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Generalization of hysteresis modeling to anisotropic materials

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Abstract

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Keywords

Magnetic anisotropy, Magnetic hysteresis, Magnetic materials, Magnetic susceptibilities

Disciplines

Electromagnetics and Photonics | Engineering Physics | Materials Science and Engineering

Comments

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Generalization of hysteresis modeling to anisotropic materials

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An extension to the model of hysteresis has been presented earlier which included the effect of anisotropy in the modeling of the anhysteretic magnetization curves of uniaxially anisotropic single crystalline materials. Further exploration of this extension shown here considers different kinds of crystal anisotropy in materials. Theory considers that the differential susceptibility at any given field is determined by the displacement of the prevailing magnetization from the anhysteretic magnetization. Thus, it has been shown that the effect of anisotropy on magnetic hysteresis in materials can be incorporated into the model of hysteresis through the anisotropic anhysteretic. This extension is likely to be particularly useful in the case of hard magnetic materials which exhibit high anisotropy. © 1997 American Institute of Physics. [S0021-8979(97)39108-7]

INTRODUCTION

In this work, the hysteresis model developed earlier¹ has been extended to include the effects of anisotropy and texture in materials. The original model considered only isotropic magnetization. This model has been shown to work well for a wide class of ferromagnetic materials.² The extension describes the differential susceptibility at any applied magnetic field strength H as a function of the displacement of the prevailing magnetization from the anisotropic anhysteretic.

Most ferromagnetic materials are anisotropic. Therefore, the applicability of the previous model to real materials is ultimately limited to those which are isotropic or have very low anisotropy. In order to model materials with a high degree of accuracy, the anisotropy of the material has to be considered. This has been accomplished by considering the effect of anisotropy on the anhysteretic magnetization of magnetic materials but otherwise retaining all the original features of the earlier model. An improvement of the model of anhysteresis, by considering the anisotropy in uniaxially anisotropic materials, has been suggested recently.³ This considered all the magnetic moments under uniaxial anisotropy and showed the variation in anhysteretic magnetization curves with the direction of the applied field. In the present article, this has been generalized to take into account magnetic moments in three dimensional space and also the different forms of crystal anisotropy, such as uniaxial anisotropy and cubic anisotropy. The variation in the anhysteretic magnetization curves with direction is then used to calculate the hysteresis loops of different kinds of materials along different crystallographic directions.

Among the many models available to describe hysteresis in a material, phenomenological models have certain advantages.^{4,5} These models require very few parameters to describe hysteresis compared with mathematical models such as the Preisach model. Also the parameters used in these phenomenological models have a physical meaning which is not present in purely mathematical models. Thus, the physical models are comparatively simple and are more easily extended to include various external effects such as applied stress or different frequencies of applied field, which are known to affect the hysteresis curves of materials.

In the case of iron which has a cubic structure, the anisotropy constant K is positive and so the easy axes are $\langle 100 \rangle$

and the hard axes are $\langle 111 \rangle$. For nickel, the reverse is true since the anisotropy constant is negative. Cobalt exhibits a uniaxial anisotropy, due to its *hcp* structure, in which the base plane is the “hard” and the c axis is the “easy” direction.

The ease with which a material is magnetized along a certain direction depends on the energy required to make a moment point along that direction. The energy of any particular magnetic moment is the sum of its field, exchange and anisotropy energies. If the material is isotropic, then the anisotropy energy can be considered zero. The field energy of a moment is then

$$E_F = -\mu_0 \mathbf{m} \cdot (\mathbf{H} + \alpha \mathbf{M}), \quad (1)$$

where α is the exchange coupling coefficient. This energy is lowest along the direction of the applied field and largest along the direction anti-parallel to the field. As a result, the fraction of moments pointing along the direction of the applied field is higher than the other directions. However, if it is anisotropic, then the anisotropy energy plays a role in perturbing the energy surfaces, the effects of which are particularly apparent at low applied magnetic fields. The anisotropy energy is low along certain crystallographic directions compared to other directions. As a result, the fraction of the moments in a material that are aligned by a magnetic field of a given strength is dependent on the direction of the applied field relative to the magnetocrystalline anisotropy. In the case of uniaxially anisotropic materials, the anisotropy energy is given by:

