Dynamic Decisions For The Cow-Calf Producer

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Abstract
In the production and marketing decisions of a cow-calf producer breeding herd investments and land acquisitions are primary. A static model could assume either fixed or infinitely variable breeding herd and land assets in the short-run or long-run respectively. Actually, the assets of a producer are neither completely fixed nor infinitely variable, but must change over time.

Disciplines
Agribusiness | Business Administration, Management, and Operations | Economic Policy | Entrepreneurial and Small Business Operations | Science and Technology Policy
DYNAMIC DECISIONS FOR THE COW-CALF PRODUCER

Greg Hertzler

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In the production and marketing decisions of a cow-calf producer breeding herd investments and land acquisitions are primary. A static model could assume either fixed or infinitely variable breeding herd and land assets in the short-run or long-run respectively. Actually, the assets of a producer are neither completely fixed nor infinitely variable, but must change over time.

The major purpose of this paper is to construct a dynamic theoretical foundation upon which a model of the cow-calf production process could be built. The key distinguishing features are the equations of motion specifying the changes over time of assets such as the breeding herd and land. These equations of motion are either differential or difference equations which constrain the adjustments cow-calf producers can make. As constraints, they have dual variables giving the derived value of the assets at each point in time.

A second purpose of this paper is to generalize the specification of a cow-calf producer's objective function. It is customary to assume producers maximize expected profits, or perhaps the utility of expected profits. However, for many ranchers the lifestyle is most important and they may be willing to pay for that lifestyle in profits foregone. Their objectives will, of course, affect their behavior and also the beef and land markets.

A third and final purpose of this paper is to briefly discuss how the basic theory could be expanded for use in empirical research and policy
making. Specifically, implications are drawn for setting the rental rate on public grazing land, and indications are given of how uncertainty and expectations could be included in aggregate models of the beef and land markets.

The Objective Function of a Cow-Calf Producer

Cow-calf producers who enjoy the amenities of ranching may have an objective such as

$$\max_{t=0}^{T} \sum_{t=0}^{T} \Delta U(q_t, m_t, b_t, l_t, l'_t, w_t) + \Delta_{T+1} U(M_{T+1} - D_{T+1}),$$

where

- $\Delta$ is a discount factor embodying the producer's rate of time preference,
- $U$ is direct utility,
- $q$ is the amount of commodities consumed,
- $m$ is money assets in dollars,
- $b$ is the number of animals in the breeding herd,
- $l$ is the animal units of land owned,
- $l'$ is the animal units of land rented,
- $w$ is the animal units of work by the rancher,
- $M$ is total wealth in dollars,
- $D$ is total debt in dollars,
- $T$ is the number of years in the producer's planning horizon,
- $t$ is a time subscript.

Cow-calf producers gain satisfaction from commodity consumption, $q$, like any other consumer, but they may also enjoy owning and operating a ranch.
It is possible they feel more secure by saving. Hence, m, b, 1, l', and w could all contribute to utility. If there is a bequest motive it might take the specific form of leaving a particular ranch to particular heirs. However, in Equation 1, the only consideration is the utility of bequeathing net equity at time T+1.

A convenient functional form for utility is

\[ U_t = \left[ (q_t - \bar{q}_t)^{\alpha_q} \left( m_t - \bar{m}_t \right)^{\alpha_m} (b_t - \bar{b}_t)^{\alpha_b} (1_t - \bar{1}_t)^{\alpha_1} \left( 1_t' - \bar{1}_t' \right)^{\alpha_1'} (\bar{w}_t - w_t)^{\alpha_w} \right]^\varepsilon, \]

where

- \( \bar{q}, \bar{m}, \bar{b}, \bar{1}, \) and \( \bar{1}' \) are minimum subsistence bundles,
- \( \bar{w} \) is maximum possible work, \( \bar{w} - w \) being leisure,
- \( \varepsilon \) is a parameter such that \( 0 < \varepsilon < +\infty \),

and all other variables are as previously defined.

Equation 2 is a modified form of the Stone-Geary utility function which is the basis of the linear expenditure system. The additive separability of the Stone-Geary utility function limits the substitution possibilities between its arguments. In spite of this empirical disadvantage, the linear expenditure system gives very intuitive interpretations in a theoretical discussion.

The usual form of the Stone-Geary utility function assumes a logarithmic transformation. The modified form in Equation 2 is transformed by the exponential parameter \( \varepsilon \). While demand functions are ordinal and invariant to monotonic transformations, marginal utility of expenditures is not. Transformations are important in discussing cardinal properties such as risk preferences.
Equations of Motions

Four difference equations will be included to describe changes over time in money, the breeding herd, land ownership, and debt.

