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ABSTRACT

The literature on contests has focused primarily on the technology of the contest and perfectly divisible private rewards valued at a constant rate. This paper extends the analysis by explicitly modeling tastes in contests over pure public goods. Since the prize is embodied in a good, its value is endogenously determined. The interaction between tastes and technology leads to nonconvexities and possibly corner solutions so that individual participation in lobbying is not guaranteed. Sufficient conditions for interior solutions involve preference restrictions. Under these conditions endogenous valuation of the prize leads to income effects that limit free riding as the group expands, causing its lobbying expenditures to increase. Total expenditure on lobbying by both groups also increases as one or both expands.

CONTESTS WITH ENDOGENOUSLY VALUED PRIZES: THE CASE OF PURE PUBLIC GOODS

1. Introduction

Economic agents can allocate resources to three broad categories of activity: consumption, production, and conflict.[1] Relative to consumption and production decisions that have been extensively explored over the last two centuries, the economic theory of contests has just begun to receive rigorous analytical development [see for example Hirshleifer (1988)]. Contest theory has forced economists to consider how agents use resources to compete over rewards in which no directly productive activities are undertaken. The prime example is rent seeking in which two or more agents compete in the political arena to secure access to monopoly rents [see Tullock (1980)].

This paper considers two less explored areas of contest theory. First, we examine the interaction between tastes and technology. The main focus of contest literature has been on technology, represented by a mapping of effort levels into probability of success. Tastes are often bypassed by either assuming prizes are in monetary equivalents or that the agent's valuations are exogenous. But there is no reason to suspect that exogenous values exist in every contest. In fact, endogenous valuation of prizes drives many contests. This paper attempts to balance the emphasis by explicitly including tastes into the analysis. The multiplicative interaction between tastes and technology generates complex behavior patterns [see Shogren (forthcoming 1992)]. We explore some implications of this interaction.

Second, we consider contests over pure public goods. In general, contest theory has focused on perfectly divisible private rewards. A noted exception is Katz et al. (1990) who consider a rent seeking contest over a pure public good. We extend their model to include endogenously valued prizes given both income and substitution effects and quantity constraints. In pure subscription models, these effects have been shown to have significant impacts on the slopes of reaction function and free riding behavior [see Cornes and Sandler (1986)].

We derive six propositions. First, the slope of the within-group reaction function is negative and is less than one in absolute value. Second, an individual's contribution to the group decreases with group size. Third, the between-group reaction functions are positively sloped and agents' effort levels increase with an increase in the rival groups' aggregate effort levels. Fourth, as group size increases, aggregate effort levels on rent seeking increase. Fifth, wealth has a generally positive impact on effort, with noted exceptions due to relative group size. Sixth, as group size expands, total rent seeking increases and this total effort is independent of the distribution of members between groups.

The general lesson that emerges from these propositions is that in contests, especially those involving public goods, a richer variety of behavior is allowed by taking preferences into account. Including preferences allows other contributions to have income effects. As in pure subscription models, the income effects serve to limit free riding by preventing the individual from fully offsetting increases in others' contributions. However, unlike subscription models the income effects do not work through increased provision. Instead, they cause individuals to accord higher value to given quantity of the prize. It is for this reason the endogenous valuation is crucial. Moreover, explicit consideration of tastes in the form of a utility function permits modeling consumption relations between public and private goods. If such relations are present, groups retaliate against higher resource commitments to conflict by their rivals. This feature, which characterizes real life contests, is not addressed by models driven by technology alone.

The next section presents the basic model. Section 3 deals with competition within interest groups while competition between interest groups is considered in section 4. Section 5 analyzes the effect of group size on its aggregate political expenditure. Section 6 deals with relationships between wealth and political effectiveness. The question of how total rent seeking by both groups changes with their sizes is addressed in section 7. The final section contains concluding remarks.

2. A Model

Assume two groups compete for a prize that is a local public good. First we define the technology of the contest then the preferences and finally, we describe the interaction between them. Technology of the contest is represented by a function $\pi(.,.)$ that maps the aggregate resources allocated by the groups to the contest into probability of their success. For example, $\pi^A = \pi^A(A,B)$ is the probability that group A wins the prize, where $\pi^A = 1-\pi^B$. An increase in group A's expenditure increases its probability of winning, whereas an increase in the rival group B's expenditures decreases it, $\pi^A_A > 0$ and $\pi^A_B < 0$. Similar conditions hold for group B. We also assume additional lobbying increases the probability of winning at a decreasing rate $\pi^A_{AA} < 0$ and $\pi^A_{BB} > 0$.

The groups are composed of individuals with identical preferences and incomes. The initial quantity of public good is fixed at Q_l. If a group wins the prize, the quantity of public good available for its consumption increases to Q_h. Prizes are payable only in quantities of public good, which are like putty-clay. Once provided, the public good cannot be resold or exchanged for another good. When the higher level of public good Q_h is available, utility is $U(x,Q_h)$; otherwise it is $\tilde{U}(x,Q_l)$. The utility functions are nonseparable. Both $U(.)$ and $\tilde{U}(.)$ represent the same preferences but different levels of public good. Note the quantity of public good is not a choice variable.

