Optimizing Design for Resilience for Risk-Averse Firms Using Expected Utility and Value-at-Risk

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Abstract
Research Problem • Determine how resilience should be integrated into a firm's design decisions • Optimize design for a risk-averse firm that incorporates resilience

Disciplines
Industrial Engineering | Industrial Technology | Manufacturing | Other Operations Research, Systems Engineering and Industrial Engineering

Comments
Optimizing Design for Resilience for Risk-Averse Firms Using Expected Utility and Value-at-Risk

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Adverse Events

Hurricane Sandy New York City Power Outage

Washington Train Derailment

Wind Turbine Failure
Risk attitude of the decision maker

Simulation optimization

Optimal design for resilience
Research Problem

- Determine how resilience should be integrated into a firm’s design decisions

- Optimize design for a risk-averse firm that incorporates resilience
Tradeoff Between Design Cost and Resilience
Resilience Definition

- Reliability: The ability of the system to stay above the failure limit

- Restoration: The ability to restore and recover a system’s performance after an adverse event occurs

Time-Dependent Resilience Analysis

Resilience: Performance > Failure limit

Firm Decision Making for Resilience

Decision Variables

- Redundancy
- Robustness
- Response Time
- Recovery Time

Monte Carlo simulation

Output

- Adverse condition
- Profit/utility resilience

Bayesian optimization
Optimization of Decision-Making Model

- Decision variables:
  - Redundancy (number of components)
  - Robustness (ability to withstand adverse event)
  - Response (time to respond after adverse event)
  - Recovery (time to recover system to functioning after failure)

- Objective function (profit or utility) can only be evaluated via Monte Carlo simulation
Decision Making Models

- Develop and solve mathematical model for risk-averse design firms
  - Expected utility
  - Value-at-risk
- Integrate risk-averse decision-making model into design-for-resilience simulation
Expected Utility Model

- Risk aversion parameter ($\gamma$) for exponential utility function
- Analyze optimal design for risk-neutral to very risk averse firms
- Optimal design maximizes expected utility

\[ U(x) = 1 - \exp(-\gamma x) \]
Value-at-Risk (VAR) Model

- VAR is defined as the largest profit $x_{var}$ such that there is a $q$ probability that the profit is less than or equal to $x_{var}$.
- The risk exposure of a firm can be limited by using VAR.
  - Firm maximizes expected profit subject to VAR.

*VAR constraint: $\text{Probability (profit < } x_{var} \text{)} < q$*
Two Illustrative Examples

- One-subsystem example

- Three-subsystems example
One-Subsystem Example
### Optimal Design of One Subsystem Example

<table>
<thead>
<tr>
<th></th>
<th>Risk neutral decision maker</th>
<th>Risk averse decision maker</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 0$ (Result of Expected Value model)</td>
<td>$\gamma = 1 \times 10^{-4}$</td>
<td>$\gamma = 5 \times 10^{-4}$</td>
</tr>
<tr>
<td><strong>Number of Components</strong></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td><strong>Mean of Robustness</strong></td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td><strong>Mean of Response Time</strong></td>
<td>7.5</td>
<td>5.2</td>
</tr>
<tr>
<td><strong>Mean of Recovery Time</strong></td>
<td>31.0</td>
<td>14.4</td>
</tr>
<tr>
<td><strong>Expected Profit</strong></td>
<td>1423</td>
<td>1385</td>
</tr>
<tr>
<td><strong>Resilience</strong></td>
<td>94.5%</td>
<td>96.9%</td>
</tr>
<tr>
<td><strong>Design Cost</strong></td>
<td>704.6</td>
<td>930.7</td>
</tr>
</tbody>
</table>
Three-Subsystems Example

- Subsystems in series
- Identical components in parallel
- Identical components in the $i$th subsystem possess the same resilience properties
# Optimal Design of three-Subsystem Example for the Expected Utility Model

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Redundancy (number of components)</th>
<th>Mean robustness</th>
<th>Mean response time</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Risk neutral</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Moderately risk averse</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
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<table>
<thead>
<tr>
<th>Mean recovery time</th>
<th>Expected profit</th>
<th>Resilience</th>
<th>Design cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsystem</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Risk neutral</td>
<td>29</td>
<td>50</td>
<td>42</td>
</tr>
<tr>
<td>Moderately risk averse</td>
<td>50</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>Very risk averse</td>
<td>1</td>
<td>49</td>
<td>1</td>
</tr>
</tbody>
</table>
Optimal Design of three-Subsystem Example for Expected Profit with VAR Constraint

<table>
<thead>
<tr>
<th>Probability (profit &lt; alpha) &lt; 0.05</th>
<th>Redundancy (number of components)</th>
<th>Mean robustness</th>
<th>Mean response time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Subsystem</td>
<td>Subsystem</td>
<td>Subsystem</td>
</tr>
<tr>
<td></td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
</tr>
<tr>
<td>Expected profit</td>
<td>3 1 2</td>
<td>-1 -13 -1</td>
<td>1 2 1</td>
</tr>
<tr>
<td>alpha = 3300</td>
<td>3 1 2</td>
<td>-1 -13 -1</td>
<td>1 10 1</td>
</tr>
<tr>
<td>alpha = 3600</td>
<td>3 1 2</td>
<td>-1 -16 -1</td>
<td>1 18 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean recovery time</th>
<th>Expected profit</th>
<th>Resilience</th>
<th>Design cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsystem</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
</tr>
<tr>
<td>Expected profit</td>
<td>29 50 42</td>
<td>3970</td>
<td>97.7%</td>
</tr>
<tr>
<td>alpha = 3300</td>
<td>6 3 4</td>
<td>3953</td>
<td>98.4%</td>
</tr>
<tr>
<td>alpha = 3600</td>
<td>16 47 1</td>
<td>3919</td>
<td>98.7%</td>
</tr>
</tbody>
</table>
Interpret Results

• As risk aversion increases
  • Firm should design more resilient systems
  • Firm should sacrifice some profit by paying more in design costs
• Having more resilience requires
  • More redundancy (components)
  • More robustness
  • Quicker response time
  • Quicker recovery time
Conclusions

• Trade off between designing a more resilient but costly system

• Framework to incorporate the risk aversion of the decision maker (expected utility and VAR)

• Solve for the optimal design for engineered systems (illustrative examples)