given by $\pi_1 - g^i(k,x^0)$. The question now is: how do the normalized prices change for a marginal change in the reference vector, say, a change in $x_j$? Totally differentiating (10) we have,

$$
d\pi_1 = g^j dx^0_j + g^i_k dk \tag{15}
$$
Since utility is held constant along the same indifference curve for a change in any $x_j^p$,

$$dU = 0 = \Sigma_i U_i x_i^p \, dk + kU_j \, dx_j^p \, \text{ or }$$

$$dk = -\left(\frac{U_j}{\Sigma_i U_i x_i^p}\right) k^2 \, dx_j^p = -\pi_j k^2 \, dx_j^p \quad (16)$$

Finally, letting $k=1$ and substituting (16) for $dk$ in (15),

$$g_{ij} = (\frac{\partial \pi_i}{\partial x_j})_U + g_0 \pi_j$$

In elasticity terms,

$$\delta_{ij}^* = \delta_{ij} - \mu_{i} \nu_j \quad (17)$$

where the starred $\delta$'s are 'compensated' flexibilities.

The Antonelli matrix of substitution effects ($A$) and the Slutsky matrix ($S$) are generalized inverses of each other (Deaton and Muellbauer 1980a)\(^5\) so that

$$S = y(SAS)$$

and

$$A = y(ASA)$$

---

\(^5\)When one of the prices is treated as the numeraire, the Antonelli and Slutsky matrices are $(n-1) \times (n-1)$ in size and $S = A^{-1}$ (Katzner 1970 p.49). In terms of elasticities, the full matrix of flexibilities is the inverse of the matrix of elasticities (Houck 1966).
Flexible Functional Forms

Traditionally, mathematically convenient forms like the Cobb-Douglas, the CES and the Leontief were used to represent utility functions. These functions however imposed severe restrictions on the nature of preferences. For instance, the Cobb-Douglas utility forces all Allen elasticities to equal unity, the Leontief forces them to be equal to zero, and the CES forces them to be a constant. None of these cases is in general likely. Consequently, a search for more general forms with fewer restrictions on the parameters have yielded a number of flexible functional forms (FFF).

A function $f(x)$ represents a second order differential approximation to a function $g(x)$ at the point $x^0$ iff

$$f(x^0) = g(x^0)$$
$$\nabla f(x^0) = \nabla g(x^0)$$
$$\nabla^2 f(x^0) = \nabla^2 g(x^0)$$

Thus the parameters of $f(x)$ can be chosen such that its function value, gradient and Hessian equal the corresponding magnitudes for any arbitrary function $g(x)$ at $x^0$. If $n$ is the dimension of the vector $x$, a FFF must have at least $1 + n + n(n+1)/2$ parameters, which is the minimum amount of parametric freedom a functional form must have to satisfy the minimality property of Barnett and Lee (1985). For the case of demand systems, the number of free parameters needed for the flexibility criteria is fewer than this because of the ordinality of the utility function. For example, if a FFF is used to approximate the direct utility function, the function value itself need not be approximated, and in addition, the parametric
representation of preferences are invariant to any monotonic transformation of the utility function. This permits arbitrary normalizations on the gradient\(^6\) and on the Hessian, bringing down the number of free parameters needed to \((n-1)(n+4)/2\).

A somewhat stronger definition of a second order approximation is that of a second order numerical approximation which is necessary and sufficient for the definition of differential approximation to hold (Barnett 1983). Thus a Taylor series approximation to a function \(f(x)\) can be interpreted as a second order differential approximation to an arbitrary function \(g(x)\) at the point \(x^0\).

The more commonly used FFFs include the Translog (Christensen, Jorgensen and Lau 1975), the quadratic, and the generalized Leontief all of which are special cases of the generalized quadratic (Blackorby, Primont and Russel 1978).\(^7\) A flexible specification of demands that is widely used today is the Almost Ideal Demand System (AIDS) model (Deaton and Muellbauer 1980b) who use a Translog FFF in the specifying a cost function that satisfies aggregation conditions.

Other FFFs that have been proposed include the Minflex Laurent system of Barnett (1985), based on the Laurent series expansion that also provides a local approximation. This model possesses more parameters than the Translog or the generalized Leontief, but most of its coefficients are

---

\(^6\)Typically, this normalization is imposed to overcome the identification problem of the parameters, as in the case of the Translog.

\(^7\)Berdnt and Khaled (1979) introduced the Generalized Box–Cox functional form which contains most other commonly used FFFs like the Translog, quadratic functional form and the Generalized Leontief as limiting or special cases.
subject to inequality constraints. Unlike the FFFs mentioned above, the Fourier flexible form of Gallant (1981), based on a nonparametric approach, can provide a global approximation. This is, however, achieved at the expense of considerable computational burden.

More recently, Diewert and Wales (1988a) proposed a demand systems derived from normalized quadratic indirect utility and expenditure functions, on which curvature conditions can be imposed (on a similar note, Lewbel (1989) also proposed a globally concave flexible expenditure function). However, only the model derived from the expenditure function retains its flexibility after the restrictions are imposed. Even in this case, imposition of curvature conditions may prove difficult for a large number of goods, lack of degrees of freedom, or computational difficulties. For such situations, Diewert and Wales (1988b) present a procedure of choosing the 'degree of flexibility' consistent with feasibility of estimation such that concavity conditions are maintained without obviously restricting the second order derivatives. The resulting functional form is termed 'semiflexible'.

Another approach to local approximation is the differential approach or the Rotterdam model. While the FFF approach can be considered an approximation in the variable space, the differential approach can be viewed as an approximation in the parameter space.8

Although there has been a considerable debate on the merits of various FFFs, the Translog has been shown to perform as well or better than most others by the criteria of statistical performance (Berdnt, Darrough, and

8The Rotterdam model is discussed in section II of the dissertation.
Diewert 1977) and tracking known functions in Monte Carlo studies (Wales 1977, Guilkey, Lovell and Sickles 1983). Another consideration is the models' tendency to violate maintained regularity conditions within the region of the data. Caves and Christensen (1980) developed a procedure for deriving and graphically displaying the regular regions of the FFFs. They found that the regularity properties of the Translog and the generalized Leontief deteriorated rapidly as elasticities were moved away from those at each model's globally well behaved special cases, i.e., when preferences are homothetic in both the cases. In a similar analysis, Barnett and Lee (1985) compare the regular regions of the Translog and the generalized Leontief to the Minflex Laurent model. They show that the Minflex Laurent model violates curvature conditions less often than the other two.

In Section I of the dissertation, the Translog functional form is used in specifying a FFF for a distance function from which the Linear Inverse Demand System is derived. The Translog is also modified to represent quasi-homothetic preferences from which an aggregable demand system is derived.

The Aggregation Problem

Schematically, figure (4) Shows the two alternatives of aggregating individual demand functions and individual preferences (Van Daal and Merkies 1984). Clearly, aggregating over individual preferences is of little practical importance since no empirical content can be attached to utility either at the individual level or at the market level. Moreover, there is the problem of inter-personal comparison which precludes the
Table 2. Aggregation in demand analysis
aggregation of individual indifference curves. Therefore, in consumption analysis, the aggregation problem is limited to the aggregation of demand functions over individuals.

The questions that remain for all practical purposes are,

(i) Under what conditions is the average (aggregate) demand a function of prices and of average incomes? These conditions permit 'exact linear aggregation' i.e. the average demands can be expressed as \( \bar{x}(p, \bar{y}) \) where \( \bar{x} = (1/H) \Sigma x^h \), \( \bar{y} = (1/H) \Sigma y^h \) and the superscript \( h \) refers to the \( h \)th individual (household); \( H \) is the total number of individuals.\(^9\)

(ii) What are the (less restrictive) conditions which allow us to express the average demand as \( \bar{x}(p, y^0) \), where \( \bar{x} \) is the average demand as before and \( y^0 \) is the 'representative' level of income which is in general a function of the distribution of income and of prices? These are the conditions which permit 'exact non-linear aggregation'.

As Deaton and Muellbauer (1980a) point out, (i) and (ii) can hold whether or not utility maximization holds at the market level. In fact, the additional requirement of utility maximization leaves the restrictions on individual Engel curves required by exact (linear and nonlinear) aggregation unchanged (Deaton and Muellbauer 1980a, p.161).

For (i) to hold for any demand system, the Engel curves have to be linear and the slopes identical across individuals so that a re-allocation of a unit of income from one individual to another leaves the total demand unchanged. In other words, the marginal propensities to consume of all

\(^9\)The additional condition \( \delta \bar{x}(p, \bar{y})/\delta y_1 = \Sigma x^h(p, \bar{y})/\delta y_1 \) is satisfied when the number of individuals is greater than the number of commodities (Schafer and Sonnenschein 1982).
individuals are identical. This means that preferences are homothetic (Eisenberg's theorem in Shafer and Sonnenschein 1982) or quasi-homothetic (Muellbauer 1975, 1976). This is the class of preferences (Gorman Polar Form) for which exact aggregation is possible.

For (ii) to hold, the marginal propensities (the slopes of the Engel curves) vary linearly with one another as total expenditure changes at constant prices. This is the case of 'generalized linearity' where the representative income level is, in general, some point in the income distribution, the position of which is determined by the degree of non-linearity in the Engel curves and by the prevailing prices. A special case occurs when the representative expenditure level is independent of prices and depends only on the distribution of income. This is the case of price independent generalized linearity (PIGL) the logarithmic form of which (PIGLOG) is specially amenable to estimation. The PIGLOG form belongs to the generalized GPF.

For empirical purposes, all the above restrictions on preferences, which make aggregation legitimate, imply restrictions on the indirect

---

10 The AIDS and a particular case of the Translog both belong to the PIGLOG class (Lewbel 1987a).

11 The entire aggregation issue can be viewed as a discussion of Engel curves. Gorman (1981) showed that demand functions of the form $x_i = \sum a_i y f_i(y)$, where $a_i$ and $f_i$ are some functions of prices and income respectively and $i$ refers to the $i$th good, can have at most three terms and discussed the possible functions that $f_i$ can take. Lewbel (1987b) characterized a subset of the above demands, where $x_i = a_i + b_i y + c_i f_i$ and showed that homothetic, quasi-homothetic, PIGL and PIGLOG preferences are special cases when $a_i = 0$. Muellbauer's (1975) class of generalized linear demands is more general than this class. Approaching the description of Engel curves from another angle, Lewbel (1987c) provided a complete characterization of fractional demand systems of which homothetic, PIGL and PIGLOG are particular cases.
utility or the cost functions, both of which can be used in deriving the direct demand systems to be estimated. In the case of inverse demands, the restrictions would apply directly to the utility or the distance function. This again means restrictions on the functional forms chosen to approximate these functions.

In the context of aggregation, Jorgensen, Lau and Stoker (1982) introduce a tractable manner of modeling heterogeneous preferences by introducing an attribute vector at the individual level. In the same vein of analysis Heineke and Shefrin (1990) analyze the conditions under which the parameters of the estimated aggregate demand system can be used to identify the parameters of the underlying individual demand systems.

Aggregation has been of interest in empirical analysis. In one such recent study Lee, Pesaran and Pierse (1990) test for aggregation bias in linear models where aggregation bias is defined in terms of the deviations of macro parameters from the averages of the corresponding micro parameters.

The two classes of preferences that are consistent with exact linear aggregation i.e. homothetic and quasi-homothetic preferences are briefly discussed below.

**Homothetic preferences**

Homothetic preferences imply linear income expansion paths that radiate from the origin. In general, the cost function that is dual to this class of preferences takes the form

\[ C(u, p) = f(u)b(p) \]  \hspace{1cm} (18)

where \( b(p) \) is homogeneous of degree one and is concave and \( f \) is an
increasing function of $u$. For income distribution to be of no consequence, $b(p)$ has to be identical for all individuals, while the utility levels can vary. The indirect utility function takes the form

$$V(y,p) = \frac{y}{b(p)} \quad (19)$$

In deriving indirect demands, Wold's identity can be directly applied to the homothetic utility function. Alternatively the distance function can be used. If the direct utility function is homothetic, the distance function is homogeneous of degree zero in $x$ and $u$. The distance function for the class of homothetic preferences is given by

$$D(u,x) = \frac{b(x)}{u}$$

where $b(x)$ homogeneous and concave in $x$.

**Quasi-homothetic preferences**

In the case of quasi-homothetic preferences the Engel curves are still linear but do not radiate from the origin. In fact, they radiate from a base 'surface' which need not necessarily lie in the consumption space (the positive orthant). When it does, the points on this surface can be interpreted as 'subsistence' bundles. Evidently, these subsistence bundles depend on prices. If the surface degenerates to a point, the underlying preferences are said to be affinely homothetic, and the subsistence bundles are independent of prices. If the point coincides with the origin, the preferences are homothetic (Blackorby, Boyce and Russel 1978). The income elasticities for quasi-homothetic preferences tend to unity as income increases.
The less restrictive assumption of quasi-homotheticity has been extensively discussed in terms of the cost and indirect utility functions. A fixed cost element is added to the cost function (18) so that the Gorman Polar Form is given by

\[ C(u,p) = a(p) + ub(p) \]

where \( a(p) \) can be interpreted as the 'subsistence' expenditure when \( u=0 \), and can vary across individuals. Both \( a(p) \) and \( b(p) \) are linear homogeneous and concave. The indirect utility function is given by

\[ V(p,y) = (y-a(p))/b(p) = y/b(p) - a(p)/b(p) \]

which can be interpreted as the real value of expenditure in excess of that required for subsistence. Clearly, \( V(p,y) \) is homogeneous of degree zero in \( p \) and \( y \).

Both the above forms give legitimate forms of the cost and indirect utility functions that can be employed in deriving market level direct demand systems. To derive inverse demands, the utility or the distance function underlying quasi-homothetic preferences have to be known. When Engel curves radiate from a single point \( \gamma=(\gamma_1, \gamma_2, \ldots, \gamma_n) \), preferences are homothetic to the point \( \gamma \) i.e., \( U(x-\gamma) \) is homogeneous in the translated variables \( x-\gamma \). When the point \( \gamma \) is fixed and is in the positive orthant, it takes on the interpretation of a subsistence bundle.\(^{12}\) In general

\(^{12}\)Dickinson (1980) uses a flexible cost function for quasi-homothetic preferences where the Engel curves are parallel (and linear) and the substitution possibilities are independent of the utility level. He does this by letting \( C=u(\Sigma p_i \gamma_i)+\Lambda(p) \), where \( \Lambda(p) \) can be approximated by one of the FFFs like the Translog.
however \( \gamma \) is a function of prices and can lie anywhere, i.e the co-
ordinates of \( \gamma \) can be positive or negative. The preferences underlying the
GPF indirect utility function are given by

\[
V(\pi, l) = \max_{x, \gamma} \left( \bar{U}(x-\gamma) + \hat{U}(\gamma) \mid \pi \cdot x = 1; x-\bar{x}+\gamma; \hat{U}(\gamma) = 0 \right)
\]

where \( \bar{U} \) is linearly homogeneous in the translated variable \((x-\gamma)\) and \( \hat{U} \) is
an arbitrary function which attains a maximum at zero (Blackorby and
Schwom 1988). \( x \) is the total consumption, \( \gamma \) is the base level consumption
quantity, and \( \bar{x} \), the consumption in excess this level. In reality, \( \gamma \)
cannot be observed and is usually (in empirical studies) assumed to be
independent of prices and treated as a parameter. This is the case in the
Linear Expenditure System (LES) derived from the Stone-Geary utility
function which takes the form of the Cobb-Douglas in the displaced
variable. This will also be the case in the aggregable empirical model to
be specified later. In all these cases, preferences are described only for
all \((x-\gamma)>0\).\textsuperscript{13}

Estimation of Systems of Equations

Demand equations derived from some flexible representation of
preferences (or which are differential approximations as in the case of the
mixed demands in Section II) involve cross equation restrictions implied by
theory. In addition, one can in general expect contemporaneous correlation

\textsuperscript{13}Income levels have to be sufficiently high if quasi-homothetic
preferences (with fixed \( \gamma \)) are assumed so that \((x-\gamma)\) is positive. In terms
of the cost function \( C(U, p) = a(p) + U(b(p)) \), this means that \( C(U, p) > a(p) \) so that
concavity is not violated.
across the equations. The demand equations are therefore treated as a set of equations and are estimated using the Seemingly Unrelated Regressions Equations (SURE) technique. To make the specified (deterministic) equations stochastic for the purpose of estimation, random error terms are appended to each of the (possibly nonlinear) demand equations.\footnote{Other ways of incorporating the random error have been considered. McElroy (1987) and Chavas and Segerson (1987) embed the error terms into the objective functions. The error terms entering the demand equations derived from such models need not necessarily be homoskedastic.} The stochastic system of equations is given by

$$w_t = f(Z_t, \theta) + u_t$$

$$t = 1, 2, \ldots, T$$

where $$w_t$$ is a Mx1 vector of expenditure shares, $$Z_t$$ is the vector of $$n$$ independent variables (quantities in the case of inverse demands) at time $$t$$ and $$\theta$$ is the vector of all coefficients to be estimated. $$t$$ indexes time series and $$u_t$$ is a Mx1 vector of random errors that satisfies:

\[
E(u_t) = 0 \tag{20}
\]

\[
E(u_t u_t') = \Omega \tag{21}
\]

\[
E(u_t u_s') = 0 \quad t \neq s \tag{22}
\]

conditions (21) and (22) imply contemporaneous correlation, i.e., nonzero covariances between the disturbances in different equations, with the same covariance at each sample point. They also imply that the disturbance variance is different for the different equations, but homoskedasticity and zero covariances are imposed within each equation. Since the shares add up to one, the parameters of the equations can be recovered from the rest of the equations. This adding up condition implies that the matrix of
covariance is singular. Hence one of the equations is dropped in estimation.