The representative individual in each group maximizes expected utility subject to an income constraint by choosing the quantity of private good and the level of effort. The aggregate group expenditure affects the probability of getting the public good. The representative individual maximization problem is presented below. We suppress the group superscript to simplify notation.

$$MAX_{ax}\ Z=\pi(a_i+\tilde{A};B)U(x,Q_h) + [1-\pi(a_i+\tilde{A};B)]\ \tilde{U}(x,Q_l) + \lambda[M-px-a] \tag{1}$$

where

x = private good,

Q_h = quantity of public good if the group wins the prize,

Q_l = quantity of public good if the group fails to win,

a = the representative individual's effort,

$\tilde{A}$ = effort of all other group members,

A = aggregate expenditure of the group,

B = aggregate expenditure of the rival group,

M = individual income,

p = price of the private good,

n = number of members in group A, and

m = number of members in group B.

Assuming Nash-Cournot behavior, the first-order conditions are

$$-px - a + M = 0 \tag{2}$$

$$\pi(a_i + \tilde{A};\ B)U_x(x,Q_h) - [1 - \pi(a_i + \tilde{A};\ B)]\tilde{U}_x(x,Q_l) - \lambda p = 0, \tag{3}$$

and

$$\pi_A(a_i + \tilde{A};\ B)\Big[U(x,Q_h) - \tilde{U}(x,Q_l\ )\Big] - \lambda = 0. \tag{4}$$

Now we turn to the interaction between tastes and technology. Although U(.) and π(.) are concave, their product is not a concave function. Therefore, the maximand is not necessarily concave, implying the possibility of a corner solution. Individuals may spend all their income either on lobbying or on the private good, x. This may explain why certain groups remain politically inactive. Now we want focus on the case where interior solutions are obtained and impose concavity. Concavity requires that

$$|\bar{H}| = \begin{vmatrix} 0 & -p & -1 \\ -p & [\pi.(U_{xx} - \tilde{U}_{xx}) + \tilde{U}_{xx}] & \pi_A.(U_x - \tilde{U}_x) \\ -1 & \pi_A.(U_x - \tilde{U}_x) & \pi_{AA}.(U - \tilde{U}) \end{vmatrix} > 0. \tag{5}$$

Expanding $|\overline{H}|$ we get

$$- p^2.\pi_{AA}.(U - \tilde{U}) + 2p\pi_A.(U_x - \tilde{U}_x) - \pi.(U_{xx} - \tilde{U}_{xx}) - \tilde{U}_{xx}. \tag{6}$$

The following conditions are sufficient, but not necessary, for this expression to be positive.[2]

$$\nabla = (U - \tilde{U}) \geq 0,\ \nabla_x = (U_x - \tilde{U}_x) \geq 0,\ \nabla_{xx} = (U_{xx} - \tilde{U}_{xx}) \leq 0. \tag{7}$$

The first condition simply requires that the additional quantity of public good must not decrease the utility level. The second requires that marginal utility of the private good, x, should increase, or at least remain constant, with higher level of public good provision. The last condition implies that at a higher level of public good provision, the marginal utility of private good declines faster, or at least at the same rate, than at a lower level. For the rest of the analysis, these inequalities are assumed to hold in strict form. Since these conditions involve restrictions on taste, they define a "preference profile." Imposing these conditions is equivalent to requiring that the representative individual should have this personality profile in terms her tastes.

3. Competition within the Interest Group

Under the Nash-Cournot assumptions, the individual makes the contribution "a" by taking as given the contributions of other members of his group $\tilde{A}$, and also the aggregate political expenditures of the rival group B. A relevant issue is how an individual contribution changes in response to parametric variation in $\tilde{A}$, the slope of the reaction function. In particular, we consider whether (i) the slope is negative , positive or zero, and (ii) the absolute value of the slope is less than, equal to, or greater than one. A negative slope indicates free riding. If the absolute value of the slope is equal to one, this would imply that an increased contribution by one member is exactly offset by decreased contributions by all other members, leaving total expenditure unchanged. By the same logic, if the absolute value of the slope is less than one, reductions in contribution would not fully offset the additional contribution by a member. The total expenditure would rise.

Totally differentiating the first-order conditions with respect to $\tilde{A}$ where

$$da/d\tilde{A} = -\frac{p\begin{vmatrix} -p & \pi_A \triangledown_x \\ -1 & \pi_{AA}\triangledown \end{vmatrix}}{p\begin{vmatrix} -p & \pi_A \triangledown_x \\ -1 & \pi_{AA}\triangledown_x \end{vmatrix} + \begin{vmatrix} -p & -(\pi_A \triangledown_{xx} + \tilde{U}_{xx}) \\ -1 & -\pi_A \triangledown_x \end{vmatrix}} = -\frac{D_1}{D_1 + D_2} < 0, \tag{8}$$

where

$$D_1 = p[-p\pi_{AA}\triangledown + \pi_A \triangledown_x] > 0$$

$$D_2 = [p\pi_A \triangledown_x - (\pi_A \triangledown_{xx} + \tilde{U}_{xx})] > 0.$$

Note that the reaction function's slope can never have an absolute value of unity. This holds since D_2 is a positive definite by assumption. This leads to the following proposition.

