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## Impact of Extreme Weather on North American Transmission System Outages

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# Impact of Extreme Weather on North American Transmission System Outages

## Abstract

The impact of weather on the power grid has been a focus of multiple studies, and its importance has grown with the number and magnitude of extreme weather events. This paper uses transmission outage and inventory data collected in Transmission Availability Data System (TADS) to identify and analyze weather related transmission events and quantify their impact on the North American Bulk Electric System. The impact of a transmission event is measured by several factors: the number of outages, affected miles and MVA, event duration, and number of groups of simultaneous outages (known as generations of outages). We analyze the largest events from 2015 to 2019, and use an event propagation metric to estimate the probability of small, medium, and large events, and track how these probabilities change from year-to-year.

## Keywords

TADS, North American Electric Reliability Corporation (NERC), extreme weather, transmission system reliability, System Event Propagation Slope Index (SEPSI)

## Disciplines

Power and Energy

## Comments

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# Impact of Extreme Weather on North American Transmission System Outages

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**Abstract**—The impact of weather on the power grid has been a focus of multiple studies, and its importance has grown with the number and magnitude of extreme weather events. This paper uses transmission outage and inventory data collected in Transmission Availability Data System (TADS) to identify and analyze weather related transmission events and quantify their impact on the North American Bulk Electric System. The impact of a transmission event is measured by several factors: the number of outages, affected miles and MVA, event duration, and number of groups of simultaneous outages (known as generations of outages). We analyze the largest events from 2015 to 2019, and use an event propagation metric to estimate the probability of small, medium, and large events, and track how these probabilities change from year-to-year.

**Index Terms**—TADS, North American Electric Reliability Corporation (NERC), extreme weather, transmission system reliability, System Event Propagation Slope Index (SEPSI)

## I. INTRODUCTION

NERC’s State of Reliability reports [1] define extreme transmission days as those with the largest MVA loss caused by transmission outages on the North American Bulk Electric System (BES). These annual reports routinely find that the top causes of extreme transmission days are weather related. In 2019, the top cause in the Eastern and Québec interconnections was Weather excluding lightning, the top causes in the ERCOT interconnection were Weather excluding lightning and Lightning, and the top causes in the Western interconnection were Fire, Weather excluding lightning, and Lightning [1]. Overall, in 2013-2018 weather related causes (Weather excluding lightning, Lightning, Fire, and Environmental) initiated almost a third of all sustained outages reported in NERC’s TADS [2].

Among the 13 major event analysis reports that NERC has published since 2011, eight deal with extreme weather events [3]. Among them are hurricanes Sandy, Harvey, and Irma that caused outages in transmission, generation, and distribution systems. Another three events initiated by cold weather greatly affected the generation fleet; among them was the 2014 Polar Vortex—the largest event on the North American generation

system [4], [5]. Multiple publications are focused on analysis, prediction, and mitigation of weather related events on the transmission system. For example, reliability and resilience of the transmission system under severe weather conditions are studied in [6], and the effect of weather on cascading is analyzed in [7]. Paper [8] introduces and applies a weather model to derive weather-specific reliability indices for more precise reliability analyses for transmission and distribution systems. Detailed analysis of severe weather impact on distribution system reliability in the U.S. was recently published in [9].

The rest of the paper is organized as follows. Section II provides an overview of the data set and introduces an algorithm to identify the weather related transmission outage events on the North American BES. Section III analyzes events by cause, quantifies their impact on the grid, and provides details about the largest 2015-2019 events. In Section IV, the System Event Propagation Slope Index is used to study the event size measured in the number of generations, and the probabilities of small, medium, and large events are derived from a fitted Zipf distribution. Conclusions complete the paper.

## II. DATA AND METHODS

### A. NERC TADS

NERC has been collecting North American automatic (momentary and sustained) outage data for transmission elements operating at 200 kV and above since January 1, 2008. Transmission BES elements reportable in TADS are: 1) AC circuit (overhead and underground); 2) transformer (excluding generator step-up units); 3) DC circuit (one pole of an overhead or underground DC line that is bound by AC/DC terminal on each end); and 4) AC/DC back-to-back converter [10]. In 2015, TADS reporting requirements changed to align with the implementation of the BES definition approved by the Federal Energy Regulation Commission [11]. Two additional voltage classes were added – namely, sustained automatic outages of elements operating at less than 100 kV and sustained automatic outages for elements operated at 100 to 199 kV. The 51,979 automatic outages collected in TADS from 2015 to 2019 for all transmission elements are included in the study.

