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# Leveraging Virtual Reality Experiences With Mixed-Integer Nonlinear Programming Visualization of Disassembly Sequence Planning Under Uncertainty

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## **Abstract**

Disassembly sequence planning at the early conceptual stage of design leads to enormous benefits including simplification of products, lower assembly and disassembly costs, and design modifications which result in increased potential profitability of end-of-life salvaging operations. However, in the early design stage, determining the best disassembly sequence is challenging. First, the required information is not readily available and very time-consuming to gather. In addition, the best solution is sometimes counterintuitive, even to those with experience and expertise in disassembly procedures. Integrating analytical models with immersive computing technology (ICT) can help designers overcome these issues. A two-stage procedure for doing so is introduced in this paper. In the first stage, a stochastic programming model together with the information obtained through immersive simulation is applied to determine the optimal disassembly sequence, while considering uncertain outcomes, such as time, cost, and the probability of causing damage. In the second stage, ICT is applied as a tool to explore alternative disassembly sequence solutions in an intuitive way. The benefit of using this procedure is to determine the best disassembly sequence, not only by solving the analytic model but also by capturing human expertise. The designer can apply the obtained results from these two stages to analyze and modify the product design. An example of a Burr puzzle is used to illustrate the application of the method.

## **Disciplines**

Applied Mechanics | Graphics and Human Computer Interfaces | Manufacturing | Systems Engineering and Multidisciplinary Design Optimization

## **Comments**

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*Disassembly sequence planning at the early conceptual stage of design leads to enormous benefits including simplification of products, lower assembly and disassembly costs, and design modifications which result in increased potential profitability of end-of-life salvaging operations. However, in the early design stage, determining the best disassembly sequence is challenging. First, the required information is not readily available and very time-consuming to gather. In addition, the best solution is sometimes counterintuitive, even to those with experience and expertise in disassembly procedures. Integrating analytical models with immersive computing technology (ICT) can help designers overcome these issues. A two-stage procedure for doing so is introduced in this paper. In the first stage, a stochastic programming model together with the information obtained through immersive simulation is applied to determine the optimal disassembly sequence, while considering uncertain outcomes, such as time, cost, and the probability of causing damage. In the second stage, ICT is applied as a tool to explore alternative disassembly sequence solutions in an intuitive way. The benefit of using this procedure is to determine the best disassembly sequence, not only by solving the analytic model but also by capturing human expertise. The designer can apply the obtained results from these two stages to analyze and modify the product design. An example of a Burr puzzle is used to illustrate the application of the method. [DOI: 10.1115/1.4026463]*

## 1 Introduction

Recent emphasis on environmental impacts across the entire product lifecycle has increased the importance of disassembly sequence planning during product design. As mentioned by Lambert [1], disassembly sequencing is an invaluable tool in concurrent engineering and plays an important role in the modern design process.

Disassembly planning has been addressed in the reverse logistics literature. Several surveys of the relevant literature are provided in Refs. [1–5]. According to Reveliotis [6], most of the studies address disassembly planning in three main steps: (1) first formalize the dynamics of the disassembly process by applying a particular representation tool such as tree representation, AND/OR graph representation, and state representation, (2) assign a “cost structure” to the representation and modeling the economic elements involved in the decision-making process; and (3) finally, apply a method to select the best disassembly sequence by means of the established framework in steps (1) and (2). Much of the existing work on optimal disassembly planning assumes a deterministic model for the underlying process dynamics and cost structure [6]. However, disassembly is a process in which uncertainty is often encountered, both in product and process characteristics.

Depending on the purpose of disassembly, the uncertainty sources are different. When the purpose is re-using, remanufacturing or recycling, uncertainty in the incoming feedstock design,

material, age and quantity creates enormous impediments to cost-effective operations. When the purpose of disassembly is maintenance, uncertainty lies in dimensional instability and the possibility of causing damage to valuable components [7]. However, regardless of the purpose of disassembly, uncertainty in outcomes such as disassembly time, cost, and other important outcomes are common.

The starting point for the work presented in this paper is the observation that the effective management of these uncertainties has not been adequately addressed in the relevant literature. The emphasis of much of the existing research in disassembly under uncertainty (e.g., Refs. [8–15]) is on product characteristics such as the uncertainty in estimating the recovery value, net profits of salvaging operations, rate of return of used products, product states, and the uncertain quality of take-back products. In contrast, the current work addresses uncertainty for disassembly process characteristics, such as disassembly time or the probability of causing component damage.

Furthermore, the studies that identify the potential undeterministic nature of disassembly planning usually deal with this issue by conducting a sensitivity analysis of a solution developed using a deterministic optimization model (e.g., Refs. [16,17]).

Behdad and Thurston [7] applied a multi-objective decision analytical approach to deal with the uncertainty associated in the disassembly process characteristics. They applied a multiattribute utility function to consider the trade-offs between two attributes: disassembly time and the probability of damage during disassembly, and the uncertainty associated with those attributes. Applying utility theory requires a time commitment by decision makers/designers to formulate and assess the utility functions.

In the current paper, a stochastic programming model for determining the best disassembly sequence is introduced. The model

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considers disassembly process outcomes (e.g., disassembly time and probability of damage during disassembly) as uncertain parameters. The stochastic model is defined in a form of chance constrained programming and is then converted to a mixed-integer nonlinear programming (MINP). However, although the model provides a complete quantitative characterization of the uncertainty, the potential unavailability of the information necessary to develop the model early in the design stage may affect its accuracy. To overcome this issue, the capabilities of ICT are applied to derive the required data for the model. Furthermore, sometimes the optimal disassembly sequence obtained from the analytical model is different from the intuitive sequence that remanufacturing or maintenance experts follow while disassembling a product during its use or at end-of-life. A two-stage procedure is presented here to address this issue. In the first stage, an analytic model is applied to determine the best disassembly sequence, and then in the second stage other potential solutions are explored by simulating the disassembly process in the virtual environment applying ICT. Finally, the results of these two stages are combined together to derive some design modifications.