Proposition 1. The slope of the *within* group reaction function is negative and less than one in absolute value.

The terms $\triangledown_x$ and $\triangledown_{xx}$ in the expression for slope of the within-group reaction function represent the consumption relation between public and private goods. To understand how changes in valuation of the prize affect the slope, assume public and private goods are unrelated in consumption, $\triangledown_x = 0 = \triangledown_{xx}$. This implies $da/d\tilde{A} = -[-p^2 \pi_{AA} \triangledown]/[-p^2 \pi_{AA} \triangledown - \tilde{U}_{xx}]$.

The slope of the within-group reaction function is still negative and less than one in absolute value. The intuition behind this result is as follows. An individual considers others' contribution to lobbying ($\tilde{A}$) as a perfect substitute for her own. If $\tilde{A}$ increases by one dollar, she reduces her contribution by exactly a dollar. But, now she has an extra dollar to spend, as if her income has increased by a dollar. This is the income effect. Since public good is supplied only by the government, the surplus dollar is spent on the private good, driving down its marginal utility

($\tilde{U} < 0$) . Marginal utility of the public good remains the same because its consumption has not changed. This implies that the valuation of the public good, as measured by its marginal rate of substitution, has increased. As the prize increases in value, stakes become higher, so the individual is willing to allocate more resources to the contest [Hirshleifer (1991) has expressed a similar view]. Therefore, the reduction in an individual's contribution is not as much as the increase in $\tilde{A}$.

Note the income effect works indirectly through the private good x. If the marginal utility of x is constant (i.e., $U_{xx} = 0$), the valuation of the prize does not change. Only the technology of the contest drives the reaction function and we get exactly the same result as Katz et al. (1990), a slope of exactly minus one.

Now it is possible to see how individual contribution changes in response to group size. First, note that n can affect a only through $\tilde{A}$. Hence $da/dn = da/d\tilde{A} \cdot d\tilde{A}/dn$. Second, all individuals in the group are identical; therefore, by symmetry $\tilde{A} = (n-1)a$. This implies $d\tilde{A}/dn = d[(n-1)a]/dn = (n-1)\, da/dn + a$.

Substituting the expression for $d\tilde{A}/dn$ into that for da/dn and collecting terms, we get

$$da/dn = \left(\frac{a}{(1+\delta)}\right) \cdot da/d\tilde{A} < 0 \qquad (9)$$

$$\text{since } \delta = (1-n) \cdot da/d\tilde{A} > 0 \text{ for } n \geq 2 \quad \text{and } da/d\tilde{A} < 0.$$

This result is presented in the following proposition.

Proposition 2. An individual's contribution to the group decreases with group size.

This result shows that the individual contribution does decline as others join the group. However, group size affects the representative individual only through an increase in others' contribution $\tilde{A}$. By Proposition 1, an increase in $\tilde{A}$ is not fully offset by a decrease in the individual's contribution. Taken together, both these propositions suggest the group's total expenditure on lobbying increases with size. This is more rigorously demonstrated in a later section.

4. Competition Between Interest Groups

Now consider the response of a group to an increase in the political expenditure by the rival group. The earlier sections considered only a single utility maximizing individual. The issue addressed in this section is how a group of identical utility maximizing individuals behaves when making contributions to lobby activities. Since the first-order conditions are in the form of implicit functions, analytical solutions for the individual's contribution, a_i, are impossible. Therefore, the expression for aggregate lobbying expenditure of the group cannot be obtained by adding the expressions for individual contributions. Instead, we will use an indirect approach.

Recall the vector of parameters ($\tilde{A}$, B; Q_h, Q_l, p, M) solves the first-order conditions, at least in principle. While $\tilde{A}$ is a parameter for the individual, it is an endogenous variable for the whole group. The reason is that $\tilde{A}$ is the aggregation of the behaviors of "everyone else" in the group except the representative individual. By making the substitution

$$\bar{A} = A - a \tag{10}$$

we eliminate a and $\tilde{A}$ from the first-order conditions (2) through (4). We still have a system with four unknowns to solve (x,λ,a, and A) and only three equations. The additional equation is provided by the definition

$$A = na. \tag{11}$$

Using this equation, a can be eliminated, with the resulting system:

$$-px - \frac{1}{n}A + M - 0 \tag{12}$$

$$\pi(A;\ B)U_x(x,Q_h) - [1 - \pi(A;\ B)]\tilde{U}_x(x,Q_l) - \lambda p = 0 \tag{13}$$

$$\pi_A(A;\ B)U(x,Q_h) - \pi_A(A;\ B)\tilde{U}(x,Q_l) - \lambda = 0. \tag{14}$$

Using Cramer's rule,

$$\frac{dA}{dB} = \frac{-p\pi_B\nabla_x + p^2\,\pi_{AB}\nabla}{(1+\frac{1}{n})p\pi_A\nabla_x - \frac{1}{n}(\pi\nabla_{xx}+\tilde{U}_{xx}) - p^2\pi_{AA}\nabla}. \tag{15}$$

