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Growing like Google: Endogenous Growth with Global Network Externalities

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Keywords

Network externalities, endogenous growth, anti-trust policies

Disciplines

Growth and Development | International Economics | Technology and Innovation

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We study endogenous growth in the presence of domestic and international network externalities. In our model, network externalities provide natural protection to first movers and incentivize disruptive innovations without the need for patent protection. Domestic and global growth depends on the extent of network externalities, international compatibility costs, and anti-trust policies. We find that traditional anti-trust policies may lead to unintended outcomes. Policies such as banning price discrimination or collusion may reduce economic growth. In particular, price discrimination and collusion could increase economic growth when network externalities are large in relation to compatibility costs.

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1 Introduction

The information technology (IT) revolution, or digital revolution, is a key driving force of economic growth (Niebel, 2018). E-commerce companies like Amazon and Alibaba have significantly reduced transaction costs. The Android operating system and iPhone Operating System (IOS) have reshaped people's lifestyles. New mobile payment platforms like PayPal and Alipay have changed the payment industry. Many of these industries face network externalities. A positive network externality refers to the idea that users of a particular platform benefit from an increasing number of users. For example, if only a few businesses adopt PayPal, the benefit of adopting PayPal is low. However, if all businesses adopt PayPal, it becomes very convenient for all users. Positive network externalities are conducive to market power and market concentration. (Ducci, 2020) In the examples above, a few large firms control most of their domestic markets.

However, such market concentration often, but not always, stops at a country's borders. Market leaders often differ across countries and regions, though network externalities do exist. For example, Amazon is the superpower of the United States' e-commerce market, yet it only has a small share of China's market. Amazon closed its domestic e-commerce business in China in April 2019. Similarly, the Chinese giant Alibaba has a small share in the U.S. market. Another example is the messaging apps market. In North America, the Facebook messenger is widely used; whereas WhatsApp is the top choice in many European countries, such as the U.K., Germany, Italy, and Spain. In China, WeChat dominates the market. However, this is not always the case. Some IT firms manage to exert market domination across borders too. For example, in almost every country, Android and IOS have the largest share of the smartphone market.

The evidence above underscores the importance of incorporating the features of IT industries such as network externalities into existing endogenous growth models. Furthermore, suppose network externalities are conducive to market concentration. In that case, it is natural to reconsider the role of traditional antitrust policies, often intended for closed economies, in fostering economic growth both domestically and internationally. Ducci (2020) contends that "*the economic and technological features of digital platform markets overall challenge the dichotomy between ex-ante and ex-post approaches to market power, and thus demand rethinking the interface between regulation and competition policy*". This is particularly important for the contemporary world. Many cases show governments using antitrust policies or even stricter policies to regulate or even ban foreign companies' activities. To provide a recent example, Tiktok, which ByteDance launched in China, was forced to cooperate with American companies by the Trump Administration. To the best of our knowledge, we are the first to incorporate network externalities into a multi-country general equilibrium model of endogenous growth. We use the model to understand the process of innovations in the IT industry and their effect on worldwide economic growth. We also study how antitrust policies can affect innovation activities and economic growth.

The features of IT technologies also provide a novel mechanism to sustain the innovation of non-rival ideas and endogenous growth, even in the absence of patent protection. Romer (1990) argues that competitive markets cannot provide compensation for the creation of "non-rival" inputs of production, or ideas. He argues that if ideas are at least partly excludable, then patent protection can provide the necessary protection, so that innovators can profit from the creation of ideas. In Romer's formulation, governments are central to economic growth. Romer's model provided the basis for the ensuing literature on endogenous growth, emphasizing purposeful efforts by profit-seeking individuals in the creation of ideas.

However, the idea that patents and government intervention are needed for growth has been challenged. In their *The Case Against Patents*, Boldrin and Levine (2013) noticed that purposeful innovations were already significant, much before strong intellectual property rights and patent protections were in place: "*the initial eruption of innovations leading to the creation of a new industry – from chemicals to cars, from radio and television to personal computers and investment banking—is seldom, if ever, born out of patent protection ...*" Boldrin and Levine (2008) argue that competitive markets can provide incentives for innovation. In their models, ideas are partly embodied in objects and are partly excludable so that those innovators can profit.

Since ideas in Boldrin and Levine's study (2008) are embodied in objects, these ideas are still rivals. We may argue that ideas are non-rivals in some respects. This is particularly true in the present economy, such as the IT industry. The idea behind Facebook can be easily copied. However, it does not prevent the creation of Facebook and the profits made by its founder. Thus, we may like to see a model in which ideas are truly non-rival. Our model provides a mechanism where innovations exist even in a world where ideas are entirely non-rival. Network externalities guarantee the profit of the innovators.

We organize this paper as follows. First, we consider the endogenous growth model with expanding varieties. Then, we turn to a growth model with quality ladders. Expanding varieties and quality ladders are two common ways to introduce endogenous innovations into growth models. We show that our mechanism works in both models. Expanding varieties model is simpler to solve and can provide us with more intuitive analytical solutions in our model. Thus, we initially use an expanding varieties model to give us the main intuitions as a benchmark model. However, some people may argue that innovations that increase the quality of a product may be more common in the modern economy, like the IT industry. Thus, we also study a quality ladder model better to describe the story of innovations in the IT industry. We show that the two models have similar results.

In both models, we first build the models in a general way that allows J countries and general productivities. We then focus on a two-country world and solve the models in this way. Here, each country produces a final good using a continuum of intermediate goods provided by domestic producers. Each intermediate goods producer is a monopolistic firm that pays license fees to either big domestic or international R&D firms for the use of their platforms. Here, an IT Titan plays a role as a big R&D firm and invests in developing new platforms. Intermediate goods producers are companies using the platform developed by IT giants to produce their goods. For example, if Amazon and Alibaba develop their platform for e-commerce companies, a small factory can choose to sell its products on either platform.

Platforms have network externalities. In particular, if intermediate goods producers in both countries use a single platform, the cost is lower. Our specification allows for extra costs of adopting foreign platforms through what we call international compatibility costs. They impose a natural barrier to the adoption of foreign technologies. Consider, for example, the messaging apps market. Many functions in WeChat are devised to be used in China, but they are useless in other countries and make WeChat complicated to use.

The key feature of our models is that we introduce network externalities and international compatibility costs in two-country growth models. International compatibility costs differentiate our study from the work of other researchers who have studied network externalities in closed economies. Because of network externalities, a bigger platform is always more productive in a closed economy. In an international context, network externalities also benefit larger platforms when seeking to enter foreign markets, but international compatibility costs hurt them. We first allow traditional antitrust policies, such that price discrimination and collusion are forbidden. We find that network externalities and compatibility costs determine whether there are two networks or one network in the world for each market, along a balanced growth path (BGP). Thus, the interplay between these two forces could explain why market power may or may not cross borders. If network externalities are small compared to compatibility costs (Case 1), two networks can co-exist, one in each country. Each intermediate goods producer in a given market uses the platform provided by its domestic R&D firms. If network externalities are intermediate compared to compatibility costs (Case 2), two networks still co-exist, but this equilibrium is problematic. Economic growth could increase if only one network were used. However, this is not achievable, given the anti-trust policies. If network externalities are large (Case 3), only one network takes over the whole market: intermediate goods producers in both countries use this platform. However, the license fee set by the winner is limited by the threat of entry by the foreign R&D firm. In this case, the expected profits of the R&D sector are still not as large as they could be, which implies that the economy may not grow at full potential.

We also study whether removing some antitrust policies could foster economic growth. Here, we only study the antitrust policies that increase competition after innovations are in place and a new platform is developed. We first study what happens when price discrimination is allowed. Only Cases 2 and 3 are affected by price discrimination policies. Price discrimination leads to dumping-like behavior: IT Titans charge higher license fees in their country while charging lower license fees in a foreign country. This is due to the compatibility cost, which increases the cost of using foreign technologies. Price discrimination allows full market control and dominance by one single platform in Case 2. In this case, the domestic IT Titan first develops a new platform and sets a license fee in its country to extract all the profits of the domestic intermediate goods producers. However, it compensates for the foreign market, owing to the competition by the foreign R&D firm. This behavior increases the size of the market, and the leader fully benefits from this outcome. In Case 3, IT Titans get higher profits and are willing invest more. In this case, technological progress is faster: price discrimination fosters growth. In the quality ladder model, we further study the effects of collusion. Collusion does not affect the results of Case 1. However, collusion along with price discrimination leads to the highest growth rate in Cases 2 and 3. This is because collusion can eliminate the competition of sales when a platform has been developed.

These findings challenge traditional views of antitrust policies such as banning price discrimination, dumping, and collusion. We find that these policies may hinder economic growth under certain circumstances when network externalities are present. It is worth mentioning that our discussion is about antitrust policies after the innovations are successful. These results are not fit for discussing antitrust policies before innovations. The logic is simple, and the story gives enough freedom so that the IT Titans who have been successful in innovation activities can obtain more profits. Thus, innovation activities are rewarded adequately, and IT Titans are willing to invest more. Simultaneously, there is no need to worry about the competition. IT Titans must keep on investing to avoid losses in upcoming innovations.

Related literature. Among the different studies on network externalities, this study is mostly related to the literature studying innovation in the presence of network externalities. Kristiansen (1998) studies how two rival firms make R&D decisions with network externalities in a three-stage game. Firms only compete once, and there is no technological progress. Shy (1996) uses an overlapping generations model to study how innovation is adopted with network externalities. Technological progress is exogenous, and switching from old technology to new technology is only caused by the changing of generations. Markovich (2008) studies the innovation dynamics in the software market in a partial equilibrium model. The quality ladder in the software market is also finite rather than infinite. Segal and Whinston (2007) allow continuing innovations in a partial equilibrium model with only one industry and a closed economy. To the best of our knowledge, our study is the first to study network externalities in a general equilibrium model of endogenous growth and international competition. Our study provides insights into how network externalities affect the innovation process and how antitrust policies may affect innovation and economic growth.

Our study is also related to others studying international economics with network externalities. Gandal and Shy (2001) and Barrett and Yang (2001) study standardization policy in international trade with the existence of network externalities. Klimenko and Saggi (2007) study FDI under network externalities and partial incompatibility between domestic and foreign technology. Janeba (2007) studies the effects of trade liberalization. Klimenko (2009) studies international agreements on policies with network externalities. None of these studies have endogenous innovation.

