NONDESTRUCTIVE EVALUATION WITH BEAMFORMING TRANSDUCER ARRAYS

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ABSTRACT

If a nondestructive evaluation system is designed to detect the presence or absence of a flaw in a material, typically one transducer may be sufficient. If, however, a characterization of the flaw is desired, then an array of transducers is in most cases required. Besides the capability of two and three dimensional imaging, array data has the advantages of increased resolution, improved signal-to-noise ratio after preprocessing and sharper focusing.

In any NDE system, the acquisition of data is only one step towards the final objective of flaw characterization. The other step is that of processing the data in order to extract the desired information. In this paper, we consider one signal processing aspect of data obtained by a linear array of transducers. Each element on the array normally operates as transmitter and receiver simultaneously, and the data is collected by exciting one transducer at a time. The measured signals, after suitable time shifting for alignment, are summed in order to focus (or beamsteer) the array at a specific point. The resolution of this summing process depends on the side lobes of the array reject response, and this in turn depends on the number of elements and spacing between elements on the array.

While summing is the simplest signal processing procedure to perform, it is however, as far as beamforming is concerned, not the most effective. The side lobe levels decrease as the number of elements N increases, and this has a lower bound of

about -14 dB as $N \rightarrow \infty$. In this paper, we introduce an additional processing step with specially designed optimum filters before summing. The design methodology for these filters will be discussed in detail, and it will be shown that these filters have a superior frequency reject response which becomes more apparent if the array has a small number of elements.

INTRODUCTION

Ultrasonic nondestructive testing is based on the propagation, scattering and reflections of ultrasonic waveforms in the material under test. The interpretation of these waveforms is very important in order to arrive at decisions regarding the existence and characterization of flaws in the material. Often, before such interpretation is possible, processing the data in order to enhance its quality is necessary so that correct signal analysis and interpretation can be made [1].

If a nondestructive evaluation system is designed to only detect the presence or absence of a flaw in the material, typically one transducer may be sufficient and the time of travel of the source pulse is the important variable. If, however, a characterization of the flaw is desired, then an array of transducers is in most cases required. The use of arrays of transducers is an attractive procedure for eliminating undesired coherent signals which tend to interfere with the desired measurement signals. An array of transducers usually can - while a single transducer cannot - provide the directivity and rejection response needed to enhance the data quality, and cancel out any coherent interference.

BEAMFORMING TRANSDUCER ARRAY

The system under consideration is essentially the same as the one described in [2]. A linear array of transducers is used as shown in Figure (1); and each element on the array normally operates as transmitter and receiver simultaneously. The measured signals after suitable time shifting for alignment are summed in order to beamsteer the array along a certain direction or arc. The resolution of the summing process depends on the side lobes of the array rejection response and this in turn depends on the ratio of the spacing between elements and the signal wavelength. Figure (2), for example, illustrates the rejection response for an 8-element array.

The system under consideration consists of inserting optimum-frequency filters between the time shifting and the summing process as shown in figure (3). The design procedure of these