The structure of this paper is as follows: Section 2 provides an overview of the two-stage procedure suggested for deriving the best disassembly sequence under uncertainty. The proposed stochastic model is introduced in Sec. 3. Section 4 presents the application of the suggested procedure through a Burr puzzle example. And finally, Sec. 5 concludes the paper.

## 2 Method

The procedure proposed in this paper for deriving the best disassembly sequence under uncertainty includes two basic stages. Stage I is the application of a stochastic programming model to derive the optimum disassembly sequence based on the input information provided to the model. Stage II is the application of the ICT technique to visualize the product and explore potential intuitive disassembly alternatives to compare to the optimum solution(s) resulting from the first stage. The stochastic model originates from mathematical models that drive the decision variables to converge to their optimal values without need of visiting the complete solution space, and the ICT technique simulates the disassembly process in the virtual environment, gathering the user's expert knowledge to find intuitive solutions. This approach integrates the ICT's visual abstraction of the physical world with the mathematical model's abstraction of the cause and effect relationships and tradeoff decisions. Each provides insights to the other.

Finally, integrating the results of both the mathematical model and the ICT can help the designer derive improved design modifications. The proposed procedure is summarized in Fig. 1 and described below.

**2.1 Stage I: Obtain the Optimum Disassembly Sequence Through a Mathematical Model.** Analytical programming methods require modeling with a high level of abstraction. Applying the mathematical model in disassembly sequence planning usually starts with the assembly drawing or a computer-aided design (CAD) file; then, a connection diagram and a set of precedence relations are derived [1]. The first step in developing the optimization model is to visualize the feasible disassembly operations by graphical networks in which the nodes represent states (resulting subassemblies) and the arcs represent disassembly operations and precedence relations. The second step after deriving the disassembly graphs is to introduce some parameter values (such as costs, disassembly time or chance of damage) that are expected to result from every feasible disassembly action and/or revenue that could be realized from every feasible resulting subassembly.

In addition to proper modeling of the problem, providing accurate input data is an important step in developing useful mathematical models. Therefore, the third step of Stage I is the

application of ICT techniques to obtain the required input data for the mathematical model. It should be noted that the focus of this paper is on disassembly under uncertainty, where disassembly time and the outcome of incurring damage to components during disassembly operations is uncertain. The potential of ICT techniques to help the designer simulate the disassembly process in a virtual environment and determine the statistical distributions of the uncertain parameters is realized. Once data are obtained through immersive simulation, a stochastic model in the form of a mixed-integer nonlinear program is applied for selecting optimum solution (s).

**2.2 Stage II: Obtain the Intuitive Disassembly Sequences Through the ICT Technique.** Disassembly is most often carried out manually, without automated robotics. Human experts often develop efficient procedures, but there is a great deal of variability in those procedures, both across experts and across the operations of a single expert. Simulating the product disassembly process using ICT techniques assists the designer in determining disassembly solutions in an intuitive way. Using the virtual environment, realistic prototyping can be performed and a designer can examine how humans interact with components [18].

Geometric reasoning that results from human interaction with the product components helps the designer generate disassembly solution(s) that might be different from the optimal disassembly sequence obtained from using only the analytical models. The immersive environment provides a space for this human interaction to occur without the need for building physical prototypes. Exploring disassembly solutions using the ICT techniques is especially important, given the variability inherent in the human-driven disassembly processes. Spending time and money on simulating and visualizing the products in the ICT environment with the aim of gathering input data or exploring intuitive disassembly sequence is especially beneficial in the early conceptual stage of design where design modifications are less costly.

The designer can compare the results of the mathematical model with intuitive solutions and apply the results to modify the product design. Design changes can be made and new data can be gathered to finalize the product design. In addition, different design alternatives can be compared and evaluated applying the proposed procedure.

## 3 Stochastic Model

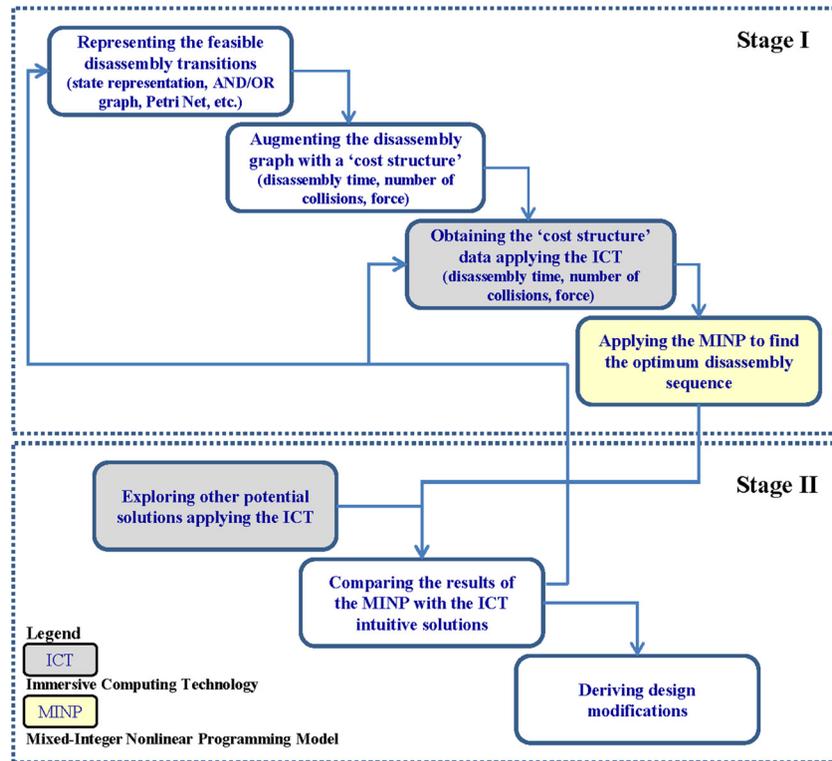
In this section, a mathematical model is proposed to identify the best disassembly sequence. A disassembly graph which includes all feasible disassembly transitions in the arcs and resulting subassemblies in the nodes is first developed. The optimization model is based on the shortest path method. The objective is to find the shortest path considering a decision criterion or objective (e.g., disassembly time, probability of damage) given that the disassembly parameters (times and probability of damage corresponding to disassembly transitions) are uncertain values.