Assuming the vector of disturbances is multinormally distributed, maximum likelihood estimation can be done. The log-likelihood function of all T sets of observations is:

$$\ell(\theta, \Omega) = -(1/2)MT \ln(2\pi) - (1/2)T \ln|\Omega| - (1/2) \sum_{t=1}^{T} u_t^\top \Omega^{-1} u_t$$

The concentrated likelihood function is given by

$$\ell(\theta) = k - \frac{1}{2} T \ln |\Omega^*|$$

where $k = (-\frac{1}{2} MT \ln(2\pi) - \frac{1}{2} MT)$, a constant that can be ignored in the maximization process. $\Omega^* = (1/T)(UU')$, where $U = (u_1, \ldots, u_T)$. Since the components of the variance-covariance matrix $\Omega$ are non-linear functions of the parameters $\theta$, a numerical optimization procedure like Davidon-Fletcher-Powell (DFP) algorithm can be used.

The Linear Inverse Demand System

The AIDS model of Deaton and Muellbauer (1980b) is one of the most commonly used demand systems in applied demand analysis. While the 'ideal' connotation of this model stems from its aggregation properties, it is arguable that one of the main reasons for its popularity is the availability of an approximate version of this system that is linear in the parameters. In fact it is this linear version of the AIDS model that is typically estimated.
A system of inverse demand equations that resembles the AIDS model can be derived from a properly specified and flexible distance function. The proposed Linear Inverse Demand System (LIDS), although similar in structure to the AIDS, does not claim the same aggregation properties. Nonetheless, the simplicity of its linear structure and its approximation abilities are likely to make this system of interest for empirical studies.

Consider the cost function $C(p,u)$ underlying the PIGLOG preferences:

$$\ln C = a(p) + ub(p) \quad (23)$$

where $a(p)$ and $b(p)$ are defined as:

$$a(p) = \alpha_0 + \sum_i \alpha_i \ln p_i + \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j$$

$$b(p) = \beta_0 \prod_k p_k^{\beta_k}$$

The cost function $C(p,u)$ is homogeneous of degree one in $p$, when the following restrictions apply: $\sum_i \alpha_i = 1$, $\sum_j \gamma_{ij} = \sum_i \gamma_{ij} = 0$, and $\sum_k \beta_k = 0$. In addition, symmetry implies $\gamma_{ij} = \gamma_{ji}$.

Appealing to the concept of "symmetric duality" discussed by Hanoch (1978), it is always possible to represent preferences (or technology) uniquely by a distance function $D(x,u)$ which behaves with respect to $(x;1/u)$ in exactly the same way as the cost function $C(p,u)$ behaves with respect to $(p;u)$. Hence, from the PIGLOG cost function of the AIDS model (23), one can immediately write down its "polar" (in Hanoch's terminology) distance function. However, this distance function is not dual to $C(p,u)$. It follows that the aggregation properties of the PIGLOG cost function are
not shared by its polar distance function.\textsuperscript{15}

Consider the following parametric specification for the distance function $D(x,u)$ which is the symmetric dual to the cost function (23):

$$\ln D = a(x) + (1/u)b(x)$$  \hspace{1cm} (24)

where $a(x)$ and $b(x)$ are quantity aggregator functions defined as:

$$a(x) = \alpha_0 + \sum_i \alpha_i \ln x_i + \frac{1}{a} \sum_i \gamma_{ij} \ln x_i \ln x_j$$  \hspace{1cm} (25)

$$b(x) = \beta_0 \prod_k \lambda_k \beta_k$$  \hspace{1cm} (26)

Because $D(x,u)$ is homogeneous of degree one in $x$, the following restrictions apply: $\sum_i \alpha_i = 1$, $\sum_j \gamma_{ij} = \sum_i \gamma_{ij} = 0$, and $\sum_i \beta_i = 0$. Once again, symmetry implies $\gamma_{ij} = \gamma_{ji}$.

The derivative property (9) applied to the distance function yields compensated inverse demands as $\pi_i = \partial D/\partial x_i = \pi_i(x,u)$, where $\pi_i$ is the normalized price of the $i^{th}$ good (the nominal price divided by total expenditure). Because at $D=1$ the distance function is an implicit form of the direct utility function, (24) implies the utility function $U = -b(x)/a(x)$. This, together with the derivative property, implies that the uncompensated inverse demand functions associated with (24) can be written in share form as:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln x_j - \beta_i \ln X$$  \hspace{1cm} (27)

\textsuperscript{15}Eales and Unnevehr (1991) independently developed a very similar model derived from a distance function that is identical in structure to the PIGLOG cost function. Although their specification is different from the distance function motivated by the concept of symmetric duality, the derived inverse demands are identical to ours. The developed model was applied to U.S. meat demand.
where $w_i = p_i x_i$ is the $i^{th}$ budget share, and $lnX$ is a quantity index defined as $lnX = a(x)$. Equations (27) and (25) together entail a nonlinear structure for the inverse demand model. In practice, however, $lnX$ can be replaced by an index $lnX^*$ constructed prior to estimation of the share system, thereby making (27) a linear system of equations. Many index formulae may be considered here. If quantities are properly scaled, one may use the geometric index $lnX^* = \sum_i w_i lnx_i$, although other indices (say superlative indices) may have better approximation properties.

The inverse demand system presented here satisfies standard flexibility properties. In particular, if $n$ is the number of goods, the resulting demand system (27) provides a local approximation to an arbitrary demand system in that its $(n-1)(n+4)/2$ free parameters can be chosen to represent at a point an arbitrary set of quantity elasticities [of which $n(n+1)-1$ are independent] and an arbitrary set of left-hand-side shares [of which $(n-1)$ are independent]. It can be verified that the distance function (24)-(26) has enough parameters to be a flexible functional form for an arbitrary distance function once it is realized that the ordinality

\[\text{An index number is said to be exact for a function } f(.) \text{ if the ratio of the value of } f(.) \text{ between any two periods is identically equal to the values of the index. An index number is said to be superlative if it is exact for a functional form } f(.) \text{ that can provide a second order approximation to an arbitrary homogeneous aggregator function (i.e., a FFF). Since the aggregator } a(x) \text{ is itself a Translog FFF, the Törnquist quantity index which is exact for a Translog } f(.) \text{ can be used. The Paasche and Laspeyres quantity and price indexes approximate the superlative indexes to the first order and for time series data provide acceptable approximations (Diewert 1987).} \]
of utility always allows one to put $\partial^2 \ln D/\partial (1/u)^2 = 0$ at a point.\(^{17}\)

**Simulation results**

To illustrate the approximation properties of the LIDS model presented here, a small simulation exercise was carried out. Shares for a 3-good system are generated using the inverse share equations of the Linear Expenditure System (LES)\(^{18}\), that is:

$$w_j = \frac{\alpha_j [x_j/(x_j - \gamma_j)]}{\sum \alpha_j [x_j/(x_j - \gamma_j)]} \quad (28)$$

The parameters and the data that are used are those employed by Wales (1984), except that the price series used by him are treated as the quantity series.\(^{19}\) In particular, the parameters used are

$\alpha = [0.2, 0.4, 0.4]$ and $\gamma = [0.2, 0.1, 0.3]$. From this structure 200 samples were generated, each with 40 observations, by appending multinormal disturbances [from the covariance matrix also used by Wales (1984)] to these shares.

---

\(^{17}\)This can be shown by differentiating the logarithmic transformation of the identity $U[x/D(x,u)] = u$ and noting that one can always find a monotonic transformation of the utility such that $\sum \partial^2 u/\partial \ln x_i \partial \ln x_j = 0$ at a point. A similar 'money metric scaling' is assumed in the AIDS model as pointed out by Diewert and Wales (1988a).

\(^{18}\)The LES is derived from the Stone-Geary utility function $U = \prod (x_i - \gamma)\alpha_i$ which belongs to the class of quasi-homothetic preferences.

\(^{19}\)The price and income series (of 40 observations) used by Wales were generated such that they tend, on average, to grow at a constant rate (higher for income than for prices, and varying among the latter), modified by random shocks. While some of the random shocks affect all the series (to reflect the role of the business cycles), some of the shocks affect individual series only. Thus the data generation process tries to replicate the real world data as closely as possible (Kiefer and MacKinnon 1976). The series are normalized to equal unity at observation 21.
With these data, the LIDS model was estimated 200 times (using the geometric index $\ln X^* = \sum w_j \ln x_j$). For the purpose of comparison, the true LES model (28) was also estimated, along with the inverse Translog (TL) system introduced by Christensen, Jorgenson, and Lau (1975), which is:

$$w_i = \frac{\alpha_i + \sum \beta_{ij} \ln x_j}{1 + \sum \beta_{ij} \ln x_j}$$  \hspace{1cm} (29)$$

with the normalization $\sum \beta_{ij} = 0$, which is justified because we are seeking a local approximation, and one can always find a monotonic transformation of utility such that $\sum \beta_{ij} \frac{\partial^2 u}{\partial \ln x_i \partial \ln x_j} = 0$ at a point. Hence, both LIDS and TL have 7 free parameters in this application.

The approximation properties of the models considered can be illustrated in terms of the quantity and scale elasticities defined earlier. These elasticities were computed at the median observation point (at which $x_i^* = 1$, $w_i$) for each of the 200 replications, and summary statistics for the own-quantity and scale elasticities are reported in Table 1 below.

The first column of this Table reports the true elasticities at this point. Then, for each of LES, LIDS, and TL the mean of the estimated elasticities are reported. The mean of the absolute deviation (MAD) of these estimates from the true elasticities, computed over the 200 replications, as well as the average MAD computed over the entire set of 12 elasticities are computed and reported.
Table 1. A comparison of inverse demand systems

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>True value</th>
<th>LES mean</th>
<th>MAD</th>
<th>TL mean</th>
<th>MAD</th>
<th>LIDS mean</th>
<th>MAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{11}$</td>
<td>-1.201</td>
<td>-1.183</td>
<td>0.085</td>
<td>-1.157</td>
<td>0.117</td>
<td>-1.154</td>
<td>0.120</td>
</tr>
<tr>
<td>$f_{22}$</td>
<td>-1.072</td>
<td>-1.061</td>
<td>0.052</td>
<td>-1.136</td>
<td>0.118</td>
<td>-1.021</td>
<td>0.097</td>
</tr>
<tr>
<td>$f_{33}$</td>
<td>-1.235</td>
<td>-1.221</td>
<td>0.067</td>
<td>-1.454</td>
<td>0.223</td>
<td>-1.361</td>
<td>0.141</td>
</tr>
<tr>
<td>$k_1$</td>
<td>-0.968</td>
<td>-0.968</td>
<td>0.012</td>
<td>-0.913</td>
<td>0.072</td>
<td>-0.873</td>
<td>0.098</td>
</tr>
<tr>
<td>$k_2$</td>
<td>-0.829</td>
<td>-0.834</td>
<td>0.025</td>
<td>-0.799</td>
<td>0.057</td>
<td>-0.820</td>
<td>0.045</td>
</tr>
<tr>
<td>$k_3$</td>
<td>-1.147</td>
<td>-1.143</td>
<td>0.019</td>
<td>-1.194</td>
<td>0.052</td>
<td>-1.195</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>average MAD</td>
<td>0.039</td>
<td>0.126</td>
<td>0.106</td>
</tr>
</tbody>
</table>

All models seem to provide a reasonable approximation. As expected, the best results are obtained by estimating the true LES model. The performance of LIDS and TL are similar, with LIDS actually being slightly better.\(^{20}\)

\(^{20}\)One might wonder if the inverse demand system with the nonlinear quantity index $\ln X$ performs better than the linear approximation using $\ln X^*$. This was checked in the simulation exercise and it was found that two systems produced virtually identical results. In addition, the simulation exercise was repeated with the income and quantity series generated from a uniform distribution instead of the trended data with the same parameter and covariance values as before. The nature of the results did not change.
An Aggregable Flexible Inverse Demand System

Inverse market demand systems such as the LIDS and the Translog satisfy the criteria of flexibility, but fail to meet the aggregation conditions necessary for the existence of market demands. A new flexible form that provides a second order approximation to an arbitrary utility function, and which belongs to the class of quasi-homothetic preferences is proposed. Inverse demand functions derived from this utility function represent valid aggregate market demands.

A translog FFF for the utility function is specified in the displaced variable \((x_{-\gamma})\) as:

\[
\ln U = \alpha_0 + \sum_i \alpha_i \ln(x_{i \gamma_1}) + (1/2) \sum_i \sum_j \alpha_{ij} \ln(x_{i \gamma_1}) \ln(x_{j \gamma_1})
\] (30)

where \(\alpha_0\), \(\alpha_i\)'s, \(\alpha_{ij}\)'s and \(\gamma_i\)'s are parameters. When homogeneity of this utility function in the displaced variable is imposed, the FFF represents quasi-homothetic preferences. This can be formally stated as below.

Proposition: Let \(U^*\) be a utility function, twice continuously differentiable at \(x^*\). Let \(\bar{U}(x_{-\gamma})\) be FFF linearly homogeneous in \((x_{-\gamma})\), where \(\gamma\) is a vector of constants. Then \(\bar{U}(x_{-\gamma})\) can provide a second order differential approximation to \(U^*\) at the point \(x^*\).\(^{22}\)

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\(^{21}\)In Gorman's (1980) words "...sufficiently smooth preferences can always be approximated quasi-homothetically in a given neighborhood, just as their Engel curves can be by straight lines."

\(^{22}\)Dievert (1980) shows that the GPF cost and indirect utility functions (for a local cardinalization of utility) can approximate arbitrary preferences to the second order and can be FFFs.
Proof: We have to choose \( \gamma \) and \( \bar{U} \), linearly homogeneous in \((x-\gamma)\), so as to satisfy the following conditions, assuming \( \bar{U} > 0 \) and \((x^{*}-\gamma) > 0\):

\[
U^*(x^*) = \bar{U}(x-\gamma) \tag{31}
\]

\[
\frac{\partial U^*(x^*)}{\partial x_i} = \frac{\partial \bar{U}(x^*-\gamma)}{\partial x_i} \quad i=1,2,\ldots,n \tag{32}
\]

\[
\frac{\partial^2 U^*(x^*)}{\partial x_i \partial x_j} = \frac{\partial^2 \bar{U}(x^*-\gamma)}{\partial x_i \partial x_j} \quad i,j=1,2,\ldots,n \tag{33}
\]

Since \( \bar{U} \) is a FFF, we can pick \( n(n-1)/2 \) elements of (33) freely. We next pick the \( n \) \( \gamma \)'s such that \((x^*-\gamma) > 0\), \( i=1,2,\ldots,n \), and solve the first \( n-1 \) equations of (32) for \( \frac{\partial \bar{U}(x^*-\gamma)}{\partial x_i} \), \( i=1,2,\ldots,n-1 \). Since \( \bar{U} \) is homogeneous in \((x-\gamma)\),

\[
\frac{\partial \bar{U}(x^*-\gamma)}{\partial x_n} = \frac{[U^*(x^*) - \Sigma_i (\partial \bar{U}(x^*-\gamma)/\partial x_i)(x_i-\gamma_i)]}{(x_n-\gamma_n)}
\]

where the summation over \( i \) is from 1 to \( n-1 \). Thus (31) is also satisfied.

Using Wold's identity, we derive the following inverse (share) demand functions from (30):

\[
\frac{x_i}{x_i/(x_i-\gamma_i)} \left[ \frac{\alpha_i + \Sigma_j \alpha_{ij} \ln(x_j-\gamma_j)}{\Sigma_i \alpha_i (x_i/(x_i-\gamma_i)) + \Sigma_j \Sigma_i \alpha_{ij} (x_i/(x_i-\gamma_i)) \ln(x_j-\gamma_j)} \right] \tag{34}
\]

Since the inverse demand functions are homogeneous of degree zero in the \( \alpha_i \)'s and \( \alpha_{ij} \)'s, the normalization \( \Sigma_i \alpha_i = 1 \) is made. (30) is homogeneous in the variable \((x-\gamma)\).\(^\text{23}\) The FFF satisfies this condition if \( \Sigma_i \alpha_{ij} = 0 \) and \( \Sigma_j \alpha_{ij} = 0 \). Symmetry implies that \( \alpha_{ij} = \alpha_{ji} \). Put together, the restrictions to

\(^{23}\) When \( \gamma = 0 \), (30) reduces to a homothetic utility function.
be imposed on the model to be estimated are:

\[ \Sigma_i \alpha_i = 1 \quad \text{and} \quad \Sigma_i \alpha_{ij} = 0 \]  
\[ \Sigma_j \alpha_{ij} = 0 \]  
\[ \alpha_{ij} = \alpha_{ji} \]  

Restrictions (35a) are the column restrictions. Given the column restriction \( \Sigma_i \alpha_{ij} = 0 \), the row restriction (35b) follow from the restrictions (35c) which arise from the symmetry of the Hessian of the utility function. Homogeneity of the demand functions as used in the direct demand sense is automatically satisfied since we use normalized prices. We cannot say anything about the homogeneity property of the inverse demand functions in the variable \( x \).  

