

11-1982

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Recommended Citation

Faden, Arnold, "The Simplest Possible Adequate Monetary Policy Model" (1982). *Economic Staff Paper Series*. 61.
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The Simplest Possible Adequate Monetary Policy Model

Abstract

I define an adequate monetary policy model as one that incorporates dynamics and uncertainty in a plausible manner. We need a state variable X that changes over time in a partially random manner, and a policy variable whose time-path is to be specified. Finally, we need an objective function.

Disciplines

Applied Statistics | Business Administration, Management, and Operations | Statistical Methodology |
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THE SIMPLEST POSSIBLE ADEQUATE
MONETARY POLICY MODEL

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November 1982

No. 123

1. I define an adequate monetary policy model as one that incorporates dynamics and uncertainty in a plausible manner. We need a state variable x that changes over time in a partially random manner, and a policy variable whose time-path is to be specified. Finally, we need an objective function.

2. Consider the stochastic differential equation

$$dx = f(x, m)dt + g(x, m)dB \quad (1)$$

governing the state variable. Here $f(x, m)$ is the deterministic drift of the process per unit time, while $g(x, m)$ is the magnitude of the random "kick" administered. dB is "white noise" or, more exactly, the differential of Brownian motion

$$dB \sim N(0, dt)$$

having variance equal to time elapsed dt . (In general, x and m are vector-valued, but in our simple model they are both real-valued.)

3. We are interested in autonomous policies of the form $m(x)$, that is, chosen m depends on states x but not on calendar time. And of these, we are interested in policies that, when substituted in (1), make the x -process ergodic. That is, as $t \rightarrow \infty$, $x(t)$ has a distribution that approaches a limit--the long-term stationary distribution--and furthermore, that this limiting distribution does not depend on the current state x .

Let $P(x)$ be this limiting distribution; it will in general depend on the policy $m(x)$ that is chosen (with probability one, $P(x)$ is also the long-run distribution of the sojourn times in the various states x).

4. Let $C(x, m)$ be the cost function to be minimized. We want it to have the following property: $\lim C(x, m) \rightarrow \infty$ as $x \rightarrow \pm\infty$, uniformly in m (2)

That is, it is bad for $|x|$ to be very large. Under condition (2), any reasonable policy $m(x)$ will try to reduce $|x|$ when $|x|$ gets large and this is precisely the condition needed for ergodicity.

5. We now make precise the sense in which $C(x,m)$ is to be minimized. Non-ergodic policies are simply ruled out as "bad". "Good" policies $m(x)$ yield limiting distributions $P(x)$, and are to be ranked by the criterion:

$$\text{Minimize: } EC(x, m(x))$$

expectation with respect to $P(x)$. (This yields $C(x, m(x))$ weighted by the fraction of time spent in the various states x).

6. Now consider the special case in which $f(x, m)$ is linear, $g(x, m) = g$ is constant and $C(x, m)$ is quadratic and strictly convex, satisfying (2)

Under these conditions, optimal $m(x)$ is also linear. Substituting in (1) yields

$$dx = (\alpha - \beta x)dt + gdB \quad (3)$$

for some constants α, β . $\beta > 0$ is the ergodicity condition.

7. Equation (3) is a Langevin equation. (A model is a spring or rubber-band subject to random forces). The stationary solution to (3) is an Ornstein-Uhlenbeck process (characterized by being stationary, gaussian and markov). with each $x(t)$ having the distribution

$$P_x \sim N\left(\frac{\alpha}{\beta}, \frac{g^2}{2\beta}\right) \quad (4)$$

8. Now enter money. The variables we shall use are those from the quantity equation

$$MV = Y = PQ \quad (5)$$

where M = money stock, V = velocity, Y = nominal GNP, P = price level, Q = real GNP, and also r , the nominal interest rate (say the prime rate).

9. The control variable will be $m = \dot{M}/M$ ($\dot{\cdot}$ denotes $\frac{d}{dt}$), the growth rate of M . (Actually, it is no more complicated to allow for the Fed missing its monetary target by random shocks. Let actual \dot{M}/M follow a law of the form (2) with the drift term being m , targeted monetary growth).