### 3.1 Index Set.

- $j$ : feasible disassembly transition (action)
- $l$ : node of disassembly graph (assembly states)
- $t$ : target node
- $J$ : the set of all feasible disassembly transitions
- $I_l$ : the set of disassembly transitions (arcs) coming to node  $l$
- $O_l$ : the set of disassembly transitions outgoing from node  $l$
- $n$ : the total number of disassembly transitions/arcs

### 3.2 Parameters.

- $\gamma_j$ : The uncertain parameter associated with transition  $j$ .
- $\alpha$ : Confidence level selected for converting stochastic constraints
- $\gamma_j$  can be the number of collisions associated with the disassembly transition  $j$ , the disassembly time of transition  $j$  or any other



**Fig. 1 A schematic view of the two-stage procedure of disassembly sequence planning**

process parameter. It is assumed that  $\gamma$  is a random variable which follows a normal distribution. The statistical parameter  $\gamma$  is obtained from the simulation, which can show the exact distribution of  $\gamma$ . When the result is not normally distributed, appropriate remedial actions can be taken to transform non-normal to normal distributions to facilitate modeling. For example, sometimes extreme values in a data set result in a skewed distribution. In this case, normality can be achieved by removing the outliers, if appropriate. Non-normality can also result when data originate from more than one process, shift or operator. Often it is possible to normalize this data by applying methods such as Box-Cox transformation, or using the sample mean and employing the central limit theorem [19].

**3.3 Decision Variable.** The decision variable is defined as  $x_j$ , the binary (0, 1) variable that indicates whether disassembly transition  $j$  is performed ( $x_j = 1$ ) or not ( $x_j = 0$ ).

Objective function:

The objective of the model is to minimize the total disassembly time, or alternatively, the number of collisions resulting from conducting disassembly transitions to reach to the target nodes (target assembly).

$$\min \sum_{j=1}^n \gamma_j x_j$$

Subject to

$$\sum_{j \in O_1} x_j = 1 \quad (\text{initial node}) \quad (1)$$

$$\sum_{j \in I_1} x_j = \sum_{j \in O_1} x_j \quad (\text{transit nodes}) \quad (2)$$

$$\sum_{j \in I_t} x_j = 1 \quad (\text{target node}) \quad (3)$$

The objective of shortest path problem is to find the path between two nodes (vertices) in a graph such that the sum of the weights of its constituent arcs (edges) is minimized.

A binary decision variable ( $x_j$ ) is assigned to each arc of the graph or disassembly transition, meaning that arc is either traversed or not. The disassembly graph can be represented by a set of node equations. The summation of the arcs exiting the first node should be equal to 1 (Eq. (1)). The summation of the arcs entering the target node (target assembly) should also be equal to 1 (Eq. (3)). For the transit nodes, the number of arcs entering each node must be equal to the number of arcs leaving a node (Eq. (2)).

The objective function includes a random parameter. To convert it to a model with a deterministic function, it will be restated as decision variable  $\bar{h}$ . Therefore, a new constraint (Eq. (4)) needs to be added [20].

Objective function

$$\text{Min } \bar{h}$$

Subject to

$$\text{pr} \left( \sum_j \gamma_j x_j \leq \bar{h} \right) \geq \alpha \quad j \in J \quad (4)$$

The objective chance constraint (4) introduces a target value  $\bar{h}$  with confidence level  $\alpha$ . Considering both the objective function and constraint (4), we are looking for the minimum value of  $\bar{h}$  that satisfies the above mentioned probability constraint. In other words

$$\bar{h} = \min \{h | \text{pr} \left( \sum_j \gamma_j x_j \leq h \right) \geq \alpha\}$$

Now, we need to convert the new constraint incorporating the probability term into corresponding crisp equivalence.