Money

The change in money assets is savings or dissavings described as

\[
m_{t+1} - m_t = (1-\omega)[p_{mt}M_t + \Pi_t + p_w w_t - I_t] + (1-\gamma)B_t - L_t - p_q q_t,
\]

where

- \( \omega \) is the tax rate on ordinary income,
- \( p_m \) is the rate of return on assets the producer is willing to accept,
- \( \Pi \) is profit from the production of offspring from the breeding herd,
- \( p_w \) is the price per animal unit of work,
- \( I \) is total interest on outstanding debt,
- \( \gamma \) is the tax rate on capital gains,
- \( B \) is the income from buying and selling breeding animals,
- \( L \) is the diversion of money assets to land equity,
- \( p_q \) is the price per unit of consumption commodity \( q \),

and all other variables are as previously defined.

Notice capital gains taxes are not computed on land. This would be the case if land is only accumulated and not sold.

Breeding herd

Investment in the breeding herd is an optimal replacement problem. One approach is to choose the optimal replacement age of each animal in the herd. However, determining the optimal age becomes very difficult because the optimum for any animal depends upon every other animal in the herd.
An alternative is to determine the number of replacement and culled animals in each year. The optimal age before replacement is, in effect, determined as some unknown function of replacement and culling decisions. The change over time of the breeding herd becomes

\[ b_{t+1} - b_t = r_t - c_t - \beta b_t, \]

where

- \( r \) is the number of replacement animals,
- \( c \) is the number of culled animals excluding death loss,
- \( \beta \) is the proportion of the breeding herd which dies unexpectedly.

Culling, at a minimum, includes all animals that failed to reproduce.

Land

The change over time of owned land is

\[ l_{t+1} - l_t = a_t, \]

where

- \( a \) is the additional animal units of land acquired.

Debt

Producers could finance both cattle and land purchases through debt. Because the dynamics of debt can become very complicated, breeding herd expansions are assumed to be financed internally. Land can be financed internally, or by long-term debt. Each loan is amortized over a given repayment period at a fixed rate of interest. The fixed interest rate assumption can be relaxed if necessary.
For a loan contracted at time $t-T$, the outstanding debt at various
times is

6i) $d_{t-T,t-T} = p_{t-T} (a_{t-T} - \bar{a}_{t-T})$,

6ii) $d_{t-T+1,t-T} = d_{t-T,t-T} - [p_{t-T} p_{t-T} (a_{t-T} - \bar{a}_{t-T}) - p_{mt-T} d_{t-T,t-T}]$,

6iii) $d_{t-T+2,t-T} = d_{t-T+1,t-T} - [p_{t-T} p_{t-T} (a_{t-T} - \bar{a}_{t-T}) - p_{mt-T} d_{t-T+1,t-T}]$,

$\vdots$

6iv) $d_{t,t-T} = d_{t-1,t-T} - [p_{t-T} p_{t-T} (a_{t-T} - \bar{a}_{t-T}) - p_{mt-T} d_{t-1,t-T}] = 0$,

where

- $d_{s,t-T}$ is debt at time $s$ for a loan contracted at time $t-T$.
- $p_1$ is the price per animal unit of land,
- $\bar{a}$ is the down payment in animal units of land,
- $p_T$ is the repayment factor defined below,
- $T$ is the length of the repayment period.

Outstanding debt plus interest is repaid in $T$ equal installments, for a repayment factor of

7) $p_{t-T} = \frac{p_{mt-T}}{1 - 1/(1+p_{mt-T})^T}$.

From 6, the decrease in debt (increase in equity) for a loan is the amount of the installment payment minus interest on outstanding debt. A more compact representation of the change in debt over time is possible by
substituting 6i into 6ii then subtracting 6ii from 6iii and combining terms, and so on for all time periods, or

6ii') \[ d_{t-T+1,t-T} - d_{t-T+1,t-T} = - (p_{tt-T} - p_{mt-T}) p_{lt-T} (a_{t-T} - a_{t-T}) , \]

6iii') \[ d_{t-T+2,t-T} - d_{t-T+1,t-T} = - (1 + p_{mt-T}) (p_{tt-T} - p_{mt-T}) p_{lt-T} (a_{t-T} - a_{t-T}) , \]

6iv') \[ d_{t,t-T} - d_{t-1,t-T} = (1 + p_{mt-T}) (d_{t-1,t-T} - d_{t-2,t-T}) \]

\[ = - (1 + p_{mt-T})^{T-1} (p_{tt-T} - p_{mt-T}) p_{lt-T} (a_{t-T} - a_{t-T}) . \]

A new loan can be contracted in each time period, making total debt at time \( t \) a function of all loans initiated between times \( t-T \) and \( t \). The amount of debt retired is the sum of the change in each individual debt,

8) \[ D_{t+1} - D_t = \sum_{s=0}^{T-1} (d_{t+1,t+s} - d_{t,t-s}) \]

\[ = - \sum_{s=0}^{T-1} (1 + p_{mt-s})^{s} (p_{tt-t-s} - p_{mt-s}) p_{lt-t-s} (a_{t-s} - a_{t-s}) , \]

where

\( s \) is a time subscript.