Since the restrictions are not independent of each other, the \( n+1 \) column restrictions along with the \( n(n-1)/2 \) symmetry restrictions imply the row restrictions (35b) leaving \( n(n+1)/2 + n-1 \) free parameters to be estimated (which is one more than the \( n-1)(n+4)/2 \) parameters in the LIDS and the Translog models).

In practice, for a large number of commodities, the denominator in (34) can be a large and unwieldy expression. One way to avoid this inconvenience would be to estimate share-ratio equations, \( w_i/w_n \), \( i=1,2,...,n-1 \), so that the denominator of these share ratio equations is the

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24 In general, for a homogeneous utility function \( U(x) \), the inverse demand functions are homogeneous of degree \( -1 \) and the inverse share demands are homogeneous of degree zero in \( x \).
simpler numerator of the $n^{th}$ share equation in (34). Each of the demand equations can be written as

$$w_i = \frac{N_i}{D}$$

where $N_i$ is the numerator of the $i^{th}$ share demand. The denominator is identical for all the equations. The flexibilities derived can be written as

$$\delta_{ik} = \frac{\gamma_k}{w_k} + \frac{\tilde{x}_k \alpha_{ik}}{N_i} - \frac{\Sigma_j \alpha_{jk} \tilde{x}_j}{D}$$

and

$$\delta_{ii} = \frac{\gamma_i}{(w_i - 1)} + \frac{\tilde{x}_i \alpha_{ii}}{N_i} - \frac{\Sigma_j \alpha_{ji} \tilde{x}_j}{D} - 1$$

where $\tilde{x} = x_i / (x_i - \gamma_i)$. The scale elasticities are given by

$$\mu_i = \Sigma_j \delta_{ij}$$

for all $i,j = 1,2,...,n$.

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$^{25}$Gallant (1987) estimates differenced logarithmic shares, i.e., $\ln w_i - \ln w_n$ for a system of direct demands derived from a Translog indirect utility function.
Summary and Conclusions

Flexible representations of inverse demand systems are explored in this section of the dissertation. After briefly laying out the theory of inverse demands, the concept of flexibility and aggregation are discussed. Two new flexible inverse demands are proposed.

The first of these is the Linear Inverse Demand System (LIDS). This specification is based on a distance function which can be interpreted as a polar form to the PIGLOG cost function underlying the AIDS model commonly used for direct demand models. The approximation properties of the model were illustrated with a simulation exercise, and the performance of this model was found to be similar to that of a Translog inverse demand system. The simplicity and linearity of the model is likely to make it a useful specification for empirical applications.

The second inverse demand system is derived from a FFF that provides a quasi-homothetic second order approximation to an arbitrary utility function. This demand system satisfies the aggregation conditions necessary for the existence of market demand functions. Further research on this functional form both in simulation studies and with actual data is likely to reveal the approximating capability of this functional form.
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SECTION II: MIXED DEMANDS: THE CANADIAN MARKET FOR MEATS

Introduction

A number of empirical studies in applied demand analysis use systems of demand equations that are specified in apparent isolation from the supply side and estimated by seemingly unrelated regression techniques. To obtain consistent estimates of the underlying preference parameters, this approach typically relies on one of two assumptions: either prices are assumed predetermined or quantities are assumed predetermined. The first of these assumptions leads to quantity dependent or direct demand functions. This is the usual representation of preferences that arises in the case of the consumer, who is typically taken as making optimal consumption decisions for given prices and income. Its use at the aggregate level is equivalent to assuming that supplies are perfectly elastic and that demands adjust to clear the market. This condition may hold for aggregate (market) data when one is modeling the demand of tradable goods in the case of a small open economy, or when prices are administratively set (e.g., public utilities). The alternative of assuming that quantities are predetermined, and that prices adjust to clear the market, leads to price dependent or inverse demand functions, an approach that may be useful when analyzing the demand for perishable products defined over a short period of time.\(^1\)

In addition to the two polar cases of direct and inverse demand

\(^1\)Section I of the dissertation deals with inverse demands.
functions, there is another class of models that allows one to sidestep the task of estimating both demand and supply functions in a simultaneous equations framework. This is the case of the "mixed demand" functions (Samuelson 1965, Chavas 1984) where the price of some goods are predetermined, such that their respective quantities demanded adjust to clear the market, while for the remaining set of goods it is the quantity supplied which is predetermined and prices must adjust to clear the market. Despite its obvious potential for applications, stemming from the fact of being a combination of the two polar cases discussed above, the mixed demand approach has been virtually ignored in empirical applications.

The aim in this section is to develop a mixed demand model suitable for empirical applications, and to use it to analyze the demand for meat in Canada. The mixed demand approach appears particularly suited to the Canadian meat demand case. First of all, there is virtually free trade between U.S. and Canada for beef and pork. Because Canada is a small country in the North-American market, the assumptions that beef and pork prices are exogenous to the Canadian market appears a tenable one. On the other hand, Canadian imports of poultry products are restricted by an import quota (Moschini and Meilke 1991). This import quota insulates the domestic market, and the internal price formation mechanism heavily depends on the institutional setting. In particular, chicken producers are organized in provincial Marketing Boards which are coordinated by the Canadian Chicken Marketing Agency. The objective of this monopoly-like

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2 Note the distinction between the mixed demand functions of interest here and the case of rationed demands (Neary and Roberts 1980), for which some markets do not clear.
organization is to guarantee producers a favorable price, and this objective is pursued by limiting the output in the market. This "supply management" is enforced by production quotas which are first allocated to each province and then to the individual producers. Hence, for chicken it seems that equilibrium is characterized by exogenously determined supply with price adjusting to clear the market.³

In specifying an empirical model of mixed demands, FFFs cannot be used as in the case of direct or inverse demands. This is because the optimization process involves knowledge of both the direct and indirect utility functions, and most FFFs do not have a closed form dual representations. The alternative of using a FFF for the 'mixed' utility function is also not possible since the derivative property cannot be implemented to derive mixed demands, as will be elucidated in the following sub-section. Given this, our strategy is to approximate the mixed demand equations directly with a differential approach. While a FFF provides an approximation to the true function in the variable space, the differential model can be viewed as an approximation in the parameter space that can also satisfy a meaningful definition of flexibility.

To do this, we first develop the theory of mixed demands using the concept of 'virtual' or shadow prices (Gorman 1976, Neary and Roberts

³The Farm Products Marketing Act of 1972 allowed the creation of marketing boards which led to the establishment of the Canadian Egg Marketing Agency in 1972, the Canadian Turkey Marketing Agency in 1973 and the Canadian Chicken Marketing Agency in 1978. Along with the Canadian Dairy Commission which existed since 1966, these agencies effectively became supply-restricting boards with considerable powers (Van Kooten 1987). For the case of chicken, import quotas were introduced in 1979, so that supply management for the chicken industry became fully operational by the end of that year.
A 'mixed' cost function is derived using which the Slutsky relations are derived for the mixed demand system. These relations expressed in terms of 'mixed' elasticities are employed in the specification of the mixed differential demand system. Finally, the proposed model is estimated using Canadian market data for beef, pork and chicken. In addition to the mixed differential demand system, the direct differential (Rotterdam) model and the mixed LES are also estimated. Finally, an ad hoc specification of the first stage of budget allocation is also estimated from which unconditional mixed elasticities are computed.

This section is organized as follows. The theory of mixed demand functions is discussed, followed by the specification of our differential mixed demand system and the other models that are estimated. The data used in the application is then described, and the estimation results are presented and discussed. The main points of this section are then summarized.

Theory of Mixed Demands

The problem can be stated as follows (Samuelson 1965, Chavas 1984). Let the n commodity bundle $x$ be partitioned into $x_A=(x_1, x_2, \ldots, x_m)$ and $x_B=(x_{m+1}, \ldots, x_n)$. $x_A$ is the vector of quantities optimally chosen given the corresponding normalized prices $\pi_A=(\pi_1, \ldots, \pi_m)$, and $\pi_B=(\pi_{m+1}, \ldots, \pi_n)$ is the optimal price vector given the quantity (supply) vector $x_B$. These $\pi$'s are normalized prices; if $p_i$ denotes the nominal price of good $i$, and $Y$ the total expenditure on all the commodities, then $\pi_i=p_i/Y$. The constrained
optimization problem is given by

\[
\text{Max} \quad U(x_A, x_B) - V(x_A, x_B) \quad \text{s.t.} \quad \pi_A x_A + \pi_B x_B = 1
\]  

where \( U \) and \( V \) are the direct and indirect utility functions, quasi-concave and quasi-convex in their respective arguments. The solution to this problem is given by the Marshallian mixed demands, \( \bar{x}_A(x_B, \pi_A, 1) \) and \( \bar{x}_B(x_B, \pi_A, 1) \). Clearly, at the optimum, \( U(\bar{x}_A(x_B, \pi_A, 1), x_B, \pi_A, 1) = V(\pi_A, \pi_B(x_B, \pi_A, 1)) - \bar{V} \) where \( \bar{V} \) is the 'mixed' utility function that is homogeneous of degree zero in \( p_A \) and \( Y \). Hence \( \bar{V}(x_B, \pi_A, 1) = \bar{V}(x_B, p_A, Y) \).

Alternatively, the optimization problem can be stated as the optimization of the sub-problems (Chavas 1984)

\[
\bar{V}(x_B, \pi_A, 1) = \text{Max} \quad U(x_A, x_B) \quad \text{s.t.} \quad \pi_A x_A + x_B \cdot \bar{x}_B(x_B, \pi_A, 1) = 1 \quad (2a)
\]

and

\[
\bar{V}(x_B, \pi_A, 1) = \text{Min} \quad V(x_A, \pi_B) \quad \text{s.t.} \quad \pi_A \cdot \bar{x}_A(x_B, \pi_A, 1) + x_B \cdot \pi_B = 1 \quad (2b)
\]

Applying the envelope theorem obtains:

\[
\bar{x}_1 = - \frac{\partial \bar{V}}{\partial p_1} = - \frac{\sum_{k=m+1}^{n} (\partial \bar{x}_k / \partial p_1) x_k}{\partial \bar{V} / \partial Y} \quad (3a)
\]

and

\[
-\bar{p}_k = - \frac{\partial \bar{V}}{\partial x_k} = \frac{\sum_{i=1}^{m} (\partial \bar{x}_i / \partial x_k) p_i}{\partial \bar{V} / \partial Y} \quad (3b)
\]

Denoting by \( \lambda \) the Lagrange multiplier for the budget constraint in (2) and (3), the marginal utility of income is denoted by \( \partial \bar{V} / \partial Y = \lambda / Y \).
The mixed demand functions \( \bar{x}_A(x_B, \pi_A, 1) \) and \( \bar{x}_B(x_B, \pi_A, 1) \) satisfy some restrictions. First of all, they satisfy the adding up condition \( \pi_A \cdot x_A + x_B \cdot \pi_B = 1 \) implied by the budget constraint. Secondly, the homogeneity condition implies that \( \bar{x}_A(x_B, \pi_A, 1) \) is homogeneous of degree zero in nominal prices and income, i.e., \( \bar{x}_A(x_B, \pi_A, 1) = \bar{x}_A(x_B, p_A, Y) \). Similarly, the optimal nominal prices for group B are homogeneous of degree one in \( (p_A, Y) \), implying that \( \bar{x}_B(x_B, \pi_A, 1) \) are homogeneous of degree zero in \( (p_A, Y) \) and hence \( \bar{x}_B(x_B, \pi_A, 1) = \bar{x}_B(x_B, p_A, Y) \).

We now define the Marshallian 'mixed elasticities' as

\[
\eta_{ij} = \left( \frac{\partial x_i}{\partial p_j} \right) \left( \frac{p_j}{x_i} \right) \quad i,j \in A
\]

\[
\psi_{ik} = \left( \frac{\partial x_i}{\partial x_k} \right) \left( \frac{x_k}{x_i} \right) \quad i \in A, \ k \in B
\]

\[
\rho_{ki} = \left( \frac{\partial p_k}{\partial p_i} \right) \left( \frac{p_i}{p_k} \right) \quad i \in A, \ k \in B
\]

\[
\theta_{ks} = \left( \frac{\partial p_k}{\partial x_s} \right) \left( \frac{x_s}{p_k} \right) \quad k, s \in B
\]

The 'mixed' income elasticities are defined by

\[
\eta_i = \left( \frac{\partial x_i}{\partial Y} \right) \left( \frac{Y}{x_i} \right) \quad i \in A
\]

\[
\theta_k = \left( \frac{\partial p_k}{\partial Y} \right) \left( \frac{Y}{p_k} \right) \quad k \in B
\]

The adding-up restrictions can now be expressed in terms of the elasticities as follows:

\[
\sum_{i \in A} w_i \eta_{ij} + \sum_{k \in B} w_k \rho_{kj} = -w_j \quad j \in A
\]

\[
\sum_{i \in A} w_i \psi_{is} + \sum_{k \in B} w_k \theta_{ks} = -w_s \quad s \in B
\]

\[
\sum_{i \in A} w_i \eta_i + \sum_{k \in B} w_k \theta_k = 1
\]

where \( w_i \) is the share of the \( i \)th commodity. The homogeneity conditions can
also be expressed in terms of elasticities as follows:

\[
\begin{align*}
\sum_{j \in A} \eta_{ij} + \eta_i &= 0 & i & \in A \\
\sum_{j \in A} \rho_{kj} + \theta_k &= 1 & k & \in B
\end{align*}
\]

From duality theory, we know that preferences can be represented by a cost function just as they can be by a utility function. To derive the 'mixed' cost function, we first consider the restricted cost function where the vector of commodities \( x_B \) are treated as 'fixed' (Gorman, 1976). Given these 'fixed' commodities, the consumer allocates his or her income optimally to the other commodities that can be chosen.

\[
C(x_B, p_A, u) = \min_{x_A} \{ p_A' x_A \text{ s.t. } U(x_A, x_B) = u \}
\]

(4)

\( C(.) \) is homogeneous of degree one and concave in \( p_A \) and convex and decreasing in \( x_B \).\(^4\) From the derivative property, the partial derivative of \( C(.) \) with respect to \( p_A \) gives the compensated mixed demands \( x_i^C(x_B, p_A, u) \)

\[
\frac{\partial C}{\partial p_i} = x_i^C(x_B, p_A, u) & \quad i \in A
\]

while the partial derivative with respect to \( x_B \) gives the compensated shadow or 'virtual' price of \( x_B \).

\[
\frac{\partial C}{\partial x_k} = - p_k^C(x_B, p_A, u) & \quad k \in B
\]

These virtual prices, at the market level, can be interpreted as those (market clearing) prices which would support the consumption bundle \( x_B \).

\(^4\)Deaton (1981) formally proves that \( C(.) \) is decreasing and convex in \( x_B \).
given prices $p^c_A$ and income $C(.)$. The negative sign in front of $p^c_k$ is because an increase in $x_k$ holding utility constant would require less of at least one commodity of $x_A$ implying a decrease of $p^c_A \cdot x_A$. The virtual prices themselves are expressed in positive terms. The virtual prices expressed as functions of $x_B$, $p_A$, and $u$ can be considered the compensated price dependent demand of $x_B$. Curvature and symmetry conditions imply that the matrix of partial derivatives $[\partial x_A / \partial p_A]$ is symmetric and negative semi-definite; the matrix of partial derivatives $[\partial p_B / \partial x_B]$ is symmetric and negative definite; and from Young's theorem, $\partial x_f / \partial x_k - \partial^2 C / \partial p_1 \partial x_k - \partial^2 C / \partial x_k \partial p_1 = -\partial p^c_k / \partial p_1$ for all $i \in A, k \in B$. These three conditions imply that the Hessian of the restricted cost function is skew symmetric.

