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Using futures and option contracts to manage price and quantity risk: A case of corn farmers in central Iowa

Li-Fen Lei
Iowa State University

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Using futures and option contracts to manage price and quantity risk: A case of corn farmers in central Iowa

Lei, Li-Fen, Ph.D.

Iowa State University, 1992
Using futures and option contracts to manage price and quantity risk:
A case of corn farmers in central Iowa

by

Li-Fen Lei

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY

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Major: Agricultural Economics

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CHAPTER I.
INTRODUCTION

Risks are pervasive in agriculture. Total risk is usually divided into two components: business risk and financial risk. Business risk is the variability in the net operating returns inherent in a farm's operations. The additional variability added to the farm's net returns that results from fixed financial claims against the farm is referred to as financial risk (Gabriel and Baker, 1980). The business risks in agricultural sector occur as unanticipated variations in production and prices, uncertainties about personnel performance, technological change, and changes in the farm's legal environment. The major source of financial risk is the use of fixed financial obligations such as the risk of cash insolvency and illiquidity. Business and financial risks combine to magnify potential losses in farmers' equity capital and create inefficiencies in the use of resource by hampering business planning.

Risk management involves the selection of methods for countering business and financial risks in order to meet a decision maker's risk-averting goals. A farmer's responses to risk generally focus on reducing the likelihood of business and financial risk, transferring risks to other agents, and increasing the farm's ability to operate within a given level
of risk. Risk management strategies can be categorized in terms of production, marketing, and financial organizations of farm businesses (Barry et al., 1988).

In production, risk responses include the selection of stable enterprises, diversification, and maintaining operating flexibility. Inventory management, sequential marketing, forward contracting, and hedging are classified as a producer's market responses to risk. Financial responses to risk involve the management of leverage and liquidity and reflect the farmer's capacity to accept risks in production and marketing, and to spread these risks among those with financial claims on the farm.

This study focuses on the management of business risk for a representative central Iowa corn producer. Specifically, the producer uses futures and/or option contracts to manage price risk and quantity risk.

Price risk is a form of uncertainty in which the prices at which transactions are completed may deviate from the prices prevailing at the time of decision making. In agricultural production, there is a significant lag between the time the decisions are made and the time outcomes are realized. In the time that passes between decisions and outcomes, the price structure may change due to market dynamics. Thus, what appeared to be an optimal choice at the time of decision making, may be turn out not to be optimal by
the time that the outcome is realized. In addition to price risk, agricultural producers are also exposed to quantity risk as well. Whenever there is the possibility of an unexpected shortfall in production there is quantity risk.

**Risk Management with Futures and Options**

Hedging is done by market participants who buy (long) or sell (short) in the cash market. Hedgers are typically primary producers, processors, marketing intermediaries who hold a product. Futures hedging is to take a futures position that is opposite to the cash position. Any gains (losses) in the cash market are offset by losses (gains) in the futures market.

Agricultural options, options on futures, give the right but not obligation, to a position in the underlying futures contract. There are two basic types of options, a "call" option and a "put" option. A call option gives the holder the right to buy the underlying futures at a specific price within a specific time period. A put option gives the holder the right to sell the underlying futures at a specific price within a specific time period.

The put option holder profits from a decline in the underlying futures price while a call option holder profits from an increase in the underlying futures price. As futures price and cash price move in the same direction, using a long
put option to hedge a long cash position would limit downside risk but not upside potential. Similarly, a short cash position hedged by a long call option would limit the risk of a price rise but not the downside potential. The most that an option buyer can lose is the option premium which is the price he has to pay at the time of purchasing.

If price risk is the only uncertainty faced by the decision maker, hedging in futures market produces less volatile profit outcome than no hedging at all (Holthausen, 1979; Feder et al., 1980). In the cases where both price risk and quantity risk are present, hedging in options could outperform the futures hedging because the holder of options has right but no obligation to exercise them. However, there is a cost associated with the added advantage of options - the premium. As such hedging in options could be rather expensive. Obviously, there is a need to examine the decision maker's optimal response to price and quantity risks when both futures and options are available.

A number of previous studies have addressed the issue of optimal marketing strategy in the presence of commodity futures markets. Traditional hedging theory emphasizes the risk avoidance potential of futures markets and argues for full hedge with a unit-for-unit position in the cash and futures markets. Johnson (1960) shows that under certain circumstances the optimal hedge may not be the traditional
hedge. Rolfe (1980), Berck (1981), and Anderson and Danthine (1983) examine the optimal hedge ratio with the context where decision makers are allowed to use futures contracts as hedging or speculative tools. However, relatively little work has been devoted to the study of optimal hedging strategy in the presence of option markets. Wolf (1987), Hanson (1988), Bullock (1989), and Lapan et al. (1990) are among them.

The selection of an optimal hedging strategy depends in part on the producer's expectations about the course of prices and harvest outputs. Knowledge about price movements and serial correlation properties of price changes is important to the decision makers. They are concerned about: (1) what the most likely price will be; and (2) how far the actual price can deviate from the most likely price and with what kind of probability. These questions can be summarized by the first two moments of the decision maker's expected probability distribution of the unknown price.

Unfortunately, a direct elicitation of probability distributions for a decision maker is often difficult, if not impossible. If the distribution is estimated by statistical techniques using time-series data, allowances for its nonsationarity property is important. There is evidence that agricultural commodity price volatility changes as markets move through various phases of economic cycles (Anderson, 1985; Gordon, 1985; Fackler, 1986). Thus, there is a need to
examine the decision maker's subjective probability assuming non-constant variance. Bollerslev (1986) suggests, the time variation of conditional second moments and/or higher moments might have important implications for the empirical performance of various asset pricing models. In addition, empirical results show that time-varying models provide a good description of the commodities prices distributions (Baillie and Myers, 1989; Myers, 1990).

Objectives

The overall objective of this study is to examine the optimal responses of a representative central Iowa corn producer to quantity and price uncertainty when both futures and option contracts are available as risk management tools. The corn farmer is selected as a representative decision maker because corn is the most important crop in Iowa.

The specific objectives are:

1. To generate price distributions of cash, futures, and, options for corn and the distribution of yield.
2. To determine optimal market positions for the utility-maximizing corn producer under three scenarios: 1) only futures available; 2) only options available; 3) both futures and options available.
3. To examine the effect of model parameters on the optimal solution.

4. To compute the access value of futures and option markets to the risk-averse corn producer.

To achieve these objectives, numerical optimization techniques are used to solve the expected utility maximization problem for the corn producer. The producer is assumed to form his subjective probability distributions about random variables by making predictions on cash, futures, and option prices and yield. The price distributions are estimated using a recent developed method, conditional heteroscedasticity model, to allow for non-constant variance of the prices. A trend equation is applied to estimate the distribution of yield. The optimal market positions for various scenarios are obtained using a nonlinear numerical optimization routine in GAMS (Brooke et al., 1988). Sensitivity analysis is conducted to assess the impact of model parameters on the optimal solution. The access values of futures and options added to the producer are computed based on the concept of certainty equivalent.

Organization of the Study

The remainder of the study is divided into five chapters. Chapter II presents the review of literature organized into three sections. The first section describes the
characteristics, and trading strategies of futures and option contracts. The second section reviews the empirical methods used to estimate price distributions, while the third section is an overview of studies related to optimal hedging strategies. Chapter III contains a discussion of the derivation of price distribution models, and estimation results. Chapter IV conveys the theoretical framework used in the analysis of optimal market positions. Optimization results and sensitivity analysis are presented in Chapter V. Finally, summary, conclusions, and limitations of the study are contained in Chapter VI.
CHAPTER II.

REVIEW OF LITERATURE

This chapter is divided into three major sections. The first section reviews the characteristics, and trading strategies of futures and option markets. Empirical methods used to estimate price distributions are given in the second section. Finally, studies related to optimal market positions are reviewed in the third section.

Characteristics and Strategies of Futures and Options

Futures contracts

A futures contract is an agreement between two parties- a buyer and a seller- to buy or sell something at a future date. The contract trades on a futures exchange and is subject to a daily settlement procedure. A trader may establish a market position by either buying or selling these contracts. It is not necessary to buy futures before selling them. To be short, a trader has sold contracts not covered by purchases. The seller simply has assumed a contractual obligation to deliver the specified commodity at a specified price and to receive the agreed-upon price. To be long, a trader has purchased contracts not covered by an equivalent amount of sales. The buyer is obligated to accept delivery and pay for the contracted amount unless he or she subsequently offsets
the long position with appropriate sales.

Assuming the trader does not make or take delivery, the cost of the futures trading includes commission charges to a broker, and the possible adverse movement of price while holding the contract. Because of the possibility of adverse price movements, traders are required to make a margin deposit. Margins vary from commodity to commodity and are typically a small percentage of the total value of the contracts. The objective is to provide protection against default by the trader. Thus, a margin is not a down payment but is more like "earnest money." Margin calls occur with adverse price movements: a price decline for the purchaser and a price increase for the seller. For example, suppose that the initial margin on a 5,000 bushel contract is 15 cents per bushel and that the maintenance margin is 10 cents per bushel. As prices fluctuate, the trader's margin fluctuates. If the trader holds a long position, a price increase is favorable and the margin is increased; a price decrease is unfavorable and the margin is decreased. Given an unfavorable price change, the trader is asked (margin call) to provide additional funds when the margin goes below the maintenance level. In this example, if price declines more than 5 cents per bushel, the trader is asked to deposit sufficient funds to bring the margin back to the initial level of 15 cents per bushel.
"Open interest" is the number of contracts remaining to be settled for a particular contract. For example, if a trader with a net zero position buys a December contract from another trader with a net zero position, the open interest in this contract has increased by one. If this trader subsequently sells a December contract, the open interest is reduced by one. The open interest is equal to the net number of long or short positions. The number of contracts long and short must be equal. The "volume of trading" is the total number of transactions in a given time period, say a day. It may be quoted as the number of contracts or the physical volume of contracts traded.

The "basis" is the difference between the current cash price and a futures price. A market is said to be in "contango" when the futures price exceeds the cash price. When the futures price is below the cash price, the market is referred to as being in backwardation. A short hedge, long cash and short futures, will receive additional profits if the basis narrows. A long hedge, short cash and long futures, will profit from a widening of the basis. The magnitude of the basis depends on several factors including time to maturity, transaction costs, storage costs, and demand and supply conditions in and across markets. Most studies relating futures hedges have assumed no basis uncertainty by using the end-of-period cash prices as the end-of-period
futures prices. In real world, as a futures contract approaches delivery, the futures prices rarely converge to cash prices. Hence, the basis risk is allowed to exist in present study.

Agricultural option contracts

Agricultural options, also called options on futures, give participants the right but not the obligation to buy or sell a futures contract at a later date at a price agreed upon today. The option purchaser, who is called the option holder and said to be long the option, pays the option writer, who is said to be short option, a market-determined price called the option premium. The agreed price is called the strike price or exercise price. The seller of either a call or put option is required to deposit an initial futures margin requirement and maintain it based on daily premium settlement because the option sold could be exercised at any time.

All options can be described as either a call or put option and either an American or European option. A call option gives the owner the right to buy the underlying futures contract at the strike price and a put option gives the owner the right to sell it at the strike price. An American option allows the owner to exercise the option at any time before the maturity date, while a European option can only be exercised on the expiration date.
Since an option contract gives the right but not the obligation to buy or sell underlying futures, there are several ways to deal with an option transaction: closing the transaction, exercising the option, and allowing option expiration. A holder of an option will close the position by making an equal but opposite transaction prior to the expiration date if the value of the option increases. In contrast, the writer of an option will close the position only when the value of the option decreases.

The option holder will exercise the option contract only when it is profitable. A call option is exercised when the strike price is below the market price of the underlying futures and a put option is exercised when the strike price is above the market price of underlying futures. When an option is exercised, the writer will incur a loss which is either partially or completely offset by the premium collected from the buyer. Alternatively, the holder of an option will allow it to expire when it is unprofitable. The writer of an option will profit by the entire premium.

An individual who exercises an option on a futures contract will acquire a futures position. That is, when a call is exercised, the option buyer is assigned a long futures position at the option strike price and the option writer is assigned the opposite short position. Conversely, a short position is assigned to option buyer and the opposite long
futures position is assigned to the option writer when a put is exercised.

If option markets are efficient (characterized by numerous well-informed participants), option premiums should represent the fair market value use of options. The assumption that option markets are perfect implies, among other things, that the market are efficient. For this reason, it is correct to use the terms option premium, option value, and option price interchangeably. A market is said to be perfect if the following conditions are satisfied: (1) there are no transaction costs; (2) information is equally and instantaneously available to all market participants; (3) there are no taxes and transaction costs; (4) securities are infinitely divisible; (5) there are no artificial restrictions on any type of trading, especially short selling; and (6) market participants can both borrow and lend at the same risk-free rate (Marshall, 1989).

An option may be in-the-money, at-the-money, or out-of-the-money depending on the relationship between the option's strike price and the current price of the underlying futures. Let \( F(t) \) denote the price of the underlying futures at time \( t \), and let \( S \) denote the strike price of the option. The conditions for call and put options may be summarized as follows.
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<td>At-the-money</td>
<td>$S = F(t)$</td>
<td>$S = F(t)$</td>
</tr>
<tr>
<td>Out-of-the-money</td>
<td>$S &gt; F(t)$</td>
<td>$S &lt; F(t)$</td>
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Sometimes the term near-the-money is used to describe an option that is in-the-money or out-of-the-money but only by a small amount. And the terms deep-in-the-money and deep-out-of-the-money are used to describe options that are substantially in-the-money or out-of-the-money, respectively.

An option premium embodies two distinct kinds of value: intrinsic value and time value. An option's intrinsic value is the dollar amount that would be realized if the option were exercised immediately. Thus, an option's intrinsic value is defined as the greater of the amount by which the option is in-the-money or zero. The intrinsic values are given by

\[
\text{MAX}\{F(t)-S, 0\} \text{ for a call option} \\
\text{and} \quad \text{MAX}\{S-F(t), 0\} \text{ for a put option.}
\]

The expression \(\text{MAX}\{F(t)-S, 0\}\) means "Take the maximum value of the two arguments, \(F(t)-S\) or zero."

If an option were at-the-money or out-of-the-money, the option would have no intrinsic value. In this case, the premium would consist solely of time value.

Assume an option expires at time \(T\) and that determination of the value of the option at time \(t\) is of interest. For a call (put) option, the price of the underlying futures
might increase (decrease) during time $t$ and the option's expiration at time $T$. Therefore the option contract has greater intrinsic value. The positive probability of a future increase in intrinsic value gives an option a value beyond its intrinsic value. This additional value is called time value. Time value at time $t$ is equal to the premium less the intrinsic value and is reflective of the amount of remaining until expiration.

There is also a positive probability that the intrinsic value of an option may decline prior to expiration. However, since the downside potential to an option's intrinsic value is generally significantly less than its upside potential, its time value is generally positive. In the case of an American option, the time value is never negative, although it can be zero. The actual source of the time value is price volatility. As the price of the underlying futures becomes more variable, the probability of an increase in intrinsic value is enforced. Since an option's time value is directly related to the price volatility of the underlying futures, it is expected to decline progressively more rapidly as time elapses. That is as an option approaches its expiration date, the option's time value decays progressively more rapidly, and the option premium approaches the option's intrinsic value.
**Similarities and differences**

The factors affecting options and futures prices have some certain similarities. First both options and futures prices are linked to the prices of the underlying instrument (commodities or futures contracts). Moreover, the prices of options and futures contracts are related to other common factors such as short-term interest rates and the income of the underlying instrument. Both options and futures have limited life, and both contracts have been established by an exchange and are guaranteed by a clearinghouse.

Some important structural differences between option and futures make them each uniquely suitable for various types of traders. The initial cash outflow which represents the maximum loss on the position is greater when buying options. However, a security deposit is required when a futures position is established. Since losses and gains are realized daily in the futures markets, a security deposit for futures contracts is required. In contrast, gains or losses are not realized for purchasers of option contracts until the positions are offset or exercised.

Options and futures can be further assessed by examining both the expected return and expected risk. Figure 2.1(a) compares the hypothetical income distribution of a commodity in a stable and in a volatile price environment. The expected income is higher in a volatile environment but the standard
Figure 2.1. Hedged and unhedged income distributions
deviation of income is also greater. Hedging with futures contracts cannot change the fundamental nature of the income distribution of the commodity as shown in Figure 2.1(b). The income distribution of a hedged position is similar to that of an unhedged cash position.

Buying put options can not only protect the value of underlying instrument, but also change the distribution of incomes favorably as presented in (c) part of Figure 2.1. The income distribution shifts to the left, reflecting the cost of buying the option, and truncates the left hand tails of the distribution. A put option contract eliminates large losses while retaining the possibility of gains. Selling call options has the reverse effect upon the distribution of unhedged income as demonstrated in Figure 2.1(d). The distribution shifts to right, reflecting the receipt of option premium, and truncates the right hand tails of distribution. Selling call options limits the upside of the return distribution but increases the probability of incomes close to the average or expected income. The upside income potential or downside income volatility can be altered by most option strategies in some way. Thus option contracts give traders a risk management tool that allows them to take advantage of a correct view of the price prospects for the underlying futures while adjusting risk to a level below what it would be if only the underlying futures was held (Schwarz et al., 1986).
Hedging and trading strategies

The objective of trading in futures and option markets can be classified as hedging, speculating, arbitrage, or some combination of the above. Hedging is a technique of establishing an approximate price for a cash commodity or in some cases ensuring that adequate supplies of some assets are available. Speculating can be defined as a risky investment made in an effort to achieve a financial profit. Arbitrage is the purchase or sale in one market for immediate sale or purchase in another market in an effort to capture profits (Seidel and Ginsberg, 1983). All three types of traders use the same technique of taking a position in markets. Their objectives in trading is the only distinguishing characteristic among the three types of traders.

Futures hedging is to take a futures position that is opposite to his cash position in the commodity to offset price risk associated with cash market. However, it creates another market position with its own price risk. The effectiveness of a futures hedge is determined by the degree to which the price risk of the futures position offsets the price risk of the cash position. Futures hedges are scarcely perfectly effective since basis usually exists and is not constant over time. Basis patterns are difficult to predict, therefore, basis risk generally remains after a hedge is established. The basis risk is defined as the variance of the basis at the
time the hedge is lifted (Marshall). The basis risk tends to decline as the futures contract approaches delivery, but this decline is not linear. Further, a futures hedge requires that trader know exactly the quantity he wants to hedge. However, he may only receive part of his bid. If he were using futures to hedge his expected position, he would be overhedged, and be subject to price risk on his futures positions.

Johnson (1960) and Ederington (1979) considered a decision maker who wants to minimize the variance of his return. The minimum variance hedge ratio was derived as the ratio of the covariance between cash and futures prices to the variance of the futures price. Benninga et al. (1984) show that the minimum variance hedge position is also an optimal hedge ratio when futures markets are assumed to be unbiased.

An optimal hedge ratio or minimum variance hedge ratio is usually defined as the proportion of a cash position that should be covered with an opposite position in a futures market. It can be estimated by regressing historical cash prices, price changes, on futures prices, price changes. The resulting slope coefficient is the estimated optimal hedge ratio (Enderington, 1979; Benninga et al., 1983). Myers (1990) criticized the conventional regression method to optimal hedge ratio estimation for two reasons: 1) it fails to include all information available to hedgers at the time hedging decisions are made; 2) it implicitly assumes
that the covariance matrix of cash and futures prices.

