1994

Review of Pertubation Methods for Engineers and Scientists

James Murdock
Iowa State University, jmurdock@iastate.edu

large number of subroutines. For the user, however, in addition to the main subroutine pltmg essentially only six others are of interest and, of course, the driving program. Very informative descriptions of these are given in Chapters 3–9, followed by a short chapter on the only machine-dependent subprograms—dealing with timing and the graphics interface. Although a reader may be overwhelmed at first by the wealth of possibilities to control and display the solution process, a careful study of each section will show that with the explanations given all the capabilities provided can be exploited with ease. While, for example, the casual reader of Chapter 2 may think that the initial triangulation would always have to be defined rather explicitly, in Chapter 3 it becomes clear that only a comparably coarse definition of the domain (*skeleton*) is needed for the built-in triangulation procedure. Subsequently, triangulations can be refined adaptively using a posteriori error estimators to a level limited only by the available memory, but they can also be unrefined and also a new *skeleton* may be formed and the domain retriangulated in order to have, for example, the grid lines aligned with contours of the solution.

To given an indication of the variety of output possibilities we just mention that fifteen different tables and curves may be displayed at any time and more than twenty functions. Here, naturally one of the output formats is PostScript.

This book is indispensable for any user of PLTMG but also for everyone who wants to see a prime example of a modern scientific computing package incorporating ideas from both computer science (data structures, graphics, etc.) and mathematics (both algebraic and analytic) that were developed by the author—partly in cooperation with others—over a number of years and may be found in full detail in his extensive list of publications with key papers listed in the bibliography of the book under review.

H. MITTELMANN
Arizona State University

Perturbation Methods for Engineers and Scientists. *By Alan W. Bush.* CRC Press, Boca Raton, FL. 303 pp. $59.95, hardcover. ISBN 0-8493-8608-X.

This is an introduction to perturbation methods, at the beginning graduate level, suitable for courses focusing on methods rather than justification. Boundary layers and fluid flow are emphasized much more than nonlinear oscillations, but given this constraint, this book may have some advantages over its nearest competitor, Nayfeh's *Introduction to Perturbation Techniques*.

The author introduces perturbation expansions with a few examples, such as motion with small friction, roots of polynomials, and integration by parts. This leads to a second chapter on order symbols, asymptotic expansions, and uniformity. The next four chapters are each devoted to one of the basic classes of perturbation methods for differential equations, strained coordinates, multiple scales, matching, and WKB. (Strained coordinates are handled mostly by renormalization, that is, computing a nonuniform straightforward expansion and then rendering it uniform by straining.) A final chapter concerns asymptotic evaluation of integrals. The chapters on strained coordinates, multiple scales, and matching each have lengthy sections treating a serious physical application at a depth that is unusual in an introductory book; for strained coordinates and matching, these concern fluid flow, while for multiple scales the application is to lubricated bearings.

From the standpoint of nonlinear oscillations, the offerings here are meager. The Lindstedt method for periodic solutions is given, and the multiple scale method is applied to the unforced Van der Pol oscillator. The method of averaging is touched on in three pages, treating only the leading order approximation for the general, unforced, nearly linear oscillator in one degree of freedom. Even here, the treatment is misleading in that the method is said to "assume" that the solution has the form $u = a\cos(\omega t + \theta)$, where a and θ are functions of time. In fact the use of this formula "assumes" nothing; it is merely a change of variables from $(u, \dot{u})$ to (a, θ). Much of the theoretical advantage of the method of averaging results from the fact that it, as opposed to the method of multiple scales, does not begin by postulating a form for the solution arbitrarily. This enables every step to be justified in a natural way, with rigorous conclusions both as to the existence and stability of periodic solutions, and as to error estimates for the approximations for both periodic and transient solutions. (Multiple scales can sometimes be justified also, but only after the approximate solution is computed; it is the solution itself that is justified, not the steps on the way to the solution.) No one would guess any of this from the presentation given here.