To sign this expression we have to know the sign of π_{AB}. Dixit (1987, p. 892) has pointed out that if there is perfect symmetry between players an interchanging of effort levels would interchange probabilities of winning. Hence $\pi_{AB} = 0$.[3] This allows us to write

$$\frac{dA}{dB} = \frac{-p\pi_B\nabla_x}{(1 + \frac{1}{n})p\pi_A\nabla_x - \frac{1}{n}(\pi\nabla_{xx} + \tilde{U}_{xx}) - p^2\pi_{AA}\nabla} > 0. \tag{16}$$

Since π_A, ∇_x, $\nabla > 0$ and π_B, ∇_{xx}, $\tilde{U}_{xx}$, $\pi_{AA} < 0$. This implies the group increases its lobbying expenditure in response to an increase in similar expenditures by the rival group. Further note,

$$\frac{\partial a}{\partial B} = \frac{1}{n}\frac{\partial A}{\partial B} > 0 \qquad\qquad since \quad a = \frac{1}{n}A\ . \tag{17}$$

This implies each individual in the group contributes more when the rival group increases its lobbying expenditure.

If public and private goods are unrelated in consumption, $\nabla_x = 0 = \nabla_{xx}$, then the numerator in equation (16) becomes zero. The group does not react to increased lobbying by its rival. This is exactly the result Dixit (1987) obtained from the assumption of exogenously valued prizes.

But in our model public and private goods are related in consumption; therefore, ∇_x and ∇_{xx} are nonzero. This leads to:

Proposition 3. (i) The *between* group reaction functions are positively sloped; (ii) Individuals' expenditures increase with an increase in the rival group's aggregate expenditure.

This suggests an action-reaction relationship between groups, such as an arms race between nations. If one side increases its military spending, the other side may do the same. Another example is deterrence against international terrorism. Suppose a group of countries faces a threatened attack from the same terrorist organization. Increasing in deterrence expenditures by one country decreases the probability of terrorist success in attacks against it and increases the probability that the terrorist would choose another country as its target. Sandler and Lapan (1988) demonstrate that this leads the other countries to increase their respective deterrence expenditures to an inefficient level. A similar situation arises in protecting against crimes such as drugs. If one city gets tough on drug dealers (by increasing the police force and imposing stiffer penalties), they move their operations to other cities, inducing those cities to adopt similar measures.

In these examples, groups react to their rival's actions. If equilibrium is to be reached, the sequence of actions and reactions must converge. This will happen if certain stability conditions are satisfied.

Sufficient conditions for stability. Stability requires $\partial A/\partial B<1$.[4] From Equation (16) above, a sufficient condition to be satisfied is $|\pi_A| = |\pi_B|$.

Dixit (1987, p. 892) has shown that if groups are symmetric, $\pi_A = -\pi_B$. Therefore, this stability condition holds. The group reaction functions have a positive slope less than 1.

5. Group Size and Aggregate Political Expenditure

The effect of an increase in group size on the amount of rent seeking by the group can be determined by differentiating equations (12) through (14) for n and using Cramer's rule:

$$dA/dn = \frac{\frac{a}{n}[p\pi_A\nabla_x - (\pi\nabla_{xx} + \bar{U}_{xx})]}{(1+\frac{1}{n})p\pi_A\nabla_x - \frac{1}{n}(\pi\nabla_{xx}+\bar{U}_{xx}) - p^2\pi_{AA}\nabla} > 0. \tag{18}$$

Equation (18) is positive since π_A, ∇_x, > 0 and ∇_{xx}, $\tilde{U}_{xx}$ < 0. Thus the group's expenditures increase with an increase in its size, given the level of such expenditures by the rival group.

But expenditures by the rival group do not remain constant when the group in question increases its own expenditures due to increased size. We demonstrate in the Appendix that the sequence of actions and reactions converges to a new equilibrium involving higher levels of group expenditures provided the stability condition is met.

Figure 1 shows the reaction functions R^A and R^B of groups A and B. In accordance with the stability condition, both curves have a slope less than one. The equilibrium is at point E_0, where the curves intersect. If group A increases in size, then spending for every level of expenditure by group B increases. The reaction function for group A shifts to the right. The new intersection is at E_1. At this point the equilibrium level of rent seeking by group A is A_1, which is higher than before. Hence:

> **Proposition 4:** As the size of the group increases, its aggregate expenditure on the contest activities also increases.