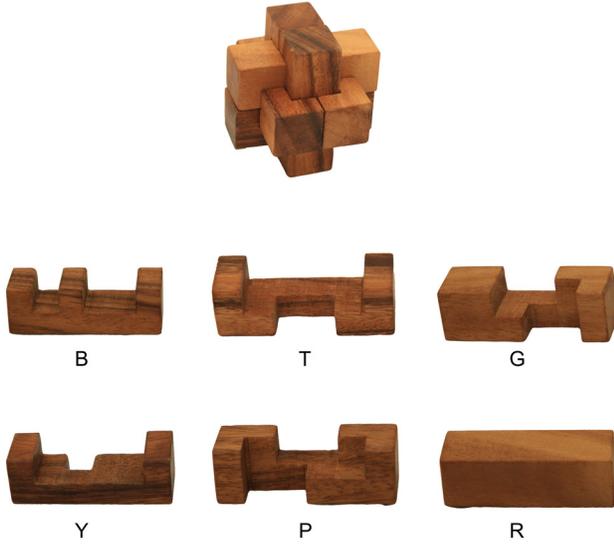


Fig. 2 A Burr puzzle with six interlocking pieces

Let us define A as follows:

$$A = \sum_j \gamma_j x_j - \bar{h} \quad (5)$$

where  $\gamma_j$  are random variables in the above expression. If  $\gamma_j$  follows a normal distribution, since the sum of independent normally distributed random variables follows normal distribution, then A follows a normal distribution and the expected value and the variance are calculated as follows:

$$E(A) = \sum_j E(\gamma_j) x_j - \bar{h} \quad (6)$$

$$\text{Var}(A) = \sum_j x_j^2 \text{Var}(\gamma_j) \quad (7)$$

Since A follows a normal distribution,  $A - E(A) / \sqrt{\text{Var}(A)}$  follows the standardized normal distribution.

Now, consider constraint (4)

$$\text{pr} \left( \sum_j \gamma_j x_j \leq \bar{h} \right) \geq \alpha \quad (8)$$

$$\text{pr} \left( \sum_j \gamma_j x_j - \bar{h} \leq 0 \right) \geq \alpha \quad (9)$$

$$\text{pr}(A \leq 0) \geq \alpha \quad (10)$$

$$\text{pr} \left( \frac{A - E(A)}{\sqrt{\text{Var}(A)}} \leq 0 - \frac{E(A)}{\sqrt{\text{Var}(A)}} \right) \geq \alpha \quad (11)$$

Let us define

$$\frac{A - E(A)}{\sqrt{\text{Var}(A)}} = \rho \quad (12)$$

Therefore, constraint (4) can be converted to

$$\text{Pr} \left( \rho \leq - \frac{E(A)}{\sqrt{\text{Var}(A)}} \right) \geq \alpha \quad (13)$$

where  $\rho \sim N(0,1)$ . The inequality is satisfied if and only if

$$\Phi^{-1}(\alpha) \leq - \frac{E(A)}{\sqrt{\text{Var}(A)}} \quad (14)$$

Then

$$\Phi^{-1}(\alpha) \sqrt{\text{Var}(A)} \leq -E(A) \quad (15)$$

Therefore, the equivalence of constraint (4) is

$$\Phi^{-1}(\alpha) \sqrt{\sum_j x_j^2 \text{Var}(\gamma_j)} \leq \bar{h} - \sum_j E(\gamma_j) x_j \quad (16)$$

The final mathematical model is in the form of a MINP in which  $\Phi^{-1}(\alpha)$ ,  $E(\gamma_j)$ , and  $\text{Var}(\gamma_j)$  are known. The mean and variance of  $\gamma$  are obtained from simulating the disassembly process using ICT.

Objective function

Min  $\bar{h}$

Subject to

$$\Phi^{-1}(\alpha) \sqrt{\sum_j x_j^2 \text{Var}(\gamma_j)} \leq \bar{h} - \sum_j E(\gamma_j) x_j \quad j \in J$$

$$\sum_{j \in O_1} x_j = 1 \quad (\text{initial node})$$

$$\sum_{j \in I_1} x_j = \sum_{j \in O_1} x_j \quad (\text{transit nodes})$$

$$\sum_{j \in I_t} x_j = 1 \quad (\text{target node})$$

We have assumed that the disassembly transitions are independent. Therefore, we can use the fact that the summation of two independent normally distributed random variables is normal, with its mean being the sum of the two means, and its variance being the sum of the two variances.

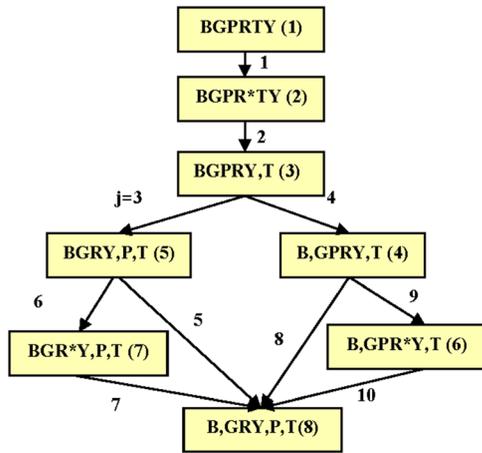
#### 4 Application of the Proposed Procedure

This section describes the application of the two-stage procedure and MINP model for a Burr puzzle. The Burr puzzle receives its name from the most traditional shape for the finished puzzle, a symmetrical set of interlocking cuboids thought to resemble a seed burr. Traditionally, they are made from wood and known in Asia and Europe since at least the 18th century [21].

Burr puzzles have unique geometric properties that make them appropriate for testing assembly/disassembly tasks. For example, only certain components can move at certain times and most movement is completely orthogonal to other movements. Figure 2 shows an example of a six-piece burr puzzle used in this project and its components. A label is assigned to each component of the burr puzzle.

The purpose here is to separate a selected set of components from the burr puzzle. This type of disassembly in which the target component(s) are given is called "selective disassembly." Applications include maintenance, and removal of high-value components prior to the shredding process often employed in material sorting and recycling operations. Here, the objective is to retrieve the "GRY" subassembly from the whole assembly. The general procedure is described below.