### B. Outage-Grouping Algorithm and Weather Related Transmission Events

For each interconnection, the 2015-2019 automatic outages are grouped together into transmission outage events based on their starting times and durations. Every outage in an event has to either start within five minutes of a previous outage in the event or overlap in duration with at least one previous outage in the event that has a difference in starting time not exceeding one hour. If an outage cannot be grouped together with any other outage, it will be placed in an event of size 1 by itself. Any transmission outage event that contains an automatic outage with a TADS initiating or sustained cause code of Fire, Weather excluding lightning, Lightning or Environmental is defined as a weather related event [10]. We call all other events non-weather related.

The idea of dividing an event into generations of outages is that groups of "parent" outages produce groups of "child" outages in the next generation. The outages that occur simultaneously, or within the same minute, are grouped into the same generation. Then, each event can be regarded as a series of generations of outages [12]. TADS reports the minute in which each outage starts, and each generation of outages contains the outages that occur in the same minute. For example, protection actions can cause several outages in the same minute that would be grouped into the same generation.

### III. ANALYSIS OF WEATHER RELATED TRANSMISSION EVENTS

#### A. 2015-2019 Transmission Events by Year

The 29,710 transmission outage events found by the grouping algorithm described in Section IIB contain 10,681 weather related events (36%) and 19,029 non-weather related events. Fig.1 breaks down the events by year. One measure of event size is its number of outages. The number of outages in events varies from one to 380 outages, with a majority of the events (71.4 %) consisting of one outage. Note that the relative changes in the number of events between consecutive years were greater for weather related events than for non-weather related events. It is not surprising, since the number and magnitude of extreme and severe weather events that affect the transmission system changes from year to year.

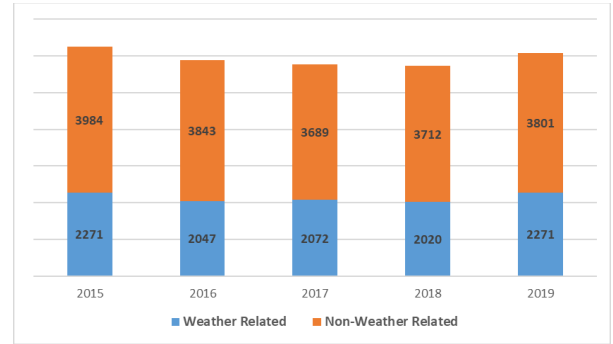


Figure 1. Transmission events by year (2015-2019)

#### B. Causes of Outages in Events of Different Sizes

To analyze initiating causes of outages by event size, we group together events by their number of outages: events of size one comprise their own (and the largest) group with 21,229 events; 8,158 events with sizes 2-9; 250 events with sizes 10-19; 48 events of sizes 20-34; and the smallest group of 25 largest events with at least 35 outages each. The last two groups contain only weather related events. The percentage of outages with different causes is shown in Fig. 2 for each group of events and for all groups combined.

The four causes used to determine weather related events—Weather excluding lightning, Lightning, Fire, and Environmental—comprise 34% of all outages, with the percentage increasing significantly as event size increases, from 30% for events of size one to 78% for events of size 35-380. Fig. 2 shows that this increase is due specifically to outages caused by Weather excluding lightning. The percent of outages caused by Weather excluding lightning decreases with event size, the percent of Environmental stays below 1% for all groups, and percent of Fire is largest for events of sizes 10-19 and 20-34 (3.1% and 3.8%, respectively). Failures of different equipment types cause 23% of all outages with the largest 29% in events of sizes 2-9 and the smallest 9% in events of sizes 35-380. Human error initiates a smaller share of outages as event size increases.