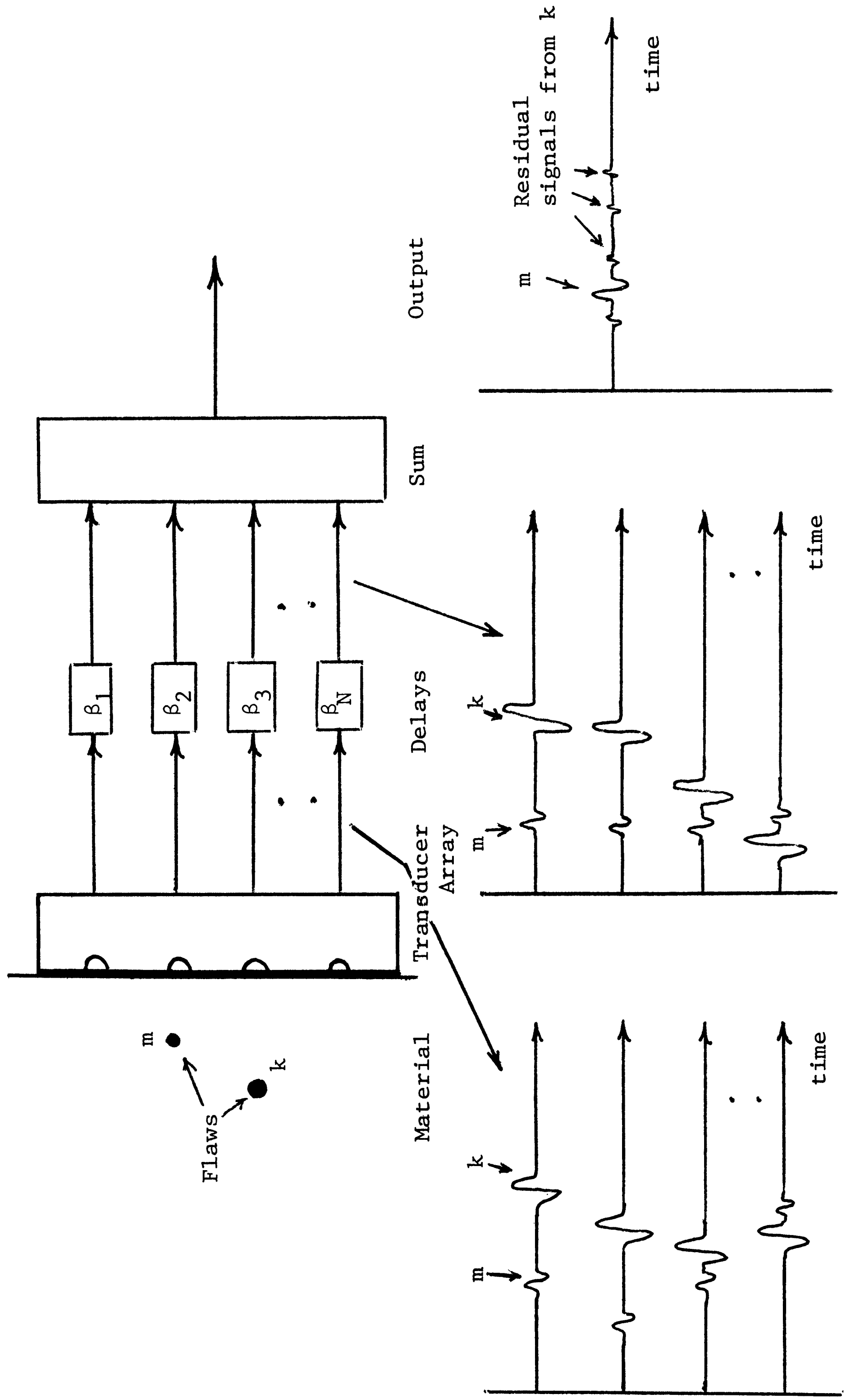

Figure 1: NDE System