Commodity options provide several risk features which are not available in futures contracts and may be used in conjunction with, or even as a substitute for, futures contracts in a farm's portfolio to manage risks. Options can be used to convert established long or short futures positions into synthetic long and short calls or puts. For example, a long put will hedge the possible loss on a long futures position while leaving unlimited gains. Similarly, a long call in combination with a short position in futures will limit the risk of an increase in prices since the call option can be exercised. Some possible synthetic positions are given in Table 2.1. However, various assumptions on the degree of substitutability between markets are required to establish the equilibrium relationships between options on futures and the underlying futures contracts. These include equality in transaction costs, ability to transact simultaneous transactions at stated parity levels, and institutional, tax, and government regulations that do not bias trader choice (Schwarz et al., 1986, p.413).

The basic option strategies include buying calls, buying puts, writing calls, and writing puts. Every advanced option strategy relies on these four basic strategies. These strategies can be broken down into its component buy or write, call or put strategies. Some more advanced strategies use
Table 2.1. Synthetic futures and options positions

<table>
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<tr>
<th>Synthetic long futures</th>
<th>Long call + Short put</th>
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<tr>
<td>Synthetic short futures</td>
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<td>Synthetic long call</td>
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<td>Synthetic long put</td>
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<td>Synthetic short put</td>
<td>Short call + Long futures</td>
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both options and futures contracts. In some strategies, the options serve as the "insurance" for the risk of holding the futures contracts and in others, the options are just another leverage-enhancement tool (Angell, 1986; Marshall, 1989; Chance, 1989). The advanced strategies can be used as hedging strategies or speculating strategies depending on the trader's objective. If risk reduction or risk transformation is the dominant explanation for an option trader's position, then that trader uses option contracts to hedge his cash positions. If the motivation of holding a position in options is to profit from a change in the option's value, then the trader is speculating. Finally, if the option position is taken in conjunction with positions in other assets to exploit a price discrepancy, then the trader uses option strategies as arbitrage strategies.

In sum, from the standpoint of the grower, the futures market serves as a risk-reduction mechanism. The only
drawback is that the hedger, seeking to limit risk, might lock out a windfall gain by contracting to sell or purchase at a fixed price. Additionally, hedgers must deal with basis risk and quantity risk. Agricultural options offer many of the opportunities available with futures: hedging, arbitrage, and speculation. Options also provide unique strategies unavailable with futures. With options, the traders can alter the distribution of returns by selling potential upside returns or buying insurance against downside losses. Option hedging also allows greater flexibility in the management of quantity risk. But option hedging is not cheap in that the hedger purchases the protection afforded by the options from the writer at a price called the option premium.

Studies Related to Price Distributions

Predicted values of economic variables serve as the primary inputs in decision making. For a hedger, the selection of a marketing strategy depends partially on his expectations about the course of prices and output harvested. This section is used to review studies related to the distributional properties of commodity prices.

The works by Mandelbrot (1963) and Fama (1965) show that the first differences of the logarithm of cotton and common stock prices generally have fatter tails than are compatible with the normal distribution. As a result, researchers
attempt to depict price changes with non-normal distributions. Mandelbrot (1963) and Fama (1965) suggest the stable Paretian family of distributions to characterize the stochastic properties of speculative prices. Praetz (1972) and Blattberg and Gonedes (1974) argue that for both stock prices and price indices, the scaled t-distribution has better descriptive validity. Clark (1973) proposes the lognormal-normal model for commodity futures prices. The general conclusion emerged from most of these studies is that price changes and rates of return appear not to be independent, but rather to be described by tranquil and volatile periods with variance changing over time and are well characterized by a unimodal symmetric distribution with fatter tails than the normal.

Engle (1982) introduces the Autoregressive Conditional Heteroskedastic (ARCH) model which allows the conditional variance to change over time as a function of past errors leaving the unconditional variance constant. The conditional variance, $h_t$, of a ARCH($q$) model is expressed as follows:

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \ldots + \alpha_q \epsilon_{t-q}^2.$$

where $\epsilon_t$ is the disturbance term of a time series. This model is in contrast to conventional time series and econometric models which operate under an assumption of constant variance. The ARCH model has been applied in modelling several different economic phenomena. Engle (1982), Engle (1983), and Engle and Kraft (1983) employ it to model
the inflation rate and recognized that the uncertainty of inflation tends to change over time. Engle et al. (1985) model the term structure of interest rates using an estimate of conditional variance as a proxy for the risk premium. The same idea is applied to the foreign exchange market in Domowitz and Hakkio (1985).

A more general class of processes, Generalized Autoregressive Conditional Heteroskedastic (GARCH), is introduced by Bollerslev in 1986. In the ARCH process the conditional variance is specified as a linear function of past sample variances only, whereas the GARCH process allows lagged conditional variances to enter as well. The conditional variance of a GARCH(p,q) model is shown in following:

\[ h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i} + \sum_{j=1}^{p} \beta_j h_{t-j}. \]

An advantage of the GARCH model, as indicated in Baillie and Myers (1989, p.2), is that very convenient assumptions about the conditional density of commodity price changes, such as the normal or t-distribution, can lead to rich models that allow for time-dependent conditional variances and fat-tailed or leptokurtosis in the unconditional distribution of price changes.

The GARCH model has proven useful in explaining the distribution of common stock prices, Bollerslev (1987), Bollerslev et al. (1988), and French et al. (1987); and also exchange rates, McCurdy and Morgan (1987), Milhoj (1987),
Diebold and Nerlove (1989), and Baillie and Bollerslev (1989). Aradhyula and Holt (1988) apply GARCH models to analyze the retail prices of beef, pork, and chicken. The estimation results indicate that the constant conditional variance assumption can be rejected. Baillie and Myers (1989) and Myers (1990) show that GARCH models are effective in describing the distribution of commodity cash and futures prices, and that they lead to a natural description of the optimal hedge ratio in commodity futures market. Brorsen and Yang (1989) also use the GARCH specification to model the distribution of futures returns. They concluded that the GARCH model explains much of the non-normality in futures price changes.

Studies Related to Optimal Hedging

There are many methods which have been used to model decision-making under uncertainty (Anderson, 1979). Despite various criticisms, the expected utility hypothesis and mean-variance analysis has been suggested by some as the appropriate ways to manage this problem.

The expected utility hypothesis was first put forth by Bernoulli in 1738, in the context of what is now known as the St. Petersburg Paradox. Its theoretical importance was not recognized until 1944 in the work of von Neumann and Morgenstern. Expected utility theory, which deals with decision-making under uncertainty, is based on the decision
maker's personal beliefs about the likelihood of uncertain outcomes and his personal valuation of the possible outcomes. If behavioral characteristics of the decision maker are consistent with the preference axioms of ordering, transitivity, continuity, and independence, then a utility function \( U(\cdot) \) exists to represent his preference ordering. Let \( f(x_i) \) be the probability distribution of uncertain outcomes associated with any choice \( a_i \), and \( U(x_i) \) be the utility index for each outcome, the expected utility for each action choice can be calculated as follows:

\[
E[U(a_i)] = \sum_{i} U(x_i) f(x_i)
\]

or

\[
E[U(a_i)] = \int_{x_1}^{x} U(x_i) f(x_i) dx_i
\]

for discrete and continuous distributions, respectively. If \( E[U(a_1)] > E[U(a_2)] \), then action \( a_1 \) is preferred by the decision maker to action choice \( a_2 \). The decision maker ranks all action choice by their expected utility index and chooses the action choice which provides the largest utility index. That is, the decision maker chooses the action choice which maximizes his expected utility.

It is generally accepted that most decision makers would prefer an action with a sure return to an action which provides a risky return \((\pi)\) when both actions provide the same expected return. This preference for avoidance of risk is known as risk aversion. Risk-averse behavior results when the
decision maker exhibits diminishing marginal utility for increases in expected return. The risk averter values a risky alternative at less than its expected monetary value. The difference between the expected monetary value and risk averter's value is the risk premium. The greater is the aversion to risk, the higher level of risk premium. Two frequently used measures of the risk aversion of a particular utility function are the coefficient of absolute risk aversion, \( A(\pi) \), and the coefficient of relative risk aversion, \( R(\pi) \). The coefficient of absolute risk aversion is defined as:

\[
(2.5) \quad A(\pi) = -\frac{U''(\pi)}{U'(\pi)}.
\]

This measure is not dimensionless and depends on the units in which income is measured, i.e., the coefficient of absolute risk aversion when income level is measured in U.S. dollars will be different from income levels measured in Japanese Yen. Thus the income level and units need to be considered when \( A(\pi) \) is applied. The coefficient of relative risk aversion is defined as:

\[
(2.6) \quad R(\pi) = -\pi \left[ \frac{U''(\pi)}{U'(\pi)} \right].
\]

It is also referred to as the elasticity of the marginal utility of income. Since \( R(\pi) \) is an elasticity, it is dimensionless but it is still valued at a particular level of income. The two measures are related in that

\[
(2.7) \quad R(\pi) = \pi A(\pi).
\]
There are several special utility functions used in decision analysis because of their tractability for particular types of problems and in some cases their desirable representation of risk preferences. The following is a list of the more frequently used utility functions and their characteristics.

1. The exponential utility function, \( U(\pi) = -\exp(-A\pi) \):
   This utility function exhibits constant absolute risk aversion. The level of relative risk aversion is an increasing function of the level of income.

2. The logarithmic utility function, \( U(\pi) = \log\pi \):
   This utility function is employed in the case of a decision maker who exhibits constant relative risk aversion. The level of absolute risk aversion is a decreasing function of income.

3. The quadratic utility function, \( U(\pi) = -(a-b\pi)^2 \):
   The expected utility can be expressed in terms of the first two moments of the risky attributes' distribution for a given action choice by using a quadratic utility function. Both absolute and relative risk aversion are increasing function of income.

4. The k\textsuperscript{th} order polynomial utility function:
   The expected utility can be represented by the first k moments of the underlying distribution of the utility function's attribute.

5. The power function, \( U(\pi) = \pi^x, \ 0<x<1 \):
The utility can be used to represent a decision maker who exhibits constant relative risk aversion and decreasing absolute risk aversion.

In empirical studies, the convention in choosing a specific utility function is to choose a utility function which is in a tractable form and reflects the general risk preference characteristics, given the problem at hand. Newberry and Stiglitz (1981, p.104) address the issue of choosing a particular utility function in the following:

"... and where it is convenient to explore the implications of a particular choice of utility function we shall variously assume constant absolute risk aversion (when the distribution between income and wealth is irrelevant)...

Hence, the negative exponential function is used to represent the preference attitudes of the decision maker.

Advocator of mean-variance analysis usually argue that expected utility is an approximation of the true unknown preference function of decision maker. In addition, actual estimates of the true probability distribution of income are difficult to obtain. The mean-variance framework focuses a decision maker's attitudes and the decision-making process on the first two moments of the underlying attribute's distribution, i.e., the mean and the variance. The popularity of the mean-variance model can be traced to the tractable theoretical results it produces and the computationally convenient empirical applications of the model. However,
Specifying an expected utility function in terms of the first two moments of the underlying attributes distribution has shown to be consistent with the expected utility hypothesis only if at least one of the following sufficient conditions are met:

1. The decision maker's utility function is quadratic;
2. The decision maker has a concave utility function and the random attribute is normally distributed;
3. The random attribute is a monotonic linear function of a single random variable (Hanson, 1988).

The straightforward nature of the mean-variance model results from some fairly restrictive assumptions that are violated when options are added to the model. The reason is that the actual income distribution is nonnormal when both futures and options are used as hedging and/or speculating tools by a decision maker. This can happen even though the end-of-period output is certain and the cash price follows a normal distribution. This is due to the income-truncating aspect that results from the inclusion of an option. The price distribution of an option is a truncated function of the underlying futures price and poses special problems in modeling.

In this study, the expected utility maximization model will be solved by numerical techniques. A mean-variance model is used to derive analytical solutions for optimal
marketing positions. No apparent analytical solutions can be derived from the expected utility maximization model because the expected utility level can not be expressed in a form that does not involve integrals.

Holthausen (1979) and Feder et al. (1980) employ general utility and density functions to initiate an extensive discussion of a risk-averse firm which uses futures contracts when faced with an uncertain output price but no basis risk. Their conclusions include independence of the production decision from the probability density of the cash price and the firm's degree of risk aversion. Antonovitz and Nelson (1988) present a general utility and density function framework to consider optimal production and marketing decisions when both forward and futures contracts are available to a risk-averse firm.

Wolf (1987) uses a generic specification of the mean vector and variance-covariance matrix of returns in a portfolio model containing inventory, futures, and options to explore the use of commodity options as risk management tools in incomplete markets. A major conclusion of Wolf's study is that changes in the expected net return on futures results in larger changes in the optimal option position as compared to the optimal futures position. The other finding is that changes in the expected net return on the option causes larger changes in the optimal option position relative to the optimal
futures position. Wolf also uses a simulation model with a hyperbolic absolute risk aversion utility function to derive the optimal market position. The results support his theoretical findings.

Hanson (1988) uses simulation experiments to analyze several questions regarding the inclusion of a put option into a hedger's portfolio. Hanson's model assumes a preharvest hedge where the individual investor has a fixed or random endowment of the cash commodity at the moment when the hedge is to be lifted. Futures and put options are used as the hedging instrument in the decision maker's portfolio. Hanson examines the consistency of mean-variance and expected utility results for the portfolio containing put options and finds that there are negligible differences between the optimal position implied by the mean-variance analysis and by expected utility maximization. Hanson also concludes that put options are of very little value to a hedger and will only be used as a speculative medium by measuring the amount of fixed monetary compensation required to provide a decision maker. The data set used by Hanson was generated using a partial factorial experimental design. An assumed maximum bias of four percent is used for deviations of market futures and option prices from their true values.

Bullock (1989) modifies the mean-variance portfolio model to investigate the informational role of option markets. The
modified model uses a statistical theorem, based on the definition of a conditional moment, to arrive at a mean vector and variance-covariance matrix of returns for the portfolio containing options. The determination of the optimal portfolio positions is found to be related to the interaction of information incorporated into the speculative and hedging components of the option and futures demand equations. When an individual investor had information only regarding the variance of prices, the "straddle" position is used to capitalize upon this information. Bullock uses actual price (spot, futures, and options) for corn, soybeans, live cattle, and live hogs.

Lapan et al. (1990) provide a further extension of the general expected utility model by allowing commodity options as a means of coping with price risk. Their work considers the simultaneous choice of a production level and of hedging levels of futures and options. The model allows for basis uncertainty, but the production process is assumed non-stochastic. The major finding is that when futures and option prices are unbiased optimal hedging requires only futures. Options are used together with futures as speculative tools when market prices are perceived as biased. They also conclude that mean-variance analysis in general is not consistent with expected utility when options are allowed.

Turvey and Baker (1990) use an expected utility-
maximizing farm-level mathematical programming model to investigate the relationships of farm's decisions to hedge with futures or options. Given the stochastic nature of the state variables used in this work, the results indicate that, on average, hedging occurs predominantly with options rather than futures. This finding is consistent with the liquidity arguments. Use of futures to hedge increases potential losses when futures prices are increasing, whereas the potential losses are limited by option use. Therefore, even though both futures and options provide cash flow when futures prices decrease, losses to profit potential are minimized when prices increase.

Schroeder and Featherstone (1990) examine optimal calf retention and marketing activities for cow-calf producers in the same fashion. The marketing alternatives include cash, hedging with futures, and a put option. The results of this study are based upon cattle price distributions and relationships present during the 1976 through 1988 period. Results show how calf retention decisions depend on current profit, expected future profit distributions, and the producer's aversion to risk. Option usage is the highest for low risk and middle risk-averse producers because they are more willing to pay the initial premium for the chance of higher profits. The more risk-averse producers substitute the less variable and lower initial cost hedges for options.
Turvey and Baker, and Schroeder and Featherstone generate distributions of cash, futures, and option prices in order to apply discrete stochastic programming model. However, these price distributions are not modelled by time-varying methods.

**Summary**

In this chapter a review of literature relating to the objectives of the present study is given. The first section reviewed the properties and trading strategies of futures and option contracts. The inclusion of commodity options in a risk management portfolio allows a decision maker to create new types of income distributions which were previously unavailable.

Since the selection of a hedging strategy depends on the expectation of the decision maker, the empirical commodity prices studies are reviewed. In the context of commodity price movements, evidence rejects the normal distribution and supports the more general time-series models that allow for time-dependent conditional variance. Studies show that the conditional heteroscedastic model is effective in describing the distributions of commodity cash and futures price movements.

Finally, methods and results of studies examining the optimal market positions are reviewed. The distribution of returns to an option position is truncated at the strike
price. The modeling of optimal marketing strategies becomes more complicated in the presence of these new types of income distributions. To derive the optimal hedging positions when option contracts are added to portfolio by mean-variance analysis could lead to erroneous conclusions and/or results. A few studies on optimal hedging have analyzed the effects of expected price distributions on the optimal positions. However, the use of a time-varying model in describing the distribution of cash, futures and option prices is not conducted in these works. Most studies on optimal hedging have been concerned under production certainty. After reviewing the existing works, the stage for the discussion of results analyzed in the present study can be set.
CHAPTER III.

PRICE DISTRIBUTIONS AND ESTIMATION RESULTS

This chapter begins with the properties of cash, futures and option prices. A proposed price distribution model is presented in the second section. The major properties of the conditional heteroskedasticity model and estimation results from fitting ARCH models to crop prices are presented in the third and the forth sections. The implications of their results for future research are discussed in the final section.

Properties of Cash, Futures, and Options Prices

The formation of cash prices

Agricultural prices observed through time are the result of a mixture of changes associated with seasonal, cyclical, trend, and irregular factors. The most common regularity observed in agricultural prices is a seasonal pattern of change. Normally, prices of storable commodities such as corn are the lowest at harvest time and then rise as the season progresses, reaching a peak prior to the next harvest time.

Storage is a form of investment in which the investor postpones selling a commodity in the hope of obtaining a higher price for it at a later date. During the time that the commodity is stored, costs are incurred which include the
interest on funds tied. If grain merchants correctly forecast future demands relative to supplies and hence store the correct quantity, the cash price will rise from a low point at harvest by cost of storage. Since the future cash price is unknown, the market will price the expectation of the cash price at time T to be equal to the current cash price plus cost of storage and interest in order to induce the individual to store the commodity. Rational individuals are generally assumed to be risk averse. Hence, a risk premium is essential to inspire someone to store a commodity. As such, under uncertainty and risk aversion, the cash price equals the expected future cash price minus the cost of storage, the interest rate, and the risk premium (Chance, 1987). The cash price at time t can be expressed as:

\[ C_t = E(C_T) - cs - r - \theta \]

where \( E(C_T) \) is expected cash price at time T, \( cs \) is the cost of storage, \( r \) is the interest rate, and \( \theta \) is the risk premium.