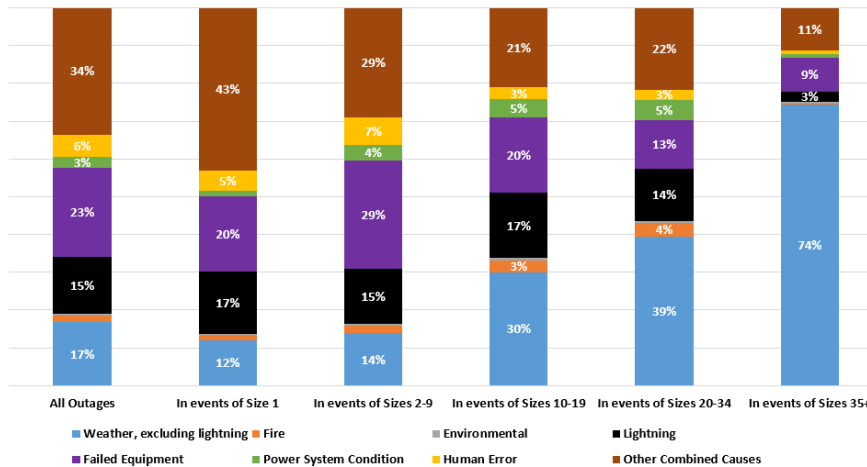


Figure 2. Initiating causes of automatic outages by event size (2015-2019)

### C. Comparative Impact of Weather related Events

Next, we determine the cumulative impact of the weather related events in the NERC-footprint over the years 2015-2019. We assess the impact of a transmission event by several important factors: the event size calculated by the number of outages and the number of generations, event duration, affected miles (the total mileage of AC and DC circuits outaged in the event), and the event MVA (the total equivalent MVA of all transmission elements outaged in the event).

TABLE I. TRANSMISSION EVENT IMPACT STATISTICS

2015-2019 Transmission Events	# Events	# Events with one outage	Mean # of outages (for events of size >1)	Mean # of Generations (for events of size >1)	Mean affected miles	Mean Duration (hours)	Mean MVA
Weather related	10681	6454	4.51	3.05	97	36	1288
Non-weather related	19029	14775	2.74	1.71	51	27	758

Table I lists basic impact statistics for weather related and non-weather related events. The distributions of event sizes significantly differ between weather related and non-weather related events (measured in the number of outages as well as the number of generations). The weather related events have a smaller share of events of size one, and overall tend to be larger. All events with more than 26 outages are weather related. Also, weather related events tend to have longer duration, affect higher equivalent MVA and affect almost twice greater line mileage. Consequently, even though weather related events comprise 36% of all events, they contain 49% of the 2015-2019 TADS automatic outages and account for 49% of MVA and 51% of miles affected by these outages. Additionally, the weather related events have the higher percent of outages with complex faults (i.e., phase-to-phase, three-phase, and multi-phase-to-ground) versus single phase-to-ground faults compared with non-weather related events: 32% and 18%, respectively. These comparisons confirm and quantify the very large effect of weather related events to the grid, which are further highlighted in the next subsection by analysis of largest events for the five years.

### D. Largest Weather related Events

Table II provides the information about the 29 largest transmission events in the 2015-2019 TADS data; the size of the events ranges from 380 to 32 automatic outages. All these events are weather related. Five events are caused by major hurricanes that hit Southeastern U.S. in 2016, 2017, and 2018. Another 13 major events were caused by extreme winter weather, often associated with low temperatures, high winds, heavy snow, hail, and blizzards. The Saddle Ridge Fire event was a result of a wildfire in California in October 2019. Other events were caused by widespread heavy thunderstorms and tornadoes, which were not limited to any particular region or season, though there are portions of regions which rarely experience these event types.

Typically, extreme weather events not only affect the transmission system, but also cause generator outages and load loss. In these cases, an assessment of the event impact requires data not only from TADS, but also from other sources including NERC's GADS, MIDAS, and Event Analysis reports.

The transmission event duration is closely correlated to the magnitude, duration, and footprint of the extreme weather that caused it. The average duration of the largest events is 20 days, with restoration for 19 events completed in less than 10 days. In the longest event, caused by thunderstorms and tornadoes in April 2017, the majority of transmission lines and transformers were restored to service in the next 2-25 days, and only one AC circuit outage lasted until January of the next year. Event MVA and affected miles are both correlated with not only the number of outages, but also with the outaged elements' types and voltages. For example, the low-ranking Saddle Ridge Fire event affected a large amount of mileage and MVA due to the high voltage of components involved in the event.