We now define the compensated mixed elasticities as follows:

\[
\eta_{i,j}^c = \left( \frac{\partial x_f}{\partial p_j} \right) \left( \frac{p_j}{x_i} \right) \quad i,j \in A
\]
\[
\phi_{i,k}^c = \left( \frac{\partial x_f}{\partial x_k} \right) \left( \frac{x_k}{x_i} \right) \quad i \in A, k \in B
\]
\[
\rho_{i,k}^c = \left( \frac{\partial p^c_k}{\partial p_1} \right) \left( \frac{p_1}{p_k} \right) \quad i \in A, k \in B
\]
\[
\theta_{k,s}^c = \left( \frac{\partial p^c_k}{\partial x_s} \right) \left( \frac{x_s}{p_k} \right) \quad k,s \in B
\]

Since the restricted cost function is homogeneous of degree one in $p_A$, the compensated demands $x_f^c$ are homogeneous of degree zero in $p_A$, while the compensated (nominal) price dependent demands are homogeneous of degree one in $p_A$. The homogeneity restrictions in terms of the compensated elasticities are:

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5 In relation to consumer demand under rationing (e.g., Neary and Roberts 1980, Deaton 1981, and Howard 1977) this is the situation when the ration 'just' bites, in the sense that the ration levels coincide with the quantities which would have been chosen by an unrationed household facing the same prices and income. Latham (1980) uses a similar shadow price interpretation.
Given the symmetry conditions: \( \sum_{j \in A} \eta_{ij} \rho_{kj} = \sum_{i \in A} \omega_{ij} \eta_{ij} \theta_{ik} \theta_{kj} = w_s \theta_{sk} \), and \( \omega_{ij} \eta_{ij} = \omega_{kj} \rho_{ki} \), the adding-up restrictions can now be expressed in terms of the compensated elasticities:

\[
\begin{align*}
\sum_{i \in A} \omega_{ij} \eta_{ij} &= 0 \\
\sum_{i \in A} \omega_{ij} \psi_{ik} &= -w_k
\end{align*}
\]

We now define the 'mixed' cost function as the sum of the restricted cost function and the value of the 'fixed' commodity bundle evaluated at its virtual price:

\[
\bar{C}(x_B, p_A, u) = C(x_B, p_A, u) + p_B^* \left( x_B, p_A, u \right) x_B
\]

This mixed cost function implies that, if \( p_A \) and \( p_B \) were exogenous, with \( p_B \) equal to the virtual price of the fixed bundle \( x_B \), given income level \( y=\bar{C}(x_B, p_A, u) \), the optimal quantity choices would be precisely \( x_A \) and \( x_B \), the solution vector and the fixed quantity bundle, respectively, of the earlier restricted optimization problem. From (5) we have,

\[
\begin{align*}
\frac{\partial \bar{C}}{\partial p_1} &= x_f^* + \sum_{k \in B} \left( \frac{\partial p_f^*}{\partial p_1} \right) x_k \\
\frac{\partial \bar{C}}{\partial x_k} &= \sum_{s \in B} \left( \frac{\partial p_s^*}{\partial x_k} \right) x_s
\end{align*}
\]

and

\[
\frac{\partial^2 \bar{C}}{\partial x_s \partial p_1} = \frac{\partial^2 \bar{C}}{\partial p_1 \partial x_s} = \frac{\partial x_f^*}{\partial x_s} + \Sigma_{k \in B} \left( \frac{\partial^2 p_f^*}{\partial x_s \partial p_1} \right) + \frac{\partial p_f^*}{\partial p_1}
\]

Let us now consider the identities:

---

6 The mixed cost function \( \bar{C} \) always exists as long as there is some \( i \in A \).
Differentiating these identities and using the relations (6), we have the following Slutsky equations:

\[ \frac{\partial x_f}{\partial p_j} = \frac{\partial x_i}{\partial p_j} + (\frac{\partial x_i}{\partial y})(\frac{\partial x_f}{\partial x_i} + \sum_{k \in B} (\frac{\partial p_k}{\partial p_j}) x_k) \]

\[ \frac{\partial x_f}{\partial x_k} = \frac{\partial x_i}{\partial x_k} + (\frac{\partial x_i}{\partial y})(\frac{\partial x_f}{\partial x_i} + \sum_{s \in B} (\frac{\partial p_s}{\partial x_k}) x_s) \]

\[ \frac{\partial p_k}{\partial p_i} = \frac{\partial p_k}{\partial x_i} + (\frac{\partial p_k}{\partial y})(\frac{\partial x_i}{\partial x_i} + \sum_{s \in B} (\frac{\partial p_s}{\partial p_i}) x_s) \]

\[ \frac{\partial p_k}{\partial x_s} = \frac{\partial p_k}{\partial x_s} + (\frac{\partial p_k}{\partial y})(\sum_{r \in B} (\frac{\partial p_r}{\partial x_s}) x_r) \]

In terms of elasticities, the Slutsky relations are:

\[ \eta_{ij} = \eta_{ij} + \eta_i (w_j + \sum_{k \in B} w_k \rho_{kj}) \quad i,j \in A \quad (7a) \]

\[ \psi_{ik} = \psi_{ik} + \eta_i (\sum_{s \in B} w_s \theta_{ks}) \quad i \in A, k \in B \quad (7b) \]

\[ \rho_{ki} = \rho_{ki} + \rho_k (w_i + \sum_{s \in B} w_s \rho_{si}) \quad i \in A, k \in B \quad (7c) \]

\[ \theta_{ks} = \theta_{ks} + \theta_k (\sum_{r \in B} w_r \theta_{rs}) \quad k,s \in B \quad (7d) \]

As in the case of direct and inverse demands, the compensated elasticities are the sum of the uncompensated elasticities and the income effect. The income effect in the mixed case is the product of the 'mixed' income elasticities and a weighted income share, where the weights are the compensated mixed elasticities themselves.
Model Specification

The theoretical formulation (1) from which mixed demands are derived is not very useful in specifying an empirical model because knowledge of both the direct and the indirect utility functions is required. This rules out using one of the commonly adopted FFFs because they do not have a closed form dual. For example, if one were to specify the direct utility function in terms of the Translog form used by Christensen, Jorgenson, and Lau (1975), there would be no closed form dual functional form that could consistently and simultaneously represent the indirect utility function. The other alternative of specifying a FFF for the 'mixed' utility function as derived in (2) is also not useful since the derivative property (3) cannot be used to derive estimable mixed demands. One possible solution to the specification of a flexible empirical model for mixed demands is to approximate the mixed (market) demands locally at some point using a differential demand system like the Rotterdam model (Barnett 1979).

One criticism that has been raised against the Rotterdam model is that it implies unitary income elasticities (Phlips 1974, p.88). Barnett (1979), however, shows that this is not true. He performs an aggregation analysis based on an infinitesimal (absolute) version of the Rotterdam model for which a closed form dual always exists is the class of self-dual additive preferences for which the corresponding direct utility function can be written in the same mathematical form as the corresponding direct utility function. However, all such preferences (with one exception, see Hicks, 1969) imply unitary income elasticities and are thus very restrictive (Houthakker 1960, Samuelson 1965). The Linear Expenditure System belongs to the generalized Bergson Family for which a closed form indirect utility function exists. For this class of preferences, the Engel curves are linear.
model with random coefficients. The result is that the Rotterdam model
with constant parameters provides a first order Taylor-series approximation
to some theoretical system of equations at the per capita level under
conditions far weaker than those necessary for aggregate integrability. 8
Theil (1980, p.178) points out that the Rotterdam can be viewed as a linear
approximation even without invoking the considerations that Barnett does.
As Deaton and Muellbauer (1980, p.73) point out, unitary income
elasticities are implied only when the differential demands are required to
be consistent demand functions in levels. More recently, Mountain (1988)
showed that the Rotterdam, like other FFFs, is a valid linear approximation
in variable space at the individual consumer level.

Constancy of parameters over the sample period is assumed to
facilitate estimation. In reply to Byron (1984) who states this as a
criticism of the Rotterdam model, Barnett (1984) points out that the
Rotterdam like the Taylor series expansions (FFFs) is after all a local
approximation. In all such models, constancy of parameters is acquired by
the device of evaluating the parameters at a 'point of approximation'. 9
Barnett, however, notes that constancy of parameters acquired in such a
manner are often not acceptable empirically, and is in general a testable
hypothesis. Constancy of the parameters, however, also raises the issue of

8Barnett (1979) dismisses Muellbauer's (1976) class of PIGL preferences
as a highly restrictive case. Keller and Van Driel (1985) however develop
a relative price version of the Rotterdam which has PIGLOG Engel curves and
in which concavity conditions can be implemented.

9This reasoning also applies to Keller and Van Driel's (1985) comment
that the Rotterdam model implies linear Engel curves. This observation
apparently stems from inferring that constant parameters implies constant
marginal budget shares, which as in the case of the LES, implies quasi-
homothetic preferences.
approximation errors. In response to Byron's statement that the Rotterdam possesses errors of the first order, Barnett (1979) points out that the remainder term is of at least second order and cites empirical tests none of which reject the hypothesis of constant parameters. Byron (1984) in a simulation study shows that the Rotterdam mimics the Translog and the LES well under the assumption of constant parameters.¹⁰

Given that the differential demand system provides a flexible approximation to an arbitrary demand system, a differential approximation for mixed demands is now proposed. Totally differentiating the mixed demands \( \bar{x}_A(x_B, p_A, Y) \) and \( \bar{p}_B(x_B, p_A, Y) \) we have the following differential 'mixed' demand system in absolute prices:

\[
\begin{align*}
\text{(8a)} & \quad w_i \delta \ln x_i = w_i \eta_i \delta \ln Y + \sum_j w_{ij} \eta_{ij} \delta \ln p_j + \sum_k w_{ik} \psi_{ik} \delta \ln x_k \\
\text{(8b)} & \quad w_k \delta \ln p_k = w_k \eta_k \delta \ln Y + \sum_j w_{kj} \rho_{kj} \delta \ln p_j + \sum_s w_{ks} \theta_{ks} \delta \ln x_s
\end{align*}
\]

for all \( i, j \in A \), and \( k, s \in B \); where \( Y \) is the total expenditure as before, and \( p \) and \( x \) are the nominal prices and quantities respectively. Using the Slutsky relations (7) of the previous section, and imposing the symmetry restrictions we have:

\[
\begin{align*}
\text{(9a)} & \quad w_i \delta \ln x_i = a_i \delta \ln y + \sum_j (a_{ij} + a_i \sum_k \gamma_{jk}) \delta \ln p_j + \sum_k (\gamma_{ik} - a_i \sum_s \beta_{ks}) \delta \ln x_k \\
\text{(9b)} & \quad w_k \delta \ln p_k = \beta_k \delta \ln y + \sum_j (-\gamma_{jk} + \beta_k \sum_s \gamma_{js}) \delta \ln p_j + \sum_s (\beta_{ks} - \beta_k \sum_t \beta_{ts}) \delta \ln x_s
\end{align*}
\]

¹⁰Barnett (1984) points out that theoretically, the magnitudes of the remainder terms in the Rotterdam and the Translog are not directly comparable since the models possess differently transformed variables. Thus any comparison must be from simulation studies.
where $d\ln y$ is equal to $d\ln Y - \sum_i \omega_i d\ln p_i$, $i=1,2\ldots m$. The parameters are defined by:

\begin{align*}
\alpha_{ij} &= \omega_i \eta_j \\quad \gamma_{ik} = \omega_i \psi_k \\
\beta_{ks} &= \omega_k \delta_s \\
\eta_i &= \omega_i \\
\delta_k &= \omega_k
\end{align*}

The model is linear in the variables but nonlinear in the parameters.

Homogeneity is satisfied when:

\begin{align*}
\sum_j \alpha_{ij} &= 0 \\
\sum_i \gamma_{ik} &= -\omega_k
\end{align*}

The adding-up conditions for the model are:

\begin{align*}
\sum_i \alpha_{ij} + \sum_k \beta_{k} &= 1 \\
\sum_i \alpha_{ij} &= 0 \\
\sum_i \gamma_{ik} &= -\omega_k
\end{align*}

The direct elasticities implied by the mixed system\footnote{The correspondence between the direct and mixed elasticities is given by $e_{AA} = \eta - \psi^{-1} \rho$, $e_{AB} = -\psi^{-1}$, $e_{BA} = -\rho$ and $e_{BB} = \rho^{-1}$. This is obtained by rearranging the mixed demand system in (8) to yield the direct differential system. $\eta, \psi$ and $\theta$ are the matrices of mixed elasticities of size $mm$, $m(m-n)$, $(n-m)xm$ and $(n-m)x(n-m)$ respectively. Dahlgran (1987) uses this identity to derive a mixed differential demand system from the direct Rotterdam.} can be compared to the direct elasticities yielded by the direct Rotterdam which is specified in the absolute price version (Deaton and Muellbauer 1980) as:
\[ w_i \text{dln}x_i = \alpha_i \text{dln}y + \sum_j \alpha_{ij} \text{dln}p_j \]

where the rate of change of real income \( \text{dln}y \) is with respect to all the prices, i.e., \( \text{dln}y = \text{dln}Y - \sum_i w_i \text{dln}p_i \), \( i = 1,2,\ldots,n \); \( \alpha_i = w_i \varepsilon_i \), where \( \varepsilon_i \) is the income elasticity of the \( i \)th good; and \( \alpha_{ij} = w_i \varepsilon_{ij} \), where \( \varepsilon_{ij} \) is the compensated price elasticity. Homogeneity implies that \( \sum_j \alpha_{ij} = 0 \); symmetry implies that \( \alpha_{ij} = \alpha_{ji} \); and adding-up implies that \( \sum_i \alpha_i = 1 \) and \( \sum_j \alpha_{ij} = 0 \).

The differential mixed demand system is also compared to the non-flexible system of mixed demands derived from the Stone-Geary utility function

\[ U(q) = \prod_{1}^{n} \ln (x_i - \gamma_i)^{\beta_i}, \quad \sum_i \beta_i = 1 \]

For the three good case under consideration, the 'mixed'(share dependent) linear expenditure system (LES) is given by:

\[
\begin{align*}
    w_1 &= \pi_1 \gamma_1 + \beta_1 (1-\sum_{i\in A} \pi_1 \gamma_1) (x_3-\gamma_3) / (x_3-\gamma_3+\gamma_3\beta_3) \quad i = 1,2 \\
    w_3 &= x_3 \beta_3 (1-\sum_{i\in A} \pi_1 \gamma_1) / (x_3-\gamma_3+\gamma_3\beta_3)
\end{align*}
\]

The variables subscripted by 1 and 2 stand for beef and pork while the variables subscripted by 3 stand for chicken.

\(^{12}\)\text{dln}y as defined in the direct Rotterdam is also equal to the Divisia quantity index. To see this, totally differentiate the budget constraint to get \( \text{d}Y = \sum_i \pi_i \text{d}x_i + \sum_i x_i \text{d}P_i \) from which \( \text{dln}Y = \sum_i w_i \text{dln}x_i + \sum_i w_i \text{dln}P_i \). The two terms on the right are the Divisia quantity and price indexes.
Quarterly Canadian data on consumption and prices (obtained from Agriculture Canada) for beef, pork and chicken from the first quarter of 1980 to the first quarter of 1990 are used. The quantity data are per capita disappearance (in kilograms) of beef, pork and chicken. The quantities were converted from carcass weights to retail weights using conversion factors supplied by Statistics Canada. The conversion factor for beef was 0.74 from 1980 to 1985 and 0.73 from 1986 to 1990. The conversion factor for pork was 0.77 from 1980 to 1982 and 0.76 from 1983 to 1990. The price data are consumer price indexes (with 1981 as base year) converted to actual prices using survey data obtained from Family Food Expenditure Surveys, Statistics Canada. This was done as follows. From the data on weekly family expenditures and quantities consumed (for all classes and all provinces) for beef, pork and chicken, prices were computed for the three commodities by dividing expenditures by quantities for the years 1974, 1976, 1978, 1982, 1984 and 1986. These prices were regressed (through the origin) on the respective annual consumer price indexes. The raw moment $R^2$ values were over 0.99 for all the three equations\(^\text{13}\) and the regression coefficients were 0.052, 0.037 and 0.029. These estimated coefficients were then used in generating prices for the entire sample period.

Additional (quarterly) data used for the first stage of budget

\(^{13}\)The raw moment $R^2$ reported in SHAZAM for regressions through the origin is computed as $1 - \text{RSS}/\text{TSS}$, where $\text{RSS}$ is the residual sum of squares, while $\text{TSS}$ is the total sum of squares without correcting for the mean.
allocation included total food expenditure and consumer price index for food provided by Agriculture Canada; the consumer price index (CPI) and personal disposable income converted to a per capita basis using quarterly estimates of Canadian population.

Estimation and Results

The discrete formulation of the mixed Rotterdam model in (9) and the direct Rotterdam model are estimated using the nonlinear estimation procedure available in SHAZAM 6.2. Since quarterly data is used, a four-period difference is taken to correct for seasonality. The shares used in multiplying each of the equations are averages of two consecutive quarters (as opposed to contiguous quarters). For example, in the beef equation, corresponding to $\log p_{bt} - \log p_{bt-1} - \log p_{bt-4}$, the share is $(w_{bt-4} + w_{bt})/2$ where $p_{bt}$ and $w_{bt}$ denote the price and expenditure share of beef and $t$ denotes time. The mixed LES model is also estimated using the nonlinear estimation procedure. Symmetry and homogeneity restrictions are maintained and the chicken equation is dropped in the estimation of all the three models. In this analysis it is assumed that the meats group is weakly separable from other commodities in the consumption bundle.