The proof for this proposition is in the Appendix. This result is consistent with McGuire (1974), who found that aggregate expenditure on public good increases with group size.[5] A similar result was also obtained by Chamberlin (1974). But this proposition has to be interpreted with caution. The increase in aggregate expenditure due to size increase does not imply that there is less free riding than before. The proposition says nothing about the relationship between size and the extent of free riding. The latter occurs because rational agents maximize individual as opposed to group welfare. For a given group size, free riding is the difference between the *actual* and the Pareto optimal aggregate expenditures.[6] The proposition merely states that the group's *actual* expenditures increase as membership grows. This does not rule out the possibility that sub-optimality, in the sense of divergence between actual and the optimum, also increases.[7]

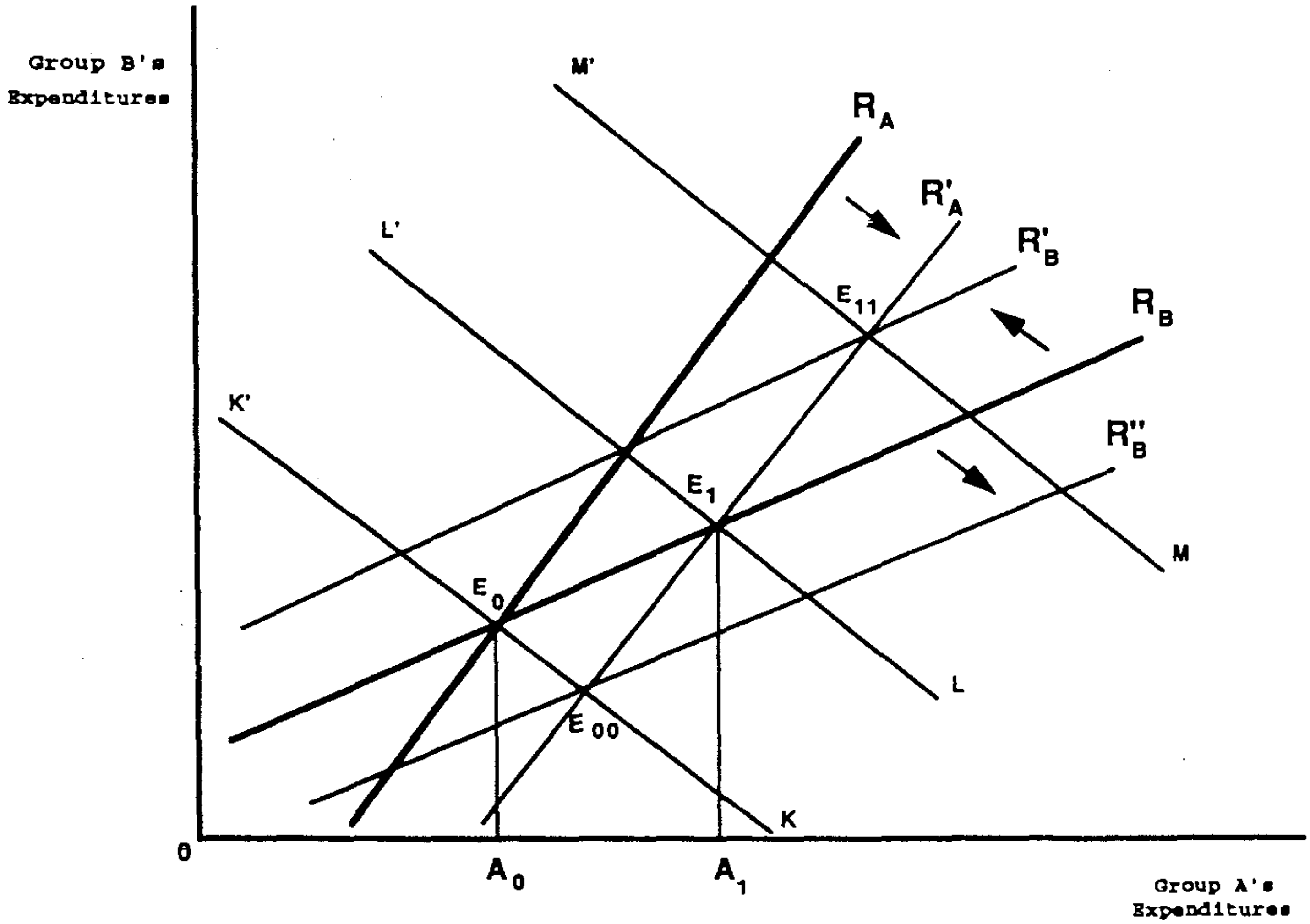

Figure 1. Competition between groups

Therefore, inasmuch as political effectiveness depends on resources committed to lobbying, groups do not become weak as they grow larger, even though free riding may increase.

6. Wealth and Political Effectiveness

This section considers the effect of increase in wealth on aggregate lobbying expenditure of the group. Suppose that incomes of all the individuals in the group increase by the same proportion. Differentiating conditions (12) through (14) for M and using Cramer's rule,

$$dA/dM = -\frac{[-p\pi_A \nabla_x + (\pi\nabla_{xx} + \tilde{U}_{xx})]}{(1+\frac{1}{n})p\pi_A\nabla_x - \frac{1}{n}(\pi\nabla_{xx} + \tilde{U}_{xx}) - p^2\pi_{AA}\nabla} > 0, \tag{19}$$

$$since\ \pi_A,\ \nabla_x, > 0\ and\ \nabla_{xx},\ \tilde{U}_{xx} < 0.$$

For a given level of a rival group's political expenditure, an increase in wealth increases political expenditure. In Figure 1, an increase in wealth shifts the reaction function of group A to the right, resulting in a higher equilibrium level of expenditure A_1.