If nonlinear oscillations are the weak point of the book, matching is its strong point. Beginning with a simple linear example, the author computes the outer solution and the exact solution, and by comparing them, discovers the existence of a boundary layer. Nice motivations are given for the choice of an inner variable and for Prandtl's matching condition for the first order approximation. Next, some examples having different behavior are given: a layer at the opposite end, a layer of thickness $\sqrt{\epsilon}$, an interior layer. Then higher-order matching is presented, using an intermediate variable; it is surprising that the author does not follow up his use of Prandtl's matching condition by a development of Van Dyke's rules, which are certainly easier to use than the intermediate variable method, although not as general in their applicability. A short section on nonlinear examples follows. Finally, there is a long (almost 40 pages) section dealing with the Navier–Stokes equation, Reynolds and Prandtl numbers, the Blasius solution, thermal boundary layers, and so forth, containing several computer programs and tables of output. Much of this material is unknown to me, but it looks quite readable and if I wanted to learn it I would certainly begin here.

Unfortunately, this book commits the canonical mistake made, as far as I am aware, by every book in this field that is intended for a strictly "applied" audience. For the sake of illustration, I have made this mistake myself when I said above that renormalizing takes a nonuniform expansion and renders it uniform. In fact, renormalization only takes a "disordered" expansion and renders it "uniformly ordered." All textbooks begin with the correct definition of uniform, which involves an estimate of the difference between the exact solution and an approximation; but they quickly lapse into the practice of "checking uniformity" by examining the sizes of successive terms of the expansion, making no reference to the (unknown) exact solution. This practice is partly responsible for the undeserved reputation that perturbation theory sometimes has among mathematicians, a reputation roughly comparable to that of tabloid newspapers. I do not ask that a book at this level provide proofs of actual uniformity. But it is not unreasonable to ask two things: first, that it make clear that uniform ordering is only a necessary, not a sufficient condition for uniformity; and second, that it make students aware that for proving actual uniformity there exists a good deal of useful theory, as well as many interesting unsolved problems. These things could be done in the book under review with the addition of only a few pages in Chapter 2 and the changing of a few words elsewhere. This much honesty would not, of course, change the reputation of the field overnight, but it might gradually elevate the level of awareness with which perturbation theory is used. I would urge those adopting this textbook to make these points clear to their students.

JAMES MURDOCK
Iowa State University

Semigroup Theory with Applications to Systems and Control. *By N. U. Ahmed.* Longman, Essex, England, 1991. 282 pp. $44.00. ISBN 0-582-06559-2.

Suppose X is a Banach space, $\Omega \subset X$, and $A : \Omega \to X$ such that if x is in X, then there is a unique function $y : [0, \infty) \to \Omega$ such that

$$(1)\quad y(0) = x, \quad y'(t) = A(y(t)), \quad t \geq 0.$$

Define T on $[0, \infty)$ so that if $s \geq 0$, then $T(s) : \Omega \to \Omega$ is defined by $T(s)x = y(s)$, where y satisfies (1). Then T has the characteristic properties

$$(2)\quad \begin{aligned} T(t)\Omega \to \Omega, \quad T(0) = I, \\ T(t)T(s) = T(t+s), \quad t, s \geq 0. \end{aligned}$$

A transformation T satisfying (2) is called a one parameter semigroup of transformations on Ω or simply a semigroup. Semigroups are classified by being linear ($T(t)$ is linear, $t \geq 0$) or else nonlinear. It is generally assumed that $T(t)$ is continuous, $t \geq 0$. A further classification rests upon whether or not $T(t)$ converges uniformly on bounded subsets of X to I as $t \to o+$. In the former case T is called continuous. If it is only true that for each $x \in \Omega$, $\lim_{t \to o+} T(t)x = x$, then T is called strongly continuous. Generally, if T arises from ordinary differential equations, T is continuous whereas if T arises from partial differential equations, then T is at most strongly continuous.

The book under review concerns mainly strongly continuous semigroups of linear transformations, although applications to nonlinear problems are a major concern. For linear strongly continuous semigroups on a Banach space X