10. From growth theory, there is a relation between growth rates and interest rates. For the short-term model in this paper, one must distinguish between the actual interest rate and the equilibrium interest rate toward which it tends. The latter is the one connected to the growth rate. Further, the real-world situation is one of a slight gap between these rates rather than equality. Thus we postulate

$$\text{equilibrium real interest rate} = \frac{\dot{Q}}{Q} + \psi \quad (6)$$

where ψ is the gap. The USA is characterized by \dot{Q}/Q having a long-term average of about .03/year, while equilibrium real interest is about .06/year. Thus $\psi \doteq .03/\text{year}$, the value we shall use.

11. Adding \dot{P}/P to both sides of (6) yields

$$\text{equilibrium } r = \frac{\dot{Y}}{Y} + .03/\text{year} \quad (7)$$

from (5). Actual r tends toward equilibrium r , and the standard partial-adjustment specification reads

$$\dot{r} = a \left[\frac{\dot{Y}}{Y} + .03 - r \right] + \text{noise} \quad (8)$$

("a" = adjustment speed; .3/year seems a ballpark estimate). Here "noise" summarizes all disturbances, and (8) is a slightly informal rendering of a stochastic differential equation.

12. Now define $x = r - .03/\text{year}$. This will be our state variable. (8) now reads

$$\dot{x} = a \left[\frac{\dot{Y}}{Y} - x \right] + \text{noise} \quad (9)$$

13. We need a relation between V and r . r measures the cost of holding money, hence V rises with r . We postulate

$$\frac{\dot{V}}{V} = br + \text{noise} \quad (10)$$

for some adjustment constant b . To estimate it, note that

$$b = \frac{dV}{dr} \frac{1}{V} = \left(\frac{dV}{dr} \frac{r}{V} \right) \frac{1}{r} \quad (11)$$

On the right, the first term is the interest-elasticity of velocity, which is about .2. Taking a "typical" r of .10/year yields an estimate $b \doteq 2$ years.

14. (10) may also be written as

$$\frac{\dot{V}}{V} = bx + \text{noise} \quad (12)$$

Substituting in (9) we get

$$\dot{x} = a \left[\frac{\dot{M}}{M} + \frac{\dot{V}}{V} - x \right] + \text{noise} = a(m + bx - x) + \text{noise}$$

yielding

$$\dot{x} = \frac{a}{1-ab} (m - x) + \text{noise} \quad (13)$$

(This derivation makes sense only if $ab < 1$, which it is with the estimates above).

15. (13) may be written more accurately as

$$dx = k(m - x)dt + gdB. \quad (14)$$

for some g , where $k = \frac{a}{1-ab} > 0$. (14) is our basic dynamical law.

16. We now come to the objective function $C(x, m)$. This represents "real" costs imposed on the economy by disturbances and inefficiencies in the monetary-financial sector. Think of $C(x, m)$ as the shortfall from some "natural" growth rate, for example.

17. There are two basic mechanisms imposing these costs. The first is from disequilibrium in the capital market. If interest rates are above their equilibrium levels, some marginal projects (especially in construction) are not worth undertaking, and the economy is in a state of low aggregate demand, low or negative growth, and recession.

The harm from below-equilibrium interest rates is less clear. The situation is one of queuing for credit and capital rationing, and some more deserving projects may be shelved while less deserving ones are undertaken. On the other hand, holding interest moderately below equilibrium may yield a Wicksellian secular boom and be beneficial.

How to represent these effects? Well, by (14), interest rates are below equilibrium when $m - x > 0$, and above equilibrium when $m - x < 0$. This suggests a cost term of the form

$$(m - x - c)^2 \quad (15)$$

Here c is a constant. $c = 0$ corresponds to over- and under-equilibrium both being harmful. $c > 0$ would allow for the asymmetry discussed above and the possible Wicksellian benefits from below-equilibrium interest.