The cost of storage with interest is defined as cost of carrying. If storage creates a net cash outflow, the cost of carrying is positive. The cost of carrying is negative if interest is large enough to offset it.
Properties of futures prices

Futures prices are determined by auction in an open outcry market. The primary determinant of a futures price is the cash price of the underlying commodity. Other variables that affect futures prices include the cost of financing, insurance, transportation. Agricultural commodities are also affected by the old crop/new crop variable. Arbitrage normally keeps the futures price within certain boundaries around the cash price of the underlying commodity. In equilibrium, the futures price equals the cash price plus the cost of carrying. Therefore, the price at time \( t \) for a futures contract expired at time \( T \) can be written as:

\[
F_t = C_t + cs + r.
\]

The cash price of a commodity is highly correlated with its futures price. For storable commodities, the cost of carrying is normally positive. This would cause the futures price to lie above the cash price. In theory, the cash and futures price should be the same during the delivery period (Black, 1976). But in practice, differences tend to exist between the cash and futures price even during the delivery period. These differences are caused by such factors as the transaction costs associated with actually meeting the delivery specification of the futures contract, and demand and supply conditions in the delivery market (Hieronymous, 1971).

As to risk premium, there are two hypotheses proposed.
The risk premium hypothesis is justified by Keynes (1930) and Hicks (1939). They argue that futures prices are biased expectations of future cash prices, with the bias attributable to the risk premium. This hypothesis can be expressed as:

\[ E(C_t) = F_t + \theta = E(F_T) \]

The idea of the existence of a risk premium is that futures and cash markets are dominated by individuals who hold long or short positions in the underlying commodities. If most of individuals hold long cash positions and desire the protection afforded by selling futures contracts, they need traders willing to take a long position in futures. To induce speculators to take long positions in futures, the futures price must be below the expected price of contract at expiration, which is the expected future cash price. On the contrary, if hedgers are predominantly short in the commodity market, futures prices would be driven up in order to attract speculators to take short positions in futures. Hence, futures prices overestimate future cash prices. Speculators who sold futures would earn a risk premium.

The no-risk-premium hypothesis is discussed by Telser (1958) and Gray (1961). They assert that on average today's futures price equals the expected price of the futures contract at expiration, or \( F_t = E(F_T) \). Since the expected futures price at expiration equals the expected cash price at expiration, \( E(F_T) = E(C_T) \), the following result is obtained:
(3.4) \( F_t = E(C_T) \).

This statement says that the futures price is the market's expectation of the future cash price, or that futures prices are unbiased expectations of future cash prices. If one wishes to obtain a forecast of the future cash price, one need only observe the futures price according to this hypothesis.

The major differences of these two hypotheses lie on the nature of the cash market and the risk aversions of market participants. The no-risk-premium hypothesis assumes that there is no opportunity to take a position in the cash market and make money. Futures traders who are risk-neutral are simply competing with one another. The risk premium hypothesis assumes that individuals are risk averse and hold a long position in commodities. If these individuals are unhedged, they expect to earn a risk premium from the cash market. They can sell futures contracts to reduce price risk and purchase insurance from other futures traders (speculators). In so doing, they transfer the risk and the risk premium from cash markets into the futures markets.

Since the risk premium in cash prices is mostly certain, the risk premium is transferred to futures traders as long as hedgers hold cash positions. Thus, a risk premium is included in futures prices. However, if cash positions hedged are insufficient or if most hedging is done by traders holding
short cash positions, a risk premium may not be observed in futures prices. Chance (p.354-58) provides more detailed discussion about this topic.

Option valuation

Option-pricing theory has a long and illustrious history, but it underwent a revolutionary change in 1973. At that time, Fischer Black and Myron Scholes present the first completely satisfactory equilibrium option-pricing model. In the same year Robert Merton extends their model in several important ways. Later, in 1975, William Sharpe suggests an intuitively appealing way to simplify many of these developments. With the advent of options trading on organized exchanges, the field has become of prime interest to practitioners as well as academics. Many theoretical developments have been adopted and quickly implemented and, in some instances, new theoretical developments have been sparked by the financial markets' growing interest in futures and options on financial and physical commodities. Black (1976) derives formulas for the values of forward contracts and commodity options in terms of the future price and other variables using assumptions like those used in deriving the original option formula. This model has received wide-spread attention among professional traders, and is used by many of them as a benchmark for evaluations of options premia. In
addition, some of the commodities exchanges also use it to calculate margin requirements for floor traders.

The Black pricing model can be described as follows. In a frictionless economy, a straightforward arbitrage argument yields a theoretical formula for the value of European futures options. The value of a call option at any time is determined by the futures price (F), the strike price of the option (S), the riskless interest rate (r), the time to maturity of the option (T), and the future standard deviation of the percentage change in the futures price (σ). The price of a European call, EC, can be calculated:

\[
(3.5) \quad EC(F, S, r, T, σ) = e^{-rT}(FN(d_1) - SN(d_2))
\]

where

\[
d_1 = \frac{\log(F/S) + \sigma^2T/2}{\sigma\sqrt{T}}
\]

\[
d_2 = d_1 - \sigma\sqrt{T}
\]

\[
N(.)=\text{cumulative normal distribution.}
\]

The put option premium EP on a futures contract can be derived from the put/call parity, \((EC - EP) = (F - S)e^{-rT}\) and defined as:

\[
(3.6) \quad EP(F, S, r, T, σ) = -e^{-rT}(FN(-d_1) - SN(-d_2)).
\]

The effects of changes in valuation factors on options premiums can be shown by using the partial derivative technique and the results are presented in Table 3.1. These effects have intuitive interpretations: as futures price increases, the expected payoff of the call option increases. With a higher exercise price, the expected payoff decreases.
Table 3.1 Effects of changes in valuation factors on premiums

<table>
<thead>
<tr>
<th>Option type</th>
<th>Futures price</th>
<th>Exercise price</th>
<th>Level of interest</th>
<th>Time to expire</th>
<th>Volatility of futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>?</td>
<td>+</td>
</tr>
<tr>
<td>Put</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>+</td>
</tr>
</tbody>
</table>

An interest rate increase will reduce the value of the call option relative to its underlying futures contract. With a larger variance of the underlying futures price, the probability of a large price change in the futures during the life of the option is greater. Call options on futures with different expiration months have different underlying instruments because they call for purchase or sale of futures contracts expiring in different months. Thus, depending on the time series of futures prices, option prices may increase or decrease with time to expiration. A put option's price is inversely affected by changes in futures price, exercise price, and interest rate. However, volatility and time to expiration have the same relationship to put price as they do to call option prices (Schwarz et al., 1986).

All the variables of Black's pricing model are directly observable with the exception of the future standard deviation, which will be observed between the current date and the option expiration date. Unfortunately, this information is
not available so an estimated standard deviation must be used. The simplest way to estimate the future standard deviation is to take futures prices over the last $N$ days and calculate an annualized standard deviation. The other method is to use the implied volatility of the pricing model. The implied volatility is calculated using those readily observable variables plus the prevailing premium level:

\[(3.7) \sigma = f(\frac{EC}{EP}, F, S, r, T).\]

However, some studies have argued that since the options currently traded are American options, pricing them by relying on principles developed for European options can be misleading. The difference arises because the early exercise privilege of American futures options has a significant effect on prices.

Authors have presented various approaches to deal with this problem (Barone-Adesi and Whaley, 1985; Whaley, 1986; Gordon and Plato, 1986). However, empirical evidences provided by Jordan et al. (1987), and Wolf and Pohlman (1987) have supported that the Black model is better than variations in some cases.

**Price Distributions Modelling**

This section attempts to derive the cash, futures, and option price distributions by using a time-series model. The results will be used in simulation analysis when expected
utility maximization framework is applied. Agricultural production is usually characterized by a lag between the time the production decision is made and the time the output actually reaches the market. Hence, the actual cash price that will be received by producers is unknown when the production decision is made. Agricultural economists usually agree that economic decisions depend not only on the observable values of variables entering an econometric model, but also on an individual's expectations about futures values of those variables. This implies economic theory cannot be discussed without expectations formulation. Based on this consideration, this study uses price predictions instead of historical data which are commonly used in other works.

Economic agents are assumed to have a correct perception of market behavior. Therefore, they use all available information in making decisions.

Rewriting equations (3.1), the result can be expressed as

\[ C_t = E(C_t | I_{t-1}) + \text{bias} \]  

where \( I_{t-1} \) is a information set obtained at time \( t \), and bias can be cost of carrying, and a risk premium.

The equation (3.8) shows that the cash price at time \( t \) is the expected cash price at time \( T \) with some bias.

Rewriting equations (3.1) and (3.2), the result can be expressed as:

\[ F_t = E(C_t | I_{t-1}) + \text{bias} \]
where bias is a risk premium.

The equation (3.9) implies that the futures price at time \( t \) deviates from the expected cash price at maturity by bias. The information set is assumed to include only lagged prices, i.e. \( I_t=(C_{t-1},...,F_{t-1},...) \). Hence, the price distributions can be written as:

(3.11.1) \( C_t=f^C(C_{t-1},...,F_{t-1},...) + \epsilon_{ct} \)

(3.11.2) \( F_t=f^F(C_{t-1},...,F_{t-1},...) + \epsilon_{ft} \)

where \( \epsilon_t \) is random shock.

In the context of commodity price movements, evidence rejects normal distributions and supports the distributions with time-dependent conditional variance. The ARCH and GARCH models are employed in order to allow price variances to change. The random shock is modelled by

(3.12) ARCH(q) model: \( E(\epsilon^2_t|X_t) = w_0 + \sum_{i=1}^{q} w_i \epsilon^2_{t-i} \) or

(3.13) GARCH(p,q) model: \( E(\epsilon^2_t|X_t) = h_t \)

\[ = w_0 + \sum_{i=1}^{q} w_i \epsilon^2_{t-i} + \sum_{j=1}^{p} b_j h_{t-j}. \]

The relationship between cash and futures prices is obvious. Empirically and theoretically, cash and futures prices move in the same direction.

The Black pricing model of a put option premium can be simplified as:

(3.14) \( BP_t = S - E(F_t|I_{t-1}) + \text{bias} \)

where bias is the time value.

For a given futures contract, there are many option contracts
traded at different strike prices. To study the relationships between cash, futures, and option prices, the observations on option prices should reflect the sum of time value and intrinsic value. Therefore, a specific option contract must be chosen according to strike prices. The preliminary results of fitting ARCH model to option prices show that ARCH model does not perform well. Since this study focuses on the expected prices at the end of period, the expected end-of-period option price is defined by intrinsic value at expiration. Option contracts expire before underlying futures contracts delivery. The maturity date of the December futures is during the third week of December and option contracts based on December futures expire in middle November when is the harvest time in present study. Therefore, it is reasonable to assume zero time value for option contracts.

**The ARCH(q)/GARCH(p,q) Process**

Let $\epsilon_t$ denote a real valued discrete-time stochastic process and $I_{t-1}$ the set of all information available through time period $t-1$. The ARCH(q) process for a normal conditional distribution is given by:

\begin{align*}
(3.15) \quad y_t | I_{t-1} &\sim N(x_{t-1}'b, h_t) \\
(3.16) \quad h_t &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \ldots + \alpha_q \epsilon_{t-q}^2.
\end{align*}

where $y_t$ is the dependent variable, $x_{t-1}$ is a vector of explanatory variables including past realizations of...
\( y_t \), and \( b \) is a vector of unknown parameters to be estimated.

The simplest and often very useful ARCH(q) model is the first-order linear model, in which case (3.16) becomes:

(3.17) \[ h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2. \]

If \( \alpha_1 = 0 \), \( y_t \) will be Gaussian white noise. If \( \alpha_1 \) is a positive number, successive observations will be dependent through higher-order moments. The condition for the variance to be finite is that \( \alpha_1 < 1 \). In general, the stationary variance for the ARCH(q) process is

(3.18) \[ \mathbb{E}(y_t^2) = \alpha_0 / (1 - \sum_{i=1}^{q} \alpha_i) \text{ with } \alpha_0 > 0, \alpha_1, \ldots, \alpha_q \geq 0. \]

The conditional variance equation for the GARCH(p,q) process is obtained by extending equation (3.16) to

(3.19) \[ h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \]

\[ = \alpha_0 + \alpha(L) \varepsilon_t^2 + \beta(L) h_t \]

where \( p \geq 0, q \geq 0 \),

\[ \alpha_0 > 0, \alpha_i \geq 0 \quad i=1, \ldots, q, \]

\[ \beta_j \geq 0 \quad j=1, \ldots, p, \]

and \( \alpha(L) \) and \( \beta(L) \) are the lag operators of order \( q \) and \( p \), respectively.

Bollerslev (1986) shows that \( \varepsilon_t \) is covariance stationary with \( \mathbb{E}(\varepsilon_t) = 0, \text{var}(\varepsilon_t) = \alpha_0 (1 - \alpha(1) - \beta(1))^{-1} \) and \( \text{cov}(\varepsilon_t, \varepsilon_s) = 0 \) for \( t \neq s \) if and only if \( \alpha(1) + \beta(1) < 1 \), or \( \alpha_1 + \beta_1 < 1 \).

The LS (Least Squares) estimator of \( b \) is consistent as \( x_{t-1} \) and \( \varepsilon_t \) are uncorrelated. However, if there are lagged
dependent variables in \( x \), the standard errors as conventionally computed will not be consistent, since the squares of the disturbances will be correlated with squares of the \( x \)'s. The ML (Maximum Likelihood) estimator is more efficient. Let \( L \) be the average log likelihood function, and \( T \) the sample size, then

\[
L = T^{-1} \sum_{t=1}^{T} (-0.5 \log(h_t) - 0.5 \epsilon_t^2 h_t^{-1})
\]

apart from constant terms.

The first and second derivatives of the log likelihood function in (3.20) with respect to the unknown parameters \( \alpha \) and \( \beta \) are outlined in Engle (1982, pp.995-6). The same approach has been applied to derive estimators of the GARCH process by Bollerslev (1986, pp. 315-6). Weiss (1986) proved asymptotic normality of quasi-maximum likelihood estimators in the ARCH process by assuming a finite fourth moment, whereas Lumsdaine (1990) showed that asymptotic normality in the GARCH process can be obtained without a finite fourth moment condition.

To test whether the disturbances follow an ARCH\((q)\) process, the Lagrange Multiplier procedure is proposed by Engle (1982). The test procedure is to run an OLS regression and save residuals. The LM statistic is computed as the number of observations times the \( R^2 \) of the regression of the squared residuals on a constant and \( q \) lags under the null hypothesis \( \alpha_1 = \ldots = \alpha_q = 0 \). The statistic will be asymptotically
distributed as chi-square with q degrees of freedom when the null hypothesis is true. Bollerslev (1986) suggests that autocorrelation and partial autocorrelation functions as applied to the squared residual series can be useful for identifying and checking the time-series behavior of the conditional variance equation of the GARCH model. He argues that the absence of serial correlation in the conditional first moments, coupled with the presence of serial correlation in the conditional second moments, is one of the implications of the GARCH process.

To accommodate the correlations among cash and futures prices, the variance-covariance (VC) matrix is essential. The price equations can be rewritten as: \( C_t - f^C(\cdot) = \varepsilon_t \) and \( F_t - f^F(\cdot) = \varepsilon_{ft} \). Let \( R_t = (\varepsilon_{At}, \varepsilon_{ft})' \), then the prediction errors have a time-varying VC matrix:

\[
(3.21) \quad H_t = E(R_tR_t' | X_t).
\]

The VC matrix is modelled through the bivariate ARCH(q) and GARCH(p,q) models,

\[
(3.22) \quad \text{vech}(H_t) = C + \sum_{i=1}^{q} A_i \text{vech}(R_{t-i}R_{t-i}')
\]

\[
(3.23) \quad \text{vech}(H_t) = C + \sum_{i=1}^{q} A_i \text{vech}(R_{t-i}R_{t-i}') + \sum_{j=1}^{p} B_j \text{vech}(H_{t-j})
\]

where \( C \) is a 6x1 vector of parameters; the \( A_i \)'s are 6x6 matrices of parameters for \( i=1,2,\ldots,q \); the \( B_j \)'s are 6x6 matrices of parameters for \( j=1,2,\ldots,p \); and vech is the column stacking operator that stacks the lower triangular portion of a symmetric matrix.
This study uses univariate ARCH/GARCH models to analyze distributions of cash and futures prices. Preliminary estimation of bivariate GARCH/ARCH model suffers from problems relating to convergence.

**Estimation Results**

The price data used in the empirical analysis are local cash prices on corn for North-Central Iowa and futures prices for corn trading on CBT. The contract month of corn futures includes March, May, July, September, and December. The cash prices are based on weekly observations on each Thursday and are averaged between the high and low price quotes. Futures price data are weekly observations taken at the Thursday close of trading. The estimation period runs from the first Thursday of 1985 through the last Thursday of 1990. All price series are obtained from the extension service at Iowa State University.

ML estimates of the model parameters are obtained by using the HET command in SHAZAM version 6.2.

OLS estimation results for the cash and futures prices of corn, along with $R^2$s, sample MAPEs (mean absolute percent errors) and Durbin $h$-statistics are presented in Table 3.2. The fits are quite good, with higher than 0.8 $R^2$s and all MAPEs less than 3.5%. The values of the first lagged dependent variable suggest that one-period ahead prices
Table 3.2. Estimation of OLS models for cash and futures price of corn

<table>
<thead>
<tr>
<th>Cash prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ P_{ct} = 0.207 + 0.946 P_{ct-1} - 0.043 P_{ct-2} ]</td>
</tr>
<tr>
<td>[ h_t = 0.024 ]</td>
</tr>
<tr>
<td>R^2 = 0.88</td>
</tr>
<tr>
<td>MAPE = 3.50</td>
</tr>
<tr>
<td>Dh = 53.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>March contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ P_{3t} = 0.325 + 0.793 P_{3t-1} + 0.072 P_{3t-2} ]</td>
</tr>
<tr>
<td>[ h_t = 0.031 ]</td>
</tr>
<tr>
<td>R^2 = 0.82</td>
</tr>
<tr>
<td>MAPE = 3.32</td>
</tr>
<tr>
<td>Dh = 51.56</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>May contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ P_{st} = 0.275 + 0.556 P_{st-1} + 0.684 P_{ct} - 0.306 P_{ct-2} ]</td>
</tr>
<tr>
<td>[ h_t = 0.010 ]</td>
</tr>
<tr>
<td>R^2 = 0.94</td>
</tr>
<tr>
<td>MAPE = 2.60</td>
</tr>
<tr>
<td>Dh = 6.16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>July contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ P_{7t} = 0.081 + 0.00006 T + 0.843 P_{7t-1} + 0.037 P_{3t-1} + 1.027 P_{ct} - 0.931 P_{ct-1} ]</td>
</tr>
<tr>
<td>[ h_t = 0.002 ]</td>
</tr>
<tr>
<td>R^2 = 0.99</td>
</tr>
<tr>
<td>MAPE = 1.45</td>
</tr>
<tr>
<td>Dh = 1.85</td>
</tr>
</tbody>
</table>

Note: \[ h_t = \text{var}(P_t) \]. MAPE = T^{-1} \sum |C_t/P_t| \times 100\% (Pankratz, 1983). Figures in parentheses are standard errors. Dh is the Durbin h-statistic. The critical value of a test for positive serial correlation at the 5% level of significance is 1.645. All prices are in dollars per bushel.
Table 3.2. (continued)

**September contract**

\[ P_{st} = 0.327 + 0.529 P_{st-1} - 0.071 P_{st-2} + 0.450 P_{st} \]

\[
\begin{align*}
\text{ht} &= 0.016 \\
&\quad (0.0013)
\end{align*}
\]

\[ R^2 = 0.90 \]

\[ \text{MAPE} = 3.48 \]

\[ \text{Dh} = 24.66 \]

**December contract**

\[ P_{12t} = 0.128 + 0.169 P_{12t-1} + 0.715 P_{12t-2} + 0.882 P_{ct} - \\
0.815 P_{st-2} \]

\[
\begin{align*}
\text{ht} &= 0.006 \\
&\quad (0.0005)
\end{align*}
\]

\[ R^2 = 0.97 \]

\[ \text{MAPE} = 2.38 \]

\[ \text{Dh} = 11.33 \]
explain much of the behavior in price movements. If the disturbance term is white noise, the OLS estimators have very desirable properties (Harvey, 1981). However, the results of Durbin h-statistic indicate that the assumption of white noise disturbances is rejected (Durbin, 1970).