Our studies follow the tradition of endogenous growth models with expanding varieties, as in Romer (1990). Then we use the tradition to study innovation first developed by Schumpeter (1934). Schumpeter first proposed the term "creative destruction" in innovations and growth. Recent studies in endogenous growth follow Schumpeter to study how innovations affect economic growth (Aghion and Howitt (1992), Grossman, Gene M., and Helpman (1991a, 1991b)). Our model follows these two approaches to study innovation activities with network externalities and international compatibility cost. However, as we stated before, our study provides a mechanism whereby conducting innovations are profitable, even with non-rival ideas and no protection from the government.

This paper is organized as follows. Section 2 sets up the expanding varieties model. We characterize the balanced growth paths (BGP) that arise under different parametric assumptions and different antitrust policies in Section 3. Section 4 describes the quality ladder model. Discussions about BGPs and antitrust policies with the quality ladder model are in Section 5. Section 6 concludes the paper.

2 The Expanding Varieties Model

There are J symmetric countries: $j = 1, 2, .., J$. Time is discrete: $t = 0, 1, ...$We do not allow price discrimination and collusion in the basic model. We will assume $J = 2$ when we solve the model.

2.1 Households

There is a representative household in every country that has a standard utility function

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{j,1-\theta} - 1}{1-\theta}$$

where β is the discount rate and c_t is the consumption in period t. Household provides all its labor inelastically every period and aggregate labor supply is L. Household faces budget constraints

$$c_t^j + b_t^j = b_{t-1}^j R_t^j + w_t^j L + \Pi_t$$

where w_t^j is the wage rate, b_t^j is the number of bonds, R_t is the interest rate, and Π_t are profits from intermediate goods producers.

Optimal bond holdings lead to the following usual Euler Equation:

$$R_{t+1}^j = \left(\frac{c_{t+1}^j}{c_t^j}\right)^\theta \frac{1}{\beta} = \left(1 + g_{t+1}^{j,c}\right)^\theta \frac{1}{\beta} \tag{1}$$

2.2 Final Goods Producer

There is one final good in each country. Let the final goods be numeraire. All consumptions, investments, and production of inputs use final goods. The representative final goods producer is in the competitive market and utilizes intermediate goods during production. There is a continuum of intermediate goods $n \in [0, M_t]$. The technology of producing final goods, Y, is

$$Y_t^j = AL^{1-\sigma} \int_0^{M_t} \left(X_t^j(n)\right)^\sigma dn, \ 0 < \sigma < 1. \tag{2}$$

where A denotes the exogenous technology of final goods producer, and $X_t^j(n)$ is the amount of variety n.

Let $P_t^j(n)$ denote the price of intermediate goods n in country j. The profit maximization problem of the final goods producer is

$$\max_{L, \left[X_t^j(n)\right]_0^{M_t}} Y_t^j - \int_0^{M_t} P_t^j(n)\, X_t^j(n)\, dn - w_t^j L$$

subject to the production function. It is standard to derive the corresponding demand function for intermediate goods n :

$$P_t^j(n) = \sigma AL^{1-\sigma} \left(X_t^j(n)\right)^{\sigma-1}. \tag{3}$$

The wage rate satisfies:

$$w_t^j = (1-\sigma)\, AL^{-\sigma} \int_0^{M_t} \left(X_t^j(n)\right)^\sigma dn.$$

2.3 Intermediate Goods Producers

Every intermediate good is produced by a monopoly in each country. The monopoly needs to adopt a platform from the available platforms to produce and pay a lifetime license fee, for its use to the platform developer, the IT Titan. Let $\mathbf{\Omega}_t$ be the set of available platforms and $\Omega \in \mathbf{\Omega}_t$ be a particular element. With a slight abuse of notation, let Ω also denote the productivity of that platform: To produce one unit of a variety n requires $1/\Omega$ units of the final output. Let $\pi(\Omega)$ be the per-period profit of using platform Ω. By using (3), $\pi(\Omega)$ is given by

$$\pi(\Omega) = \max_X \sigma A L^{1-\sigma} (X)^{\sigma} - \frac{X}{\Omega} \tag{4}$$

The first order condition gives

$$X^*(\Omega) = \left(\Omega\sigma^2 A\right)^{\frac{1}{1-\sigma}} L \tag{5}$$

Plug (5) into (4) we get one period profit

$$\pi(\Omega) = BL\Omega^{\frac{\sigma}{1-\sigma}} \tag{6}$$

where $B \equiv \sigma(1-\sigma) A^{\frac{1}{1-\sigma}} \sigma^{\frac{2\sigma}{1-\sigma}}$.

Let $V_t(\Omega)$ the present value of these profits

$$V_t(\Omega) = \pi(\Omega) + V_{t+1}/R_{t+1}.$$

Given a set of licenses-fees, $\Phi_t(\Omega)$, the monopolist chooses the platform that maximizes $V_t(\Omega) - \Phi_t(\Omega)$ when a new variety is created. Let

$$\widehat{\Omega} \in \arg\max_{\Omega \in \boldsymbol{\Omega}} \{V_t(\Omega) - \Phi_t(\Omega)\}$$

2.4 R&D Firms

R&D firms create new varieties (platforms) and sell the license to the intermediate goods producers. R&D firms spend η unit of final products to create a new platform. We assume that the other R&D firms in domestic and foreign countries can freely copy the idea of the leader, but copying an idea is time-consuming. Thus, the innovator first enters the market, while the copiers follow.

Let platform $\Omega_t^j(i, N, n)$ denote a platform developed in country i, available in country j to produce variety n, for a mass N of total users. The set of available platforms in country j variety n is given by $\Omega_t^j(n) = \left[\Omega_t^j(i, N, n)\right]_{i,N}$. As before, $\Omega_t^j(i, N, n)$ also describes the productivity of such a platform.

However, n does not affect productivity; therefore, we omit it across the remaining parts of our paper. Hence, the productivity is just $\Omega_t^j(i, N)$.

Assumption. i. $\partial\Omega_t^j(i, N)/\partial N > 0$ (positive network externalities); (ii) $\Omega_t^j(j, N) > \Omega_t^j(i, N)$ if $j \neq i$ (local platforms are more productive - international compatibility costs).

The positive network externalities and the time cost of copying an idea ensure the innovator has a leader advantage in its domestic market. No domestic competitor exists in this model in the equilibrium. However, in the foreign market, a foreign competitor can build a more suitable network to the foreign market, so there is no international compatibility cost. Thus, the first foreign copier may survive though the network externalities exist. Foreign intermediate goods producers may choose to wait for the small foreign Platform, rather than use the bigger platform if the international compatibility costs are high.

The worldwide number of users of that platform created is

$$N_t^j(n) = \sum_{j=1}^{J} 1_t^j(i, n)$$

where $1_t^j(i, n)$ is an indicator function which is 1 when country j uses the platform developed by IT Titan in country j to create intermediate goods n at period t. The zero-profit condition for the leader then require the license fee of the leader is

$$\sum_{j=1}^{J} 1_t^j(i, n)\Phi_{t+1}(\Omega_{t+1}) = \eta \tag{7}$$

2.5 Equilibrium

Let ΔM_t^j is the new varieties first created by country j.

A competitive equilibrium in the expanding varieties model is defined as a sequence of allocations $\left\{c_t^j, b_t^j, X_t^j(n), Y_t^j, \Pi_t, [N_t^i(n)]_{n=0}^{M_t}, \Delta M_t^j\right\}$, prices

$\left\{w_t^j, R_t^j, \left[P_t^j(n)\right]_{n=0}^{M_t}, \Phi_t(\Omega)\right\}$, and time paths $\{M_t, \mathbf{\Omega}_t, N_t^i(n)\}$, such that households in all countries maximize their utility, and final goods producers and intermediate goods producers solve the profit maximization problems we defined before. Zero-profit conditions (7) are satisfied for IT Titans who first develop a platform and the followers that set zero license fee. Market clearing conditions are satisfied which are goods markets are clearing

$$Y_t^j = c_t^j + \Delta M_t^j \eta + \int_0^{M_t} \frac{X_t^j(n)}{\widehat{\Omega}_t^j(n)} dn \tag{8}$$

and bonds market are clearing

$$b_t^j = 0$$

The size of every platform first created by an IT Titan in country i follows

$$N_t^i(n) = \sum_{j=1}^{J} 1_t^j(i, n)$$

while total varieties follow

$$M_{t+1} = M_t + \sum_{j=1}^{J} \Delta M_t^j$$

3 Characterization of the equilibrium

Now, we characterize the equilibria along the BGP. We focus on a two-country world, which means $J = 2$. It is convenient to use the following notation to denote domestic and foreign variables:

$$\Omega(N) \equiv \Omega^j(j, N);\ \Omega^*(N) \equiv \Omega^j(-j, N);$$
$$V(N) \equiv V\left(\Omega^j(j, N)\right); V^*(N) \equiv V\left(\Omega^j(-j, N)\right).$$

The profits of an intermediate goods producer using a domestic platform, a platform that may or may not be used abroad, are given according to (6), by

$$\pi(N)=\begin{cases} BL\Omega(1)^{\frac{\sigma}{1-\sigma}}, & \text{if } N=L(\text{only local})\\ BL\Omega(2)^{\frac{\sigma}{1-\sigma}}, & \text{if } N=2L(\text{global})\end{cases} \tag{9}$$

Here, we omit n because this function is the same for all industries. Moreover, the present discount value of such a firm, along with BGP, is

$$V(N)=\begin{cases} V(\Omega(1))=\frac{R\pi(\Omega(1))}{R-1}, & \text{if } N=L \text{ (only local)}\\ V(\Omega(2))=\frac{R\pi(\Omega(2))}{R-1}, & \text{if } N=2L \text{ (global)}\end{cases} \tag{10}$$

The profits and values of an intermediate goods producer using a foreign platform are, respectively

$$\pi^*(2)=\pi(\Omega^*(2))=BL\Omega^*(2L)^{\frac{\sigma}{1-\sigma}}. \tag{11}$$

$$V^*(2)=V(\Omega^*(2))=\frac{R\pi(\Omega^*(2L))}{R-1} \tag{12}$$

The world present value of revenues of intermediate producers is either $2V(\Omega(L))$ when two domestic platforms co-exist or $V(\Omega(2L))+V(\Omega^*(2L))$ with only one global platform of a particular variety n. In the absence of compatibility costs, $V(\Omega(2L))+V(\Omega^*(2L))=2V(\Omega(2L))>2V(\Omega(L))$ so that having only one global network is more valuable, for the world, than having two domestic networks. If compatibility costs are large enough, then $V(\Omega(2L))+V(\Omega^*(2L))<2V(\Omega(L))$ so that having two domestic networks is more valuable. To characterize the equilibria, we now turn to the determination of the license fees.