(One might guess $c = .20/\text{year}$; remarkably enough, optimal monetary policy turns out not to depend on c at all, so we needn't worry about estimating it).

18. The second cost arises from monetary inefficiency. People treat money as a scarce resource though it is not from the viewpoint of society. Thus, they develop financial intermediaries, ship it from country to country, make frequent trips to the bank, etc. This effect depends on the level of interest rates and suggests a cost term of the form

$$(x - d)^2 \quad (16)$$

Here d is a constant. Probably d is close to zero, and is unlikely to be more than, say, $.04/\text{year}$.

19. Putting (15) and (16) together yields

$$C(x, m) = \theta(m - x - c)^2 + (1 - \theta)(x - d)^2 \quad (17)$$

This is the objective function. Here θ is a constant between 0 and 1 representing the relative importance of capital-market disequilibrium and monetary inefficiency. The former seems to be much more important than the latter, so we take θ to be (at least) $.9$.

20. The problem of minimizing (17) subject to (14) may now be solved.

$C(x, m)$ is quadratic and strictly convex, $f(x, m)$ is linear, and $g(x, m)$ is constant. Under these conditions, optimal policy is linear (see ¶ 6). So write

$$m = \lambda + \mu x \quad (18)$$

and let us determine optimal λ and μ in terms of the parameters g , k , c , d , and θ .

21. (18) includes a number of standard policy recommendations. $\mu = 0$ is the Friedman policy: expand M as a constant rate regardless of other conditions. ($\lambda = .04/\text{year}$ is also Friedman's recommendation, but this is less crucial).

On the other hand, $\mu = 1$ may be described as "accommodating the needs of trade" by sustaining the current level of interest rates.

22. Substituting (18) into (14) yields the Langevin equation

$$dx = k(\lambda + \mu x - x)dt + gdB \quad (19)$$

23. Comparing (19) with (3) we see immediately that the necessary and sufficient condition for ergodicity is $\mu < 1$. Any policy with $\mu \geq 1$ is "bad" and will lead to interest rates wandering arbitrarily far, leading to very high monetary inefficiency costs. (But we cannot yet exclude, say, $\mu = .99$).

24. Taking $\mu < 1$, then, we obtain the stationary distribution for x from (4)

$$P_x \sim N \left[\frac{\lambda}{1-\mu}, \frac{g^2}{2k(1-\mu)} \right] \quad (20)$$

25. Under policy (18), the cost function is

$$\theta \left((1-\mu)x + c - \lambda \right)^2 + (1-\theta)(x-d)^2 \quad (21)$$

The expectation of (21) under distribution (20) is

$$\theta c^2 + (1 - \theta) \left[\frac{\lambda}{1-\mu} - d \right]^2 + \frac{g^2}{2k} \left[\theta(1-\mu) + \frac{1-\theta}{1-\mu} \right] \quad (22)$$

The minimum of (22) over λ, μ occurs at

$$1 - \hat{\mu} = \sqrt{\frac{1-\theta}{\theta}}, \quad \frac{\hat{\lambda}}{1-\hat{\mu}} = d \quad (23)$$

yielding a minimum value of

$$\theta c^2 + \frac{g^2}{k} \sqrt{\theta - \theta^2} \quad (24)$$

26. We have a number of remarkable conclusions:

- a. Parameters g, k, c play no role in determining optimal monetary policy.
- b. Policy should be chosen so that the long-term average value of x ($= r - .03/\text{year}$) equals \underline{d} . (If $d = 0$, choose $\hat{\lambda} = 0$: monetary expansion is proportional to x).
- c. The optimal interest-sensitivity parameter $\hat{\mu}$ depends only on the relative importance of capital-market disequilibrium vs. monetary inefficiency. If $\theta = .9$, then $\hat{\mu} = 2/3$ (if $\theta = .99$, then $\hat{\mu} = .9$). The Friedman rule $\hat{\mu} = 0$ is correct only if $\theta = .5$. If $\theta = .9$ it yields rather excessive capital-market disequilibrium costs (this forms the basis for a scenario on what's wrong with monetarism).