The results of LM tests for the ARCH effects are presented in Table 3.3. The LM test for the first-order ARCH effect for the futures prices of March contract is not significant. However, testing for the second-order ARCH process, the Chi-square statistic with two degrees of freedom is 14.94, indicating highly significant.

The results of LM test for the higher-order ARCH effects are significant in the case of other contracts, but the stationarity conditions for the conditional variance equations are not satisfied. GARCH(1,1) and GARCH(2,1) processes are applied to all price series as they are parsimonious and are often the most likely candidates in applied analysis. The results showed that no price series can be explained well by a GARCH process. The ARCH results are presented in later tables.

The ML estimates of the ARCH regression models for the cash and futures price series of corn are reported in Table 3.4. The stationarity conditions and nonnegativity requirements for the estimated parameters in the conditional variance equations are satisfied in each instance. Checks of
Table 3.3. Results of Lagrange Multiplier Tests for cash and futures prices of corn

<table>
<thead>
<tr>
<th>LM test statistic</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash prices</td>
<td>76.40</td>
</tr>
<tr>
<td>March contract</td>
<td>1.74</td>
</tr>
<tr>
<td></td>
<td>14.94</td>
</tr>
<tr>
<td>May contract</td>
<td>149.46</td>
</tr>
<tr>
<td>July contract</td>
<td>11.13</td>
</tr>
<tr>
<td>September contract</td>
<td>9.64</td>
</tr>
<tr>
<td>December contract</td>
<td>28.28</td>
</tr>
</tbody>
</table>

Note: Chi-square distribution at one degree of freedom and 5% (1%) level of significance is 3.84 (6.64) and at two degrees of freedom and 5% (1%) level of significance is 5.99 (9.21).
Table 3.4. MLE estimation of ARCH models for cash and futures prices of corn

<table>
<thead>
<tr>
<th>Cash prices</th>
<th>R²=0.87</th>
<th>MAPE=2.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pₜ=cₜ= 0.018 + 1.110 Pₜ₋₁ - 0.118 Pₜ₋₂</td>
<td>(0.015) (0.057)</td>
<td>(0.056)</td>
</tr>
<tr>
<td>hₜ= 0.002 + 0.501 ε²ₜ₋₁ + 0.115 ε²ₜ₋₂</td>
<td>(0.0003) (0.116)</td>
<td>(0.056)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>March contract</th>
<th>R²=0.80</th>
<th>MAPE=2.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pₜₚₜ= 0.037 + 0.990 Pₜ₋₁ - 0.006 Pₜ₋₂</td>
<td>(0.030) (0.078)</td>
<td>(0.078)</td>
</tr>
<tr>
<td>hₜ= 0.008 + 0.165 ε²ₜ₋₁</td>
<td>(0.0007) (0.082)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>May contract</th>
<th>R²=0.89</th>
<th>MAPE=2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pₜₚₖₜ= 0.062 + 0.930 Pₜ₋₁ + 0.279 Pₜ - 0.231 Pₜ₋₂</td>
<td>(0.018) (0.020)</td>
<td>(0.030) (0.028)</td>
</tr>
<tr>
<td>hₜ= 0.002 + 0.537 ε²ₜ₋₁</td>
<td>(0.0002) (0.132)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>July contract</th>
<th>R²=0.97</th>
<th>MAPE=1.40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pₜₖₜ= 0.075 + 0.00006 T + 0.801 Pₜ₋₁ + 0.100 Pₜ₋₁ + 0.715 Pₜ - 0.638 Pₜ₋₁</td>
<td>(0.014) (0.00002) (0.024) (0.015) (0.025)</td>
<td></td>
</tr>
<tr>
<td>hₜ= 0.001 + 0.489 ε²ₜ₋₁</td>
<td>(0.0001) (0.123)</td>
<td></td>
</tr>
</tbody>
</table>

Note: MAPE=T⁻¹Σ|εₜ/Pₜ|×100% (Pankratz, 1983). Figures in parentheses are standard errors. The condition for an ARCH(1) model to have a finite fourth-order moment is α₁ < 3⁻¹/₂ and 3(α₁⁺α₂²α₂⁺α₂²⁻α₂⁴)/1-α₂ <1 for an ARCH(2) model (Milhoj, 1985). All prices are in dollars per bushel.
Table 3.4. (continued)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>September contract</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{gt} = 0.032 + 0.961 P_{gt-1} - 0.007 P_{gt-2} + 0.036 P_{ct}$</td>
<td>$P_{gt} = 0.032 + 0.961 P_{gt-1} - 0.007 P_{gt-2} + 0.036 P_{ct}$</td>
<td>$P_{gt} = 0.032 + 0.961 P_{gt-1} - 0.007 P_{gt-2} + 0.036 P_{ct}$</td>
<td>$P_{gt} = 0.032 + 0.961 P_{gt-1} - 0.007 P_{gt-2} + 0.036 P_{ct}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_t = 0.002 + 0.572 \epsilon^2_{t-1}$</td>
<td>$h_t = 0.002 + 0.572 \epsilon^2_{t-1}$</td>
<td>$h_t = 0.002 + 0.572 \epsilon^2_{t-1}$</td>
<td>$h_t = 0.002 + 0.572 \epsilon^2_{t-1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2 = 0.82$</td>
<td>$R^2 = 0.82$</td>
<td>$R^2 = 0.82$</td>
<td>$R^2 = 0.82$</td>
</tr>
<tr>
<td>MAPE=2.37</td>
<td>MAPE=2.37</td>
<td>MAPE=2.37</td>
<td>MAPE=2.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>December contract</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{12t} = 0.089 + 0.560 P_{12t-1} + 0.379 P_{12t-2} + 0.435 P_{ct}$</td>
<td>$P_{12t} = 0.089 + 0.560 P_{12t-1} + 0.379 P_{12t-2} + 0.435 P_{ct}$</td>
<td>$P_{12t} = 0.089 + 0.560 P_{12t-1} + 0.379 P_{12t-2} + 0.435 P_{ct}$</td>
<td>$P_{12t} = 0.089 + 0.560 P_{12t-1} + 0.379 P_{12t-2} + 0.435 P_{ct}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_t = 0.002 + 0.368 \epsilon^2_{t-1}$</td>
<td>$h_t = 0.002 + 0.368 \epsilon^2_{t-1}$</td>
<td>$h_t = 0.002 + 0.368 \epsilon^2_{t-1}$</td>
<td>$h_t = 0.002 + 0.368 \epsilon^2_{t-1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2 = 0.93$</td>
<td>$R^2 = 0.93$</td>
<td>$R^2 = 0.93$</td>
<td>$R^2 = 0.93$</td>
</tr>
<tr>
<td>MAPE=2.09</td>
<td>MAPE=2.09</td>
<td>MAPE=2.09</td>
<td>MAPE=2.09</td>
</tr>
</tbody>
</table>
the estimated ARCH parameters show that the fourth-order moment condition is satisfied. Hence, following Weiss (1986), the asymptotic properties of ML estimates are established for the cash and futures prices of corn.

Generally, the standard errors of the estimates of the parameters in the OLS equations are reduced by the inclusion of ARCH assumption. The standard errors of OLS estimates are 3% to 63% greater than the standard errors of ARCH model estimates, with some exceptions where the standard errors actually falls by 2% to 116%. As mentioned earlier, however, the least squared estimates are biased when there are lagged dependent variables. Therefore ML estimation of the regression models with ARCH errors may give more efficient estimates and information about the conditional variance as well as the conditional mean.

The reported $R^2$s and MAPEs in Table 3.4 indicate that the estimated parameters associated with the conditional means of the estimated ARCH models do a good job of explaining historical movements. Moreover, ARCH models provide more information about the precision of the forecast performance of the means of the stochastic process. That is, there is a tendency for large and small forecast errors to cluster together.

To show the characterization of changing variance processes, confidence intervals for the one-step-ahead within
sample forecasts derived from the ARCH(1) model are computed for cash prices and futures prices of December contract of corn. Engle and Bollerslev (1986) have presented that the one-step ahead forecast of the conditional mean and conditional variance of $y_{t+1}$, evaluated at time $t$, can be expressed as:

(3.24) $E(y_{t+1} | I_t) = x_t' b$

(3.25) $V(y_{t+1} | I_t) = \alpha_0 + \alpha_1 (y_t - x_t' b)^2$.

The 99% confidence intervals for OLS and ARCH model using cash prices of corn, along with the actual price series of 1989-1990, are presented in Figure 3.1 and Figure 3.2. The results in Figure 3.2 show that the confidence intervals are much smaller during the period of relatively stable prices. The finding supports the idea that the forecast variance may change over time and can be predicted by past forecast errors. Standard time-series models do not give such intuitively appealing results because the width of the confidence interval remains constant as shown in Figure 3.1. Similar plots for the OLS and ARCH models using futures prices of December contract are illustrated in Figure 3.3 and 3.4, respectively. The December contract is used in the expected utility maximization model. As with cash prices, the forecast intervals for futures prices are widest during May through August in 1989 and same months of 1990 and are relatively stable during the rest of period.
Figure 3.1. 99% Confidence intervals of OLS, cash price of corn
Figure 3.2. 99% Confidence intervals of ARCH(1), cash price of corn
Figure 3.3. 99% Confidence intervals of OLS, futures price of December contract
Figure 3.4. 99% Confidence intervals of ARCH(1), futures price of December contract.
Conclusions

For time-series analysis, the ARCH model proposed by Engle (1982), offers one interesting characterization of changing variance processes. In ARCH the conditional variance at each point in time is assumed to depend on the currently available information set. Hence, it is endogenous in the sense that the current variance is a random variable generated by the past realizations of the series.

In this study, the ARCH process is applied to corn prices. The estimated model replicates historical movements in these price series adequately. Confidence intervals derived from the conditional forecast variances change substantially over the sample period as illustrated in Figure 3.2 and Figure 3.4. As discovered in this study, crop price volatility changes as markets move through cycles of high and low uncertainty about future economic conditions. Hence, care should be taken in modelling the conditional variances for time-series data. The normality assumption associated with the conditional distribution can be replaced with other distributions such as t-distribution.

The results of this study show that recent advances in the econometrics literature may be successfully applied to agricultural data. A wider application of conditional heteroscedasticity process (ARCH/GARCH) could be beneficial to future research in agricultural area.
This chapter presents an expected utility maximization model which will be applied to study optimal risk management strategy for a representative corn producer facing price and quantity uncertainties. The decision maker is allowed to use both futures and option contracts to hedge against the risk associated with his cash position. The first section discusses the income distribution which enters the producer's utility function. As discussed in Chapter II, the income distribution requires special attentions because it is truncated in the case where options are included in the model. The second section presents the expected utility maximization model, followed by a discussion of the method of measuring the value of futures and options to the producer which is similar to the concept of "value of information."

Income Distribution

Consider a two-period production/marketing model where the producer plants his crops in period 0 and market his outputs in period 1. The producer maximizes his expected utility of the end-of-period income at the beginning period by choosing futures and options position. The purpose of using futures and option contracts is to provide added income in the
event that the end-of-period cash prices should turn out to be unfavorable. In choosing his options position, the producer is assumed to consider only the put options because a "synthetic call" can always be constructed through using futures and puts together (see Appendix B for more details). The producer holds his futures and options positions until period 1 when the positions have to be closed. For simplicity, the model assumes no margin calls, commission fees, and other transaction costs.

Total output is defined as yield per acre times a fixed planted acreage. The random variables in the model include the end-of-period cash price, end-of-period futures price, end-of-period option price, and yield per acre. Total production cost, defined as a constant cost per acre times the planted acreage, is assumed to be known. Conditional on his information set at the initial time, the producer forms his subjective probability distributions for the above random variables on which the maximization process is based. The random end-of-period income, $\tilde{\pi}_1$, can be written as:

\begin{equation}
\tilde{\pi}_1 = \tilde{C}_1 \tilde{y} - PCH + (\tilde{F}_1 - F_0) R + (\tilde{P}_1 - P_0) Z
\end{equation}

where

- $H$: cropland in acres,
- $\tilde{y}$: random yield per acre,
- $\tilde{C}_1$: random cash price at period 1,
- $PC$: production cost per acre,
F_0: futures price at period 0,
F_1: random futures price at period 1,
R: the futures quantity, >0 bought, <0 sold,
P_0: put premium,
F_1: random terminal put price,
Z: the put option quantity, >0 bought, <0 sold.

Depending on whether the put option is in-the-money or
out-of-the-money at maturity, the end-of-period income
distribution differs. If the put option is in-the-money at
maturity, the producer exercises the put and buys back the
existing futures position at the prevailing market price. In
this case, equation (4.1) can be written as:

\[ (4.2.1) \pi_1 = \bar{C}_1 \bar{H}Y - PCH + \left( \bar{F}_1 - F_0 \right) R + \left( S - \bar{F}_1 - P_0 \right) Z \quad \text{if } F_1 < S \]

where \( F_1 \) is a realization of the \( \bar{F}_1 \), and \( S \) is the
strike price of the put options.

On the other hand, if the put option is out-of-the-money at
maturity, the producer simply let the option expire and
equation (4.1) becomes:

\[ (4.2.2) \pi_1 = \bar{C}_1 \bar{H}Y - PCH + \left( \bar{F}_1 - F_0 \right) R - P_0 Z \quad \text{if } F_1 \geq S \]

In comparing (4.2.1) and (4.2.2) with (4.1), it is clear that
the end-of-period option price is simply the intrinsic value
of the option at the time and can be derived from the end-of-
period futures price according to: \( \bar{F}_1 - \text{MAX}\{0, S - \bar{F}_1\} \). This is
intuitive because the option expired at (or near) the end of
period and, hence, its time value is zero (or negligible).

In the empirical analysis, the producer is assumed to choose the strike price which renders the put option at-the-money at the time of purchasing. Presumably, the choice of strike price can be made endogenous. However, this complicates the analysis and data requirements. As the option is at-the-money at the purchasing time, \( S = F_0 \), in the case of price decline as depicted by (4.2.1) the losses in the futures market will be exactly offset by the gains in option market. On the other hand, in the case of rising futures prices as depicted by (4.2.2), the losses in the option premium will be exactly offset by the gains in futures market.

If there is no basis risk (i.e. \( C_t = F_t \)), the only price uncertainty faced by the producer is the cash price and hence, the distribution of income can be derived from the distribution of cash prices. Hanson has discussed a variety of income distributions for different positions in the futures and option markets and concludes that the distribution of income becomes rather complex when both futures and options positions are taken simultaneously. If the decision maker participates only in the futures market and the underlying cash price is normally distributed, the assumption of income normality may be acceptable. However, in the case where options are included, the set of income distributions available to the agent is no longer normal distributions.
**Expected Utility Maximization Model**

The producer is assumed to face the income function (4.1) and have a von Newmann-Morgenstern utility function, $U$, defined over the end-of-period income. The utility function is assumed to be increasing, strictly concave and twice differentiable. Given this setup, the producer chooses $(R,Z)$ to maximize the expected utility:

$$E[U(\tilde{\pi})]=\int\int\int U(C,F,P,y)f(C,F,P,y)dCdFdPdy.$$  

The first order conditions can be expressed as:

$$E[U'(\tilde{F}_1-P_0)]=\int\int\int U'(\tilde{F}_1-P_0)f(C,F,P,y)dCdFdPdy=0$$

$$E[\Pi'(\tilde{P}_1-P_0)]=\int\int\int \Pi'(\tilde{P}_1-P_0)f(C,F,P,y)dCdFdPdy=0$$

where $U'=dU/d\pi$.

The second order conditions are satisfied because of the strictly concave assumption. The optimal market positions $(R,Z)$ can be obtained by solving the first order conditions. There are no apparent analytical solutions to equations (4.4.1) and (4.4.2) because they involves taking integral of the derivative of the expected utility function. Consequently, solution techniques involving numerical simulation-optimization procedures are needed. The expected utility function for the producer is specified as a negative exponential function:

$$U(\tilde{\pi}) = - \exp (-\lambda \tilde{\pi})$$

where $\lambda$ is the Arrow-Pratt coefficient of absolute risk aversion.
The simulation-optimization procedure used to solve for the optimal futures and options positions includes the following steps:

Step 1: Stochastic simulation procedure

The purpose of the stochastic simulation is to randomly draw a desired number (N) of states of nature for each random variable, based on their estimated distributions. Since all the N states of nature for each random variable are drawn from the estimated distribution, they have equal probability (1/N) of being realized. A more detailed discussion on the stochastic simulation procedure can be found in Chapter V.

Step 2: Numerical integration

Compute the end-of-period income under each state of nature using (4.1) and then integrate over all states of nature with equal probability to arrive at a discrete representation of the negative exponential expected utility function:

$$E[U(R,Z)] = \sum_{i=1}^{N} w_i (-\exp(-\lambda \pi_{1i}))$$

where $\pi_{1i}$ is the end-of-period income under state i, and $w_i$ is the probability of state of nature i ($w_i = 1/N$, for all i).

Step 3: Numerical optimization

Maximize the objective function in step 2 with respect
to the decision variables \((R,Z)\) using a nonlinear numerical optimization routine. For more details, see Appendix C for the GAMS file used in the empirical analysis.

In solving the optimal solutions, three scenarios will be considered: 1) the producer is allowed to use only the futures; 2) the producer is allowed to use only options; 3) the producer is allowed to use both futures and options as risk management tools. By comparing results among different scenario, insights toward the optimal decision-making structure can be obtained. Sensitivity analysis on the optimal solution will also be conducted with respect to a change in model parameters such as the distributions of random variables, the strike price, the size of farm (acreage of planted), and the producer's risk aversion coefficient.

A Measure of the Value of Futures and Options Markets

The optimal market positions under different scenarios can be directly compared. However, the value of risk management tools to the producer is difficult to determined. The introduction of an option will never lower the value of the decision maker's portfolio by an application of the LeChatelier Principle. Options add value to a portfolio through an expanded set of possible return structure. However, Hanson concludes, based on his analytical results and
simulation findings, that options have little value added to the decision maker who already uses futures. He also finds that the primary factors determining the value of options are the end-of-period output, the variance of the cash price, and the level of risk aversion. On the other hand, Bullock shows that an option adds statistically significant value to the decision maker even when futures market is already in use and concludes that information about the mean of the end-of-period cash price appears to have a major effect on the value of an option.

This section is devoted to discuss the procedure that will be used to determine the value of futures and options to the decision maker. The measure is constructed by using a certainty equivalent concept of the theoretical model of Antonovitz and Roe (1986). Comparative static results are not derived in this section but will be presented in empirical Chapter.

Following Antonovitz and Roe, a money metric for access value $V$ can be constructed as:

\[
E[U(\pi_b^*)] = E[U(\pi_a^* + V)]
\]

where $\pi_b^*$ is the ex ante profit from holding the optimal futures and options positions; $\pi_a^*$ is the ex ante profit from holding only the optimal futures position; and $V$ is the access value of options added to the producer.
A certainty equivalent (CE) is defined as the risk-free level of income that is equivalent in expected utility to a risky lottery. Using CE, equation (4.6) can be rewritten as

\[(4.7) \quad CE'_b = CE'_a + V.\]

where \(CE'_a\) is the certainty equivalent under optimization.