Note that this result has been derived by keeping all other variables, including n, constant. Therefore, if we want to conclude that wealthier groups do more rent seeking, we must limit our attention to groups of equal size.

Also note that individual rent seeking is 1/n times the group rent seeking; that is, a = (1/n) A. Therefore,

$$\frac{\partial a}{\partial M} = \frac{1}{n}\frac{\partial A}{\partial M} > 0 \qquad since\ a = \frac{1}{n}A\ . \tag{20}$$

We have not considered the more plausible case where the wealthy group is smaller. In this situation the *wealth effect* tends to increase aggregate lobbying expenditure, as shown above. But it follows from Proposition 4 that the smallness of the group would tend to reduce its lobbying expenditure. Consequently, the net result is ambiguous and depends on the relative strengths of the *wealth* and *size* effects. These results are summarized in the fifth proposition:

Proposition 5. (i) If two groups are of equal size, then the wealthier group will spend more on lobbying activities. (ii) Given the size of the group, individual's contribution to lobbying increases with an increase in wealth. (iii) If the wealthy group is smaller, then it is ambiguous whether its expenditure on lobbying is more than, equal to, or less than the poor group.

This proof is also in the Appendix. Katz et al. (1990) have also concluded that the richer group does more rent seeking. In this regard, there are two points worth noting. First, to get this result they allowed the rate at which individuals value the public good to change with the level of wealth, thus departing from their earlier assumption that individuals value the public good at a constant rate α. In this respect they come somewhat closer to our more general case where the individual valuation of the public good is variable and is endogenously determined. Second, in contrast to Proposition 5(iii) above, their result would not change even if the wealthy group is smaller because in their model rent seeking by the group is independent of its size.

Proposition 5 implies that smaller groups can be effective if they are considerably wealthier. Alternatively, poor groups can also be effective, provided they are considerably larger, free riding due to size not withstanding. For example, there is a larger disparity in per capita incomes among the group of countries called the "super powers" than there is in their comparative defense capabilities. Numbers compensate for lower individual wealth levels to a certain point. France has a high per capita income, China a large population, yet military capabilities are similar.

7. Total Rent Seeking by Both Groups

Rent seeking involves channeling scarce resources into unproductive uses. Therefore, the total magnitude of social waste from this activity, and how it changes with group size, is of great interest to economists. Now that we have shown how rent seeking by a single group changes as it expands and how groups react to rent seeking by their rivals, we have the building blocks for the answer.

First, we assume our point of departure is a symmetric Nash equilibrium involving two equally sized groups. This implies both groups expend equal resources on lobbying. Second, we consider three cases of size change:

Case 1. Only group A increases in size, $dn > 0$, $dm = 0$.

Case 2. Both groups increase in size by attracting equal number of new members, $dn = dm > 0$.

Case 3. Members leave one group to join the other group, $dn = - dm$.

Case 1. From equation (18), this implies that political expenditures, A, increase for each level of expenditure by the rival group. Figure 1 illustrates that the reaction function of group A shifts to the right. The new equilibrium is established at E_1. The line L'L is drawn with a slope of 45° and passes through E_1. The horizontal intercept of this line represents the sum of rent seeking done by both groups at E_1. This line completely dominates the similar 45° line K'K, drawn through the initial equilibrium E_0. Therefore, the sum of rent seeking done by both groups is higher at E_1 than at E_0.

Case 2. As in Case 1, group A's reaction function shifts to the right and group B's reaction function (which is a mirror image of group A's reaction function) shifts to the left. The new equilibrium is at E_{11}. The 45° line through E_{11} is M'M. This line not only dominates K'K but also L'L. Thus rent seeking increases when both groups increase in size. Further, the magnitude of increase is greater when both groups expand as compared to the case when only one group expands.

Case 3. Suppose some members leave group B to join group A, $dn = -dm$. Group A's reaction function shifts to the right to $R^{A'}$. Since members leave group B, its political expenditures decline for all levels of such expenditures by group A. Group B's reaction function also shifts to the right. The new reaction function for group B is

$R^{B'}$. Equilibrium is at E_{00}. The new equilibrium lies below the E_0 along the 45° line through that point. Hence the sum of rent seeking done by both groups remains same.

Proposition 6. (i) Assuming an initial, symmetric Nash equilibrium, involving groups of equal size, an increase in membership of one group causes the sum of rent seeking done by *both* groups to rise. (ii) Assume a symmetric equilibrium involving groups of equal size. If both groups expand by gaining an equal number of members, then the sum of rent seeking done by them increases. Furthermore, the magnitude of this increase is larger as compared to the increase which results from expansion of a single group. (iii) The sum of rent seeking done by both groups is independent of the distribution of members between them.

This proof is in the Appendix. Proposition 6 underscores the significance of numbers to rent seeking for pure public goods. It is somewhat discouraging to suggest social waste due to rent seeking cannot be reduced even by redistributing members between groups. But this result is due to the pure publicness of the prize. If some form of impurity (e.g., congestion) is introduced, the total amount of rent seeking may no longer remain independent of the distribution of members between groups [see Cornes and Sandler (1986)].