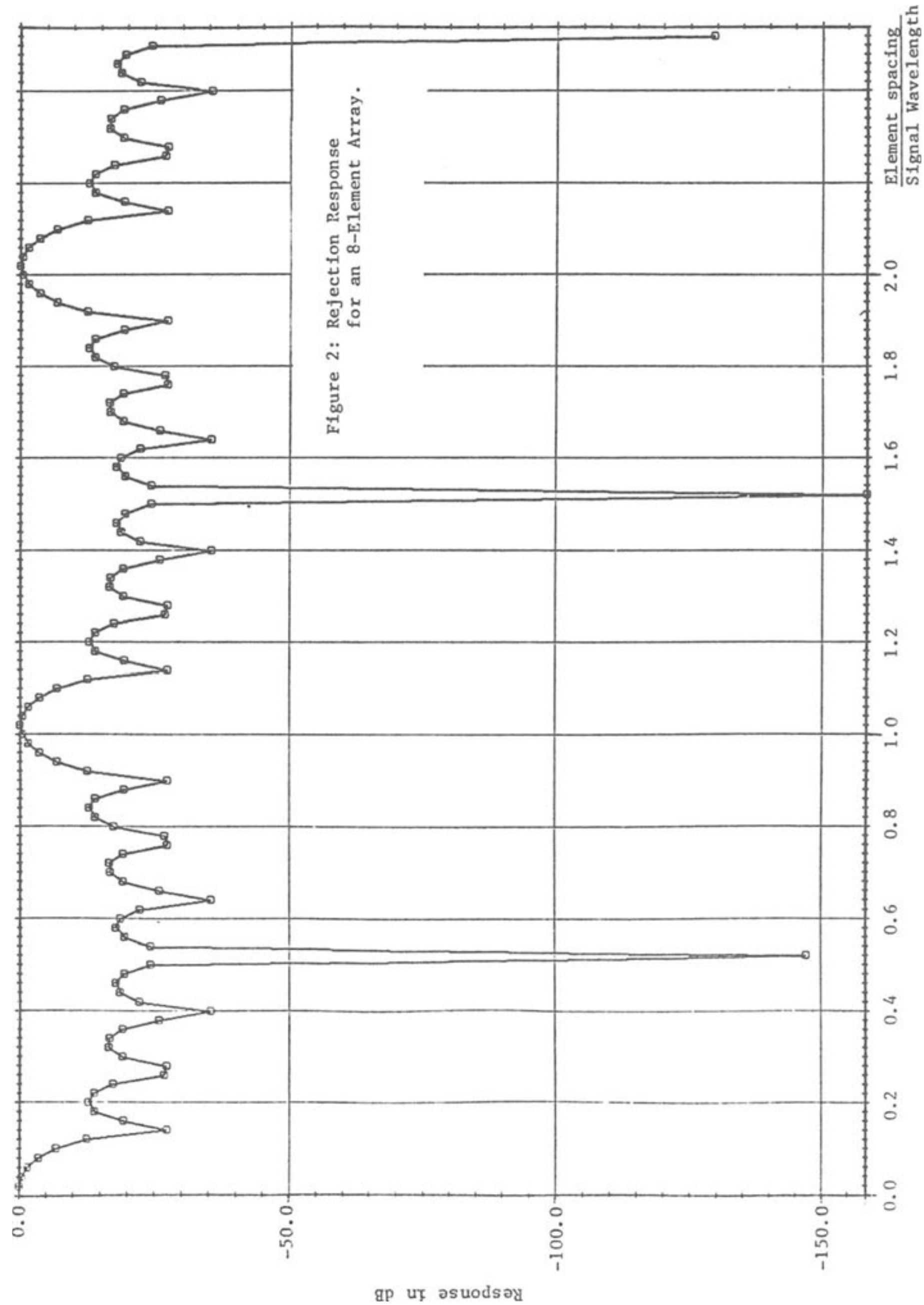

Figure 2: Rejection Response for an 8-Element Array.