Since the utility function of the decision maker is represented by a negative exponential function, the CE in (4.7) can be expressed by the following Taylor series expansion:

\[(4.8) \quad CE = E[\pi] - (\lambda/2)E[(\pi - E(\pi))^2] + (\lambda^2/6)E[(\pi - E(\pi))^3] + \ldots\]

For practical purposes, it is important to determine at what point to cut off the Taylor series as an approximation to the CE. Models that consider only the first two terms of the expansion in (4.8) are able to provide less ambiguous comparative static results. Bullock and Hayes (1992) have used the second-order CE to study the private value of accessing to options market.

Since the inclusion of options into the maximization model skews the income distribution, higher-order CE should also be considered. Both the second-order CE and the third-order CE will be computed to determine the best approximation of the CE for different scenarios.
CHAPTER V.

EMPIRICAL OPTIMIZATION RESULTS

In the expected utility maximization model, the producer is assumed to own 400 acres of land on which he plants corn. He plants his crops on the first week of April and harvests them in early November. There are 32 weeks lie between the time of planting and harvesting. The farmer has to make a decision as how to hedge the risk associated with his cash position by choosing December futures contracts and/or at-the-money put options on futures. The producer holds his hedging positions for 32 weeks till the crops are harvested.

The remainder of this chapter is divided into four sections. The first section provides a detailed description of the stochastic simulation procedure which is necessary to operationalize the negative exponential expected utility maximization problem discussed in Chapter IV. The optimization results under various scenarios regarding whether the model entertains futures and/or options as risk management tools are presented in the second section. The third section reports the sensitivity analysis which assesses the impacts of model parameter such as farm size, risk attitudes, price levels, and price variances on the optimal solution. The fourth section presents the access values of futures and options to the corn producer.
Stochastic Simulation Procedure

As discussed in Chapter IV, the maximization of the expected utility function involves a simulation of random variables based on the estimated equations. The stochastic simulation procedure is used to generate a set of the end-of-period prices and yield outcomes which are viewed as discrete representations of true random variables. Once the end-of-period distributions are obtained, numerical integration and optimization techniques can be employed to solve the negative exponential expected utility maximization problem.

Simulation of cash, futures, and options prices

The estimated ARCH price equations are used to simulate the price distributions for the end-of-period cash and December futures prices. In Chapter III, the price equations were estimated using the whole sample (January 1985 through December 1990). In optimization model, the price equations are re-estimated using data only up to March 1990 since the time of decision making is on the first week of April 1990. The re-estimated ARCH equations for corn cash and December futures prices are:

(5.1.1) \( P_{ct} = 0.022 + 1.185 P_{ct-1} - 0.196 P_{ct-2} + \epsilon_{ct} \)

(5.1.2) \( h_{ct} = 0.003 + 0.542 \epsilon_{ct-1}^2 \)

(5.1.3) \( P_{ft} = 0.084 + 0.615 P_{ft-1} + 0.335 P_{ft-2} + 0.375 \hat{P}_{ct} - 0.359 P_{te-2} + \epsilon_{ft} \)
The shock for the simulation procedure is based on the unconditional variance. The relationship between $\epsilon_t$ (the disturbance term) and $h_t$ (conditional variance) is

$$\epsilon_t = h_t^{1/2}e_t$$

where $e_t$ is a fundamental disturbance term driving the ARCH process and is normally distributed with zero mean and variance one.

At any given time $t$, the producer's information set is assumed to contain lagged prices and lagged disturbance terms in the ARCH equations. Given the information at the initial time (the first week of April 1990), equation (5.1.2) (or (5.1.4)) is used to compute $h_t$ for the first period. An $e_t$ is randomly drawn from $N(0,1)$ and equation (5.2) is used to compute the corresponding $\epsilon_t$. Lagged prices and $\epsilon_t$ are fed into the price equation (5.1.1) (or (5.1.3)) to forecast the first period price. The above forecasting procedure is applied recursively to obtain a 32-step-ahead forecast for the end-of-period price. This forecast is defined as one possible state of nature of the price at the end of period. Through repeatedly shocking $e_t$ and conducting the forecasting procedure, a desired number of states of nature with equal probability for the price can be simulated.

Three hundred cash prices and futures prices are simulated by the above procedure. The choice of 300
simulations was based on time considerations. To obtain a joint price distribution of cash price and futures price, the simulated end-of-period cash and futures prices are ordered by their magnitudes, i.e., the largest cash price accompanies to largest futures price. A bivariate ARCH model or a bivariate GARCH model which allows for the variances and the covariances of cash and futures prices to be treated as random variables presumably would provide a better description of the joint distribution. However, as mentioned in Chapter III, estimates for such a process was unable to obtain.

Given the strike price of the put option, the 300 simulated futures prices are used to derive the corresponding end-of-period put prices as discussed in Chapter III.

Simulation of yield estimates

In previous studies of optimal hedging under price and quantity uncertainty, yield distribution is often assumed to be a function of time trend (Rolfo, 1980; Miller, 1986; Grant, 1989). Hence, the simulation of yield is based on an estimated trend equation. To estimate the trend equation, average yield for Northern-Central Iowa from 1950 through 1990 are obtained from various issues of Iowa Agricultural Statistics. Three hundred yield estimates are simulated from equation (5.3):

\[ Y = 48.821 + 2.184 \, T + U \]
where \( T \) is the time trend variable (\( T=41 \) in 1990) and \( U \) is normally distributed with mean zero and estimated variance 113.53.

In agricultural production, high prices are most often associated with small harvests and vice-versa. Hence, a negative relationship between cash price and yield is assumed in this study. To account for the negative correlation between cash price and production, the simulated yield estimates are ordered in the reverse order of cash and futures prices.

**Results of Base Solution**

As a negative exponential utility function is used, the agent's coefficient of risk aversion needs to be decided. King and Robison (1981) suggest that the levels of absolute risk aversion should be concentrated in the range of \(-0.0001\) to \(0.001\) because actual measurements for most individuals tended to fall in that interval and several empirical decision studies indicate that optimal solutions are strongly affected by changes in risk aversion coefficient within this range.

Other model parameters include production cost, strike price, beginning-period prices (cash, futures, and options), and the three hundred sets of simulated prices (cash, futures, and options) as well as yield estimates. The production cost is obtained from the extension service at Iowa State
University while the strike price and premium for put option from Chicago Board of Trade. The expected utility maximization model is solved using MINOS 5.2. Three scenarios are considered: 1) the producer is allowed to hold only cash and futures positions (CF); 2) the producer is allowed to hold only cash and put option positions (CO); 3) the producer is allowed to hold cash, futures, and put option positions (CFO).

Table 5.1 presents the optimal market positions under the three scenarios for various levels of risk aversion. The changes in the optimal market positions as the level of risk aversion increases are significant in the interval of 0.000075 and 0.00025. The average simulated yield estimates is 138 bushels per acre with the expected output about 55,200 bushels.

The results of the three scenarios may be interpreted, to some extent, using the mean-variance model put forth in Appendix A. The following discussions are based on the results derived from the mean-variance model.

**CF scenario**

As model 1 in Appendix A suggests, the optimal futures position consists of a hedging and a speculative component. The hedging component is the futures position that minimizes the variance of the end-of-period income, while the speculative component is the futures position that allows the
Table 5.1. Optimal market positions under the CF, CO, and CFO scenarios, with and without certain output

<table>
<thead>
<tr>
<th>Level of risk aversion</th>
<th>Futures position</th>
<th>Options position</th>
<th>Futures position</th>
<th>Options position</th>
</tr>
</thead>
<tbody>
<tr>
<td>without certain output</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000075</td>
<td>-111862.45</td>
<td>81934.14</td>
<td>-140044.95</td>
<td>-71392.25</td>
</tr>
<tr>
<td>0.0001</td>
<td>-103334.20</td>
<td>79549.17</td>
<td>-122574.39</td>
<td>-49234.46</td>
</tr>
<tr>
<td>0.000125</td>
<td>-98005.89</td>
<td>78012.32</td>
<td>-111775.25</td>
<td>-35472.44</td>
</tr>
<tr>
<td>0.00015</td>
<td>-94320.51</td>
<td>77034.40</td>
<td>-104396.42</td>
<td>-26025.49</td>
</tr>
<tr>
<td>0.000175</td>
<td>-91613.61</td>
<td>76443.97</td>
<td>-99015.89</td>
<td>-19071.63</td>
</tr>
<tr>
<td>0.0002</td>
<td>-89560.98</td>
<td>76123.11</td>
<td>-94923.70</td>
<td>-13727.57</td>
</tr>
<tr>
<td>0.000225</td>
<td>-87970.13</td>
<td>75985.95</td>
<td>-91716.65</td>
<td>-9496.54</td>
</tr>
<tr>
<td>0.00025</td>
<td>-86723.73</td>
<td>75970.64</td>
<td>-89136.18</td>
<td>-6037.47</td>
</tr>
<tr>
<td>with certain output</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000075</td>
<td>-127873.53</td>
<td>91825.50</td>
<td>-156473.63</td>
<td>-75280.59</td>
</tr>
<tr>
<td>0.0001</td>
<td>-119016.17</td>
<td>88716.32</td>
<td>-138716.23</td>
<td>-53054.61</td>
</tr>
</tbody>
</table>

Note: CF indicates futures only; CO indicates options only; CFO indicates both futures and options. Futures >0 = long position, <0 = short position. Options >0 = long position, <0 = short position.
Table 5.1. (continued)

<table>
<thead>
<tr>
<th>Level of risk aversion</th>
<th>Futures position</th>
<th>Options position</th>
<th>Futures position</th>
<th>Options position</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000125</td>
<td>-113377.58</td>
<td>86654.77</td>
<td>-127721.56</td>
<td>-39362.05</td>
</tr>
<tr>
<td>0.00015</td>
<td>-109407.74</td>
<td>85285.74</td>
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<td>-29982.80</td>
</tr>
<tr>
<td>0.000175</td>
<td>-106441.63</td>
<td>84400.03</td>
<td>-114704.07</td>
<td>-23132.74</td>
</tr>
<tr>
<td>0.0002</td>
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<td>83848.08</td>
<td>-110555.68</td>
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</tr>
<tr>
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<td>-107291.21</td>
<td>-13840.58</td>
</tr>
<tr>
<td>0.00025</td>
<td>-100870.97</td>
<td>83367.69</td>
<td>-104685.36</td>
<td>-10538.75</td>
</tr>
</tbody>
</table>

with certain output
producer to take advantage of his expected bias in the futures market without having any cash position. More specifically, the producer hedges his crops by selling futures according to the hedge ratio defined as the covariance between cash market revenues and futures prices over the variance of futures prices. He speculates according to the expected bias in the futures market adjusted by the variance of futures price and the level of risk aversion. The expected bias in the futures market is negative and equals to $0.19 which is the difference between the expected value of the end-of-period futures price ($2.43) and the beginning-period price ($2.62).

As shown in Table 5.1, the optimal futures position is much larger than the expected output, for example, it is twice of the expected output in the lowest risk aversion case. The variances of the simulated cash price and futures price are 0.2 and 0.06, respectively, indicating the futures market is relatively more stable than the cash market. Since the producer expects the futures market to be more stable than the cash market, he overhedges in the futures. The results in Table 5.1 also show that the net futures position decreases by 22.47% as the level of risk aversion increases from 0.000075 to 0.00025. This result is consistent with the mean-variance model that the speculative component decreases as the level of risk aversion increases.
CO scenario

As implied by the mean-variance model 2 in Appendix A, the option market plays both the hedging and speculative roles under the CO scenario. The producer hedges his crops by purchasing put options and speculates according to the expected bias in option market adjusted by the variance of option price and the level of risk aversion. The expected bias in the option market is positive and equals to $0.026 which is the difference between the expected value of the end-of-period option price ($0.18) and the option premium ($0.154). Since the end-of-period option price is defined as the intrinsic value at expiration, the volatility of the option price depends on that of the futures price. Hence, the option market is more stable than the cash market. Consequently, the producer overhedges his crops in the option market as indicated in Table 5.1. The option position decreases by 7.28% as the level of risk aversion increases from 0.000075 to 0.00025. This result is again consistent with the mean-variance model. However, notice that the rate of decreasing in the options position is less than that in the futures position. The distinction is due to a smaller market bias in the option market than in the futures market.
CFO scenario

As suggested by the mean-variance model 3 in Appendix A, the producer hedges his output in both markets and speculates in each market according to the level of expected bias in each market adjusted by the income distribution and the level of risk aversion. However, the futures position under the CFO scenario is close to the futures position under the CF scenario. This result shows that the producer hedges in the futures market using a short position and uses a short straddle to hedge the risk associated with the futures position. The short (synthetic) straddle is obtained by selling a short futures and two short puts.

By selling a put, the premium income is offset, in part, any downward movement in prices- a sort of profit cushion below the strike price. The benefits of this type of position are quite obvious when the conditional nature of the option position is taken into account. The option hedge allows the producer to capture speculative price gains without the offsetting position (option is allowed to expire) and covers speculative losses with an offsetting position (option is exercised).

However, the option position decreases rapidly as the level of risk aversion increases. The producer may hedge in the option market but in a trivial amount. The theoretical results in Lapan et al. (1990) show that options are used
together with futures as speculative tools when bias is expected in markets. Options therefore are more useful as speculative tools to exploit private information on the price distribution rather than as an alternative hedging instrument when futures market is already in use. Lapan et al. concludes that straddles are used to speculate on expected variance of prices and to hedge the futures position which is used to speculate the mean of the futures price. These properties derived from a model with non-stochastic production process will be examined in the next section for the case where production is stochastic. Table 5.1 shows that the futures position decreases as the level of risk aversion increases. Further, the straddle position decreases exponentially as the level of risk aversion increases.

Certain output

The optimal market positions under three scenarios, CF, CO, CFO, in the case of certain output are also presented in Table 5.1. The results are similar to and are larger than those obtained from the uncertain output case for all three scenarios. This result implies that the producer uses less futures and/or options when he faces both yield and price uncertainty.

The results under CF scenario show that the producer takes a short position which is greater than the expected
output. The speculative component exists because of the market bias. Also, the optimal futures position decreases as the level of risk aversion increase.

The optimal options position under the CO scenario indicates that the producer takes a long position. The options position decreases at a decreasing rate as the level of risk aversion increases.

Under the CFO scenario, the producer take short positions in both markets. The futures position is near the position in CF scenario but the option position becomes relatively smaller. Both optimal positions decrease as the level of risk aversion increases.

Results similar to the above have been obtained by Hanson (1988). Hanson concluded that the producer hedges his output in the futures market and speculates in both markets according to his expected market bias, risk aversion level, and perception of the income distribution under the CFO scenario. Under the CO scenario, the option market plays both hedging and speculative roles.

**Ex post profits**

It is of interesting to examine the ex post profits derived from the above ex ante optimal market positions. Presumably, an ex ante utility maximization model, in conjunction with a good forecasting system, should produce
"reasonable" ex post profits. Ex post profits under each scenario are presented in Table 5.2. The actual end-of-period cash, futures, and option prices are $2.01, $2.22, and $0.38, respectively. The actual yield is 136 bushel per acre. The ex post profit under the CF scenario is the largest among the three scenarios. However, if margin calls had been included into the model, the profit of the CF scenario could have been reduced. The profit from the futures market is partially offset by loss in the option market under the CFO scenario. The loss in the option market is due to the downward movement in the futures price. The profit under the CO scenario is the smallest among the three scenarios because of the option premium.

Comparative Static Analysis

This section examines the changes in the optimal market positions arising from the changes in the following model parameters: farm size (H), strike price (S), the mean of the cash price (MC), the variance of the cash price (VC), the mean of the futures prices (MF), and the variance of the futures prices (VF). The change of production cost is expected to have no impact on the optimal market positions because of the constant cost (per acre) assumption. But production cost has an effect on certainty equivalent which will be discussed in the next section. In the sensitivity analysis, the model
Table 5.2. *Ex post* profits under the CF, CO, and CFO scenarios, without certain output

<table>
<thead>
<tr>
<th>Level of risk aversion</th>
<th>CF</th>
<th>Marketing Scenario</th>
<th>CO</th>
<th>CFO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dollars</td>
<td>dollars</td>
<td>dollars</td>
<td>dollars</td>
</tr>
<tr>
<td>0.000075</td>
<td>66996.98</td>
<td>40769.12</td>
<td>62135.33</td>
<td></td>
</tr>
<tr>
<td>0.0001</td>
<td>63585.68</td>
<td>40230.11</td>
<td>60154.77</td>
<td></td>
</tr>
<tr>
<td>0.000125</td>
<td>61454.36</td>
<td>39882.78</td>
<td>58945.33</td>
<td></td>
</tr>
<tr>
<td>0.00015</td>
<td>59980.20</td>
<td>39661.77</td>
<td>58128.81</td>
<td></td>
</tr>
<tr>
<td>0.000175</td>
<td>58897.44</td>
<td>39528.34</td>
<td>57548.17</td>
<td></td>
</tr>
<tr>
<td>0.0002</td>
<td>58076.39</td>
<td>39455.82</td>
<td>57119.05</td>
<td></td>
</tr>
<tr>
<td>0.000225</td>
<td>57440.05</td>
<td>39424.83</td>
<td>57065.44</td>
<td></td>
</tr>
<tr>
<td>0.00025</td>
<td>56941.49</td>
<td>39421.37</td>
<td>56542.00</td>
<td></td>
</tr>
</tbody>
</table>

Note: CF indicates futures only; CO indicates options only; CFO indicates both futures and options.
parameter is increased by 5% and 10%, and is decreased by 5% and 10%. The optimal market positions are solved for each parameter change and then graphed. Figures 5.1 through 5.5 illustrate the changes of H, MC, VC, MF, and VF on the futures position under the CF scenario. Figures 5.6 through 5.11 show the changes in the options positions under the CO scenario when H, S, MC, VC, MF, and VF change. The changes of H, S, MC, VC, MF, and VF on the futures and option positions under the CFO scenario are presented in Figures 5.12a through 5.17b. In each figure, the optimal positions are presented by absolute value.

Comparative static under the CF scenario

Result 1: The futures position increases as farm size increases.

Figure 5.1 shows that the futures position increases as farm size increases. An increase in farm size leads to an increase in the expected output. As shown by the mean-variance model 1 in Appendix A, the producer is expected to increase futures position when output increases.

The impacts on the optimal market position arising from a change in the mean of the cash price, the variance of the cash price, the mean of the futures price, and the variance of the futures price are examined based on normal distribution
assumption. The normal distribution is used because the disturbance term of the ARCH model is assumed to be conditionally normally distributed. The means and variances of cash and futures prices obtained from base solution are increased by 5% and 10%, and are decreased by 5% and 10% to examine the changes in parameters of price distribution on the optimal market positions. However, the simulation procedure suffers from a small sample problem in that the optimal market positions obtained from base solution are not within the range of the optimal market positions obtain from current sensitivity analysis. Therefore, the base solution is not included in the following graphs and only four curves (depicting +10%, +5%, -5%, and -10% cases) are exhibited.