3.1 With Traditional Antitrust Policies

First, we consider equilibrium allocations and growth, in an environment with traditional antitrust policies banning price discrimination and collusion. There are three cases, owing to the relationships between network effects and international compatibility cost.

Case 1: $2\Omega(L)^{\frac{\sigma}{1-\sigma}}>\Omega(2L)^{\frac{\sigma}{1-\sigma}}+\Omega^*(2L)^{\frac{\sigma}{1-\sigma}}$, which implies $2V(\Omega(L))>V(\Omega(2L))+V(\Omega^*(2L))$.

Case 2: $\frac{4}{3}\Omega^*(2L)^{\frac{\sigma}{1-\sigma}} < 2\Omega(L)^{\frac{\sigma}{1-\sigma}} < \Omega(2L)^{\frac{\sigma}{1-\sigma}} + \Omega^*(2L)^{\frac{\sigma}{1-\sigma}}$, which implies $\frac{4}{3}V(\Omega^*(2L)) < 2V(\Omega(L)) < V(\Omega(2L)) + V(\Omega^*(2L))$.

Case 3: $3\Omega(L)^{\frac{\sigma}{1-\sigma}} \leq 2\Omega^*(2L)^{\frac{\sigma}{1-\sigma}}$, that implies $3V(\Omega(L)) \leq 2V(\Omega^*(2L))$.

Figure 1 helps us to understand these three cases. If the compatibility cost is high compared to network effects, the economy is case 1. If network effects are big compared to compatibility cost, it is case 3. Case 2 is in the middle, when network effects are big, but not too big compared to the compatibility cost. Soon we will see case 1 where two platforms are built and efficient for growth, while in case 3, only one platform is built in an industry. Case 2 is where two networks are built, but one network, if achievable, is better for growth. The following figures 2, 3, and 4 illustrate cases 1, 2, and 3 respectively in the study by connecting network size and firm value.

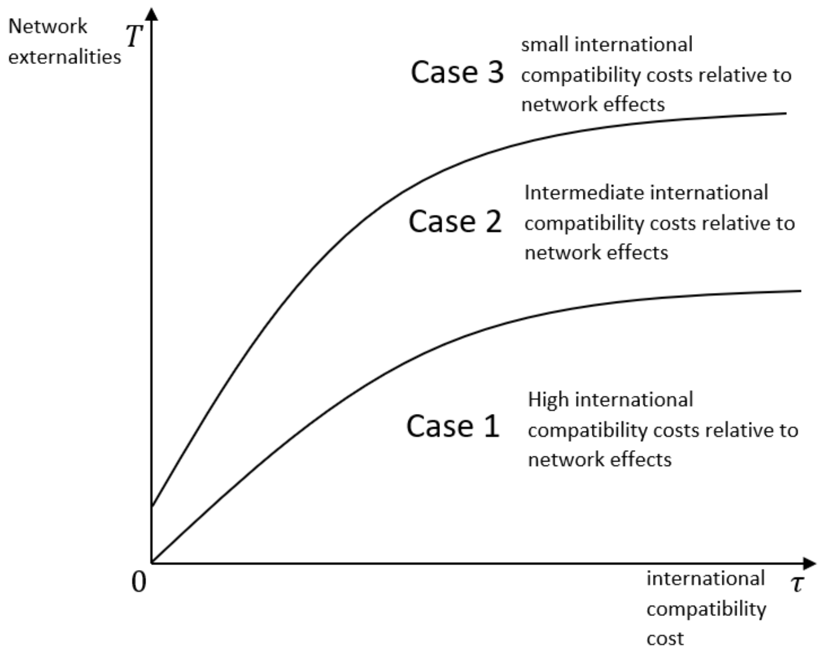

Figure 1: Three cases which are determined by network externalities and international compatibility cost

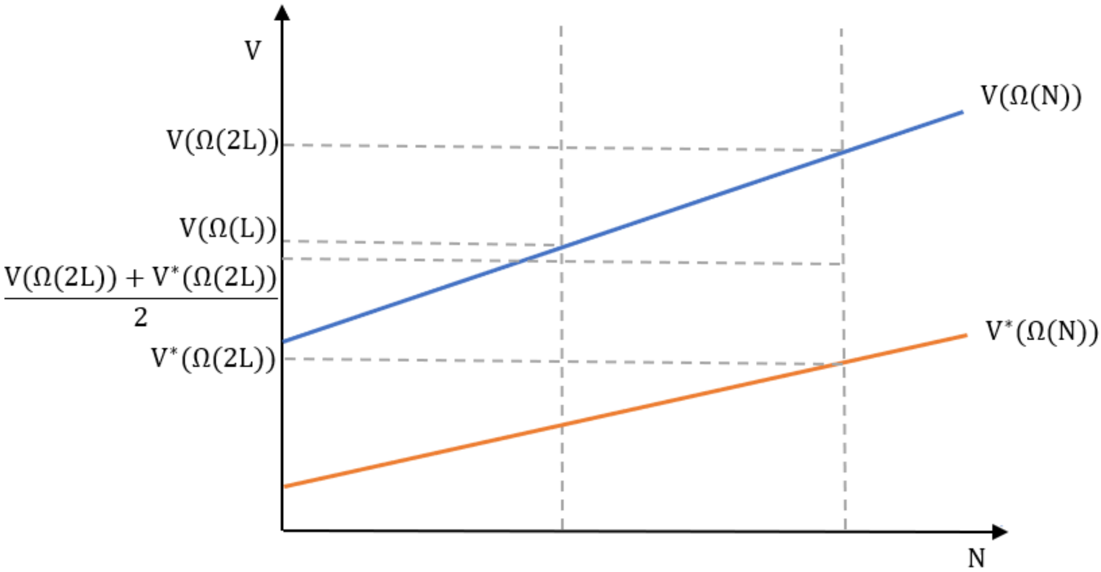

Figure 2: Case 1: High compatibility costs relative to network effects

3.1.1 Case 1: High compatibility costs relative to network effects

In this case, $2V(\Omega(L)) > V(\Omega(2L)) + V(\Omega^*(2L))$. Without loss of generality, we assume the R&D firm in country j is the one to create the new variety n. If it decides to only get its domestic market, it can set a license fee $\Phi(\Omega(L))$ equals to $V(\Omega(L))$, which is the expected profit of an intermediate goods producer in their country. If it wants to also compete in the foreign market, it has to set the license fee not more than $V(\Omega^*(2L))$, otherwise the intermediate goods producer in the country $-j$ will face a negative expected profit and opt not to join. However, the R&D firm in the other country can easily take over the market with any price which is lesser than the license fee charged by the firm in country j plus Ξ, where $\Xi = V(\Omega(L)) - V(\Omega^*(2L)) > 0$. Thus, even if the R&D firm in the country j sets the license fee as 0; it still cannot win in the country $-j$. For this reason, there are two platforms, and the license fee in the leader's country is

$$\Phi(\Omega(L)) = \frac{RBL\Omega(L)^{\frac{\sigma}{1-\sigma}}}{R-1} = \eta \tag{13}$$

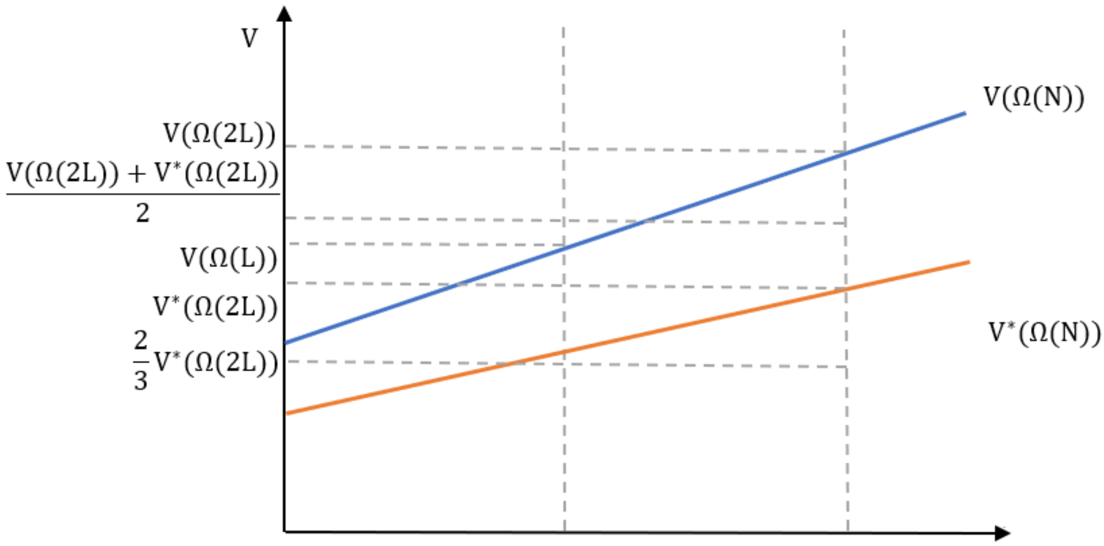

Figure 3: Case 2: Intermediate compatibility cost relative to network effects

While the license fee in foreign countries is 0, owing to the zero-profit condition.

By using (13),

$$R = \frac{\eta}{\eta - BL\Omega\left(L\right)^{\frac{\sigma}{1-\sigma}}}$$

From (1),

$$1 + g_{case1} = \left(R\beta\right)^{\frac{1}{\theta}} = \left(\frac{\eta\beta}{\eta - BL\Omega\left(L\right)^{\frac{\sigma}{1-\sigma}}}\right)^{\frac{1}{\theta}}$$

is the growth rate along the BGP.

Notice the condition $2V\left(\Omega\left(L\right)\right) > V\left(\Omega\left(2L\right)\right) + V\left(\Omega^{*}\left(2L\right)\right)$ implies that the total profits made using one platform is higher than the profits made using two platforms. Moreover, R&D firms extract all profits with two platforms. By creating two platforms, the total revenues extracted by the R&D sectors are maximal. There is no room to increase economic growth by changing policies.