Retired debt is not the same as the change in total debt which equals

\[ \sum_{s=0}^{T-1} (d_{t+1,t+1-s} - d_{t,t-s}) = D_{t+1} - D_t + d_{t+1,t+1} . \]
Other Functional Forms

It remains to specify functional forms for wealth, M, profit from production, \( \Pi \), interest expense, I, capital gains income from sale of the breeding herd, B, and money converted to land equity, L. Tax rates on ordinary and capital gains income, \( \omega \) and \( \gamma \) respectively will be assumed constant, although \( \omega \) especially, is a function of income and depreciation.

**Wealth**

In dollar terms, total wealth of the cow-calf producer is

\[
M_t = m_t + p_b b_t + p_l l_t
\]

where

\( p_b \) is a weighted average price over all breeding animals of different ages.

**Profit**

As the primary source of income, the cow-calf producer's profit from the sale of offspring is

\[
\Pi_t = p_r F(b_{t-\delta}, l_t + l_t', w_t) - p_{mt} p_b b_t - p_{mt} (p_l l_t + p_l' l_t') - p_{wt} w_t
\]

where

\( p_r \) is the price per animal of offspring either sold or used as breeding replacements,

\( F \) is the production function giving animals produced as a function of animal unit inputs,
\( \sigma \) is the time lag between breeding and sale, 
\( p_1 \), is the price per animal unit of rental land.

The producer must pay rent on breeding animals, on both owned and rented land, and must pay wages. Of course, returns to the breeding herd, land, and wages are sources of income in Equation 3. Only rental land is an out-of-pocket expense.

Production function \( F \) is difficult to adequately specify. Biological growth models from resource economics where harvesting is imperfect and population dynamics allow the breeding stock to expand into the environment are of little use in beef production where harvesting is perfect and population dynamics are completely controlled. When land and work are limiting factors, reproduction and growth may be less than maximum with some substitutions possible between breeding animals, land, and work. However, when the number of breeding animals is limiting and land and work exist in sufficient quantities for proper reproduction and growth a biological plateau may be reached where increasing land or work will not increase production. On this plateau, calves produced will be proportional to the size of the breeding herd.

One possible approach is to specify

\[
F_t = \nu(b_{t-\sigma}, l_t + l_t {'}^t, w_t)b_{t-\sigma},
\]

where

\( \nu \) is the reproduction rate as a function of the breeding herd, land, and work.
The rate of reproduction can be considered constant when there is sufficient or excess land and work for a given size herd. Otherwise, \( v \) is decreasing in \( b \), increasing in \( 1 + l \), and increasing in \( w \). Overall, \( F \) is concave but not strictly concave.

**Interest expense**

Total interest paid on outstanding debt equals total monthly payments minus that portion of the payments which go toward retiring debt (increasing equity) or,

\[
I_t = \sum_{s=0}^{T-1} (p_{t-s} p_{l-s} (a_{t-s} - \bar{a}_{t-s})) + D_{t+1} - D_t,
\]

because \(-(D_{t+1} - D_t)\) is the increase in equity.

**Capital gains**

Gain from investing in the breeding herd is

\[
R_t = -p_{r+T} t + \frac{\lambda_{t+T}}{\lambda_t} \beta_2 \frac{t+T}{\tau} p_{r+t} c_{t+T},
\]

where

- \( \lambda \) is the discounted marginal utility of expenditures,
- \( \beta_2 \) is the rate at which the suitability for slaughter of a breeding animal depreciates over time,
- \( \tau \) is the age of the breeding animals being culled.

Current investment in replacement animals subtracts from current income. Over time these invested animals will depreciate as slaughter animals while the future income from their sale must be discounted to its present value by the ratio of marginal utilities.
Land equity

Land equity acquired in any period is the amount of debt retired on previous loans plus any current down payment,

14) \[ L_t = -(D_{t+1} - D_t) + p_{lt} a_t. \]

Constraints on the Decision Process

Several feasibility constraints exist, but only those which are apt to be binding are explicitly imposed. These are

15) \[ \frac{\bar{m}_t}{m_t} \leq m_t \] (liquidity)

16) \[ D_t \leq m_t \] (collateral)

17) \[ 0 \leq r_t \] (replacement)

18) \[ \beta \beta_t \leq c_t \leq b_t \] (culling)

19) \[ \bar{l}' \leq l_t' \leq \bar{l}' \] (land rental)

20) \[ 0 \leq \bar{a}_t \leq a_t \] (land acquired)

where

\( \beta_{3} \) is the proportion of the breeding herd which fails to successfully reproduce,

\( \bar{l}' \) is the maximum animal units of land available for rent.