The mixed differential model is estimated with an intercept and correction for first order autocorrelation\(^{14}\), where the coefficient of

\(^{14}\)The model was also estimated with single period differencing but with an AR(4) error process and dummy variables to account for seasonality. This model yielded elasticities close to those of the model with four-period differencing.
correlation was also estimated. The intercept in this differential model is a trend coefficient since the first difference of the trend variable yields a vector of ones. The intercept can be interpreted as change in the expenditure share and from the budget constraint, these share changes must add up to zero.

The estimates of the mixed demand system are presented in Table 1. The coefficients \( \alpha_{11} \) and \( \beta_{33} \) (which are weighted compensated elasticities of beef and chicken, respectively) have the expected negative signs. The income coefficients are positive. The intercepts in the beef and pork equations are negative implying that the budget shares of beef and pork have been falling while the share of chicken has been rising. From the eigenvalues of the sub-matrix of the coefficients of the mixed system concavity conditions were found to be satisfied.

The estimated mixed compensated elasticities obtained by dividing the estimated coefficients by the relevant mean shares, and their asymptotic standard errors, are reported in Table 2. The ratios of the elasticities to their respective standard errors are asymptotically normally distributed. Beef and pork are net substitutes while chicken is a

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15 The model was estimated with seasonal dummies for the quarters. The dummies were found to be insignificant indicating that the four-period differencing was also effective in removing seasonality from the data. A likelihood ratio test showed that the model without dummies could not be rejected at the 5 percent level of significance.

16 The left-hand-side of the \( i^{th} \) (quantity) dependent equation is \( w_i \text{dln} x_i \), or \( (p_i x_i/Y)(dx_i/x_i) \) which can be written as \( (p_i/Y)dx_i \) which is the change in the \( i^{th} \) expenditure share since \( p_i \) and \( Y \) are exogenous.
Table 1. Estimates of the mixed demand system

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.5441</td>
<td>0.0468</td>
<td>11.63</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.2879</td>
<td>0.0419</td>
<td>6.87</td>
</tr>
<tr>
<td>$\alpha_{11}$</td>
<td>-0.0946</td>
<td>0.0276</td>
<td>-3.43</td>
</tr>
<tr>
<td>$\gamma_{13}$</td>
<td>-0.1485</td>
<td>0.0236</td>
<td>-6.31</td>
</tr>
<tr>
<td>$\beta_{33}$</td>
<td>-0.2692</td>
<td>0.0526</td>
<td>-5.12</td>
</tr>
<tr>
<td>rho</td>
<td>0.5236</td>
<td>0.1146</td>
<td>4.57</td>
</tr>
</tbody>
</table>

Trend

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>beef</td>
<td>-0.0083</td>
<td>0.0035</td>
<td>-2.34</td>
</tr>
<tr>
<td>pork</td>
<td>-0.0034</td>
<td>0.0033</td>
<td>-1.02</td>
</tr>
</tbody>
</table>

Log likelihood function: 257.8646
Table 2. Compensated and Marshallian mixed elasticities at the mean

<table>
<thead>
<tr>
<th>Compensated mixed elasticities</th>
<th>P_B</th>
<th>P_P</th>
<th>X_C</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_B</td>
<td>-0.1923</td>
<td>0.1923</td>
<td>-0.3019</td>
</tr>
<tr>
<td></td>
<td>(0.0561)</td>
<td>(0.0561)</td>
<td>(0.0479)</td>
</tr>
<tr>
<td>X_P</td>
<td>0.2929</td>
<td>-0.2929</td>
<td>-0.1129</td>
</tr>
<tr>
<td></td>
<td>(0.0855)</td>
<td>(0.0855)</td>
<td>(0.0729)</td>
</tr>
<tr>
<td>P_C</td>
<td>0.8028</td>
<td>0.1972</td>
<td>-1.4550</td>
</tr>
<tr>
<td></td>
<td>(0.1273)</td>
<td>(0.1273)</td>
<td>(0.2842)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marshallian mixed elasticities</th>
<th>P_B</th>
<th>P_P</th>
<th>X_C</th>
<th>Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_B</td>
<td>-0.9007</td>
<td>-0.2053</td>
<td>-0.0042</td>
<td>1.1059</td>
</tr>
<tr>
<td></td>
<td>(0.0999)</td>
<td>(0.0612)</td>
<td>(0.0731)</td>
<td>(0.0951)</td>
</tr>
<tr>
<td>X_P</td>
<td>-0.2781</td>
<td>-0.6133</td>
<td>0.1269</td>
<td>0.8914</td>
</tr>
<tr>
<td></td>
<td>(0.1309)</td>
<td>(0.0916)</td>
<td>(0.0797)</td>
<td>(0.1298)</td>
</tr>
<tr>
<td>P_C</td>
<td>0.2213</td>
<td>-0.1292</td>
<td>-1.2106</td>
<td>0.9079</td>
</tr>
<tr>
<td></td>
<td>(0.1833)</td>
<td>(0.1108)</td>
<td>(0.2102)</td>
<td>(0.1946)</td>
</tr>
</tbody>
</table>

*a standard errors of the elasticities are reported in parentheses.*
substitute to both beef and pork. For instance, the mixed elasticity \( \psi_{PC} \) of \(-0.1129\) shows that a one percentage increase in the supply of chicken causes about one-tenth of a percent decrease in the consumption of pork. The Marshallian mixed elasticities are also reported in Table 2. Beef and pork are found to be gross complements. Chicken is found to be a substitute for pork but a complement to beef. The own 'quantity' elasticity of chicken is greater than one in absolute value indicating that a one percent rise in the supply of chicken would, ceteris paribus, decrease the price of chicken by more than a percent.

The mixed expenditure elasticities are close to unity for all the three commodities. For beef and pork, they indicate the usual change in consumption of the commodity due to a change in total expenditure. For chicken, however, the expenditure elasticity indicates how much more (or less) consumers (at the market level) are willing to pay for chicken when income increases by one percent. For a normal good, one would expect this elasticity to be positive as seems to be the case for chicken.

To compare the computed mixed elasticities with the more familiar direct elasticities estimated in other studies, the direct compensated Marshallian elasticities and the direct expenditure elasticities are retrieved from the mixed elasticities. Looking at the retrieved Marshallian elasticities in Table 3, we see that the own price elasticities of beef and chicken, at \(-0.9014\) and \(-0.8260\) are higher than that of pork which is \(-0.6269\). The expenditure elasticities of beef and pork are close

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17Note that, in general, substitutability defined in terms of the 'mixed' compensated elasticities need not be equivalent to either p-substitutability defined in terms of the direct system, nor the q-substitutability defined in terms of the inverse system.
Table 3. Direct elasticities at the mean retrieved from the mixed system

Compensated direct elasticities

<table>
<thead>
<tr>
<th></th>
<th>$P_B$</th>
<th>$P_P$</th>
<th>$P_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_B$</td>
<td>-0.3588</td>
<td>0.1514</td>
<td>0.2075</td>
</tr>
<tr>
<td></td>
<td>(0.0859)*</td>
<td>(0.0607)</td>
<td>(0.0553)</td>
</tr>
<tr>
<td>$X_P$</td>
<td>0.2306</td>
<td>-0.3082</td>
<td>0.0776</td>
</tr>
<tr>
<td></td>
<td>(0.0924)</td>
<td>(0.0871)</td>
<td>(0.0505)</td>
</tr>
<tr>
<td>$X_C$</td>
<td>0.5518</td>
<td>0.1355</td>
<td>-0.6873</td>
</tr>
<tr>
<td></td>
<td>(0.1469)</td>
<td>(0.0882)</td>
<td>(0.1343)</td>
</tr>
</tbody>
</table>

Marshallian direct elasticities

<table>
<thead>
<tr>
<th></th>
<th>$P_B$</th>
<th>$P_P$</th>
<th>$P_C$</th>
<th>Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_B$</td>
<td>-0.9014</td>
<td>-0.2048</td>
<td>0.0034</td>
<td>1.1028</td>
</tr>
<tr>
<td></td>
<td>(0.1035)</td>
<td>(0.0628)</td>
<td>(0.0608)</td>
<td>(0.1054)</td>
</tr>
<tr>
<td>$X_P$</td>
<td>-0.2548</td>
<td>-0.6269</td>
<td>-0.1049</td>
<td>0.9866</td>
</tr>
<tr>
<td></td>
<td>(0.1264)</td>
<td>(0.0885)</td>
<td>(0.0601)</td>
<td>(0.1469)</td>
</tr>
<tr>
<td>$X_C$</td>
<td>0.1828</td>
<td>-0.1067</td>
<td>-0.8260</td>
<td>0.7499</td>
</tr>
<tr>
<td></td>
<td>(0.1644)</td>
<td>(0.0899)</td>
<td>(0.1434)</td>
<td>(0.1603)</td>
</tr>
</tbody>
</table>

*Standard errors of the estimates are reported in parenthesis
to unity, while that of chicken is 0.7499. In fact a likelihood ratio test fails to reject the null hypothesis of homotheticity.

These elasticities are compared to the elasticities of the direct Rotterdam whose parameter estimates are presented in Table 4. The model was estimated with an intercept and correction for first order autocorrelation. Once again beef and pork have negative time trends, and chicken, a positive trend. The compensated and Marshallian elasticities from the direct Rotterdam are reported in Table 5.

The beef and pork own price (compensated and Marshallian) and expenditure elasticities from the direct Rotterdam are somewhat similar to the retrieved direct elasticities, but the absolute value of the own price elasticity of chicken can be seen to be much lower in the direct demand system. However, the chicken (own price) elasticity from the direct demand system is comparable to those from other studies on Canadian meat demand. For instance, Young (1987) using single equation models (for unconditional demands) for the period 1967-84 reports own price elasticities ranging from -0.31 to -0.48 for beef, -0.55 to -0.67 for pork and -0.22 to -0.47 for chicken. The income elasticities ranged from 0.14 to 0.91 for beef, 0.21 to 0.39 for pork and 0.26 to 0.28 for chicken.

However, the higher own-price elasticity of chicken is supported by Thurman (1986) and Shonkwiler and Taylor (1984) who show that a least squares estimation of quantity dependent demand system underestimates demand elasticities when prices are in fact endogenous. It may, therefore, be possible that the direct elasticities from the mixed system reflect the

18 Homotheticity is rejected in the direct system both at the one and five percent significant levels.
Table 4. Estimates of the direct demand system

<table>
<thead>
<tr>
<th>Equation</th>
<th>dlny</th>
<th>dlnP_B</th>
<th>dlnP_P</th>
<th>dlnP_C</th>
<th>Intercept</th>
<th>Rho^a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beef</td>
<td>0.5696</td>
<td>-0.1314</td>
<td>0.0858</td>
<td>0.0456</td>
<td>-0.0069</td>
<td>0.4007</td>
</tr>
<tr>
<td></td>
<td>(0.0452)</td>
<td>(0.0283)</td>
<td>(0.0232)</td>
<td>(0.0146)</td>
<td>(0.0024)</td>
<td>(0.1139)</td>
</tr>
<tr>
<td>Pork</td>
<td>0.3276</td>
<td>0.0858</td>
<td>-0.0992</td>
<td>0.0135</td>
<td>-0.0009</td>
<td>0.4007</td>
</tr>
<tr>
<td></td>
<td>(0.0474)</td>
<td>(0.0232)</td>
<td>(0.0241)</td>
<td>(0.0107)</td>
<td>(0.0024)</td>
<td>(0.1139)</td>
</tr>
</tbody>
</table>

^aRho=coefficient of autocorrelation
^b=coefficients
^c=standard errors
^d=t='t' ratios
Table 5. Compensated and Marshallian elasticities
at the mean from the direct system

Compensated direct elasticities

<table>
<thead>
<tr>
<th></th>
<th>$P_B$</th>
<th>$P_P$</th>
<th>$P_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_B$</td>
<td>-0.2669</td>
<td>0.1743</td>
<td>0.0927</td>
</tr>
<tr>
<td></td>
<td>(0.0576)$^a$</td>
<td>(0.0471)</td>
<td>(0.0297)</td>
</tr>
<tr>
<td>$X_P$</td>
<td>0.2655</td>
<td>-0.3072</td>
<td>0.0417</td>
</tr>
<tr>
<td></td>
<td>(0.0718)</td>
<td>(0.0745)</td>
<td>(0.0332)</td>
</tr>
<tr>
<td>$X_C$</td>
<td>0.2465</td>
<td>0.0728</td>
<td>-0.3193</td>
</tr>
<tr>
<td></td>
<td>(0.0791)</td>
<td>(0.0579)</td>
<td>(0.0608)</td>
</tr>
</tbody>
</table>

Marshallian direct elasticities

<table>
<thead>
<tr>
<th></th>
<th>$P_B$</th>
<th>$P_P$</th>
<th>$P_C$</th>
<th>Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_B$</td>
<td>-0.8366</td>
<td>-0.1996</td>
<td>-0.1215</td>
<td>1.1576</td>
</tr>
<tr>
<td></td>
<td>(0.0819)</td>
<td>(0.0500)</td>
<td>(0.0334)</td>
<td>(0.0919)</td>
</tr>
<tr>
<td>$X_P$</td>
<td>-0.2335</td>
<td>-0.6348</td>
<td>-0.1459</td>
<td>1.0142</td>
</tr>
<tr>
<td></td>
<td>(0.1114)</td>
<td>(0.0799)</td>
<td>(0.0435)</td>
<td>(0.1468)</td>
</tr>
<tr>
<td>$X_C$</td>
<td>-0.0270</td>
<td>-0.1068</td>
<td>-0.4221</td>
<td>0.5559</td>
</tr>
<tr>
<td></td>
<td>(0.1047)</td>
<td>(0.0630)</td>
<td>(0.0644)</td>
<td>(0.1178)</td>
</tr>
</tbody>
</table>

$^a$Standard errors of the elasticities are reported in parenthesis.
true elasticities implying that the demand for chicken in the Canadian market is more elastic than suggested by previous studies.

The mixed LES is estimated with a time trend and dummy variables to correct for seasonality. We can now compare the performance of this non-flexible functional form to the differential approximation of mixed demands. The estimates of the mixed LES are reported in Table 6 below.

<table>
<thead>
<tr>
<th>equation</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beef</td>
<td>-0.0043</td>
<td>0.0196</td>
<td>0.0209</td>
<td>-0.0022</td>
</tr>
<tr>
<td></td>
<td>(-1.32)</td>
<td>(5.19)</td>
<td>(5.60)</td>
<td>(-11.21)</td>
</tr>
<tr>
<td>Pork</td>
<td>-0.0059</td>
<td>-0.0315</td>
<td>-0.0309</td>
<td>-0.0002</td>
</tr>
<tr>
<td></td>
<td>(-1.99)</td>
<td>(-9.60)</td>
<td>(-9.43)</td>
<td>(-1.43)</td>
</tr>
</tbody>
</table>

\(^a\)figures in parenthesis are 't' ratios

Once again the time trend in the beef and pork equations are negative. The estimated $\gamma$'s are less than the respective quantities for all the observations indicating that concavity conditions are not violated. The mixed Marshallian and expenditure elasticities are presented in Table 7 along with the retrieved direct elasticities. Except for the mixed income
Table 7. Mixed elasticities and retrieved elasticities from the mixed LES

### LES Marshallian mixed elasticities

<table>
<thead>
<tr>
<th></th>
<th>$P_B$</th>
<th>$P_F$</th>
<th>$X_C$</th>
<th>Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_B$</td>
<td>-0.8958</td>
<td>-0.1709</td>
<td>0.0281</td>
<td>1.0667</td>
</tr>
<tr>
<td></td>
<td>(0.0735)</td>
<td>(0.0283)</td>
<td>(0.0289)</td>
<td>(0.1004)</td>
</tr>
<tr>
<td>$X_P$</td>
<td>-0.0954</td>
<td>-0.6527</td>
<td>0.0197</td>
<td>0.7482</td>
</tr>
<tr>
<td></td>
<td>(0.0671)</td>
<td>(0.0565)</td>
<td>(0.0204)</td>
<td>(0.0985)</td>
</tr>
<tr>
<td>$P_C$</td>
<td>-0.1791</td>
<td>-0.2250</td>
<td>-1.1718</td>
<td>1.4042</td>
</tr>
<tr>
<td></td>
<td>(0.1419)</td>
<td>(0.0439)</td>
<td>(0.1259)</td>
<td>(0.1819)</td>
</tr>
</tbody>
</table>

### LES retrieved direct Marshallian elasticities

<table>
<thead>
<tr>
<th></th>
<th>$P_B$</th>
<th>$P_F$</th>
<th>$P_C$</th>
<th>Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_B$</td>
<td>-0.9001</td>
<td>-0.1763</td>
<td>-0.0239</td>
<td>1.1004</td>
</tr>
<tr>
<td></td>
<td>(0.0712)</td>
<td>(0.0294)</td>
<td>(0.0221)</td>
<td>(0.1151)</td>
</tr>
<tr>
<td>$X_P$</td>
<td>-0.0985</td>
<td>-0.6565</td>
<td>-0.0168</td>
<td>0.7718</td>
</tr>
<tr>
<td></td>
<td>(0.0693)</td>
<td>(0.0523)</td>
<td>(0.0156)</td>
<td>(0.1063)</td>
</tr>
<tr>
<td>$X_C$</td>
<td>-0.1529</td>
<td>-0.1920</td>
<td>-0.8534</td>
<td>1.1983</td>
</tr>
<tr>
<td></td>
<td>(0.1104)</td>
<td>(0.0480)</td>
<td>(0.0917)</td>
<td>(0.2387)</td>
</tr>
</tbody>
</table>

*standard errors of the elasticities are reported in parentheses.*
elasticity of chicken from the mixed LES which is higher, the LES mixed elasticities are comparable to those of the mixed differential model. Looking at the retrieved elasticities, we see that once again the income elasticity of chicken is higher than before while the own price elasticities are comparable to those retrieved from the mixed differential system.