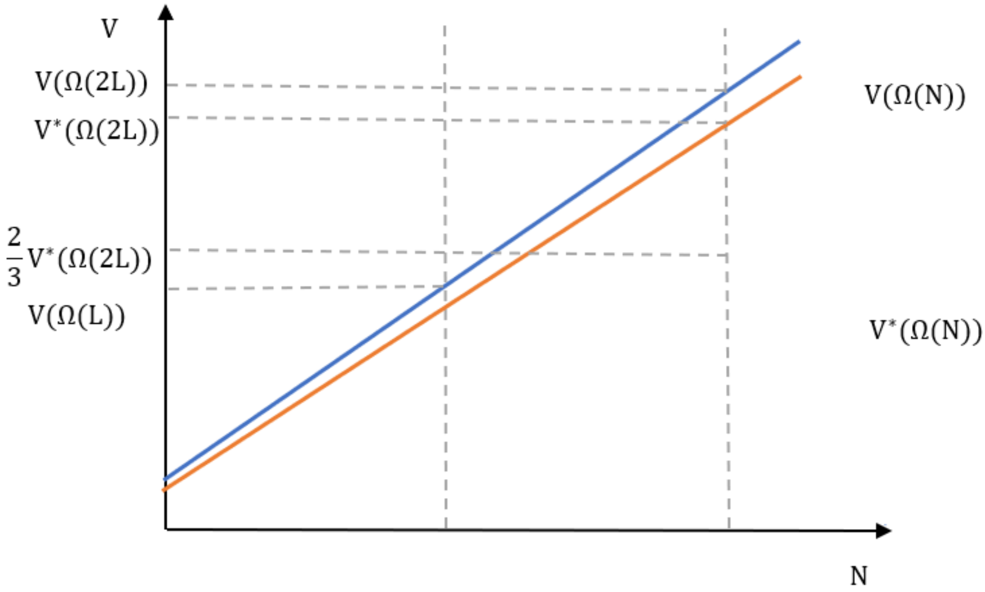

Figure 4: Case 3: Small compatibility cost relative to network effects

3.1.2 Case 2: Intermediate compatibility cost relative to network effects

In this case, $\frac{4}{3}V(\Omega^*(2L)) < 2V(\Omega(L)) < V(\Omega(2L)) + V(\Omega^*(2L))$. The profit of an intermediate goods producer that is made when two firms are using two platforms separately is $V(\Omega(L))$ which means total profits of the world is $2V(\Omega(1))$. The total profit of one platform is $V(\Omega(2L)) + V(\Omega^*(2L))$. However, without the price discrimination, the R&D firm can set its price as $V(\Omega^*(2L))$ at most. Otherwise, the foreign intermediate goods producer will get a negative profit so they will not buy the license. The foreign IT Titan can also build its network and offer a license fee lesser than the leader's license fee, plus $(V(\Omega(L)) - V(\Omega^*(2L)))$ and take over the foreign market. Only when the leader compensates the foreign market and sets the license fee as $-(V(\Omega(L)) - V(\Omega^*(2L)))$, can the potential entry be prevented. Because a foreign IT Titan cannot set a nonnegative license fee to take over the foreign market, so the optimal choice is not to build a network. However, the total profit for the leader is now $-2(V(\Omega(L)) - V(\Omega^*(2L)))$. Since $\frac{4}{3}V(\Omega^*(2L)) < 2V(\Omega(L))$, $-2(V(\Omega(L)) - V(\Omega^*(2L))) < V(\Omega(L))$.

Thus, the total profit for getting the whole world's market is less than just getting the domestic market. The R&D firm which first develops a new platform would like to only have its domestic market. For this reason, the result comes back to case 1 without price discrimination.

However, $2V(\Omega(L)) < V(\Omega(2L)) + V(\Omega^*(2L))$. For the whole world, one platform provides more profits in total. If IT Titans can obtain all these profits, they may wish to invest further in innovations. However, this is not achievable in business. A world with intermediate network effects and compatibility costs can benefit from using a larger network. Unfortunately, the network effect is not big enough to ensure the success of a solitary network, in a business sense. Later we will study what happens when antitrust policies are eliminated.

3.1.3 Case 3: Small compatibility cost relative to network effects

In this case, $3V(\Omega(L)) \leq 2V(\Omega^*(2L))$. Without loss of generality, we assume the R&D firm in country j is the first to create a new platform. If it decides only to target its domestic market, it can set a license fee equal to $V(\Omega(L))$ which is the expected profit of an intermediate goods producer in their country. If it wants to compete in the foreign market as well, it has to set a license fee not more than $V(\Omega^*(2L))$, otherwise the intermediate goods producer in country $-j$ will face a negative expected profit and will not want to buy the license.

If the R&D firm in country j decides to open a business in country $-j$, the R&D firm in country $-j$ has two choices. It can either choose to build a second network or give up the market. If it chooses to build the second network, it can at most get its domestic market. To prevent potential entry, the leader sets the license fee at $V(\Omega^*(2L)) - V(\Omega(L))$. From the condition $3V(\Omega(L)) \leq 2V(\Omega^*(2L))$, $2(V(\Omega^*(2L)) - V(\Omega(L))) \geq V(\Omega(L))$. The profits of operating in the whole world market are higher than just having the domestic market. Thus, the leader will set a license fee $\Phi(\Omega(2L)) =$

$\Phi\left(\Omega^{*}\left(2L\right)\right)=V\left(\Omega^{*}\left(2L\right)\right)-V\left(\Omega\left(L\right)\right)$ to get the foreign market. Now

$$V\left(\Omega^{*}\left(2L\right)\right)-V\left(\Omega\left(L\right)\right)=\frac{RBL\left(\Omega^{*}\left(2L\right)^{\frac{\sigma}{1-\sigma}}-\Omega\left(L\right)^{\frac{\sigma}{1-\sigma}}\right)}{R-1}$$

It is worth mentioning that the intermediate goods producer in country j has an expected profit in this situation. It transfers the profit to the household.

We can also calculate what is the total production of a country compared to two networks. Like what we did in case 1, we use the zero-profit condition,

$$\eta=\frac{RBL\left(\Omega^{*}\left(2L\right)^{\frac{\sigma}{1-\sigma}}-\Omega\left(L\right)^{\frac{\sigma}{1-\sigma}}\right)}{R-1}$$

$$R=\frac{\eta}{\eta-BL\left(\Omega^{*}\left(2L\right)^{\frac{\sigma}{1-\sigma}}-\Omega\left(L\right)^{\frac{\sigma}{1-\sigma}}\right)}$$

From (1),

$$1+g=\left(R\beta\right)^{\frac{1}{\theta}}=\left(\frac{\eta\beta}{\eta-BL\left(\Omega^{*}\left(2L\right)^{\frac{\sigma}{1-\sigma}}-\Omega\left(L\right)^{\frac{\sigma}{1-\sigma}}\right)}\right)^{\frac{1}{\theta}}$$

In case 3, network effects are big enough and dominate the compatibility cost. Thus, one network can be successful enough for a business. However, since the leader has to compensate for the foreign market and decreases the license fee, its revenue is much less than the benefit it creates. For this reason, IT Titans, in this particular case, do not invest as much as they invest when they get all the benefits they create.

3.2 Price Discrimination

In this section, we study whether relaxing some antitrust policies can affect economic growth. We consider allowing price discrimination in this section. Owing to the presence of numerous firms that compete in the market, collusion is difficult. We do not allow collusion here. Later in the quality ladder model, we also study what happens with collusion.

Since in case 1, a foreign intermediate goods producer achieves higher productivity by using the platform provided by its country, and the total profits of using two networks are greater than using one network. Price discrimination cannot change case 1. We focus on cases 2 and 3.

3.2.1 Case 2 with price discrimination

Without loss of generality, we assume the R&D firm in country j is the first to create a new platform. With price discrimination, the first developer can set the license fee in their country, which equals to

$$\Phi\left(\Omega\left(2L\right)\right) = V\left(\Omega\left(2L\right)\right)$$

Simultaneously, it is interesting to see what the R&D firm does in the country $-j$. A natural answer is to set the license fee as $V\left(\Omega^*\left(2L\right)\right)$. However, this is not true. The R&D firm in $-j$ has an incentive to develop a new network, and we have shown that the R&D firm in $-j$ sets the license fee as 0. For this reason, R&D firm in country j must set a license fee equal to

$$\Phi\left(\Omega^*\left(2L\right)\right) = V\left(\Omega^*\left(2L\right)\right) = V\left(\Omega^*\left(2L\right)\right) - V\left(\Omega\left(L\right)\right)$$

This is interesting, because the conditions of case 2 do not ensure $V\left(\Omega^*\left(2L\right)\right) - V\left(\Omega\left(L\right)\right)$ is positive. Surprisingly, the first developer would like to accept a negative license fee in the foreign country. However, we can see the total profit

$$\begin{aligned}\Phi\left(\Omega\left(2L\right)\right) + \Phi\left(\Omega^*\left(2L\right)\right) &= V\left(\Omega\left(2L\right)\right) + V\left(\Omega^*\left(2L\right)\right) - V\left(\Omega\left(L\right)\right) \\ &> V\left(\Omega\left(L\right)\right)\end{aligned}$$

by using the condition $2V\left(\Omega\left(L\right)\right) < V\left(\Omega\left(2L\right)\right) + V\left(\Omega^*\left(2L\right)\right)$. This result means it is still possible to profit by paying the foreign market to increase the size of the network, so it is still a rational behavior. Some may argue that it is difficult to imagine this behavior transpiring in reality. This is because we do not assume there is any cost in selling its platform to consumers. If there is a cost, then the price may be positive again. It is worth mentioning that the negative price does occur in some IT industries. An example would be online games. For some online games, the marginal cost of extra players are little. Players are free to enter the game and can even exchange "gold," which they earn during the game for some cash, via the game company. However, if they want to have some excellent equipment for their character in the game, they have to pay a lot of real money. If there are more players in the game, the top players feel a heightened sense of accomplishment; this is also an instance of network externality. Thus, by paying to attract more players who want to earn cash, the online game firm induces some wealthy players to pay more. That is how they make money by providing a negative price.