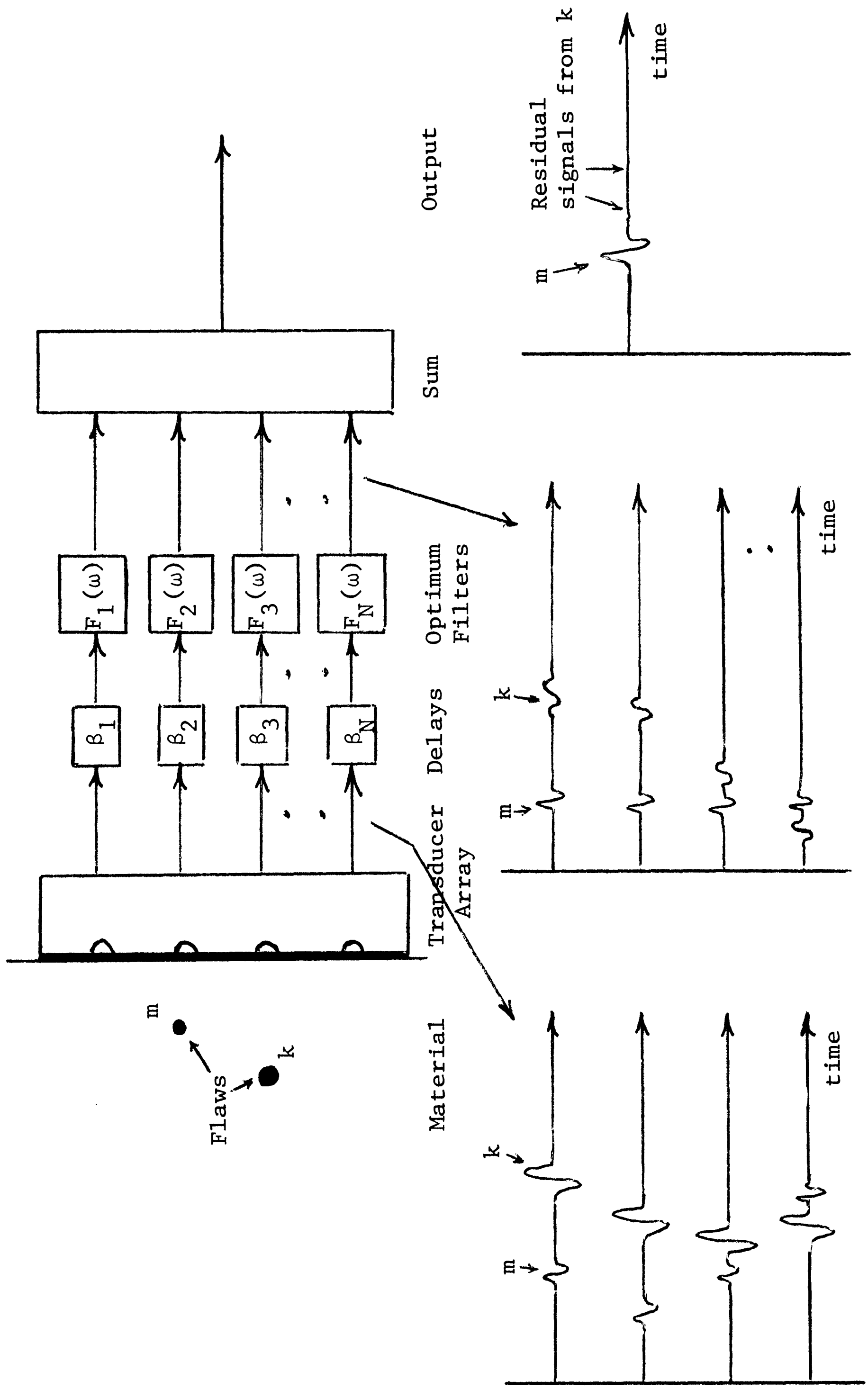

Figure 3: NDE System with Optimum Filters

filters will be discussed in the next section. Their purpose however, is to produce a more effective beamsteering of the data than the simple summing process.

Consider an array of N transducers. The presence of M flaws in the material will produce measurements of the form:

$$y_i(t) = \sum_{k=1}^{M} a_{ik}\, u_k(t - \beta_{ik}) \qquad i = 1, \ldots . N \quad (1)$$

where $u_k(t)$ is the reflection from the k^{th} flaw, β_{ik} is the two-way travel time between k^{th} flaw and i^{th} transducer and a_{ik} is a scaling constant describing the relative amplitude of the signal.

After proper time shifting to align with respect to the m^{th} flaw, and if we define $z_{im}(t) = y_i(t + \beta_{im})$ and $\tau_{ik} = \beta_{im} - \beta_{ik}$ then:

$$z_{im}(t) = a_{im}\, u_m(t) + \sum_{\substack{k=1 \\ k\neq m}}^{M} a_{ik}\, u_k(t - \tau_{ik}). \qquad (2)$$