**First stage of budget allocation**

The mixed and direct elasticities reported and discussed are 'conditional' elasticities, i.e., conditional on the first stage of budget allocation where expenditure shares among various groups of commodities (like the meat group) are optimally determined. For the typical case of the individual consumer who makes optimal quantity decisions given prices, when the utility function is weakly separable between the groups of commodities (and if the Engel curves within each sub-group utility are linear) the first stage budgeting is possible using group quantity and price indices and total expenditure on all groups. When sub-group utility maximization involves choosing optimal quantities (prices) within groups, the resulting first stage would be a direct (inverse) demand system for the first stage involving group price (quantity) indices and total expenditure. If optimal quantity decisions are made for some groups, and optimal price decisions for the others, the first stage would in fact be a 'mixed' demand system involving both quantity and price indices as

---

19 The Rotterdam model can be used to approximate the first stage of budget allocation where optimal group expenditures are functions of total expenditures and two sets of price indices for each group, assuming only weak separability (Deaton and Muellbauer 1980).
In the present case of the mixed meat demand system, the first stage of budget allocation to the meats group is given by

\[ Y_M = f(p_B, p_P, x_C, p_{OF}, p_G, Y) \]  

This formulation implies that the utility function is assumed to be weakly separable in the meats (M) group and homothetically weakly separable in the groups 'other food' (OF) and 'all other goods' (G). Hence price indices are used for these groups (\( p_{OF} \) and \( p_G \)) while actual prices for beef and pork (\( p_B \) and \( p_P \)) and actual consumption quantities of chicken (\( x_C \)) are used. \( Y \) now stands for total disposable income. \( Y_M \) the expenditure on meats is homogeneous of degree one in prices and total income \( Y \).

The price index for 'other food' is calculated as follows. Let the 'food' group consist of beef (B), pork (P), chicken (C) and 'other food' (OF). The Laspeyres price index for food is then given by

\[ P_{Ft} = \frac{\sum_i P_{it}x_{i0}}{\sum_i P_{i0}x_{i0}} \text{ } i=B,P,C,OF \]

The subscript 't' indexes time, and 'o' the base period. The denominator \( \sum_i P_{i0}x_{i0} \) is the food expenditure in the base period. Multiplying and dividing the numerator by \( p_{i0} \), we have

\[ P_{Ft} = \sum_i \left( \frac{p_{it}}{p_{i0}} \right) S_{i0} \text{ } i=B,P,C,OF \]

where \( S_{i0} \) is the budget share of the \( i^{th} \) commodity in total food expenditure in the base period. Using quarterly data on total food expenditure and the consumer price index for food, the price index for 'other food' is generated as
The price index for 'all other goods' \( G \) is similarly generated using data on personal disposable income, total expenditure on food, price index for food and the consumer price index, CPI,

\[
P_G = \frac{(P_{Gt}/P_{Go}) - (CPI - (P_{Ft}/P_{Fo})S_{Fo})}{S_{Go}}
\]

where \( S_{Fo} \) and \( S_{Go} \) are budget shares of the food and non-food groups in total disposable income in the base period.

The double log form of equation (10) was estimated with the homogeneity condition imposed and a trend coefficient included. The estimated equation showed a fairly good fit with an \( R^2 \) of 0.952. The elasticities of meat expenditure with respect to prices of 'other food' and 'all other goods' were 0.7599 and -0.5189 implying that the meat group and 'other food' are substitutes while the non-food and meat groups are complements. The elasticities of meat expenditure with respect to prices of beef, pork, quantity of chicken\(^\text{20} \) and total disposable income were respectively 0.4839, 0.1973, 0.0257 and 0.0777. Using these, the unconditional mixed elasticities were computed for beef, pork and chicken and are presented below in Table 8.

\(^\text{20} \)Although the partial derivative of the mixed cost function with respect to the quantity of chicken can be expected to be negative (from 6b) there is no reason for the elasticity of \( Y_M \) with respect to \( X_C \) to be negative. Differentiating the expression \( Y_M = p_BX_B(p_B,p_F,X_C,p_{OF},p_G,Y) + p_PX_P(.) + X_{CPG}(.) \), we see that this elasticity can be of either sign.
Table 8. Unconditional mixed elasticities

<table>
<thead>
<tr>
<th></th>
<th>$P_B$</th>
<th>$P_P$</th>
<th>$X_C$</th>
<th>Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_B$</td>
<td>-0.3656</td>
<td>-0.0129</td>
<td>0.0242</td>
<td>0.0859</td>
</tr>
<tr>
<td>$X_P$</td>
<td>0.1532</td>
<td>-0.4374</td>
<td>0.1498</td>
<td>0.0693</td>
</tr>
<tr>
<td>$P_C$</td>
<td>0.2180</td>
<td>0.0499</td>
<td>-1.1873</td>
<td>0.0705</td>
</tr>
</tbody>
</table>

These are unconditional mixed elasticities and cannot be compared to the unconditional direct elasticities from other studies.21

Summary and Conclusions

A useful empirical framework is developed in this paper to study the case of mixed demands, where, at the market level, quantities (demanded) of some commodities are optimally determined given their prices while for the other commodities, the prices are optimally determined given their quantities.

A 'mixed' cost function is derived using the concept of 'virtual' prices. Adding-up and homogeneity conditions are derived in elasticity terms. The equivalent of the Slutsky equation in terms of elasticities is also derived for the mixed demands. These relations are used to specify a

21Direct unconditional elasticities cannot be retrieved from this matrix since it is only a part of the full matrix of unconditional mixed elasticities.
Rotterdam-type differential model for mixed demands.

The empirical context is provided by the Canadian market for meats where the chicken market is subjected to 'supply management', while there is free trade in beef and pork with the U.S. Quarterly Canadian data for beef, pork and chicken are used to estimate the model. The estimated mixed elasticities are compared to the elasticities of the mixed LES model that is also estimated. Direct compensated elasticities are retrieved from the estimated mixed compensated elasticities which are compared to direct Rotterdam model that is also estimated. Finally, the first stage of budget allocation to the meats group is estimated, and the unconditional mixed demands are computed.

The direct elasticities from the direct system are close to estimates from other studies of the Canadian meat demand and to those from the mixed system except for the own price elasticity of chicken which is higher in the mixed demand system indicating that demand for chicken is elastic in the Canadian market for meats.
References


Hicks, J.R. "Direct and Indirect Additivity." Econometrica, 37(1969):353-


SECTION III. FLEXIBLE FUNCTIONAL FORMS AND WEAK SEPARABILITY:
A MONTE CARLO STUDY

Introduction

Separability of some groups of commodities from others in a utility (or production) function is a commonly maintained hypothesis in a lot of empirical work. This is because the concept of separability allows a significant advantage in modeling what might otherwise be an intractable problem. For example, weak separability of the utility function is both a necessary and sufficient condition (Deaton and Muellbauer 1980) for the existence of conditional demand functions such as the meat demands of Section II. In addition, homothetic weak separability allows decentralization of the budget allocation so that the first stage of expenditure allocation to various groups can be carried out using consistent group price and quantity indices. Moreover, empirical and theoretical demand studies abstract from some allocation problems like labor-leisure or intertemporal allocation decisions. These abstractions can also be rationalized by appropriate separability restrictions.

Separability also has implications for the functional structure of dual representations of preferences. Thus homothetic weak separability also implies indirect homothetic weak separability. Another interesting result is that simultaneous direct and indirect strong separability (additivity) of the utility function implies unitary income elasticities
Typically, separability is maintained in most empirical work. It is, however, a strong maintained hypothesis and should ideally be tested. Since Flexible Functional Forms (FFFs) have widely replaced the use of rigid functions in empirical analysis, it is of interest to test separability using FFFs. However, as Blackorby, Primont, and Russel (BPR 1978) have shown, most of the commonly used FFFs are 'separability-inflexible', i.e., the FFFs lose their flexibility when (global) separability is imposed on them. The alternative to imposing separability globally is to impose it locally at a point (Denny and Fuss 1977, Jorgensen and Lau 1975). Testing separability however remains a problem, first, because of the functional specification and second, because of the testing procedures used.

Recently, Moschini (1990) proposed a Flexible Model that is globally separable but is not nested in the general, non-separable model. This section of the dissertation presents a Monte Carlo study of the abilities of the above globally separable model and the locally separable model in providing correct inferences about separability. The data used is generated through the (globally) separable W-S branch utility function used.

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There is one exception to this result. Hicks (1969) shows that the general form of the exception is when the utility function takes the form $U = u(x_1) + \alpha_2 \ln x_2 + \ldots + \alpha_n \ln x_n$, where $u$ is any well behaved utility function of the single quantity $x_1$. For this form alone, direct and indirect additivity does not imply unitary income elasticities.

Woodland (1979) avoided the inflexibility problem for the special case of constant returns to scale and one separable input on the production side. More recently, Pope and Hallam (1988) proposed rank tests of separability that can be implemented using standard nested hypothesis testing procedures.
by Barnett and Choi (1989). Tests of local separability are conducted using the Likelihood ratio test while the non-nested hypothesis tests proposed by Davidson and Mackinnon (1982) are used to test for global separability.

This section is organized as follows. First, the concept of separability is briefly reviewed. The problem of 'separability-inflexibility' is discussed and models under consideration are then presented. This is followed by a description of the data generating process. The testing procedures are explained followed by a discussion of the results. The concluding sub-section contains a summary and conclusions.

Separability

Separability can be characterized in terms of preference ordering (Deaton and Muellbauer 1980). 'Conditional ordering' on a sub-group (vector) of commodities $x_A$ is defined as the ordering on $x_A$ given arbitrary fixed quantities of commodities outside the group. When the conditional ordering on the goods in the group is independent of consumption levels outside the group, the group is said to be separable and can be represented by a sub-utility function. When the whole commodity vector ($x$) can be partitioned into $G$ such groups, we have direct weak separability which takes the form (Goldman and Uzawa 1964):

$$U = f(U_A(x_A), U_B(x_B), \ldots, U_G(x_G))$$

where $x_A$, $x_B$, ..., $x_G$ are subvectors and $f$ is some function which is
increasing in all its arguments. It is obvious that weak separability has implications for commodity groupings, a notion that is particularly important in empirical analysis. A separable utility function of this form satisfies the original Leontief–Sono separability condition:

\[ \frac{\partial (U_i(x)/U_j(x))}{\partial x_k} = 0 \quad \text{for all } i, j \in A \text{ and } k \in B \]  

(1a)

where \( A \) and \( B \) are any two sub-groups. An alternative, differential formulation of the above condition is:

\[ \frac{U_{1k}(x)}{U_1(x)} = \frac{U_{jk}(x)}{U_j(x)} \]  

(1b)

where \( U_1 \) is the partial derivative of \( U \) with respect to the \( i \)th variable and \( U_{1k} \) is the cross partial derivative.

An immediate implication of direct weak separability is the existence of conditional demands where the demand for any good depends on prices within the group and the total group expenditure, i.e., \( x_i(p_A, y_A) \) where \( i \in A \). Thus the decision making process of the consumer can be broken into sequential steps for analytic purposes. In the first stage, expenditure allocation is made between groups, while in the second stage, allocation is made between commodities in a group (Strotz 1957, Gorman 1959).

Separability also has consequences on the degree of (Hicksian) substitutability between goods in different groups. Consider the change in the consumption of \( x_i, i \in A \), due to a change in \( p_j, j \in B \), holding utility

---

3 An alternate rationale for commodity aggregation is that provided by the composite commodity theorem by which a group of commodities can be treated as a single good if the corresponding prices move proportionately. However, by definition, the usefulness of this theorem is limited for most empirical applications.
constant. This effect can be only through the group expenditure, $y_A$:

$$s_{ij} = \left( \frac{\partial x_i}{\partial y_A} \right) \left( \frac{\partial y_A}{\partial p_j} \right) |_{U=\text{const}}$$

It can be shown that (Goldman and Uzawa 1964)

$$s_{ji} - s_{ij} = \mu_{AB} \left( \frac{\partial x_i}{\partial y} \right) \left( \frac{\partial x_j}{\partial y} \right)$$

where $\mu_{AB}$ is a constant that is independent of the $i$ and $j$. The above expression which summarizes the interrelation between the two groups is both a necessary and sufficient condition for weak separability.\(^4\)

Separability has also been defined on the dual representations of preferences. For example, implicit or quasi-separability is weak separability defined on the cost function. This is identical to separability defined on the distance function. A stronger definition of separability is that of strong separability where the sub-utility functions are additively separable from each other. While this condition is sufficient for all the implications for aggregation (over commodities) and two-stage budgeting that follow from weak separability, it is not necessary. The differential implications of strong separability are that conditions (1) hold for $i \in A$, $j \in B$ and $k \in A, B$. In terms of the restriction (2) on elasticities, strong separability implies that the constant $\mu_{AB}$ is independent of the groups to which $i$ and $j$ belong and is thus equal to some constant $\mu$ for any pair of groups.

\(^4\)BPR (1978) give more general definitions of separability, including that of asymmetric separability which do not require differentiability.
Separability and Flexible Functional Forms

A FFF can be interpreted in two ways. It can be viewed as a local approximation at a point as afforded by any of the FFFs, or it can be interpreted as an exact representation of the true function. If the former view is taken, separability can be imposed (and tested) locally at the point of approximation. The idea is that the approximation to a separable function need not itself be separable although it will satisfy the differential implication of separability at the point of approximation. If the latter interpretation is taken, the FFF must be globally separable. As BPR (1978) show, this narrows down the possible functional structures that the macro and the aggregator functions simultaneously take. For instance, if the macro function is a Translog in the aggregators, the aggregators are themselves Cobb-Douglas. The aggregators themselves can be Translog if and only if the macro function is a Cobb-Douglas in the aggregators.\(^5\)

Clearly, BPR require that the separable model be parametrically nested in the unrestricted model.

Moschini (1990) suggests that such a requirement may not be necessary and proposes that both the macro and the aggregator functions be modeled to the same degree of approximation such that the FFF criteria is satisfied.\(^6\) To this end, a macro function that is a Translog in the aggregators is

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\(^5\)BPR (1978 p.301) show that the generalized quadratic is separability-inflexible from which the inflexibility of other commonly used FFF's follows.

\(^6\)Along a similar vein of thought Blackorby, Schworm and Fisher (1986) present a procedure (using a generalized symmetric Barnett function) for generating functional forms that remain flexible when separability is imposed.
proposed, where the aggregators are themselves Translogs.

This analysis looks at homothetic separability for the case of three goods. In general, separability of the utility function does not imply nor is implied by the separability of the indirect utility function except in the case of homothetic separability (BPR 1978 p.89). Hence homothetic separability of the utility function is equivalent to the homothetic separability of the indirect utility function in the corresponding variables. In terms of the indirect utility function, the structure of preferences to be tested is

\[ V(V_A(\pi_1, \pi_2), \pi_3) \]

where \( V_A \) is the aggregator that is homogeneous in its arguments.

The Translog specification for the three good indirect utility function is given by

\[ \ln V = \alpha_0 + \Sigma_1 \alpha_1 \ln \pi_1 + \frac{1}{2} \Sigma_1 \Sigma_j \alpha_{1j} \ln \pi_1 \ln \pi_j \]

for all \( i, j = 1, 2, 3 \). The \( \pi_i \)'s are the normalized prices \( p_i/y \) where \( y \) is the total expenditure. Roy's identity yields the following Marshallian demand functions in share form.

\[ w_i = \frac{\alpha_i + \Sigma_j \alpha_{ij} \ln \pi_j}{\Sigma_i \alpha_i + \Sigma_j \Sigma_i \alpha_{ij} \ln \pi_j} \quad i = 1, 2, 3. \]  

(4a)

Since the share equations are homogeneous of degree zero in all the parameters the normalization \( \Sigma_i \alpha_i = 1 \) is adopted. The homogeneity
restrictions $\Sigma_i \alpha_{ij} = 0$ are imposed in addition to the following separability restriction equivalent to (1) for the translog specification (3):

$$\alpha_1 \alpha_{23} = \alpha_2 \alpha_{13} \quad (4b)$$

Since separability imposed on a utility function that is homogeneous implies homothetic separability (Denny and Fuss 1977), the resulting demand functions represent a homothetically separable utility function. Symmetry is maintained. The symmetry and homogeneity restrictions imply that the denominator of (4a) simplifies to unity. The total number of parameters in this system is now four.