Does the price discrimination policy increase the growth rate? Zero profit condition requires

$$\begin{aligned}&\Phi(\Omega(2L)) + \Phi(\Omega^*(2L)) \\ &= \frac{RBL\left(\Omega(2L)^{\frac{\sigma}{1-\sigma}} + \Omega^*(2L)^{\frac{\sigma}{1-\sigma}} - \Omega(L)^{\frac{\sigma}{1-\sigma}}\right)}{R-1} \\ &= \eta\end{aligned}$$

Combining this equation with (1), we get the growth rate

$$\begin{aligned}1 + g^D_{case2} &= \left(\frac{\eta\beta}{\eta - BL\left(\Omega(2L)^{\frac{\sigma}{1-\sigma}} + \Omega^*(2L)^{\frac{\sigma}{1-\sigma}} - \Omega(L)^{\frac{\sigma}{1-\sigma}}\right)}\right)^{\frac{1}{\theta}} \\ &> 1 + g_{case2}\end{aligned}$$

Thus, price discrimination increases the growth rate.

3.2.2 Case 3 with price discrimination

It is easy to know that with price discrimination, an R&D firm in the country j is able to set $\Phi(\Omega(2L)) = V(\Omega(2L))$, $\Phi(\Omega^*(2L)) = V(\Omega^*(2L)) - V(\Omega(L))$. Now the license fee is

$$\Phi\left(\Omega\left(2L\right)\right)+\Phi\left(\Omega^{*}\left(2L\right)\right)$$
$$=\frac{RBL\left(\Omega\left(2L\right)^{\frac{\sigma}{1-\sigma}}+\Omega^{*}\left(2L\right)^{\frac{\sigma}{1-\sigma}}-\Omega\left(L\right)^{\frac{\sigma}{1-\sigma}}\right)}{R-1}$$

Zero profit condition and (1) give us

$$1+g_{case3}^{D}=\left(\frac{\eta\beta}{\eta-BL\left(\Omega\left(2L\right)^{\frac{\sigma}{1-\sigma}}+\Omega^{*}\left(2L\right)^{\frac{\sigma}{1-\sigma}}-\Omega\left(L\right)^{\frac{\sigma}{1-\sigma}}\right)}\right)^{\frac{1}{\theta}}$$
$$>1+g_{case3}$$

4 The Quality Ladder Model

We now turn to the quality ladder model which may be more appropriate to discuss IT Titans. The settings of households are the same as before. We assume the number of countries is $J=2$ directly for simplicity, and directly give them the form of investment technology. The settings of the model are almost the same as with $J>2$. As before, no discrimination and collusion are allowed in the basic model.

4.1 Final Goods Producer

For simplicity, we assume the total number of varieties is fixed. There is a continuum of intermediate goods $n\in[0,1]$. The technology of producing final goods is

$$Y_{t}^{j}=AL^{1-\sigma}\int_{n=0}^{1}\left(q^{\iota(n)}X_{t}^{j}\left(n\right)\right)^{\sigma}dn,\ 0<\sigma<1. \tag{14}$$

where A still denotes the exogenous technology of the final goods producer. We follow the tradition of Schumpeterian models like Aghion and Howitt (1992) and Grossman and Helpman (1991b). Potential grades of intermediate goods follow quality ladders, and the rungs of quality ladders are spaced proportionately at interval $q > 1$. Thus, the available grade of goods n are $1,\ q,\ q^2,\ ...,\ q^{\iota(n)},\ ...$. Here $\iota(n) = \iota,\ \iota = 1,\ 2,\ 3,\ ....$

Let $P_t^j(n)$ denote the price of intermediate goods n in the country j. The profit maximization problem of the final goods producer is

$$\max_{L,\left[X_t^j(n)\right]_0^{M_t}} Y_t^j - \int_{n=0}^{1} P_t^j(n)\, X_t^j(n)\, dn - w_t^j L$$

subject to the production function. It is standard to derive the corresponding demand function for intermediate goods n :

$$P_t^j(n) = \sigma A L^{1-\sigma} \left(q^{\iota(n)}\right)^{\sigma} \left(X_t^j(n)\right)^{\sigma-1}. \tag{15}$$

The wage rate satisfies:

$$w_t^j = (1-\sigma)\, A L^{-\sigma} \int_{n=0}^{1} \left(q^{\iota(n)} X_t^j(n)\right)^{\sigma} dn.$$

4.2 Intermediate Goods Producers

Generally speaking, the setting for intermediate goods producers is similar to before. The only difference is if there are new technologies, which means a higher quality of the intermediate goods, the incumbent producer must buy the license of the new platform. Otherwise, other firms can buy the license and take over its market if the expected profit is not negative. Thus, a license cannot guarantee profits interminably. When the platform is still the most advanced, by using (15), the profit maximization problem of a platform with quality ι is

$$\pi(\Omega_\iota) = \max_X \sigma A L^{1-\sigma} \left(q^{\iota}\right)^{\sigma} (X)^{\sigma-1} X - \frac{X}{\Omega_\iota} \tag{16}$$

First order condition gives us

$$X^*(\Omega_\iota) = \left(\Omega_\iota \sigma^2 A \left(q^{\iota}\right)^{\sigma}\right)^{\frac{1}{1-\sigma}} L \tag{17}$$

Plug (17) into (16)

$$\pi(\Omega_\iota) = BL\, (\Omega_\iota)^{\frac{\sigma}{1-\sigma}} \left(q^{\iota}\right)^{\frac{\sigma}{1-\sigma}} \tag{18}$$

If a platform with quality $\iota + 1$ has been created, $\pi(\Omega_\iota) = 0$.

Again, let $V_t(\Omega)$ the present value of these profits

$$V_t(\Omega) = \pi(\Omega) + V_{t+1}/R_{t+1}.$$

Given a set of licenses-fees, $\Phi_t(\Omega_\iota)$, the monopolist chooses the platform that maximizes $V_t(\Omega_\iota) - \Phi_t(\Omega_\iota)$ when a new quality is created. Let

$$\Omega_\iota^* \in \arg\max_{\Omega_\iota \in \mathbf{\Omega}} \{V_t(\Omega_\iota) - \Phi_t(\Omega_\iota)\}$$

4.3 IT Titans (R&D Firms)

We assume there are big R&D firms (IT Titans) in both countries. Here, "big" means a firm is involved in industries with a measure greater than 0. Without loss of generality, firm 1 is involved in the intermediate goods market $n \in \left[0, \frac{1}{I}\right]$, firm 2 is involved in market $n \in \left(\frac{1}{I}, \frac{2}{I}\right]$, ..., firm I is involved in market $n \in \left(\frac{I-1}{I}, 1\right]$. The foreign market has the same setting[1]. Let us imagine that the quality in market n is $\iota - 1$, now the R&D firms in both countries want to develop a new platform that can create quality ι. They need to invest $\eta_\iota^{j,n}$ to have a chance to succeed in the next period. As with other Schumpeterian models, we assume that the investment of R&D firms affects the probability of innovation success. The probability of success for the R&D firm in country j during every period is

$$\delta_\iota^{j,n} = f\left(\frac{\eta_\iota^{j,n}}{\left(q^{\iota(n)}\right)^{\frac{\sigma}{1-\sigma}}}\right)$$

Where $\eta_\iota^{j,n}$ is country j's amount of final goods invested in innovation for platform n with quality ι. $f(0) = 0$, $f(\infty) = 1$, $f'(.) > 0$, $f'(0) = \infty$, $f''(\cdot) < 0$. We also assume the elasticity of $f(x)$ concerning x is less or equal to 1, which means $\frac{f'(x)x}{f(x)} \leq 1$. Because of the symmetry of the two countries, the domestic firm and the foreign firm have the same investment

[1]The number of firms and the share of each firm do not change any result. We only assume this for simplicity.

in equilibrium, which can be seen in the next section. Thus, $\eta_{\iota}^{j,n} = \eta_{\iota}^{-j,n} = \eta_{\iota}^{n}$. We define $\delta_{\iota}^{n} = F(\frac{\eta_{\iota}^{n}}{\left(q^{\iota(n)}\right)^{\frac{\sigma}{1-\sigma}}}) = 1-\left(1-\delta_{\iota}^{j,n}\right)^{2}$. We can easily prove that $F\left(x\right)$ has the same properties as $f\left(x\right)$.

As before, we assume that if the domestic R&D firm first develops a platform with quality ι, it has an advantage in its country. One foreign R&D firm which is developing ι can imitate it if it wants to develop it into a real product, and vice versa. In the next paragraph, we discuss how this foreign R&D firm is determined.

We need to discuss more R&D firms' behavior. We assume investment in R&D activities has additional benefits besides it gives an R&D firm a chance to develop a new platform first. Investment increases technological preparation for using a new platform. Although there are several R&D firms in one country, only the R&D firm with the most investment in a particular country has the first chance to develop it when a foreign R&D firm first develops a new platform. This is realistic. If Huawei in China provides a new 5G technology innovation first, Qualcomm in the US has an advantage over a small firm that never invests in developing 5G. Thus, the zero-profit condition must be satisfied in equilibrium for the incumbent R&D firm to prevent potential entry. By adding this condition, we exclude the free-rider problem that an R&D firm does not invest.

As before, let platform $\Omega_{t}^{j}\left(i,N,n,\iota\right)$ denote a platform developed in the country i, available in the country j to produce variety n, for a mass N of total users. The set of available platforms in the country j variety n is given by $\Omega_{t}^{j}(n) = \left[\Omega_{t}^{j}\left(i,N,n\right)\right]_{i,N}$. $\Omega_{t}^{j}\left(i,N,n,\iota\right)$ also describes the productivity of such a platform. However, n and ι do not affect the productivity, we will omit them in most of the remaining part.

Assumption. i. $\partial\Omega_{t}^{j}\left(i,N\right)/\partial N > 0$ (positive network effect); (ii) $\Omega_{t}^{j}\left(j,N\right) > \Omega_{t}^{j}\left(i,N\right)$ if $j \neq i$ (local platforms are more productive - international compatibility costs).