**4.1 Stage I.** This stage includes four steps that help the designer obtain a disassembly sequence applying the mathematical model.

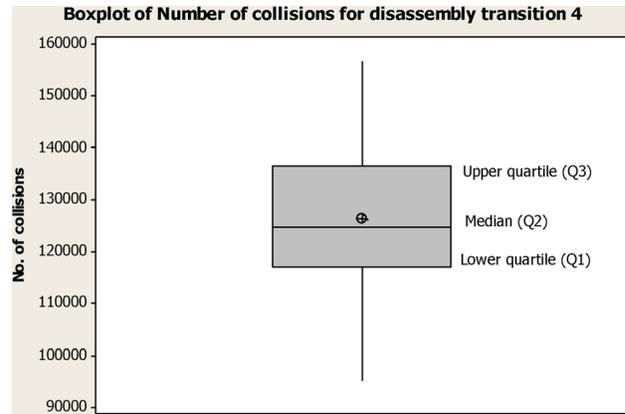


**Fig. 3 Feasible disassembly operations of six-piece Burr puzzle in the form of disassembly graph**

**4.1.1 Step 1: Representing the Feasible Disassembly Transitions.** The first step is to identify feasible disassembly operations from assembly drawings and present them in the form of a disassembly graph or network. Different methods have been developed to represent disassembly sequences, including AND/OR graphs, Petri net methods, undirected graphs and digraphs [22]. For complex products, the feasible disassembly operations can be listed in the form of a matrix called the transition matrix [23]. For the purpose of the burr puzzle example, the feasible disassembly operations and possible paths to reach the GRY subassembly is listed in the disassembly graph shown in Fig. 3. They are all feasible disassembly operations that leave GRY intact. Each arc of the graph indicates a single disassembly operation. The resulting subassemblies are listed in each node. There are three disassembly operations (1, 6, and 9) in which no component is removed from the assembly; instead components are repositioned slightly. The \* notation in the resulting nodes shows the component repositioned.

Disassembly graphs are constructed by defining the precedence relationships of the disassembly steps. The larger the problem, the greater the graph complexity and computational cost. Luckily, all possible combinations and permutations rarely need to be considered, since most real products are designed and assembled in such a way that the number of possible disassembly sequences is far fewer than the number that would result were there no precedence relationships. For example, a computer hard drive cannot be disassembled before it is first removed from the computer. All possible disassembly sequences must be included in the graph. For more complex products, the set of disassembly operations can be organized in the form of a matrix instead of a disassembly graph. This matrix called Transition matrix. In fact, the disassembly graph and transition matrix are two different ways of representing the same information. Several studies have already used transition matrices to represent the set of feasible disassembly transitions (e.g., Refs. [17,22,23]). For the shortest path modeling here, the disassembly graph has been used, which is easier to understand by users, compared to the transition matrix. Moreover, disassembly graphs may particularly be used for evaluation and training of disassembly sequences. Future work includes disassembly graph visualizations in the ICT environment to evaluate potential product disassembly sequences and provide input for redesign.

**4.1.2 Step 2: Augmenting the Graph With a Cost Structure.** After identifying feasible disassembly alternatives, the next step is to define the decision criterion or objectives. The criterion can be minimizing disassembly time, cost, probability of damage or any other criterion involved in disassembly operations. The aim of this example is to find the disassembly sequence with the minimum number of collisions between parts, as a proxy for the probability of incurring damage during the disassembly process. Therefore, in



**Fig. 4 The boxplot of the number of collision for disassembly transition 4**

this example the random variable  $\gamma_j$  refers to the number of collisions. Our interest here is in examining the amount of damage occurring during disassembly; however, any cost measure could be accommodated by this method.

**4.1.3 Step 3: Obtaining Cost Structure Data Applying the ICT.** The third step is to estimate the “cost” or other impacts incurred by carrying out each feasible disassembly operation (or transitioning each arc in the network). In the early design stages, estimating these values can be difficult. The potential of ICT can be exploited to overcome this difficulty. In the ICT environment, a user can virtually disassemble the puzzle and collect the data needed to make these estimates.

For this example, the numbers of collisions are recorded for approximately 30 trials of each feasible disassembly sequence conducted in the ICT environment. Each component is modeled as a collection of volume elements or voxels. Collisions are calculated on a voxel-to-voxel basis. When a user moves one component in contact with another, several thousand voxel collisions may occur. The user manually disassembles the puzzle in the ICT environment. Our rationale is that asking someone to actually disassemble the product will produce relatively reliable damage estimates. As the complexity of the assembly increases, the time to gather the damage estimates will increase. Additionally, this time also increases as the disassembly opportunities (graph) grow in complexity (independently of the number of components). The computational demands are minimal in calculating this cost.

Figure 4 shows the boxplot of the number of collisions recorded for transition 4 (dismantling component B from subassembly BGPRY). Two of four feasible disassembly sequences include transition 4, therefore the data are recorded for 60 trials of disassembly transition 4.