The liquidity constraint requires a nonnegative bank balance at all times while the collateral constraint prevents further borrowing when equity vanishes. The replacement constraint makes investment in the breeding herd
irreversible, and culling is bounded below and above by the number of animals that have failed to reproduce and the total breeding herd. Land rental must be above a minimum level, but cannot exceed total rental land available. Land acquisition is restricted to be non-negative because capital gains taxes are not computed for land, and the down payment cannot exceed total land purchased.

Optimality Conditions

The Lagrangian (not Hamiltonian) for the cow-calf producer's decision problem is

\[ J = \sum_{t=0}^{T} \left( \Delta U_t + \lambda_t \left( (1-\omega) \left( \pi_t^M + \pi_t^W - I_t^t + \rho_t^M \nu_t^M + \rho_t^W \nu_t^W - I_t^t \right) + (1-\gamma) b_t^t - L_t^t + p_{qt}^t q_t^t - m_{t+1}^t + m_t^t \right) \\
+ \theta_t \left( r_t^t - c_t^t - b_t^t + (1-\beta)^t b_t^t \right) + \varnothing_t \left[ a_t^t - 1_{t+1}^t + 1_t^t \right] \\
+ \psi_t \sum_{s=0}^{T-1} (1 + p_{mt-s}^t)^s \left( p_{Ts-s}^t - p_{mt-s}^t \right) \left( a_t-s - \bar{a}_t-s \right) - D_{t+1}^t + D_t^t \\
+ \mu_{mt}^t (m_t^t - m_t^t) + \mu_{Dt}^t (M_t^t - D_t^t) + \mu_{rt}^t r_t^t + \mu_{lc}^t (c_t - \beta c_t^t) + \mu_{ct}^t (b_t - c_t^t) \\
+ \mu_{1lt}^t (1_t^t - \bar{1}_t^t) + \mu_{2lt}^t (\bar{1}_t^t - 1_t^t) + \mu_{tat}^t \bar{a}_t^t + \mu_{2at}^t (a_t - \bar{a}_t^t) \right) \\
+ \Delta_{T+1}^t U_{T+1}^t \]

where

\theta is the value of holding breeding animals for future production,
\varnothing is the value of holding land for future production,
\[ \psi \text{ is the value of holding debt for the future,} \]

\[ \mu \text{'s are Lagrange multipliers.} \]

The state variables associated with the equations of motion in Lagrangian \( J \) are \( m, b, l, \) and \( D. \) These state variables have corresponding costate variables \( \lambda, \theta, \phi, \) and \( \psi. \) The variables available to control the system are \( q, r, c, l', a, \bar{a}, \) and \( w. \) Because of assumptions about functions \( U \) and \( F, \) \( J \) is continuous and concave in \( m, b, l, q, l', \) and \( w, \) but is linear in \( D, r, c, a, \) and \( \bar{a}. \) Taking derivatives,

\[ \frac{\partial J}{\partial m_t} = 0 = \frac{\Delta_t \varepsilon U_t}{\alpha} + \lambda_t (1-\omega) P_{mt} - \lambda_{t-1} + \lambda_t + \mu_{mt} + \mu_{Dt}, \]

\[ \frac{\partial J}{\partial b_t} = 0 = \frac{\Delta_t \varepsilon U_t}{\beta} + \lambda_t (1-\omega) P_{rt+\sigma} (\nu_{t+\sigma} \frac{\partial \nu_{t+\sigma}}{\partial b} b_t) \]

\[ - \theta_{t-1} + (1-\beta_1) \theta_t + \mu_{Dt} b_t - \mu_{1ct} 3 + \mu_{2ct}, \]

\[ \frac{\partial J}{\partial l_t} = 0 = \frac{\Delta_t \varepsilon U_t}{\nu} + \lambda_t (1-\omega) P_{rt} \nu_t \frac{\partial \nu_t}{\partial l_t} b_{t-\sigma} \]

\[ - \phi_{t-1} + \phi_t + \mu_{Dt} P_{1t}, \]

\[ \frac{\partial J}{\partial D_t} = 0 = \omega (\lambda_{t-1} - \lambda_t) - \psi_t - \mu_{Dt}, \]

\[ \frac{\partial J}{\partial q_t} = 0 = \frac{\Delta_t \varepsilon U_t}{\alpha} - \lambda_t P_{qt}, \]