8. Concluding Remarks

The objectives of this paper were twofold. First, we explored contests with prizes that were embodied in goods and services and, therefore, had an endogenously determined value. It is shown that behavior is no longer determined by contest technology alone. The decisions of economic agents are outcomes of complex interaction between tastes and technology. Second, we extend the analysis of rent seeking contests over pure public goods to include both substitution and income effects. Our model gives significantly different results with the inclusion of this structure.

Other implications include that, although individual contributions decline with an increase in group size, the aggregate lobbying expenditure of the group increases. Rival groups react to increases in each other's expenditures by raising their own expenditure. Income increases lead to higher individual contributions to lobbying. For groups of equal size, higher member incomes also increase aggregate lobbying expenditures; if groups are of unequal size, the result is ambiguous. The sum of rent seeking by both groups increases even when only one group expands. If two groups are identical and they expand at the same rate, the increase is twice as much. Total rent seeking by both groups is independent of the distribution of members.

An important consequence is the issue of nonconvexity. In order to ensure interior solutions, some preference restrictions were imposed. These restrictions define a personality profile of politically active individuals. The individuals whose preferences do not fit this profile may choose to remain politically inactive. Frequently, the literature on political economy attributes the inactivity of certain groups to higher costs of organization (e.g., due to large numbers or geographic dispersal). In our view, nonconvexities provide an alternative explanation.

APPENDIX

The comparative static results presented were derived on the assumption that political expenditure of the rival group remained the same. Therefore, changes in group expenditure indicated by these results were not equilibrium changes but merely shifts in the reaction functions. The resulting changes in equilibrium were only shown graphically.

Now we are going to derive expression for these changes mathematically. This will be done by allowing the rival group's expenditure to change in response to initial change in expenditure by the first group.

Recall that

$$a = a(\tilde{A},\ B;\ Q_h,\ Q_l,\ p,\ M). \tag{A-1}$$

$$But \quad \tilde{A} = (n-1)a \quad , \quad A = a+\tilde{A}.$$

Therefore,

$$A = A(n,\ B;\ Q_h,\ Q_l,\ p,\ M)$$

or,

$$A = A(\theta_1\ ,\ B;\ .) \qquad where\ \theta_1 = (n,\ Q_h\ ,\ Q_l\ ,\ p,\ M). \tag{A-2}$$

Similarly,

$$B = B(\theta_2\ ,\ A;\ .\ ) \qquad where\ \theta_2 = (m,\ Q_h\ ,\ Q_l\ ,\ p,\ M). \tag{A-3}$$

Note that 'A' depends on 'B' and vice versa. In principle, these equations can be solved simultaneously to give reduced forms for 'A' and 'B'. But since we are dealing with general functions and not specific functional forms, this is impossible. We could still substitute the general expression for 'B' into that for 'A'. Of course 'B', in turn, depends on 'A' and therefore repeating this process generates an infinite loop. The equation below represents this loop.

$$A = A[\theta_{1i}, B(A(\theta_{1i} B(...(...); \theta_{2j}))] \tag{A-4}$$

A derivative of 'A' with respect to any parameter say, θ_{1i} would involve sum of an infinite series. We use the following lemma to prove the comparative static results. The lemma illustrates the sum of an infinite series

__Lemma:__ The comparative static for aggregate group expenditures can be written as

$$(\frac{dA}{d\theta_{1i}}) = \frac{(\frac{\partial A}{\partial \theta_{1i}}|_B)}{1 - (\frac{\partial A}{\partial B})^2} \tag{A-5}$$

where θ_{1i} is a component of the parameter vector θ_1.

PROOF: Differentiating equation (A-4) with respect to θ_{1i} yields

$$\frac{\partial A}{\partial \theta_{1i}} = \frac{\partial A}{\partial \theta_{1i}}|_B + \frac{\partial A}{\partial B}\cdot\frac{\partial B}{\partial A}\left[\frac{\partial A}{\partial \theta_{1i}}|_B + \frac{\partial A}{\partial B}\cdot\frac{\partial B}{\partial A}(.....)\right]. \tag{A-6}$$

Since the two groups are identical and the equilibrium is symmetric,

$$\frac{\partial A}{\partial B} = \frac{\partial B}{\partial A} \quad \rightarrow \quad \frac{\partial A}{\partial B} \cdot \frac{\partial B}{\partial A} = (\frac{\partial A}{\partial B})^2.$$

Substitute this in (A-6) and

$$\begin{aligned} &let \quad \frac{\partial A}{\partial \theta_{1i}}|_B = c \qquad \frac{\partial A}{\partial B} = r \\ &\therefore \quad \frac{\partial A}{\partial \theta_{1i}} = c + r^2 (c + r^2 (c + r^2 (.... \\ &\qquad\qquad = c (1 + r^2 + r^4 + ... \end{aligned} \tag{A-7}$$

The series converges to c $/(1-r^2)$, provided $r^2 < 1$. Therefore,

$$\frac{\partial A}{\partial \theta_{1i}} = \frac{\frac{\partial A}{\partial n}\Big|_B}{1 - (\frac{\partial A}{\partial B})^2} \qquad if \quad \frac{\partial A}{\partial B} < 1 \ .$$

Q.E.D.