The number of collisions associated with disassembly operations are considered to be an uncertain parameter because these operations are performed manually, and will vary from operator to operator, and can also vary even for the same operator. Random influences result in a statistical distribution for the number of collisions. Statistical properties of  $\gamma$  have been investigated using Input Analyzer in ARENA simulation software. Table 1 lists the average, variance and the statistical distribution of the number of collisions generated from the ICT simulation of all disassembly transitions specified in the disassembly graph. Both “Chi square goodness of fit” and “Kolmogorov-Smirnov” tests have been conducted on the data to describe how well the distributions fit the set of data collected. The exact statistical distributions expressions and the corresponding P-values of “Chi square” and Kolmogorov-Smirnov are listed in the second, third, and fourth columns of Table 2, respectively. In cases in which the P-value is less than the significance level ( $\alpha$ ), the distribution fitted is not acceptable and more data points are needed to specify the exact distribution

**Table 1 The average and variance of the number of collisions for each disassembly transition generated from the ICT simulation**

Disassembly transition	Sample mean $\bar{y}_j$	Sample variance $s_{y_j}^2$	No. of data points	Distribution
1 (node 1 to 2)	2482	1,978,173	121	Weibull
2 (node 2 to 3)	2623	1,927,1275	118	Exponential
3 (node 3 to 5)	2775	1,174,823	61	Exponential
4 (node 3 to 4)	126,249	195,660,077	60	Normal
5 (node 5 to 8)	87,455	1268,367,196	31	Uniform
6 (node 5 to 7)	2994	4,658,269	29	Weibull
7 (node 7 to 8)	103,465	137,735,293	30	Triangular
8 (node 4 to 8)	8746	127,596,936	30	Weibull
9 (node 4 to 6)	2990	1,562,574	30	Exponential
10 (node 6 to 8)	2745	558,210	30	Exponential

Note: The outliers have been removed from the data set.

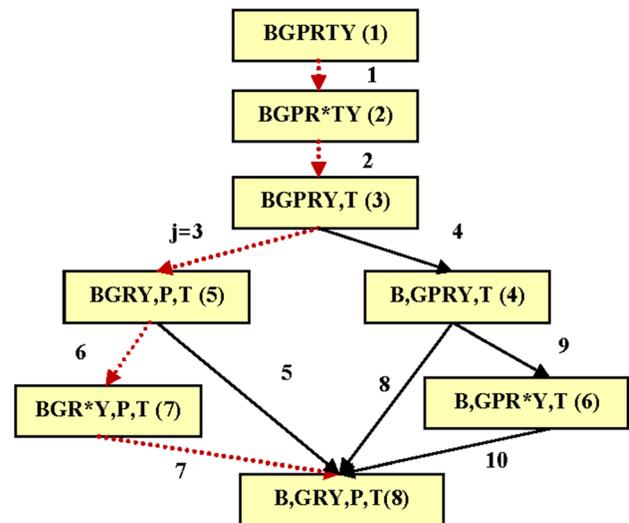
**Table 2 The average and variance of the number of collisions for each disassembly transition after transforming data to normal data**

Transition	Initial distributions	Chi square test P-value	Kolmogorov-Smirnov test P-value	Mean and St. Dev of transformed data $X \sim \text{Normal}(\mu, \sigma)$	Linear transformation of normal data derived from Box-Cox $Y = a * x \sim \text{Normal}(\mu', \sigma')$
1 (node 1 to 2)	959 + WEIB( $1.58 \times 10^3$ , 1.11)	0.395	> 0.15	(0.021880, 0.004799)	(2482, 544.4)
2 (node 2 to 3)	221 + EXPO( $1.97 \times 10^3$ )	0.0186	0.013	(0.12326, 0.03131)	(2624, 666.5)
3 (node 3 to 5)	$1.63 \times 10^3 + \text{EXPO}(1.15 \times 10^3)$	0.573	> 0.15	(0.000402, 0.000117)	(2775, 808)
4 (node 3 to 4)	NORM( $1.26 \times 10^5$ , $1.39 \times 10^4$ )	0.229	> 0.15	(126249, 13988)	(126,249, 13,988)
5 (node 5 to 8)	UNIF( $3.55 \times 10^4$ , $1.5 \times 10^5$ )	0.669	> 0.15	(289.4, 62.1)	(87,443, 18,758)
6 (node 5 to 7)	$612 + \text{WEIB}(2.69 \times 10^3, 0.793)$	< 0.005	> 0.15	(7.792, 0.662)	(2995.1, 254.6)
7 (node 7 to 8)	TRIA( $8.25 \times 10^4$ , $9.83 \times 10^4$ , $0.35 \times 10^5$ )	0.194	> 0.15	(11.541, 0.110)	(103466, 983)
8 (node 4 to 8)	$1.31 \times 10^3 + \text{WEIB}(4.5 \times 10^3, 0.535)$	< 0.005	> 0.15	(0.01665, 0.00736)	(8746, 3868)
9 (node 4 to 6)	$1.71 \times 10^3 + \text{EXPO}(1.28 \times 10^3)$	0.541	> 0.15	(0.000381, 0.000123)	(2990, 966)
10 (node 6 to 8)	$1.86 \times 10^3 + \text{EXPO}(890)$	0.365	> 0.15	(0.000388, 0.000092)	(2743, 654)

Note: The subgroup size is set to 1 for Box-Cox transformation.

of the data. As can be seen from the second column of Table 2, the normality assumption does not hold for  $\gamma$ . Therefore, the non-normal data have been transformed to normal data using Box-Cox transformation in MINITAB software. The fifth column of Table 2 shows the statistical properties of the number of collisions data after the data have been transformed to normal data. Finally, since the normal distribution is closed under linear transformations, if random variable X is normally distributed, then a linear transform  $aX + b$  is also normally distributed, therefore linear transformation has been used to rescale the normal data based on initial sample means. Rescaling the data makes them appropriate for comparison of the number of collisions of different disassembly transitions. The last column of Table 2 represents the linear transformation of the normal data. In the case of sufficiently large number of sample size, the central limit theorem can be applied instead of Box-Cox transformation and sample means can be calculated to be used as an input to the model.