\[ \frac{\partial J}{\partial r_t} = 0 = -\lambda_t (1-\gamma) P_{rt} + \theta_t + \mu_{rt}. \]
\[
\begin{align*}
\frac{\partial J}{\partial c_t} &= 0 = \lambda_t (1-\gamma)\beta_2 p_{rt} - \theta_t + \mu_{1ct} - \mu_{2ct}, \\
\frac{\partial J}{\partial l_t} &= 0 = \frac{\alpha t}{1_t - l_t} + \Lambda_t (1-\omega) (p_{rt}^\prime \frac{\partial v_t}{\partial \alpha_t} b_{t-o} - p_{mt}^\prime p_{lt}^\prime), \\
&\quad + \mu_{1l't} - \mu_{2l't}, \\
\frac{\partial J}{\partial a_t} &= 0 = -(1-\omega)p_{tt}^\prime p_{lt}^\prime \sum_{s=0}^{T-1} \lambda_{t+s} + \varnothing_t \\
&\quad - (p_{tt}^\prime - p_{mt}^\prime) p_{lt}^\prime \sum_{s=0}^{T-1} (1+p_{mt}^\prime)^s \psi_{t+s} + \mu_{2at}, \\
\frac{\partial J}{\partial a_t} &= 0 = (1-\omega)p_{tt}^\prime p_{lt}^\prime \sum_{s=0}^{T-1} \lambda_{t+s} - \lambda_t p_{lt}^\prime \\
&\quad + (p_{tt}^\prime - p_{mt}^\prime) p_{lt}^\prime \sum_{s=0}^{T-1} (1+p_{mt}^\prime)^s \psi_{t+s} + \mu_{lat} - \mu_{2at}, \\
\frac{\partial J}{\partial w_t} &= 0 = \frac{-\Delta_t \epsilon_U t \alpha}{w_t - w_t} + \lambda_t (1-\omega)p_{rt}^\prime \frac{\partial v_t}{\partial w_t} b_{t-o}.
\end{align*}
\]

Terminal conditions are
\[
\begin{align*}
\frac{\partial J}{\partial m_{T+1}} &= 0 = -\lambda_T + \Delta_{T+1} \frac{\partial U_{T+1}}{\partial M_{T+1}}, \\
\frac{\partial J}{\partial b_{T+1}} &= 0 = -\theta_T + \Delta_{T+1} \frac{\partial U_{T+1}}{\partial M_{T+1}} p_{bT+1} = -\Theta_T + \lambda_T p_{bT+1}, \\
\frac{\partial J}{\partial l_{T+1}} &= 0 = -\varnothing_T + \Delta_{T+1} \frac{\partial U_{T+1}}{\partial M_{T+1}} p_{1T+1} = -\Theta_T + \lambda_T p_{1T+1},
\end{align*}
\]
The above first order conditions are a generalization of the usual separation of consumption from production decisions. If $\alpha_m = \alpha_b = \alpha_l = \alpha_w = 0$ and money is constant with $m_{t+1} = m_t$, the above first order conditions collapse to those obtained by maximizing the utility of consumption subject to a budget constraint and separately maximizing the utility of net income (which equals expenditures) subject to equations of motion and other constraints. For example, the discounted marginal utility of expenditures is $\lambda_t = \Delta_t (\partial U/\partial E_t)$ which can be substituted into condition 22 to obtain

$$\lambda_t (1-\gamma)(1-\beta_2) p_{rt} = \mu_{rt} + \mu_{1ct} - \mu_{2ct}.$$  

Because the left-hand side of 37 is always positive, one or both of $\mu_r$ and $\mu_{1c}$ must be positive, with either or both $r$ and $c$ at their respective lower bounds. It would be possible, in some cases, for $r$ to be at its lower bound and $c$ at its upper bound if the outlook for the beef market is so grim. 

Demand for Consumption and Amenities

The first step in specifying the demand system is determining relationships for the costates $\theta$ and $\phi$. Equating 27 and 28,

$$\frac{\partial J}{\partial D_{t+1}} = 0 = \omega \lambda_T - \psi_T - \Delta_{t+1} \frac{\partial U}{\partial M_{t+1}} = -\psi_T - (1-\omega) \lambda_T.$$ 

36)

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or cash flow such a problem the only alternative is to completely liquidate the herd. In a more normal circumstance, \( c \) would be at its lower bound and \( r \) at an interior solution. Then, in 27, the discounted value of holding stock for future production, \( \theta \), would equal the discounted current after tax value of a replacement animal.

The value of holding land is found by adding 30 to 31,

\[
\theta_t = \lambda_t p_{1t} - \mu_{lat}.
\]

At an interior solution, the discounted value of holding land for the future equals the discounted current value of land. If land acquisition is constrained to be non-negative with \( \mu_{la} \) positive, the value of holding land can be less than the current value.