The worldwide number of users of that platform created is

$$N_t^i(n) = \sum_{j=1}^{J} 1_t^j(i,n)$$

where $1_t^j(i,n)$ is an indicator function which is 1 when the country j uses the platform developed by IT Titan in the country j to create intermediate goods n at period t. With this definition, zero profit condition can be explained mathematically as

$$\eta_\iota^{j,n} = \frac{1}{JR_{t+1}} F\left(\frac{\eta_\iota^{j,n}}{\left(q^{\iota(n)}\right)^{\frac{\sigma}{1-\sigma}}}\right) \sum_{j=1}^{J} 1_t^j(i,n)\Phi_t\left(\Omega_\iota\right) \tag{19}$$

4.4 Competitive Equilibrium

A competitive equilibrium in the quality ladder model is defined as a sequence of allocations $\left\{c_t^j, b_t^j, X_t^j\left(n\right), \quad \eta_\iota^{j,n}, Y_t^j, \Pi_t\right\}$, prices $\left\{w_t^j, R_t^j, P_t^j\left(n\right), \Phi_t\left(\Omega_\iota\right)\right\}$, and time paths $\{\mathbf{\Omega}_t, N_t^i(n)\}$ such that households in both countries maximize their utility, final goods producers and intermediate goods producers solve their profit maximization problems we defined before. Zero profit conditions (19) must be satisfied in the R&D sector. Moreover, market clearing conditions are satisfied which are goods markets are clearing

$$Y_t^j = c_t^j + \int_{n=0}^{1} \eta_t^{j,n} dn + \int_0^1 \frac{X_t^j\left(n\right)}{\Omega_t^{j*}(n)} dn \tag{20}$$

and bonds markets are clearing

$$b_t^j = 0$$

The size of every platform first created by an IT Titan in the country i follows

$$N_t^i(n) = \sum_{j=1}^{J} 1_t^j(i,n)$$

5 Analysis of The Quality Ladder Model

As the expanding varieties model, it is convenient to use the following notation to denote domestic and foreign variables:

$$\Omega(N) \equiv \ \Omega^j(j,N)\,; \ \Omega^*(N) \equiv \ \Omega^j(-j,N)\,;$$
$$V(N) \equiv V\left(\Omega^j(j,N)\right); V^*(N) \equiv V\left(\Omega^j(-j,N)\right).$$

5.1 With Traditional Antitrust Policies

In this section, price discrimination and collusion are not allowed. We first view δ_ι^n as given. From (18), if an intermediate goods producer n in the country j tries to use the platform with quality ι provided by the R&D firm in the country j, its profit every period along the BGP is

$$\pi(\iota,\Omega(N)) = \begin{cases} BL\Omega(1)^{\frac{\sigma}{1-\sigma}}\left(q^\iota\right)^{\frac{\sigma}{1-\sigma}}, & \text{if } N=L \text{ (only local)} \\ BL\Omega(2)^{\frac{\sigma}{1-\sigma}}\left(q^\iota\right)^{\frac{\sigma}{1-\sigma}}, & \text{if } N=2L \text{ (global)} \end{cases} \tag{21}$$

Thus, the expected total profit along the BGP is

$$V(\iota,\Omega(N)) = \begin{cases} \frac{R\pi(\iota,\Omega(1))}{R+\delta_\iota^n-1}, & \text{if } N=L \text{ (only local)} \\ \frac{R\pi(\iota,\Omega(2))}{R+\delta_\iota^n-1}, & \text{if } N=2L \text{ (global)} \end{cases} \tag{22}$$

If the country j is not the leader of the new platform and it tries to use the platform provided by the other country, we only need to consider the case both countries use the country $-j$'s platform.

$$\pi(\iota,\Omega^*(2L)) = BL\Omega^*(2L)^{\frac{\sigma}{1-\sigma}}\left(q^\iota\right)^{\frac{\sigma}{1-\sigma}} \tag{23}$$

The expected total revenue of the whole world is

$$V(\iota,\Omega^*(2L)) = \frac{R\pi(\iota,\Omega^*(2L))}{R+\delta-1} \tag{24}$$

The expected total revenue of intermediate producers along the BGP is either $2V(\iota,\Omega(L))$ when there are two networks or $V(\iota,\Omega(2L))+V(\iota,\Omega^*(2L))$

when there is one network in one industry. As the expanding varieties model, there are still three cases. The conditions are the same.

Case 1: $2\Omega(L)^{\frac{\sigma}{1-\sigma}} > \Omega(2L)^{\frac{\sigma}{1-\sigma}} + \Omega^*(2L)^{\frac{\sigma}{1-\sigma}}$, which implies $2V(\iota, \Omega(L)) > V(\iota, \Omega(2L)) + V(\iota, \Omega^*(2L))$.

Case 2: $\frac{4}{3}\Omega^*(2L)^{\frac{\sigma}{1-\sigma}} < 2\Omega(L)^{\frac{\sigma}{1-\sigma}} < \Omega(2L)^{\frac{\sigma}{1-\sigma}} + \Omega^*(2L)^{\frac{\sigma}{1-\sigma}}$, which implies $\frac{4}{3}V(\iota, \Omega^*(2L)) < 2V(\iota, \Omega(L)) < V(\iota, \Omega(2L)) + V(\iota, \Omega^*(2L))$.

Case 3: $3\Omega(L)^{\frac{\sigma}{1-\sigma}} \leq 2\Omega^*(2L)^{\frac{\sigma}{1-\sigma}}$, which implies $3V(\iota, \Omega(L)) \leq 2V(\iota, \Omega^*(2L))$.

Before we move on, we will first provide a proposition that will help us to show the existence and uniqueness of the BGP in all three cases. We can also use this proposition to study the effect of antitrust policies in the next section.

Proposition 1 *There is a unique positive solution in an equation that has the following form*

$$x = \frac{F(x)C}{\left(F(x)\left(q^{\frac{\sigma}{1-\sigma}} - 1\right) + 1\right)^{\theta} \frac{1}{\beta} + F(x) - 1} \tag{25}$$

Here $C > 0$ is a parameter, $x \in (0, +\infty)$. Greater C leads to a greater positive solution of x.

Proof. See Appendix A. ■

Although we leave the proof of proposition in the appendix, Figure 5 helps us understand some of the proof and economic intuition of this proposition. The horizontal axis is x, whereas the vertical axis is the function value. Let left-hand side (LHS) and right-hand side (RHS) of equation (25) be functions of x. The black 45° line is the graph of the LHS, whereas the red curve and blue curve are graphs of RHS with different values of C ($C2 > C1$). We first show that although LHS and RHS start from 0, RHS has a higher slope initially but has a lower value in infinity. Thus, there is at least a positive solution. Then, we use the properties of probability function $F(x)$ to show only one intersection, so the solution is unique. Thus, the slope of RHS must be smaller than the slope of LHS at the solution. As figure 5 shows, greater C leads to a greater solution of x.

Later in the study, we define the investment over quality as x while the C represents some terms without investment. Thus, the LHS is the cost of investment for the new platform, whereas the RHS benefits of the investment. The benefit of investment initially increases faster than cost. However, the marginal return of investment decreases according to the definition of $F(x)$. Increasing investment also decreases the expected life of one new platform, which is reflected in the denominator. We can see this more clearly later when we discuss case 1. Thus, the marginal return of investment decreases even more. The marginal cost, however, is constant. Thus, there is a unique solution when the marginal cost of investment is equal to the marginal benefit. Higher C increases the benefit and postpones the intersection. Thus, firms like to invest more.

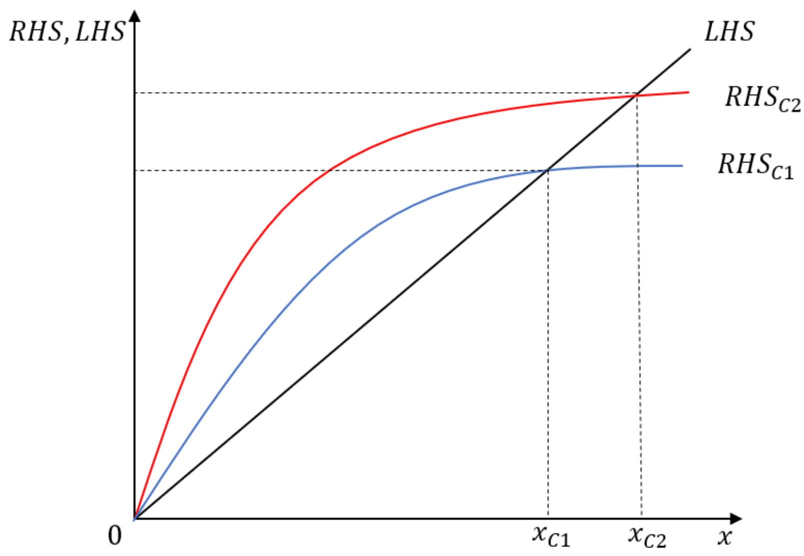

Figure 5: Explanation of proposition

5.1.1 Case 1: small network effects compared to compatibility cost

In this case $2\Omega(L)^{\frac{\sigma}{1-\sigma}} > \Omega(2L)^{\frac{\sigma}{1-\sigma}} + \Omega^*(2L)^{\frac{\sigma}{1-\sigma}}$, which means $2V(\iota, \Omega(L)) > V(\iota, \Omega(2L)) + V(\iota, \Omega^*(2L))$. Without loss of generality, we assume the R&D

firm in the country j is the first to create the new platform. If it decides to only capture its domestic market, it can set a license fee $\Phi(\iota, \Omega(L))$ equal to $V(\iota, \Omega(L))$, which is the expected profit of an intermediate goods producer in their country. If it wants to also compete in the foreign market, it has to set the license fee not more than $V(\iota, \Omega^*(2L))$, otherwise the intermediate goods producer in the country $-j$ will face a negative expected profit and not join. However, the R&D firm in the other country can easily take over the market with any price less than the license fee plus Ξ, where $\Xi = V(\iota, \Omega(L)) - V(\iota, \Omega^*(2L)) > 0$. Thus even the R&D firm in the country j sets 0 license fee, it still cannot win in the country $-j$. For this reason, there are two platforms and the license fee in both countries are

$$\Phi(\iota, \Omega(1)) = \frac{RBL\Omega(L)^{\frac{\sigma}{1-\sigma}} (q^{\iota})^{\frac{\sigma}{1-\sigma}}}{R + \delta_{\iota}^{n} - 1} \tag{26}$$