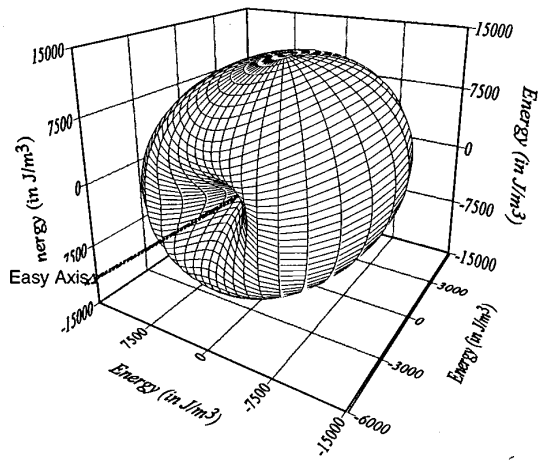
$$E_{\text{anis}} = K_0 + K_1 \sin^2 \phi, \quad (2)$$

where ϕ is the angle which the magnetization in a domain makes with the unique axis. When $K_1 > 0$, the unique axis is the easy axis and when $K_1 < 0$, the unique axis is the hard axis. Figure 1(a) shows the anisotropy energy surface of a uniaxial material when $K_1 > 0$.

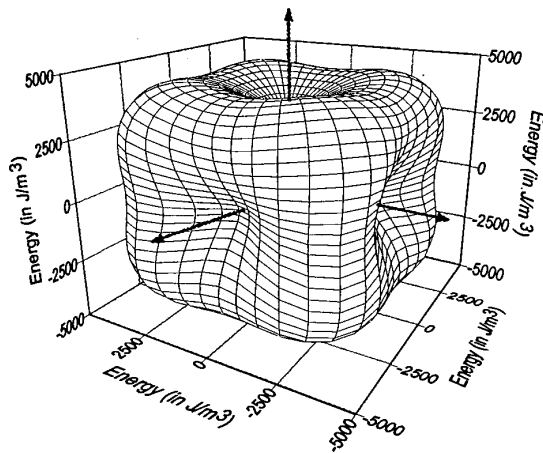
In case of materials with a cubic structure, the anisotropy energy is given as

$$E_{\text{anis}} = K_0 + K_1 (\alpha_1^2 \alpha_2^2 + \alpha_2^2 \alpha_3^2 + \alpha_3^2 \alpha_1^2), \quad (3)$$

where the α 's are the direction cosines of the moment with respect to the crystallographic axes. The anisotropy energy surface in this case is shown in Fig. 1(b) and it can be seen that the energy is lowest along $\langle 100 \rangle$ and highest along $\langle 111 \rangle$.



(a)



(b)

FIG. 1. Energy surface of a material in the absence of an applied magnetic field for the case of (a) uniaxial anisotropy and (b) cubic anisotropy.

ANHYSERETIC MODELING FOR ANISOTROPIC MATERIALS

The anhysteretic magnetization curve of a material represents the global equilibrium state of magnetization of a ferromagnet. In the case of isotropic materials, a modified form of the Langevin expression has been used to represent the anhysteretic curve which is given as:

$$M_{an}(H_e) = M_s [\coth(H_e/a) - H_e/a], \quad (4)$$

where a is a parameter with dimensions of magnetic field and H_e is the effective magnetic field. Thus, the anhysteretic in the case of an isotropic material can be modeled with three parameters: a , α , M_s . In the case of anisotropic materials, this expression does not take the form of the Langevin expression because the anisotropy energy is different along different directions. In such a case the magnetization M_{an} can only be calculated by the summation of the contribution to magnetization of magnetic moments along the field direction. This can be given by the expression,