First order conditions 26, 22, 23, 24, 29, and 32 are the demand system for consumption and amenities. The value of holding stock from 27 is substituted into 23, and the value of holding land from 38 is substituted into 24. Each of the first order conditions is then solved for its corresponding \( \alpha \),

\[
\begin{align*}
\alpha_q &= \frac{\lambda_t}{\Delta_e U_t} p_{qt} (q_t - \bar{q}_t), \\
\alpha_m &= \frac{\lambda_t}{\Delta_e U_t} p_{mt} (m_t - \bar{m}_t), \\
\alpha_b &= \frac{\lambda_t}{\Delta_e U_t} p_{bt} (b_t - \bar{b}_t), \\
\alpha_1 &= \frac{\lambda_t}{\Delta_e U_t} p_{1t} (1_t - \bar{1}_t),
\end{align*}
\]
\[ a_{1t'} = \frac{\lambda_t}{\Delta \epsilon U_t} P_{1t'} (1_{t'} - \bar{1}_{t'}) , \]

\[ a_{w} = \frac{\lambda_t}{\Delta \epsilon U_t} P_{w} (\bar{w}_t - w_t) , \]

where

\[ p_{qt} \] is as before, and

\[ 0 \leq P_{mt} = \frac{\lambda_{t-1}}{\lambda_t} - (1 + (1 - \omega)p_{mt}) - \frac{\mu_{mt}}{\lambda_t} - \frac{\mu_{Dt}}{\lambda_t} , \]

\[ 0 \leq P_{bt} = -(1 - \omega)p_{rt + \sigma} \left( v_{t+\sigma} + \frac{\partial v_{t+\sigma}}{\partial b_{t+\sigma}} b_t \right) + \frac{\lambda_{t-1}}{\lambda_t} (1 - \gamma)p_{rt - \lambda_t} \]

\[ - (1 - \beta_1)(1 - \gamma)p_{rt - \lambda_t} - \frac{\mu_{rt - \lambda_t}}{\lambda_t} + (1 - \beta_1) \frac{\mu_{rt}}{\lambda_t} \]

\[ - \frac{\mu_{Dt}}{\lambda_t} P_{bt} + \frac{\mu_{lt \epsilon t}}{\lambda_t} \beta_3 - \frac{\mu_{2ct}}{\lambda_t} , \]

\[ 0 \leq P_{lt} = -(1 - \omega)p_{rt} \left( \frac{\partial v_{t}}{\partial l_{t}} b_{t - \sigma} + \frac{\lambda_{t-1}}{\lambda_t} p_{lt - \lambda_t - 1} - P_{lt} \right) \]

\[ - \frac{\mu_{1at - \lambda_t}}{\lambda_t} + \frac{\mu_{lat}}{\lambda_t} - \frac{\mu_{Dt}}{\lambda_t} P_{lt} , \]

\[ 0 \leq P_{l't} = -(1 - \omega)(p_{rt} \frac{\partial v_{t}}{\partial l_{t}} b_{t - \sigma} - p_{mt} p_{l't}) - \frac{\mu_{11't}}{\lambda_t} + \frac{\mu_{21't}}{\lambda_t} , \]

\[ 0 \leq P_{wt} = (1 - \omega)p_{rt} \frac{\partial v_{t}}{\partial w_{t}} b_{t - \sigma} . \]
If any $\alpha$ (excluding $\alpha_q$) is zero the corresponding amount paid in profits foregone, $P$, must be zero. As long as cow-calf producers can meet liquidity, collateral, and other constraints, they can consume amenities and accept less than profit maximizing returns to their assets and labor.

Normally, cardinality can be purged from a demand system such as 39-44 by noting $\alpha_q + \alpha_m + \alpha_b + \alpha_1 + \alpha_l + \alpha_w = 1$, adding up all equations and solving for discounted marginal utility as

$$\lambda_t = \frac{\Delta_t e U_t}{E_t - \bar{E}_t} = \frac{\alpha_q \alpha_m \alpha_b \alpha_1 \alpha_l \alpha_w}{P_{qt} P_{mt} P_{bt} P_{lt} P_{l't} P_{wt}(E_t - \bar{E}_t)^{1-c}},$$

where

$$E_t - \bar{E}_t = P_{qt}(q_t - \bar{q}_t) + P_{mt}(m_t - \bar{m}_t) + P_{bt}(b_t - \bar{b}_t)$$

$$+ P_{lt}(l_t - \bar{l}_t) + P_{l't}(l'_t - \bar{l}'_t) + P_{wt}(w_t - \bar{w}_t),$$

which are expenditures above subsistence. Then each of the $\alpha$'s becomes a function of expenditures on a single good divided by expenditures on all goods. Upon rearranging, Equations 39-44 would become a demand system.

Unfortunately, $P_m, P_b, P_1,$ and $P_{l'}$ still depend upon cardinal discounted marginal utility, $\lambda$, especially if a constraint is binding. Only $P_w$ is directly observable from known technologies, with producers paying for leisure whenever more work would increase production. Unless all other $P$'s are known, the demand system cannot be estimated.
Observing Payments for Amenities

The keys to identifying the amounts paid for amenities are 1) decide what constraints are binding, and 2) make some assumption which allows the ratio of discounted marginal utilities, \( \lambda_{t-1}/\lambda_t \), to be observed as a function of known parameters. Only two constraints will be important in many situations, the lower bound on culling with \( \mu_{1ct} \) positive, and perhaps the upper bound on rental land available with \( \mu_{21t} \) positive. An assumption about the \( \alpha \)'s will allow the ratio of marginal utilities to be observed.