TABLE II. 2015-2019 LARGEST TRANSMISSION EVENTS

Year	Extreme/Severe Weather	# Outages	# Generations	Miles Affected	Event Duration (Days)	Total MVA
2017	Hurricane Irma	380	271	6744	19.5	131415
2016	Hurricane Matthew	198	147	5660	58.8	73431
2015	Strong wind storms	143	106	4844	5.9	45578
2017	Widespread thunderstorms and tornadoes	103	55	3321	246	39253
2018	Hurricane Michael	73	48	1507	28.2	22589
2015	Widespread rains and snowstorms	64	53	2157	1.5	24331
2018	Blizzard, Severe thunderstorms and tornadoes	63	47	1362	1.7	21076
2019	Strong winter storms with high winds	60	33	2415	10.4	25078
2018	Nor'easter	55	14	734	2.8	21670
2018	Hurricane Michael	55	40	1360	4.8	17347
2016	Heavy snow and freezing rains	53	38	1996	0.7	22156
2019	Storm system with high winds, snow, sleet, and ice	50	35	1896	81	34821
2018	Nor'easter	48	31	840	7.2	16573
2017	Widespread rain and thunderstorms; heavy, wet snow showers	47	32	1779	3.3	12499
2015	Strong thunderstorms and tornadoes	46	32	1363	4.2	20528
2019	Lightning storm	45	15	2232	8.6	30240
2015	Strong storms with high winds	44	38	672	6.2	7841
2018	Strong storms with high winds	43	36	1039	4.4	15278
2018	Extreme cold weather	42	37	1004	3.4	18744
2018	Hurricane Florence	42	31	1567	21.3	16246
2018	Storm system with blizzard	40	33	1681	5.9	17824
2019	Heavy snowfall and blizzard	38	20	1692	1	17629
2015	Thunderstorms with damaging winds and hail	36	27	688	5.6	14370
2019	Winter Windstorm	35	27	870	19.8	7167
2018	Winter Windstorm	35	27	796	4.4	10840
2017	Tornadoes	34	28	1047	10.1	10889
2018	Widespread thunderstorms	33	17	619	1.3	9750
2019	Saddle Ridge Fire	32	13	2150	1	30144
2015	Strong thunderstorms	32	20	558	17.3	11691

### E. Event Outage Visualization Tool

In its current state the algorithm described in Section II (B) can act as an automated screen to identify likely outage groupings, which are largely accurate to identify events as they occur on the system. An additional manual check is also conducted on events of interest as well as those that appear anomalous. By making use of the algorithm's output in combination with a customized program, it is possible to create

useful visual representations of the events and how they developed. Fig. 3 shows one such representation, with the black lines representing connected TADS elements that did not experience an outage, the red lines representing those elements that did experience outages, and the numbers on the lines representing the outage's generation. Currently, the program only accepts AC circuits and displays the highest generation for a given line; however, further development will allow a more holistic representation. Due to limitations in the data collected in TADS, the visualization shows the connections, but the layout is not topographically accurate. The visual representation is particularly useful in further event analysis.

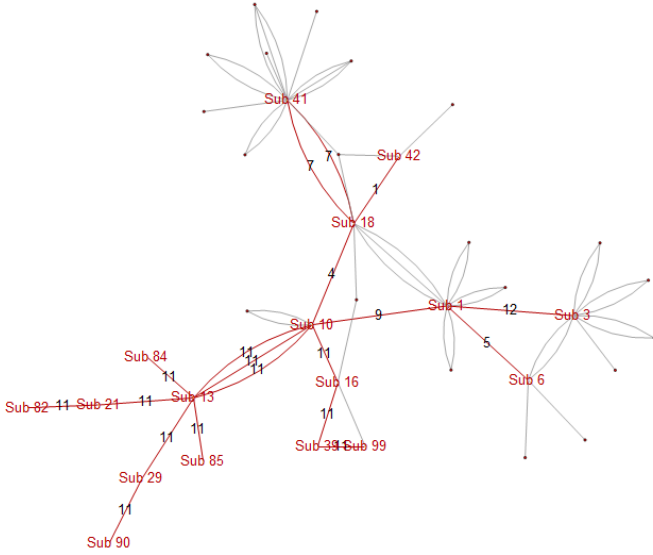


Figure 3. Visual representation of a large event. This figure replaces the substation names by numbers to make the data anonymous.