The globally separable model (Moschini 1990) is given by

$$\ln V^* = \alpha_\sigma + \alpha_A \ln V_A^* + \alpha_3 \ln \pi_3 + \frac{1}{2} \alpha_{33} (\ln \pi_3)^2$$

$$+ (1/2) \alpha_{AA} (\ln V_A^*)^2 + \alpha_{3A} \ln \pi_3 \ln V_A^*$$

$$\ln V_A^* = \beta_\sigma + \Sigma_i \beta_1 \ln \pi_i + (1/2) \Sigma_i \Sigma_j \beta_{1j} \ln \pi_i \ln \pi_j$$

where $\Sigma_i \beta_{1j} = \Sigma_j \beta_{1i} = 0$ because of the homogeneity of the aggregator $V_A^*$ in the $\pi_i$'s. The normalization $\Sigma_i \beta_i = 1$ is made. Since the parameter $\beta_\sigma$ is not estimable from the data, we set $\beta_\sigma = 0$ which means that $V_A$ is defined up to a multiplicative factor. The usual symmetry conditions $\alpha_{3A} = \alpha_{A3}$ and $\beta_{12} = \beta_{21}$ apply. The Marshallian demands from this model are

$$w_1 = (\alpha_A + \alpha_{AA} \ln V_A^* + \alpha_{3A} \ln \pi_3)(\beta_1 + \beta_{11} \ln \pi_1 + \beta_{12} \ln \pi_2) / D \quad (5a)$$

$$w_2 = (\alpha_A + \alpha_{AA} \ln V_A^* + \alpha_{3A} \ln \pi_3)(\beta_2 + \beta_{12} \ln \pi_1 + \beta_{22} \ln \pi_2) / D \quad (5b)$$
$$w_3 = (\alpha_3 + \alpha_{3A} \ln V_A^* + \alpha_{33} \ln \pi_3) / D$$  \hspace{1cm} (5c)$$

where $D = \alpha_A + \alpha_3 + (\alpha_{AA} + \alpha_{3A}) \ln V_A^* + (\alpha_{3A} + \alpha_{33}) \ln \pi_3$. The adding-up restriction for this system of equations is given by $\alpha_A + \alpha_3 = 1$. As in the case of the locally separable model, homogeneity is imposed on the macro function so that $\alpha_{AA} = \alpha_{33}$ and $\alpha_{AA} = -\alpha_{3A}$ and once again the denominator reduces to unity. The number of parameters to be estimated in this model is also equal to four.

Neither of the models (4) and (5) is nested in the other. Each of the models constitutes a specification for the null hypothesis of separability and can be tested against the alternative hypothesis that the utility function has the general form $U(\pi_1, \pi_2, \pi_3)$ whose Translog approximation is given by (3). The locally separable model is nested in (3) while the globally separable model is not. It is now interesting to see which specification of separability provides a stronger inference about separability.

**Data Generation**

We use Barnett's (1977) homothetic W-S branch utility tree as the underlying model that generates the Monte Carlo data (as in Barnett and Choi 1989). In this model both the macro and the aggregator functions are generalized quadratic mean of order $\rho$ which is of the following form

$$U(q_1, \ldots, q_m) = A \left( \Sigma_i \Sigma_j B_{ij} q_i^{\rho} q_j^{\rho} \right)^{1/2\rho}$$
where $\rho < 1$; $B_{ij} > 0$ for all $i,j$; $\sum_i \sum_j B_{ij} = 1$; $B_{ii} = B_{jj}$ for all $i = j$; and $A > 0$. These conditions ensure the monotonicity and quasi-concavity of the function.

The generalized quadratic mean of order $\rho$ form of the direct utility function is

$$U = A \left( B_{11} q^{2\rho} + 2B_{12} q^\rho x_3^\rho + B_{22} x_3^{2\rho} \right)^{1/2\rho}$$

where

$$q = (A_{11} x_1^{2\delta} + 2A_{12} x_1^\delta x_2^\delta + A_{22} x_2^{2\delta})^{1/2\delta}$$

Without loss of generality, the parameter $A$ can be set equal to one. The elasticity of substitution takes the form,

$$\sigma(q, x_3) = 1/(1 - \rho + R)$$

where $R = -\rho (B_{11} B_{22} - B_{12}^2)/(B_{11} Q^\rho + B_{12})(B_{12} + B_{22} Q^\rho)$ and $Q = x_3/q$. The elasticity of substitution formula is homogeneous of degree zero in the quantities and can also be applied to the aggregator. The generalized quadratic mean of order $\rho$ is by construction homogeneous in its arguments. Hence as in the case of the models to be estimated, the above model is more restrictive than homothetic separability since the macro function is also forced to be homothetic.

Given that there are not more than two groups and two commodities in each of the two groups, the W-S branch utility model is a flexible functional form and thus can achieve arbitrary elasticities at any particular data point (Caves and Christensen 1980, Barnett and Lee 1985), i.e. for any arbitrary set of elasticities, given a data point we can always solve for the parameters of the flexible functional form.
Following Barnett and Choi (1989) we solve for the parameters at the data point (x) = 20, (p) = 1 (and hence y, the total expenditure is 60) for a certain subset of elasticities. Because of homothetic separability there are only two independent elasticities, $\sigma_{12}$ and $\sigma_{13}$, where $\sigma_{13} = \sigma_{23}$. The values of these elasticities are varied for the different experiments. The parameters are solved for, using the three inverse demands derived from the above model with the restrictions given below. The inverse demands are,

$$
\begin{align*}
    p_1/y &= \frac{(B_{11} q^2 - 2 \delta + B_{12} x_3 \rho - 2) (A_{11} x_1^2 \delta - 1 + A_{12} x_1 \delta - 1 - x_2 \delta)}{D} \\
    p_2/y &= \frac{(B_{11} q^2 - 2 \delta + B_{12} x_3) \rho - 2) (A_{12} x_1 \delta - 1 + A_{22} x_2 \delta - 1)}{D} \\
    p_3/y &= \frac{(B_{12} q^2 x_3 \rho - 1 + B_{22} x_3 \rho - 1)}{D}
\end{align*}
$$

where the denominator

$$
D = (B_{12} q^2 x_3 \rho + B_{22} x_3 \rho) + (B_{11} q^2 + B_{12} x_3 q^2)\rho.
$$

The restrictions on the parameters are

$$
\begin{align*}
    A_{12} &= 0.1 \\
    B_{12} &= 0.1 \\
    A_{11} + A_{12} + A_{22} &= 1 \\
    B_{11} + B_{12} + B_{22} &= 1
\end{align*}
$$

$$
\begin{align*}
    \frac{1}{\sigma_{12}} - 1 &= \frac{(2A_{11}^2 - 1.6 A_{11} - 0.08)}{(0.8 A_{11} - A_{11}^2 + 0.09)} \cdot \delta \\
    \frac{1}{\sigma_{13}} - 1 &= \frac{(2B_{11}^2 - 1.6 B_{11} - 0.08)}{(0.8 B_{11} - B_{11}^2 + 0.09)} \cdot \rho
\end{align*}
$$

(7) and (8) are restrictions assumed in addition to the regularity conditions (9) and (10) on the parameters. (11) and (12) are the
elasticity restrictions where the pre-selected values of the elasticities appear on the left-hand-side.

There are, in all, eight independent equations (only two of the three demand functions are independent because of the adding-up condition) in eight unknowns. In solving for the parameters at the stated data point, the following strategy is adopted. Dividing equation (6a) by (6b) and using (7) and (9), we solve for $A_{11}$ and $A_{22}$. From equation (11) we get $\delta$.\(^7\) Now $B_{11}$ can be obtained from either of the two inverse demands (6a, b or $B_{22}$ from 6c) and $\rho$ from (12).

The first 60 observations on the quantities and expenditure for the three commodity groups, perishables, semi-durables and services from Barnett (1981) are used. This data is normalized to yield the data point used above at the median. Using the parameter values, prices are then generated. Finally, noise is added to the quantity data without altering the total expenditure as in Barnett and Choi (1989). This is done as follows. Two vectors $z_1$ and $z_2$ are drawn from the normal distribution with variance 0.25. The new quantity series (say $X_1$, $X_2$ and $X_3$) were generated as follows.

\[
\begin{align*}
X_1 &= x_1 + z_1 \\
X_2 &= x_2 - \left( s_{2p} x_1 \right)/p_2 + z_2 \\
X_3 &= \left( y-p_1 x_1-p_2 x_2 \right)/p_3
\end{align*}
\]

\(^7\)When $\sigma_{12} = 1$, $\delta = 0$, the limiting form of $q$ as $\delta \to 0$ is used. This is given by $\ln q = (A_{11}+A_{12})\ln x_1 + (A_{12}+A_{22})\ln x_2$. However, when $\sigma_{13} = 1$, the condition $\rho = 0$ is imposed on the inverse demands. The resulting inverse demands are identical to those derived from the limiting form of the macro function as $\rho \to 0$.\)
where \( s_2 = \frac{p_2x_2}{(p_2x_2 + p_3x_3)} \). The various models are then estimated and the separability tests conducted. This step is repeated 100 times. Hence there are 100 data sets for every set of elasticities chosen. The random number generator is initialized with the same number for all the experiments so that the noise added across the experiments is the same.

### Tests of Separability

For the chosen combinations of elasticities, two sets of experiments are performed. In the first, the locally separable models are tested against the unrestricted (non-separable) homothetic translog using the asymptotic log-likelihood ratio test statistic

\[
LR = 2 \log L_1 - 2 \log L_0
\]

which has a \( \chi^2 \) distribution under the null with one (the number of restrictions) degree of freedom. \( L_1 \) and \( L_0 \) are the likelihood function values of the model under the alternate and null hypotheses respectively. In the second set, the globally separable models are tested against the unrestricted homothetic translog functions using non-nested tests.

An econometric model, \( H_0 \) is said to be nested within an alternative model \( H_1 \) if \( H_1 \) can be reduced to \( H_0 \) by imposing one or more restrictions on

---

\(^8\)Noise added to quantities in this way implies nonzero covariance of error terms in the share dependent equations. Specifically, if \( e_1 \) and \( e_2 \) are the error terms in the share equations of goods 1 and 2, the \( \text{var}(e_1) = 0.25(\Sigma t \pi_1^2 t) \), \( \text{var}(e_2) = 0.25 \Sigma t (\pi_2^2 t + \pi_2^2 t) \), and \( \text{cov}(e_1, e_2) = -0.25(\Sigma t \pi_2^2 t) \) where \( t \) indexes the observations. Alternatively, noise could have been added directly to the shares as was done for the simulation experiment in Section I, with chosen elements of the variance-covariance matrix.
its parameters. 'Nested' hypothesis testing, based on the likelihood ratio principle (such as testing for local separability in the present case) are common in econometric practice. These tests, however, do not recognize the possibility that the model being tested is only one of several models to explain the same data. For example, one may specify an alternative formulation of $H_0$ (such as the globally separable model) that is not nested within $H_1$ and which does not nest $H_1$ within itself. In this case $H_0$ and $H_1$ are said to be non-nested.

The earliest procedure to test non-nested hypothesis is the Cox test based on the 'modified' likelihood ratio principle. As MacKinnon (1983) points out, the basic idea of this procedure is that the validity of a null hypothesis $H_0$ as a representation of the data generation process can be tested by comparing the observed ratio of the values of the likelihood for $H_0$ and for some non-nested alternative $H_1$, with an estimate of this likelihood ratio if $H_0$ were true. If $H_1$ fits either better or worse than it should if $H_0$ were true, $H_0$ must be false. The Cox test and modifications of it to test nonlinear and multivariate cases (Pesaran and Deaton 1978) are however not easily implementable. Other tests based on the likelihood ratio principle have been proposed. For example, Vuong (1989) proposes likelihood ratio based statistic for testing the null hypothesis that competing models are equally close to the true data generating process against the alternative hypothesis that one model is closer. This is a more general approach that applies whether or not the models are nested. More recently Woolridge (1990) proposed a test for non-nested models that is particularly convenient to apply. This test is based on the correlation between the residuals under the null and the gradient of
the alternative regression function.

The non-nested tests used in this study are the P-tests based on artificial regressions. Proposed by Davidson and MacKinnon (1981, 1982), these tests can be used to test multivariate non-linear models (such as the system of equations of the present case). Let the null hypothesis and the alternate against which it is tested be

\[ H_0 : y_{it} = f_{it}(x_{it}, \beta) + e_{it}^0 \quad (13) \]
\[ H_1 : y_{it} = g_{it}(z_{it}, \gamma) + e_{it}^1 \quad (14) \]

where \(i(-1, \ldots, m)\) is the index of equation and \(t(-1, \ldots, n)\) is the index of observation. For given \(t\), the \(e_{it} (0,1)\) are assumed to be multivariate normal with covariance matrix \(\Omega_0\) or \(\Omega_1\), and serially independent.

The first step in the construction of a P test is to nest \(H_0\) and \(H_1\) in an artificial compound model. Two possibilities which lead to two variants of the P tests are considered. The \(P_0\) test is given by the t-test of \(\alpha = 0\) in the following regression

\[ y - \hat{f} = \hat{F}b + \alpha(\hat{g} - \hat{f}) + \epsilon \]

where \(y, \hat{f}, \hat{g}\) and \(\epsilon\) denote vectors of length \(mn\) formed by stacking the \(y_{it}'s, \hat{f}_{it}'s, \hat{g}_{it}'s\) and \(\epsilon_{it}'s\) respectively, \(\hat{F}\) is an \(mn \times k\) matrix formed by stacking the derivatives of \(f_{it}\) with respect to \(\beta\), evaluated at \(\beta\). \(\beta\) and \(\gamma\) are the Maximum Likelihood estimates of \(\beta\) and \(\gamma\) respectively, and \(\hat{f}\) and \(\hat{g}\) denote \(f(\hat{\beta})\) and \(g(\hat{\gamma})\). \(b\) is a \(k\)-vector of coefficients and \(\alpha\) is a single coefficient to be estimated. Under \(H_0\), the vector \(\epsilon\) is distributed as \(N(0, \Omega_0 \otimes I_n)\).

The \(P_1\) test utilizes the testing regression
\[ y - \hat{f} = \hat{b} + \lambda (\hat{\mu}_0 \hat{H}_1^{-1} \Theta \hat{I}_n) (\hat{g} - \hat{f}) + \epsilon \]

where the notation is as defined above. The \( t \)-test of \( \lambda = 0 \) is the \( P_1 \) test which is asymptotically equal to a Cox test. Intuitively, we can see that what the \( P \) test examines is that if \( H_0 \) is a true model, its residuals should be uncorrelated with the difference between the fitted values from \( H_0 \) and \( H_1 \), which is what is exactly what the Cox test also examines.

**Results**

The estimation was carried out using the non-linear regression procedure available on SHAZAM 6.1. The experiments were conducted for 36 combinations of the true elasticities \( \sigma_{12} \) and \( \sigma_{13} \) as each of them were fixed at 0.1, 0.3, 0.5, 1, 1.5 and 3. For each set of the true elasticities, estimation and tests of separability were carried out 100 times.

In the tests of both local and global separability, the alternate hypothesis was always the non-separable homothetic translog. The null hypotheses were the models with true (T) separability \( V(V_A(\pi_1, \pi_2, \pi_3)) \), the false separability (F) \( V(V_A(\pi_1, \pi_3), \pi_2) \), and the false separability (K) \( V(V_A(\pi_2, \pi_3), \pi_1) \) imposed. When the null is T, the number of times the null is rejected gives us the size of the test, and when the null is F or K, the power of the test.

Results of the (nested) tests of local separability are organized in three tables. Table 1 contains the results of cases when \( \sigma_{12} = \sigma_{13} \), Table 2 reports the results when either \( \sigma_{12} \) or \( \sigma_{13} \) is 'low', i.e. 0.1 or 0.3 and
Table 3 contains all other cases. Similarly, Tables 4, 5 and 6 report the results of the (non-nested) tests of global separability. In all the cases, the rejection (of the null) percentages are reported for 1, 5 and 10 percent significance levels.

Looking at Table 1, for the cases 4, 5 and 6 (when the true elasticities are 1, 1.5 and 3), the size (column T) and power (columns F and K) are low and similar. This is to be expected since given homothetic separability, if \( \sigma_{12} = \sigma_{13} \), all the three hypotheses must be true. These results are consistent with those of Barnett and Choi (1989). As in their study, for low levels of \( \sigma_{12} \) and \( \sigma_{13} \), this pattern of size and power does not emerge (cases 1, 2 and 3 of Table 1 when the true elasticities are 0.1, 0.3 and 0.5). In fact, we see that for these cases, the power when F is the null is low, while the power when K is the null is high. This same pattern is evident in Table 2, when one of the elasticities is 0.1 and the other is 0.3 (cases 1 and 6). In general unexpected results surface again in Table 2. We see that when either \( \sigma_{12} \) or \( \sigma_{13} \) is 'low' even when the other is high, the power (F and K) is high but so is the size (the exception is when the other elasticity is equal to one; however, in case 12 of Table 2, the power is also fairly low). Cases 2 and 7 suggest that the elasticity level 0.5 may be a cut off point between the 'high' and 'low' elasticities.