In this equilibrium, the expected profit of intermediate goods producers is zero. The next question is to calculate the R, $\delta = \delta_{\iota} = \delta_{\iota}^{n}$ and growth rate g along the BGP by using zero profit condition

$$\eta_{\iota}^{n} = \frac{\delta}{R}\Phi(\iota, \Omega(L))$$

We can omit the industry number along the BGP. Thus

$$\eta_{\iota} = \frac{\delta}{R}\Phi(\iota, \Omega(L)) \tag{27}$$

We know $\delta_{\iota} = F(\frac{\eta_{\iota}}{(q^{\iota(n)})^{\frac{\sigma}{1-\sigma}}})$, we know the need to calculate the BGP growth rate. From (20), along the BGP $1 + g_Y = 1 + g_c = 1 + g$. We first assume $\delta = \delta_{\iota} = F(\frac{\eta_{\iota}}{(q^{\iota(n)})^{\frac{\sigma}{1-\sigma}}})$ for all ι, which means investment makes every industry have the same probability of succeeding. Soon we find that it is true that every industry has the same probability to succeed. We do not track every industry in the model. Since there is a continuum of industries, the law of large number works. Every period, there is a fraction of δ of all industries get new platforms while the remaining industries remain the same. We rewrite the final production as

$$
\begin{aligned}
Y_t &= \delta A L^{1-\sigma} \int_{n=0}^{1} \left(q^{\iota(n)_{t-1}+1} q^{\frac{\sigma}{1-\sigma}} X_{t-1}^n \right)^{\sigma} dn + (1-\delta) Y_{t-1} \\
&= \delta q^{\frac{\sigma}{1-\sigma}} A L^{1-\sigma} \int_{n=0}^{1} \left(q^{\iota(n)_{t-1}} X_{t-1}^n \right)^{\sigma} dn + (1-\delta) Y_{t-1} \\
&= \delta q^{\frac{\sigma}{1-\sigma}} Y_{t-1} + (1-\delta) Y_{t-1}
\end{aligned}
$$

Thus,

$$
1 + g_{case1} = \frac{Y_t}{Y_{t-1}} = \delta q^{\frac{\sigma}{1-\sigma}} + (1-\delta) = F\left(\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}\right) \left(q^{\frac{\sigma}{1-\sigma}} - 1 \right) + 1
$$

The success probability of the investment determines this growth rate. If R&D firms invest more, new technologies come to life in a shorter time, the productivity of the economy increases faster. From (1), we now get the interest rate

$$
R = (1+g)^{\theta} \frac{1}{\beta} = \left(F\left(\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}\right) \left(q^{\frac{\sigma}{1-\sigma}} - 1 \right) + 1 \right)^{\theta} \frac{1}{\beta}
$$

By using interest rate, (26) and (27), we have an equation

$$
\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}} = \frac{F\left(\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}\right) B L \Omega (L)^{\frac{\sigma}{1-\sigma}}}{\left(F\left(\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}\right) \left(q^{\frac{\sigma}{1-\sigma}} - 1 \right) + 1 \right)^{\theta} \frac{1}{\beta} + F\left(\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}\right) - 1} \tag{28}
$$

It is an equation with one unknown $\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}$. Let $x = \frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}$, $C = C_{case1} = BL\Omega(L)^{\frac{\sigma}{1-\sigma}}$. Proposition 1 shows the existence and uniqueness of the $\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}$ along the BGP. Thus, we can solve equation (28) to get $\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}$. Since $\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}$ is same for all ι, the assumption $\delta = \delta_\iota = F\left(\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}\right)$ is correct. We can also get g_{case1}, R, δ along the BGP since we have $\frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}$.

Now we get the result of case 1. In case 1, network effects are small compared to compatibility cost. Notice $2V(\iota, \Omega(1)) > V(\iota, \Omega(2)) + V(\iota, \Omega^*(2))$ implies that the total profits using one platform is higher than using two platforms. Moreover, R&D firms extract all profits with two platforms. By creating two platforms, the total revenue extracted by R&D sectors is maximized. There is no room to increase economic growth by changing policies. For this reason, the benefit of using one platform cannot compensate the high international compatibility cost. Thus, every country uses the platform provided by the domestic IT Titans. What we have now is efficient for economic growth.

5.1.2 Case 2: middle network effects compared to compatibility Cost

In this case, $\frac{4}{3}\Omega^*(2L)^{\frac{\sigma}{1-\sigma}} < 2\Omega(L)^{\frac{\sigma}{1-\sigma}} < \Omega(2L)^{\frac{\sigma}{1-\sigma}} + \Omega^*(2L)^{\frac{\sigma}{1-\sigma}}$, that means $\frac{4}{3}V(\iota, \Omega^*(2L)) < 2V(\iota, \Omega(L)) < V(\iota, \Omega(2L)) + V(\iota, \Omega^*(2L))$. The profit of an intermediate goods producer that two firms are using two networks separately is $V(\iota, \Omega(L))$. The total profit of one network is $V(\iota, \Omega(2L)) + V(\iota, \Omega^*(2L))$. However, without the price discrimination, the R&D firm can at most set its price as $V(\iota, \Omega^*(2L))$. Otherwise, the foreign intermediate goods producer will have a loss. The foreign IT Titan can also build its network and offer a license fee less than the leader's license fee plus $(V(\iota, \Omega(L)) - V(\iota, \Omega(2L)))$ and take over the foreign market. Only when the leader compensates the foreign market and set license fee as $-(V(\iota, \Omega(L)) - V(\iota, \Omega(2L)))$, the potential entry can be prevented. Because the foreign IT Titan cannot set a license fee higher than zero, so the optimal choice is not to build a network. However, the total profit for the leader is now $-2(V(\iota, \Omega(L)) - V(\iota, \Omega(2L))) < V(\iota, \Omega(L))$ with the condition $\frac{4}{3}V(\iota, \Omega^*(2L)) < 2V(\iota, \Omega(L))$. The R&D firm which first develops a new platform would like only to have its domestic market. For this reason, the result comes back to case 1 without price discrimination.

It is worth mentioning $2V(\iota, \Omega(L)) < V(\iota, \Omega(2L)) + V(\iota, \Omega^*(2L))$ in case 2. If an IT Titan can get all the profit from one platform, the expected profit of R&D activities is higher, and an IT Titan would like to invest more. This will result in a higher growth rate. However, this result is not achievable in our basic model with traditional antitrust policies. A world with intermediate network effects and compatibility costs can benefit from using a bigger network. Unfortunately, the network effect is not big enough to ensure one network will be successful in a business sense. Later, when we discuss the effects of antitrust policies, we will show that price discrimination and collusion can lead to this higher growth rate.

5.1.3 Case 3: big network effects compared to compatibility cost

In this case, $3\Omega\left(L\right)^{\frac{\sigma}{1-\sigma}} \leq 2\Omega^{*}\left(2L\right)^{\frac{\sigma}{1-\sigma}}$, that means $3V\left(\iota,\Omega\left(L\right)\right) \leq 2V\left(\iota,\Omega^{*}\left(2L\right)\right)$. Without loss of generality, we assume the R&D firm in the country j is the first to create the new platform. If it decides only to get its domestic market, it can set a license fee equal to $V\left(\iota,\Omega\left(L\right)\right)$ which is the expected profit of an intermediate goods producer in their country. If it wants to compete in the foreign market as well, it has to set a license fee not more than $V\left(\iota,\Omega^{*}\left(2L\right)\right)$, otherwise, the intermediate goods producer in the country $-j$ will face a negative expected profit and does not want to buy the license.

If the R&D firm in the country j decides to open a business in the country $-j$, the R&D firm in the country $-j$ has two choices. It can either choose to build a second network or give up the market. If it chooses to build the second network, it can at most get its domestic market. To prevent potential entry, the leader sets the license fee at $\Phi\left(\iota,\Omega\left(2L\right)\right) = V\left(\iota,\Omega^{*}\left(2L\right)\right) - V\left(\iota,\Omega\left(L\right)\right)$. Since from the condition of case 3, $2\left(V\left(\iota,\Omega^{*}\left(2L\right)\right) - V\left(\iota,\Omega\left(L\right)\right)\right) \geq V\left(\iota,\Omega\left(L\right)\right)$. The profits of operating in the whole world market are higher than just having the domestic market. Thus, the leader will set the license fee $\Phi\left(\iota,\Omega\left(2L\right)\right) = V\left(\iota,\Omega^{*}\left(2L\right)\right) - V\left(\iota,\Omega\left(L\right)\right)$ to get the foreign market. Now

$$\begin{aligned}\Phi\left(\iota,\Omega\left(2L\right)\right) &= V\left(\iota,\Omega^{*}\left(2L\right)\right) - V\left(\iota,\Omega\left(L\right)\right) \\ &= \frac{RBL\left(\Omega^{*}\left(2L\right)^{\frac{\sigma}{1-\sigma}} - \Omega\left(L\right)^{\frac{\sigma}{1-\sigma}}\right)}{R+\delta-1}\end{aligned}$$

It is worth mentioning that the intermediate goods producer in the country j has an expected profit in this situation. It transfers the profit to the household.

We can also calculate what is the total production of a country compared with two networks. Like what we did in case 1, we use the zero-profit condition,

$$\eta_\iota = \frac{1}{2}\frac{\delta}{R}2\Phi\left(\iota, \Omega\left(2L\right)\right) + \frac{1}{2} * 0 = \frac{\delta}{R}\Phi\left(\iota, \Omega\left(2L\right)\right)$$

Here, the RHS has two terms because an IT Titan does not know whether it can be the leader or not. It has a 50% chance to be the leader and get the license fees from both countries. It also has a 50% chance to get nothing. Similar to case 1, we can get a function

$$\begin{aligned} &\frac{\eta_\iota}{\left(q^\iota\right)^{\frac{\sigma}{1-\sigma}}} \\ &= \frac{F(\frac{\eta_\iota}{\left(q^\iota\right)^{\frac{\sigma}{1-\sigma}}})BL\left(\Omega^*\left(2L\right)^{\frac{\sigma}{1-\sigma}} - \Omega\left(L\right)^{\frac{\sigma}{1-\sigma}}\right)}{\left(F(\frac{\eta_\iota}{\left(q^\iota\right)^{\frac{\sigma}{1-\sigma}}})\left(q^{\frac{\sigma}{1-\sigma}} - 1\right) + 1\right)^{\theta}\frac{1}{\beta} + F(\frac{\eta_\iota}{\left(q^\iota\right)^{\frac{\sigma}{1-\sigma}}}) - 1} \end{aligned} \tag{29}$$

Let $x = \frac{\eta_\iota}{\left(q^i\right)^{\frac{\sigma}{1-\sigma}}}$, $C_{case3} = BL\left(\Omega^*\left(2L\right)^{\frac{\sigma}{1-\sigma}} - \Omega\left(L\right)^{\frac{\sigma}{1-\sigma}}\right)$. Proposition 1 ensures there is a unique solution in the equation (29). Thus, we can get g, R, δ. Later we show that price discrimination leads to a higher growth rate than this result.