**Result 2:** The futures position decreases as the mean of the cash price increases.

Figure 5.2 shows that the futures position increases as the mean of the end-of-period cash price decreases. An increase in the mean of the cash price makes the cash market relatively more certain than the base case, given the common variance in both cases. Hence, the producer reduces the usage of futures. The futures position is decreased by 3.5% as the mean of the cash price increases from the lowest value to the highest value.
Result 3: The futures position decreases as the mean of the futures price increases. In Figure 5.3, the futures position decreases as the mean of the end-of-period futures prices increases. As shown in the mean-variance model 1 in appendix A, the producer speculates according to his expected market bias adjusted by the variance of futures price and the level of risk aversion. The expected futures market bias is difference between the expected value of the end-of-period futures price and the beginning-period futures price. Given the negative market bias as mentioned before, an increase in the expected futures price has the effect of making the bias to be less negative (to be positive) which leads to a smaller short speculative position (to a long speculative position). Consequently, the net short futures position decreases.

Result 4: The futures position increases as the variance of the cash price increases.

The futures position increases as the variance of the end-of-period cash price increases as exhibited in Figure 5.4. An increase in the variance of the cash price makes the cash market relatively more uncertain than the futures market. Therefore, the producer uses more futures to reduce the price risk in the cash market. The futures position is increased by
11% as the variance of the cash price increases from the lowest value to the highest value.

Result 5: The futures position decrease as the variance of the futures price increases.

Figure 5.5 shows that the futures position decreases as the variance of the end-of-period futures price increases. An increase in the variance of the futures price makes the future market relatively uncertain than the cash market. Hence, hedging in futures is less effective and speculation in futures market becomes more risky than base case. The result is consistent with the mean-variance model 1 in Appendix A that an increase in the variance of the futures price induces decreases in both hedging and speculative components.

Comparative static under the CO scenario

Result 1: The options position increases as farm size increases.

Figure 5.6 shows that the options position increases as farm size increases. The expected output increases as farm size increases. The producer needs more options to hedge his crops. The findings from Figure 5.1 and 5.6 imply a positive relationship between optimal usage of futures or options and farm size. Further study may be helpful for agricultural enterprises to select hedging strategies.
Figure 5.1. Change in the futures position as farm size changes using the CF strategy

0.000075 0.0001 0.000125 0.00015 0.000175 0.0002 0.000225 0.00025
Level of risk aversion

□ H=400  + H=300  ◊ H=350  ▲ H=450  × H=500

Futures position, bushel (Thousands)
Figure 5.2. Change in the futures position as the mean of end-of-period cash price changes using the CF strategy.
Figure 5.3. Change in the futures position as the mean of end-of-period futures price changes using the CF strategy
Figure 5.4. Change in the futures position as the variance of end-of-period cash price changes using the CF strategy.
Figure 5.5. Change in the futures position as the variance of end-of-period futures price changes using the CF strategy.
Result 2: The options position decreases as the strike price increases.

Figure 5.7 shows that the producer uses less options when the strike price increases. An increase in strike price is equivalent to an increase in intrinsic value of the option at expiration and hence, hedging in puts position is more effective than base case. On the other hand, a decreases in the strike price leads to a decrease in intrinsic value of the option as expiration and hedging in puts position is less effective than base case. Therefore, the options position decreases as the strike price increases.

Result 3: The options position decreases as the mean of the cash price increases.

As shown in Figure 5.8, the options position decreases as the mean of the cash price increases. The cash market is now relatively less risky compared to base case, given the common variance in both cases. Therefore, the producer hedges less options than base case. The option position is decreased by 3% as the mean of the cash price increases from the lowest value to the highest value.

Result 4: The options position decreases as the mean of the futures price increases.

As the option price is defined as the intrinsic value at
expiration, a change in the expected futures price affects the end-of-period option prices. An increase in the mean of the futures price leads to a decrease in the expected value of the end-of-period option price and hence, to a decrease in the market bias. Given the positive bias in base case, an increase in the mean of the futures price has a effect of making the bias to be less positive (to be negative) which leads to a decrease in the long speculative position (to a short speculative position). Consequently, the net options position decreases as presented in Figure 5.9.

Result 5: The options position increases as the variance of the cash price increases.

Figure 5.10 shows that the options position increases as the variance of the cash price increases. An increase in the variance of the cash price makes the cash market relatively more risky than the option market. Therefore, the producer hedges more options than base case. The options position is increased by 12% as the variance of the cash price increases from the lowest value to the highest value.

Result 6: The options position decreases as the variance of the futures price increases.

The exponential objective function overflowed for the case where the variance of the futures price is decreased by
10% and the optimal options position was not obtained. Only three curves are exhibited in Figure 5.11. The options position decreases as the variance of the futures price increases. Since the end-of-period option price is defined as the intrinsic value at expiration, a change in the variance of futures price affects the variance of the end-of-period option price. That is, an increase in the variance of the futures price leads to an increase in the variance of the option price, making the option market relatively more uncertain than the cash market. Therefore, the options position decreases as the variance of the futures price increases.

**Comparative static under the CFO scenario**

**Result 1: The futures position increases and the options position decreases as farm size increases.**

Figure 5.12a and 5.12b show how the producer responds to changes in farm size when both futures and options are available as risk management tools. The futures position increase as farm size increases. However, the options position decreases slightly as farm size increases. Since the futures market plays a major hedging role under the CFO scenario, the changes in the futures position arising from the change in farm size is similar to that under the CF scenario. As suggested in the mean-variance model 3 in Appendix A, the optimal market positions appear to be adjusted in such a way
Figure 5.6. Change in the options position as farm size changes using the CO strategy
Figure 5.7. Change in the options position as strike price changes using the CO strategy
Figure 5.8. Change in the options position as the mean of end-of-period cash price changes using the CO strategy
Figure 5.9. Change in the options position as the mean of end-of-period futures price changes using the CO strategy.
Figure 5.10. Change in the options position as the variance of end-of-period cash price changes using the CO strategy.
Figure 5.11. Change in the options position as the variance of end-of-period futures price changes using the CO strategy.
that an equilibrium is maintained between the returns to speculation and the reduction in risk due to the hedging. A speculative tradeoff occurs between the futures and the options positions in that an increase in the futures position is associated with a decrease in the options position.

Result 2: The futures position increases as the strike price increases. The options position decreases as the strike price increases.

The futures position increases when strike price increases as shown in Figure 5.13a. The option position decreases as the change in the strike price. But notice that the change in the options position does not show in a pattern. The short straddle seems to play both hedging and speculative roles in the case of changing the strike price. A speculative tradeoff occurs when the straddle is used to speculate. On the other hand, when the straddle is used to hedge the futures position, an increase in the futures position is associated with an increase in the options position.

Result 3: The futures position decreases as the mean of the cash price increases. The change in the mean of the cash price does not affect the options position.

Figure 5.14a and 5.14b shows that the futures position
has slightly decreased when the mean of the cash price increases, while the options position is not affected. An increase in the mean of the cash price leads to a relatively less risky cash market compared to base case, given the common variance in both cases. As the futures market plays a major hedging role, the producer reduces usage of futures. The options position could increase if a speculative tradeoff occurs between the futures and options positions. On the other hand, the options position decreases if the options are used to hedge futures position. Since the change in the futures position is at a relatively small magnitude compared to previous cases, the options position appears to no change as the mean of the cash price increases.

Result 4: The futures position decreases as the mean of the futures price increases. The options position seems to increase as the mean of the futures price increases.

Figure 5.15a shows that the futures position decreases as the mean of the futures price increases. However, the change in the options position is ambiguous though is appears to increase as depicted in Figure 5.15b. An increase in the mean of futures price leads to a less negative bias in the futures market and a less positive bias in the option market from the sensitivity analysis under the CF and CO scenarios. The
futures position decreases as that under the Cf scenario. The options position increases since a speculative tradeoff occurs between two markets.

**Result 5:** The futures position increases and the options position decreases as the variance of the cash price increases.

Figure 5.16a and 5.16b illustrates how the producer's optimal market positions change as the variance of the cash price changes. The futures position increases and the options position decreases as the variance of cash price increases. But the changes in the futures and option positions are at small magnitudes. An increase in the variance of the cash price makes the cash market more risky than the futures and option markets. The producer uses more futures because the futures market play a major hedging role. The optimal market positions are adjusted to maintain an equilibrium between the returns to speculation and the risk reduction from hedging. A speculative tradeoff occurs between the futures position and the options position in that an increase in the futures position is associated with a decrease in the options position.

**Result 6:** The futures and options positions decrease as the variance of the futures price increases.
Figure 5.17a and 5.17b shows that the futures position and options position decrease as the variance of futures price increases. The futures position decreases as the variance of the futures price increases because the futures market is relative more risky than the cash market. Since option prices are defined as the intrinsic value at expiration, a change in the variance of the futures price affects the variance of the option price. That is, an increase in the variance of the futures price leads to an increase in the variance of the option price. An increase in the variance of the option price makes the option market relatively more uncertain than the cash market. Therefore, both the put option and futures position decrease as the variance of the futures prices increases.

In general, the producer hedges his crops in the futures market with a major position and hedges in the option market with a minor position. He speculates in both markets. Short put positions are used to hedge futures position which is used to speculate the change in the variance of futures price. The producer uses put options to speculate the futures position which is used to speculate the mean of the cash price, and the mean of futures price. When a put option is used to hedge the future position, an increase in the futures position is associated with an increase in put options position. If put options are used as speculative tools, a speculative tradeoff
Figure 5.12. Change in the futures and options position as farm size changes using CFO strategy
(a) futures position
Figure 5.12. (continued) (b) options position
Figure 5.13. Change in the futures and options position as strike price changes using CFO strategy
(a) futures position
Figure 5.13. (continued) (b) options position
Figure 5.14. Change in the futures and options position as the mean of end-of-period cash price changes using the CFO strategy (a) futures position
Figure 5.14. (continued) (b) options position
Figure 5.15. Change in the futures and option position as the mean of end-of-period futures price changes using the CFO strategy (a) futures position.
Figure 5.15. (continued) (b) option position
Figure 5.16. Change in the futures and options position as the variance of end-of-period cash price changes using the CFO strategy
(a) futures position
Figure 5.16. (continued) (b) options position
Figure 5.17. Change in the futures and options position as the variance of end-of-period futures price changes using the CFO strategy (a) futures position
Figure 5.17. (continued) (b) option position
occurs between the futures and put positions.

**Certainty Equivalent and Access Values**

The certainty equivalent under each scenario and the access value of futures and options are computed and presented in this section. The impacts of a change in model parameters on the certainty equivalent and the access value are also reported.

Since options positions can make the end-of-period income distribution to be quite skewed, the certainty equivalent has been expanded to the third order Taylor series for each scenario. Comparisons of the second-order and the third-order measures are presented in Table 5.3. The difference between the two measures are negligible under the CF and CFO scenarios, but it is significant under the CO scenario. Therefore, the second-order measure is used to measure the certainty equivalent under the CF scenario and the third-order measure is used under the CO and CFO scenarios. The certainty equivalent under the CF and CFO scenarios decreases 4.6% and 10.18%, respectively, as the level of risk aversion increases from 0.000075 to 0.00025. Under the CO scenario, it increases 39.50% as the level of risk aversion increases from 0.000075 to 0.00025.

The access values of futures, options, and futures-and-options to the producer are measured by computing the
Table 5.3. A comparison of the second-order and the third-order certainty equivalent

<table>
<thead>
<tr>
<th>Level of risk aversion</th>
<th>CF</th>
<th>CO</th>
<th>CFO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000075</td>
<td>CE1 55574.80</td>
<td>36459.08</td>
<td>59902.23</td>
</tr>
<tr>
<td></td>
<td>CE2 55481.50</td>
<td>38710.84</td>
<td>59201.55</td>
</tr>
<tr>
<td>0.0001</td>
<td>CE1 54812.43</td>
<td>34962.86</td>
<td>57432.01</td>
</tr>
<tr>
<td></td>
<td>CE2 54735.58</td>
<td>39157.91</td>
<td>57891.07</td>
</tr>
<tr>
<td>0.000125</td>
<td>CE1 54316.32</td>
<td>33455.10</td>
<td>56605.93</td>
</tr>
<tr>
<td></td>
<td>CE2 54242.87</td>
<td>40199.71</td>
<td>56142.86</td>
</tr>
<tr>
<td>0.00015</td>
<td>CE1 53950.74</td>
<td>31947.97</td>
<td>55686.22</td>
</tr>
<tr>
<td></td>
<td>CE2 53871.35</td>
<td>41831.85</td>
<td>55276.06</td>
</tr>
<tr>
<td>0.000175</td>
<td>CE1 53658.68</td>
<td>30449.78</td>
<td>54976.37</td>
</tr>
<tr>
<td></td>
<td>CE2 53565.57</td>
<td>44042.60</td>
<td>54601.30</td>
</tr>
<tr>
<td>0.0002</td>
<td>CE1 53413.45</td>
<td>28964.78</td>
<td>54399.12</td>
</tr>
<tr>
<td></td>
<td>CE2 53299.77</td>
<td>46817.32</td>
<td>54049.26</td>
</tr>
<tr>
<td>0.000225</td>
<td>CE1 53200.90</td>
<td>27493.96</td>
<td>53581.49</td>
</tr>
<tr>
<td></td>
<td>CE2 53060.48</td>
<td>50141.80</td>
<td>53581.49</td>
</tr>
<tr>
<td>0.00025</td>
<td>CE1 53013.36</td>
<td>26036.38</td>
<td>53485.83</td>
</tr>
<tr>
<td></td>
<td>CE2 52840.58</td>
<td>54004.00</td>
<td>53173.92</td>
</tr>
</tbody>
</table>

Note: CF indicates futures only; CO indicates options only; CFO indicates both futures and options.

CE1 = E[π] - 1/2λ E[(π - E(π))^2].

differences between the certainty equivalent under all three scenarios and the certainty equivalent in the case of cash-only position. The results are exhibited in Table 5.4.

Adding a futures or both futures and options to the cash-only position clearly provides more value to the expected utility maximizing producer than adding an option. The difference between the access value of futures and option is 188.07% for the lowest level of risk aversion and is 6.9% for the highest level of risk aversion. The difference between the access value of futures-and-option and option is 236.64% for the lowest level of risk aversion and is 7.76% for the highest level of risk aversion. The access value of futures-and-option position is the largest among the three scenarios because the largest opportunity set is available to the producer under the CFO scenario. The access value of the options position is the smallest among the three scenarios because of the premium. At equilibrium, the access value of the futures is expected to be equal to the access value of the options if margin calls or interest on margin funds are included in the model. This may leave for further study. The access value of futures, option, and futures-and-option increases as the level of risk aversion increases. This finding implies that the usage of futures and option markets is more valuable to more risk averse individuals.

The market is unbiased if the expected values of the end-
Table 5.4. Access values of futures, options, and futures-and-options over the cash-only position

<table>
<thead>
<tr>
<th>Level of risk aversion</th>
<th>CF vs. Cash</th>
<th>CO vs. Cash</th>
<th>CFO vs. Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base solution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000075</td>
<td>25830.60</td>
<td>8966.64</td>
<td>30158.03</td>
</tr>
<tr>
<td></td>
<td>(0.47)</td>
<td>(0.16)</td>
<td>(0.55)</td>
</tr>
<tr>
<td>0.0001</td>
<td>29734.16</td>
<td>14079.64</td>
<td>32812.80</td>
</tr>
<tr>
<td></td>
<td>(0.54)</td>
<td>(0.26)</td>
<td>(0.59)</td>
</tr>
<tr>
<td>0.000125</td>
<td>33635.21</td>
<td>19518.60</td>
<td>35924.82</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
<td>(0.35)</td>
<td>(0.65)</td>
</tr>
<tr>
<td>0.00015</td>
<td>37396.33</td>
<td>25277.44</td>
<td>39131.81</td>
</tr>
<tr>
<td></td>
<td>(0.68)</td>
<td>(0.46)</td>
<td>(0.71)</td>
</tr>
<tr>
<td>0.000175</td>
<td>40962.21</td>
<td>31346.13</td>
<td>42279.90</td>
</tr>
<tr>
<td></td>
<td>(0.74)</td>
<td>(0.57)</td>
<td>(0.77)</td>
</tr>
<tr>
<td>0.0002</td>
<td>44305.59</td>
<td>37709.46</td>
<td>45291.26</td>
</tr>
<tr>
<td></td>
<td>(0.80)</td>
<td>(0.68)</td>
<td>(0.82)</td>
</tr>
<tr>
<td>0.000225</td>
<td>47412.31</td>
<td>44353.21</td>
<td>47792.90</td>
</tr>
<tr>
<td></td>
<td>(0.86)</td>
<td>(0.80)</td>
<td>(0.87)</td>
</tr>
</tbody>
</table>

Note: Cash indicates cash-only position; CF indicates futures only; CO indicates options only; CFO indicates both futures and options only. Figures in parentheses are the access values measured by production quantity, 55200 bushels. The access value of options is greater than those of futures and futures-and-options in the case of risk aversion level 0.00025.
Table 5.4. (continued)

<table>
<thead>
<tr>
<th>Level of risk aversion</th>
<th>CF vs. Cash</th>
<th>CO vs. Cash</th>
<th>CFO vs. Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unbiased market</td>
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<tr>
<td>0.000075</td>
<td>9532.84</td>
<td>3105.50</td>
<td>10974.77</td>
</tr>
<tr>
<td>(0.17)</td>
<td>(0.06)</td>
<td>(0.20)</td>
<td></td>
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<tr>
<td>0.0001</td>
<td>13926.51</td>
<td>7430.89</td>
<td>15564.59</td>
</tr>
<tr>
<td>(0.25)</td>
<td>(0.13)</td>
<td>(0.28)</td>
<td></td>
</tr>
<tr>
<td>0.000125</td>
<td>18051.41</td>
<td>12043.97</td>
<td>19881.37</td>
</tr>
<tr>
<td>(0.33)</td>
<td>(0.22)</td>
<td>(0.36)</td>
<td></td>
</tr>
<tr>
<td>0.00015</td>
<td>21905.85</td>
<td>16943.08</td>
<td>23923.42</td>
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<tr>
<td>(0.40)</td>
<td>(0.31)</td>
<td>(0.43)</td>
<td></td>
</tr>
<tr>
<td>0.000175</td>
<td>25491.54</td>
<td>22129.89</td>
<td>27692.44</td>
</tr>
<tr>
<td>(0.46)</td>
<td>(0.40)</td>
<td>(0.50)</td>
<td></td>
</tr>
<tr>
<td>0.0002</td>
<td>28807.89</td>
<td>27603.85</td>
<td>31187.87</td>
</tr>
<tr>
<td>(0.52)</td>
<td>(0.50)</td>
<td>(0.56)</td>
<td></td>
</tr>
<tr>
<td>0.000225</td>
<td>31854.90</td>
<td>31364.94</td>
<td>34409.69</td>
</tr>
<tr>
<td>(0.58)</td>
<td>(0.57)</td>
<td>(0.62)</td>
<td></td>
</tr>
</tbody>
</table>
of-period prices are equal to the beginning-period prices. The speculative component is therefore suppressed from the optimal market position in this case and the access values reflect the values added to the pure hedging producer. In Table 5.4, the access values of futures, option and futures-and-option are presented for the case where markets are unbiased. The beginning-period prices and historical variances of cash and futures prices are used as the mean and variance for a normal distribution. The option premium is adjusted to be equal to the expected value of the end-of-period option price (see Hanson for a more detailed procedure). The access values of futures-and-option are the largest among the three scenarios. The options add 3 to 4 cent per bushel to the producer than only futures market in use. This result indicates that the put option adds hedging benefits to the producer. However, the approach used to arrive at markets unbiased is not consistent with the stochastic simulation procedure discussed earlier. Further study using simulation stochastic procedure to examine the case of unbiased market is required in order to obtain results which are consistent with base solution.