One possible explanation of these results when either \( \sigma_{12} \) or \( \sigma_{13} \) is low the estimated models lie outside the globally well-behaved regions of the Translog (Christensen and Caves 1980). Indeed, from Table 7 of Barnett and Choi (1989), we see that the regularity violation percentage is 54.8 when \( \sigma_{12} = 0.1 \) and \( \sigma_{13} = 0.3 \). When \( \sigma_{12} \) increases to one, this percentage
Table 1. Tests of local Separability: Rejection rates of the true and false hypotheses when the elasticities are similar

<table>
<thead>
<tr>
<th>True elasticities</th>
<th>1% level</th>
<th>5% level</th>
<th>10% level</th>
</tr>
</thead>
<tbody>
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<td>$\sigma_{13}$</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
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<td>0.1</td>
<td>100</td>
<td>0</td>
</tr>
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<td>1.0</td>
<td>4</td>
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</tr>
<tr>
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Table 2. Tests of local Separability: Rejection rates of the true and false hypotheses when one of the elasticities is low

<table>
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<th>10% level</th>
</tr>
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<td>$\sigma_{12}$</td>
<td>$\sigma_{13}$</td>
<td>T</td>
</tr>
<tr>
<td>1</td>
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<td>0.3</td>
<td>100</td>
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<td>0.3</td>
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<td>59</td>
</tr>
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<td>100</td>
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<td>100</td>
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<td>87</td>
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</table>
Table 3. Tests of local Separability: Rejection rates of the true and false hypotheses when both the elasticities are high

<table>
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<tr>
<th>True elasticities</th>
<th>1 % level</th>
<th>5 % level</th>
<th>10 % level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{12}$</td>
<td>$\sigma_{13}$</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>1 0.5 1.0</td>
<td></td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>2 0.5 1.5</td>
<td></td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>3 0.5 3.0</td>
<td></td>
<td>14</td>
<td>100</td>
</tr>
<tr>
<td>4 1.0 0.5</td>
<td></td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>5 1.0 1.5</td>
<td></td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>6 1.0 3.0</td>
<td></td>
<td>1</td>
<td>100</td>
</tr>
<tr>
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<td>99</td>
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<tr>
<td>9 1.5 3.0</td>
<td></td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>10 3.0 0.5</td>
<td></td>
<td>16</td>
<td>100</td>
</tr>
<tr>
<td>11 3.0 1.0</td>
<td></td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>12 3.0 1.5</td>
<td></td>
<td>6</td>
<td>100</td>
</tr>
</tbody>
</table>
drops to 25. When both $\sigma_{12}$ and $\sigma_{13}$ are high (1.5 and 3 respectively), the regularity violation is 16 percent. That the rejection rates are as expected when at least one of the true elasticities is one is not surprising since the Translog is globally well behaved when $\sigma_{12} = \sigma_{13} = 1$.

Table 3 presents the rejection rates of the null when $\sigma_{12}$ and $\sigma_{13}$ are 'high' (0.5, 1, 1.5, 3). In all the cases and for all three significance levels, the power (F and K) is high. Rejection rates of the null when it is true are low except in cases 3, 7 and 10 where one of the elasticities is still a fairly low 0.5.

The results of the nested tests of local separability are treated as a benchmark to which the non-nested tests of (global) separability are compared. Tables 4, 5 and 6 present results of both the $P_0$ and $P_1$ tests for all three significance levels. Once again, when the true $\sigma_{12}$ and $\sigma_{13}$ are equal and 'high' (here cases 3 to 6 of Table 4), the size and power (F and K) are similar and fairly low. However, when the elasticities are 0.1 and 0.3, the results are different. In fact, in case 1, the results of the $P_0$ test are drastically different from the results of the $P_1$ test.

In Tables 5a and 5b, when either $\sigma_{12}$ or $\sigma_{13}$ is 0.1, results vary for the $P_0$ and $P_1$ tests (in fact, from Table 5a, we see that the powers F and K vary for the $P_0$ test in cases 1-6 and 11). However, when either $\sigma_{12}$ or $\sigma_{13}$ is 0.3 and the other higher than or equal to 0.5, the power (F and K) is consistently high and the size relatively low for both the $P_0$ and $P_1$ tests. The discrepancy between the $P_0$ and the $P_1$ tests and between the rejection rates of the hypotheses F and K completely disappears for all $\sigma_{12}$ and $\sigma_{13}$ greater than or equal to 0.5, as can be seen in Tables 6a and 6b. In fact for all the cases in these tables the tests exhibit fairly good size and
Table 4. Tests of global Separability: Rejection rates of the true and false hypotheses when the elasticities are similar

<table>
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<th>5% level</th>
<th>10% level</th>
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<td>σ₁₃</td>
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<td>1.0</td>
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<tr>
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<td>3.0</td>
<td>10</td>
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</table>

Results of the P₁ test

<table>
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<th>10% level</th>
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<td>σ₁₃</td>
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</tr>
<tr>
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<td>0.1</td>
<td>0</td>
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<tr>
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Table 5a. Tests of global Separability: Rejection rates of the true and false hypotheses for the $P_o$ test when one of the elasticities is low

<table>
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<th>True elasticities</th>
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<th>5 % level</th>
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Table 5b. Tests of global Separability: Rejection rates of the true and false hypotheses for the $F_1$ test when one of the elasticities is low

<table>
<thead>
<tr>
<th>True elasticities</th>
<th>1 % level</th>
<th>5 % level</th>
<th>10 % level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{12}$</td>
<td>$\sigma_{13}$</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>1 0.1 0.3</td>
<td>100 100 100</td>
<td>100 100 100</td>
<td>100 100 100</td>
</tr>
<tr>
<td>2 0.1 0.5</td>
<td>100 100 100</td>
<td>100 100 100</td>
<td>100 100 100</td>
</tr>
<tr>
<td>3 0.1 1.0</td>
<td>55 100 100</td>
<td>61 100 100</td>
<td>66 100 100</td>
</tr>
<tr>
<td>4 0.1 1.5</td>
<td>100 100 100</td>
<td>100 100 100</td>
<td>100 100 100</td>
</tr>
<tr>
<td>5 0.1 3.0</td>
<td>100 100 100</td>
<td>100 100 100</td>
<td>100 100 100</td>
</tr>
<tr>
<td>6 0.3 0.1</td>
<td>100 100 100</td>
<td>100 100 100</td>
<td>100 100 100</td>
</tr>
<tr>
<td>7 0.3 0.5</td>
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<td>6 100 100</td>
</tr>
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<td>6 100 100</td>
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</tr>
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<td>23 100 100</td>
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<td>31 100 100</td>
</tr>
<tr>
<td>10 0.3 3.0</td>
<td>20 100 100</td>
<td>26 100 100</td>
<td>30 100 100</td>
</tr>
<tr>
<td>11 0.5 0.1</td>
<td>98 100 100</td>
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<td>100 100 100</td>
</tr>
<tr>
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<tr>
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<td>84 100 100</td>
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<tr>
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</tr>
<tr>
<td>15 0.5 0.3</td>
<td>15 100 100</td>
<td>23 100 100</td>
<td>33 100 100</td>
</tr>
<tr>
<td>16 1.0 0.3</td>
<td>8 100 100</td>
<td>12 100 100</td>
<td>21 100 100</td>
</tr>
<tr>
<td>17 1.5 0.3</td>
<td>5 100 100</td>
<td>8 100 100</td>
<td>17 100 100</td>
</tr>
<tr>
<td>18 3.0 0.3</td>
<td>7 100 100</td>
<td>11 100 100</td>
<td>14 100 100</td>
</tr>
</tbody>
</table>
Table 6a. Tests of global Separability: Rejection rates of the true and false hypotheses for the $P_0$ when both the elasticities are high

<table>
<thead>
<tr>
<th>True elasticities</th>
<th>$\sigma_{12}$</th>
<th>$\sigma_{13}$</th>
<th>1% level</th>
<th>5% level</th>
<th>10% level</th>
</tr>
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<td>$T$</td>
<td>$F$</td>
<td>$K$</td>
<td>$T$</td>
<td>$F$</td>
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<td>2</td>
<td>0.5</td>
<td>1.5</td>
<td>2</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
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<td>0.5</td>
<td>3.0</td>
<td>5</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>0.5</td>
<td>6</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>1.5</td>
<td>5</td>
<td>100</td>
<td>99</td>
</tr>
<tr>
<td>6</td>
<td>1.0</td>
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<td>7</td>
<td>1.5</td>
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<td>6</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>8</td>
<td>1.5</td>
<td>1.0</td>
<td>11</td>
<td>95</td>
<td>100</td>
</tr>
<tr>
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<td>1.5</td>
<td>3.0</td>
<td>5</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>3.0</td>
<td>0.5</td>
<td>9</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>11</td>
<td>3.0</td>
<td>1.0</td>
<td>9</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>12</td>
<td>3.0</td>
<td>1.5</td>
<td>12</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>
Table 6b. Tests of global Separability: Rejection rates of the true and false hypotheses for the $P_1$ when both the elasticities are high

<table>
<thead>
<tr>
<th>True elasticities</th>
<th>1 % level</th>
<th>5 % level</th>
<th>10 % level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{12}$</td>
<td>$\sigma_{13}$</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>1 0.5</td>
<td>1.0</td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>2 0.5</td>
<td>1.5</td>
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</tr>
<tr>
<td>3 0.5</td>
<td>3.0</td>
<td>8</td>
<td>100</td>
</tr>
<tr>
<td>4 1.0</td>
<td>0.5</td>
<td>6</td>
<td>100</td>
</tr>
<tr>
<td>5 1.0</td>
<td>1.5</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>6 1.0</td>
<td>3.0</td>
<td>2</td>
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</tr>
<tr>
<td>7 1.5</td>
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</tr>
<tr>
<td>8 1.5</td>
<td>1.0</td>
<td>11</td>
<td>95</td>
</tr>
<tr>
<td>9 1.5</td>
<td>3.0</td>
<td>6</td>
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</tr>
<tr>
<td>10 3.0</td>
<td>0.5</td>
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<td>100</td>
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<tr>
<td>11 3.0</td>
<td>1.0</td>
<td>9</td>
<td>100</td>
</tr>
<tr>
<td>12 3.0</td>
<td>1.5</td>
<td>12</td>
<td>100</td>
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</tbody>
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power.

From the above discussion, tests of local separability seem to perform poorly when one of the true elasticities is equal to or lower than 0.5, while the tests of global separability seem to do well for elasticities greater than or equal to 0.5. This suggests that the regular region of the globally separable model may possibly be larger than that of the Translog. In general, however, the non-nested tests of global separability seem to perform as well as the tests for local separability, and in fact marginally better since they do well for the cases when one of the elasticities is 'low' at 0.5. The P_0 and P_1 tests both seem to perform similarly except for the extreme cases when at least one of the true elasticities is low. Thus the globally separable model seems to do well in providing correct inferences about separability. This is in line with the results of Moschini (1990) who tests local and global models of two specifications of separability using the test proposed by Vuong (1989).

**Summary and Conclusions**

Weak separability of a group of commodities from the others in a utility (or production) function is a commonly maintained hypothesis in a lot of empirical work. Ideally, such an assumption should be tested before it is maintained. However, imposing separability parametrically on a FFF renders it inflexible by restricting the structural form of either the macro or the aggregator functions. This is the case when the restricted (separable) model is required to be parametrically nested in the unrestricted model. This study looks at a recently proposed globally
separable flexible functional form that models the macro and the aggregator functions to the same degree of flexibility. This is achieved by using the Translog to model the macro and the aggregator functions.

The locally separable Translog is compared to the globally separable model using Monte Carlo techniques. Data is generated from a globally separable utility function and noise is added to the quantities. The locally and globally separable models are then estimated. When the locally and globally separable models are tested against the unrestricted Translog using the Likelihood ratio test and non-nested tests (the $P_0$ and the $P_1$ tests) respectively, we find that the tests exhibit high power and low rejection rates of the true hypothesis for true elasticities that are not low in value.

In conclusion, the globally separable model seems to do as well and in fact slightly better than the locally separable model in making the correct inference about separability. It is, however, of interest to see how these results change when other non-nested tests are employed.
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*Econometrica*

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Wooldridge, J.M. "An Encompassing Approach to Conditional Mean Tests with
GENERAL SUMMARY

In this study, the empirical examination of consumer demand systems was investigated within the framework afforded by duality in consumer theory. The focus of attention has been on theoretically valid empirical specification of demand systems that reflect particular market situations or preference structures. A common tool of empirical analysis that runs through all three sections of this dissertation is that of a flexible functional form (FFF). Such functional forms which provide local second order approximations to unknown functions are widely used in applied econometrics today. Since these FFFs impose no prior restrictions on estimated demand elasticities, considerable attention has been devoted in recent years to the development of less restrictive and more well behaved FFFs that meet a wider criteria of acceptability.

This study first examined the market situation where supplies are inelastic and prices must adjust to clear the market. Such market situations arise for most agricultural commodities, particularly for perishable commodities. A price dependent demand system can reflect such a market situation. Two flexible inverse demand systems were proposed. The first of these, a linear inverse demand system (LIDS) was derived using a functional form that is a symmetric dual to the PIGLOG cost function of the AIDS model. This new model does not embody the aggregation properties of the AIDS model. It does, however, present a particularly convenient linear system of equations that may carry a significant advantage over the available nonlinear flexible inverse demand systems, especially when the number of goods is large. A simulation exercise revealed that the LIDS
performs as well and in fact slightly better than the inverse Translog system.

In specifying such market demands however, a problem that needs attention is that of aggregation. This issue, in demand analysis, is cast in terms of aggregating individual quantity dependent demands. Whether the market demands to be estimated are inverse or direct, the aggregation conditions need to hold. The aggregation problem was discussed and a new FFF was proposed that satisfies such aggregation conditions. For the class of quasi-homothetic preferences considered (at the individual level), the market demands satisfy integrability conditions without any additional restrictions. Such a market demand system can be derived from a specification of some hypothetical utility function such as the FFF proposed. This specification yields a system of share dependent nonlinear equations.

The Canadian market for meat poses an interesting problem for economic analysis. While the country trades freely in beef and pork with the U.S., so that it is a price taker for these commodities, the Chicken market in Canada is subjected to supply restricting activities of domestic marketing boards. This in essence creates the situation of an inelastic supply of chicken where prices must adjust to clear the market while the converse is true for beef and pork.

Such a market situation calls for a mixed demand system where the prices of some, and quantities of other commodities are exogenous. However, the existing theoretical framework for mixed demands does not allow a convenient transition to an empirical model. The concept of 'virtual' or 'shadow' price was used in developing a theoretical framework
that does allow the specification of an empirical mixed demand system.

The notion of 'virtual' price was used in developing a 'mixed' cost function. Using this, the Slutsky equations were derived, the elasticity forms of which were easily implemented in a differential approximation of the mixed demand system. Such a differential demand system also satisfies a meaningful and an alternate criterion of local approximation.

This model was estimated using Canadian data. The elasticities from this system compared well with those from the mixed LES which was also estimated. However, the direct elasticities retrieved from the mixed system differed from those of the direct differential system with regard to the own price elasticity of chicken. The mixed system suggests that the demand for chicken in Canada may be more elastic than implied by estimated direct systems. An ad hoc specification of the first stage of budget allocation was also estimated and the unconditional elasticities computed.

The notion of separability is frequently invoked in applied demand analysis. Indeed, it is only with such an assumption that we can specify a demand system for meats as was done for the Canadian situation. In general, separability is a strong maintained hypothesis and should ideally be tested before it is maintained. However, testing separability using FFFs has been problematic. While the alternative of imposing local separability of a FFF exists, imposing global separability parametrically on most of the commonly used FFFs renders them inflexible.

In Section III, a recently proposed globally separable FFF is considered. Its ability, as well as that of the locally separable model, in providing correct inferences about separability are examined in a Monte Carlo study. Tests of local separability were carried out using the
likelihood ratio test while tests of global separability were conducted using non-nested P tests. Results revealed that both the locally and globally separable models fared well for certain regions of the true elasticity values. When the true elasticities were fairly 'high', i.e., around or greater than one, all three tests exhibited low size and high power. Rejection patterns however became unpredictable when elasticities were low. However, the globally separable model seemed to perform well for a few cases of 'low' elasticity values as well.

A number of interesting questions were raised in this study. To begin with, one might wonder about the large sample behavior of the LIDS model. Further research on the aggregable inverse demand is also called for. Its ability in replicating the true data generation process can be examined in simulation studies. Using this functional form, the hypothesis of quasi-homothetic preferences can be tested using non-nested hypothesis tests. The empirical mixed demand system presents a framework of analysis for the particular market situation it reflects. Thus, other issues, such as the debate on structural change in preferences for meat, can be examined in this framework. Finally, the performance of the globally flexible functional form can be examined using other non-nested tests. This would show if the results of the present study are specific to the tests used in this study.
ACKNOWLEDGEMENTS

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