If we define $z_m(t) = \sum_{i=1}^{N} \frac{1}{a_{im}} z_{im}(t) * f_i(t)$ where $f_i(t)$ is is the time-domain, yet unknown, filter applied to the i^{th} transducer, then:

$$z_m(t) = u_m(t) * \sum_{i=1}^{N} f_i(t) + \sum_{\substack{k=1 \\ k\neq m}}^{M} \sum_{i=1}^{N} \frac{a_{ik}}{a_{im}} u_k(t - \tau_{ik}) * f_i(t), \qquad (3)$$

which in the frequency domain translates to:

$$Z_m(\omega) = U_m(\omega) + \sum_{\substack{k=1 \\ k\neq m}}^{M} \sum_{i=1}^{N} c_{ik}\, U_k(\omega)\, F_i(\omega)\, e^{-j\omega\tau_{ik}} \qquad (4)$$

where $c_{ik} = \frac{a_{ik}}{a_{im}}$ and provided the constraint $\sum_{i=1}^{N} F_i(\omega) = 1$ is imposed. The quantity

$$R_k(\omega) = \sum_{i=1}^{N} c_{ik}\, F_i(\omega)\, e^{-j\omega\tau_{ik}} \qquad (5)$$

is the rejection response of the array with respect to the kth flaw. Equation (4) can now be written as

$$Z_m(\omega) = U_m(\omega) + \sum_{\substack{k=1 \\ k\neq m}}^{M} R_k(\omega)\, U_k(\omega). \tag{6}$$

In order to minimize the residual signal $\sum_{k=1}^{M} R_k(\omega)\, U_k(\omega)$ in equation 6, we will minimize the expression.

$$E(\omega) = \sum_{i=1}^{N} \Big| \sum_{\substack{k=1 \\ k\neq m}}^{M} R_k(\omega) - R_{ik}(\omega) \Big|^2 \tag{7}$$

where $R_{ik} = c_{ik}\, F_i(\omega)\, e^{-j\omega\tau_{ik}}$. The interpretation of this criterion is that the rejection response of each individual channel on the array is being made as close as possible to the reject response of the overall filtering scheme. This will occur when the rejection response $|R_k(\omega)|$ is as close as possible to an ideal rejection response which consists of a train of impulses located at frequencies which correspond to integer multiples of the ratio of the element spacing on the array to the signal wavelength [3].

COMPUTATION OF THE FILTER COEFFICIENTS

We will illustrate the computation of the filter coefficients for the case where M = 2. The generalization to the case where M is larger is straightforward except the algebraic manipulations are even more involved. Since there is only one flaw in addition to the reference flaw, the subscript k in the expressions (3) - (7) will be dropped (in particular τ_{ik} and c_{ik} will be replaced by τ_i and c_i). If we write the filter coefficients in rectangular form as follows:

$$F_i(\omega) = A_i(\omega) + jB_i(\omega) \quad \text{for } i = 1, \ldots N, \tag{8}$$

and if we use the following vector notation:

$$c(\omega) = \begin{bmatrix} c_1\cos\omega\tau_1 \\ \vdots \\ \vdots \\ c_N\cos\omega\tau_N \end{bmatrix}, \quad s(\omega) = \begin{bmatrix} c_1\sin\omega\tau_1 \\ \vdots \\ \vdots \\ c_N\sin\omega\tau_N \end{bmatrix}, \quad e_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftarrow i^{th} \text{ element} \tag{9}$$

$$A(\omega) = \begin{bmatrix} A_1(\omega) \\ A_2(\omega) \\ \cdot \\ \cdot \\ \cdot \\ A_N(\omega) \end{bmatrix} \quad \text{and} \quad B(\omega) = \begin{bmatrix} B_1(\omega) \\ B_2(\omega) \\ \cdot \\ \cdot \\ \cdot \\ B_N(\omega) \end{bmatrix} \qquad (10)$$

Then the expression for $E(\omega)$ in (7) can be written in the form.

$$E(\omega) = \sum_{i=1}^{N} [p_i'(\omega)A(\omega) + r_i'(\omega)B(\omega)]^2 + [p_i'(\omega)B(\omega) - r_i'(\omega)A(\omega)]^2$$

where the $N \times 1$ vectors $p_i(\omega)$ and $r_i(\omega)$ are defined as:

$$p_i(\omega) = c(\omega) - e_i c'(\omega) e_i$$

and (11)

$$r_i(\omega) = s(\omega) - e_i s'(\omega) e_i .$$

and where ' denotes transpose. The contraint $\sum_{i=1}^{N} F_i(\omega) = 1$ reduces to:

$$\sum_{i=1}^{N} A_i(\omega) = 1 \text{ and } \sum_{i=1}^{N} B_i(\omega) = 0 . \qquad (12)$$