In case 3, network effects are big enough and dominate the compatibility cost. Thus, one network can be successful enough for business. However, since the leader has to compensate for the foreign market and decreases the license fee, its revenue is much less than the benefit it creates. For this reason, IT Titans, in this case, do not invest as much as they invest when they get all the benefits they create. We will see this result in the next section.

5.2 Price Discrimination

This subsection studies how price discrimination affects our model. As the expanding varieties model, price discrimination cannot change case 1. We focus on cases 2 and 3.

5.2.1 Case 2 with price discrimination

Without loss of generality, we assume the R&D firm in the country j is the first to create the new platform. With price discrimination, the first developer can set the license fee in their country equals to

$$\Phi(\iota, \Omega(2L)) = V(\iota, \Omega(2L))$$

For the foreign market, the R&D firm in $-j$ has an incentive to develop a new network. For this reason, the R&D firm in the country j must set a license fee equal to

$$\Phi(\iota, \Omega^*(2L)) = V(\iota, \Omega^*(2L)) - V(\iota, \Omega(L))$$

As the expanding varieties model, the conditions of case 2 does not ensure $V(\iota, \Omega^*(2L)) - V(\iota, \Omega(L))$ is positive. We can see the total profit

$$\begin{aligned} &\Phi(\iota, \Omega(2L)) + \Phi(\iota, \Omega^*(2L)) \\ &= V(\iota, \Omega(2L)) + V(\iota, \Omega^*(2L)) - V(\iota, \Omega(L)) \\ &> V(\iota, \Omega(L)) \end{aligned}$$

This result means it is still profitable by paying the foreign market to increase the size of the network, so it is still a rational behavior. The logic is the same as the expanding varieties model, so we do not discuss it again.

Does price discrimination policy increase the growth rate? The answer is maybe. We turn both with and without price discrimination cases in (25) and $x = \frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}$, the constant with price discrimination, with expected profit as $\frac{\Phi(\iota,\Omega(2L))+\Phi(\iota,\Omega^*(2L))}{2}$,is

$$C^D_{case2} = \frac{BL\left(\Omega(2L)^{\frac{\sigma}{1-\sigma}} + \Omega^*(2L)^{\frac{\sigma}{1-\sigma}} - \Omega(L)^{\frac{\sigma}{1-\sigma}}\right)}{2}$$

while the constant without price discrimination, with expected profit as

$V(\iota, \Omega(L))$, is

$$C_{case2} = BL\Omega(L)^{\frac{\sigma}{1-\sigma}}$$

It is undetermined whether C^D_{Case2} is greater than C_{Case2} or not. Thus, proposition 1 shows the investment of R&D is lower with one network when $C^D_{Case2} < C_{Case2}$ and higher when $C^D_{Case2} > C_{Case2}$. This is different from the result in the expanding varieties model. Here, price discrimination may hurt or help economic growth. It is easy to understand the benefit of one network owing to the network externalities. However, why would having one network hurt the economy? Although one network increases the total productivity in the intermediate goods sector, the expected return of R&D activities may be lower owing to sales competition. However, we show later that allowing collusion changes this result. This is the only difference with the expanding varieties model.

5.2.2 Case 3 with price discrimination

It is easy to know that with price discrimination, the R&D firm in the country j is able to set domestic license fee as $\Phi(\iota, \Omega(2L)) = V(\iota, \Omega(2L))$ and foreign license fee at $\Phi(\iota, \Omega^*(2L)) = V(\iota, \Omega^*(2L)) - V(\iota, \Omega(L))$. Now the expected profit is

$$\frac{1}{2}\frac{\delta}{R}\left(\Phi(\iota, \Omega(2L)) + \Phi(\iota, \Omega^*(2L))\right)$$

We turn this case and the case without price discrimination in the form of (25) and $x = \frac{\eta_\iota}{(q^\iota)^{\frac{\sigma}{1-\sigma}}}$. With price discrimination, the constant is

$$C^D_{case3} = \frac{BL\left(\Omega(2L)^{\frac{\sigma}{1-\sigma}} + \Omega^*(2L)^{\frac{\sigma}{1-\sigma}} - \Omega(L)^{\frac{\sigma}{1-\sigma}}\right)}{2}$$

Thus,

$$C^D_{case3} > C_{case3}$$

By using proposition 1, we know price discrimination increases investment in R&D activities and increases growth. The intuition is simple; price discrimination helps the R&D firm extract more profits. Thus, the expected revenue of R&D increases. R&D firms prefer higher investment in innovations. However, R&D firms still do not extract all the profits since a leader has to prevent foreign entry.

5.3 Collusion

What happens if collusion is allowed when the R&D activities are completed on a new platform? For case 1, the total profits of using two networks are greater than one network. Thus, R&D firms have no incentive to collude. For cases 2 and 3, things are different. Since the R&D firms are involved in many industries, rather than only one industry, they can collude to increase profit. If they promise to only develop networks when they are the first developer, the lucky one does not need to compensate the foreign country when price discrimination is also allowed. Now the license fee is

$$\Phi\left(\iota, \Omega\left(2L\right)\right) = V\left(\iota, \Omega\left(2L\right)\right)$$

in domestic country and

$$\Phi\left(\iota, \Omega^*\left(2L\right)\right) = V\left(\iota, \Omega^*\left(2L\right)\right)$$

in the foreign market. The expected profit is now

$$\frac{\delta}{R}\frac{\Phi\left(\iota, \Omega\left(2L\right)\right) + \Phi\left(\iota, \Omega^*\left(2L\right)\right)}{2}$$

Again, we turn this case into (25). Now the constant is

$$C^{CD} = \frac{1}{2}BL\left(\Omega\left(2L\right)^{\frac{\sigma}{1-\sigma}} + \Omega^*\left(2L\right)^{\frac{\sigma}{1-\sigma}}\right)$$

From the conditions of case 2, we get $C^{CD} > C_{case2}$ and $C^{CD} > C^D_{case2}$. By proposition 1, we know they invest more here, than in the case without price discrimination and in the case with only price discrimination. Thus, price discrimination with collusion makes the economy grow fastest in case

2. From the conditions of case 3, $C^{CD} > C^{D}_{case3}$. Collusion also gives the economy in case 3 the fastest growth. The intuition is simple. Collusion eliminates the competition of sales and helps the winner extract all profits. Thus, the incentive to invest is greater, and the probability of success is higher.

6 Conclusion

Here, we investigate endogenous economic growth in a world of IT Titans. We find that network externalities and international compatibility costs determine the number of giants that can co-exist in an industry in the world. These two features are enough to generate endogenous non-rival innovations. We also find that traditional antitrust policies may hurt economic growth when network externalities are sufficiently large compared to international compatibility costs. The reward of innovation is enough to encourage competition in innovations. Repealing some international antitrust policies after innovations are successful in IT industries may help economic growth worldwide.

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A Proof of Proposition 1

Equation (25) is

$$x = \frac{F(x)C}{\left(F(x)\left(q^{\frac{\sigma}{1-\sigma}} - 1\right) + 1\right)^{\theta} \frac{1}{\beta} + F(x) - 1}$$

We first rewrite the equation as

$$\left(F(x)\left(q^{\frac{\sigma}{1-\sigma}} - 1\right) + 1\right)^{\theta} \frac{1}{\beta} + F(x) - 1 = \frac{F(x)C}{x}$$

When $x \to 0$,

$$\lim_{x\to 0} LHS = \frac{1}{\beta} - 1 < \infty = F^{'}(0)C = \lim_{x\to 0} RHS$$

when $x \to \infty$,

$$\lim_{x\to\infty} LHS = \frac{1}{\beta} q^{\frac{\sigma\theta}{1-\sigma}} > 0 = \frac{C}{x} = \lim_{x\to 0} RHS$$

For this reason, there is at least a positive solution.

The first order condition of the LHS is

$$\theta \left(F(x)\left(q^{\frac{\sigma}{1-\sigma}} - 1\right) + 1\right)^{\theta-1} \frac{1}{\beta} \left(q^{\frac{\sigma}{1-\sigma}} - 1\right) F^{'}(x) + 1 > 0$$

The first order condition of the RHS is

$$C\frac{F^{'}(x) - F(x)x}{x^2} = C\frac{\frac{F^{'}(x)}{F(x)x} - 1}{x^2} \leq 0$$

Thus, the solution is unique.

We now turn to the original equation. Since the LHS of the original equation is 0 when $x \to 0$, is ∞ when $x \to \infty$. The RHS of the original equation is 0 when $x \to 0$, is $\frac{C}{q^{\frac{\sigma\theta}{1-\sigma}}\frac{1}{\beta}} < \infty$ when $x \to \infty$. Moreover, the first order condition of the original equation at $x = 0$ is

$$FOCLHS = 1$$

$$\begin{aligned}
&FOCRHS \\
&= C \frac{\begin{array}{c} F'(0)\left[\left(F(0)\left(q^{\frac{\sigma}{1-\sigma}}-1\right)+1\right)^{\theta}\frac{1}{\beta}+F(0)-1\right] \\ -F(0)\left[\theta\left(F(0)\left(q^{\frac{\sigma}{1-\sigma}}-1\right)+1\right)^{\theta-1}\frac{1}{\beta}\left(q^{\frac{\sigma}{1-\sigma}}-1\right)F'(0)+1\right] \end{array}}{\left[\left(F(0)\left(q^{\frac{\sigma}{1-\sigma}}-1\right)+1\right)^{\theta}\frac{1}{\beta}+F(0)-1\right]^{2}} \\
&= C \frac{F'(0)\left[\frac{1}{\beta}-1\right]}{\left[\frac{1}{\beta}-1\right]^{2}} \\
&= \infty \\
&> 1
\end{aligned}$$

Thus, the slope of the RHS of the original equation must be smaller than 1 at the solution.

Since the RHS of the original equation is $\frac{F(x)C}{\left(F(x)\left(q^{\frac{\sigma}{1-\sigma}}-1\right)+1\right)^{\theta}\frac{1}{\beta}+F(x)-1}$ which means $\frac{dRHS}{dC} > 0$ for given x. Thus, the intersection with the LHS must have greater x. For this reason, greater C leads to greater solution.