**4.1.4 Step 4: Applying the MINP to Identify the Optimum Disassembly Sequence.** The Tomlab/minlpBB solver has been employed to solve the current optimization problem. Using the data provided in step 3 and solving the mixed-integer nonlinear programming model gives the optimal (lowest number of collisions) route from node 1 to node 8. Route 1-2-3-5-7-8 shown in Fig. 5 with a dashed line is the optimal disassembly sequence for reaching subassembly GRY, which is our target subassembly. The first step is to move the “R” component toward the operator (node 1 to 2), next “T” component is vertically removed from the assembly (node 2 to 3). From node 3 to 5, the “P” component is removed through one horizontal motion to the right. The “R” component is moved in the direction away from the operator until it is halfway exposed on the backside of the puzzle (node 5 to 7). Finally disassembly is complete by removing the “B” component through two orthogonal movements (node 7 to 8).



**Fig. 5 The disassembly graph and the optimum sequence derived from the mathematical model**

**4.2 Stage II.** The optimum disassembly sequence shown in Fig. 5 includes several operations that may not be intuitive. The first operation involves the movement of the “R” component toward the operator. This action causes the lower portion of the puzzle to be visually obscured by the “R” component making it difficult to evaluate the assembly. The removal of the “T” component is challenging, as the majority of the piece cannot be seen. At this point of the disassembly, node 3, there are two opportunities to remove components. Both the “B” and “P” components appear

physically constrained in similar ways. In actuality, the removal processes for these pieces are very different. The “P” component may be removed through a single horizontal manipulation, while the “B” component requires two distinct manipulations across two orthogonal axes; however, this constraint is not visually apparent. Following the optimal sequence, the “P” component is removed. Node 5 affords two disassembly operations. First, in efforts of wanting to complete the disassembly sequence, an operator may remove the “B” component. While this seemingly simple one-piece removal appears to be the intuitive choice, the optimal sequence instead calls for an intermediate operation. Instead of removing the “B” component, the “R” component is reoriented in the opposite direction away from the operator. This disassembly operation results in the exposure of the “B” component. From this perspective an operator has gained additional understanding as to the physical constraints holding the “B” component in the assembly. From this view it is apparent that the “B” component requires manipulations in two directions. The “B” component is removed and the disassembly objective is accomplished.

**4.2.1 Comparing the Results of MINP With the ICT Intuitive Solutions.** The intuitive disassembly path, however, diverts from the optimum path at node 5. While the optimal path calls for the reorientation of the “R” component, this appears to be an illogical operation considering the final objective (an assembly containing “G,” “R,” and “Y”). Seeking a path of lesser resistance, it may be more likely that, in efforts to reach the objective, an operator will attempt to remove the “B” component without cognizance of physical constraints.

This conflict between the optimal path and the intuitive path (via operator intuition) provides an opportunity for product redesign. The realignment of the “R” component, node 5 to node 7, presents the operator a stronger visual perspective of the “B” component’s interconnectedness within the assembly. Utilizing this new information, the operator may remove the “B” component while respecting physical constraints and minimizing potential damage. In efforts to serve both the disassembly objectives (minimize cost, minimize damage) and leverage operator intuition, the product may be redesigned to make the interconnectedness and physical constraints, more apparent to the operator during disassembly.

The second stage shows that sometimes the normative model results in a solution that is counterintuitive, even to those with expertise in disassembly procedures. Exploring the potential disassembly solutions in the ICT environment simulates the real world. In reality, disassembly sequence planning relies on expert qualitative judgment. Users conduct disassembly processes based on their particular knowledge about causality, disassembly time and constraints rather than on quantitative estimation of values. The result of the second stage can assist designers in modifying the product’s design. Therefore, each stage of the proposed method provides insights to the other.

## 5 Conclusion

A new procedure for disassembly sequence planning under uncertainty has been presented in this paper. The aim is to help designers determine the best sequence for product disassembly while considering uncertain disassembly process outcomes such as time, cost, or the probability of incurring damage. The proposed procedure consists of two main stages. In the first stage a stochastic programming model in the form of a mixed-integer nonlinear program incorporating data collected using ICT has been developed to determine an optimum disassembly sequence. Then, in the second stage the ICT has been applied to explore intuitive solutions. Finally, the results of both mathematical model and the ICT simulation can be combined to modify the product design.

The proposed procedure was tested on a six-piece Burr puzzle example. The objective was to find the disassembly sequence with

the lowest number of collisions between components. The data for the number of collisions were gathered through ICT simulation. Using the gathered data, the MINP model was solved and a solution was derived. Then, other intuitive solutions were explored using ICT tools.

We could have just assumed a distribution for the damage without using ICT. However, by employing the ICT to gather data to estimate the distribution, insights were gained and used to explore other potential disassembly methods that were not necessarily evident as a result of the mathematical model.

The current research can be extended to consider uncertainties in several attributes simultaneously, such as disassembly time and the probability of damage. One method for dealing with this is to convert and aggregate multiple attributes to a single one such as cost, and then minimize total expected cost. For example, disassembly time could be converted to labor cost, and the probability of damage could be converted to the cost of repairing the damage. Another option would be to explicitly consider tradeoffs among several attributes using normative decision analytic methods such as multiattribute utility analysis.

Future work includes studying the systematic cognitive biases that may happen while using immersive computing technologies to explore intuitive solutions. In addition, more detailed case studies of various products with high design modification and material recovery potential are needed to show the benefits of the proposed method.