Applying the Lagrange multiplier method, the solution for the filter coefficients can be obtained at every ω by minimizing the Lagrangian function:

$$\hat{E}(\omega) = E(\omega) + \lambda_1(\omega)[\sum_{i=1}^{N} A_i(\omega) - 1] + \lambda_2(\omega)\ [\sum_{i=1}^{N} B_i(\omega)] \qquad (13)$$

where $\lambda_1(\omega)$ and $\lambda_2(\omega)$ are scalar Lagrange multipliers. The conditions for minimization of $\hat{E}(\omega)$ can be easily derived by setting:

$$\frac{\partial \hat{E}(\omega)}{\partial A_i(\omega)} = \frac{\partial \hat{E}(\omega)}{\partial B_i(\omega)} = 0 \quad \text{for } i=1,\ldots N$$

and (14)

$$\frac{\partial \hat{E}(\omega)}{\partial \lambda_1(\omega)} = \frac{\partial \hat{E}(\omega)}{\partial \lambda_2(\omega)} = 0.$$

After lengthy algebraic manipulations, the above conditions will lead to the following system of matrix equations:

$$\left[\begin{array}{c|c} P(\omega) & -Q(\omega) \\ \hline Q(\omega) & P(\omega) \end{array}\right] \cdot \left[\begin{array}{c} A(\omega) \\ \hline B(\omega) \end{array}\right] = \left[\begin{array}{c} \mu\lambda_1(\omega) \\ \hline \mu\lambda_2(\omega) \end{array}\right] \tag{15}$$

$$\sum_{i=1}^{N} A_i(\omega) = 1 \quad \text{and} \quad \sum_{i=1}^{N} B_i(\omega) = 0,$$

where $P(\omega)$ and $Q(\omega)$ are ω-dependent N x N matrices given by:

$$P(\omega) = \sum_{i=1}^{N} [p_i(\omega)p_i'(\omega) + r_i(\omega)r_i'(\omega)] \tag{16}$$

and

$$Q(\omega) = \sum_{i=1}^{N} [r_i(\omega)p_i'(\omega) - p_i(\omega)r_i'(\omega)]. \tag{17}$$

In order to determine the filter coefficients, all equations in (15) must be solved simultaneously at every frequency point. Clearly, the critical step in this solution is the determination of the inverse of the 2N x 2N matrix:

$$M(\omega) = \left[\begin{array}{c|c} P(\omega) & -Q(\omega) \\ \hline Q(\omega) & P(\omega) \end{array}\right]. \tag{18}$$

Because of the skew-symmetric nature of $M(\omega)$, its inverse can be easily shown to be of the form:

$$M^{-1}(\omega) = \left[\begin{array}{c|c} R(\omega) & -S(\omega) \\ \hline S(\omega) & R(\omega) \end{array}\right] \tag{19}$$

and once the N x N matrices $R(\omega)$ and $S(\omega)$ are determined, Equations (15) can be solved to give:

$$A_i(\omega) = \frac{\gamma(\omega)}{\gamma^2(\omega) + \gamma^2(\omega)} \sum_{j=1}^{N} R_{ij}(\omega) + \frac{\sigma(\omega)}{\gamma^2(\omega) + \sigma^2(\omega)} \sum_{j=1}^{N} S_{ij}(\omega),$$

$$\text{for } i = 1, \ldots N \tag{20}$$

$$B_i(\omega) = -\frac{\sigma(\omega)}{\gamma^2(\omega) + \gamma^2(\omega)} \sum_{j=1}^{N} R_{ij}(\omega) - \frac{\gamma(\omega)}{\gamma^2(\omega) + \sigma^2(\omega)} \sum_{j=1}^{N} S_{ij}(\omega),$$

$$\text{for } i = , \ldots N \qquad (21)$$

where $\gamma(\omega) = \sum_{i=1}^{N} \sum_{j=1}^{N} R_{ij}(\omega)$ and $\sigma(\omega) = \sum_{i=1}^{N} \sum_{j=1}^{N} S_{ij}(\omega)$, which are frequency dependent scalars.