The access value of futures and options added to the cash-and-option position and cash-and-futures position is listed in Table 5.5. Adding futures is of value to the producer who currently uses only options. The access value of
Table 5.5. Access value of futures and options derived from the futures and options scenario

<table>
<thead>
<tr>
<th>Level of risk aversion</th>
<th>Access value of options</th>
<th>Access value of futures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dollars</td>
<td>dollars</td>
</tr>
<tr>
<td>0.000075</td>
<td>3626.75</td>
<td>20490.17</td>
</tr>
<tr>
<td>0.0001</td>
<td>2529.58</td>
<td>18184.10</td>
</tr>
<tr>
<td>0.000125</td>
<td>1826.54</td>
<td>15943.15</td>
</tr>
<tr>
<td>0.00015</td>
<td>1325.32</td>
<td>13444.21</td>
</tr>
<tr>
<td>0.000175</td>
<td>942.62</td>
<td>10558.70</td>
</tr>
<tr>
<td>0.0002</td>
<td>635.81</td>
<td>7231.94</td>
</tr>
<tr>
<td>0.000225</td>
<td>380.59</td>
<td>3439.69</td>
</tr>
</tbody>
</table>

Note: The access value of options is greater than that of futures in the case of risk aversion level 0.00025.
futures is greater than the access value of option by 803.78% for the lowest level of risk aversion and by 464.97% for the highest level of risk aversion. The seeming small value of the options to the producer appears to reflect the superior hedging ability of futures. The access value decreases as the level of risk aversion increases. This finding indicates that the more risk averse individuals prefers to use only futures or only options.

The impacts of the changes in model parameters on certainty equivalent are presented in Table 5.6 and Table 5.7 for the level of risk aversion equals to 0.000075 and 0.00025, respectively. Farm size, and the mean of the end-of-period cash price have positive effects on certainty equivalent for all scenarios but production cost and the mean of the end-of-period futures price have negative effects for all three scenarios. The variance of the cash price has a positive effect under the CF and CFO scenarios and a negative effect on certainty equivalent under the CO scenario. The variance of the futures price has a negative effect under the CF and CFO scenarios and a positive effect under the CO scenario. The effect of the changes in the strike price is ambiguous. Certainty equivalent decreases as the level of risk aversion increases under the CF and CFO scenarios while increases under the CO scenario.

The impacts of the change in model parameters on the
Table 5.6. Comparative static on certainty equivalent:
Impacts of farm size, production cost, strike
price, and price distributions, level
of risk aversion = 0.000075

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>CF</th>
<th>CO</th>
<th>CFO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dollars</td>
<td>dollars</td>
<td>dollars</td>
</tr>
<tr>
<td>Farm size</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300 acres</td>
<td>42405.14</td>
<td>29071.61</td>
<td>46168.93</td>
</tr>
<tr>
<td>350 acres</td>
<td>48993.86</td>
<td>33858.59</td>
<td>52694.81</td>
</tr>
<tr>
<td>400 acres</td>
<td>55574.80</td>
<td>38710.84</td>
<td>59201.55</td>
</tr>
<tr>
<td>450 acres</td>
<td>62147.45</td>
<td>43660.05</td>
<td>65688.06</td>
</tr>
<tr>
<td>500 acres</td>
<td>68711.96</td>
<td>48738.01</td>
<td>72156.42</td>
</tr>
<tr>
<td>Production cost</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>46702.80</td>
<td>29838.84</td>
<td>50329.48</td>
</tr>
<tr>
<td>+ 5%</td>
<td>51138.79</td>
<td>34274.84</td>
<td>54765.44</td>
</tr>
<tr>
<td>0%</td>
<td>55574.80</td>
<td>38710.84</td>
<td>59201.55</td>
</tr>
<tr>
<td>- 5%</td>
<td>60010.80</td>
<td>43146.84</td>
<td>63637.50</td>
</tr>
<tr>
<td>-10%</td>
<td>64442.79</td>
<td>47578.84</td>
<td>68069.24</td>
</tr>
<tr>
<td>Strike price</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2.4</td>
<td>N/A</td>
<td>39357.76</td>
<td>56289.95</td>
</tr>
<tr>
<td>$2.5</td>
<td>N/A</td>
<td>40035.42</td>
<td>57220.78</td>
</tr>
<tr>
<td>$2.6</td>
<td>N/A</td>
<td>38710.84</td>
<td>59201.55</td>
</tr>
<tr>
<td>$2.7</td>
<td>N/A</td>
<td>40133.41</td>
<td>59081.22</td>
</tr>
<tr>
<td>$2.9</td>
<td>N/A</td>
<td>42045.66</td>
<td>58844.83</td>
</tr>
</tbody>
</table>

Note: CF indicates futures only; CO indicates options only; CFO indicates both futures and options. + indicates an increase, and - indicates a decrease. N/A indicates not available.
Table 5.6. (continued)

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>CF</th>
<th>CO</th>
<th>CFO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dollars</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean of cash price</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>66033.49</td>
<td>49881.24</td>
<td>68864.44</td>
</tr>
<tr>
<td>+ 5%</td>
<td>60038.07</td>
<td>43762.44</td>
<td>62865.35</td>
</tr>
<tr>
<td>- 5%</td>
<td>46957.11</td>
<td>30412.69</td>
<td>49776.36</td>
</tr>
<tr>
<td>-10%</td>
<td>40961.64</td>
<td>24294.17</td>
<td>43777.22</td>
</tr>
<tr>
<td>Mean of futures price</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>31471.65</td>
<td>23973.23</td>
<td>36787.32</td>
</tr>
<tr>
<td>+ 5%</td>
<td>41264.07</td>
<td>29206.52</td>
<td>44959.85</td>
</tr>
<tr>
<td>- 5%</td>
<td>68609.19</td>
<td>48280.42</td>
<td>71039.48</td>
</tr>
<tr>
<td>-10%</td>
<td>88239.22</td>
<td>60995.26</td>
<td>91802.27</td>
</tr>
<tr>
<td>Variance of cash price</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>55255.96</td>
<td>36245.72</td>
<td>58109.86</td>
</tr>
<tr>
<td>+ 5%</td>
<td>54994.79</td>
<td>36375.19</td>
<td>57902.31</td>
</tr>
<tr>
<td>- 5%</td>
<td>54208.33</td>
<td>36765.23</td>
<td>57277.40</td>
</tr>
<tr>
<td>-10%</td>
<td>53945.20</td>
<td>36895.36</td>
<td>57068.34</td>
</tr>
<tr>
<td>Variance of futures price</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>52987.29</td>
<td>37280.90</td>
<td>54895.99</td>
</tr>
<tr>
<td>+ 5%</td>
<td>53695.56</td>
<td>36965.69</td>
<td>56109.06</td>
</tr>
<tr>
<td>- 5%</td>
<td>55323.32</td>
<td>36287.89</td>
<td>59055.95</td>
</tr>
<tr>
<td>-10%</td>
<td>56264.42</td>
<td>N/A</td>
<td>60854.61</td>
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</table>
Table 5.7. Comparative static on certainty equivalent: Impacts of farm size, production cost, strike price, and price distributions, level of risk aversion = 0.00025

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>CF</th>
<th>CO</th>
<th>CFO</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>dollars</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Farm size</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300 acres</td>
<td>40148.56</td>
<td>34020.28</td>
<td>40734.34</td>
</tr>
<tr>
<td>350 acres</td>
<td>46595.04</td>
<td>43102.36</td>
<td>46980.34</td>
</tr>
<tr>
<td>400 acres</td>
<td>53013.36</td>
<td>54004.00</td>
<td>53173.92</td>
</tr>
<tr>
<td>450 acres</td>
<td>59407.35</td>
<td>67021.23</td>
<td>59318.39</td>
</tr>
<tr>
<td>500 acres</td>
<td>65783.55</td>
<td>82449.83</td>
<td>65415.87</td>
</tr>
<tr>
<td><strong>Production cost</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>44141.36</td>
<td>45132.00</td>
<td>44301.82</td>
</tr>
<tr>
<td>+5%</td>
<td>48577.33</td>
<td>49568.00</td>
<td>48737.89</td>
</tr>
<tr>
<td>0%</td>
<td>53013.36</td>
<td>54004.00</td>
<td>53173.92</td>
</tr>
<tr>
<td>-5%</td>
<td>57449.33</td>
<td>58440.00</td>
<td>57608.43</td>
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<tr>
<td>-10%</td>
<td>61881.04</td>
<td>62872.00</td>
<td>62041.54</td>
</tr>
<tr>
<td><strong>Strike price</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2.4</td>
<td>N/A</td>
<td>62661.25</td>
<td>53173.92</td>
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<tr>
<td>$2.5</td>
<td>N/A</td>
<td>59320.36</td>
<td>52872.54</td>
</tr>
<tr>
<td>$2.6</td>
<td>N/A</td>
<td>54004.00</td>
<td>53173.92</td>
</tr>
<tr>
<td>$2.7</td>
<td>N/A</td>
<td>50005.58</td>
<td>53135.35</td>
</tr>
<tr>
<td>$2.9</td>
<td>N/A</td>
<td>43688.36</td>
<td>53051.93</td>
</tr>
</tbody>
</table>

Note: CF indicates futures only; CO indicates options only; CFO indicates both futures and options. + indicates an increase, and - indicates a decrease. N/A indicates not available.
Table 5.7. (continued)

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>CF</th>
<th>CO</th>
<th>CFO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dollars</td>
<td>dollars</td>
<td>dollars</td>
</tr>
<tr>
<td>Mean of cash price</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>62906.65</td>
<td>52766.60</td>
<td>62573.61</td>
</tr>
<tr>
<td>+ 5%</td>
<td>56909.33</td>
<td>46826.09</td>
<td>56573.99</td>
</tr>
<tr>
<td>- 5%</td>
<td>43822.68</td>
<td>33882.96</td>
<td>43481.69</td>
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<tr>
<td>-10%</td>
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<td>27959.00</td>
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<td>Mean of futures price</td>
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<td></td>
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<td>30710.40</td>
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<tr>
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<td>35458.19</td>
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<td>72544.38</td>
<td>55786.85</td>
<td>71838.04</td>
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<td>Variance of cash price</td>
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<td></td>
</tr>
<tr>
<td>+10%</td>
<td>51671.11</td>
<td>42703.39</td>
<td>51218.74</td>
</tr>
<tr>
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<td>51480.19</td>
<td>42129.11</td>
<td>51049.23</td>
</tr>
<tr>
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<td>50895.82</td>
<td>40675.05</td>
<td>50535.05</td>
</tr>
<tr>
<td>-10%</td>
<td>50697.04</td>
<td>40275.03</td>
<td>50361.72</td>
</tr>
<tr>
<td>Variance of futures price</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>49984.56</td>
<td>41890.81</td>
<td>49602.13</td>
</tr>
<tr>
<td>+ 5%</td>
<td>50515.75</td>
<td>41518.87</td>
<td>50125.73</td>
</tr>
<tr>
<td>- 5%</td>
<td>51721.21</td>
<td>40680.33</td>
<td>51356.48</td>
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<tr>
<td>-10%</td>
<td>52408.85</td>
<td>N/A</td>
<td>52084.40</td>
</tr>
</tbody>
</table>
access value of options are presented in Table 5.8. The access values in some cases are negative when the level of risk aversion is equal to 0.00025. These numbers have been replaced with zero by application of LeChatelier Principle. The results are summarized as follows:

1. The farm size has a positive impact on the access value of options.
2. Production costs have no effect on the access value of options.
3. The mean of the end-of-period cash price appears to have a positive effect upon the access value of options.
4. The impact of the mean of the end-of-period futures price on the access value of options is ambiguous.
5. The variances of the cash price and the futures price have a negative effect on the access value of options.
6. The level of risk aversion has a negative effect on access value of options.
Table 5.8. Comparative static on access values of options: Impacts of farm size, production cost, strike price, and price distributions, level of risk aversion = 0.000075 and 0.00025

<table>
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<th>0.00025</th>
</tr>
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<tr>
<td><strong>Farm size</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300 acres</td>
<td>3763.79</td>
<td>585.78</td>
</tr>
<tr>
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<td>3700.95</td>
<td>385.30</td>
</tr>
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<td>3626.75</td>
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<tr>
<td>450 acres</td>
<td>3540.61</td>
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</tr>
<tr>
<td>500 acres</td>
<td>3444.46</td>
<td>0.00&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>Production cost</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>3626.68</td>
<td>160.46</td>
</tr>
<tr>
<td>+5%</td>
<td>3626.65</td>
<td>160.56</td>
</tr>
<tr>
<td>0%</td>
<td>3626.75</td>
<td>160.56</td>
</tr>
<tr>
<td>-5%</td>
<td>3626.70</td>
<td>159.10</td>
</tr>
<tr>
<td>-10%</td>
<td>3626.71</td>
<td>160.50</td>
</tr>
<tr>
<td><strong>Mean of cash price</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>2830.95</td>
<td>0.00&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>+5%</td>
<td>2827.28</td>
<td>0.00&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Note: + indicates an increase, and - indicates a decrease.<sup>a</sup> The access value is negative. By application of LeChatelletier principle, the number is replaced by zero.
Table 5.8. (continued)

<table>
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<th>Model parameters</th>
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<th>0.00025</th>
</tr>
</thead>
<tbody>
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<td>dollars</td>
<td></td>
</tr>
<tr>
<td>Mean of cash price</td>
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<td></td>
</tr>
<tr>
<td>- 5%</td>
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<td>0.00</td>
</tr>
<tr>
<td>-10%</td>
<td>2815.58</td>
<td>0.00</td>
</tr>
<tr>
<td>Mean of futures price</td>
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<td></td>
</tr>
<tr>
<td>+10%</td>
<td>5315.67</td>
<td>249.47</td>
</tr>
<tr>
<td>+ 5%</td>
<td>3695.78</td>
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<td>2430.29</td>
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<td>3563.05</td>
<td>0.00</td>
</tr>
<tr>
<td>Variance of cash price</td>
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<td></td>
</tr>
<tr>
<td>+10%</td>
<td>2853.90</td>
<td>0.00</td>
</tr>
<tr>
<td>+ 5%</td>
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<td>3123.14</td>
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<td>1908.70</td>
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<tr>
<td>+ 5%</td>
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<td>-10%</td>
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<td>0.00</td>
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</table>
CHAPTER VI.
SUMMARY AND CONCLUSIONS

Futures and options can be used to transfer price risk commonly faced by agricultural producers. Option contracts have added flexibilities to producers' hedging strategies. However, this advantage incurs cost--the option premium. The choice of a hedging strategy depends, in part, on the decision maker's preferences and risk attitudes. It also depends on the agent's prediction about the end-of-period random prices and production.

The overall objective of this study is to examine the optimal hedging strategy for a representative central Iowa corn producer who is allowed to used both futures and options. The present study is conducted within a context where cash, futures, and option prices and production are stochastic. An expected utility maximization model is employed to solve the optimal market positions under three scenarios: 1) the producer is allowed to use only futures; 2) the producer is allowed to use only put options; 3) the producers is allowed to use both futures and put options to hedge the risk associated with his cash position. Traditional mean-variance analysis in general is not consistent with expected utility maximization when options are allowed since the income distribution is truncated.
Cash, and futures prices are generated by ARCH models to provide a better description of the non-constant variance nature of the prices. The use of ARCH models has been supported by previous research findings that commodity futures prices are not normally distributed. The price distribution of put options is defined as intrinsic value at expiration since the options position is assumed to hold till the time of harvesting. Actually, the option prices is derived from the estimated futures prices for a given strike price. A trend equation is estimated to generate random yield estimates.

The comparative static behavior of the expected utility maximizing producer is investigated through changing model parameters such as farm size, strike price, mean of the end-of-period cash price, mean of the end-of-period futures price, variance of the end-of-period cash price, and variance of the end-of-period futures price. The access value of futures, options, and futures-and-options added to the producer are examined by the concept of certainty equivalent. Comparative static results of a change in farm size, production cost, and parameters of price distributions on the access value of option are also obtained.

The results of applying ARCH model to corn cash and futures prices replicate historical movements in these price series adequately. In addition, the confidence intervals, derived from the conditional forecast variances indicate that
variance changes substantially over the sample period. These findings suggest that a wider application of the non-constant variance model could be beneficial to future research in the field of agricultural prices analysis.

The expected utility maximization results under the scenario that the producer is allowed to use both futures and options as risk management tools show that the agent takes short positions in the futures and put options markets. Since a short put and a short futures construct a synthetic short straddle, this result indicates that the producer hedges his crops by a short futures position and uses a short synthetic straddle to hedge the risk associated with the futures positions. However, comparative static results show that the synthetic short straddle is used to hedge the futures position which is used to speculate the variance of the futures price. The straddle is used to speculate on the expected value of the cash price, and the expected value of the futures price. A speculative tradeoff occurs between the futures position and the put position in that obtaining additional short futures contracts tends to be associated with reducing put options. From a pure risk minimizing standpoint, the options offer no added advantage in risk management if the futures market is already in use. However, the put option plays both the hedging and speculative roles when the producer uses only options as risk management tools.
Comparative static results for the optimal futures and options positions are:

(a) Farm size has a positive effect on both the futures options positions.
(b) The mean of the cash price has a negative effect on both the futures and options positions.
(c) The mean of the futures price has a negative effect on both the futures and options positions.
(d) The variance of the cash price has a positive effect on both the futures and options positions.
(e) The variance of the futures price has a positive effect on both the futures and options positions.
(f) The strike price has a negative effect on the options position.

The results of the access values show that there is a positive value to the producer by adding a futures position to a cash position or to a cash-and-option position. There is also a positive value to the producer by adding an option position to a cash position. However, adding an option position to a cash-and-futures position adds relative smaller value than a futures position does. The primary factors determining the access value of options are farm size, the variability of prices, and level of risk aversion. The level of farm size has a positive effect on the access value of options, while the variances of the cash and futures price
exert a negative effect. The level of risk aversion appears to have a negative effect on access value of options.

The expected utility maximization model used in this study is based on several simplifying assumptions. Thus, it is an abstract reflection of the reality and the results should be interpreted with the limitations in mind.

The risk attitude of the decision maker is an important determinant of his optimal market position. Futures are an ideal tool for hedging risk when risk is measured as the variance of return, while options are more versatile when risk is defined as downside risk which is negative deviations from an expected return. This study assumes the producer possesses a constant absolute risk aversion utility function. Specifying a preference function with different risk aversion characteristics, such as the target deviation model, may result in different optimal market behavior from those obtained in this study.

The analysis also ignores the transaction costs of establishing a hedging position such as margin calls and commission fees. In addition, the strike price of the option and the farm production cost are treated as exogenous to the maximization model. Definitely, the relaxing of one or more these assumptions will complicate the analysis and potentially affect the results of study. For example, in a market where major and sustained price trends are present, significant
margin calls can be involved and hence, the futures contracts become a costly hedging tools. In this case, taking an option position may be preferred by decision makers who would have problems of financing margin calls.

The adoption of univariate ARCH models also presents a limitation for the study. Even though a procedure is used to account for the joint probability function of the random prices, a multivariate ARCH/GARCH model presumably can provide more accurate simulated values for random prices. Option premium can be generated using the Black model with consistently estimated variance-variance matrix in the ARCH/GARCH model. Also, the correlation between crop prices and yield should be further investigated.