#### IV. EVENT SIZE DISTRIBUTION AND PROPAGATION METRIC

##### A. Method Overview and Results

Another measure of event size is the number of generations of outages in the event. If we consider all the events in the TADS data, by counting the number of events with 1, 2, 3, ... generations and dividing by the total number of events, we obtain the empirical probability distribution of the number of generations plotted on a log-log scale as shown by the dots in Fig. 4. The empirical probability distribution of the number of generations has the notable pattern [13] that it can be approximated as lying on the gray straight line shown in Fig. 4. The probability distribution on the number of generations that perfectly lies on a straight line on a log-log plot is called the Zipf distribution or the zeta distribution [14]. The gray line is obtained as the best fit of the data to a Zipf distribution using Goldstein's method in [15]. Here we use the distribution of the number of generations of outages rather than the number of outages because it shows better linearity on the log-log plot.

The Zipf distribution is heavy-tailed, which implies that large events are not vanishingly rare as is the case with many common distributions, but are rarer events that will occur occasionally. The slope of the Zipf distribution shows how much successive generations propagate and the balance between small and large events. A steeper slope means that

there is less propagation of events into larger numbers of generations and fewer large events. A shallower slope means that there is more propagation into larger numbers of generations and more large events. Indeed, [13] proposes the absolute value of the slope as the System Event Propagation Slope Index or SEPSI. Thus, a larger SEPSI indicates a steeper slope, less propagation of events into further generations, and relatively fewer large events.

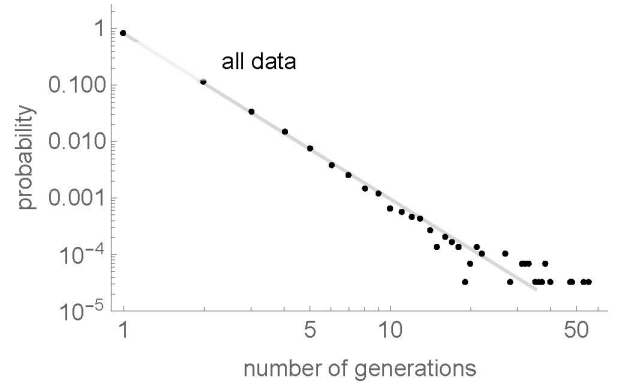


Figure 4. Log-log plot of distribution of number of generations for all the events (dots) with gray line showing the slope of the fitted Zipf distribution.

The TADS events all together have SEPSI 2.93 (the slope of the gray line in Fig. 4 is -2.93). We divide the TADS data into weather related and non-weather related, plot their respective event sizes in Fig. 5, and fit the straight lines shown to find that weather events have substantially more propagation into large events (SEPSI 2.41) than non-weather events (SEPSI 3.54). These results are consistent with Table I that confirms the larger size of weather related events than non-weather related events, with the size measured in either the number of outages or the number of generations of outages.

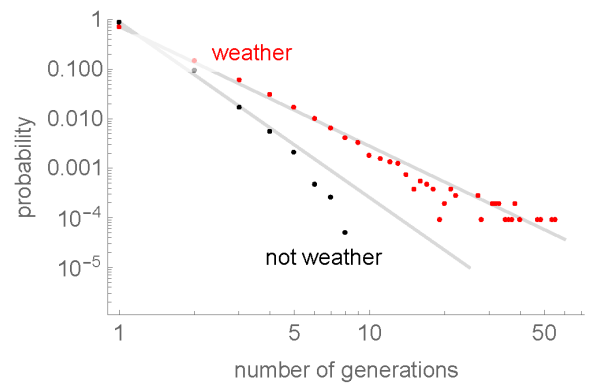


Figure 5. Log-log plot of distribution of number of generations for weather (red dots) and non-weather related events (black dots) with gray lines showing the slopes of fitted Zipf distributions