The filter coefficients $A_i(\omega)$ and $B_i(\omega)$ for $i = 1, \ldots N$ can therefore be computed provided the matrix $M^{-1}(\omega)$ is nonsingular and analytically computable. In the theorem to follow, we discuss this issue and derive expressions for the matrices $R(\omega)$ and $S(\omega)$.

Theorem: The matrix $M(\omega)$ in (18) is nonsingular for all ω. Its inverse is as in (19) where the $N \times N$ matrices $R(\omega)$ and $S(\omega)$ are given by the expressions:

$$R(\omega) = \begin{bmatrix} \alpha_{11} & & -\alpha_{ij} \cos\omega(\tau_i - \tau_j) \\ & \ddots & \\ -\alpha_{ij} \cos\omega(\tau_i - \tau_j) & & \alpha_{NN} \end{bmatrix} \qquad (22)$$

and

$$S(\omega) = \begin{bmatrix} 0 & & \alpha_{ij} \sin\omega(\tau_i - \tau_j) \\ & \ddots & \\ \alpha_{ij} \sin\omega(\tau_i - \tau_j) & & 0 \end{bmatrix} \qquad (23)$$

where

$$\alpha_{ij} = \begin{cases} \dfrac{N^2 - 3N + 3}{(N-1)^2 c_i^2} & \text{for } i = j \\[2ex] \dfrac{N - 2}{(N-1)^2 c_i c_j} & \text{for } i \neq j \end{cases} \qquad \text{for } i, j = 1, \ldots N \qquad (24)$$

Proof: Details for the proof can be found in [4]. In summary, however, it can be shown that the matrix $M(\omega)$ can be written in the form

$$M(\omega) = D + (N-2)V(\omega)V'(\omega) \qquad (25)$$

where D is a constant 2N x 2N diagonal matrix given by:

$$D = \mathrm{diag}\ [c_1^2, \ldots, c_N^2, c_1^2, \ldots c_N^2] \tag{26}$$

and $V(\omega)$ is a 2N x 2 matrix of the form:

$$V(\omega) = \left[\begin{array}{c|c} c(\omega) & s(\omega) \\ \hline -s(\omega) & c(\omega) \end{array}\right] . \tag{27}$$

Now applying the Sherman-Morrison Modification formula [5] to (25) we get:

$$M^{-1}(\omega) = D^{-1} - \frac{N-2}{(N-1)^2} D^{-1} V(\omega) V'(\omega) D^{-1}$$

which can be simplified using (26) and (27) to the form given in (19).

When the expressions for $R(\omega)$ and $S(\omega)$ in (22) and (23) are substituted in (20) and (21), and if we assume equal reflection constants $c_{ij} = c_o$ (which is practically always true), then the filter coefficients can be derived as follows

$$F_i(\omega) = \frac{(N-1)^2 - (N-2) \sum_{m=1}^{N} e^{-j\omega(\tau_k - \tau_m)}}{N(N-1)^2 - (N-2) \sum_{n=1}^{N} \sum_{m=1}^{N} \cos\omega(\tau_n - \tau_m)} . \tag{28}$$

Note that, in order to design these filters, only the time shifts τ_i are needed. Furthermore, if N = 2, the filters become frequency independent and are equivalent to simple averaging. Thus, in order to take advantage of the attenuation power of the filters N has to be strictly larger than 2. Note also that the expression for the filter coefficients (28) satisfies $F_i(-\omega) = F_i^*(\omega)$ which essentially means that in the time domain, the filters' impulse responses are real valued functions.