The results presented here could form the basis for future analysis. Further work should concentrate on relaxing the assumptions and restrictions applied in this study. Additionally, more effort is in need to develop an analytical framework to study the optimal usage of futures and options as risk management tools in the context that multiple random prices and production uncertainty are admitted.
REFERENCES


Avery, D. "Have Options Passed their Three-Year Test?" Futures. 10(1985):72-73.


Kahl, K. H. "Determination of the Recommended Hedging Ratio." 


King, R. R. and L. J. Robison. "An Interval Approach to the 

Lapan, H., G. Moschini, and S. D. Hanson. "Production, 
Hedging, and Speculative Decisions with Options and 
Futures Markets." Paper presented at the AAEA Annual 
Meeting, Vancouver, 1990.

Lumsdaine, R. L. "Asymptotic Properties of the Quasi-Maximum 
Likelihood Estimator in GARCH(1,1) and IGARCH(1,1) 
Models." Harvard University, mimeo.

Mandelbrot, B. "The Variation of Certain Speculative Prices." 

Marshall, J. F. Futures and Options Contracting. Cincinnati: 

Hypothesis for Foreign Currency Futures with Time Varying 

Milhoj, A. "A Conditional Variance Model for Daily Deviations 


Myers, R. "Estimating Time-Varying Optimal Hedge Ratios on 
Futures Markets." Working Paper, Michigan State 
University, 1990.


Newberry, D. M. and J. E. Stiglitz. The Theory of Commodity 
Price Stabilization. New York: Oxford University Press, 
1981.

Pankratz, A. Forecasting with Univariate Box-jenkin Models. 

Praetz, P. D. "The Distribution of Share Price Changes." 


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Finally, to my family, whose endless love I have enjoyed through all stages of my academic endeavors, I am deeply indebted. So it is to them, with love that I dedicate this dissertation.
APPENDIX A.

ANALYTICAL RESULTS DERIVED FROM THE MEAN-VARIANCE MODEL

Hanson (1988) shows that the standard mean-variance model can be used by the decision maker to provide estimates of the optimal market positions as an alternative to solving the expected utility maximization problem. Although, the mean-variance model can provide analytical results useful in describing the general behavioral characteristics of the expected utility model, some care must be taken in interpreting the analytical results of the mean-variance model. The comparative static results of the mean-variance model are not always consistent with the comparative static results of the expected utility model. To demonstrate the analytical results of the mean-variance model, three examples of the optimal market positions derived based on three scenarios are presented. The first model illustrates the results that occur when a decision maker is allowed to use only futures. The second model examines the results of using only options markets. The third model considers the optimal market positions when a decision maker is allowed to use both futures and options markets. All three models are studied for the case of uncertain output.

To be consistent with the assumptions in the general expected utility model discussed earlier, the following
assumptions are made:

(1) The decision maker maximizes the expected utility of the end-of-period income which can be represented by the mean-variance model;

(2) No call options are available to the decision maker;

(3) The strike price of options is exogenous;

(4) The farm produces a single output;

(5) Input level is fixed;

(6) The futures and option contract units are perfectly divisible.

The model uses the following definitions:

\begin{align*}
A & = \text{random end-of-period cash price;} \\
F & = \text{random end-of-period futures price;} \\
P & = \text{random end-of-period put price;} \\
Y & = \text{random yield;} \\
H & = \text{cropland in acres;} \\
C & = \text{production cost in dollars per acre;} \\
F_0 & = \text{current futures price;} \\
P_0 & = \text{current option premium;} \\
R & = \text{the amount of futures used;} \\
Z & = \text{the amount of options used.}
\end{align*}

The mean-variance model is specified as

\begin{equation}
\text{MAX } E[U(\Pi)] = \mu_r - (\lambda/2) \sigma^2_r
\end{equation}

where \( \mu_r \) is the expected value of the end-of-period income; \( \lambda \) is Arrow-Pratt measure of absolute risk
aversion; and $\sigma^2_{\tau}$ is the variance of the end-of-period income.

Model 1: Futures only

The income function is shown as follows:

$$\pi = AHY - CH + R(F - F_0)$$

The optimal futures positions using the mean-variance model in equation (A.1) and the equation (A.2) can be expressed as

$$R = - H(\sigma_{\tau}, \tau) + \frac{(\mu_t - E_t)}{\lambda(\sigma^2_{\tau})}$$

The second order condition is satisfied since the coefficient of risk aversion is positive. The optimal futures position consists of a hedging and a speculative component. The hedging component depends on the covariance between cash market revenue and futures price, and the variance of the futures price. The speculative component is the futures position that results if the decision maker does not have any cash position and obtains his entire end-of-period income from the futures market. The producer speculates according to his expected market bias adjusted by the variance of the futures price and the level of risk aversion. The expected futures market bias is the difference between the expected end-of-period futures price and the beginning-period futures price. A negative bias induces the producer to speculate short futures while a positive bias long futures. The speculative
component disappears when the futures market is unbiased. If the decision maker becomes more risk averse he tends to speculate less. As the decision maker becomes infinitely risk averse the speculative component will vanish.

Model 2: Options only

Model 2 looks at the optimal market position for the case of the decision maker hedging his output in the options market. The income function can be written as

\[ \pi = AHY - CH + Z(P - P_0) \]

Using the mean-variance model, the optimal options position is

\[ Z = - \frac{H(c_{ay,p})}{\sigma_p^2} \left[ \mu_p - P_0 \right] + \frac{\lambda \sigma_p^2}{\sigma_p^2} \]

The optimal position consists of a hedging and a speculative component. The hedging component depends on the covariance between cash market revenue and option price, and the variance of the option price. If options market is biased the decision maker modifies the options position by his expected bias adjusted by the level of risk aversion and variability in the end-of-period options value. The expected bias in the option market is the difference between the expected end-of-period option price and the option premium at the purchasing time.

Model 3: Futures and Option

The income function under the scenario of using both
futures and options is shown as:

\[(A.6) \pi = \Delta H - CH + R(F - F_0) + Z(P - P_0)\]

The optimal futures and options positions using equation (A.6) and the mean-variance model in equation (A.1) can be expressed as:

\[(A.7.1) R = \frac{H(\sigma_p^2 g_{xy} + \sigma_y^2 g_{tp})}{\lambda (\sigma_p^2 + \sigma_{tp}^2)} + \frac{(F_0 - \mu_t) \sigma_{tp}}{\lambda (\sigma_p^2 + \sigma_{tp}^2)}\]

\[(A.7.2) Z = \frac{H(\sigma_p^2 g_{xy} + \sigma_y^2 g_{tp})}{\lambda (\sigma_p^2 + \sigma_{tp}^2)} + \frac{(F_0 - \mu_t) \sigma_{tp}}{\lambda (\sigma_p^2 + \sigma_{tp}^2)}\]

The second order conditions are satisfied if \((\sigma_p^2 + \sigma_{tp}^2) > 0\).

This condition is satisfied as long as the correlation coefficient between futures and options price is not equal to negative one, i.e., the strike price is not set at an infinite value. The optimal futures and options positions consist of a hedging and a speculative component. Thus even if both the futures and options markets are unbiased, the decision maker will hedge in both the futures and options market. The decision maker takes a hedge position in each market which minimize the variance of the end-of-period income and speculates according to his expected bias in futures and option markets. Tradeoffs occur between the futures and option markets for changes in a given level of bias. If there is a positive increase in the bias in the futures market, the decision maker will increase his futures position and decrease his options position. The futures position has become
relatively more profitable than the options position so that the decision maker increase his position in futures market. However, as the futures position increase, the speculative risk increases and the decision maker adjusted for the increase in risk by reducing the relatively less profitable option position. Likewise, a positive increase in the bias in the option market will result in an increase in options position and a decrease in futures position.
APPENDIX B.
A COMPARISON OF FUTURES AND SYNTHETIC FUTURES CONSTRUCTED BY CALL AND PUT USING THE CERTAINTY EQUIVALENT

Call price is obtained from Chicago Board of Trade. The call options and put options have the same strike price. The expected call price at the end of period is defined as intrinsic value at expiration. The optimal market position under the call and put (CCP) scenario is shown in Table B.1. The producer owns a short call position and a long put position. The call position is at least twice larger than the put position. This implies that the producer uses a synthetic futures to hedge the short call position. The synthetic short futures is constructed by selling one call and buying one put. The call position decreases as the level of risk aversion increases while the put position increases as the level of risk aversion increases.

The second-order and the third-order certainty equivalent (CE) have been computed. The results show that the third-order CE is larger than the second-order CE by 8.17% in the case of certain output and by 1.8% in the case of uncertain output. The second-order CE is used to compute the certainty equivalent under the CCP scenario. Table B.2 presents the access values of futures and call-and-put added to the producer for the case of uncertain output. The access values of call-and-put are larger than the access values of futures.
The difference may result from the extra short call position.

The case of unbiased market is also examined and presented in Table B.2. The unbiased case is obtained by generating cash and futures prices from a normal distribution with the beginning-period prices and historical variances. The beginning-period option prices are adjusted to be equal to the means of the end-of-period call and put prices. The access values are smaller than base case. The access value of call-and-put is equal to the access value of futures when measured by expected output. Further study in synthetic strategies may lead to more useful conclusions.
Table B.1. Optimal market positions under the cash, call, and put scenario, with and without certain output

<table>
<thead>
<tr>
<th>Level of risk aversion</th>
<th>Market positions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Call</td>
</tr>
<tr>
<td>&lt;-----------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>with certain output</td>
<td></td>
</tr>
<tr>
<td>0.000075</td>
<td>-198354.35</td>
</tr>
<tr>
<td>0.0001</td>
<td>-173629.34</td>
</tr>
<tr>
<td>0.000125</td>
<td>-158779.49</td>
</tr>
<tr>
<td>0.00015</td>
<td>-148893.57</td>
</tr>
<tr>
<td>0.000175</td>
<td>-141865.04</td>
</tr>
<tr>
<td>0.0002</td>
<td>-136639.04</td>
</tr>
<tr>
<td>0.000225</td>
<td>-132625.58</td>
</tr>
<tr>
<td>0.00025</td>
<td>-129467.32</td>
</tr>
<tr>
<td>without certain output</td>
<td></td>
</tr>
<tr>
<td>0.000075</td>
<td>-196013.52</td>
</tr>
<tr>
<td>0.0001</td>
<td>-171166.81</td>
</tr>
<tr>
<td>0.000125</td>
<td>-156225.17</td>
</tr>
<tr>
<td>0.00015</td>
<td>-146302.07</td>
</tr>
<tr>
<td>0.000175</td>
<td>-139268.13</td>
</tr>
<tr>
<td>0.0002</td>
<td>-134057.43</td>
</tr>
<tr>
<td>0.000225</td>
<td>-130087.26</td>
</tr>
<tr>
<td>0.00025</td>
<td>-126978.72</td>
</tr>
</tbody>
</table>

Note: Call < 0 indicates short positions; Put > 0 indicates long positions.
Table B.2. Access values of futures and call-and-put over the cash-only position

<table>
<thead>
<tr>
<th>Level of risk aversion</th>
<th>CF vs. Cash</th>
<th>CCP vs. Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dollars</td>
<td>dollars</td>
</tr>
<tr>
<td>Base solution</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000075</td>
<td>25830.60</td>
<td>37080.76</td>
</tr>
<tr>
<td></td>
<td>(0.47)</td>
<td>(0.68)</td>
</tr>
<tr>
<td>0.0001</td>
<td>29734.16</td>
<td>37824.71</td>
</tr>
<tr>
<td></td>
<td>(0.54)</td>
<td>(0.69)</td>
</tr>
<tr>
<td>0.000125</td>
<td>33635.21</td>
<td>39590.76</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
<td>(0.72)</td>
</tr>
<tr>
<td>0.00015</td>
<td>37396.33</td>
<td>41744.18</td>
</tr>
<tr>
<td></td>
<td>(0.68)</td>
<td>(0.76)</td>
</tr>
<tr>
<td>0.000175</td>
<td>40962.21</td>
<td>44013.36</td>
</tr>
<tr>
<td></td>
<td>(0.74)</td>
<td>(0.80)</td>
</tr>
<tr>
<td>0.0002</td>
<td>44305.59</td>
<td>46259.63</td>
</tr>
<tr>
<td></td>
<td>(0.80)</td>
<td>(0.84)</td>
</tr>
<tr>
<td>0.000225</td>
<td>47412.31</td>
<td>48407.21</td>
</tr>
<tr>
<td></td>
<td>(0.86)</td>
<td>(0.88)</td>
</tr>
<tr>
<td>0.00025</td>
<td>50274.72</td>
<td>50405.51</td>
</tr>
<tr>
<td></td>
<td>(0.91)</td>
<td>(0.91)</td>
</tr>
</tbody>
</table>

Note: Cash indicates cash-only position; CF indicates futures only; CCP indicates calls and puts. Figures in the parentheses are access values measured by the expected output, 55200 bushels.
Table B.2. (continued)

<table>
<thead>
<tr>
<th>Level of risk aversion</th>
<th>CF vs. Cash</th>
<th>CCP vs. Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Unbiased market</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000075</td>
<td>9532.84</td>
<td>9613.77</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>0.0001</td>
<td>13926.51</td>
<td>14054.34</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>0.000125</td>
<td>18051.41</td>
<td>18394.24</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td>(0.33)</td>
</tr>
<tr>
<td>0.00015</td>
<td>21905.85</td>
<td>22318.32</td>
</tr>
<tr>
<td></td>
<td>(0.40)</td>
<td>(0.40)</td>
</tr>
<tr>
<td>0.000175</td>
<td>25491.54</td>
<td>25573.64</td>
</tr>
<tr>
<td></td>
<td>(0.46)</td>
<td>(0.46)</td>
</tr>
<tr>
<td>0.0002</td>
<td>28807.89</td>
<td>28973.63</td>
</tr>
<tr>
<td></td>
<td>(0.52)</td>
<td>(0.52)</td>
</tr>
<tr>
<td>0.000225</td>
<td>31854.90</td>
<td>32211.90</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
<td>(0.58)</td>
</tr>
<tr>
<td>0.00025</td>
<td>34032.59</td>
<td>34405.80</td>
</tr>
<tr>
<td></td>
<td>(0.62)</td>
<td>(0.62)</td>
</tr>
</tbody>
</table>
APPENDIX C.

A "GAMS" INPUT FILE

***********************************************************
* GAMS file to demonstrate expected utility maximization  *
* using negative exponential utility function with simulated *
* prices and yield estimates.                               *
***********************************************************
set I state of nature /1*300/;
set T time index /1*32/;
alias (i,j);
parameter C(T) cash price at time T;
parameter F(T) futures price at time T;
parameter CS(I) simulated cash price at T=32 for state=I;
parameter FS(I) simulated futures price at T=32 for state=I;
parameter PS(I) simulated put option price for state=I;
parameter YS(I) simulated yield estimate for state=I;
parameter PR(I) probability of state=I;
PR(I)=1/300;
scalars
  CO beginning-period cash price /2.43/
  FO beginning-period futures price /2.62/
  S strike price /2.6/
  PO option premium /0.154/
  PC production cost dollar per acre /307.75/
  LA cropland in acres /400/
  RA coefficient of risk aversion /0.000075/
  TR index in trend equation
  ECINI initial value of forecast error for cash price
  EFINI initial value of forecast error for futures price
  CINI1 initial value of cash price at time T-1
  CINI2 initial value of cash price at time T-2
  FINI1 initial value of futures price at time T-1
  FINI2 initial value of futures price at time T-2
  GENCA random generator for cash price
  GENFU random generator for futures price
  RMSCA random shock for cash price
  RMSFU random shock for futures price;
***********************************************************
* To simulate cash and futures prices using ARCH(1) model  *
***********************************************************
loop(I,
  ECINI=0.01291;
  EFINI=0.00074;
  CINI1=2.43;
  CINI2=2.34;
  FINI1=2.62;
  FINI2=2.55;
  loop(T,
GENCA=NORMAL(0,1);
GENFU=NORMAL(0,1);
RMSCA=SQRT(0.003+0.542*(ECINI)**2)*GENCA;
RMSFU=SQRT(0.002+0.489*(EFINI)**2)*GENFU;
C(T)=0.022+1.185*CINI1-0.196*CINI2+RMSCA;
F(T)=0.084+0.615*FINIl+0.335*FINI2+0.375*C(T)
    -0.359*CINI2+RMSFU;
CINI2=CINI1;
CINI1=C(T);
FINI2=FINI1;
FINI1=F(T);
ECINI=RMSCA;
EFINI=RMSFU;
CS(I)=C('32');
FS(I)=F('32');

************************************************************
* To simulate yield estimates using a trend equation *
************************************************************

loop(I,
   TR=41;
   YS(I)=48.821+2.184*TR+NORMAL(0, 10.65) );

************************************************************
* To order cash and futures prices, and yield estimates *
************************************************************

scalars
   CC control constant for cash price
   CF control constant for futures price
   CY control constant for yield estimate;

loop(J,
   CS(J)=SMAX(I, CS(I));
   FS(J)=SMAX(I, FS(I));
   YS(J)=SMAX(I, YS(I));
   CC=0;
   CF=0;
   CY=0;

loop(I,
   CC$( CS(I) EQ CS(J) )=CC+1;
   CF$( FS(I) EQ FS(J) )=CF+1;
   CY$( YS(I) EQ YS(J) )=CY+1;
   CS(I)$ ( CS(I) EQ CS(J) AND CC EQ 1 )=-1;
   FS(I)$ ( FS(I) EQ FS(J) AND CF EQ 1 )=-1;
   YS(I)$ ( YS(I) EQ YS(J) AND CY EQ 1 )=1000); ));

************************************************************
* To generate option price using intrinsic value at *
* expiration *
************************************************************

PS(I)=MAX(0, S-FS(I));
To solve the expected utility maximization model:

**variables**
- NE objective function variable
- R amount of futures contracts
- Z amount of option contracts;

**equation** NEUTILITY negative exponential utility function;

\[ NE = \text{SUM}(I, \text{PR}(I) \cdot (-\exp(-RA \cdot (CS(I) \cdot YS(I) \cdot LA - PC \cdot LA + (FS(I) - F0) \cdot R + (PS(I) - P0) \cdot Z$( PS(I) \gt P0) - P0 \cdot Z$( PS(I) \leq P0))))) \]

**model** NEUEXP /NEUTILITY/;

**solve** NEUEXP maximizing NE using NLP;

**display** R.L, Z.L;

To compute certainty equivalent:

**parameter** PROFIT(I) ex ante profit;

**scalars**
- MPROFI mean of ex ante profit
- VPROFI variance of ex ante profit
- SPROFI skewness of ex ante profit
- CE certainty equivalent;

\[ PROFIT = \text{SUM}(I, \text{PR}(I) \cdot PROFIT(I)) \]
\[ MPROFI = \text{SUM}(I, \text{PR}(I) \cdot PROFIT(I)) \]
\[ VPROFI = \text{SUM}(I, \text{PR}(I) \cdot (PROFIT(I) - MPROFI)^2) \]
\[ SPROFI = \text{SUM}(I, \text{PR}(I) \cdot (PROFIT(I) - MPROFI)^3) \]
\[ CE = MPROFI - 0.5 \cdot VPROFI + RA^2 / 6 \cdot SPROFI \]

**display** CE;