The SEPSI can be associated to probabilities that an event has a given range of sizes. For example, suppose that one defines a small event as 1 to 3 generations, a medium event as 4 to 9 generations, and a large event as 10 or more generations. The Zipf distribution has the explicit formula in terms of SEPSI for the probability of  $k$  generations:  $p_k = k^{-SEPSI} / \zeta(SEPSI)$ , where  $\zeta$  is the Riemann zeta function. Therefore, we can estimate probabilities of small, medium, and large events as:

$p_{small} = \sum_{i=1}^3 p_i$ ,  $p_{medium} = \sum_{i=4}^9 p_i$ , and  $p_{large} = 1 - p_{small} - p_{medium}$ . These event size probabilities are shown in Table III for all years combined and for weather events by year. The probability of large events can change by a large factor with a change in SEPSI. In particular, Table III shows that when a weather related event happens, it has approximately 20 times greater probability to become a large event compared with a non-weather related event. This phenomena is related to the “bunching” of outages as the forced outage rates increase dramatically during the event [8]. The weather is a common environmental cause of the increased outages.

TABLE III. 2015-2019 TRANSMISSION EVENT PROBABILITY BY SIZE

Transmission Events	SEPSI	Small Events	Medium Events	Large Events
All events	2.93	0.963	0.032	0.005
Non-weather	3.54	0.986	0.013	0.001
Weather events	2.41	0.914	0.065	0.021
Weather 2015	2.47	0.922	0.06	0.018
Weather 2016	2.41	0.913	0.065	0.021
Weather 2017	2.39	0.91	0.067	0.023
Weather 2018	2.38	0.908	0.068	0.023
Weather 2019	2.42	0.915	0.065	0.021

The weather events show some small variation in SEPSI from year to year. For example, the 2019 SEPSI corresponds to the second steepest slope of the fitted Zipf distribution after 2015, and is similar to the 2016 SEPSI. Consequently, large weather events had the lowest probability of 0.018 in 2015, and equal probabilities in 2019 and 2016.

We performed the SEPSI-based analysis for summer events versus non-summer events to detect a possible seasonal difference. SEPSI for summer and non-summer events were identical. Moreover, we found no significant difference in event size for summer and non-summer events. Further analysis based on regional level could be necessary to detect seasonal differences. Note that SEPSI and event size do not reflect event frequency; indeed, events occur more frequently in the summer.

## V. CONCLUSIONS

We analyze all TADS outages reported to NERC from 2015 to 2019 by grouping outages into events according to their start times and their overlaps in time. The algorithm grouping outages into events is generally successful in automatically grouping together the outages for the largest weather events. This processing of outage data enables statistical analysis of the events, which we do in this paper, as well as providing automated initial groupings of outages for further engineering analysis. TADS also provides network connectivity information and it is useful to automatically display the event outages as they develop in the network to facilitate the insights from further engineering analysis.

The analysis of the TADS-reported outage causes in events of different sizes reveals a dramatic increase in the percentage of weather-initiated outages as the event size increases. Approximately 3/4 of the outages in events with more than 34 outages are weather-initiated. The TADS outage cause codes are used to distinguish the weather related events. We compare and contrast the event sizes in weather related events versus non-weather events. We can measure event size either by the

number of outages or the number of generations of outages, where each generation of outages occurs in the same minute. The weather related events tend to contain more outages as well as more generations than non-weather events. All 2015-2019 events with more than 26 outages are weather related. Moreover, measures of event impact such as affected miles, affected MVA, and duration are notably larger for weather related events.

The System Event Propagation Slope Index or SEPSI is a bulk statistical measure of how much events propagate to a large size in terms of number of generations. SEPSI measures the slope of a log-log plot of the distribution of event size and can be related to the probability that an event becomes large. We find that SEPSI for weather related events implies a much higher probability of an event being large compared to non-weather events.

All our analyses point to the significance of weather for the largest events that, although rarer, have the highest impact on the transmission grid and consequently substantial risk.

## ACKNOWLEDGMENTS

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