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## References

- [1] Lambert, A. J. D., 2003, “Disassembly Sequencing: A Survey,” *Int. J. Prod. Res.*, **41**(16), pp. 3721–3759.
- [2] O’Shea, B., Grewal, S. S., and Kaebemick, H., 1998, “State of the Art Literature Survey on Disassembly Planning,” *Concurrent Eng.: Res. Appl.*, **6**(4), pp. 345–357.
- [3] Ying, T., MengChu, Z., Zussman, E., and Caudill, R., 2002, “Disassembly Modeling, Planning, and Application,” *J. Manuf. Syst.*, **21**(3), pp. 200–217.
- [4] Dong, J., and Arndt, G., 2003, “A Review of Current Research on Disassembly Sequence Generation and Computer Aided Design for Disassembly,” *Proc. Inst. Mech. Eng., Part B (J. Eng. Manuf.)*, **217**(B3), pp. 299–312.
- [5] Kang, J.-G., and Xirouchakis, P., 2006, “Disassembly Sequencing for Maintenance: A Survey,” *Proc. Inst. Mech. Eng., Part B: J. Eng. Manuf.*, **220**(10), pp. 1697–1716.
- [6] Reveliotis, S. A., 2007, “Uncertainty Management in Optimal Disassembly Planning Through Learning-Based Strategies,” *IIE Trans.*, **39**(6), pp. 645–658.
- [7] Behdad, S., and Thurston, D., 2012, “Disassembly and Reassembly Sequence Planning Tradeoffs Under Uncertainty for Product Maintenance,” *ASME J. Mech. Des.*, **134**(4), p. 041011.
- [8] Zussman, E., and Meng Chu, Z., 2000, “Design and Implementation of an Adaptive Process Planner for Disassembly Processes,” *IEEE Trans. Rob. Autom.*, **16**(2), pp. 171–179.
- [9] Zuidwijk, R., and Krikke, H., 2001, “Disassembly for Recovery Under Uncertainty,” *Proceedings of Environmentally Conscious Manufacturing II*, Oct. 28–29, SPIE-International Society for Optics and Eng., pp. 44–53.
- [10] Gupta, S. M., and Kongar, E., 2006, “Disassembly to Order System Under Uncertainty,” *Omega*, **34**(6), pp. 550–561.
- [11] Ying, T., and Turowski, M., 2007, “Adaptive Fuzzy System for Disassembly Process Planning With Uncertainty,” *J. Chin. Inst. Ind. Eng.*, **24**(1), pp. 20–29.
- [12] Tripathi, M., Agrawal, S., Pandey, M. K., Shankar, R., and Tiwari, M. K., 2009, “Real World Disassembly Modeling and Sequencing Problem: Optimization by Algorithm of Self-Guided Ants (ASGA),” *Rob. Comput.-Integr. Manuf.*, **25**(3), pp. 483–496.
- [13] Xanthopoulos, A., and Iakovou, E., 2009, “On the Optimal Design of the Disassembly and Recovery Processes,” *Waste Manage.*, **29**(5), pp. 1702–1711.
- [14] Behdad, S., Williams, A. S., and Thurston, D., 2012, “End-of-Life Decision Making With Uncertain Product Return Quantity,” *ASME J. Mech. Des.*, **134**(10), p. 100902.
- [15] Ruijun, L., Guangdong, T., Xueyi, Z., Anyan, Z., Xiaolan, W., and Qingning, N., 2011, “Disassembly Sequence Optimization for Automotive Product Based on Probabilistic Planning Method,” *Proceedings of 2011 International*

- Conference on Consumer Electronics, Communications and Networks, CECNet 2011, Apr. 16–18, IEEE Computer Society, pp. 284–288.
- [16] Erdos, G., Kis, T., and Xirouchakis, P., 2001, “Modelling and Evaluating Product End-of-Life Options,” *Int. J. Prod. Res.*, **39**(6), pp. 1203–1220.
- [17] Lambert, A. J. D., 2002, “Determining Optimum Disassembly Sequences in Electronic Equipment,” *Comput. Ind. Eng.*, **43**(3), pp. 553–575.
- [18] Kelsick, J., Vance, J. M., Buhr, L., and Moller, C., 2003, “Discrete Event Simulation Implemented in a Virtual Environment,” *ASME J. Mech. Des.*, **125**(3), pp. 428–433.
- [19] Box, G. E., and Cox, D. R., 1964, *An Analysis of Transformations*, Journal of the Royal Statistical Society. Series B (Methodological), Royal Statistical Society, Wiley, New York. Available at: <http://www.jstor.org/stable/2984418>
- [20] Alexander Shapiro, D. D., and Andrzej Ruszczyński, 2009, “Lectures on Stochastic Programming: Modeling and Theory,” *Society for Industrial and Applied Mathematics*, P. Toint, ed., Philadelphia, PA.
- [21] Burr Puzzle, 2013. Available at: [http://en.wikipedia.org/wiki/Burr\\_puzzle](http://en.wikipedia.org/wiki/Burr_puzzle)
- [22] Behdad, S., Kwak, M., Kim, H., and Thurston, D., 2010, “Simultaneous Selective Disassembly and End-of-Life Decision Making for Multiple Products That Share Disassembly Operations,” *Am. Soc. Mech. Eng.*, **132**(4), p. 041002.
- [23] Lambert, A. J. D., 1999, “Linear Programming in Disassembly/Clustering Sequence Generation,” *Comput. Ind. Eng.*, **36**(4), pp. 723–738.