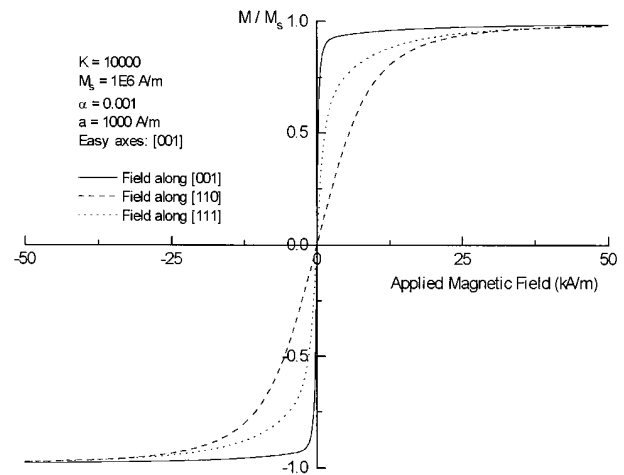


FIG. 2. Variation of the anhysteretic magnetization curves for a uniaxially anisotropic material along different directions.

$$M_{an} = M_s \frac{\sum_{\text{all moments}} e^{-E/k_B T} \cos \theta}{\sum_{\text{all moments}} e^{-E/k_B T}}, \quad (5)$$

where E is the total energy of the magnetic moment, which is the sum of field and anisotropy energies, θ is the angle between the direction of the magnetic moment and the direction of the applied magnetic field, and $k_B T$ represents the Boltzmann, or thermal, energy. This expression represents the anhysteretic magnetization curve of an anisotropic material and requires four parameters: a , α , M_s , K_a , where K_a is the anisotropy constant. In addition, a complete description now requires the direction of the applied magnetic field.

This expression was used to calculate the anhysteretic curves for a uniaxially anisotropic material along different directions as shown in Fig. 2. The variation of the anhysteretic curves with the direction of the field in the case of a cubic material is shown in Fig. 3.

EXTENSION OF THE MODEL FOR ANISOTROPY

The initial magnetization curve of a material always lies below the anhysteretic and approaches it near saturation. Considering the energy of all the pinning sites to be equal,

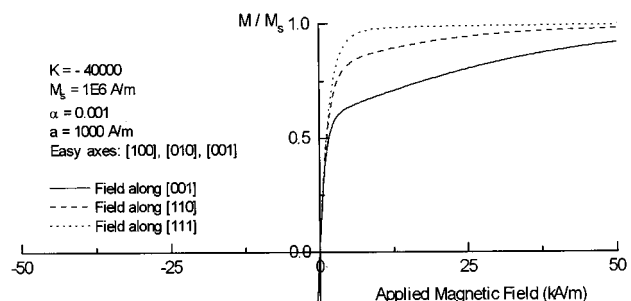


FIG. 3. Variation of the anhysteretic magnetization curves for a material with cubic anisotropy along different directions.

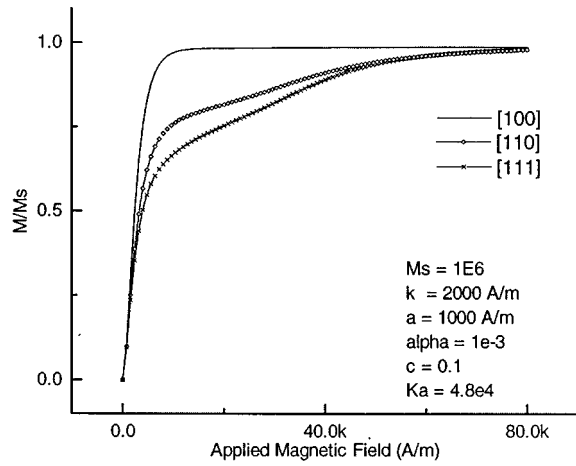


FIG. 4. Initial magnetization curves of a cubic material.

the differential susceptibility of the material at any given field is a function of the displacement of the magnetization from the anhysteretic, as given by the equation⁵

$$\frac{dM}{dH} = (1-c) \frac{M_{an} - M_{irr}}{k\delta - \alpha(M_{an} - M_{irr})} + c \frac{dM_{an}}{dH}, \quad (6)$$

where c is the reversible component of domain wall motion. Thus, it can be seen that the hysteresis behaviour is strongly dependent on the anhysteretic magnetization at any given applied field. Incorporating the effects of anisotropy into the anhysteretic function allows the hysteresis curves of materials with different kinds of anisotropy to be calculated.