First suppose the only nonzero Lagrange multiplier is \( \mu_{1ct} \). Also suppose money does not contribute to utility, i.e., there is no "saving for a rainy day," making \( \alpha_m = P_m = 0 \). Then relationship 37 determines \( \mu_{1ct} \) and \( \lambda_{t-1}/\lambda_t \) can be solved from 45 to give

\[
0 = P_m = \frac{\lambda_{t-1}}{\lambda_t} - (1 + (1-\omega)p_{mt})
\]

\[
0 \leq P_{bt} = -(1-\omega)p_{rt+\sigma}^{(y_t+\sigma) + \frac{\delta y_t}{\delta b_t+\sigma} b_t} + (1 + (1-\omega)p_{mt})(1-\gamma)p_{t-1}
\]

\[
- (1-\beta_1)(1-\gamma)p_{rt} + (1-\gamma)(1-\beta_2)p_{rt-\beta_3}.
\]

\[
0 \leq P_{lt} = - (1-\omega)p_{rt} \frac{\delta y_t}{\delta l_t} b_t - (1 + (1-\omega)p_{mt})p_{lt-1} - p_{lt},
\]

\[
0 \leq p_{lt} = - (1-\omega)(p_{rt} \frac{\delta y_t}{\delta l_t} b_t - p_{mt}p_{lt}),
\]

Relation 48' holds only when the rental rate of rented land is set sufficiently high to ensure an interior solution. For some ranchers renting
public lands, this may or may not be the case. If there is excess demand for rented land, \( \nu_{21}t \) will be positive and 48 becomes unobservable.

Suppose it is possible to assume a rancher is willing to pay equally for either rented or owned land. This is an extremely strong assumption, but allows \( P_1 \) to be substituted for \( P_1 \), in the estimation process without involving relationship 48.

If any other constraints are binding, further restrictions will, regrettably, be necessary. Once estimation of the ordinal demand system is done, measurement of the cardinal parameter \( \lambda \) is possible by an experiment to determine risk preferences. In a model with certainty, a more correct term would be variation preferences, but the term risk preferences will be used in anticipation of generalizing the model to the uncertain case. Risk preferences are measured by

\[
\rho = -\frac{\partial \ln \lambda_t}{\partial \ln E_t} = (1-\varepsilon) \frac{E_t}{E_t - E_t},
\]

where

\( \rho \) is the expenditure elasticity of the discounted marginal utility of expenditures.

Elasticity \( \rho \) is also known as the Arrow-Pratt coefficient of relative risk aversion. Once it is found by experimental elicitation, \( \varepsilon \) is known. Producers have risk aversion, neutrality, or affinity if \( \rho > 0 \) and \( 0 < \varepsilon < 1 \), \( \rho = 0 \) and \( \varepsilon = 1 \), \( \rho < 0 \) and \( \varepsilon > 1 \) respectively. If \( \varepsilon \) were fixed with \( \rho \) and \( E \) allowed to vary, producers would display decreasing relative risk aversion as expenditures increased above subsistence.
Risk preferences are especially important at corner solutions where a positive Lagrange multiplier is divided by $\lambda$. In 50, more risk averse producers will have a smaller $\lambda$, compounding the effect of a binding constraint. Risk preferences also play a role in determining interest rates. If discounting is exponential with $\Delta_t = 1/(1 + \delta)^t$, then 50 substituted into 45' yields

$$U = (1+\delta)(\frac{E_t}{U_t}) - (1 + (1-\omega)p_{mt}),$$

where $\delta$ is the rate of time preference.

The classic result holding the after tax interest rate to equal the rate of time preference requires a steady-state with constant $U$ and $E$ under the maintained assumption $\alpha_m = 0$. Instability allows risk preferences to affect the interest rate because $\epsilon$ is a parameter of $U$.

Implications for Public Land Policy

When there is excess demand for renting public land with $\mu_21't > 0$ and other multipliers zero, and assuming $\alpha_m = 0$ and $p_{1t} = p_{1't}$, 47' equated to 48 provides the relationship

$$\mu_21't = (1-\omega)(p_{mt}p_{lt-1} - p_{mt}p_{1't}) + p_{lt-1} - p_{lt}.$$

If the price of private land was constant, $p_{lt-1} = p_{lt}$, and the rate of return on private land, $p_{mt}p_{lt-1}$, exceeds the rental rate on public land, $p_{mt}p_{1't}$. At the point where excess demand vanishes, $\mu_21't = 0$ and
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\( P_{mt}^{P_{1t-1}} = P_{mt}^{P_{1t}} \) for constant land prices. When the price of private land has been increasing, the rate of return on owned land exceeds the rental rate on public land, even when excess demand vanishes. This is because the value to the producer of holding land for the future has increased. Of course the opposite is true when land prices have been declining.