Figure (3) illustrates the system with the optimum filters applied between the delays and summing steps. The effect of these filters is to suppress interference from other adjacent flaws when a particular flaw is being characterized by beamforming the array and focusing it at the point of interest. The reject response of the system, for an 8-transducer array is shown in figure 4. When compared with figure 2, the effect of the filters

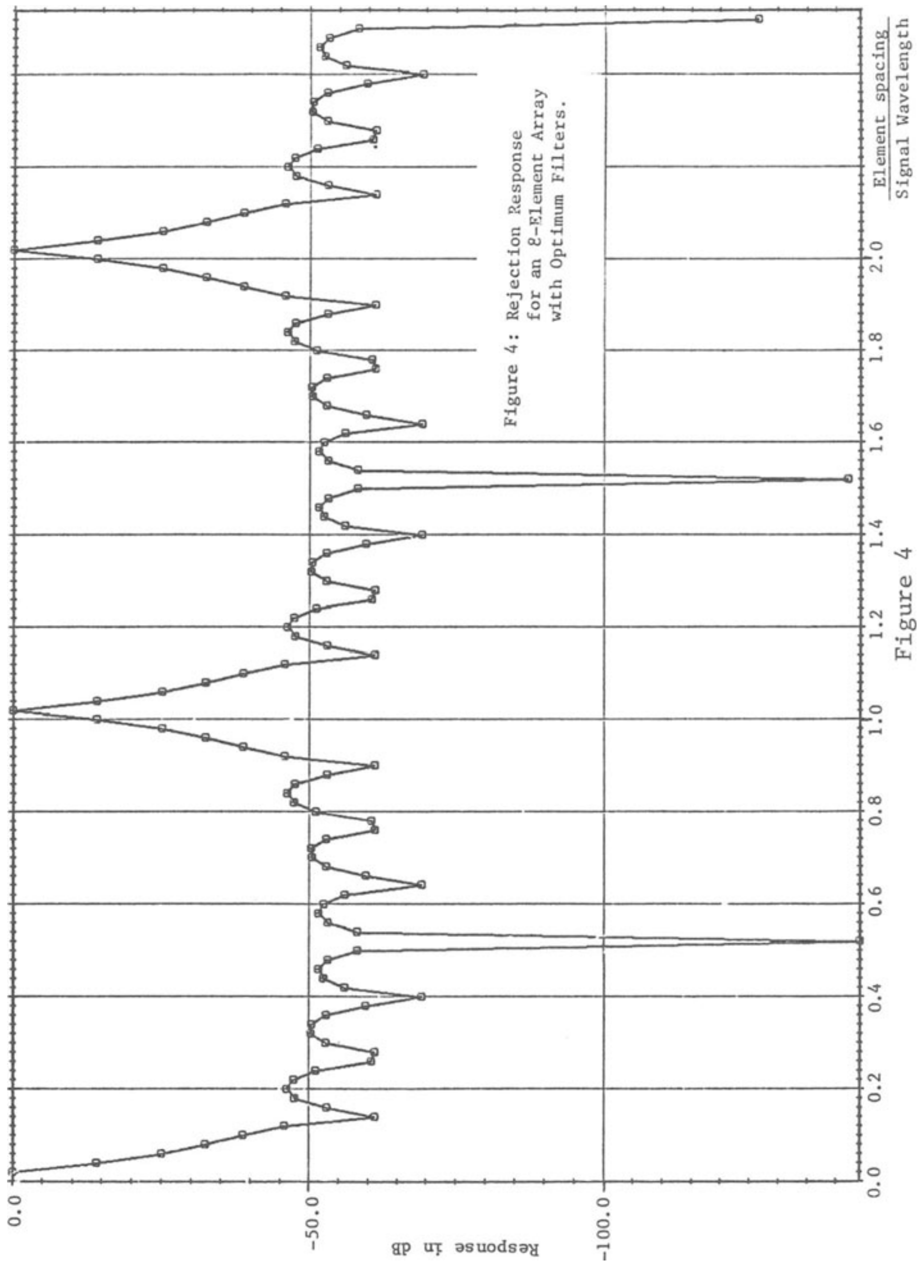

Figure 4: Rejection Response for an 8-Element Array with Optimum Filters.

Figure 4

is approximately 25 dB additional suppression on the average over the frequency spectrum. Also the width of the main lobes has decreased. A full image can be created by perpendicularly scanning across the array; and with this method, the net result is, therefore, increased resolution of the final image.

CONCLUSION

In this paper, we have discussed a method for processing transducer-array data in NDE applications. This method consists of applying transducer measurements in order to beamsteer the array to a specific point. It is shown that these filters yield increased resolution and hence improved reliability of the final NDE image.

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