Figure 4 shows the initial magnetization curves along different directions of a cubically anisotropic material with anisotropy constant $K=4.8 \times 10^4 \text{ J/m}^3$, and hysteresis parameters $a=1000 \text{ A/m}$, $k=2000 \text{ A/m}$, $\alpha=.001$, and $c=0.1$. It can be seen that the material has high initial permeability along [100] direction, which is one of the easy directions and a lower initial permeability along the [111] direction. The

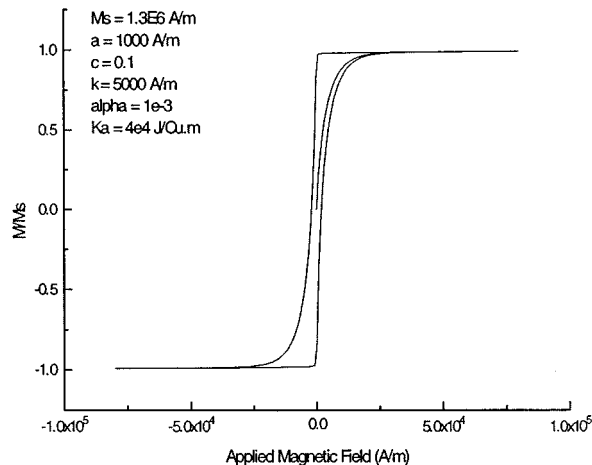


FIG. 5. Hysteresis loop of a uniaxially anisotropic material magnetized along the easy direction. Model parameters are: $M_s=1.3e6 \text{ A/m}$; $a=1000 \text{ A/m}$; $k=5000 \text{ A/m}$; $c=0.1$; $\alpha=0.001$; $K=4e4 \text{ J/Cu.m}$.

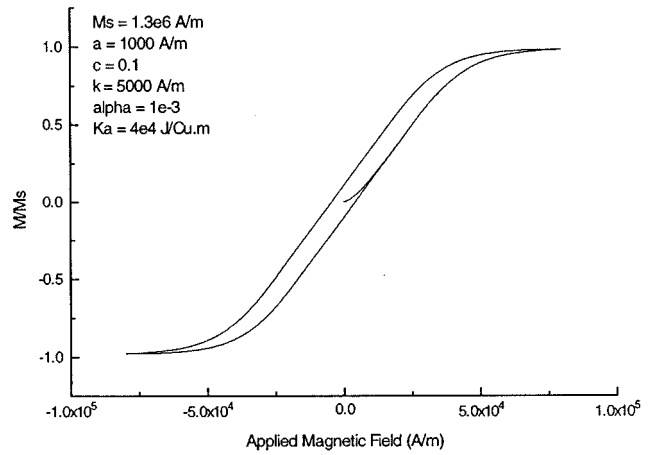


FIG. 6. Hysteresis loop of a uniaxial material magnetized along one of its hard direction. Model parameters are: $M_s=1.3e6 \text{ A/m}$; $a=1000 \text{ A/m}$; $k=5000 \text{ A/m}$; $c=0.1$; $\alpha=0.001$; $K=4e4 \text{ J/m}^3$.

difference in the permeability is made greater as the anisotropy coefficient increases. Thus, the effect of anisotropy on the initial magnetization curves can be incorporated for modeling of hysteresis in anisotropic materials. The hysteresis curves of a uniaxial material magnetized along different directions is shown in Figs. 5 and 6. While the switching from positive to negative magnetization is rapid when it is magnetized along [001], it is more gradual as the direction of magnetization gets closer to a ‘‘hard axis.’’ As a result, there is a significant difference in the calculated magnetic properties such as coercivity, remanence, initial permeability, and hysteresis loss along different crystallographic directions.

CONCLUSIONS

The existing hysteresis model has been extended to include the effects of magnetocrystalline anisotropy in the calculation of magnetic properties of materials. These effects have been incorporated into the model through the anisotropic anhysteretic magnetization. The variations in the anhysteretic magnetization curves with the direction of the applied field have been shown. These changes in the anhysteretic can be used to calculate the hysteresis, and the variations in the magnetic properties of the material, along different directions. Furthermore, the model can accommodate the complicated situation of texture in polycrystalline materials. This extension of the model is particularly useful in the calculation of hysteresis curves in hard magnetic materials⁶ because these materials have high anisotropy.

ACKNOWLEDGMENT

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