A policy maker wishing to have all public lands rented, but also wishing to set the rental rate as high as possible cannot necessarily equate the rental rate on public land to the rate of return on private land, even if producers are willing to pay equally for either private or public land. Rental rates on public land should be determined by changes in land values. If producers were willing to pay less for public than private land, the rental rate would be reduced by an unknown amount.

Uncertainty in Cow-Calf Producer Decisions

The introduction of uncertainty and expectations of the future into producer decisions is difficult and tedious. This paper will present only a basic introduction to the features which distinguish expectations in a dynamic optimization model from previous expectations mechanisms as they have been applied to essentially static models. The interested reader is referred to Hertzler, 1982.

The difference between expectations based on dynamic decision making and ad hoc expectations can be introduced by an analogy. Producers following an extrapolative expectation scheme and preparing for the future are assumed to be looking backward to the past, and taking a step backwards into the
future. Producers who are planning optimally, take the past and present as given, look to the future, and step forward.

The key variables in forming expectations of the future are the costates. Using terminal condition 34 and the relationship for the change over time of the value of holding breeding stock in 23, the value of holding stock, $\theta$ at time $t$ can be solved backward from time $T$ as a function of future variables. Upon recursive substitution,

$$\theta_t = (1-\beta_1)^{T-t} \lambda_T p_{T+1} T \sum_{s=t+1}^{T} (1-\beta_1)^{s-(t+1)} \frac{\Delta_s \epsilon U_s \alpha_s}{b_s - \beta_s}$$

$$+ \lambda_s (1-\omega) p_{rs+\sigma} (\nu_s + \frac{\partial \nu_s}{\partial \beta_s} b_s)$$

$$+ \frac{\mu_{ds}}{\lambda_s} p_{rs} - \frac{\mu_{1cs}}{\lambda_s} \beta_3 + \frac{\mu_{2cs}}{\lambda_s})].$$

The value of holding land, $\psi$, can be solved from terminal condition 35 and recursive relationship 24, or

$$\psi_t = \lambda_T p_{1T+1} + \sum_{s=t+1}^{T} \frac{\Delta_s \epsilon U_s \alpha_s}{b_s - \beta_s}$$

$$+ \lambda_s (1-\omega) p_{rs} \frac{\partial \psi_s}{\partial \beta_s} b_s - \frac{\mu_{ds}}{\lambda_s} p_{1s}].$$

The value of holding debt from 36 and 25 becomes

$$\psi_t = -\lambda_T + \omega \lambda_t - \sum_{s=t+1}^{T} \mu_{ds}. $$
A producer at time \( t \) wishing to maximize Lagrangian \( J \) in 21, but faced with uncertainty about the future must take the expectation of \( J \) conditioned upon currently available information. If all current variables are adequately observed, the first order conditions for time \( t \) involve expectations only of the costates, \( \theta \), \( \phi \), and \( \psi \), and also of future discounted marginal utilities, \( \lambda \).

Demand relationship 41 can be rearranged to yield \( \Delta_t \varepsilon U_t / (b_t - \bar{b}_t) = \lambda_t \beta_{bt} \). Assuming the only positive Lagrange multiplier in 55 is \( \mu_{lct} \) determined by 37, then \( P_b \) from 46 converts the value of holding stock, \( \theta \), in 55 into an alternate specification

\[
\bar{\theta}_t = (1 - \beta_t)^{T-t} \lambda_t \beta_{bt+1} + \sum_{s=t+1}^{T} (1 - \beta_s)^{s-(t+1)} \lambda_s ((1 + (1-\omega)p_{ms})(1-\gamma)p_{rs-1} - (1-\beta_1)(1-\gamma)p_{rs}).
\]

Expectations of the value of holding stock involve uncertain future interest rates, prices for investment animals, and discounted marginal utilities of expenditures. Discounted marginal utility of expenditures, \( \lambda \), is itself a costate variable defined by terminal condition 33 and relationship 22.

The value of holding land also has an alternate specification found by applying 42 and 47 to 56

\[
\phi_t = \lambda_t \beta_{lT+1} + \sum_{s=t+1}^{T} \lambda_s (1 + (1-\omega)p_{ms}p_{ls-1} - p_{ls}).
\]

At an interior solution, the value of holding land depends upon uncertain
future interest rates and land prices, and uncertain future discounted marginal utilities of expenditures.

Summary

A dynamic theoretical model of the cow-calf producer's decision problem was presented using a utility function to specify producer's objectives. A practical method for measuring the amenities of a lifestyle was proposed. Implications for public land policy were drawn, and suggestions for including uncertainty given.

Further extensions of the model might include aggregation of the single producer model into capital, beef, and land markets by including price equations for interest rates, beef and land prices. The model could be altered to allow estimation of behavioral relationships for breeding herd investment and culling and land acquisition by the inclusion of adjustment costs. Dynamic phenomena such as long-run instability in the beef market could then be examined.

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