For a change in an arbitrary parameter θ_{1i}, the lemma shows that (i) the total derivative $dA/d\theta_{1i}$ has the same sign as the partial derivative $(\partial A/\partial \theta_{1i})|_B$ and (ii) the expression for total derivative can be obtained from expressions for the partial derivative and the slope of the group reaction function $(\partial A/\partial B)$.

Now it is a simple matter to provide proofs for the Propositions 4-6.

Proof Proposition 4: Let θ_{1i} =n. Apply lemma.

Q.E.D.

Proof Proposition 5(i): Let θ_{1i} =M. Apply lemma.

Q.E.D.

(ii) Follows from Proposition 5(i)

(iii) Follows from Proposition 5(i) and Proposition 4.

Proof Proposition 6(i):

$$\frac{\partial(A+B)}{\partial n}\Big|_{n=m} = \frac{\partial(A)}{\partial n} + \frac{\partial B}{\partial A} \cdot \frac{\partial A}{\partial n} \qquad \text{(A-8)}$$

$$= \frac{\partial(A+B)}{\partial n}\left(1 + \frac{\partial B}{\partial A}\right)$$

Q.E.D.

$$= \frac{\frac{\partial A}{\partial n}|_B}{1 - (\frac{\partial A}{\partial B})^2}(1 + \frac{\partial B}{\partial A}) > 0 \qquad \text{(A-9)}$$

By lemma and $\frac{\partial A}{\partial n}|_B > 0$, $\frac{\partial A}{\partial B} > 0$.

Q.E.D.

(ii) The first term on the R.H.S is the effect on total rent seeking, A+B, of change in size of group A (i.e., n) and the second is the effect of change in group B's size (i,e., m). Further, due to symmetry, the slopes of the group reaction functions, as well as the effect of size on aggregate lobbying expenditure, are equal:

$\partial A/\partial n = \partial B/\partial m$ and $\partial A/\partial B = \partial B/\partial A$.

Therefore, by proposition 6(i)

$$\frac{\partial(A+B)}{\partial n}\Big|_{\substack{n=m \\ dn=dm}} = 2\left[\frac{\partial(A)}{\partial n} + \frac{\partial B}{\partial A}\cdot\frac{\partial A}{\partial n}\right] > 0.$$

Q.E.D.

(iii) Because of symmetry,

$$\frac{\partial(A+B)}{\partial n}\Big|_{dn=dm}^{n=m} = \left[\frac{\partial(A)}{\partial n} + \frac{\partial B}{\partial A}\cdot\frac{\partial A}{\partial n}\right] - \left[\frac{\partial(A)}{\partial n} + \frac{\partial(A)}{\partial n} + \frac{\partial B}{\partial A}\cdot\frac{\partial A}{\partial n}\right]$$

$$= 0 .$$

Q.E.D

ENDNOTES

1. Refers to forcible appropriation of resources under control of other economic agents.

2. Let $h = Q_h - Q_l$. As $h \to 0$, the first condition implies $\lim_{h \to 0} [\{U(x,Q_l+h) - U(x,Q_l)\}/h] = U_Q \geq 0$. Similarly the second condition implies $\lim_{h \to 0} [\{U_x(x,Q_l+h) - U_x(x,Q_l)\}/h] = U_{xQ} \geq 0$. Finally, from the last condition, we have $\lim_{h \to 0} [\{U_{xx}(x,Q_l+h) - U_{xx}(x,Q_l)\}/h] = U_{xxQ} \leq 0$. Therefore, for an individual to be politically active (which would be the case if there are interior solutions) she must have a certain preference profile. This profile is depicted by restrictions on preferences stated above.

3. Dixit's argument is as follows. If there is perfect symmetry between players, interchanging of effort levels interchanges the probabilities of their success, so that $\pi(B,A) = 1 - \pi(A,B)$. Differentiating with respect to A, $\pi_B(B,A) = - \pi_A(A,B)$. Again, differentiating with respect to B, we get $\pi_{BA}(B,A) = - \pi_{AB}(A,B)$. In a symmetric Nash equilibrium, involving groups of equal size, $A^* = B^* = Y$. Hence, $\pi_{BA}(Y,Y) = - \pi_{AB}(Y,Y)$. But $\pi_{BA}(Y,Y) = \pi_{AB}(Y,Y)$. Therefore, $\pi_{AB} = 0$.

4. See lemma in appendix.

5. See McGuire (1974) p. 112. McGuire also demonstrated that as group size increases to infinity, group expenditure approaches a finite limiting value. However, this is not the same as Katz et al. assertion that group expenditures are constant no matter what the group size.

6. Optimality is viewed from groups' perspective.

7. If the objective is to see how free riding is affected by change in group size, a ratio of the differences between the actual and the optimum expenditures at the respective group sizes may provide a crude index [see Cornes and Sandler (1986) p. 80].

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